Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.10-c+d-x-^m-a+b-sin-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [348]. This is test number [66].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (348)	% 0. (0)
Mathematica	% 100. (348)	% 0. (0)
Maple	% 75.86 (264)	% 24.14 (84)
Maxima	% 55.17 (192)	% 44.83 (156)
Fricas	% 92.53 (322)	% 7.47 (26)
Sympy	% 28.45 (99)	% 71.55 (249)
Giac	% 44.83 (156)	% 55.17 (192)

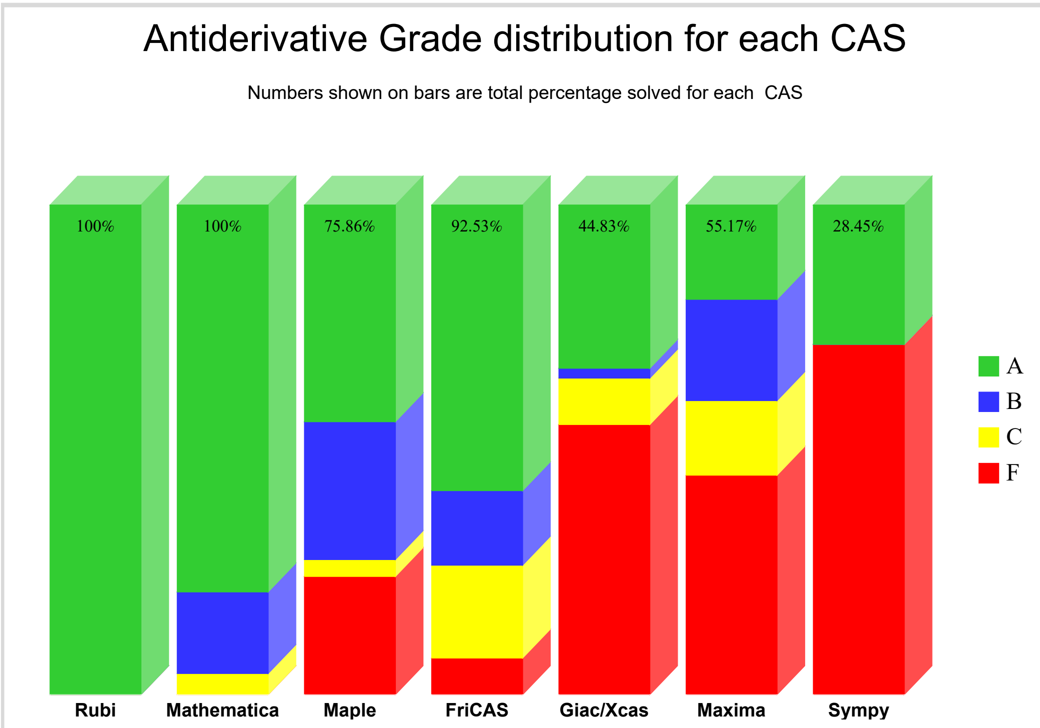
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

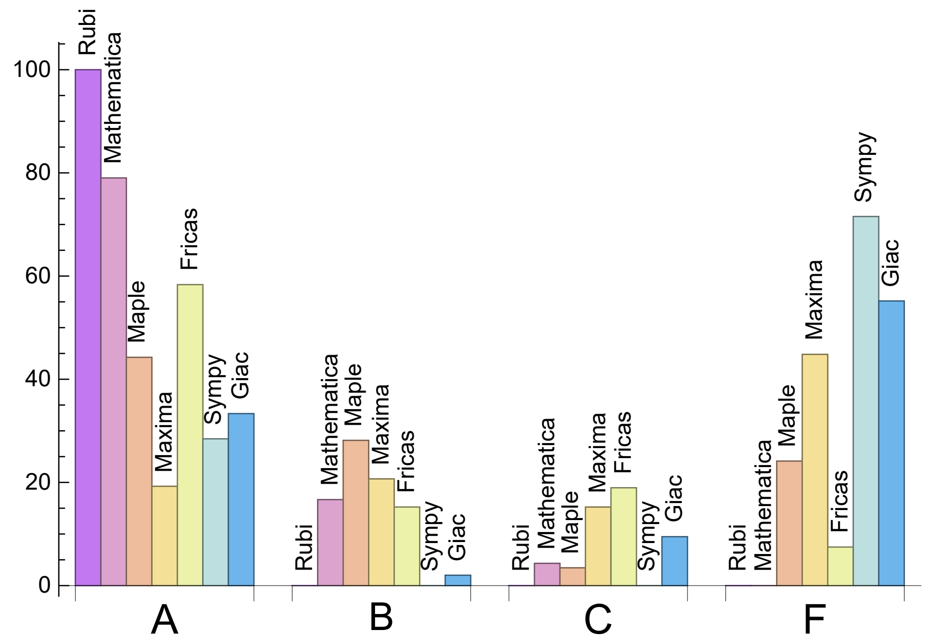
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	79.02	16.67	4.31	0.
Maple	44.25	28.16	3.45	24.14
Maxima	19.25	20.69	15.23	44.83
Fricas	58.33	15.23	18.97	7.47
Sympy	28.45	0.	0.	71.55
Giac	33.33	2.01	9.48	55.17

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.4	218.88	0.8	117.5	1.
Mathematica	6.25	476.14	1.26	122.5	0.95
Maple	0.46	321.39	1.73	158.5	1.44
Maxima	1.55	786.39	4.1	318.	3.06
Fricas	2.13	1794.07	5.01	463.	3.33
Sympy	4.45	246.05	2.38	0.	0.
Giac	0.79	864.74	7.68	83.	1.46

1.4 list of integrals that has no closed form antiderivative

{26, 27, 31, 32, 36, 37, 65, 66, 71, 75, 76, 110, 111, 115, 116, 120, 121, 137, 138, 142, 143, 144, 145, 149, 150, 166, 167, 171, 172, 173, 177, 178, 183, 184, 189, 190, 195, 196, 201, 202, 207, 208, 213, 214, 215, 216, 217, 218, 219, 240, 241, 242, 243, 244, 255, 256, 273, 274, 279, 280, 285, 286, 290, 291, 292, 293, 314, 315, 316, 317, 318}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {23, 28, 29, 197, 203, 209, 210, 220, 221, 222, 224, 226, 228, 229, 230, 232, 234, 236, 237, 238, 245, 246, 247, 248, 249, 250, 269, 270, 275, 276, 282, 298, 299, 300, 304, 308, 310, 311, 312, 320, 321, 324, 325, 327, 329, 330, 331, 333, 335, 339, 341, 342, 343, 347}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

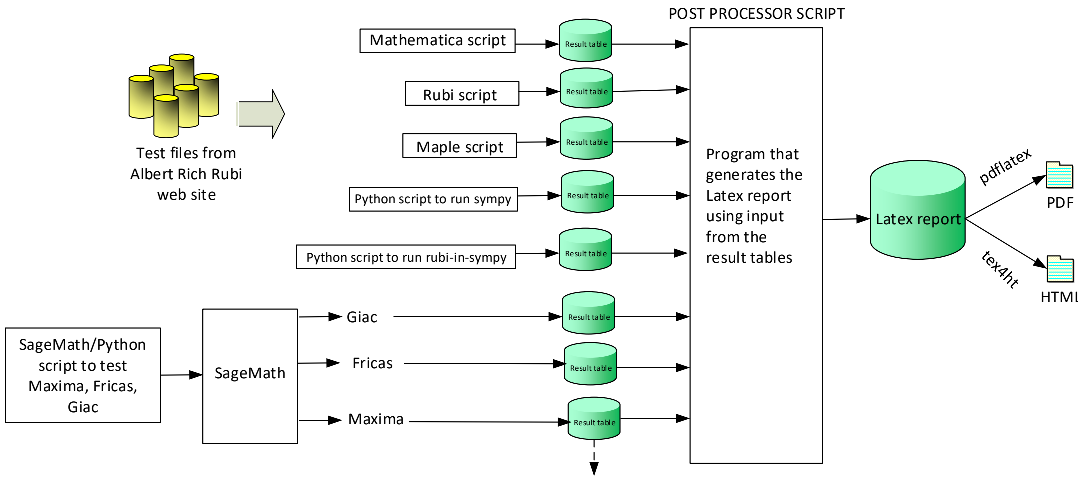
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 32, 33, 36, 37, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 183, 184, 186, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 200, 201, 202, 206, 207, 208, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 233, 235, 236, 237, 239, 240, 241, 242, 243, 244, 251, 252, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 272, 273, 274, 276, 277, 278, 279, 280, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 305, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 325, 326, 328, 332, 333, 334, 335, 336, 340, 342, 343, 344, 348 }

B grade: { 28, 29, 34, 35, 52, 59, 181, 182, 185, 187, 192, 199, 203, 204, 205, 209, 210, 211, 226, 228, 234, 238, 245, 246, 247, 248, 249, 250, 253, 260, 270, 271, 275, 281, 282, 283, 300, 301, 302, 303, 304, 306, 307, 308, 312, 323, 324, 327, 329, 330, 331, 337, 338, 339, 341, 345, 346, 347 }

C grade: { 38, 39, 40, 41, 42, 43, 44, 60, 61, 62, 63, 64, 68, 132, 133 }

F grade: { }

2.1.3 Maple

A grade: { 4, 5, 6, 7, 12, 13, 14, 15, 19, 20, 21, 22, 26, 27, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 71, 75, 76, 98, 99, 100, 104, 105, 106, 110, 111, 115, 116, 120, 121, 137, 138, 142, 143, 144, 145, 149, 150, 154, 155, 156, 160, 161, 162, 166, 167, 171, 172, 173, 177, 178, 182, 183, 184, 188, 189, 190, 195, 196, 200, 201, 202, 206, 207, 208, 212, 213, 214, 215, 216, 217, 218, 219, 223, 227, 235, 239, 240, 241, 242, 243, 244, 245, 248, 254, 255, 256, 259, 261, 262, 265, 266, 267, 268, 272, 273, 274, 276, 278, 279, 280, 284, 285, 286, 290, 291, 292, 293, 297, 305, 309, 313, 314, 315, 316, 317, 318, 328, 332, 340, 348 }

B grade: { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 23, 24, 25, 28, 29, 33, 34, 35, 95, 96, 97, 101, 102, 103, 107, 108, 109, 112, 113, 114, 117, 118, 119, 151, 152, 153, 157, 158, 159, 165, 170, 179, 180, 181, 185, 186, 187, 191, 192, 193, 194, 197, 198, 199, 203, 204, 205, 209, 210, 211, 222, 226, 230, 231, 234, 238, 251, 252, 253, 257, 258, 260, 263, 264, 269, 270, 271, 275, 277, 281, 282, 283, 296, 300, 301, 304, 308, 312, 320, 323, 327, 331, 335, 336, 339, 343, 344, 347 }

C grade: { 77, 78, 79, 80, 81, 82, 83, 122, 123, 124, 319, 322 }

F grade: { 67, 68, 69, 70, 72, 73, 74, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 146, 147, 148, 163, 164, 168, 169, 174, 175, 176, 220, 221, 224, 225, 228, 229, 232, 233, 236, 237, 246, 247, 249, 250, 287, 288, 289, 294, 295, 298, 299, 302, 303, 306, 307, 310, 311, 321, 324, 325, 326, 329, 330, 333, 334, 337, 338, 341, 342, 345, 346 }

2.1.4 Maxima

A grade: { 4, 19, 26, 27, 36, 37, 65, 66, 71, 75, 76, 103, 115, 116, 137, 138, 142, 143, 144, 145, 149, 150, 159, 166, 167, 171, 172, 173, 177, 178, 182, 200, 215, 216, 217, 218, 219, 240, 241, 242, 243, 244, 253, 254, 255, 256, 264, 265, 266, 272, 273, 274, 279, 284, 290, 291, 297, 305, 309, 314, 315, 316, 317, 318, 332, 340, 348

B grade: { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 23, 24, 25, 28, 29, 30, 33, 34, 35, 95, 96, 97, 101, 102, 107, 108, 109, 112, 113, 114, 117, 118, 119, 151, 152, 153, 157, 158, 179, 180, 181, 185, 186, 187, 188, 194, 197, 198, 199, 203, 204, 205, 206, 210, 211, 212, 251, 252, 257, 258, 259, 260, 263, 269, 270, 271, 275, 276, 277, 278, 282, 283 }

C grade: { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 98, 99, 100, 104, 105, 106, 154, 155, 156, 160, 161, 162, 261, 262, 267, 268 }

F grade: { 31, 32, 67, 68, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 110, 111, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 146, 147, 148, 163, 164, 165, 168, 169, 170, 174, 175, 176, 183, 184, 189, 190, 191, 192, 193, 195, 196, 201, 202, 207, 208, 209, 213, 214, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 245, 246, 247, 248, 249, 250, 280, 281, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 310, 311, 312, 313, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 26, 27, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 110, 111, 114, 115, 116, 120, 121, 137, 138, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 171, 172, 173, 174, 175, 176, 177, 178, 182, 183, 184, 188, 189, 190, 193, 194, 195, 196, 201, 202, 207, 208, 213, 214, 215, 216, 217, 218, 219, 223, 227, 231, 235, 240, 241, 242, 243, 244, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 277, 278, 279, 280, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 297, 301, 305, 309, 313, 314, 315, 316, 317, 318, 319, 328, 332, 336, 340, 344, 348 }

B grade: { 7, 14, 15, 22, 25, 29, 35, 52, 106, 108, 109, 113, 118, 119, 165, 170, 180, 181, 186, 187, 192, 199, 200, 205, 206, 211, 212, 222, 226, 230, 234, 238, 239, 245, 248, 253, 271, 276, 283, 296, 300, 304, 308, 312, 320, 322, 323, 327, 331, 335, 339, 343, 347 }

C grade: { 23, 24, 28, 33, 34, 107, 112, 117, 163, 164, 168, 169, 179, 185, 191, 197, 198, 203, 204, 209, 210, 220, 221, 224, 225, 228, 229, 232, 233, 236, 237, 246, 247, 249, 250, 251, 252, 269, 270, 275, 281, 282, 294, 295, 298, 299, 302, 303, 306, 307, 310, 311, 321, 324, 325, 326, 329, 330, 333, 334, 337, 338, 341, 342, 345, 346 }

F grade: { 67, 68, 70, 91, 92, 93, 94, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 144 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 26, 27, 31, 32, 36, 37, 60, 61, 62, 63, 64, 65, 66, 71, 75, 76, 95, 96, 97, 101, 102, 103, 109, 110, 111, 114, 115, 116, 119, 120, 121, 137, 138, 142, 143, 144, 149, 150, 151, 152, 153, 157, 158, 159, 166, 177, 181, 182, 183, 187, 188, 193, 194, 201, 202, 207, 208, 213, 214, 216, 217, 218, 241, 242, 243, 254, 255, 257, 258, 259, 260, 264, 265, 266, 273, 274, 279, 280, 285, 286, 290, 291, 292, 297, 315, 316, 317 }

B grade: { }

C grade: { }

F grade: { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 28, 29, 30, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 67, 68, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 98, 99, 100, 104, 105, 106, 107, 108, 112, 113, 117, 118, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 145, 146, 147, 148, 154, 155, 156, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 184, 185, 186, 189, 190, 191, 192, 195, 196, 197, 198, 199, 200, 203, 204, 205, 206, 209, 210, 211, 212, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 256, 261, 262, 263, 267, 268, 269, 270, 271, 272, 275, 276, 277, 278, 281, 282, 283, 284, 287, 288, 289, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348 }

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A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 26, 27, 31, 32, 36, 37, 65, 66, 71, 75, 76, 95, 96, 97, 101, 102, 103, 110, 111, 115, 116, 120, 121, 137, 138, 142, 143, 144, 145, 149, 150, 151, 152, 153, 157, 158, 159, 166, 167, 171, 172, 173, 177, 178, 182, 183, 184, 188, 189, 190, 194, 195, 196, 200, 201, 202, 206, 212, 215, 216, 217, 218, 219, 223, 227, 231, 235, 239, 240, 241, 242, 243, 244, 254, 255, 256, 260, 266, 272, 273, 274, 278, 280, 284, 286, 290, 291, 292, 293, 297, 301, 305, 309, 313, 314, 315, 316, 317, 318, 328, 332, 336, 340, 348 }

B grade: { 30, 109, 114, 119, 181, 277, 344 }

C grade: { 5, 6, 7, 12, 13, 14, 15, 20, 38, 39, 40, 41, 45, 46, 47, 48, 53, 54, 55, 56, 60, 61, 62, 98, 99, 100, 104, 154, 155, 156, 160, 261, 267 }

F grade: { 21, 22, 23, 24, 25, 28, 29, 33, 34, 35, 42, 43, 44, 49, 50, 51, 52, 57, 58, 59, 63, 64, 67, 68, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 105, 106, 107, 108, 112, 113, 117, 118, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 146, 147, 148, 161, 162, 163, 164, 165, 168, 169, 170, 174, 175, 176, 179, 180, 185, 186, 187, 191, 192, 193, 197, 198, 199, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 220, 221, 222, 224, 225, 226, 228, 229, 230, 232, 233, 234, 236, 237, 238, 245, 246, 247, 248, 249, 250, 251, 252, 253, 257, 258, 259, 262, 263, 264, 265, 268, 269, 270, 271, 275, 276, 279, 281, 282, 283, 285, 287, 288, 289, 294, 295, 296, 298, 299, 300, 302, 303, 304, 306, 307, 308, 310, 311, 312, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 333, 334, 335, 337, 338, 339, 341, 342, 343, 345, 346, 347 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	77	551	662	348	311	231
normalized size	1	1.	0.84	5.99	7.2	3.78	3.38	2.51
time (sec)	N/A	0.091	0.339	0.009	1.124	1.632	2.921	1.12

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	62	308	385	230	202	150
normalized size	1	1.	0.87	4.34	5.42	3.24	2.85	2.11
time (sec)	N/A	0.065	0.205	0.007	1.079	1.667	1.356	1.119

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	45	148	190	138	112	88
normalized size	1	1.	0.9	2.96	3.8	2.76	2.24	1.76
time (sec)	N/A	0.039	0.171	0.006	1.031	1.682	0.636	1.153

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	52	72	70	46	42
normalized size	1	1.	0.96	1.86	2.57	2.5	1.64	1.5
time (sec)	N/A	0.016	0.07	0.007	1.012	1.664	0.235	1.102

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	73	190	200	0	806
normalized size	1	1.	0.96	1.43	3.73	3.92	0.	15.8
time (sec)	N/A	0.098	0.095	0.009	1.269	1.611	0.	1.21

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	66	107	221	302	0	4131
normalized size	1	1.	0.92	1.49	3.07	4.19	0.	57.38
time (sec)	N/A	0.109	0.21	0.01	1.326	1.643	0.	1.295

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	87	145	269	471	0	7731
normalized size	1	1.	0.84	1.39	2.59	4.53	0.	74.34
time (sec)	N/A	0.139	0.664	0.008	1.525	1.838	0.	1.555

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	132	1030	992	593	660	300
normalized size	1	1.	0.82	6.4	6.16	3.68	4.1	1.86
time (sec)	N/A	0.103	0.638	0.048	1.146	1.771	6.157	1.166

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	106	587	597	394	456	207
normalized size	1	1.	0.79	4.38	4.46	2.94	3.4	1.54
time (sec)	N/A	0.074	0.42	0.007	1.07	1.7	3.24	1.136

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	77	289	313	247	264	127
normalized size	1	1.	0.81	3.04	3.29	2.6	2.78	1.34
time (sec)	N/A	0.054	0.302	0.007	1.025	1.776	1.461	1.124

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	52	112	130	130	126	65
normalized size	1	1.	0.95	2.04	2.36	2.36	2.29	1.18
time (sec)	N/A	0.027	0.146	0.006	1.009	1.644	0.629	1.118

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	65	105	216	238	0	826
normalized size	1	1.	0.83	1.35	2.77	3.05	0.	10.59
time (sec)	N/A	0.168	0.104	0.01	1.231	1.702	0.	1.233

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	75	156	231	325	0	3976
normalized size	1	1.	0.93	1.93	2.85	4.01	0.	49.09
time (sec)	N/A	0.139	0.399	0.009	1.322	1.819	0.	1.343

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	101	193	278	509	0	6940
normalized size	1	1.	0.89	1.71	2.46	4.5	0.	61.42
time (sec)	N/A	0.192	1.14	0.009	1.524	1.833	0.	1.616

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	122	229	346	733	0	10573
normalized size	1	1.	0.75	1.41	2.14	4.52	0.	65.27
time (sec)	N/A	0.181	1.218	0.008	1.886	1.93	0.	1.772

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	150	1023	1261	764	772	474
normalized size	1	1.	0.67	4.55	5.6	3.4	3.43	2.11
time (sec)	N/A	0.25	0.999	0.035	1.224	1.77	9.804	1.138

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	127	560	730	495	495	312
normalized size	1	1.	0.73	3.2	4.17	2.83	2.83	1.78
time (sec)	N/A	0.159	0.926	0.009	1.088	1.741	5.233	1.141

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	86	265	365	298	284	185
normalized size	1	1.	0.7	2.15	2.97	2.42	2.31	1.5
time (sec)	N/A	0.096	0.406	0.007	1.072	1.634	2.593	1.134

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	59	95	140	153	126	93
normalized size	1	1.	0.79	1.27	1.87	2.04	1.68	1.24
time (sec)	N/A	0.042	0.173	0.007	1.055	1.639	1.114	1.151

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	102	167	370	406	0	8500
normalized size	1	1.	0.84	1.38	3.06	3.36	0.	70.25
time (sec)	N/A	0.245	0.243	0.009	1.365	1.662	0.	1.759

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	175	240	406	595	0	0
normalized size	1	1.	1.21	1.66	2.8	4.1	0.	0.
time (sec)	N/A	0.242	1.051	0.01	1.818	1.997	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	221	313	454	919	0	0
normalized size	1	1.	1.2	1.7	2.47	4.99	0.	0.
time (sec)	N/A	0.354	0.796	0.01	1.973	1.98	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	185	185	221	633	953	2056	0	0
normalized size	1	1.	1.19	3.42	5.15	11.11	0.	0.
time (sec)	N/A	0.137	0.457	0.095	1.512	2.112	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	148	361	529	1330	0	0
normalized size	1	1.	1.2	2.93	4.3	10.81	0.	0.
time (sec)	N/A	0.088	0.32	0.048	1.327	1.938	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	134	151	235	721	0	0
normalized size	1	1.	2.	2.25	3.51	10.76	0.	0.
time (sec)	N/A	0.039	0.075	0.032	1.326	1.867	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	6.016	0.058	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	6.952	0.048	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	478	541	2228	1756	0	0
normalized size	1	1.	4.23	4.79	19.72	15.54	0.	0.
time (sec)	N/A	0.213	6.909	0.085	1.602	2.046	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	181	276	749	1026	0	0
normalized size	1	1.	2.18	3.33	9.02	12.36	0.	0.
time (sec)	N/A	0.136	4.827	0.043	1.462	1.795	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	52	39	293	119	0	1689
normalized size	1	1.	1.79	1.34	10.1	4.1	0.	58.24
time (sec)	N/A	0.028	0.084	0.007	1.005	1.703	0.	2.028

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	6.267	0.178	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	6.353	0.339	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	528	1056	5234	4091	0	0
normalized size	1	1.	1.71	3.42	16.94	13.24	0.	0.
time (sec)	N/A	0.226	5.103	0.125	6.517	2.684	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	471	548	2611	2379	0	0
normalized size	1	1.	2.62	3.04	14.51	13.22	0.	0.
time (sec)	N/A	0.136	7.439	0.077	2.349	2.26	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	292	246	1044	1191	0	0
normalized size	1	1.	2.68	2.26	9.58	10.93	0.	0.
time (sec)	N/A	0.067	1.822	0.054	1.646	1.984	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	31.357	2.163	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	34.8	3.402	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	124	233	891	467	0	1376
normalized size	1	1.	0.64	1.19	4.57	2.39	0.	7.06
time (sec)	N/A	0.434	0.111	0.012	1.881	1.774	0.	1.259

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	125	188	855	394	0	764
normalized size	1	1.	0.74	1.11	5.03	2.32	0.	4.49
time (sec)	N/A	0.242	0.1	0.007	1.83	1.796	0.	1.252

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	123	145	779	327	0	332
normalized size	1	1.	0.87	1.02	5.49	2.3	0.	2.34
time (sec)	N/A	0.176	0.094	0.007	1.834	1.765	0.	1.182

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	121	99	714	269	0	227
normalized size	1	1.	1.03	0.85	6.1	2.3	0.	1.94
time (sec)	N/A	0.133	0.054	0.014	1.777	1.719	0.	1.125

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	148	140	632	362	0	0
normalized size	1	1.	1.06	1.01	4.55	2.6	0.	0.
time (sec)	N/A	0.204	0.313	0.014	1.322	1.964	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	162	180	632	510	0	0
normalized size	1	1.	0.96	1.07	3.76	3.04	0.	0.
time (sec)	N/A	0.238	0.63	0.009	1.311	2.261	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	208	220	632	682	0	0
normalized size	1	1.	1.08	1.14	3.27	3.53	0.	0.
time (sec)	N/A	0.297	0.459	0.007	1.314	2.368	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	194	242	940	614	0	1419
normalized size	1	1.	0.84	1.05	4.07	2.66	0.	6.14
time (sec)	N/A	0.442	2.144	0.017	1.855	2.296	0.	1.348

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	175	197	899	478	0	772
normalized size	1	1.	0.86	0.97	4.43	2.35	0.	3.8
time (sec)	N/A	0.36	1.696	0.013	1.824	2.157	0.	1.26

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	149	150	826	367	0	331
normalized size	1	1.	0.94	0.95	5.23	2.32	0.	2.09
time (sec)	N/A	0.285	0.532	0.013	1.82	2.192	0.	1.208

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	126	108	747	281	0	220
normalized size	1	1.	0.97	0.83	5.75	2.16	0.	1.69
time (sec)	N/A	0.234	0.226	0.015	1.851	2.058	0.	1.183

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	149	145	640	340	0	0
normalized size	1	1.	1.1	1.07	4.74	2.52	0.	0.
time (sec)	N/A	0.254	0.384	0.013	1.323	2.121	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	158	189	644	502	0	0
normalized size	1	1.	0.93	1.11	3.79	2.95	0.	0.
time (sec)	N/A	0.328	1.399	0.014	1.291	2.331	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	244	230	644	745	0	0
normalized size	1	1.	1.13	1.06	2.98	3.45	0.	0.
time (sec)	N/A	0.336	2.003	0.013	1.302	2.455	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	C	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	661	273	644	941	0	0
normalized size	1	1.	2.68	1.11	2.61	3.81	0.	0.
time (sec)	N/A	0.419	4.638	0.015	1.294	2.895	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	542	476	1868	923	0	2726
normalized size	1	1.	1.32	1.16	4.56	2.25	0.	6.65
time (sec)	N/A	1.127	3.152	0.013	2.209	2.764	0.	1.629

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	389	384	1790	761	0	1513
normalized size	1	1.	1.1	1.08	5.06	2.15	0.	4.27
time (sec)	N/A	0.972	1.634	0.013	2.23	2.419	0.	1.493

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	266	296	1651	645	0	659
normalized size	1	1.	0.88	0.97	5.43	2.12	0.	2.17
time (sec)	N/A	0.498	0.804	0.01	2.144	2.32	0.	1.288

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	202	210	1527	552	0	446
normalized size	1	1.	0.79	0.82	5.94	2.15	0.	1.74
time (sec)	N/A	0.405	0.56	0.015	2.119	2.172	0.	1.221

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	300	288	1264	707	0	0
normalized size	1	1.	1.11	1.07	4.68	2.62	0.	0.
time (sec)	N/A	0.564	1.008	0.013	1.49	2.453	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	496	368	1265	963	0	0
normalized size	1	1.	1.7	1.26	4.33	3.3	0.	0.
time (sec)	N/A	0.71	2.426	0.011	1.519	2.663	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	1429	450	1265	1269	0	0
normalized size	1	1.	4.01	1.26	3.55	3.56	0.	0.
time (sec)	N/A	0.797	6.398	0.012	1.482	3.43	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	60	87	423	192	117	286
normalized size	1	1.	0.69	1.	4.86	2.21	1.34	3.29
time (sec)	N/A	0.109	0.013	0.01	1.762	2.293	126.238	1.155

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	69	65	379	149	85	238
normalized size	1	1.	1.06	1.	5.83	2.29	1.31	3.66
time (sec)	N/A	0.058	0.011	0.009	1.734	2.344	3.118	1.154

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	59	42	344	101	54	184
normalized size	1	1.	1.28	0.91	7.48	2.2	1.17	4.
time (sec)	N/A	0.034	0.008	0.007	1.665	2.043	1.381	1.156

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	64	60	231	149	80	0
normalized size	1	1.	1.	0.94	3.61	2.33	1.25	0.
time (sec)	N/A	0.065	0.023	0.007	1.157	2.229	7.55	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	111	79	231	189	114	0
normalized size	1	1.	1.28	0.91	2.66	2.17	1.31	0.
time (sec)	N/A	0.093	0.085	0.006	1.174	2.281	145.307	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	15.374	0.059	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	14.608	0.05	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.408	0.119	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	185	0	0	0	0	0
normalized size	1	1.	2.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	4.194	0.103	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	35	0	0	117	0	0
normalized size	1	1.	0.83	0.	0.	2.79	0.	0.
time (sec)	N/A	0.06	0.405	0.096	0.	1.723	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	58	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.585	0.128	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.721	0.299	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	251	0	0	459	0	0
normalized size	1	1.	0.94	0.	0.	1.72	0.	0.
time (sec)	N/A	0.303	9.809	0.207	0.	1.908	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	211	0	0	340	0	0
normalized size	1	1.	1.3	0.	0.	2.1	0.	0.
time (sec)	N/A	0.217	0.612	0.138	0.	1.81	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	121	0	0	219	0	0
normalized size	1	1.	0.95	0.	0.	1.72	0.	0.
time (sec)	N/A	0.088	0.049	0.059	0.	1.826	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	5.716	0.04	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	1.115	0.048	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	454	0	149	0	0
normalized size	1	1.	1.	5.75	0.	1.89	0.	0.
time (sec)	N/A	0.077	0.019	0.119	0.	1.811	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	353	0	149	0	0
normalized size	1	1.	1.	4.71	0.	1.99	0.	0.
time (sec)	N/A	0.073	0.016	0.063	0.	1.712	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	290	0	149	0	0
normalized size	1	1.	1.	3.67	0.	1.89	0.	0.
time (sec)	N/A	0.071	0.016	0.063	0.	1.719	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	378	0	132	0	0
normalized size	1	1.	1.	5.04	0.	1.76	0.	0.
time (sec)	N/A	0.066	0.014	0.062	0.	1.79	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	63	426	0	138	0	0
normalized size	1	1.	0.91	6.17	0.	2.	0.	0.
time (sec)	N/A	0.068	0.02	0.065	0.	1.693	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	65	529	0	149	0	0
normalized size	1	1.	0.92	7.45	0.	2.1	0.	0.
time (sec)	N/A	0.071	0.019	0.072	0.	1.82	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	599	0	149	0	0
normalized size	1	1.	1.	7.58	0.	1.89	0.	0.
time (sec)	N/A	0.072	0.016	0.079	0.	1.766	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	118	0	0	235	0	0
normalized size	1	1.	1.22	0.	0.	2.42	0.	0.
time (sec)	N/A	0.162	0.329	0.072	0.	1.832	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	120	0	0	235	0	0
normalized size	1	1.	1.17	0.	0.	2.28	0.	0.
time (sec)	N/A	0.143	0.311	0.069	0.	1.773	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	116	0	0	235	0	0
normalized size	1	1.	1.17	0.	0.	2.37	0.	0.
time (sec)	N/A	0.143	0.305	0.125	0.	1.743	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	120	0	0	203	0	0
normalized size	1	1.	1.17	0.	0.	1.97	0.	0.
time (sec)	N/A	0.134	0.269	0.084	0.	1.759	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	99	0	0	193	0	0
normalized size	1	1.	1.19	0.	0.	2.33	0.	0.
time (sec)	N/A	0.13	0.231	0.113	0.	1.747	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	117	0	0	227	0	0
normalized size	1	1.	1.16	0.	0.	2.25	0.	0.
time (sec)	N/A	0.137	0.311	0.07	0.	1.737	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	121	0	0	235	0	0
normalized size	1	1.	1.25	0.	0.	2.42	0.	0.
time (sec)	N/A	0.173	0.364	0.072	0.	1.811	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	29	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	0.487	0.092	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	87	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	0.556	0.082	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	29	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.447	0.085	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	57	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	2.228	0.089	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	123	482	624	362	264	212
normalized size	1	1.	1.37	5.36	6.93	4.02	2.93	2.36
time (sec)	N/A	0.118	0.843	0.016	1.036	1.791	1.865	1.167

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	81	241	323	228	151	128
normalized size	1	1.	1.19	3.54	4.75	3.35	2.22	1.88
time (sec)	N/A	0.088	0.499	0.012	1.013	1.743	0.864	1.124

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	51	90	126	126	68	63
normalized size	1	1.	1.13	2.	2.8	2.8	1.51	1.4
time (sec)	N/A	0.042	0.356	0.011	0.969	1.77	0.344	1.096

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	54	96	231	234	0	961
normalized size	1	1.	0.84	1.5	3.61	3.66	0.	15.02
time (sec)	N/A	0.15	0.294	0.014	1.207	1.661	0.	1.257

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	110	141	265	332	0	4251
normalized size	1	1.	1.25	1.6	3.01	3.77	0.	48.31
time (sec)	N/A	0.214	0.493	0.014	1.306	1.829	0.	1.341

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	104	177	358	517	0	8312
normalized size	1	1.	0.85	1.44	2.91	4.2	0.	67.58
time (sec)	N/A	0.257	0.672	0.017	1.442	1.753	0.	1.519

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	216	1135	1308	752	779	458
normalized size	1	1.	0.91	4.79	5.52	3.17	3.29	1.93
time (sec)	N/A	0.295	1.343	0.022	1.108	1.885	4.784	1.157

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	182	567	686	447	456	279
normalized size	1	1.	1.08	3.38	4.08	2.66	2.71	1.66
time (sec)	N/A	0.192	0.614	0.02	1.022	1.785	2.151	1.144

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	80	219	277	228	219	144
normalized size	1	1.	0.68	1.86	2.35	1.93	1.86	1.22
time (sec)	N/A	0.104	1.048	0.02	0.996	1.737	0.844	1.11

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	114	192	452	468	0	9516
normalized size	1	1.	0.79	1.32	3.12	3.23	0.	65.63
time (sec)	N/A	0.371	0.277	0.021	1.313	1.758	0.	1.624

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	206	274	500	682	0	0
normalized size	1	1.	1.27	1.69	3.09	4.21	0.	0.
time (sec)	N/A	0.333	0.587	0.026	1.514	2.013	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	353	347	641	1049	0	0
normalized size	1	1.	1.57	1.54	2.85	4.66	0.	0.
time (sec)	N/A	0.506	0.901	0.023	1.951	2.071	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	126	484	1315	2134	0	0
normalized size	1	1.	0.85	3.27	8.89	14.42	0.	0.
time (sec)	N/A	0.306	1.038	0.141	1.477	2.022	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	94	254	421	1196	0	0
normalized size	1	1.	0.83	2.25	3.73	10.58	0.	0.
time (sec)	N/A	0.218	0.654	0.07	1.353	1.814	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	122	228	251	272	940
normalized size	1	1.	0.85	2.03	3.8	4.18	4.53	15.67
time (sec)	N/A	0.064	0.15	0.043	0.985	1.77	1.125	1.241

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	4.765	0.133	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	4.619	0.325	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	257	807	4834	3872	0	0
normalized size	1	1.	0.83	2.61	15.64	12.53	0.	0.
time (sec)	N/A	0.377	1.904	0.641	4.041	2.695	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	175	421	1123	2074	0	0
normalized size	1	1.	0.72	1.73	4.62	8.53	0.	0.
time (sec)	N/A	0.287	2.202	0.495	2.214	2.064	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	225	233	1229	495	1246	4177
normalized size	1	1.	1.52	1.57	8.3	3.34	8.42	28.22
time (sec)	N/A	0.089	1.082	0.197	1.057	1.683	2.561	1.833

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	13.946	3.104	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	14.801	4.749	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	124	484	1326	2133	0	0
normalized size	1	1.	0.84	3.29	9.02	14.51	0.	0.
time (sec)	N/A	0.295	1.129	0.132	1.496	1.976	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	92	254	427	1195	0	0
normalized size	1	1.	0.82	2.27	3.81	10.67	0.	0.
time (sec)	N/A	0.212	0.723	0.087	1.351	1.878	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	47	123	228	251	272	941
normalized size	1	1.	0.8	2.08	3.86	4.25	4.61	15.95
time (sec)	N/A	0.066	0.151	0.06	1.01	1.632	1.105	1.289

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	4.902	0.136	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	4.718	0.322	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	108	145	0	0	0	0
normalized size	1	1.	0.9	1.21	0.	0.	0.	0.
time (sec)	N/A	0.141	0.288	0.103	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	92	119	0	0	0	0
normalized size	1	1.	0.94	1.21	0.	0.	0.	0.
time (sec)	N/A	0.103	0.21	0.054	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	76	93	0	0	0	0
normalized size	1	1.	1.31	1.6	0.	0.	0.	0.
time (sec)	N/A	0.068	0.154	0.053	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	83	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	0.162	0.229	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	117	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.153	0.3	0.056	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	153	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	0.327	0.054	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	231	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.23	1.113	0.064	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	191	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	0.822	0.039	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	113	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.681	0.036	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	127	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.276	0.662	0.036	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	226	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.3	0.916	0.037	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	295	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.376	0.876	0.036	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	306	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.249	0.766	0.178	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	245	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	0.594	0.064	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	231	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	1.524	0.06	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	3.04	0.046	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.747	0.049	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	691	691	455	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.354	2.807	0.036	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	352	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.235	2.031	0.039	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	308	0	0	0	0	0
normalized size	1	1.	1.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	2.545	0.038	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	32.865	0.039	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	17.471	0.036	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	2.97	0.073	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	1.096	0.329	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	376	0	0	934	0	0
normalized size	1	1.	0.84	0.	0.	2.08	0.	0.
time (sec)	N/A	0.605	0.845	0.279	0.	2.135	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	260	0	0	636	0	0
normalized size	1	1.	0.87	0.	0.	2.13	0.	0.
time (sec)	N/A	0.369	0.285	0.193	0.	1.979	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	199	0	0	319	0	0
normalized size	1	1.	1.34	0.	0.	2.16	0.	0.
time (sec)	N/A	0.144	2.714	0.093	0.	1.899	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.881	0.1	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	9.125	0.137	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	124	482	624	362	264	212
normalized size	1	1.	1.38	5.36	6.93	4.02	2.93	2.36
time (sec)	N/A	0.123	0.432	0.011	1.05	1.697	1.754	1.463

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	84	241	323	228	151	128
normalized size	1	1.	1.24	3.54	4.75	3.35	2.22	1.88
time (sec)	N/A	0.086	0.314	0.008	0.994	1.69	0.814	1.499

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	90	126	126	68	63
normalized size	1	1.	0.96	2.	2.8	2.8	1.51	1.4
time (sec)	N/A	0.042	0.109	0.006	0.973	1.627	0.343	1.751

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	57	96	231	234	0	961
normalized size	1	1.	0.89	1.5	3.61	3.66	0.	15.02
time (sec)	N/A	0.124	0.15	0.01	1.238	1.77	0.	1.924

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	72	141	265	332	0	4251
normalized size	1	1.	0.82	1.6	3.01	3.77	0.	48.31
time (sec)	N/A	0.155	0.349	0.012	1.28	1.832	0.	1.472

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	94	177	358	517	0	8312
normalized size	1	1.	0.76	1.44	2.91	4.2	0.	67.58
time (sec)	N/A	0.19	0.814	0.01	1.466	2.163	0.	1.466

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	232	1125	1295	805	779	501
normalized size	1	1.	0.93	4.5	5.18	3.22	3.12	2.
time (sec)	N/A	0.267	1.298	0.016	1.135	2.175	4.599	1.121

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	249	561	678	495	456	309
normalized size	1	1.	1.37	3.08	3.73	2.72	2.51	1.7
time (sec)	N/A	0.192	0.726	0.013	1.05	2.084	2.068	1.126

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	96	216	273	252	219	161
normalized size	1	1.	0.83	1.86	2.35	2.17	1.89	1.39
time (sec)	N/A	0.098	0.673	0.013	0.995	2.037	0.846	1.101

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	134	213	451	482	0	9986
normalized size	1	1.	0.86	1.37	2.89	3.09	0.	64.01
time (sec)	N/A	0.324	0.292	0.019	1.288	2.197	0.	1.602

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	232	301	498	693	0	0
normalized size	1	1.	1.27	1.64	2.72	3.79	0.	0.
time (sec)	N/A	0.334	0.596	0.018	1.501	2.264	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	395	374	640	1057	0	0
normalized size	1	1.	1.61	1.53	2.61	4.31	0.	0.
time (sec)	N/A	0.424	1.229	0.023	1.877	2.509	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	495	495	401	0	0	5222	0	0
normalized size	1	1.	0.81	0.	0.	10.55	0.	0.
time (sec)	N/A	0.969	0.235	0.401	0.	3.675	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	296	0	0	3729	0	0
normalized size	1	1.	0.81	0.	0.	10.16	0.	0.
time (sec)	N/A	0.822	0.191	0.283	0.	3.143	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	182	492	0	2461	0	0
normalized size	1	1.	0.78	2.1	0.	10.52	0.	0.
time (sec)	N/A	0.453	0.041	0.104	0.	3.589	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.402	0.063	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.333	0.073	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	925	925	742	0	0	11266	0	0
normalized size	1	1.	0.8	0.	0.	12.18	0.	0.
time (sec)	N/A	1.655	3.228	1.56	0.	7.286	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	671	671	530	0	0	7017	0	0
normalized size	1	1.	0.79	0.	0.	10.46	0.	0.
time (sec)	N/A	1.205	1.675	1.273	0.	4.367	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	236	641	0	3549	0	0
normalized size	1	1.	0.77	2.1	0.	11.64	0.	0.
time (sec)	N/A	0.55	0.988	0.789	0.	3.468	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	32.793	3.118	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	94.238	5.75	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.933	0.375	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	607	607	415	0	0	1045	0	0
normalized size	1	1.	0.68	0.	0.	1.72	0.	0.
time (sec)	N/A	0.764	5.657	0.285	0.	2.171	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	268	0	0	662	0	0
normalized size	1	1.	0.84	0.	0.	2.08	0.	0.
time (sec)	N/A	0.392	3.932	0.198	0.	1.972	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	138	0	0	319	0	0
normalized size	1	1.	0.93	0.	0.	2.16	0.	0.
time (sec)	N/A	0.149	0.188	0.073	0.	1.827	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.387	0.078	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	3.429	0.189	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	261	526	1766	2414	0	0
normalized size	1	1.	1.59	3.21	10.77	14.72	0.	0.
time (sec)	N/A	0.34	1.83	0.182	2.087	2.249	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	213	282	545	1378	0	0
normalized size	1	1.	1.65	2.19	4.22	10.68	0.	0.
time (sec)	N/A	0.257	1.252	0.108	1.896	1.986	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	199	446	369	363	466	1042
normalized size	1	1.	2.62	5.87	4.86	4.78	6.13	13.71
time (sec)	N/A	0.095	0.497	0.076	1.472	1.792	1.954	1.53

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	72	41	68	142	90	43
normalized size	1	1.	2.57	1.46	2.43	5.07	3.21	1.54
time (sec)	N/A	0.038	0.108	0.023	1.424	1.74	1.55	1.125

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	8.577	0.173	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	8.375	0.238	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	1314	748	6209	2984	0	0
normalized size	1	1.	5.32	3.03	25.14	12.08	0.	0.
time (sec)	N/A	0.472	3.001	0.296	3.069	2.598	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	295	408	815	1674	0	0
normalized size	1	1.	1.57	2.17	4.34	8.9	0.	0.
time (sec)	N/A	0.348	2.568	0.336	2.458	2.183	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	236	216	2379	481	2086	0
normalized size	1	1.	2.13	1.95	21.43	4.33	18.79	0.
time (sec)	N/A	0.16	0.787	0.116	1.578	1.727	4.422	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	85	64	174	186	478	104
normalized size	1	1.	1.89	1.42	3.87	4.13	10.62	2.31
time (sec)	N/A	0.082	0.139	0.026	1.457	1.661	3.377	1.101

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	8.563	0.369	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	9.674	0.639	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	538	974	0	3534	0	0
normalized size	1	1.	1.41	2.55	0.	9.25	0.	0.
time (sec)	N/A	0.621	2.628	0.18	0.	2.783	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	830	538	0	1960	0	0
normalized size	1	1.	2.99	1.94	0.	7.05	0.	0.
time (sec)	N/A	0.493	2.867	0.447	0.	2.322	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	298	662	0	625	4869	0
normalized size	1	1.	1.89	4.19	0.	3.96	30.82	0.
time (sec)	N/A	0.22	1.469	0.149	0.	1.884	9.982	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	117	163	286	247	1127	123
normalized size	1	1.	1.56	2.17	3.81	3.29	15.03	1.64
time (sec)	N/A	0.062	0.198	0.027	1.468	1.792	7.614	1.101

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	6.205	0.657	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	5.845	0.973	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	352	352	443	1151	3750	6904	0	0
normalized size	1	1.	1.26	3.27	10.65	19.61	0.	0.
time (sec)	N/A	0.469	2.569	0.285	3.41	3.22	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	330	643	1904	4084	0	0
normalized size	1	1.	1.33	2.58	7.65	16.4	0.	0.
time (sec)	N/A	0.33	2.03	0.137	1.812	2.618	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	300	245	698	1654	0	0
normalized size	1	1.	2.24	1.83	5.21	12.34	0.	0.
time (sec)	N/A	0.157	1.064	0.144	1.453	2.101	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	48	40	69	293	0	51
normalized size	1	1.	1.26	1.05	1.82	7.71	0.	1.34
time (sec)	N/A	0.054	0.066	0.036	1.005	1.604	0.	1.18

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	10.959	3.595	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	12.578	7.668	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	463	463	1013	1705	10242	10928	0	0
normalized size	1	1.	2.19	3.68	22.12	23.6	0.	0.
time (sec)	N/A	0.776	10.745	0.307	15.895	3.97	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	693	942	5003	6126	0	0
normalized size	1	1.	2.12	2.88	15.3	18.73	0.	0.
time (sec)	N/A	0.509	8.34	0.187	4.048	2.812	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	396	351	1727	2284	0	0
normalized size	1	1.	2.34	2.08	10.22	13.51	0.	0.
time (sec)	N/A	0.189	1.706	0.185	2.008	2.343	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	57	77	151	433	0	119
normalized size	1	1.	1.12	1.51	2.96	8.49	0.	2.33
time (sec)	N/A	0.077	0.186	0.042	0.991	1.843	0.	1.174

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	23.375	3.372	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	43.655	7.221	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	600	600	1485	2257	0	17747	0	0
normalized size	1	1.	2.48	3.76	0.	29.58	0.	0.
time (sec)	N/A	1.108	31.369	0.286	0.	5.571	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	392	392	951	1215	8266	9528	0	0
normalized size	1	1.	2.43	3.1	21.09	24.31	0.	0.
time (sec)	N/A	0.722	17.498	0.231	13.379	3.546	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	484	468	2817	3522	0	0
normalized size	1	1.	2.24	2.17	13.04	16.31	0.	0.
time (sec)	N/A	0.283	3.575	0.227	4.081	2.375	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	85	115	212	636	0	151
normalized size	1	1.	1.04	1.4	2.59	7.76	0.	1.84
time (sec)	N/A	0.09	0.511	0.05	0.985	1.811	0.	1.138

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	73.533	7.305	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	150.7	9.902	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	7.328	0.325	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	1.739	0.134	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.763	0.105	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	5.396	0.083	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	9.897	0.106	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	544	544	956	0	0	5493	0	0
normalized size	1	1.	1.76	0.	0.	10.1	0.	0.
time (sec)	N/A	0.968	3.295	0.843	0.	3.319	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	408	408	445	0	0	3943	0	0
normalized size	1	1.	1.09	0.	0.	9.66	0.	0.
time (sec)	N/A	0.859	2.056	0.664	0.	3.06	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	299	548	0	2583	0	0
normalized size	1	1.	1.12	2.05	0.	9.67	0.	0.
time (sec)	N/A	0.585	1.587	0.141	0.	3.139	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	59	70	0	510	0	104
normalized size	1	1.	1.04	1.23	0.	8.95	0.	1.82
time (sec)	N/A	0.068	0.11	0.	0.	1.892	0.	1.103

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	643	643	1020	0	0	6095	0	0
normalized size	1	1.	1.59	0.	0.	9.48	0.	0.
time (sec)	N/A	1.176	6.673	0.894	0.	4.403	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	479	531	0	0	4311	0	0
normalized size	1	1.	1.11	0.	0.	9.	0.	0.
time (sec)	N/A	1.038	3.187	0.976	0.	3.451	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	311	311	709	625	0	2770	0	0
normalized size	1	1.	2.28	2.01	0.	8.91	0.	0.
time (sec)	N/A	0.551	6.734	0.296	0.	3.259	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	96	0	609	0	134
normalized size	1	1.	0.95	1.28	0.	8.12	0.	1.79
time (sec)	N/A	0.106	0.18	0.027	0.	1.875	0.	1.121

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	802	802	1923	0	0	6724	0	0
normalized size	1	1.	2.4	0.	0.	8.38	0.	0.
time (sec)	N/A	1.341	5.506	0.207	0.	5.399	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	592	592	1166	0	0	4691	0	0
normalized size	1	1.	1.97	0.	0.	7.92	0.	0.
time (sec)	N/A	1.18	4.439	0.388	0.	4.033	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	382	382	752	710	0	2938	0	0
normalized size	1	1.	1.97	1.86	0.	7.69	0.	0.
time (sec)	N/A	0.666	8.038	0.43	0.	3.596	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	97	216	0	768	0	204
normalized size	1	1.	0.91	2.02	0.	7.18	0.	1.91
time (sec)	N/A	0.186	0.247	0.028	0.	2.272	0.	1.12

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	732	732	894	0	0	8263	0	0
normalized size	1	1.	1.22	0.	0.	11.29	0.	0.
time (sec)	N/A	1.117	2.603	0.864	0.	4.776	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	528	528	573	0	0	5696	0	0
normalized size	1	1.	1.09	0.	0.	10.79	0.	0.
time (sec)	N/A	0.946	1.69	0.642	0.	3.985	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	325	325	764	660	0	3510	0	0
normalized size	1	1.	2.35	2.03	0.	10.8	0.	0.
time (sec)	N/A	0.615	6.392	0.169	0.	3.939	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	77	69	0	698	0	112
normalized size	1	1.	1.15	1.03	0.	10.42	0.	1.67
time (sec)	N/A	0.083	0.07	0.001	0.	2.755	0.	1.495

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	882	882	1680	0	0	10330	0	0
normalized size	1	1.	1.9	0.	0.	11.71	0.	0.
time (sec)	N/A	1.551	43.034	2.592	0.	7.455	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	639	639	911	0	0	6947	0	0
normalized size	1	1.	1.43	0.	0.	10.87	0.	0.
time (sec)	N/A	1.206	11.921	2.214	0.	4.929	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	370	370	933	766	0	4132	0	0
normalized size	1	1.	2.52	2.07	0.	11.17	0.	0.
time (sec)	N/A	0.616	11.258	0.194	0.	4.488	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	111	109	0	944	0	176
normalized size	1	1.	1.34	1.31	0.	11.37	0.	2.12
time (sec)	N/A	0.128	0.445	0.001	0.	2.705	0.	1.269

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	7.663	0.296	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.798	0.154	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.306	0.067	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	36.304	0.079	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	6.386	0.108	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	574	574	2141	750	0	3528	0	0
normalized size	1	1.	3.73	1.31	0.	6.15	0.	0.
time (sec)	N/A	1.617	15.405	0.917	0.	4.064	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1106	1106	3757	0	0	7002	0	0
normalized size	1	1.	3.4	0.	0.	6.33	0.	0.
time (sec)	N/A	2.557	24.825	1.519	0.	4.888	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1512	1512	5444	0	0	11271	0	0
normalized size	1	1.	3.6	0.	0.	7.45	0.	0.
time (sec)	N/A	3.066	21.997	1.612	0.	7.113	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	751	751	2408	1084	0	5449	0	0
normalized size	1	1.	3.21	1.44	0.	7.26	0.	0.
time (sec)	N/A	2.955	15.461	1.699	0.	5.517	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1584	1584	13567	0	0	12407	0	0
normalized size	1	1.	8.57	0.	0.	7.83	0.	0.
time (sec)	N/A	5.943	25.016	3.147	0.	7.469	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	2348	2348	11204	0	0	22152	0	0
normalized size	1	1.	4.77	0.	0.	9.43	0.	0.
time (sec)	N/A	8.374	22.235	1.711	0.	13.021	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	276	679	689	1189	0	0
normalized size	1	1.	1.83	4.5	4.56	7.87	0.	0.
time (sec)	N/A	0.234	1.407	0.184	1.395	2.03	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	221	421	396	772	0	0
normalized size	1	1.	1.94	3.69	3.47	6.77	0.	0.
time (sec)	N/A	0.211	0.99	0.131	1.674	1.796	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	246	203	157	425	0	0
normalized size	1	1.	3.11	2.57	1.99	5.38	0.	0.
time (sec)	N/A	0.125	0.507	0.148	1.365	1.949	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	19	24	39	24	26
normalized size	1	1.	1.	1.19	1.5	2.44	1.5	1.62
time (sec)	N/A	0.025	0.011	0.012	0.996	1.718	0.491	1.149

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	3.064	0.181	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	3.91	0.217	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	102	436	721	329	1232	0
normalized size	1	1.	1.03	4.4	7.28	3.32	12.44	0.
time (sec)	N/A	0.144	0.643	0.066	1.634	1.668	12.786	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	74	215	417	209	770	0
normalized size	1	1.	0.99	2.87	5.56	2.79	10.27	0.
time (sec)	N/A	0.115	0.423	0.059	1.563	1.756	8.523	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	53	78	204	117	439	0
normalized size	1	1.	1.04	1.53	4.	2.29	8.61	0.
time (sec)	N/A	0.064	0.505	0.058	1.523	1.602	5.38	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	97	43	70	38	119	46
normalized size	1	1.	5.11	2.26	3.68	2.	6.26	2.42
time (sec)	N/A	0.042	0.139	0.048	1.592	1.598	3.262	1.117

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	58	102	220	230	0	967
normalized size	1	1.	0.81	1.42	3.06	3.19	0.	13.43
time (sec)	N/A	0.201	0.275	0.054	1.321	1.736	0.	1.359

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	80	132	232	315	0	0
normalized size	1	1.	0.84	1.39	2.44	3.32	0.	0.
time (sec)	N/A	0.2	0.416	0.053	1.417	1.611	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	132	737	772	570	0	0
normalized size	1	1.	0.6	3.37	3.53	2.6	0.	0.
time (sec)	N/A	0.243	1.145	0.063	1.196	1.697	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	95	339	390	327	1705	0
normalized size	1	1.	0.59	2.11	2.42	2.03	10.59	0.
time (sec)	N/A	0.173	0.796	0.062	1.095	1.726	17.459	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	52	114	154	167	787	0
normalized size	1	1.	0.57	1.25	1.69	1.84	8.65	0.
time (sec)	N/A	0.091	0.886	0.054	1.036	1.633	10.783	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	24	28	34	61	158	34
normalized size	1	1.	0.75	0.88	1.06	1.91	4.94	1.06
time (sec)	N/A	0.046	0.042	0.016	1.01	1.636	7.359	1.147

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	105	161	378	412	0	6518
normalized size	1	1.	0.82	1.26	2.95	3.22	0.	50.92
time (sec)	N/A	0.296	0.386	0.051	1.407	1.72	0.	2.105

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	203	230	414	610	0	0
normalized size	1	1.	1.16	1.31	2.37	3.49	0.	0.
time (sec)	N/A	0.334	0.572	0.049	1.619	1.926	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	502	502	865	1265	5164	4439	0	0
normalized size	1	1.	1.72	2.52	10.29	8.84	0.	0.
time (sec)	N/A	0.488	8.759	0.26	5.559	3.157	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	278	278	670	677	2596	2646	0	0
normalized size	1	1.	2.41	2.44	9.34	9.52	0.	0.
time (sec)	N/A	0.267	8.018	0.181	2.286	2.646	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	655	303	986	1339	0	0
normalized size	1	1.	3.81	1.76	5.73	7.78	0.	0.
time (sec)	N/A	0.139	2.933	0.214	1.625	2.111	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	54	63	163	0	78
normalized size	1	1.	0.81	1.46	1.7	4.41	0.	2.11
time (sec)	N/A	0.051	0.037	0.052	0.982	1.699	0.	1.208

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	13.839	1.774	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	22.11	3.25	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	475	475	1117	1124	6893	3767	0	0
normalized size	1	1.	2.35	2.37	14.51	7.93	0.	0.
time (sec)	N/A	0.594	8.759	0.338	6.673	3.117	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	343	343	637	573	1800	2186	0	0
normalized size	1	1.	1.86	1.67	5.25	6.37	0.	0.
time (sec)	N/A	0.378	6.61	0.236	2.926	2.357	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	231	466	1505	417	0	8986
normalized size	1	1.	1.52	3.07	9.9	2.74	0.	59.12
time (sec)	N/A	0.145	1.052	0.172	1.134	1.738	0.	3.821

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	45	70	174	131	0	90
normalized size	1	1.	1.07	1.67	4.14	3.12	0.	2.14
time (sec)	N/A	0.051	0.052	0.05	1.004	1.603	0.	1.171

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	18.86	3.027	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	24.846	5.532	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	698	698	1901	2161	0	6093	0	0
normalized size	1	1.	2.72	3.1	0.	8.73	0.	0.
time (sec)	N/A	0.735	9.986	0.325	0.	4.698	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	431	431	1468	1119	7104	3775	0	0
normalized size	1	1.	3.41	2.6	16.48	8.76	0.	0.
time (sec)	N/A	0.398	8.926	0.361	40.691	3.19	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	1171	483	2665	2080	0	0
normalized size	1	1.	4.86	2.	11.06	8.63	0.	0.
time (sec)	N/A	0.191	6.595	0.378	4.788	2.483	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	75	90	123	336	0	130
normalized size	1	1.	0.97	1.17	1.6	4.36	0.	1.69
time (sec)	N/A	0.08	0.103	0.058	0.966	1.748	0.	1.235

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	34.35	5.279	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	51.992	1.749	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	405	0	0	861	0	0
normalized size	1	1.	0.9	0.	0.	1.92	0.	0.
time (sec)	N/A	0.643	4.717	0.199	0.	1.99	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	253	0	0	468	0	0
normalized size	1	1.	0.91	0.	0.	1.69	0.	0.
time (sec)	N/A	0.319	2.511	0.19	0.	2.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	220	0	0	306	0	0
normalized size	1	1.	1.43	0.	0.	1.99	0.	0.
time (sec)	N/A	0.177	0.984	0.116	0.	1.81	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	7.704	0.088	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.333	0.	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	9.267	0.126	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	13.544	0.165	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	432	432	410	0	0	4316	0	0
normalized size	1	1.	0.95	0.	0.	9.99	0.	0.
time (sec)	N/A	0.608	0.186	0.921	0.	2.985	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	302	0	0	3082	0	0
normalized size	1	1.	0.94	0.	0.	9.63	0.	0.
time (sec)	N/A	0.513	0.172	0.716	0.	2.673	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	197	1006	0	1986	0	0
normalized size	1	1.	0.93	4.75	0.	9.37	0.	0.
time (sec)	N/A	0.285	0.049	0.162	0.	2.876	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	42	41	26
normalized size	1	1.	1.	1.06	1.33	2.33	2.28	1.44
time (sec)	N/A	0.026	0.007	0.	0.953	1.574	0.58	1.216

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	618	618	1025	0	0	5505	0	0
normalized size	1	1.	1.66	0.	0.	8.91	0.	0.
time (sec)	N/A	1.068	3.474	1.177	0.	3.825	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	460	460	536	0	0	3931	0	0
normalized size	1	1.	1.17	0.	0.	8.55	0.	0.
time (sec)	N/A	0.929	2.623	0.956	0.	3.145	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	298	298	716	1123	0	2564	0	0
normalized size	1	1.	2.4	3.77	0.	8.6	0.	0.
time (sec)	N/A	0.535	6.919	0.337	0.	3.222	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	398	142	0	498	0	128
normalized size	1	1.	5.69	2.03	0.	7.11	0.	1.83
time (sec)	N/A	0.116	2.123	0.001	0.	1.852	0.	1.149

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	737	737	2452	0	0	6086	0	0
normalized size	1	1.	3.33	0.	0.	8.26	0.	0.
time (sec)	N/A	0.881	10.121	1.418	0.	4.347	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	548	548	2397	0	0	4139	0	0
normalized size	1	1.	4.37	0.	0.	7.55	0.	0.
time (sec)	N/A	0.733	5.02	1.442	0.	3.335	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	351	351	2165	1750	0	2533	0	0
normalized size	1	1.	6.17	4.99	0.	7.22	0.	0.
time (sec)	N/A	0.409	14.457	0.773	0.	3.219	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	72	74	128	0	76
normalized size	1	1.	0.89	1.18	1.21	2.1	0.	1.25
time (sec)	N/A	0.068	0.077	0.001	0.946	1.737	0.	1.185

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	937	937	2496	0	0	7549	0	0
normalized size	1	1.	2.66	0.	0.	8.06	0.	0.
time (sec)	N/A	1.618	9.836	0.914	0.	3.97	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	667	667	1561	0	0	5157	0	0
normalized size	1	1.	2.34	0.	0.	7.73	0.	0.
time (sec)	N/A	1.143	5.548	0.7	0.	3.329	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	413	413	2743	861	0	3094	0	0
normalized size	1	1.	6.64	2.08	0.	7.49	0.	0.
time (sec)	N/A	0.635	16.59	0.254	0.	3.779	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	64	76	86	158	0	96
normalized size	1	1.	0.85	1.01	1.15	2.11	0.	1.28
time (sec)	N/A	0.081	0.061	0.002	0.946	2.256	0.	1.192

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	923	923	1438	0	0	9469	0	0
normalized size	1	1.	1.56	0.	0.	10.26	0.	0.
time (sec)	N/A	1.937	9.375	2.507	0.	7.736	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	659	659	1122	0	0	6276	0	0
normalized size	1	1.	1.7	0.	0.	9.52	0.	0.
time (sec)	N/A	1.434	7.901	3.475	0.	5.222	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	349	349	842	1542	0	3087	0	0
normalized size	1	1.	2.41	4.42	0.	8.85	0.	0.
time (sec)	N/A	0.795	9.709	0.359	0.	3.955	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	152	117	0	684	0	144
normalized size	1	1.	1.81	1.39	0.	8.14	0.	1.71
time (sec)	N/A	0.101	0.263	0.002	0.	2.182	0.	1.657

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	4.552	0.243	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	3.3	0.151	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.11	0.	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	139.158	0.079	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	6.208	0.129	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	73	194	0	736	0	0
normalized size	1	1.	0.95	2.52	0.	9.56	0.	0.
time (sec)	N/A	0.072	0.446	0.889	0.	2.076	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	280	280	311	606	0	3263	0	0
normalized size	1	1.	1.11	2.16	0.	11.65	0.	0.
time (sec)	N/A	0.528	3.089	0.892	0.	3.851	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	418	418	446	0	0	5299	0	0
normalized size	1	1.	1.07	0.	0.	12.68	0.	0.
time (sec)	N/A	0.885	2.373	1.598	0.	3.71	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	112	349	0	1364	0	0
normalized size	1	1.	0.97	3.01	0.	11.76	0.	0.
time (sec)	N/A	0.097	1.115	1.747	0.	2.033	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	1104	946	0	5252	0	0
normalized size	1	1.	3.09	2.65	0.	14.71	0.	0.
time (sec)	N/A	0.611	14.813	1.967	0.	4.332	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	753	753	2311	0	0	10630	0	0
normalized size	1	1.	3.07	0.	0.	14.12	0.	0.
time (sec)	N/A	1.274	19.517	1.497	0.	6.018	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	765	765	1194	0	0	7471	0	0
normalized size	1	1.	1.56	0.	0.	9.77	0.	0.
time (sec)	N/A	1.426	2.241	2.293	0.	3.967	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	557	557	607	0	0	5239	0	0
normalized size	1	1.	1.09	0.	0.	9.41	0.	0.
time (sec)	N/A	1.19	1.75	1.974	0.	3.303	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	351	351	812	1207	0	3267	0	0
normalized size	1	1.	2.31	3.44	0.	9.31	0.	0.
time (sec)	N/A	0.66	6.798	0.306	0.	3.314	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	90	137	0	640	0	127
normalized size	1	1.	1.2	1.83	0.	8.53	0.	1.69
time (sec)	N/A	0.184	0.112	0.003	0.	2.053	0.	2.167

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	763	763	4014	0	0	7961	0	0
normalized size	1	1.	5.26	0.	0.	10.43	0.	0.
time (sec)	N/A	1.359	10.63	4.033	0.	5.837	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	566	566	1834	0	0	5391	0	0
normalized size	1	1.	3.24	0.	0.	9.52	0.	0.
time (sec)	N/A	1.122	9.363	3.634	0.	3.794	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	379	379	2209	1721	0	3194	0	0
normalized size	1	1.	5.83	4.54	0.	8.43	0.	0.
time (sec)	N/A	0.631	14.832	1.253	0.	3.548	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	68	73	131	0	76
normalized size	1	1.	0.9	1.15	1.24	2.22	0.	1.29
time (sec)	N/A	0.108	0.075	0.073	0.967	1.967	0.	1.258

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1138	1138	1181	0	0	9441	0	0
normalized size	1	1.	1.04	0.	0.	8.3	0.	0.
time (sec)	N/A	2.107	6.635	3.191	0.	7.698	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	825	825	1254	0	0	6433	0	0
normalized size	1	1.	1.52	0.	0.	7.8	0.	0.
time (sec)	N/A	1.633	4.984	3.208	0.	5.15	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	524	524	934	1901	0	3862	0	0
normalized size	1	1.	1.78	3.63	0.	7.37	0.	0.
time (sec)	N/A	0.899	11.814	1.073	0.	4.032	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	143	334	0	853	0	247
normalized size	1	1.	1.15	2.69	0.	6.88	0.	1.99
time (sec)	N/A	0.28	0.281	0.083	0.	3.043	0.	2.088

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	852	852	2974	0	0	9106	0	0
normalized size	1	1.	3.49	0.	0.	10.69	0.	0.
time (sec)	N/A	1.782	46.364	2.659	0.	5.069	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	616	616	1833	0	0	6120	0	0
normalized size	1	1.	2.98	0.	0.	9.94	0.	0.
time (sec)	N/A	1.387	14.007	2.075	0.	3.539	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	386	386	2314	1732	0	3592	0	0
normalized size	1	1.	5.99	4.49	0.	9.31	0.	0.
time (sec)	N/A	0.784	14.746	0.381	0.	3.487	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	72	77	166	0	97
normalized size	1	1.	0.9	1.2	1.28	2.77	0.	1.62
time (sec)	N/A	0.123	0.094	0.003	0.974	1.951	0.	1.157

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1144	1144	3860	0	0	10645	0	0
normalized size	1	1.	3.37	0.	0.	9.31	0.	0.
time (sec)	N/A	2.656	42.502	3.043	0.	8.591	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	840	840	951	0	0	7205	0	0
normalized size	1	1.	1.13	0.	0.	8.58	0.	0.
time (sec)	N/A	2.152	10.812	3.44	0.	5.342	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	517	517	1019	1890	0	4292	0	0
normalized size	1	1.	1.97	3.66	0.	8.3	0.	0.
time (sec)	N/A	1.142	11.949	1.136	0.	4.273	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	146	249	0	981	0	298
normalized size	1	1.	1.4	2.39	0.	9.43	0.	2.87
time (sec)	N/A	0.27	0.802	0.076	0.	2.953	0.	1.819

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1432	1432	3944	0	0	11439	0	0
normalized size	1	1.	2.75	0.	0.	7.99	0.	0.
time (sec)	N/A	2.947	44.867	4.713	0.	9.775	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1051	1051	5156	0	0	7509	0	0
normalized size	1	1.	4.91	0.	0.	7.14	0.	0.
time (sec)	N/A	2.241	13.575	4.864	0.	5.624	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	641	641	2504	2485	0	4313	0	0
normalized size	1	1.	3.91	3.88	0.	6.73	0.	0.
time (sec)	N/A	1.225	15.217	2.569	0.	4.428	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	86	124	123	317	0	142
normalized size	1	1.	0.9	1.29	1.28	3.3	0.	1.48
time (sec)	N/A	0.155	0.192	0.073	0.983	2.17	0.	2.194

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [249] had the largest ratio of [0.6154]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	2	1.	14	0.143
2	A	4	2	1.	14	0.143
3	A	3	2	1.	14	0.143
4	A	2	2	1.	12	0.167
5	A	3	3	1.	14	0.214
6	A	4	4	1.	14	0.286
7	A	5	4	1.	14	0.286
8	A	6	4	1.	16	0.25
9	A	4	3	1.	16	0.188
10	A	4	4	1.	16	0.25
11	A	2	1	1.	14	0.071
12	A	5	4	1.	16	0.25
13	A	5	5	1.	16	0.312
14	A	7	6	1.	16	0.375
15	A	7	7	1.	16	0.438
16	A	12	4	1.	16	0.25
17	A	8	4	1.	16	0.25
18	A	6	4	1.	16	0.25
19	A	3	3	1.	14	0.214
20	A	8	4	1.	16	0.25
21	A	8	4	1.	16	0.25
22	A	12	5	1.	16	0.312
23	A	9	5	1.	14	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	7	4	1.	14	0.286
25	A	5	3	1.	12	0.25
26	A	0	0	0.	0	0.
27	A	0	0	0.	0	0.
28	A	6	6	1.	16	0.375
29	A	5	5	1.	16	0.312
30	A	2	2	1.	14	0.143
31	A	0	0	0.	0	0.
32	A	0	0	0.	0	0.
33	A	15	8	1.	16	0.5
34	A	9	6	1.	16	0.375
35	A	6	4	1.	14	0.286
36	A	0	0	0.	0	0.
37	A	0	0	0.	0	0.
38	A	8	6	1.	16	0.375
39	A	7	6	1.	16	0.375
40	A	6	6	1.	16	0.375
41	A	5	5	1.	16	0.312
42	A	6	6	1.	16	0.375
43	A	7	6	1.	16	0.375
44	A	8	6	1.	16	0.375
45	A	10	9	1.	18	0.5
46	A	9	8	1.	18	0.444
47	A	8	7	1.	18	0.389
48	A	7	6	1.	18	0.333
49	A	7	7	1.	18	0.389
50	A	9	8	1.	18	0.444
51	A	9	9	1.	18	0.5
52	A	11	8	1.	18	0.444
53	A	23	8	1.	18	0.444
54	A	20	8	1.	18	0.444
55	A	14	7	1.	18	0.389

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	12	6	1.	18	0.333
57	A	12	6	1.	18	0.333
58	A	18	7	1.	18	0.389
59	A	19	8	1.	18	0.444
60	A	4	3	1.	12	0.25
61	A	3	3	1.	12	0.25
62	A	2	2	1.	12	0.167
63	A	3	3	1.	12	0.25
64	A	4	3	1.	12	0.25
65	A	0	0	0.	0	0.
66	A	0	0	0.	0	0.
67	A	2	1	1.	25	0.04
68	A	3	2	1.	29	0.069
69	A	2	1	1.	28	0.036
70	A	3	1	1.	28	0.036
71	A	0	0	0.	0	0.
72	A	8	3	1.	16	0.188
73	A	5	3	1.	16	0.188
74	A	3	2	1.	14	0.143
75	A	0	0	0.	0	0.
76	A	0	0	0.	0	0.
77	A	3	2	1.	12	0.167
78	A	3	2	1.	12	0.167
79	A	3	2	1.	12	0.167
80	A	3	2	1.	10	0.2
81	A	3	2	1.	12	0.167
82	A	3	2	1.	12	0.167
83	A	3	2	1.	12	0.167
84	A	5	3	1.	14	0.214
85	A	5	3	1.	14	0.214
86	A	5	3	1.	14	0.214
87	A	5	3	1.	12	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	5	3	1.	14	0.214
89	A	5	3	1.	14	0.214
90	A	5	3	1.	14	0.214
91	A	4	2	1.	28	0.071
92	A	7	5	1.	32	0.156
93	A	4	2	1.	28	0.071
94	A	5	2	1.	28	0.071
95	A	6	3	1.	18	0.167
96	A	5	3	1.	18	0.167
97	A	4	3	1.	16	0.188
98	A	5	4	1.	18	0.222
99	A	6	5	1.	18	0.278
100	A	7	5	1.	18	0.278
101	A	10	6	1.	20	0.3
102	A	9	7	1.	20	0.35
103	A	6	4	1.	18	0.222
104	A	9	5	1.	20	0.25
105	A	9	5	1.	20	0.25
106	A	15	8	1.	20	0.4
107	A	7	7	1.	20	0.35
108	A	6	6	1.	20	0.3
109	A	3	3	1.	18	0.167
110	A	0	0	0.	0	0.
111	A	0	0	0.	0	0.
112	A	10	9	1.	20	0.45
113	A	9	9	1.	20	0.45
114	A	4	4	1.	18	0.222
115	A	0	0	0.	0	0.
116	A	0	0	0.	0	0.
117	A	7	7	1.	21	0.333
118	A	6	6	1.	21	0.286
119	A	3	3	1.	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	0	0	0.	0	0.
121	A	0	0	0.	0	0.
122	A	5	3	1.	18	0.167
123	A	4	3	1.	18	0.167
124	A	3	3	1.	16	0.188
125	A	4	4	1.	18	0.222
126	A	5	5	1.	18	0.278
127	A	6	5	1.	18	0.278
128	A	9	5	1.	18	0.278
129	A	7	5	1.	18	0.278
130	A	4	4	1.	16	0.25
131	A	9	5	1.	18	0.278
132	A	9	5	1.	18	0.278
133	A	13	6	1.	18	0.333
134	A	10	6	1.	18	0.333
135	A	8	5	1.	18	0.278
136	A	6	4	1.	16	0.25
137	A	0	0	0.	0	0.
138	A	0	0	0.	0	0.
139	A	16	9	1.	18	0.5
140	A	10	7	1.	18	0.389
141	A	7	5	1.	16	0.312
142	A	0	0	0.	0	0.
143	A	0	0	0.	0	0.
144	A	0	0	0.	0	0.
145	A	0	0	0.	0	0.
146	A	12	5	1.	20	0.25
147	A	9	5	1.	20	0.25
148	A	5	3	1.	18	0.167
149	A	0	0	0.	0	0.
150	A	0	0	0.	0	0.
151	A	6	3	1.	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	5	3	1.	18	0.167
153	A	4	3	1.	16	0.188
154	A	5	4	1.	18	0.222
155	A	6	5	1.	18	0.278
156	A	7	5	1.	18	0.278
157	A	10	6	1.	20	0.3
158	A	9	7	1.	20	0.35
159	A	6	4	1.	18	0.222
160	A	10	5	1.	20	0.25
161	A	11	7	1.	20	0.35
162	A	14	8	1.	20	0.4
163	A	12	7	1.	20	0.35
164	A	10	6	1.	20	0.3
165	A	8	5	1.	18	0.278
166	A	0	0	0.	0	0.
167	A	0	0	0.	0	0.
168	A	22	9	1.	20	0.45
169	A	18	10	1.	20	0.5
170	A	11	8	1.	18	0.444
171	A	0	0	0.	0	0.
172	A	0	0	0.	0	0.
173	A	0	0	0.	0	0.
174	A	18	5	1.	20	0.25
175	A	10	5	1.	20	0.25
176	A	5	3	1.	18	0.167
177	A	0	0	0.	0	0.
178	A	0	0	0.	0	0.
179	A	9	9	1.	26	0.346
180	A	8	8	1.	26	0.308
181	A	5	4	1.	24	0.167
182	A	2	2	1.	19	0.105
183	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	0	0	0.	0	0.
185	A	14	11	1.	28	0.393
186	A	12	10	1.	28	0.357
187	A	8	6	1.	26	0.231
188	A	4	4	1.	21	0.19
189	A	0	0	0.	0	0.
190	A	0	0	0.	0	0.
191	A	19	13	1.	28	0.464
192	A	17	13	1.	28	0.464
193	A	11	7	1.	26	0.269
194	A	2	2	1.	21	0.095
195	A	0	0	0.	0	0.
196	A	0	0	0.	0	0.
197	A	17	10	1.	26	0.385
198	A	14	11	1.	26	0.423
199	A	9	7	1.	24	0.292
200	A	3	3	1.	19	0.158
201	A	0	0	0.	0	0.
202	A	0	0	0.	0	0.
203	A	24	10	1.	28	0.357
204	A	20	11	1.	28	0.393
205	A	12	7	1.	26	0.269
206	A	5	5	1.	21	0.238
207	A	0	0	0.	0	0.
208	A	0	0	0.	0	0.
209	A	40	13	1.	28	0.464
210	A	30	13	1.	28	0.464
211	A	19	8	1.	26	0.308
212	A	6	6	1.	21	0.286
213	A	0	0	0.	0	0.
214	A	0	0	0.	0	0.
215	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	0	0	0.	0	0.
217	A	0	0	0.	0	0.
218	A	0	0	0.	0	0.
219	A	0	0	0.	0	0.
220	A	14	9	1.	26	0.346
221	A	12	8	1.	26	0.308
222	A	10	6	1.	24	0.25
223	A	4	4	1.	19	0.21
224	A	19	11	1.	28	0.393
225	A	16	10	1.	28	0.357
226	A	13	8	1.	26	0.308
227	A	6	6	1.	21	0.286
228	A	24	13	1.	28	0.464
229	A	21	13	1.	28	0.464
230	A	16	9	1.	26	0.346
231	A	6	6	1.	21	0.286
232	A	22	9	1.	26	0.346
233	A	18	8	1.	26	0.308
234	A	14	7	1.	24	0.292
235	A	5	5	1.	19	0.263
236	A	29	11	1.	28	0.393
237	A	24	12	1.	28	0.429
238	A	17	9	1.	26	0.346
239	A	7	7	1.	21	0.333
240	A	0	0	0.	0	0.
241	A	0	0	0.	0	0.
242	A	0	0	0.	0	0.
243	A	0	0	0.	0	0.
244	A	0	0	0.	0	0.
245	A	21	9	1.	24	0.375
246	A	30	11	1.	26	0.423
247	A	36	10	1.	26	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
248	A	48	11	1.	24	0.458
249	A	73	16	1.	26	0.615
250	A	92	14	1.	26	0.538
251	A	6	6	1.	26	0.231
252	A	5	5	1.	26	0.192
253	A	4	4	1.	24	0.167
254	A	2	2	1.	19	0.105
255	A	0	0	0.	0	0.
256	A	0	0	0.	0	0.
257	A	6	4	1.	28	0.143
258	A	5	4	1.	28	0.143
259	A	4	3	1.	26	0.115
260	A	2	2	1.	21	0.095
261	A	5	5	1.	28	0.179
262	A	6	6	1.	28	0.214
263	A	10	8	1.	28	0.286
264	A	7	5	1.	28	0.179
265	A	6	6	1.	26	0.231
266	A	2	1	1.	21	0.048
267	A	9	6	1.	28	0.214
268	A	11	7	1.	28	0.25
269	A	22	13	1.	26	0.5
270	A	13	10	1.	26	0.385
271	A	10	8	1.	24	0.333
272	A	4	3	1.	19	0.158
273	A	0	0	0.	0	0.
274	A	0	0	0.	0	0.
275	A	20	12	1.	28	0.429
276	A	16	12	1.	28	0.429
277	A	7	7	1.	26	0.269
278	A	3	3	1.	21	0.143
279	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	0	0	0.	0	0.
281	A	32	16	1.	28	0.571
282	A	17	12	1.	28	0.429
283	A	11	7	1.	26	0.269
284	A	4	3	1.	21	0.143
285	A	0	0	0.	0	0.
286	A	0	0	0.	0	0.
287	A	14	6	1.	28	0.214
288	A	9	6	1.	28	0.214
289	A	5	4	1.	28	0.143
290	A	0	0	0.	0	0.
291	A	0	0	0.	0	0.
292	A	0	0	0.	0	0.
293	A	0	0	0.	0	0.
294	A	11	6	1.	26	0.231
295	A	9	5	1.	26	0.192
296	A	7	4	1.	24	0.167
297	A	2	2	1.	19	0.105
298	A	18	11	1.	28	0.393
299	A	15	10	1.	28	0.357
300	A	12	8	1.	26	0.308
301	A	5	5	1.	21	0.238
302	A	21	14	1.	28	0.5
303	A	16	10	1.	28	0.357
304	A	13	10	1.	26	0.385
305	A	3	2	1.	21	0.095
306	A	29	10	1.	26	0.385
307	A	24	9	1.	26	0.346
308	A	19	8	1.	24	0.333
309	A	6	4	1.	19	0.21
310	A	29	13	1.	28	0.464
311	A	24	14	1.	28	0.5

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
312	A	15	11	1.	26	0.423
313	A	5	5	1.	21	0.238
314	A	0	0	0.	0	0.
315	A	0	0	0.	0	0.
316	A	0	0	0.	0	0.
317	A	0	0	0.	0	0.
318	A	0	0	0.	0	0.
319	A	4	4	1.	24	0.167
320	A	9	6	1.	26	0.231
321	A	11	7	1.	26	0.269
322	A	6	6	1.	24	0.25
323	A	12	9	1.	26	0.346
324	A	19	11	1.	26	0.423
325	A	33	14	1.	32	0.438
326	A	27	13	1.	32	0.406
327	A	21	11	1.	30	0.367
328	A	6	6	1.	25	0.24
329	A	34	17	1.	34	0.5
330	A	26	13	1.	34	0.382
331	A	22	13	1.	32	0.406
332	A	4	3	1.	27	0.111
333	A	53	18	1.	34	0.529
334	A	41	18	1.	34	0.529
335	A	31	14	1.	32	0.438
336	A	6	6	1.	27	0.222
337	A	48	19	1.	34	0.559
338	A	37	17	1.	34	0.5
339	A	28	15	1.	32	0.469
340	A	4	3	1.	27	0.111
341	A	66	20	1.	36	0.556
342	A	53	22	1.	36	0.611
343	A	38	16	1.	34	0.471

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
344	A	6	6	1.	29	0.207
345	A	85	21	1.	36	0.583
346	A	60	20	1.	36	0.556
347	A	45	17	1.	34	0.5
348	A	4	3	1.	29	0.103

Chapter 3

Listing of integrals

3.1 $\int (c+dx)^4 \sin(a+bx)\,dx$

Optimal. Leaf size=92

$$-\frac{24d^3(c+dx)\sin(a+bx)}{b^4}+\frac{12d^2(c+dx)^2\cos(a+bx)}{b^3}+\frac{4d(c+dx)^3\sin(a+bx)}{b^2}-\frac{24d^4\cos(a+bx)}{b^5}-\frac{(c+dx)^4\cos(a}{b}$$

```
[Out] (-24*d^4*Cos[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*Cos[a + b*x])/b^3 - ((c +
d*x)^4*Cos[a + b*x])/b - (24*d^3*(c + d*x)*Sin[a + b*x])/b^4 + (4*d*(c + d*
x)^3*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.0914089, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3296, 2638}

$$-\frac{24d^3(c+dx)\sin(a+bx)}{b^4}+\frac{12d^2(c+dx)^2\cos(a+bx)}{b^3}+\frac{4d(c+dx)^3\sin(a+bx)}{b^2}-\frac{24d^4\cos(a+bx)}{b^5}-\frac{(c+dx)^4\cos(a}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sin[a + b*x],x]
```

```
[Out] (-24*d^4*Cos[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*Cos[a + b*x])/b^3 - ((c +
d*x)^4*Cos[a + b*x])/b - (24*d^3*(c + d*x)*Sin[a + b*x])/b^4 + (4*d*(c + d*
x)^3*Sin[a + b*x])/b^2
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4\sin(a+bx)\,dx &= -\frac{(c+dx)^4\cos(a+bx)}{b}+\frac{(4d)\int (c+dx)^3\cos(a+bx)\,dx}{b}\\ &= -\frac{(c+dx)^4\cos(a+bx)}{b}+\frac{4d(c+dx)^3\sin(a+bx)}{b^2}-\frac{\left(12d^2\right)\int (c+dx)^2\sin(a+bx)\,dx}{b^2}\\ &= \frac{12d^2(c+dx)^2\cos(a+bx)}{b^3}-\frac{(c+dx)^4\cos(a+bx)}{b}+\frac{4d(c+dx)^3\sin(a+bx)}{b^2}-\frac{\left(24d^3\right)\int (c+\cdots}{\cdots}\\ &= \frac{12d^2(c+dx)^2\cos(a+bx)}{b^3}-\frac{(c+dx)^4\cos(a+bx)}{b}-\frac{24d^3(c+dx)\sin(a+bx)}{b^4}+\frac{4d(c+dx)^3\cdots}{b\cdots}\\ &= -\frac{24d^4\cos(a+bx)}{b^5}+\frac{12d^2(c+dx)^2\cos(a+bx)}{b^3}-\frac{(c+dx)^4\cos(a+bx)}{b}-\frac{24d^3(c+dx)\sin(\cdots}{b^4}\end{aligned}$$

Mathematica [A] time = 0.339251, size = 77, normalized size = 0.84

$$\frac{4bd(c+dx)\sin(a+bx)\left(b^2(c+dx)^2-6d^2\right)-\cos(a+bx)\left(-12b^2d^2(c+dx)^2+b^4(c+dx)^4+24d^4\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sin[a + b*x],x]
```

```
[Out] (-((24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x]) + 4*b*
d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Sin[a + b*x])/b^5
```

Maple [B] time = 0.009, size = 551, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^4*d^4*(-(b*x+a)^4*cos(b*x+a)+4*(b*x+a)^3*sin(b*x+a)+12*(b*x+a)^2*c
os(b*x+a)-24*cos(b*x+a)-24*(b*x+a)*sin(b*x+a))-4/b^4*a*d^4*(-(b*x+a)^3*cos(
b*x+a)+3*(b*x+a)^2*sin(b*x+a)-6*sin(b*x+a)+6*(b*x+a)*cos(b*x+a))+4/b^3*c*d^
3*(-(b*x+a)^3*cos(b*x+a)+3*(b*x+a)^2*sin(b*x+a)-6*sin(b*x+a)+6*(b*x+a)*cos(
b*x+a))+6/b^4*a^2*d^4*(-(b*x+a)^2*cos(b*x+a)+2*cos(b*x+a)+2*(b*x+a)*sin(b*x
+a))-12/b^3*a*c*d^3*(-(b*x+a)^2*cos(b*x+a)+2*cos(b*x+a)+2*(b*x+a)*sin(b*x+a
))+6/b^2*c^2*d^2*(-(b*x+a)^2*cos(b*x+a)+2*cos(b*x+a)+2*(b*x+a)*sin(b*x+a))-
4/b^4*a^3*d^4*(sin(b*x+a)-(b*x+a)*cos(b*x+a))+12/b^3*a^2*c*d^3*(sin(b*x+a)-
(b*x+a)*cos(b*x+a))-12/b^2*a*c^2*d^2*(sin(b*x+a)-(b*x+a)*cos(b*x+a))+4/b*c^
3*d*(sin(b*x+a)-(b*x+a)*cos(b*x+a))-1/b^4*a^4*d^4*cos(b*x+a)+4/b^3*a^3*c*d^
3*cos(b*x+a)-6/b^2*a^2*c^2*d^2*cos(b*x+a)+4/b*a*c^3*d*cos(b*x+a)-c^4*cos(b*
x+a))
```

Maxima [B] time = 1.12448, size = 662, normalized size = 7.2

$$-\frac{c^4\cos(bx+a)-\frac{4ac^3d\cos(bx+a)}{b}+\frac{6a^2c^2d^2\cos(bx+a)}{b^2}-\frac{4a^3cd^3\cos(bx+a)}{b^3}+\frac{a^4d^4\cos(bx+a)}{b^4}+\frac{4((bx+a)\cos(bx+a)-\sin(bx+a))c^3d}{b}-\frac{12((bx+ \cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -(c^4*cos(b*x + a) - 4*a*c^3*d*cos(b*x + a)/b + 6*a^2*c^2*d^2*cos(b*x + a)/
b^2 - 4*a^3*c*d^3*cos(b*x + a)/b^3 + a^4*d^4*cos(b*x + a)/b^4 + 4*((b*x + a
)*cos(b*x + a) - sin(b*x + a))*c^3*d/b - 12*((b*x + a)*cos(b*x + a) - sin(b
*x + a))*a*c^2*d^2/b^2 + 12*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a^2*c*d
^3/b^3 - 4*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a^3*d^4/b^4 + 6*(((b*x +
 a)^2 - 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*c^2*d^2/b^2 - 12*(((b*x
 + a)^2 - 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*a*c*d^3/b^3 + 6*(((b*
x + a)^2 - 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*a^2*d^4/b^4 + 4*(((b
*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - 3*((b*x + a)^2 - 2)*sin(b*x + a))*c
*d^3/b^3 - 4*(((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - 3*((b*x + a)^2 - 2
)*sin(b*x + a))*a*d^4/b^4 + (((b*x + a)^4 - 12*(b*x + a)^2 + 24)*cos(b*x +
a) - 4*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^4/b^4)/b
```

Fricas [A] time = 1.63171, size = 348, normalized size = 3.78

$$-\frac{\left(b^4d^4x^4+4\,b^4cd^3x^3+b^4c^4-12\,b^2c^2d^2+24\,d^4+6\left(b^4c^2d^2-2\,b^2d^4\right)x^2+4\left(b^4c^3d-6\,b^2cd^3\right)x\right)\cos\left(bx+a\right)-4\left(b^3d^4x^3\right.}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -((b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*(b
^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a) -
 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2
- 2*b*d^4)*x)*sin(b*x + a))/b^5
```

Sympy [A] time = 2.92093, size = 311, normalized size = 3.38

$$\begin{cases} -\frac{c^4\cos(a+bx)}{b}-\frac{4c^3dx\cos(a+bx)}{b}-\frac{6c^2d^2x^2\cos(a+bx)}{b}-\frac{4cd^3x^3\cos(a+bx)}{b}-\frac{d^4x^4\cos(a+bx)}{b}+\frac{4c^3d\sin(a+bx)}{b^2}+\frac{12c^2d^2x\sin(a+bx)}{b^2}+\frac{12cd^3x^2\sin}{b^2} \\ \left(c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5}\right)\sin(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**4*cos(a + b*x)/b - 4*c**3*d*x*cos(a + b*x)/b - 6*c**2*d**2*x
**2*cos(a + b*x)/b - 4*c*d**3*x**3*cos(a + b*x)/b - d**4*x**4*cos(a + b*x)/
b + 4*c**3*d*sin(a + b*x)/b**2 + 12*c**2*d**2*x*sin(a + b*x)/b**2 + 12*c*d*
*3*x**2*sin(a + b*x)/b**2 + 4*d**4*x**3*sin(a + b*x)/b**2 + 12*c**2*d**2*co
s(a + b*x)/b**3 + 24*c*d**3*x*cos(a + b*x)/b**3 + 12*d**4*x**2*cos(a + b*x)
/b**3 - 24*c*d**3*sin(a + b*x)/b**4 - 24*d**4*x*sin(a + b*x)/b**4 - 24*d**4
*cos(a + b*x)/b**5, Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3
+ c*d**3*x**4 + d**4*x**5/5)*sin(a), True))
```

Giac [A] time = 1.12011, size = 231, normalized size = 2.51

$$-\frac{\left(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2+4\,b^4c^3dx+b^4c^4-12\,b^2d^4x^2-24\,b^2cd^3x-12\,b^2c^2d^2+24\,d^4\right)\cos\left(bx+a\right)}{b^5}+\frac{4\left(b^3d^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c
^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*cos(b*x + a
)/b^5 + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*
b*d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

3.2 $\int (c+dx)^3 \sin(a+bx)\,dx$

Optimal. Leaf size=71

$$\frac{6d^2(c+dx)\cos(a+bx)}{b^3}+\frac{3d(c+dx)^2\sin(a+bx)}{b^2}-\frac{6d^3\sin(a+bx)}{b^4}-\frac{(c+dx)^3\cos(a+bx)}{b}$$

```
[Out] (6*d^2*(c + d*x)*Cos[a + b*x])/b^3 - ((c + d*x)^3*Cos[a + b*x])/b - (6*d^3*
Sin[a + b*x])/b^4 + (3*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.0652611, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3296, 2637}

$$\frac{6d^2(c+dx)\cos(a+bx)}{b^3}+\frac{3d(c+dx)^2\sin(a+bx)}{b^2}-\frac{6d^3\sin(a+bx)}{b^4}-\frac{(c+dx)^3\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x],x]
```

```
[Out] (6*d^2*(c + d*x)*Cos[a + b*x])/b^3 - ((c + d*x)^3*Cos[a + b*x])/b - (6*d^3*
Sin[a + b*x])/b^4 + (3*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^3 \sin(a+bx)\,dx = -\frac{(c+dx)^3\cos(a+bx)}{b} + \frac{(3d)\int (c+dx)^2\cos(a+bx)\,dx}{b}$$
$$= -\frac{(c+dx)^3\cos(a+bx)}{b} + \frac{3d(c+dx)^2\sin(a+bx)}{b^2} - \frac{\left(6d^2\right)\int (c+dx)\sin(a+bx)\,dx}{b^2}$$
$$= \frac{6d^2(c+dx)\cos(a+bx)}{b^3} - \frac{(c+dx)^3\cos(a+bx)}{b} + \frac{3d(c+dx)^2\sin(a+bx)}{b^2} - \frac{\left(6d^3\right)\int \cos(a}{b^3}$$
$$= \frac{6d^2(c+dx)\cos(a+bx)}{b^3} - \frac{(c+dx)^3\cos(a+bx)}{b} - \frac{6d^3\sin(a+bx)}{b^4} + \frac{3d(c+dx)^2\sin(a+b}{b^2}$$

Mathematica [A] time = 0.205382, size = 62, normalized size = 0.87

$$\frac{3d\sin(a+bx)\left(b^2(c+dx)^2-2d^2\right)-b(c+dx)\cos(a+bx)\left(b^2(c+dx)^2-6d^2\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x],x]
```

```
[Out] (-(b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x]) + 3*d*(-2*d^2 + b^2
*(c + d*x)^2)*Sin[a + b*x])/b^4
```

Maple [B] time = 0.007, size = 308, normalized size = 4.3

$$\frac{1}{b}\left(\frac{d^3\left(-(bx+a)^3\cos(bx+a)+3\,(bx+a)^2\sin(bx+a)-6\,\sin(bx+a)+6\,(bx+a)\cos(bx+a)\right)}{b^3} - 3\,\frac{ad^3\left(-(bx+a)^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^3*d^3*(-(b*x+a)^3*cos(b*x+a)+3*(b*x+a)^2*sin(b*x+a)-6*sin(b*x+a)+6
*(b*x+a)*cos(b*x+a))-3/b^3*a*d^3*(-(b*x+a)^2*cos(b*x+a)+2*cos(b*x+a)+2*(b*x
+a)*sin(b*x+a))+3/b^2*c*d^2*(-(b*x+a)^2*cos(b*x+a)+2*cos(b*x+a)+2*(b*x+a)*s
in(b*x+a))+3/b^3*a^2*d^3*(sin(b*x+a)-(b*x+a)*cos(b*x+a))-6/b^2*a*c*d^2*(sin
(b*x+a)-(b*x+a)*cos(b*x+a))+3/b*c^2*d*(sin(b*x+a)-(b*x+a)*cos(b*x+a))+1/b^3
*a^3*d^3*cos(b*x+a)-3/b^2*a^2*c*d^2*cos(b*x+a)+3/b*a*c^2*d*cos(b*x+a)-c^3*c
os(b*x+a))
```

Maxima [B] time = 1.07908, size = 385, normalized size = 5.42

$$-\frac{c^3\cos(bx+a) - \frac{3\,ac^2d\cos(bx+a)}{b} + \frac{3\,a^2cd^2\cos(bx+a)}{b^2} - \frac{a^3d^3\cos(bx+a)}{b^3} + \frac{3\,((bx+a)\cos(bx+a)-\sin(bx+a))c^2d}{b} - \frac{6\,((bx+a)\cos(bx+a)-\sin(bx+a))a}{b^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -(c^3*cos(b*x + a) - 3*a*c^2*d*cos(b*x + a)/b + 3*a^2*c*d^2*cos(b*x + a)/b^
2 - a^3*d^3*cos(b*x + a)/b^3 + 3*((b*x + a)*cos(b*x + a) - sin(b*x + a))*c^
2*d/b - 6*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a*c*d^2/b^2 + 3*((b*x + a
)*cos(b*x + a) - sin(b*x + a))*a^2*d^3/b^3 + 3*(((b*x + a)^2 - 2)*cos(b*x +
 a) - 2*(b*x + a)*sin(b*x + a))*c*d^2/b^2 - 3*(((b*x + a)^2 - 2)*cos(b*x +
a) - 2*(b*x + a)*sin(b*x + a))*a*d^3/b^3 + (((b*x + a)^3 - 6*b*x - 6*a)*cos
(b*x + a) - 3*((b*x + a)^2 - 2)*sin(b*x + a))*d^3/b^3)/b
```

Fricas [A] time = 1.66707, size = 230, normalized size = 3.24

$$-\frac{\left(b^3d^3x^3 + 3\,b^3cd^2x^2 + b^3c^3 - 6\,bcd^2 + 3\left(b^3c^2d - 2\,bd^3\right)x\right)\cos(bx+a) - 3\left(b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d - 2\,d^3\right)\sin(bx+a}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b
*d^3)*x)*cos(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)
*sin(b*x + a))/b^4
```

Sympy [A] time = 1.35605, size = 202, normalized size = 2.85

$$\begin{cases} -\frac{c^3\cos(a+bx)}{b} - \frac{3c^2dx\cos(a+bx)}{b} - \frac{3cd^2x^2\cos(a+bx)}{b} - \frac{d^3x^3\cos(a+bx)}{b} + \frac{3c^2d\sin(a+bx)}{b^2} + \frac{6cd^2x\sin(a+bx)}{b^2} + \frac{3d^3x^2\sin(a+bx)}{b^2} + \frac{6cd^2\cos(a+bx}{b^3} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**3*cos(a + b*x)/b - 3*c**2*d*x*cos(a + b*x)/b - 3*c*d**2*x**2
*cos(a + b*x)/b - d**3*x**3*cos(a + b*x)/b + 3*c**2*d*sin(a + b*x)/b**2 + 6
*c*d**2*x*sin(a + b*x)/b**2 + 3*d**3*x**2*sin(a + b*x)/b**2 + 6*c*d**2*cos(
a + b*x)/b**3 + 6*d**3*x*cos(a + b*x)/b**3 - 6*d**3*sin(a + b*x)/b**4, Ne(b
, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a), True
))
```

Giac [A] time = 1.11934, size = 150, normalized size = 2.11

$$-\frac{\left(b^{3}d^{3}x^{3}+3\,b^{3}cd^{2}x^{2}+3\,b^{3}c^{2}dx+b^{3}c^{3}-6\,bd^{3}x-6\,bcd^{2}\right)\cos\left(bx+a\right)}{b^{4}}+\frac{3\left(b^{2}d^{3}x^{2}+2\,b^{2}cd^{2}x+b^{2}c^{2}d-2\,d^{3}\right)\sin\left(bx+a\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b
*c*d^2)*cos(b*x + a)/b^4 + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d
^3)*sin(b*x + a)/b^4
```

3.3 $\int (c+dx)^2 \sin(a+bx)\,dx$

Optimal. Leaf size=50

$$\frac{2d(c+dx)\sin(a+bx)}{b^2}+\frac{2d^2\cos(a+bx)}{b^3}-\frac{(c+dx)^2\cos(a+bx)}{b}$$

```
[Out] (2*d^2*Cos[a + b*x])/b^3 - ((c + d*x)^2*Cos[a + b*x])/b + (2*d*(c + d*x)*Si
n[a + b*x])/b^2
```

Rubi [A] time = 0.0389583, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3296, 2638}

$$\frac{2d(c+dx)\sin(a+bx)}{b^2}+\frac{2d^2\cos(a+bx)}{b^3}-\frac{(c+dx)^2\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x],x]
```

```
[Out] (2*d^2*Cos[a + b*x])/b^3 - ((c + d*x)^2*Cos[a + b*x])/b + (2*d*(c + d*x)*Si
n[a + b*x])/b^2
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^2\sin(a+bx)\,dx = -\frac{(c+dx)^2\cos(a+bx)}{b} + \frac{(2d)\int (c+dx)\cos(a+bx)\,dx}{b}$$
$$= -\frac{(c+dx)^2\cos(a+bx)}{b} + \frac{2d(c+dx)\sin(a+bx)}{b^2} - \frac{\left(2d^2\right)\int \sin(a+bx)\,dx}{b^2}$$
$$= \frac{2d^2\cos(a+bx)}{b^3} - \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{2d(c+dx)\sin(a+bx)}{b^2}$$

Mathematica [A] time = 0.170771, size = 45, normalized size = 0.9

$$\frac{2bd(c+dx)\sin(a+bx) - \cos(a+bx)\left(b^2(c+dx)^2 - 2d^2\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x],x]
```

```
[Out] (-((-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x]) + 2*b*d*(c + d*x)*Sin[a + b*x])
/b^3
```

Maple [B] time = 0.006, size = 148, normalized size = 3.

$$\frac{1}{b}\left(\frac{d^2\left(-(bx+a)^2\cos(bx+a) + 2\cos(bx+a) + 2(bx+a)\sin(bx+a)\right)}{b^2} - 2\frac{ad^2(\sin(bx+a) - (bx+a)\cos(bx+a))}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^2*d^2*(-(b*x+a)^2*cos(b*x+a)+2*cos(b*x+a)+2*(b*x+a)*sin(b*x+a))-2/
b^2*a*d^2*(sin(b*x+a)-(b*x+a)*cos(b*x+a))+2/b*c*d*(sin(b*x+a)-(b*x+a)*cos(b
*x+a))-1/b^2*a^2*d^2*cos(b*x+a)+2/b*a*c*d*cos(b*x+a)-c^2*cos(b*x+a))
```

Maxima [B] time = 1.03117, size = 190, normalized size = 3.8

$$-\frac{c^2\cos(bx+a) - \frac{2acd\cos(bx+a)}{b} + \frac{a^2d^2\cos(bx+a)}{b^2} + \frac{2((bx+a)\cos(bx+a)-\sin(bx+a))cd}{b} - \frac{2((bx+a)\cos(bx+a)-\sin(bx+a))ad^2}{b^2} + \frac{\left(\left((bx+a)^2-2\right)\cdots\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -(c^2*cos(b*x + a) - 2*a*c*d*cos(b*x + a)/b + a^2*d^2*cos(b*x + a)/b^2 + 2*
((b*x + a)*cos(b*x + a) - sin(b*x + a))*c*d/b - 2*((b*x + a)*cos(b*x + a) -
 sin(b*x + a))*a*d^2/b^2 + (((b*x + a)^2 - 2)*cos(b*x + a) - 2*(b*x + a)*si
n(b*x + a))*d^2/b^2)/b
```

Fricas [A] time = 1.6819, size = 138, normalized size = 2.76

$$-\frac{\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - 2\,d^2\right)\cos\left(bx + a\right) - 2\left(bd^2x + bcd\right)\sin\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) - 2*(b*d^2*x +
 b*c*d)*sin(b*x + a))/b^3
```

Sympy [A] time = 0.636426, size = 112, normalized size = 2.24

$$\begin{cases} -\frac{c^2\cos\left(a+bx\right)}{b} - \frac{2cdx\cos\left(a+bx\right)}{b} - \frac{d^2x^2\cos\left(a+bx\right)}{b} + \frac{2cd\sin\left(a+bx\right)}{b^2} + \frac{2d^2x\sin\left(a+bx\right)}{b^2} + \frac{2d^2\cos\left(a+bx\right)}{b^3} & \text{for}\ b \neq 0 \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**2*cos(a + b*x)/b - 2*c*d*x*cos(a + b*x)/b - d**2*x**2*cos(a
+ b*x)/b + 2*c*d*sin(a + b*x)/b**2 + 2*d**2*x*sin(a + b*x)/b**2 + 2*d**2*co
s(a + b*x)/b**3, Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a), True
))
```

Giac [A] time = 1.15278, size = 88, normalized size = 1.76

$$-\frac{\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}-2\,d^{2}\right)\cos\left(bx+a\right)}{b^{3}}+\frac{2\left(bd^{2}x+bcd\right)\sin\left(bx+a\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)/b^3 + 2*(b*d^2*
x + b*c*d)*sin(b*x + a)/b^3
```

3.4 $\int (c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}$$

```
[Out] -(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.0162482, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3296, 2637}

$$\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x],x]
```

```
[Out] -(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sin(a+bx)\,dx &= -\frac{(c+dx)\cos(a+bx)}{b}+\frac{d\int \cos(a+bx)\,dx}{b}\\ &= -\frac{(c+dx)\cos(a+bx)}{b}+\frac{d\sin(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0703575, size = 27, normalized size = 0.96

$$\frac{d\sin(a+bx)-b(c+dx)\cos(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x],x]
```

```
[Out] (-(b*(c + d*x)*Cos[a + b*x]) + d*Sin[a + b*x])/b^2
```

Maple [A] time = 0.007, size = 52, normalized size = 1.9

$$\frac{1}{b}\left(\frac{d\left(\sin\left(bx+a\right)-\left(bx+a\right)\cos\left(bx+a\right)\right)}{b}+\frac{da\cos\left(bx+a\right)}{b}-c\cos\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b*d*(sin(b*x+a)-(b*x+a)*cos(b*x+a))+1/b*d*a*cos(b*x+a)-c*cos(b*x+a))
```

Maxima [A] time = 1.01222, size = 72, normalized size = 2.57

$$-\frac{c\cos\left(bx+a\right)-\frac{ad\cos(bx+a)}{b}+\frac{((bx+a)\cos(bx+a)-\sin(bx+a))d}{b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -(c*cos(b*x + a) - a*d*cos(b*x + a)/b + ((b*x + a)*cos(b*x + a) - sin(b*x +
 a))*d/b)/b
```

Fricas [A] time = 1.66359, size = 70, normalized size = 2.5

$$-\frac{\left(bdx+bc\right)\cos\left(bx+a\right)-d\sin\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -((b*d*x + b*c)*cos(b*x + a) - d*sin(b*x + a))/b^2
```

Sympy [A] time = 0.235433, size = 46, normalized size = 1.64

$$\begin{cases} -\frac{c\cos(a+bx)}{b} - \frac{dx\cos(a+bx)}{b} + \frac{d\sin(a+bx)}{b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sin(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a),x)
```

```
[Out] Piecewise((-c*cos(a + b*x)/b - d*x*cos(a + b*x)/b + d*sin(a + b*x)/b**2, Ne
(b, 0)), ((c*x + d*x**2/2)*sin(a), True))
```

Giac [A] time = 1.1022, size = 42, normalized size = 1.5

$$-\frac{(bdx + bc)\cos(bx + a)}{b^2} + \frac{d\sin(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*d*x + b*c)*cos(b*x + a)/b^2 + d*sin(b*x + a)/b^2
```

3.5 $\int \frac{\sin(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=51

$$\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] (CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d + (Cos[a - (b*c)/d]*SinInte
gral[(b*c)/d + b*x])/d
```

Rubi [A] time = 0.0982246, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3303, 3299, 3302}

$$\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x),x]
```

```
[Out] (CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d + (Cos[a - (b*c)/d]*SinInte
gral[(b*c)/d + b*x])/d
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{c+dx}\,dx = \cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx$$
$$=\frac{\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

Mathematica [A] time = 0.0949448, size = 49, normalized size = 0.96

$$\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)+\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(c + d*x),x]
```

```
[Out] (CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral
[(b*c)/d + b*x])/d
```

Maple [A] time = 0.009, size = 73, normalized size = 1.4

$$\frac{1}{d}\text{Si}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)-\frac{1}{d}\text{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x+c),x)
```

```
[Out] Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d
+b*c)/d)/d
```

Maxima [C] time = 1.26869, size = 190, normalized size = 3.73

$$-\frac{b\left(iE_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-iE_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\right.}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_int
egral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b*(ex
p_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(1, -(I*
b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/(b*d)
```

Fricas [A] time = 1.61087, size = 200, normalized size = 3.92

$$\frac{\left(\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+2\,\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*sin(-
(b*c - a*d)/d) + 2*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)/(c + d*x), x)
```

Giac [C] time = 1.21009, size = 806, normalized size = 15.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - i
mag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*sin_
integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*real_part(cos_integra
l(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(b*x
 + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-b*x - b*
c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - imag_part(cos_integral(b*x + b*c/d))*ta
n(1/2*a)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - 2*sin_int
egral((b*d*x + b*c)/d)*tan(1/2*a)^2 + 4*imag_part(cos_integral(b*x + b*c/d)
)*tan(1/2*a)*tan(1/2*b*c/d) - 4*imag_part(cos_integral(-b*x - b*c/d))*tan(1
/2*a)*tan(1/2*b*c/d) + 8*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b
*c/d) - imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)^2 + imag_part(c
os_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - 2*sin_integral((b*d*x + b*c)/
d)*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 2
*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a) - 2*real_part(cos_integra
l(b*x + b*c/d))*tan(1/2*b*c/d) - 2*real_part(cos_integral(-b*x - b*c/d))*ta
n(1/2*b*c/d) + imag_part(cos_integral(b*x + b*c/d)) - imag_part(cos_integra
l(-b*x - b*c/d)) + 2*sin_integral((b*d*x + b*c)/d))/(d*tan(1/2*a)^2*tan(1/2
*b*c/d)^2 + d*tan(1/2*a)^2 + d*tan(1/2*b*c/d)^2 + d)
```

3.6 $\int \frac{\sin(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=72

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\sin(a+bx)}{d(c+dx)}$$

```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 - Sin[a + b*x]/(d*(c +
d*x)) - (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2
```

Rubi [A] time = 0.108748, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3297, 3303, 3299, 3302}

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\sin(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x)^2,x]
```


```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 - Sin[a + b*x]/(d*(c +
d*x)) - (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2
```


Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx &= -\frac{\sin(a+bx)}{d(c+dx)}+\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}\\ &= -\frac{\sin(a+bx)}{d(c+dx)}+\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}-\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}\\ &= \frac{b\cos\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\sin(a+bx)}{d(c+dx)}-\frac{b\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d^2}\end{aligned}$$

Mathematica [A] time = 0.210034, size = 66, normalized size = 0.92

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)-b\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(b\left(\frac{c}{d}+x\right)\right)-\frac{d\sin(a+bx)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(c + d*x)^2,x]
```

```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - (d*Sin[a + b*x])/(c + d*x) -
 b*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)])/d^2
```

Maple [A] time = 0.01, size = 107, normalized size = 1.5

$$b\left(-\frac{\sin(bx+a)}{((bx+a)d-da+cb)d}+\frac{1}{d}\left(\frac{1}{d}\operatorname{Si}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)+\frac{1}{d}\operatorname{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x+c)^2,x)
```

```
[Out] b*(-sin(b*x+a)/((b*x+a)*d-d*a+c*b)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)
/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)
```

Maxima [C] time = 1.3256, size = 221, normalized size = 3.07

$$-\frac{b^2\left(i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin}{2\left(bcd+(bx+a)d^2-ad^2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*(I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^2
*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(2,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b*c*d + (b*x +
a)*d^2 - a*d^2)*b)
```

Fricas [A] time = 1.64332, size = 302, normalized size = 4.19

$$-\frac{2\,(bdx+bc)\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)-\left((bdx+bc)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+2\,d\sin\left(bx+\right.}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*d*x + b*c)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - (
(b*d*x + b*c)*cos_integral((b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-(
b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) + 2*d*sin(b*x + a))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)/(c + d*x)**2, x)
```

Giac [C] time = 1.29518, size = 4131, normalized size = 57.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2
*tan(1/2*b*c/d)^2 + b*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x
)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(b*x + b*
c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b*d*x*imag_part(cos_in
tegral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 4*b*d*x*
sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) +
2*b*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(
1/2*b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2
*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/
2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + b*c*real_part(cos_integral(b*x + b*c
/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b*c*real_part(cos_integ
ral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b*d*x*rea
l_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - b*d*x*real_
part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b*d*x*real
_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) +
 4*b*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*ta
n(1/2*b*c/d) - 2*b*c*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*ta
n(1/2*a)^2*tan(1/2*b*c/d) + 2*b*c*imag_part(cos_integral(-b*x - b*c/d))*tan
(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 4*b*c*sin_integral((b*d*x + b*c)/
d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - b*d*x*real_part(cos_integra
l(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - b*d*x*real_part(cos_integ
ral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 2*b*c*imag_part(cos_in
tegral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b*c*ima
g_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)
^2 + 4*b*c*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*
b*c/d)^2 + b*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*
b*c/d)^2 + b*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2
*b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan
```

```
(1/2*a) + 2*b*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(
1/2*a) - 4*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a) -
b*c*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - b*c*
real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b*d*x
*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 2*b*d
*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 4*
b*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 4*b*c*r
eal_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d
) + 4*b*c*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*t
an(1/2*b*c/d) - 2*b*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*t
an(1/2*b*c/d) + 2*b*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*
tan(1/2*b*c/d) - 4*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2
*b*c/d) - b*c*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b
*c/d)^2 - b*c*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*
b*c/d)^2 + 2*b*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*
b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2
*b*c/d)^2 + 4*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d)
^2 + b*c*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2
 + b*c*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2
+ b*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2 + b*d*x*real_pa
rt(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2 - 2*b*c*imag_part(cos_integra
l(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*imag_part(cos_integral(-b
*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) - 4*b*c*sin_integral((b*d*x + b*c)/d
)*tan(1/2*b*x)^2*tan(1/2*a) - b*d*x*real_part(cos_integral(b*x + b*c/d))*ta
n(1/2*a)^2 - b*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 + 2*b
*c*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 2*b
*c*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 4*
b*c*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 4*b*d*x*r
eal_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 4*b*d*x*rea
l_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - 2*b*c*imag_p
art(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b*c*imag_par
t(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 4*b*c*sin_integ
ral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) - b*d*x*real_part(cos_inte
gral(b*x + b*c/d))*tan(1/2*b*c/d)^2 - b*d*x*real_part(cos_integral(-b*x - b
*c/d))*tan(1/2*b*c/d)^2 + 2*b*c*imag_part(cos_integral(b*x + b*c/d))*tan(1/
2*a)*tan(1/2*b*c/d)^2 - 2*b*c*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2
*a)*tan(1/2*b*c/d)^2 + 4*b*c*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1
/2*b*c/d)^2 + 4*d*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*d*tan(1/2*
b*x)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b*c*real_part(cos_integral(b*x + b*c/d
))*tan(1/2*b*x)^2 + b*c*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^
2 - 2*b*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 2*b*d*x*imag_
part(cos_integral(-b*x - b*c/d))*tan(1/2*a) - 4*b*d*x*sin_integral((b*d*x +
 b*c)/d)*tan(1/2*a) - b*c*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2
 - b*c*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 + 2*b*d*x*imag_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) - 2*b*d*x*imag_part(cos_integr
```

```
al(-b*x - b*c/d))*tan(1/2*b*c/d) + 4*b*d*x*sin_integral((b*d*x + b*c)/d)*ta
n(1/2*b*c/d) + 4*b*c*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/
2*b*c/d) + 4*b*c*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b
*c/d) - b*c*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)^2 - b*c*rea
l_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 + b*d*x*real_part(cos_i
ntegral(b*x + b*c/d)) + b*d*x*real_part(cos_integral(-b*x - b*c/d)) - 2*b*c
*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 2*b*c*imag_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*a) - 4*b*c*sin_integral((b*d*x + b*c)/d)*tan(1/
2*a) + 4*d*tan(1/2*b*x)^2*tan(1/2*a) + 4*d*tan(1/2*b*x)*tan(1/2*a)^2 + 2*b*
c*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) - 2*b*c*imag_part(cos
_integral(-b*x - b*c/d))*tan(1/2*b*c/d) + 4*b*c*sin_integral((b*d*x + b*c)/
d)*tan(1/2*b*c/d) - 4*d*tan(1/2*b*x)*tan(1/2*b*c/d)^2 - 4*d*tan(1/2*a)*tan(
1/2*b*c/d)^2 + b*c*real_part(cos_integral(b*x + b*c/d)) + b*c*real_part(cos
_integral(-b*x - b*c/d)) - 4*d*tan(1/2*b*x) - 4*d*tan(1/2*a))/(d^3*x*tan(1/
2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + c*d^2*tan(1/2*b*x)^2*tan(1/2*a)^2*
tan(1/2*b*c/d)^2 + d^3*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + d^3*x*tan(1/2*b*x)^2
*tan(1/2*b*c/d)^2 + d^3*x*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + c*d^2*tan(1/2*b*x
)^2*tan(1/2*a)^2 + c*d^2*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + c*d^2*tan(1/2*a)
^2*tan(1/2*b*c/d)^2 + d^3*x*tan(1/2*b*x)^2 + d^3*x*tan(1/2*a)^2 + d^3*x*tan
(1/2*b*c/d)^2 + c*d^2*tan(1/2*b*x)^2 + c*d^2*tan(1/2*a)^2 + c*d^2*tan(1/2*b
*c/d)^2 + d^3*x + c*d^2)
```

3.7 $\int \frac{\sin(a+bx)}{(c+dx)^3} dx$

Optimal. Leaf size=104

$$-\frac{b^2 \sin\left(a-\frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{2d^3}-\frac{b^2 \cos\left(a-\frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d}+bx\right)}{2d^3}-\frac{b\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin(a+bx)}{2d(c+dx)^2}$$

```
[Out] -(b*Cos[a + b*x])/(2*d^2*(c + d*x)) - (b^2*CosIntegral[(b*c)/d + b*x]*Sin[a
 - (b*c)/d])/(2*d^3) - Sin[a + b*x]/(2*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/
d]*SinIntegral[(b*c)/d + b*x])/(2*d^3)
```

Rubi [A] time = 0.139031, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3297, 3303, 3299, 3302}

$$-\frac{b^2 \sin\left(a-\frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{2d^3}-\frac{b^2 \cos\left(a-\frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d}+bx\right)}{2d^3}-\frac{b\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x)^3,x]
```

```
[Out] -(b*Cos[a + b*x])/(2*d^2*(c + d*x)) - (b^2*CosIntegral[(b*c)/d + b*x]*Sin[a
 - (b*c)/d])/(2*d^3) - Sin[a + b*x]/(2*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/
d]*SinIntegral[(b*c)/d + b*x])/(2*d^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx &= -\frac{\sin(a+bx)}{2d(c+dx)^2}+\frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{2d}\\
&= -\frac{b\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin(a+bx)}{2d(c+dx)^2}-\frac{b^2\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2}\\
&= -\frac{b\cos(a+bx)}{2d^2(c+dx)}-\frac{\sin(a+bx)}{2d(c+dx)^2}-\frac{\left(b^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{2d^2}-\frac{\left(b^2\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{2d^2}\\
&= -\frac{b\cos(a+bx)}{2d^2(c+dx)}-\frac{b^2\operatorname{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{2d^3}-\frac{\sin(a+bx)}{2d(c+dx)^2}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{2d^3}
\end{aligned}$$

Mathematica [A] time = 0.663772, size = 87, normalized size = 0.84

$$-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(b\left(\frac{c}{d}+x\right)\right)+\frac{d(b(c+dx)\cos(a+bx)+d\sin(a+bx))}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(c + d*x)^3,x]
```

```
[Out] -(b^2*CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a + b
*x] + d*Sin[a + b*x]))/(c + d*x)^2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b*(c/
d + x)])/(2*d^3)
```

Maple [A] time = 0.008, size = 145, normalized size = 1.4

$$b^{2}\left(-\frac{\sin\left(bx+a\right)}{2\left(\left(bx+a\right)d-da+cb\right)^{2}d}+\frac{1}{2d}\left(-\frac{\cos\left(bx+a\right)}{\left(\left(bx+a\right)d-da+cb\right)d}-\frac{1}{d}\left(\frac{1}{d}\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)-\frac{1}{d}\mathrm{Ci}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x+c)^3,x)
```

```
[Out] b^2*(-1/2*sin(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-d*
a+c*b)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)
*sin((-a*d+b*c)/d)/d)/d)/d)
```

Maxima [C] time = 1.52538, size = 269, normalized size = 2.59

$$-\frac{b^{3}\left(iE_{3}\left(\frac{ibc+i(bx+a)d-iad}{d}\right)-iE_{3}\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^{3}\left(E_{3}\left(\frac{ibc+i(bx+a)d-iad}{d}\right)+E_{3}\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\sin}{2\left(b^{2}c^{2}d-2abcd^{2}+(bx+a)^{2}d^{3}+a^{2}d^{3}+2\left(bcd^{2}-ad^{3}\right)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*(I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^3
*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(3,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b^2*c^2*d - 2*a
*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [B] time = 1.83805, size = 471, normalized size = 4.53

$$-\frac{2d^{2}\sin\left(bx+a\right)+2\left(b^{2}d^{2}x^{2}+2b^{2}cdx+b^{2}c^{2}\right)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)+2\left(bd^{2}x+bcd\right)\cos\left(bx+a\right)+\left(\left(b^{2}d^{2}x^{2}+2b^{2}\right.\right.}{4\left(d^{5}x^{2}+2cd^{4}x+c^{2}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*sin(b*x + a) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b*
c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 2*(b*d^2*x + b*c*d)*cos(b*x + a
) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x + b*c)/d) +
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x + b*c)/d))*sin(-
(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)/(c + d*x)**3, x)
```

Giac [C] time = 1.55544, size = 7731, normalized size = 74.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d)
)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*sin_integral
((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*
x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/
2*b*c/d) + 2*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)
^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*b^2*d^2*x^2*real_part(cos_integral(b*x +
 b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*real_pa
rt(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 +
 2*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)
^2*tan(1/2*b*c/d)^2 - 2*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan
(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d*x*sin_integral((b*d*x
 + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^2*d^2*x^2*imag_
part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + b^2*d^2*x^2*i
mag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*d^
```

```
2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b^2*d^2
*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2
*b*c/d) - 4*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^
2*tan(1/2*a)*tan(1/2*b*c/d) + 8*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*t
an(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 4*b^2*c*d*x*real_part(cos_integra
l(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 4*b^2*c*d*x*re
al_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c
/d) - b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1
/2*b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x
)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*
b*x)^2*tan(1/2*b*c/d)^2 - 4*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*
tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 4*b^2*c*d*x*real_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + b^2*d^2*x^
2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^2*
d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2
 + 2*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d)^
2 + b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^
2*tan(1/2*b*c/d)^2 - b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*
b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*c^2*sin_integral((b*d*x + b*c)
/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*real_part(
cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*d^2*x^2*real_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) - 2*b^2*c*d*x*ima
g_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b^2*c*d*x
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 4*b^2*
c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*d^2
*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 2
*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b
*c/d) + 8*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan
(1/2*a)*tan(1/2*b*c/d) - 8*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*
tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 16*b^2*c*d*x*sin_integral((b*d*x
 + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 2*b^2*d^2*x^2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2*d^2*x^2*r
eal_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2*c^
2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*
b*c/d) + 2*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan
(1/2*a)^2*tan(1/2*b*c/d) - 2*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))
*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(-b*x
- b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 4*b^2*c*d*x*sin_integral((b*d*x
 + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_in
tegral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*real_part(
cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*c^2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 -
2*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*t
an(1/2*b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*
a)^2*tan(1/2*b*c/d)^2 - 2*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*t
```

```
an(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*ta
n(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2
*b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2
 - b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2 + 2*b^2
*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2 + 4*b^2*c*d*x*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 4*b^2*c*d*x*real_
part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) - b^2*d^2*x^2*im
ag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 + b^2*d^2*x^2*imag_part(cos
_integral(-b*x - b*c/d))*tan(1/2*a)^2 - 2*b^2*d^2*x^2*sin_integral((b*d*x +
 b*c)/d)*tan(1/2*a)^2 - b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/
2*b*x)^2*tan(1/2*a)^2 + b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1
/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*
x)^2*tan(1/2*a)^2 - 4*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/
2*b*x)^2*tan(1/2*b*c/d) - 4*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))
*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 4*b^2*d^2*x^2*imag_part(cos_integral(b*x +
 b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - 4*b^2*d^2*x^2*imag_part(cos_integral(-
b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 8*b^2*d^2*x^2*sin_integral((b*d*x
 + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d) + 4*b^2*c^2*imag_part(cos_integral(b*x
 + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*b^2*c^2*imag_part(c
os_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 8*b^2
*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)
 + 4*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*
c/d) + 4*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1
/2*b*c/d) - b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)
^2 + b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - 2
*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*c/d)^2 - b^2*c^2*imag_
part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + b^2*c^2*i
mag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^
2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 4*b^2
*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 4
*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^
2 + b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d
)^2 - b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*
c/d)^2 + 2*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d
)^2 + 2*b*c*d*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*im
ag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2 - 2*b^2*c*d*x*imag_part(c
os_integral(-b*x - b*c/d))*tan(1/2*b*x)^2 + 4*b^2*c*d*x*sin_integral((b*d*x
 + b*c)/d)*tan(1/2*b*x)^2 + 2*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/
d))*tan(1/2*a) + 2*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/
2*a) + 2*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/
2*a) + 2*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1
/2*a) - 2*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 + 2*b
^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - 4*b^2*c*d*x*s
in_integral((b*d*x + b*c)/d)*tan(1/2*a)^2 + 2*b*d^2*x*tan(1/2*b*x)^2*tan(1/
2*a)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)
```

```
- 2*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d) - 2*b^
2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) -
2*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/
d) + 8*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*
c/d) - 8*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2
*b*c/d) + 16*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c
/d) + 2*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b
*c/d) + 2*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/
2*b*c/d) - 2*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)^
2 + 2*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - 4*
b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*c/d)^2 - 2*b*d^2*x*tan(1/
2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*
tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*c^2*real_part(cos_integral(-b*x - b*c/d
))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 8*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*
b*c/d)^2 - 2*b*d^2*x*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^2*d^2*x^2*imag_part(
cos_integral(b*x + b*c/d)) - b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/
d)) + 2*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d) + b^2*c^2*imag_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*b*x)^2 - b^2*c^2*imag_part(cos_integral(-b*x
- b*c/d))*tan(1/2*b*x)^2 + 2*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*
b*x)^2 + 4*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 4*b^
2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a) - b^2*c^2*imag_par
t(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 + b^2*c^2*imag_part(cos_integral(
-b*x - b*c/d))*tan(1/2*a)^2 - 2*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1
/2*a)^2 + 2*b*c*d*tan(1/2*b*x)^2*tan(1/2*a)^2 - 4*b^2*c*d*x*real_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*b*c/d) - 4*b^2*c*d*x*real_part(cos_integral(-
b*x - b*c/d))*tan(1/2*b*c/d) + 4*b^2*c^2*imag_part(cos_integral(b*x + b*c/d
))*tan(1/2*a)*tan(1/2*b*c/d) - 4*b^2*c^2*imag_part(cos_integral(-b*x - b*c/
d))*tan(1/2*a)*tan(1/2*b*c/d) + 8*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan
(1/2*a)*tan(1/2*b*c/d) - b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(1
/2*b*c/d)^2 + b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^
2 - 2*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*c/d)^2 - 2*b*c*d*tan(
1/2*b*x)^2*tan(1/2*b*c/d)^2 - 8*b*c*d*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*b*c/d
)^2 - 4*d^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b*c*d*tan(1/2*a)
^2*tan(1/2*b*c/d)^2 - 4*d^2*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*
b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d)) - 2*b^2*c*d*x*imag_part(cos_
integral(-b*x - b*c/d)) + 4*b^2*c*d*x*sin_integral((b*d*x + b*c)/d) - 2*b*d
^2*x*tan(1/2*b*x)^2 + 2*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/
2*a) + 2*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a) - 8*b*d^2
*x*tan(1/2*b*x)*tan(1/2*a) - 2*b*d^2*x*tan(1/2*a)^2 - 2*b^2*c^2*real_part(c
os_integral(b*x + b*c/d))*tan(1/2*b*c/d) - 2*b^2*c^2*real_part(cos_integral
(-b*x - b*c/d))*tan(1/2*b*c/d) + 2*b*d^2*x*tan(1/2*b*c/d)^2 + b^2*c^2*imag_
part(cos_integral(b*x + b*c/d)) - b^2*c^2*imag_part(cos_integral(-b*x - b*c
/d)) + 2*b^2*c^2*sin_integral((b*d*x + b*c)/d) - 2*b*c*d*tan(1/2*b*x)^2 - 8
*b*c*d*tan(1/2*b*x)*tan(1/2*a) - 4*d^2*tan(1/2*b*x)^2*tan(1/2*a) - 2*b*c*d*
tan(1/2*a)^2 - 4*d^2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*b*c*d*tan(1/2*b*c/d)^2 +
```

```
 4*d^2*tan(1/2*b*x)*tan(1/2*b*c/d)^2 + 4*d^2*tan(1/2*a)*tan(1/2*b*c/d)^2 +
2*b*d^2*x + 2*b*c*d + 4*d^2*tan(1/2*b*x) + 4*d^2*tan(1/2*a))/(d^5*x^2*tan(1
/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*x)^2*tan(1/2*
a)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + d^5*x^2*tan(1
/2*b*x)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + c^2*d^
3*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*x)^2*t
an(1/2*a)^2 + 2*c*d^4*x*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2
*a)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^2 + d^5*x^2*tan(1/2*a)^2 + c^
2*d^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + d^5*x^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(
1/2*b*x)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*c*d
^4*x*tan(1/2*b*x)^2 + 2*c*d^4*x*tan(1/2*a)^2 + 2*c*d^4*x*tan(1/2*b*c/d)^2 +
 d^5*x^2 + c^2*d^3*tan(1/2*b*x)^2 + c^2*d^3*tan(1/2*a)^2 + c^2*d^3*tan(1/2*
b*c/d)^2 + 2*c*d^4*x + c^2*d^3)
```

3.8 $\int (c+dx)^4 \sin^2(a+bx)\,dx$

Optimal. Leaf size=161

$$-\frac{3d^3(c+dx)\sin^2(a+bx)}{2b^4}+\frac{3d^2(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b^3}+\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}-\frac{3d^4\sin(a+bx)\cos(a+b}{4b^5}$$

```
[Out] (3*d^4*x)/(4*b^4) - (d*(c + d*x)^3)/(2*b^2) + (c + d*x)^5/(10*d) - (3*d^4*C
os[a + b*x]*Sin[a + b*x])/(4*b^5) + (3*d^2*(c + d*x)^2*Cos[a + b*x]*Sin[a +
 b*x])/(2*b^3) - ((c + d*x)^4*Cos[a + b*x]*Sin[a + b*x])/(2*b) - (3*d^3*(c
+ d*x)*Sin[a + b*x]^2)/(2*b^4) + (d*(c + d*x)^3*Sin[a + b*x]^2)/b^2
```

Rubi [A] time = 0.102801, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 32, 2635, 8}

$$-\frac{3d^3(c+dx)\sin^2(a+bx)}{2b^4}+\frac{3d^2(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b^3}+\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}-\frac{3d^4\sin(a+bx)\cos(a+b}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sin[a + b*x]^2,x]
```

```
[Out] (3*d^4*x)/(4*b^4) - (d*(c + d*x)^3)/(2*b^2) + (c + d*x)^5/(10*d) - (3*d^4*C
os[a + b*x]*Sin[a + b*x])/(4*b^5) + (3*d^2*(c + d*x)^2*Cos[a + b*x]*Sin[a +
 b*x])/(2*b^3) - ((c + d*x)^4*Cos[a + b*x]*Sin[a + b*x])/(2*b) - (3*d^3*(c
+ d*x)*Sin[a + b*x]^2)/(2*b^4) + (d*(c + d*x)^3*Sin[a + b*x]^2)/b^2
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^4 \sin^2(a+bx)\,dx = -\frac{(c+dx)^4\cos(a+bx)\sin(a+bx)}{2b} + \frac{d(c+dx)^3\sin^2(a+bx)}{b^2} + \frac{1}{2}\int (c+dx)^4\,dx - \frac{\left(3d^2\right)\int$$

$$= \frac{(c+dx)^5}{10d} + \frac{3d^2(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b^3} - \frac{(c+dx)^4\cos(a+bx)\sin(a+bx)}{2b} - \frac{3d^3$$

$$= -\frac{d(c+dx)^3}{2b^2} + \frac{(c+dx)^5}{10d} - \frac{3d^4\cos(a+bx)\sin(a+bx)}{4b^5} + \frac{3d^2(c+dx)^2\cos(a+bx)\sin(a+$$

$$= \frac{3d^4x}{4b^4} - \frac{d(c+dx)^3}{2b^2} + \frac{(c+dx)^5}{10d} - \frac{3d^4\cos(a+bx)\sin(a+bx)}{4b^5} + \frac{3d^2(c+dx)^2\cos(a+bx)\mathrm{si}$$

Mathematica [A] time = 0.63842, size = 132, normalized size = 0.82

$$\frac{-10\sin(2(a+bx))\left(-6b^2d^2(c+dx)^2+2b^4(c+dx)^4+3d^4\right)-20bd(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)+8b^5x\left(10\right.}{80b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sin[a + b*x]^2,x]
```

```
[Out] (8*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) - 20
*b*d*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] - 10*(3*d^4 -
6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sin[2*(a + b*x)])/(80*b^5)
```

Maple [B] time = 0.048, size = 1030, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^4*d^4*((b*x+a)^4*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-(b*x+a
)^3*cos(b*x+a)^2+3*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/2*
(b*x+a)*cos(b*x+a)^2-3/4*cos(b*x+a)*sin(b*x+a)-3/4*b*x-3/4*a-(b*x+a)^3-2/5*
(b*x+a)^5)-4/b^4*a*d^4*((b*x+a)^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a
)-3/4*(b*x+a)^2*cos(b*x+a)^2+3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x
+1/2*a)-3/8*(b*x+a)^2-3/8*sin(b*x+a)^2-3/8*(b*x+a)^4)+4/b^3*c*d^3*((b*x+a)^
3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/4*(b*x+a)^2*cos(b*x+a)^2+3/2
*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/8*(b*x+a)^2-3/8*sin(b*
x+a)^2-3/8*(b*x+a)^4)+6/b^4*a^2*d^4*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+
1/2*b*x+1/2*a)-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1
/4*a-1/3*(b*x+a)^3)-12/b^3*a*c*d^3*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1
/2*b*x+1/2*a)-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/
4*a-1/3*(b*x+a)^3)+6/b^2*c^2*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2
*b*x+1/2*a)-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*
a-1/3*(b*x+a)^3)-4/b^4*a^3*d^4*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x
+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2)+12/b^3*a^2*c*d^3*((b*x+a)*(-1/2*cos
(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2)-12/b^2*a*
c^2*d^2*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2+1
/4*sin(b*x+a)^2)+4/b*c^3*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2
*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2)+1/b^4*a^4*d^4*(-1/2*cos(b*x+a)*sin(b*x+
a)+1/2*b*x+1/2*a)-4/b^3*a^3*c*d^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a
)+6/b^2*a^2*c^2*d^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-4/b*a*c^3*d*
(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+c^4*(-1/2*cos(b*x+a)*sin(b*x+a)+
1/2*b*x+1/2*a))
```

Maxima [B] time = 1.14591, size = 992, normalized size = 6.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/40*(10*(2*b*x + 2*a - sin(2*b*x + 2*a))*c^4 - 40*(2*b*x + 2*a - sin(2*b*x
 + 2*a))*a*c^3*d/b + 60*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^2*c^2*d^2/b^2 -
40*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^3*c*d^3/b^3 + 10*(2*b*x + 2*a - sin(2
*b*x + 2*a))*a^4*d^4/b^4 + 20*(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a)
 - cos(2*b*x + 2*a))*c^3*d/b - 60*(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x +
```

```
2*a) - cos(2*b*x + 2*a))*a*c^2*d^2/b^2 + 60*(2*(b*x + a)^2 - 2*(b*x + a)*si
n(2*b*x + 2*a) - cos(2*b*x + 2*a))*a^2*c*d^3/b^3 - 20*(2*(b*x + a)^2 - 2*(b
*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a^3*d^4/b^4 + 10*(4*(b*x + a)^
3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*
c^2*d^2/b^2 - 20*(4*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x
+ a)^2 - 1)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 10*(4*(b*x + a)^3 - 6*(b*x + a)
*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a^2*d^4/b^4 + 1
0*(2*(b*x + a)^4 - 3*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^
3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))*c*d^3/b^3 - 10*(2*(b*x + a)^4 - 3*(2*(b*
x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x
+ 2*a))*a*d^4/b^4 + (4*(b*x + a)^5 - 10*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2
*b*x + 2*a) - 5*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*sin(2*b*x + 2*a))*d^4/b
^4)/b
```

Fricas [A] time = 1.77142, size = 593, normalized size = 3.68

$$\frac{2\,b^5d^4x^5 + 10\,b^5cd^3x^4 + 10\left(2\,b^5c^2d^2 + b^3d^4\right)x^3 + 10\left(2\,b^5c^3d + 3\,b^3cd^3\right)x^2 - 10\left(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 2\,b^3c^3d - 3\,bcd^3 + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^5*d^4*x^5 + 10*b^5*c*d^3*x^4 + 10*(2*b^5*c^2*d^2 + b^3*d^4)*x^3 +
 10*(2*b^5*c^3*d + 3*b^3*c*d^3)*x^2 - 10*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 +
 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 - 5*
(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2
*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)
*sin(b*x + a) + 5*(2*b^5*c^4 + 6*b^3*c^2*d^2 - 3*b*d^4)*x)/b^5
```

Sympy [A] time = 6.1568, size = 660, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**4*x*sin(a + b*x)**2/2 + c**4*x*cos(a + b*x)**2/2 + c**3*d*x**
2*sin(a + b*x)**2 + c**3*d*x**2*cos(a + b*x)**2 + c**2*d**2*x**3*sin(a + b*
```

```
x)**2 + c**2*d**2*x**3*cos(a + b*x)**2 + c*d**3*x**4*sin(a + b*x)**2/2 + c*
d**3*x**4*cos(a + b*x)**2/2 + d**4*x**5*sin(a + b*x)**2/10 + d**4*x**5*cos(
a + b*x)**2/10 - c**4*sin(a + b*x)*cos(a + b*x)/(2*b) - 2*c**3*d*x*sin(a +
b*x)*cos(a + b*x)/b - 3*c**2*d**2*x**2*sin(a + b*x)*cos(a + b*x)/b - 2*c*d*
*3*x**3*sin(a + b*x)*cos(a + b*x)/b - d**4*x**4*sin(a + b*x)*cos(a + b*x)/(
2*b) + c**3*d*sin(a + b*x)**2/b**2 + 3*c**2*d**2*x*sin(a + b*x)**2/(2*b**2)
 - 3*c**2*d**2*x*cos(a + b*x)**2/(2*b**2) + 3*c*d**3*x**2*sin(a + b*x)**2/(
2*b**2) - 3*c*d**3*x**2*cos(a + b*x)**2/(2*b**2) + d**4*x**3*sin(a + b*x)**
2/(2*b**2) - d**4*x**3*cos(a + b*x)**2/(2*b**2) + 3*c**2*d**2*sin(a + b*x)*
cos(a + b*x)/(2*b**3) + 3*c*d**3*x*sin(a + b*x)*cos(a + b*x)/b**3 + 3*d**4*
x**2*sin(a + b*x)*cos(a + b*x)/(2*b**3) - 3*c*d**3*sin(a + b*x)**2/(2*b**4)
 - 3*d**4*x*sin(a + b*x)**2/(4*b**4) + 3*d**4*x*cos(a + b*x)**2/(4*b**4) -
3*d**4*sin(a + b*x)*cos(a + b*x)/(4*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x
**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**2, True))
```

Giac [A] time = 1.16633, size = 300, normalized size = 1.86

$$\frac{1}{10}\,d^4x^5+\frac{1}{2}\,cd^3x^4+c^2d^2x^3+c^3dx^2+\frac{1}{2}\,c^4x-\frac{\left(2\,b^3d^4x^3+6\,b^3cd^3x^2+6\,b^3c^2d^2x+2\,b^3c^3d-3\,bd^4x-3\,bcd^3\right)\cos\left(2\,bx\right.}{4\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*d^4*x^5 + 1/2*c*d^3*x^4 + c^2*d^2*x^3 + c^3*d*x^2 + 1/2*c^4*x - 1/4*(2
*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x + 2*b^3*c^3*d - 3*b*d^4*x
- 3*b*c*d^3)*cos(2*b*x + 2*a)/b^5 - 1/8*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 +
12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d
^3*x - 6*b^2*c^2*d^2 + 3*d^4)*sin(2*b*x + 2*a)/b^5
```

3.9 $\int (c + dx)^3 \sin^2(a + bx)\,dx$

Optimal. Leaf size=134

$$\frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{3d^3\sin^2(a+bx)}{8b^4}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}-$$

```
[Out] (-3*c*d^2*x)/(4*b^2) - (3*d^3*x^2)/(8*b^2) + (c + d*x)^4/(8*d) + (3*d^2*(c
+ d*x)*Cos[a + b*x]*Sin[a + b*x])/(4*b^3) - ((c + d*x)^3*Cos[a + b*x]*Sin[a
 + b*x])/(2*b) - (3*d^3*Sin[a + b*x]^2)/(8*b^4) + (3*d*(c + d*x)^2*Sin[a +
b*x]^2)/(4*b^2)
```

Rubi [A] time = 0.0742275, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3311, 32, 3310}

$$\frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{3d^3\sin^2(a+bx)}{8b^4}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]^2,x]
```

```
[Out] (-3*c*d^2*x)/(4*b^2) - (3*d^3*x^2)/(8*b^2) + (c + d*x)^4/(8*d) + (3*d^2*(c
+ d*x)*Cos[a + b*x]*Sin[a + b*x])/(4*b^3) - ((c + d*x)^3*Cos[a + b*x]*Sin[a
 + b*x])/(2*b) - (3*d^3*Sin[a + b*x]^2)/(8*b^4) + (3*d*(c + d*x)^2*Sin[a +
b*x]^2)/(4*b^2)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)^3\sin^2(a+bx)\,dx = -\frac{(c+dx)^3\cos(a+bx)\sin(a+bx)}{2b} + \frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2} + \frac{1}{2}\int (c+dx)^3\,dx - \frac{\left(3d^2\right.}{}$$

$$= \frac{(c+dx)^4}{8d} + \frac{3d^2(c+dx)\cos(a+bx)\sin(a+bx)}{4b^3} - \frac{(c+dx)^3\cos(a+bx)\sin(a+bx)}{2b} - \frac{3d}{}$$

$$= -\frac{3cd^2x}{4b^2} - \frac{3d^3x^2}{8b^2} + \frac{(c+dx)^4}{8d} + \frac{3d^2(c+dx)\cos(a+bx)\sin(a+bx)}{4b^3} - \frac{(c+dx)^3\cos(a+b}{2b}$$

Mathematica [A] time = 0.419724, size = 106, normalized size = 0.79

$$\frac{-2b(c+dx)\sin(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)-3d\cos(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+2b^4x\left(6c^2dx+4c^3+4cd^2x^2+\right.}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]^2,x]
```

```
[Out] (2*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(-d^2 + 2*b^2*(c
 + d*x)^2)*Cos[2*(a + b*x)] - 2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Si
n[2*(a + b*x)])/(16*b^4)
```

Maple [B] time = 0.007, size = 587, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^3*d^3*((b*x+a)^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/4*(b
*x+a)^2*cos(b*x+a)^2+3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-
3/8*(b*x+a)^2-3/8*sin(b*x+a)^2-3/8*(b*x+a)^4)-3/b^3*a*d^3*((b*x+a)^2*(-1/2*
cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a
)*sin(b*x+a)+1/4*b*x+1/4*a-1/3*(b*x+a)^3)+3/b^2*c*d^2*((b*x+a)^2*(-1/2*cos(
b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*si
n(b*x+a)+1/4*b*x+1/4*a-1/3*(b*x+a)^3)+3/b^3*a^2*d^3*((b*x+a)*(-1/2*cos(b*x+
a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2)-6/b^2*a*c*d^2*
((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b
*x+a)^2)+3/b*c^2*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*
(b*x+a)^2+1/4*sin(b*x+a)^2)-1/b^3*a^3*d^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b
*x+1/2*a)+3/b^2*a^2*c*d^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/b*a*
c^2*d*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+c^3*(-1/2*cos(b*x+a)*sin(b
*x+a)+1/2*b*x+1/2*a))
```

Maxima [B] time = 1.06971, size = 597, normalized size = 4.46

$$\frac{4\,(2\,bx+2\,a-\sin(2\,bx+2\,a))c^3-\frac{12\,(2\,bx+2\,a-\sin(2\,bx+2\,a))ac^2d}{b}+\frac{12\,(2\,bx+2\,a-\sin(2\,bx+2\,a))a^2cd^2}{b^2}-\frac{4\,(2\,bx+2\,a-\sin(2\,bx+2\,a))a^3d^3}{b^3}+\frac{6\,(2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(2*b*x + 2*a - sin(2*b*x + 2*a))*c^3 - 12*(2*b*x + 2*a - sin(2*b*x
+ 2*a))*a*c^2*d/b + 12*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^2*c*d^2/b^2 - 4*(
2*b*x + 2*a - sin(2*b*x + 2*a))*a^3*d^3/b^3 + 6*(2*(b*x + a)^2 - 2*(b*x + a
)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*c^2*d/b - 12*(2*(b*x + a)^2 - 2*(b*x
 + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a*c*d^2/b^2 + 6*(2*(b*x + a)^2 -
 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a^2*d^3/b^3 + 2*(4*(b*x +
 a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*
a))*c*d^2/b^2 - 2*(4*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x
 + a)^2 - 1)*sin(2*b*x + 2*a))*a*d^3/b^3 + (2*(b*x + a)^4 - 3*(2*(b*x + a)^
2 - 1)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))
*d^3/b^3)/b
```

Fricas [A] time = 1.70047, size = 394, normalized size = 2.94

$$\frac{b^4d^3x^4+4\,b^4cd^2x^3+3\left(2\,b^4c^2d+b^2d^3\right)x^2-3\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3\right)\cos\left(bx+a\right)^2-2\left(2\,b^3d^3x^3+6\,b^3cd^2x\right.}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 3*(2*b^4*c^2*d + b^2*d^3)*x^2 - 3*(2*b
^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 - 2*(2*b^3*d
^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*
x)*cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3 + 3*b^2*c*d^2)*x)/b^4
```

Sympy [A] time = 3.23952, size = 456, normalized size = 3.4

$$\begin{cases} \frac{c^3x\sin^2(a+bx)}{2} + \frac{c^3x\cos^2(a+bx)}{2} + \frac{3c^2dx^2\sin^2(a+bx)}{4} + \frac{3c^2dx^2\cos^2(a+bx)}{4} + \frac{cd^2x^3\sin^2(a+bx)}{2} + \frac{cd^2x^3\cos^2(a+bx)}{2} + \frac{d^3x^4\sin^2(a+bx)}{8} + \frac{d^3x^4 \cdots \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*x*sin(a + b*x)**2/2 + c**3*x*cos(a + b*x)**2/2 + 3*c**2*d*x
**2*sin(a + b*x)**2/4 + 3*c**2*d*x**2*cos(a + b*x)**2/4 + c*d**2*x**3*sin(a
 + b*x)**2/2 + c*d**2*x**3*cos(a + b*x)**2/2 + d**3*x**4*sin(a + b*x)**2/8
+ d**3*x**4*cos(a + b*x)**2/8 - c**3*sin(a + b*x)*cos(a + b*x)/(2*b) - 3*c*
*2*d*x*sin(a + b*x)*cos(a + b*x)/(2*b) - 3*c*d**2*x**2*sin(a + b*x)*cos(a +
 b*x)/(2*b) - d**3*x**3*sin(a + b*x)*cos(a + b*x)/(2*b) + 3*c**2*d*sin(a +
b*x)**2/(4*b**2) + 3*c*d**2*x*sin(a + b*x)**2/(4*b**2) - 3*c*d**2*x*cos(a +
 b*x)**2/(4*b**2) + 3*d**3*x**2*sin(a + b*x)**2/(8*b**2) - 3*d**3*x**2*cos(
a + b*x)**2/(8*b**2) + 3*c*d**2*sin(a + b*x)*cos(a + b*x)/(4*b**3) + 3*d**3
*x*sin(a + b*x)*cos(a + b*x)/(4*b**3) - 3*d**3*sin(a + b*x)**2/(8*b**4), Ne
(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**2,
 True))
```

Giac [A] time = 1.13585, size = 207, normalized size = 1.54

$$\frac{1}{8}d^3x^4 + \frac{1}{2}cd^2x^3 + \frac{3}{4}c^2dx^2 + \frac{1}{2}c^3x - \frac{3\left(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d - d^3\right)\cos(2bx + 2a)}{16b^4} - \frac{\left(2b^3d^3x^3 + 6b^3cd^2x^2 + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*d^3*x^4 + 1/2*c*d^2*x^3 + 3/4*c^2*d*x^2 + 1/2*c^3*x - 3/16*(2*b^2*d^3*x
^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(2*b*x + 2*a)/b^4 - 1/8*(2*b^3*d
^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^
2)*sin(2*b*x + 2*a)/b^4
```

3.10 $\int (c+dx)^2 \sin^2(a+bx)\,dx$

Optimal. Leaf size=95

$$\frac{d(c+dx)\sin^2(a+bx)}{2b^2}+\frac{d^2\sin(a+bx)\cos(a+bx)}{4b^3}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}$$

```
[Out] -(d^2*x)/(4*b^2) + (c + d*x)^3/(6*d) + (d^2*Cos[a + b*x]*Sin[a + b*x])/(4*b
^3) - ((c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*(c + d*x)*Sin[a +
b*x]^2)/(2*b^2)
```

Rubi [A] time = 0.0538203, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 32, 2635, 8}

$$\frac{d(c+dx)\sin^2(a+bx)}{2b^2}+\frac{d^2\sin(a+bx)\cos(a+bx)}{4b^3}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]^2,x]
```



```
[Out] -(d^2*x)/(4*b^2) + (c + d*x)^3/(6*d) + (d^2*Cos[a + b*x]*Sin[a + b*x])/(4*b
^3) - ((c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*(c + d*x)*Sin[a +
b*x]^2)/(2*b^2)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^2\sin^2(a+bx)\,dx = -\frac{(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}+\frac{1}{2}\int (c+dx)^2\,dx-\frac{d^2\int \sin^2}{2}$$
$$=\frac{(c+dx)^3}{6d}+\frac{d^2\cos(a+bx)\sin(a+bx)}{4b^3}-\frac{(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{d(c+dx)\sin^2}{2b^2}$$
$$=-\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}+\frac{d^2\cos(a+bx)\sin(a+bx)}{4b^3}-\frac{(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{d(c+}{}$$

Mathematica [A] time = 0.301784, size = 77, normalized size = 0.81

$$\frac{-3\sin(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)-6bd(c+dx)\cos(2(a+bx))+4b^3x\left(3c^2+3cdx+d^2x^2\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]^2,x]
```

```
[Out] (4*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 6*b*d*(c + d*x)*Cos[2*(a + b*x)] - 3
*(-d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(24*b^3)
```

Maple [B] time = 0.007, size = 289, normalized size = 3.

$$\frac{1}{b}\left(\frac{d^2}{b^2}\left((bx+a)^2\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)(\cos(bx+a))^2}{2}+\frac{\cos(bx+a)\sin(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^2*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/2*(b
*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a-1/3*(b*x+a)^3)-2
/b^2*a*d^2*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^
2+1/4*sin(b*x+a)^2)+2/b*c*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/
2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2)+1/b^2*a^2*d^2*(-1/2*cos(b*x+a)*sin(b*x
+a)+1/2*b*x+1/2*a)-2/b*a*c*d*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+c^2
*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [B] time = 1.02537, size = 313, normalized size = 3.29

$$\frac{6\,(2\,bx+2\,a-\sin(2\,bx+2\,a))c^2 - \frac{12\,(2\,bx+2\,a-\sin(2\,bx+2\,a))acd}{b} + \frac{6\,(2\,bx+2\,a-\sin(2\,bx+2\,a))a^2d^2}{b^2} + \frac{6\left(2\,(bx+a)^2-2\,(bx+a)\sin(2\,bx+2\,a)-\cos\right.}{b}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(6*(2*b*x + 2*a - sin(2*b*x + 2*a))*c^2 - 12*(2*b*x + 2*a - sin(2*b*x
+ 2*a))*a*c*d/b + 6*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^2*d^2/b^2 + 6*(2*(b*
x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*c*d/b - 6*(2*(b
*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a*d^2/b^2 + (4
*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b
*x + 2*a))*d^2/b^2)/b
```

Fricas [A] time = 1.7763, size = 247, normalized size = 2.6

$$\frac{2\,b^3d^2x^3 + 6\,b^3cdx^2 - 6\left(bd^2x + bcd\right)\cos(bx+a)^2 - 3\left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos(bx+a)\sin(bx+a) + 3\left(2\right.}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 - 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 -
3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)*sin(b*x + a)
 + 3*(2*b^3*c^2 + b*d^2)*x)/b^3
```

Sympy [A] time = 1.46087, size = 264, normalized size = 2.78

$$\begin{cases} \frac{c^2x\sin^2(a+bx)}{2} + \frac{c^2x\cos^2(a+bx)}{2} + \frac{cdx^2\sin^2(a+bx)}{2} + \frac{cdx^2\cos^2(a+bx)}{2} + \frac{d^2x^3\sin^2(a+bx)}{6} + \frac{d^2x^3\cos^2(a+bx)}{6} - \frac{c^2\sin(a+bx)\cos(a+bx)}{2b} - \frac{cdx\sin \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**2*x*sin(a + b*x)**2/2 + c**2*x*cos(a + b*x)**2/2 + c*d*x**2*s
in(a + b*x)**2/2 + c*d*x**2*cos(a + b*x)**2/2 + d**2*x**3*sin(a + b*x)**2/6
 + d**2*x**3*cos(a + b*x)**2/6 - c**2*sin(a + b*x)*cos(a + b*x)/(2*b) - c*d
*x*sin(a + b*x)*cos(a + b*x)/b - d**2*x**2*sin(a + b*x)*cos(a + b*x)/(2*b)
+ c*d*sin(a + b*x)**2/(2*b**2) + d**2*x*sin(a + b*x)**2/(4*b**2) - d**2*x*c
os(a + b*x)**2/(4*b**2) + d**2*sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)
), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**2, True))
```

Giac [A] time = 1.12363, size = 127, normalized size = 1.34

$$\frac{1}{6}d^2x^3 + \frac{1}{2}cdx^2 + \frac{1}{2}c^2x - \frac{(bd^2x + bcd)\cos(2bx + 2a)}{4b^3} - \frac{(2b^2d^2x^2 + 4b^2cdx + 2b^2c^2 - d^2)\sin(2bx + 2a)}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*d^2*x^3 + 1/2*c*d*x^2 + 1/2*c^2*x - 1/4*(b*d^2*x + b*c*d)*cos(2*b*x + 2
*a)/b^3 - 1/8*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*sin(2*b*x + 2
*a)/b^3
```

3.11 $\int (c+dx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{cx}{2}+\frac{dx^2}{4}$$

```
[Out] (c*x)/2 + (d*x^2)/4 - ((c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*Sin[
a + b*x]^2)/(4*b^2)
```

Rubi [A] time = 0.0268548, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3310}

$$\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{cx}{2}+\frac{dx^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]^2,x]
```

```
[Out] (c*x)/2 + (d*x^2)/4 - ((c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*Sin[
a + b*x]^2)/(4*b^2)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sin^2(a+bx)\,dx&=-\frac{(c+dx)\cos(a+bx)\sin(a+bx)}{2b}+\frac{d\sin^2(a+bx)}{4b^2}+\frac{1}{2}\int (c+dx)\,dx\\&=\frac{cx}{2}+\frac{dx^2}{4}-\frac{(c+dx)\cos(a+bx)\sin(a+bx)}{2b}+\frac{d\sin^2(a+bx)}{4b^2}\end{aligned}$$

Mathematica [A] time = 0.146112, size = 52, normalized size = 0.95

$$\frac{2b(-(c+dx)\sin(2(a+bx))+2ac+bx(2c+dx))-d\cos(2(a+bx))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x]^2,x]
```

```
[Out] (-(d*Cos[2*(a + b*x)]) + 2*b*(2*a*c + b*x*(2*c + d*x) - (c + d*x)*Sin[2*(a
+ b*x)]))/(8*b^2)
```

Maple [B] time = 0.006, size = 112, normalized size = 2.

$$\frac{1}{b}\left(\frac{d}{b}\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}+\frac{(\sin(bx+a))^2}{4}\right)-\frac{da}{b}\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^
2+1/4*sin(b*x+a)^2)-1/b*d*a*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+c*(-
1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [B] time = 1.00875, size = 130, normalized size = 2.36

$$\frac{2(2bx+2a-\sin(2bx+2a))c-\frac{2(2bx+2a-\sin(2bx+2a))ad}{b}+\frac{\left(2(bx+a)^2-2(bx+a)\sin(2bx+2a)-\cos(2bx+2a)\right)d}{b}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(2*b*x + 2*a - sin(2*b*x + 2*a))*c - 2*(2*b*x + 2*a - sin(2*b*x + 2*
a))*a*d/b + (2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a
))*d/b)/b
```

Fricas [A] time = 1.64362, size = 130, normalized size = 2.36

$$\frac{b^2dx^2 + 2\,b^2cx - d\cos\left(bx + a\right)^2 - 2\left(bdx + bc\right)\cos\left(bx + a\right)\sin\left(bx + a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*x^2 + 2*b^2*c*x - d*cos(b*x + a)^2 - 2*(b*d*x + b*c)*cos(b*x + a
)*sin(b*x + a))/b^2
```

Sympy [A] time = 0.629207, size = 126, normalized size = 2.29

$$\begin{cases} \frac{cx\sin^2(a+bx)}{2} + \frac{cx\cos^2(a+bx)}{2} + \frac{dx^2\sin^2(a+bx)}{4} + \frac{dx^2\cos^2(a+bx)}{4} - \frac{c\sin(a+bx)\cos(a+bx)}{2b} - \frac{dx\sin(a+bx)\cos(a+bx)}{2b} + \frac{d\sin^2(a+bx)}{4b^2} & \text{for } b \\ \left(cx + \frac{dx^2}{2}\right)\sin^2(a) & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c*x*sin(a + b*x)**2/2 + c*x*cos(a + b*x)**2/2 + d*x**2*sin(a + b
*x)**2/4 + d*x**2*cos(a + b*x)**2/4 - c*sin(a + b*x)*cos(a + b*x)/(2*b) - d
*x*sin(a + b*x)*cos(a + b*x)/(2*b) + d*sin(a + b*x)**2/(4*b**2), Ne(b, 0)),
 ((c*x + d*x**2/2)*sin(a)**2, True))
```

Giac [A] time = 1.1177, size = 65, normalized size = 1.18

$$\frac{1}{4}\,dx^2 + \frac{1}{2}\,cx - \frac{d\cos\left(2\,bx + 2\,a\right)}{8\,b^2} - \frac{\left(bdx + bc\right)\sin\left(2\,bx + 2\,a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*d*x^2 + 1/2*c*x - 1/8*d*cos(2*b*x + 2*a)/b^2 - 1/4*(b*d*x + b*c)*sin(2*
b*x + 2*a)/b^2
```

3.12 $\int \frac{\sin^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=78

$$-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}$$

```
[Out] -(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Log[c + d*x]
/(2*d) + (Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rubi [A] time = 0.168239, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3312, 3303, 3299, 3302}

$$-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x),x]
```

```
[Out] -(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Log[c + d*x]
/(2*d) + (Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(a+bx)}{c+dx}\,dx &= \int \left(\frac{1}{2(c+dx)}-\frac{\cos(2a+2bx)}{2(c+dx)}\right)dx\\ &= \frac{\log(c+dx)}{2d}-\frac{1}{2}\int \frac{\cos(2a+2bx)}{c+dx}\,dx\\ &= \frac{\log(c+dx)}{2d}-\frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx+\frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx\\ &= -\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}\end{aligned}$$

Mathematica [A] time = 0.104045, size = 65, normalized size = 0.83

$$\frac{-\cos\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right)+\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x),x]
```

```
[Out] (-(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d]) + Log[c + d*x] + Si
n[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(2*d)
```

Maple [A] time = 0.01, size = 105, normalized size = 1.4

$$\frac{\ln\left((bx+a)\,d-da+cb\right)}{2\,d}-\frac{1}{2\,d}\mathrm{Si}\left(2\,bx+2\,a+2\,\frac{-da+cb}{d}\right)\sin\left(2\,\frac{-da+cb}{d}\right)-\frac{1}{2\,d}\mathrm{Ci}\left(2\,bx+2\,a+2\,\frac{-da+cb}{d}\right)\cos\left(2\,-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c),x)
```

```
[Out] 1/2*ln((b*x+a)*d-d*a+c*b)/d-1/2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*
c)/d)/d-1/2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d
```

Maxima [C] time = 1.23145, size = 216, normalized size = 2.77

$$\frac{b\left(E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_1\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(-i\,E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+i\,E_1\left(-\frac{2i\,bc+2i\,(bx+a)d-}{d}\right.\right.}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + exp_in
tegral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c - a*d)/d
) + b*(-I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + I*ex
p_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c - a*
d)/d) + 2*b*log(b*c + (b*x + a)*d - a*d))/(b*d)
```

Fricas [A] time = 1.70157, size = 238, normalized size = 3.05

$$-\frac{\left(\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-2\,\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)-2\,\log\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/4*((cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*
cos(-2*(b*c - a*d)/d) - 2*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c
)/d) - 2*log(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2/(c + d*x), x)
```

Giac [C] time = 1.2331, size = 826, normalized size = 10.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(2*log(abs(d*x + c))*tan(a)^2*tan(b*c/d)^2 - real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - real_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(a)^2*tan(b*c/d) - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*ta
n(b*c/d) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) - 2*imag_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 2*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b
*c)/d)*tan(a)*tan(b*c/d)^2 + 2*log(abs(d*x + c))*tan(a)^2 + real_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(a)^2 + real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(a)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)
 - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 2*log(ab
s(d*x + c))*tan(b*c/d)^2 + real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c
/d)^2 + real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 + 2*imag_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(a) - 2*imag_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(a) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a) - 2*imag_part(
cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 2*imag_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*c/d) - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 2*
log(abs(d*x + c)) - real_part(cos_integral(2*b*x + 2*b*c/d)) - real_part(co
s_integral(-2*b*x - 2*b*c/d)))/(d*tan(a)^2*tan(b*c/d)^2 + d*tan(a)^2 + d*ta
n(b*c/d)^2 + d)
```

3.13 $\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=81

$$\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sin^2(a+bx)}{d(c+dx)}$$

```
[Out] (b*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d^2 - Sin[a + b*x]^
2/(d*(c + d*x)) + (b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d
^2
```

Rubi [A] time = 0.139409, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3313, 12, 3303, 3299, 3302}

$$\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sin^2(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] (b*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d^2 - Sin[a + b*x]^
2/(d*(c + d*x)) + (b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d
^2
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx &= -\frac{\sin^2(a+bx)}{d(c+dx)} + \frac{(2b)\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx}{d} \\
&= -\frac{\sin^2(a+bx)}{d(c+dx)} + \frac{b\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d} \\
&= -\frac{\sin^2(a+bx)}{d(c+dx)} + \frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} + \frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} \\
&= \frac{b\,\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2} - \frac{\sin^2(a+bx)}{d(c+dx)} + \frac{b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.398654, size = 75, normalized size = 0.93

$$\frac{b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right)-\frac{d\sin^2(a+bx)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] (b*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - (d*Sin[a + b*x]^2)
/(c + d*x) + b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^2
```

Maple [A] time = 0.009, size = 156, normalized size = 1.9

$$\frac{1}{b}\left(-\frac{b^2}{(2\,(bx+a)\,d-2\,da+2\,cb)\,d}-\frac{b^2}{4}\left(-2\,\frac{\cos(2\,bx+2\,a)}{((bx+a)\,d-da+cb)\,d}-2\,\frac{1}{d}\left(2\,\frac{1}{d}\mathrm{Si}\left(2\,bx+2\,a+2\,\frac{-da+cb}{d}\right)\cos\left(2\,\frac{-da+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] 1/b*(-1/2*b^2/((b*x+a)*d-d*a+c*b)/d-1/4*b^2*(-2*cos(2*b*x+2*a)/((b*x+a)*d-d
*a+c*b)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*
x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d))
```

Maxima [C] time = 1.32235, size = 231, normalized size = 2.85

$$\frac{16\,b^2\left(E_2\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_2\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^2\left(16i\,E_2\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-16i\,E_2\left(-\frac{2i\,bc+2}{}\right.\right.}{64\left(bcd+(bx+a)d^2-ad^2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(16*b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) +
exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c -
a*d)/d) - b^2*(16*I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)
/d) - 16*I*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin
(-2*(b*c - a*d)/d) - 32*b^2)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A] time = 1.81944, size = 325, normalized size = 4.01

$$\frac{2\,d\cos(bx+a)^2+2\,(bdx+bc)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+\left((bdx+bc)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*cos(b*x + a)^2 + 2*(b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*sin_integra
l(2*(b*d*x + b*c)/d) + ((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*
d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) - 2*d)/(
d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2/(c + d*x)**2, x)
```

Giac [C] time = 1.34291, size = 3976, normalized size = 49.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan
(b*c/d)^2 - b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(
a)^2*tan(b*c/d)^2 + 2*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(
a)^2*tan(b*c/d)^2 + 2*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(a)^2*tan(b*c/d) + 2*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b*d*x*real_part(cos_integral(2*b*x + 2
*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b*d*x*real_part(cos_integral(-2
*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b*c*imag_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b*c*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b*c*sin_inte
gral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b*d*x*imag_part(
```

```
cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + b*d*x*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b*d*x*sin_integral(2*(b*d
*x + b*c)/d)*tan(b*x)^2*tan(a)^2 + 4*b*d*x*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b*d*x*imag_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 8*b*d*x*sin_integral(2*(b*d*x
 + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d) + 2*b*c*real_part(cos_integral(2*b*
x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b*c*real_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - b*d*x*imag_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + b*d*x*imag_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b*d*x*sin_integral(2*(b
*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 2*b*c*real_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b*c*real_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b*d*x*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - b*d*x*imag_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b*d*x*sin_integral(2*(b*d*x
+ b*c)/d)*tan(a)^2*tan(b*c/d)^2 + 2*b*d*x*real_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(a) + 2*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(b*x)^2*tan(a) - b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x
)^2*tan(a)^2 + b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(a)^2 - 2*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 - 2*b*d*x
*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 2*b*d*x*r
eal_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 4*b*c*imag
_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b*c*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 8*b
*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d) + 2*b*d*x*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*b*d*x*real_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) - b*c*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + b*c*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b*c*sin_integral(2*(b
*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 2*b*d*x*real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b*d*x*real_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(a)*tan(b*c/d)^2 + b*c*imag_part(cos_integral(2*b*x + 2*b*c/
d))*tan(a)^2*tan(b*c/d)^2 - b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(a)^2*tan(b*c/d)^2 + 2*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b
*c/d)^2 + b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 - b*d*x
*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 + 2*b*d*x*sin_integra
l(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 2*b*c*real_part(cos_integral(2*b*x + 2*b*
c/d))*tan(b*x)^2*tan(a) + 2*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(b*x)^2*tan(a) - b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2
+ b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*b*d*x*sin_in
tegral(2*(b*d*x + b*c)/d)*tan(a)^2 - 2*b*c*real_part(cos_integral(2*b*x + 2
*b*c/d))*tan(b*x)^2*tan(b*c/d) - 2*b*c*real_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(b*x)^2*tan(b*c/d) + 4*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/
d))*tan(a)*tan(b*c/d) - 4*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(a)*tan(b*c/d) + 8*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d
```

```
) + 2*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*
b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) - b*d*x*i
mag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 + b*d*x*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 2*b*d*x*sin_integral(2*(b*d*x +
b*c)/d)*tan(b*c/d)^2 - 2*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a
)*tan(b*c/d)^2 - 2*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan
(b*c/d)^2 + b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 - b*c*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 + 2*b*c*sin_integral(2*
(b*d*x + b*c)/d)*tan(b*x)^2 + 2*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/
d))*tan(a) + 2*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a) - b*c
*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + b*c*imag_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*b*c*sin_integral(2*(b*d*x + b*c)/d)*ta
n(a)^2 - 2*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*b*
d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b*c*imag_part(
cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 4*b*c*imag_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 8*b*c*sin_integral(2*(b*d*x + b
*c)/d)*tan(a)*tan(b*c/d) - b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan
(b*c/d)^2 + b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 2*
b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 - 2*d*tan(b*x)^2*tan(b*c/d
)^2 - 4*d*tan(b*x)*tan(a)*tan(b*c/d)^2 - 2*d*tan(a)^2*tan(b*c/d)^2 + b*d*x*
imag_part(cos_integral(2*b*x + 2*b*c/d)) - b*d*x*imag_part(cos_integral(-2*
b*x - 2*b*c/d)) + 2*b*d*x*sin_integral(2*(b*d*x + b*c)/d) + 2*b*c*real_part
(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*b*c*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(a) - 2*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b
*c/d) - 2*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + b*c*im
ag_part(cos_integral(2*b*x + 2*b*c/d)) - b*c*imag_part(cos_integral(-2*b*x
- 2*b*c/d)) + 2*b*c*sin_integral(2*(b*d*x + b*c)/d) - 2*d*tan(b*x)^2 - 4*d*
tan(b*x)*tan(a) - 2*d*tan(a)^2)/(d^3*x*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + c
*d^2*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + d^3*x*tan(b*x)^2*tan(a)^2 + d^3*x*t
an(b*x)^2*tan(b*c/d)^2 + d^3*x*tan(a)^2*tan(b*c/d)^2 + c*d^2*tan(b*x)^2*tan
(a)^2 + c*d^2*tan(b*x)^2*tan(b*c/d)^2 + c*d^2*tan(a)^2*tan(b*c/d)^2 + d^3*x
*tan(b*x)^2 + d^3*x*tan(a)^2 + d^3*x*tan(b*c/d)^2 + c*d^2*tan(b*x)^2 + c*d^
2*tan(a)^2 + c*d^2*tan(b*c/d)^2 + d^3*x + c*d^2)
```

3.14 $\int \frac{\sin^2(a+bx)}{(c+dx)^3} dx$

Optimal. Leaf size=113

$$\frac{b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b^2 \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b \sin(a+bx) \cos(a+bx)}{d^2(c+dx)} - \frac{\sin^2(a+bx)}{2d(c+dx)^2}$$

```
[Out] (b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^3 - (b*Cos[a +
b*x]*Sin[a + b*x])/(d^2*(c + d*x)) - Sin[a + b*x]^2/(2*d*(c + d*x)^2) - (b^
2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rubi [A] time = 0.191813, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3314, 31, 3312, 3303, 3299, 3302}

$$\frac{b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b^2 \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b \sin(a+bx) \cos(a+bx)}{d^2(c+dx)} - \frac{\sin^2(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] (b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^3 - (b*Cos[a +
b*x]*Sin[a + b*x])/(d^2*(c + d*x)) - Sin[a + b*x]^2/(2*d*(c + d*x)^2) - (b^
2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx = -\frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\sin^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\int\frac{1}{c+dx}\,dx}{d^2} - \frac{\left(2b^2\right)\int\frac{\sin^2(a+bx)}{c+dx}\,dx}{d^2}$$

$$= \frac{b^2\log(c+dx)}{d^3} - \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{\left(2b^2\right)\int\left(\frac{1}{2(c+dx)} - \frac{\cos(2a+2bx)}{2(c+dx)}\right)dx}{d^2}$$

$$= -\frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\sin^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\int\frac{\cos(2a+2bx)}{c+dx}\,dx}{d^2}$$

$$= -\frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\sin^2(a+bx)}{2d(c+dx)^2} + \frac{\left(b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d^2} - \frac{\left(b^2\sin\left(2a-\right.\right.}{}$$

$$= \frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^3} - \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}}{d^3}$$

Mathematica [A] time = 1.1399, size = 101, normalized size = 0.89

$$-\frac{-2b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+2b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+\frac{d\left(b(c+dx)\sin(2(a+bx))+d\sin^2(a+bx)\right)}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] -(-2*b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + (d*(d*Sin[a
+ b*x]^2 + b*(c + d*x)*Sin[2*(a + b*x)]))/(c + d*x)^2 + 2*b^2*Sin[2*a - (2*
b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(2*d^3)
```

Maple [A] time = 0.009, size = 193, normalized size = 1.7

$$\frac{1}{b}\left(-\frac{b^3}{4\left((bx+a)d-da+cb\right)^2d}-\frac{b^3}{4}\left(-\frac{\cos(2bx+2a)}{\left((bx+a)d-da+cb\right)^2d}-\frac{1}{d}\left(-2\frac{\sin(2bx+2a)}{((bx+a)d-da+cb)d}+2\frac{1}{d}\left(2\frac{1}{d}\text{Si}\left(2bx+2a\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] 1/b*(-1/4*b^3/((b*x+a)*d-d*a+c*b)^2/d-1/4*b^3*(-cos(2*b*x+2*a)/((b*x+a)*d-d
*a+c*b)^2/d-(-2*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*Si(2*b*x+2*a+2*(-
a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*
d+b*c)/d)/d)/d)/d))
```

Maxima [C] time = 1.52385, size = 278, normalized size = 2.46

$$\frac{16\,b^3\left(E_3\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_3\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^3\left(16i\,E_3\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-16i\,E_3\left(-\frac{2i\,bc+2\right.\right.}{64\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\left(bcd^2-ad^3\right)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(16*b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) +
exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c -
```

```
a*d)/d) - b^3*(16*I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)
/d) - 16*I*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin
(-2*(b*c - a*d)/d) - 16*b^3)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 +
a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [B] time = 1.83332, size = 509, normalized size = 4.5

$$\frac{d^2 \cos(bx+a)^2 - 2\left(bd^2x + bcd\right)\cos(bx+a)\sin(bx+a) - 2\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right) - \cdots}{2\left(d^5x^2 + 2\,cd^4x + \cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(d^2*cos(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - 2
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-2*(b*c - a*d)/d)*sin_integral(2
*(b*d*x + b*c)/d) - d^2 + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integr
al(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(
-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**2/(c + d*x)**3, x)
```

Giac [C] time = 1.61643, size = 6940, normalized size = 61.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)
^2*tan(b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x
+ 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b^2*d^2*x^2*si
n_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^
2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 -
 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*
tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan
(a)*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan
(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*real_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - b^2*d^2*x^2*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b^2*d^2*x^2*real_part(cos_
integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 4*b^2*d^2*x^2*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b^2
*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c
/d) + 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(
a)^2*tan(b*c/d) - 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*ta
n(a)^2*tan(b*c/d) - b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(b*x)^2*tan(b*c/d)^2 - b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(b*x)^2*tan(b*c/d)^2 + 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b
*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 4*b^2*c*d*x*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 8*b^2*c*d*x*sin_integra
l(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b^2*d^2*x^2*real_part
(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*d^2*x^2*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*c^2*real_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + b^2*c^2
*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2
 - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)
 + 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a
) - 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 2*b^2
*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^2
*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 2*b^
2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) -
2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/
d) + 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) +
8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(
b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*t
an(a)*tan(b*c/d) - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*t
an(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(a)^2*tan(b*c/d) - 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(a
```

```
)^2*tan(b*c/d) - 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x
)^2*tan(a)^2*tan(b*c/d) + 2*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)/
d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^2*c*d*x*real_part(cos_integral(2*b*
x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b^2*c*d*x*real_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*imag_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_integra
l(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*b^2*c^2*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*imag_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 4*b^2*c^2*sin
_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*b^2*c*d*x*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c*d*x
*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*d^2*
x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + b^2*d^2*x^2*real_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 4*b^2*c*d*x*imag_part(cos
_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^2*c*d*x*imag_part(cos_i
ntegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 8*b^2*c*d*x*sin_integral(2*(
b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - b^2*d^2*x^2*real_part(cos_integral(2*b*
x + 2*b*c/d))*tan(a)^2 - b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(a)^2 - b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*
tan(a)^2 - b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(a)^2 + 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan
(b*c/d) - 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*
tan(b*c/d) + 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c
/d) + 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c
/d) + 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*
c/d) + 4*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)
*tan(b*c/d) + 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^
2*tan(a)*tan(b*c/d) - 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*
tan(a)^2*tan(b*c/d) + 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(a)^2*tan(b*c/d) - 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2
*tan(b*c/d) - b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/
d)^2 - b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 -
 b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 -
 b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2
+ 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2
- 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2
 + 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*b*d^
2*x*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b^2*c^2*real_part(cos_integral(2*b*x +
 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*c^2*real_part(cos_integral(-2*b*x -
2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b*d^2*x*tan(b*x)*tan(a)^2*tan(b*c/d)^2
+ 2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 2*b^2*c
*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 2*b^2*d^2*x^2*i
mag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*b^2*d^2*x^2*imag_part(co
```

```
s_integral(-2*b*x - 2*b*c/d))*tan(a) - 4*b^2*d^2*x^2*sin_integral(2*(b*d*x
+ b*c)/d)*tan(a) - 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b
*x)^2*tan(a) + 2*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)
^2*tan(a) - 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 2
*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 2*b^2*c*d*x*
real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 + 2*b^2*d^2*x^2*imag_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*b^2*d^2*x^2*imag_part(cos_i
ntegral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b^2*d^2*x^2*sin_integral(2*(b*d*x
 + b*c)/d)*tan(b*c/d) + 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*
tan(b*x)^2*tan(b*c/d) - 2*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(b*x)^2*tan(b*c/d) + 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)
^2*tan(b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)
*tan(b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*
tan(b*c/d) - 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*ta
n(b*c/d) + 2*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan
(b*c/d) - 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) - 2
*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 2*b^2*c*
d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 + 2*b^2*c^2*imag
_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*imag_p
art(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*b^2*c^2*sin_int
egral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*b*c*d*tan(b*x)^2*tan(a)*ta
n(b*c/d)^2 + 2*b*c*d*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + b^2*d^2*x^2*real_part
(cos_integral(2*b*x + 2*b*c/d)) + b^2*d^2*x^2*real_part(cos_integral(-2*b*x
 - 2*b*c/d)) + b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2
+ b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 4*b^2*c*d*
x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 4*b^2*c*d*x*imag_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(a) - 8*b^2*c*d*x*sin_integral(2*(b*d*x +
 b*c)/d)*tan(a) + 2*b*d^2*x*tan(b*x)^2*tan(a) - b^2*c^2*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(a)^2 - b^2*c^2*real_part(cos_integral(-2*b*x - 2*
b*c/d))*tan(a)^2 + 2*b*d^2*x*tan(b*x)*tan(a)^2 + 4*b^2*c*d*x*imag_part(cos_
integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 4*b^2*c*d*x*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(b*c/d) + 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)
*tan(b*c/d) + 4*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan
(b*c/d) + 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*
c/d) - b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - b^2*
c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 2*b*d^2*x*tan(
b*x)*tan(b*c/d)^2 - 2*b*d^2*x*tan(a)*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(c
os_integral(2*b*x + 2*b*c/d)) + 2*b^2*c*d*x*real_part(cos_integral(-2*b*x -
 2*b*c/d)) - 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*
b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a) - 4*b^2*c^2*sin_in
tegral(2*(b*d*x + b*c)/d)*tan(a) + 2*b*c*d*tan(b*x)^2*tan(a) + 2*b*c*d*tan(
b*x)*tan(a)^2 + 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/
d) - 2*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b^2
*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) - 2*b*c*d*tan(b*x)*tan(b*c/
d)^2 - d^2*tan(b*x)^2*tan(b*c/d)^2 - 2*b*c*d*tan(a)*tan(b*c/d)^2 - 2*d^2*ta
```

```
n(b*x)*tan(a)*tan(b*c/d)^2 - d^2*tan(a)^2*tan(b*c/d)^2 + b^2*c^2*real_part(
cos_integral(2*b*x + 2*b*c/d)) + b^2*c^2*real_part(cos_integral(-2*b*x - 2*
b*c/d)) - 2*b*d^2*x*tan(b*x) - 2*b*d^2*x*tan(a) - 2*b*c*d*tan(b*x) - d^2*ta
n(b*x)^2 - 2*b*c*d*tan(a) - 2*d^2*tan(b*x)*tan(a) - d^2*tan(a)^2)/(d^5*x^2*
tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^2*tan(a)^2*tan(b*c/d)
^2 + d^5*x^2*tan(b*x)^2*tan(a)^2 + d^5*x^2*tan(b*x)^2*tan(b*c/d)^2 + d^5*x^
2*tan(a)^2*tan(b*c/d)^2 + c^2*d^3*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*c*d^
4*x*tan(b*x)^2*tan(a)^2 + 2*c*d^4*x*tan(b*x)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan
(a)^2*tan(b*c/d)^2 + d^5*x^2*tan(b*x)^2 + d^5*x^2*tan(a)^2 + c^2*d^3*tan(b*
x)^2*tan(a)^2 + d^5*x^2*tan(b*c/d)^2 + c^2*d^3*tan(b*x)^2*tan(b*c/d)^2 + c^
2*d^3*tan(a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^2 + 2*c*d^4*x*tan(a)^2 + 2
*c*d^4*x*tan(b*c/d)^2 + d^5*x^2 + c^2*d^3*tan(b*x)^2 + c^2*d^3*tan(a)^2 + c
^2*d^3*tan(b*c/d)^2 + 2*c*d^4*x + c^2*d^3)
```

3.15 $\int \frac{\sin^2(a+bx)}{(c+dx)^4} \, dx$

Optimal. Leaf size=162

$$-\frac{2b^3 \sin\left(2a-\frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^3 \cos\left(2a-\frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^2 \sin^2(a+bx)}{3d^3(c+dx)}-\frac{b \sin(a+bx) \cos(}{3d^2(c+dx)}$$

```
[Out] -b^2/(3*d^3*(c + d*x)) - (2*b^3*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2
*b*c)/d])/(3*d^4) - (b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2*(c + d*x)^2) - Sin
[a + b*x]^2/(3*d*(c + d*x)^3) + (2*b^2*Sin[a + b*x]^2)/(3*d^3*(c + d*x)) -
(2*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rubi [A] time = 0.180895, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3314, 32, 3313, 12, 3303, 3299, 3302}

$$-\frac{2b^3 \sin\left(2a-\frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^3 \cos\left(2a-\frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^2 \sin^2(a+bx)}{3d^3(c+dx)}-\frac{b \sin(a+bx) \cos(}{3d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x)^4,x]
```

```
[Out] -b^2/(3*d^3*(c + d*x)) - (2*b^3*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2
*b*c)/d])/(3*d^4) - (b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2*(c + d*x)^2) - Sin
[a + b*x]^2/(3*d*(c + d*x)^3) + (2*b^2*Sin[a + b*x]^2)/(3*d^3*(c + d*x)) -
(2*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx = -\frac{b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} - \frac{\sin^2(a+bx)}{3d(c+dx)^3} + \frac{b^2\int \frac{1}{(c+dx)^2}\,dx}{3d^2} - \frac{\left(2b^2\right)\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx}{3d^2}$$

$$= -\frac{b^2}{3d^3(c+dx)} - \frac{b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} - \frac{\sin^2(a+bx)}{3d(c+dx)^3} + \frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)} - \frac{\left(4b^3\right)\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx}{3d^3}$$

$$= -\frac{b^2}{3d^3(c+dx)} - \frac{b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} - \frac{\sin^2(a+bx)}{3d(c+dx)^3} + \frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)} - \frac{\left(2b^3\right)\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{3d^3}$$

$$= -\frac{b^2}{3d^3(c+dx)} - \frac{b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} - \frac{\sin^2(a+bx)}{3d(c+dx)^3} + \frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)} - \frac{\left(2b^3\cos\left(2a-\frac{2bc}{d}\right)\right)}{3d^3}$$

$$= -\frac{b^2}{3d^3(c+dx)} - \frac{2b^3\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{3d^4} - \frac{b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} - \frac{\sin^2(a+bx)}{3d(c+dx)^3} + \frac{2b^2}{3d}$$

Mathematica [A] time = 1.21809, size = 122, normalized size = 0.75

$$-\frac{4b^3\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + \frac{d\left(\cos(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+d(b(c+dx)\sin(2(a+bx))+d)\right)}{(c+dx)^3} + 4b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c}{d}\right.}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x)^4,x]
```

```
[Out] -(4*b^3*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*((-d^2 + 2
*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + d*(d + b*(c + d*x)*Sin[2*(a + b*x)])))
/(c + d*x)^3 + 4*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(
6*d^4)
```

Maple [A] time = 0.008, size = 229, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{b^4}{6\,((bx+a)d-da+cb)^3\,d} - \frac{b^4}{4}\left(-\frac{2\,\cos(2\,bx+2\,a)}{3\,((bx+a)d-da+cb)^3\,d} - \frac{2}{3\,d}\left(-\frac{\sin(2\,bx+2\,a)}{((bx+a)d-da+cb)^2\,d} + \frac{1}{d}\left(-2\,\frac{\cos(2\,b}{((bx+a)d}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^4,x)
```

```
[Out] 1/b*(-1/6*b^4/((b*x+a)*d-d*a+c*b)^3/d-1/4*b^4*(-2/3*cos(2*b*x+2*a)/((b*x+a)
*d-d*a+c*b)^3/d-2/3*(-sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d+(-2*cos(2*b*x+
2*a)/((b*x+a)*d-d*a+c*b)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*
c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)/d))
```

Maxima [C] time = 1.88609, size = 346, normalized size = 2.14

$$\frac{3\,b^4\left(E_4\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_4\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^4\left(3i\,E_4\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-3i\,E_4\left(-\frac{2i\,bc+2i\,(b\cdots}{}\right.\right.}{12\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\left(bcd^3-ad^4\right)(bx+a)^2+3\left(b^2c^2d^2-2\,abcd^3+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^4*(exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + e
xp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b^4*(3*I*exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d
) - 3*I*exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2
*(b*c - a*d)/d) - 2*b^4)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b
*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2
- 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Fricas [B] time = 1.92985, size = 733, normalized size = 4.52

$$\frac{b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-d^3-\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3\right)\cos\left(bx+a\right)^2-\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)\sin\left(bx\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - d^3 - (2*b^2*d^3*x^2 + 4*b^2
*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 - (b*d^3*x + b*c*d^2)*cos(b*x
+ a)*sin(b*x + a) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*
c^3)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - ((b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d
) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(
-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*
```

```
d^5*x + c^3*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**2/(c + d*x)**4, x)
```

Giac [C] time = 1.77156, size = 10573, normalized size = 65.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/3*(b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
)^2*tan(b*c/d)^2 - b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)
/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^3*d^3*x^3*real_part
(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^3*d^3
*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^
2 - 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(
a)*tan(b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*
tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 3*b^3*c*d^2*x^2*imag_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*sin_in
tegral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^3*d^3*x^3*im
ag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + b^3*d^3*x^3*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^3*d^3*x^3
*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 + 4*b^3*d^3*x^3*imag_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b^3*d^3
*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)
 + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/
```

```
d) + 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*ta
n(a)^2*tan(b*c/d) + 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - b^3*d^3*x^3*imag_part(cos_integral(2*b*
x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*sin_integral(2*(
b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 6*b^3*c*d^2*x^2*real_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 6*b^3*c*d^2*x^2*re
al_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b^
3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 -
b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2
 + 2*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + 3*
b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*ta
n(b*c/d)^2 - 3*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*
x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*
tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b^3*d^3*x^3*real_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 3*b^3*c*d^2*x^2*imag_part(cos_integral(
2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 6*b^3*c*d^2*x^2*sin_integral(2
*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 - 2*b^3*d^3*x^3*real_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 2*b^3*d^3*x^3*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 12*b^3*c*d^2*x^2*imag_part
(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 12*b^3*c*d^2
*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)
 + 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b
*c/d) + 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan
(b*c/d) + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*
tan(b*c/d) + 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x
)^2*tan(a)^2*tan(b*c/d) + 6*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 3*b^3*c*d^2*x^2*imag_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 6*b^3*c*d^2*x^2*si
n_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^3*d^3*x^3*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 6*b^3*c^2*d*x*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 6
*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*ta
n(b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a
)^2*tan(b*c/d)^2 - 3*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*t
an(a)^2*tan(b*c/d)^2 + b^2*d^3*x^2*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + b^3*c
^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^
2 - b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*t
an(b*c/d)^2 + 2*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2
*tan(b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
```

```
x)^2 - b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 + 2
*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 6*b^3*c*d^2*x^2*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 6*b^3*c*d^2*x^2
*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - b^3*d^3*x^3*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + b^3*d^3*x^3*imag_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*b^3*d^3*x^3*sin_integral(2*(b*d
*x + b*c)/d)*tan(a)^2 - 3*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/
d))*tan(b*x)^2*tan(a)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(b*x)^2*tan(a)^2 - 6*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*
tan(b*x)^2*tan(a)^2 - 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/
d))*tan(b*x)^2*tan(b*c/d) - 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 4*b^3*d^3*x^3*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b
*c)/d)*tan(a)*tan(b*c/d) + 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 24*b^3*c^2*d*x*sin_inte
gral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d) + 6*b^3*c*d^2*x^2*real
_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) + 6*b^3*c*d^2*x^2*
real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*b^3*c^3*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*
b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b
*c/d) - b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 +
 b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 2*b^3
*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 - 3*b^3*c^2*d*x*imag_
part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 3*b^3*c^2*d*x
*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 6*b^3*
c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 6*b^3*c*d
^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 6*b^3
*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 -
2*b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*
c/d)^2 - 2*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(a)*tan(b*c/d)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*t
an(a)^2*tan(b*c/d)^2 - 3*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*t
an(a)^2*tan(b*c/d)^2 + 2*b^2*c*d^2*x*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 3*b
^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 - 3*b^3*c*
d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 + 6*b^3*c*d^2*
x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 2*b^3*d^3*x^3*real_part(co
s_integral(2*b*x + 2*b*c/d))*tan(a) + 2*b^3*d^3*x^3*real_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(a) + 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(a) + 6*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(b*x)^2*tan(a) - 3*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x +
 2*b*c/d))*tan(a)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(a)^2 - 6*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2 +
```

```
b^2*d^3*x^2*tan(b*x)^2*tan(a)^2 - b^3*c^3*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(a)^2 + b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(b*x)^2*tan(a)^2 - 2*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*
x)^2*tan(a)^2 - 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*c/d) - 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)
 - 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*
c/d) - 6*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*t
an(b*c/d) + 12*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a
)*tan(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(a)*tan(b*c/d) + 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*
tan(b*c/d) + 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*
tan(a)*tan(b*c/d) - 4*b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(a)*tan(b*c/d) + 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b
*x)^2*tan(a)*tan(b*c/d) + 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*
c/d))*tan(a)^2*tan(b*c/d) + 6*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(a)^2*tan(b*c/d) - 3*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(b*c/d)^2 - 6*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*t
an(b*c/d)^2 - b^2*d^3*x^2*tan(b*x)^2*tan(b*c/d)^2 - b^3*c^3*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + b^3*c^3*imag_part(cos_i
ntegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b^3*c^3*sin_integral
(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*b^2*d^3*x^2*tan(b*x)*t
an(a)*tan(b*c/d)^2 - b^2*d^3*x^2*tan(a)^2*tan(b*c/d)^2 + b^3*c^3*imag_part(
cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - b^3*c^3*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^3*c^3*sin_integra
l(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + b^2*c^2*d*tan(b*x)^2*tan(a)^2*
tan(b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d)) - b^3*d
^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d)) + 2*b^3*d^3*x^3*sin_integr
al(2*(b*d*x + b*c)/d) + 3*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/
d))*tan(b*x)^2 - 3*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2 + 6*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 6*b^3
*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 6*b^3*c*d^2*x^
2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a) + 2*b^3*c^3*real_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b^3*c^3*real_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 3*b^3*c^2*d*x*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(a)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(a)^2 - 6*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*
tan(a)^2 + 2*b^2*c*d^2*x*tan(b*x)^2*tan(a)^2 - 6*b^3*c*d^2*x^2*real_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 6*b^3*c*d^2*x^2*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(b*c/d) - 2*b^3*c^3*real_part(cos_integral(2*b*
x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 2*b^3*c^3*real_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 12*b^3*c^2*d*x*imag_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integr
```

```
al(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 24*b^3*c^2*d*x*sin_integral(2*(b*
d*x + b*c)/d)*tan(a)*tan(b*c/d) + 2*b^3*c^3*real_part(cos_integral(2*b*x +
2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*b^3*c^3*real_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(a)^2*tan(b*c/d) - 3*b^3*c^2*d*x*imag_part(cos_integral(2*b*x +
 2*b*c/d))*tan(b*c/d)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(b*c/d)^2 - 6*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c
/d)^2 - 2*b^2*c*d^2*x*tan(b*x)^2*tan(b*c/d)^2 - 2*b^3*c^3*real_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^3*c^3*real_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 8*b^2*c*d^2*x*tan(b*x)*tan(a)*
tan(b*c/d)^2 - b*d^3*x*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b^2*c*d^2*x*tan(a
)^2*tan(b*c/d)^2 - b*d^3*x*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + 3*b^3*c*d^2*x^2
*imag_part(cos_integral(2*b*x + 2*b*c/d)) - 3*b^3*c*d^2*x^2*imag_part(cos_i
ntegral(-2*b*x - 2*b*c/d)) + 6*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d
) - b^2*d^3*x^2*tan(b*x)^2 + b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d
))*tan(b*x)^2 - b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^
2 + 2*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 6*b^3*c^2*d*x*re
al_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 6*b^3*c^2*d*x*real_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(a) - 4*b^2*d^3*x^2*tan(b*x)*tan(a) - b^2*d
^3*x^2*tan(a)^2 - b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2
 + b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*b^3*c^3*s
in_integral(2*(b*d*x + b*c)/d)*tan(a)^2 + b^2*c^2*d*tan(b*x)^2*tan(a)^2 - 6
*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 6*b^3*c^
2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b^3*c^3*imag
_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 4*b^3*c^3*imag_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 8*b^3*c^3*sin_integra
l(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d) + b^2*d^3*x^2*tan(b*c/d)^2 - b^3*c^3
*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 + b^3*c^3*imag_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 2*b^3*c^3*sin_integral(2*(b*
d*x + b*c)/d)*tan(b*c/d)^2 - b^2*c^2*d*tan(b*x)^2*tan(b*c/d)^2 - 4*b^2*c^2*
d*tan(b*x)*tan(a)*tan(b*c/d)^2 - b*c*d^2*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - b
^2*c^2*d*tan(a)^2*tan(b*c/d)^2 - b*c*d^2*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + 3
*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d)) - 3*b^3*c^2*d*x*imag_
part(cos_integral(-2*b*x - 2*b*c/d)) + 6*b^3*c^2*d*x*sin_integral(2*(b*d*x
+ b*c)/d) - 2*b^2*c*d^2*x*tan(b*x)^2 + 2*b^3*c^3*real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a) + 2*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(a) - 8*b^2*c*d^2*x*tan(b*x)*tan(a) - b*d^3*x*tan(b*x)^2*tan(a) - 2*b^2
*c*d^2*x*tan(a)^2 - b*d^3*x*tan(b*x)*tan(a)^2 - 2*b^3*c^3*real_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*b^3*c^3*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(b*c/d) + 2*b^2*c*d^2*x*tan(b*c/d)^2 + b*d^3*x*tan(b*x)*ta
n(b*c/d)^2 + b*d^3*x*tan(a)*tan(b*c/d)^2 + b^2*d^3*x^2 + b^3*c^3*imag_part(
cos_integral(2*b*x + 2*b*c/d)) - b^3*c^3*imag_part(cos_integral(-2*b*x - 2*
b*c/d)) + 2*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d) - b^2*c^2*d*tan(b*x)^2
- 4*b^2*c^2*d*tan(b*x)*tan(a) - b*c*d^2*tan(b*x)^2*tan(a) - b^2*c^2*d*tan(a
)^2 - b*c*d^2*tan(b*x)*tan(a)^2 + b^2*c^2*d*tan(b*c/d)^2 + b*c*d^2*tan(b*x)
*tan(b*c/d)^2 + d^3*tan(b*x)^2*tan(b*c/d)^2 + b*c*d^2*tan(a)*tan(b*c/d)^2 +
```

```
 2*d^3*tan(b*x)*tan(a)*tan(b*c/d)^2 + d^3*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c*d
^2*x + b*d^3*x*tan(b*x) + b*d^3*x*tan(a) + b^2*c^2*d + b*c*d^2*tan(b*x) + d
^3*tan(b*x)^2 + b*c*d^2*tan(a) + 2*d^3*tan(b*x)*tan(a) + d^3*tan(a)^2)/(d^7
*x^3*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 3*c*d^6*x^2*tan(b*x)^2*tan(a)^2*tan
(b*c/d)^2 + d^7*x^3*tan(b*x)^2*tan(a)^2 + d^7*x^3*tan(b*x)^2*tan(b*c/d)^2 +
 d^7*x^3*tan(a)^2*tan(b*c/d)^2 + 3*c^2*d^5*x*tan(b*x)^2*tan(a)^2*tan(b*c/d)
^2 + 3*c*d^6*x^2*tan(b*x)^2*tan(a)^2 + 3*c*d^6*x^2*tan(b*x)^2*tan(b*c/d)^2
+ 3*c*d^6*x^2*tan(a)^2*tan(b*c/d)^2 + c^3*d^4*tan(b*x)^2*tan(a)^2*tan(b*c/d
)^2 + d^7*x^3*tan(b*x)^2 + d^7*x^3*tan(a)^2 + 3*c^2*d^5*x*tan(b*x)^2*tan(a)
^2 + d^7*x^3*tan(b*c/d)^2 + 3*c^2*d^5*x*tan(b*x)^2*tan(b*c/d)^2 + 3*c^2*d^5
*x*tan(a)^2*tan(b*c/d)^2 + 3*c*d^6*x^2*tan(b*x)^2 + 3*c*d^6*x^2*tan(a)^2 +
c^3*d^4*tan(b*x)^2*tan(a)^2 + 3*c*d^6*x^2*tan(b*c/d)^2 + c^3*d^4*tan(b*x)^2
*tan(b*c/d)^2 + c^3*d^4*tan(a)^2*tan(b*c/d)^2 + d^7*x^3 + 3*c^2*d^5*x*tan(b
*x)^2 + 3*c^2*d^5*x*tan(a)^2 + 3*c^2*d^5*x*tan(b*c/d)^2 + 3*c*d^6*x^2 + c^3
*d^4*tan(b*x)^2 + c^3*d^4*tan(a)^2 + c^3*d^4*tan(b*c/d)^2 + 3*c^2*d^5*x + c
^3*d^4)
```

3.16 $\int (c+dx)^4 \sin^3(a+bx)\,dx$

Optimal. Leaf size=225

$$-\frac{8d^3(c+dx)\sin^3(a+bx)}{27b^4}-\frac{160d^3(c+dx)\sin(a+bx)}{9b^4}+\frac{80d^2(c+dx)^2\cos(a+bx)}{9b^3}+\frac{4d^2(c+dx)^2\sin^2(a+bx)\cos(a+}{9b^3}$$

```
[Out] (-488*d^4*Cos[a + b*x])/(27*b^5) + (80*d^2*(c + d*x)^2*Cos[a + b*x])/(9*b^3
) - (2*(c + d*x)^4*Cos[a + b*x])/(3*b) + (8*d^4*Cos[a + b*x]^3)/(81*b^5) -
(160*d^3*(c + d*x)*Sin[a + b*x])/(9*b^4) + (8*d*(c + d*x)^3*Sin[a + b*x])/(
3*b^2) + (4*d^2*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^3) - ((c + d*
x)^4*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) - (8*d^3*(c + d*x)*Sin[a + b*x]^3)/
(27*b^4) + (4*d*(c + d*x)^3*Sin[a + b*x]^3)/(9*b^2)
```

Rubi [A] time = 0.250075, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 3296, 2638, 2633}

$$-\frac{8d^3(c+dx)\sin^3(a+bx)}{27b^4}-\frac{160d^3(c+dx)\sin(a+bx)}{9b^4}+\frac{80d^2(c+dx)^2\cos(a+bx)}{9b^3}+\frac{4d^2(c+dx)^2\sin^2(a+bx)\cos(a+}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sin[a + b*x]^3,x]
```


```
[Out] (-488*d^4*Cos[a + b*x])/(27*b^5) + (80*d^2*(c + d*x)^2*Cos[a + b*x])/(9*b^3
) - (2*(c + d*x)^4*Cos[a + b*x])/(3*b) + (8*d^4*Cos[a + b*x]^3)/(81*b^5) -
(160*d^3*(c + d*x)*Sin[a + b*x])/(9*b^4) + (8*d*(c + d*x)^3*Sin[a + b*x])/(
3*b^2) + (4*d^2*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^3) - ((c + d*
x)^4*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) - (8*d^3*(c + d*x)*Sin[a + b*x]^3)/
(27*b^4) + (4*d*(c + d*x)^3*Sin[a + b*x]^3)/(9*b^2)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4 \sin^3(a+bx)\,dx &= -\frac{(c+dx)^4\cos(a+bx)\sin^2(a+bx)}{3b} + \frac{4d(c+dx)^3\sin^3(a+bx)}{9b^2} + \frac{2}{3}\int (c+dx)^4\sin(a+b\\
&= -\frac{2(c+dx)^4\cos(a+bx)}{3b} + \frac{4d^2(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{9b^3} - \frac{(c+dx)^4\cos(a+bx)\,\mathrm{s}}{3b}\\
&= \frac{8d^2(c+dx)^2\cos(a+bx)}{9b^3} - \frac{2(c+dx)^4\cos(a+bx)}{3b} + \frac{8d(c+dx)^3\sin(a+bx)}{3b^2} + \frac{4d^2(c+d}{}\\
&= -\frac{8d^4\cos(a+bx)}{27b^5} + \frac{80d^2(c+dx)^2\cos(a+bx)}{9b^3} - \frac{2(c+dx)^4\cos(a+bx)}{3b} + \frac{8d^4\cos^3(a+b}{81b^5}\\
&= -\frac{56d^4\cos(a+bx)}{27b^5} + \frac{80d^2(c+dx)^2\cos(a+bx)}{9b^3} - \frac{2(c+dx)^4\cos(a+bx)}{3b} + \frac{8d^4\cos^3(a+}{81b^5}\\
&= -\frac{488d^4\cos(a+bx)}{27b^5} + \frac{80d^2(c+dx)^2\cos(a+bx)}{9b^3} - \frac{2(c+dx)^4\cos(a+bx)}{3b} + \frac{8d^4\cos^3(a+}{81b^5}\end{aligned}$$

Mathematica [A] time = 0.999195, size = 150, normalized size = 0.67

$$\frac{-243\cos(a+bx)\left(-12b^2d^2(c+dx)^2+b^4(c+dx)^4+24d^4\right)+\cos(3(a+bx))\left(-36b^2d^2(c+dx)^2+27b^4(c+dx)^4+8d^4\right)-}{324b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sin[a + b*x]^3,x]
```

```
[Out] (-243*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x] + (8
*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*x)] - 24*b
*d*(c + d*x)*(242*d^2 - 39*b^2*(c + d*x)^2 + (-2*d^2 + 3*b^2*(c + d*x)^2)*C
os[2*(a + b*x)])*Sin[a + b*x])/(324*b^5)
```

Maple [B] time = 0.035, size = 1023, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^4*d^4*(-1/3*(b*x+a)^4*(2+sin(b*x+a)^2)*cos(b*x+a)+8/3*(b*x+a)^3*si
n(b*x+a)+8*(b*x+a)^2*cos(b*x+a)-160/9*cos(b*x+a)-160/9*(b*x+a)*sin(b*x+a)+4
/9*(b*x+a)^3*sin(b*x+a)^3+4/9*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)-8/27*(b
*x+a)*sin(b*x+a)^3-8/81*(2+sin(b*x+a)^2)*cos(b*x+a))-4/b^4*a*d^4*(-1/3*(b*x
+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2*(b*x+a)^2*sin(b*x+a)-40/9*sin(b*x+a)+4*
(b*x+a)*cos(b*x+a)+1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*
cos(b*x+a)-2/27*sin(b*x+a)^3)+4/b^3*c*d^3*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*
cos(b*x+a)+2*(b*x+a)^2*sin(b*x+a)-40/9*sin(b*x+a)+4*(b*x+a)*cos(b*x+a)+1/3*
(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x
+a)^3)+6/b^4*a^2*d^4*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/3*cos(b*
x+a)+4/3*(b*x+a)*sin(b*x+a)+2/9*(b*x+a)*sin(b*x+a)^3+2/27*(2+sin(b*x+a)^2)*
cos(b*x+a))-12/b^3*a*c*d^3*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/3*
cos(b*x+a)+4/3*(b*x+a)*sin(b*x+a)+2/9*(b*x+a)*sin(b*x+a)^3+2/27*(2+sin(b*x+
a)^2)*cos(b*x+a))+6/b^2*c^2*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)
+4/3*cos(b*x+a)+4/3*(b*x+a)*sin(b*x+a)+2/9*(b*x+a)*sin(b*x+a)^3+2/27*(2+sin
(b*x+a)^2)*cos(b*x+a))-4/b^4*a^3*d^4*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x
+a)+1/9*sin(b*x+a)^3+2/3*sin(b*x+a))+12/b^3*a^2*c*d^3*(-1/3*(b*x+a)*(2+sin(
b*x+a)^2)*cos(b*x+a)+1/9*sin(b*x+a)^3+2/3*sin(b*x+a))-12/b^2*a*c^2*d^2*(-1/
3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin(b*x+a)^3+2/3*sin(b*x+a))+4/b*
c^3*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin(b*x+a)^3+2/3*sin(b*
x+a))-1/3/b^4*a^4*d^4*(2+sin(b*x+a)^2)*cos(b*x+a)+4/3/b^3*a^3*c*d^3*(2+sin(
b*x+a)^2)*cos(b*x+a)-2/b^2*a^2*c^2*d^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/3/b*a*
c^3*d*(2+sin(b*x+a)^2)*cos(b*x+a)-1/3*c^4*(2+sin(b*x+a)^2)*cos(b*x+a))
```

Maxima [B] time = 1.22366, size = 1261, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/324*(108*(cos(b*x + a)^3 - 3*cos(b*x + a))*c^4 - 432*(cos(b*x + a)^3 - 3*
cos(b*x + a))*a*c^3*d/b + 648*(cos(b*x + a)^3 - 3*cos(b*x + a))*a^2*c^2*d^2
/b^2 - 432*(cos(b*x + a)^3 - 3*cos(b*x + a))*a^3*c*d^3/b^3 + 108*(cos(b*x +
 a)^3 - 3*cos(b*x + a))*a^4*d^4/b^4 + 36*(3*(b*x + a)*cos(3*b*x + 3*a) - 27
*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) + 27*sin(b*x + a))*c^3*d/b - 108
*(3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*
a) + 27*sin(b*x + a))*a*c^2*d^2/b^2 + 108*(3*(b*x + a)*cos(3*b*x + 3*a) - 2
7*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) + 27*sin(b*x + a))*a^2*c*d^3/b^
3 - 36*(3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a) - sin(3*b*
x + 3*a) + 27*sin(b*x + a))*a^3*d^4/b^4 + 18*((9*(b*x + a)^2 - 2)*cos(3*b*x
 + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a)
+ 162*(b*x + a)*sin(b*x + a))*c^2*d^2/b^2 - 36*((9*(b*x + a)^2 - 2)*cos(3*b
*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a
) + 162*(b*x + a)*sin(b*x + a))*a*c*d^3/b^3 + 18*((9*(b*x + a)^2 - 2)*cos(3
*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3
*a) + 162*(b*x + a)*sin(b*x + a))*a^2*d^4/b^4 + 12*(3*(3*(b*x + a)^3 - 2*b*
x - 2*a)*cos(3*b*x + 3*a) - 81*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - (
9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 243*((b*x + a)^2 - 2)*sin(b*x + a))*c
*d^3/b^3 - 12*(3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 81*((b*x
+ a)^3 - 6*b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) +
 243*((b*x + a)^2 - 2)*sin(b*x + a))*a*d^4/b^4 + ((27*(b*x + a)^4 - 36*(b*x
 + a)^2 + 8)*cos(3*b*x + 3*a) - 243*((b*x + a)^4 - 12*(b*x + a)^2 + 24)*cos
(b*x + a) - 12*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) + 972*((b*x +
 a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^4/b^4)/b
```

Fricas [A] time = 1.7696, size = 764, normalized size = 3.4

$$\frac{\left(27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 27\,b^4c^4 - 36\,b^2c^2d^2 + 8\,d^4 + 18\left(9\,b^4c^2d^2 - 2\,b^2d^4\right)x^2 + 36\left(3\,b^4c^3d - 2\,b^2cd^3\right)x\right)\cos\left(bx + a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/81*((27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 + 8
*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)*
x)*cos(b*x + a)^3 - 3*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 25
2*b^2*c^2*d^2 + 488*d^4 + 18*(9*b^4*c^2*d^2 - 14*b^2*d^4)*x^2 + 36*(3*b^4*c
```

```
^3*d - 14*b^2*c*d^3)*x)*cos(b*x + a) + 12*(21*b^3*d^4*x^3 + 63*b^3*c*d^3*x^
2 + 21*b^3*c^3*d - 122*b*c*d^3 - (3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c
^3*d - 2*b*c*d^3 + (9*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + (63*b^3*c^
2*d^2 - 122*b*d^4)*x)*sin(b*x + a))/b^5
```

Sympy [A] time = 9.80385, size = 772, normalized size = 3.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)/b - 2*c**4*cos(a + b*x)**3/(3
*b) - 4*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)/b - 8*c**3*d*x*cos(a + b*x)**
3/(3*b) - 6*c**2*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)/b - 4*c**2*d**2*x**
2*cos(a + b*x)**3/b - 4*c*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)/b - 8*c*d*
*3*x**3*cos(a + b*x)**3/(3*b) - d**4*x**4*sin(a + b*x)**2*cos(a + b*x)/b -
2*d**4*x**4*cos(a + b*x)**3/(3*b) + 28*c**3*d*sin(a + b*x)**3/(9*b**2) + 8*
c**3*d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 28*c**2*d**2*x*sin(a + b*x)*
*3/(3*b**2) + 8*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**2 + 28*c*d**3*x
**2*sin(a + b*x)**3/(3*b**2) + 8*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**2/b
**2 + 28*d**4*x**3*sin(a + b*x)**3/(9*b**2) + 8*d**4*x**3*sin(a + b*x)*cos(
a + b*x)**2/(3*b**2) + 28*c**2*d**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) +
 80*c**2*d**2*cos(a + b*x)**3/(9*b**3) + 56*c*d**3*x*sin(a + b*x)**2*cos(a
+ b*x)/(3*b**3) + 160*c*d**3*x*cos(a + b*x)**3/(9*b**3) + 28*d**4*x**2*sin(
a + b*x)**2*cos(a + b*x)/(3*b**3) + 80*d**4*x**2*cos(a + b*x)**3/(9*b**3) -
 488*c*d**3*sin(a + b*x)**3/(27*b**4) - 160*c*d**3*sin(a + b*x)*cos(a + b*x
)**2/(9*b**4) - 488*d**4*x*sin(a + b*x)**3/(27*b**4) - 160*d**4*x*sin(a + b
*x)*cos(a + b*x)**2/(9*b**4) - 488*d**4*sin(a + b*x)**2*cos(a + b*x)/(27*b*
*5) - 1456*d**4*cos(a + b*x)**3/(81*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x
**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**3, True))
```

Giac [A] time = 1.13816, size = 474, normalized size = 2.11

$$\frac{\left(27\,b^4d^4x^4+108\,b^4cd^3x^3+162\,b^4c^2d^2x^2+108\,b^4c^3dx+27\,b^4c^4-36\,b^2d^4x^2-72\,b^2cd^3x-36\,b^2c^2d^2+8\,d^4\right)\cos\left(3\,bx+3\right.}{324\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c
^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*
d^4)*cos(3*b*x + 3*a)/b^5 - 3/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*
d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^
2*c^2*d^2 + 24*d^4)*cos(b*x + a)/b^5 - 1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^
2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*sin(3*b*x + 3*a)
/b^5 + 3*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b
*d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

3.17 $\int (c+dx)^3 \sin^3(a+bx)\,dx$

Optimal. Leaf size=175

$$\frac{40d^2(c+dx)\cos(a+bx)}{9b^3}+\frac{2d^2(c+dx)\sin^2(a+bx)\cos(a+bx)}{9b^3}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2d(c+dx)^2\sin(a+bx)}{b^2}-$$

```
[Out] (40*d^2*(c + d*x)*Cos[a + b*x])/(9*b^3) - (2*(c + d*x)^3*Cos[a + b*x])/(3*b
) - (40*d^3*Sin[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Sin[a + b*x])/b^2 + (2
*d^2*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^3) - ((c + d*x)^3*Cos[a +
b*x]*Sin[a + b*x]^2)/(3*b) - (2*d^3*Sin[a + b*x]^3)/(27*b^4) + (d*(c + d*x)
^2*Sin[a + b*x]^3)/(3*b^2)
```

Rubi [A] time = 0.158704, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 3296, 2637, 3310}

$$\frac{40d^2(c+dx)\cos(a+bx)}{9b^3}+\frac{2d^2(c+dx)\sin^2(a+bx)\cos(a+bx)}{9b^3}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2d(c+dx)^2\sin(a+bx)}{b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]^3,x]
```


```
[Out] (40*d^2*(c + d*x)*Cos[a + b*x])/(9*b^3) - (2*(c + d*x)^3*Cos[a + b*x])/(3*b
) - (40*d^3*Sin[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Sin[a + b*x])/b^2 + (2
*d^2*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^3) - ((c + d*x)^3*Cos[a +
b*x]*Sin[a + b*x]^2)/(3*b) - (2*d^3*Sin[a + b*x]^3)/(27*b^4) + (d*(c + d*x)
^2*Sin[a + b*x]^3)/(3*b^2)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)^3\sin^3(a+bx)\,dx = -\frac{(c+dx)^3\cos(a+bx)\sin^2(a+bx)}{3b}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\int (c+dx)^3\sin(a+bx$$

$$= -\frac{2(c+dx)^3\cos(a+bx)}{3b}+\frac{2d^2(c+dx)\cos(a+bx)\sin^2(a+bx)}{9b^3}-\frac{(c+dx)^3\cos(a+bx)\,\mathrm{si}}{3b}$$

$$= \frac{4d^2(c+dx)\cos(a+bx)}{9b^3}-\frac{2(c+dx)^3\cos(a+bx)}{3b}+\frac{2d(c+dx)^2\sin(a+bx)}{b^2}+\frac{2d^2(c+dx}{}$$

$$= \frac{40d^2(c+dx)\cos(a+bx)}{9b^3}-\frac{2(c+dx)^3\cos(a+bx)}{3b}-\frac{4d^3\sin(a+bx)}{9b^4}+\frac{2d(c+dx)^2\sin(a}{b^2}$$

$$= \frac{40d^2(c+dx)\cos(a+bx)}{9b^3}-\frac{2(c+dx)^3\cos(a+bx)}{3b}-\frac{40d^3\sin(a+bx)}{9b^4}+\frac{2d(c+dx)^2\sin(}{b^2}$$

Mathematica [A] time = 0.92572, size = 127, normalized size = 0.73

$$\frac{-162b(c+dx)\cos(a+bx)\left(b^2(c+dx)^2-6d^2\right)+6b(c+dx)\cos(3(a+bx))\left(3b^2(c+dx)^2-2d^2\right)-4d\sin(a+bx)\left(\cos(2(\right.}{216b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]^3,x]
```

```
[Out] (-162*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + 6*b*(c + d*x)*(
-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] - 4*d*(242*d^2 - 117*b^2*(c +
d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(216*
```

```
b^4)
```

Maple [B] time = 0.009, size = 560, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^3*d^3*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2*(b*x+a)^2*sin(
b*x+a)-40/9*sin(b*x+a)+4*(b*x+a)*cos(b*x+a)+1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*
(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3)-3/b^3*a*d^3*(-1/3*(b
*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/3*cos(b*x+a)+4/3*(b*x+a)*sin(b*x+a)+2
/9*(b*x+a)*sin(b*x+a)^3+2/27*(2+sin(b*x+a)^2)*cos(b*x+a))+3/b^2*c*d^2*(-1/3
*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/3*cos(b*x+a)+4/3*(b*x+a)*sin(b*x+a
)+2/9*(b*x+a)*sin(b*x+a)^3+2/27*(2+sin(b*x+a)^2)*cos(b*x+a))+3/b^3*a^2*d^3*
(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin(b*x+a)^3+2/3*sin(b*x+a))-
6/b^2*a*c*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin(b*x+a)^3+2/
3*sin(b*x+a))+3/b*c^2*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin(b
*x+a)^3+2/3*sin(b*x+a))+1/3/b^3*a^3*d^3*(2+sin(b*x+a)^2)*cos(b*x+a)-1/b^2*a
^2*c*d^2*(2+sin(b*x+a)^2)*cos(b*x+a)+1/b*a*c^2*d*(2+sin(b*x+a)^2)*cos(b*x+a
)-1/3*c^3*(2+sin(b*x+a)^2)*cos(b*x+a))
```

Maxima [B] time = 1.08757, size = 730, normalized size = 4.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/108*(36*(cos(b*x + a)^3 - 3*cos(b*x + a))*c^3 - 108*(cos(b*x + a)^3 - 3*c
os(b*x + a))*a*c^2*d/b + 108*(cos(b*x + a)^3 - 3*cos(b*x + a))*a^2*c*d^2/b^
2 - 36*(cos(b*x + a)^3 - 3*cos(b*x + a))*a^3*d^3/b^3 + 9*(3*(b*x + a)*cos(3
*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) + 27*sin(b*x + a
))*c^2*d/b - 18*(3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a) -
 sin(3*b*x + 3*a) + 27*sin(b*x + a))*a*c*d^2/b^2 + 9*(3*(b*x + a)*cos(3*b*x
 + 3*a) - 27*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) + 27*sin(b*x + a))*a
```

```
^2*d^3/b^3 + 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)
*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) + 162*(b*x + a)*sin(b*x + a))*
c*d^2/b^2 - 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*
cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) + 162*(b*x + a)*sin(b*x + a))*a
*d^3/b^3 + (3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 81*((b*x + a
)^3 - 6*b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 24
3*((b*x + a)^2 - 2)*sin(b*x + a))*d^3/b^3)/b
```

Fricas [A] time = 1.74108, size = 495, normalized size = 2.83

$$\frac{3\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3-2\,bcd^2+\left(9\,b^3c^2d-2\,bd^3\right)x\right)\cos\left(bx+a\right)^3-9\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3-14\,bcd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/27*(3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*c
^2*d - 2*b*d^3)*x)*cos(b*x + a)^3 - 9*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*
b^3*c^3 - 14*b*c*d^2 + (9*b^3*c^2*d - 14*b*d^3)*x)*cos(b*x + a) + (63*b^2*d
^3*x^2 + 126*b^2*c*d^2*x + 63*b^2*c^2*d - 122*d^3 - (9*b^2*d^3*x^2 + 18*b^2
*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/b^4
```

Sympy [A] time = 5.23282, size = 495, normalized size = 2.83

$$\begin{cases} -\frac{c^3\sin^2(a+bx)\cos(a+bx)}{b}-\frac{2c^3\cos^3(a+bx)}{3b}-\frac{3c^2dx\sin^2(a+bx)\cos(a+bx)}{b}-\frac{2c^2dx\cos^3(a+bx)}{b}-\frac{3cd^2x^2\sin^2(a+bx)\cos(a+bx)}{b}-\frac{2cd^2x^2\cos^3(a+b}{b} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**3*sin(a + b*x)**2*cos(a + b*x)/b - 2*c**3*cos(a + b*x)**3/(3
*b) - 3*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)/b - 2*c**2*d*x*cos(a + b*x)**
3/b - 3*c*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)/b - 2*c*d**2*x**2*cos(a +
b*x)**3/b - d**3*x**3*sin(a + b*x)**2*cos(a + b*x)/b - 2*d**3*x**3*cos(a +
b*x)**3/(3*b) + 7*c**2*d*sin(a + b*x)**3/(3*b**2) + 2*c**2*d*sin(a + b*x)*c
os(a + b*x)**2/b**2 + 14*c*d**2*x*sin(a + b*x)**3/(3*b**2) + 4*c*d**2*x*sin
```

```
(a + b*x)*cos(a + b*x)**2/b**2 + 7*d**3*x**2*sin(a + b*x)**3/(3*b**2) + 2*d
**3*x**2*sin(a + b*x)*cos(a + b*x)**2/b**2 + 14*c*d**2*sin(a + b*x)**2*cos(
a + b*x)/(3*b**3) + 40*c*d**2*cos(a + b*x)**3/(9*b**3) + 14*d**3*x*sin(a +
b*x)**2*cos(a + b*x)/(3*b**3) + 40*d**3*x*cos(a + b*x)**3/(9*b**3) - 122*d*
*3*sin(a + b*x)**3/(27*b**4) - 40*d**3*sin(a + b*x)*cos(a + b*x)**2/(9*b**4
), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a
)**3, True))
```

Giac [A] time = 1.1414, size = 312, normalized size = 1.78

$$\frac{\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+9\,b^3c^2dx+3\,b^3c^3-2\,bd^3x-2\,bcd^2\right)\cos\left(3\,bx+3\,a\right)}{36\,b^4}-\frac{3\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3-\right.}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3
*x - 2*b*c*d^2)*cos(3*b*x + 3*a)/b^4 - 3/4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 - 1/108*
(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a)/b^4
 + 9/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)/b^4
```

3.18 $\int (c + dx)^2 \sin^3(a + bx)\,dx$

Optimal. Leaf size=123

$$\frac{2d(c+dx)\sin^3(a+bx)}{9b^2} + \frac{4d(c+dx)\sin(a+bx)}{3b^2} - \frac{2d^2\cos^3(a+bx)}{27b^3} + \frac{14d^2\cos(a+bx)}{9b^3} - \frac{2(c+dx)^2\cos(a+bx)}{3b} - (c$$

```
[Out] (14*d^2*Cos[a + b*x])/(9*b^3) - (2*(c + d*x)^2*Cos[a + b*x])/(3*b) - (2*d^2
*Cos[a + b*x]^3)/(27*b^3) + (4*d*(c + d*x)*Sin[a + b*x])/(3*b^2) - ((c + d*
x)^2*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) + (2*d*(c + d*x)*Sin[a + b*x]^3)/(9
*b^2)
```

Rubi [A] time = 0.0960181, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 3296, 2638, 2633}

$$\frac{2d(c+dx)\sin^3(a+bx)}{9b^2} + \frac{4d(c+dx)\sin(a+bx)}{3b^2} - \frac{2d^2\cos^3(a+bx)}{27b^3} + \frac{14d^2\cos(a+bx)}{9b^3} - \frac{2(c+dx)^2\cos(a+bx)}{3b} - (c$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]^3,x]
```

```
[Out] (14*d^2*Cos[a + b*x])/(9*b^3) - (2*(c + d*x)^2*Cos[a + b*x])/(3*b) - (2*d^2
*Cos[a + b*x]^3)/(27*b^3) + (4*d*(c + d*x)*Sin[a + b*x])/(3*b^2) - ((c + d*
x)^2*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) + (2*d*(c + d*x)*Sin[a + b*x]^3)/(9
*b^2)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2 \sin^3(a+bx)\,dx &= -\frac{(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{3b} + \frac{2d(c+dx)\sin^3(a+bx)}{9b^2} + \frac{2}{3}\int (c+dx)^2\sin(a+bx) \\ &= -\frac{2(c+dx)^2\cos(a+bx)}{3b} - \frac{(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{3b} + \frac{2d(c+dx)\sin^3(a+bx)}{9b^2} + \\ &= \frac{2d^2\cos(a+bx)}{9b^3} - \frac{2(c+dx)^2\cos(a+bx)}{3b} - \frac{2d^2\cos^3(a+bx)}{27b^3} + \frac{4d(c+dx)\sin(a+bx)}{3b^2} - \\ &= \frac{14d^2\cos(a+bx)}{9b^3} - \frac{2(c+dx)^2\cos(a+bx)}{3b} - \frac{2d^2\cos^3(a+bx)}{27b^3} + \frac{4d(c+dx)\sin(a+bx)}{3b^2} - \end{aligned}$$

Mathematica [A] time = 0.405903, size = 86, normalized size = 0.7

$$\frac{-81\cos(a+bx)\left(b^2(c+dx)^2-2d^2\right)+\cos(3(a+bx))\left(9b^2(c+dx)^2-2d^2\right)-6bd(c+dx)(\sin(3(a+bx))-27\sin(a+bx))}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]^3,x]
```

```
[Out] (-81*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + (-2*d^2 + 9*b^2*(c + d*x)^2)
*Cos[3*(a + b*x)] - 6*b*d*(c + d*x)*(-27*Sin[a + b*x] + Sin[3*(a + b*x)]))/
(108*b^3)
```

Maple [B] time = 0.007, size = 265, normalized size = 2.2

$$\frac{1}{b}\left(\frac{d^2}{b^2}\left(-\frac{(bx+a)^2\left(2+(\sin(bx+a))^2\right)\cos(bx+a)}{3}+\frac{4\cos(bx+a)}{3}+\frac{(4bx+4a)\sin(bx+a)}{3}+\frac{(2bx+2a)(\sin(bx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^2*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/3*cos(b*x+a)+4
/3*(b*x+a)*sin(b*x+a)+2/9*(b*x+a)*sin(b*x+a)^3+2/27*(2+sin(b*x+a)^2)*cos(b*
x+a))-2/b^2*a*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin(b*x+a)^
3+2/3*sin(b*x+a))+2/b*c*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin
(b*x+a)^3+2/3*sin(b*x+a))-1/3/b^2*a^2*d^2*(2+sin(b*x+a)^2)*cos(b*x+a)+2/3/b
*a*c*d*(2+sin(b*x+a)^2)*cos(b*x+a)-1/3*c^2*(2+sin(b*x+a)^2)*cos(b*x+a))
```

Maxima [B] time = 1.07171, size = 365, normalized size = 2.97

$$\frac{36\left(\cos(bx+a)^3-3\cos(bx+a)\right)c^2-\frac{72\left(\cos(bx+a)^3-3\cos(bx+a)\right)acd}{b}+\frac{36\left(\cos(bx+a)^3-3\cos(bx+a)\right)a^2d^2}{b^2}+\frac{6(3(bx+a)\cos(3bx+3a)-27(}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/108*(36*(cos(b*x + a)^3 - 3*cos(b*x + a))*c^2 - 72*(cos(b*x + a)^3 - 3*co
s(b*x + a))*a*c*d/b + 36*(cos(b*x + a)^3 - 3*cos(b*x + a))*a^2*d^2/b^2 + 6*
(3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a
) + 27*sin(b*x + a))*c*d/b - 6*(3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)
*cos(b*x + a) - sin(3*b*x + 3*a) + 27*sin(b*x + a))*a*d^2/b^2 + ((9*(b*x +
a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a
)*sin(3*b*x + 3*a) + 162*(b*x + a)*sin(b*x + a))*d^2/b^2)/b
```

Fricas [A] time = 1.63428, size = 298, normalized size = 2.42

$$\frac{\left(9b^2d^2x^2+18b^2cdx+9b^2c^2-2d^2\right)\cos(bx+a)^3-3\left(9b^2d^2x^2+18b^2cdx+9b^2c^2-14d^2\right)\cos(bx+a)+6\left(7bd^2x+\right.}{27b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/27*((9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^3 - 3
*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 14*d^2)*cos(b*x + a) + 6*(7*b*
d^2*x + 7*b*c*d - (b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/b^3
```

Sympy [A] time = 2.5926, size = 284, normalized size = 2.31

$$\begin{cases} -\frac{c^2\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2c^2\cos^3(a+bx)}{3b} - \frac{2cdx\sin^2(a+bx)\cos(a+bx)}{b} - \frac{4cdx\cos^3(a+bx)}{3b} - \frac{d^2x^2\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2d^2x^2\cos^3(a+bx)}{3b} + \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**2*sin(a + b*x)**2*cos(a + b*x)/b - 2*c**2*cos(a + b*x)**3/(3
*b) - 2*c*d*x*sin(a + b*x)**2*cos(a + b*x)/b - 4*c*d*x*cos(a + b*x)**3/(3*b
) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)/b - 2*d**2*x**2*cos(a + b*x)**3/
(3*b) + 14*c*d*sin(a + b*x)**3/(9*b**2) + 4*c*d*sin(a + b*x)*cos(a + b*x)**
2/(3*b**2) + 14*d**2*x*sin(a + b*x)**3/(9*b**2) + 4*d**2*x*sin(a + b*x)*cos
(a + b*x)**2/(3*b**2) + 14*d**2*sin(a + b*x)**2*cos(a + b*x)/(9*b**3) + 40*
d**2*cos(a + b*x)**3/(27*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/
3)*sin(a)**3, True))
```

Giac [A] time = 1.13405, size = 185, normalized size = 1.5

$$\frac{\left(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 - 2\,d^2\right)\cos\left(3\,bx + 3\,a\right)}{108\,b^3} - \frac{3\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - 2\,d^2\right)\cos\left(bx + a\right)}{4\,b^3} - \frac{\left(bd^2x + bcd\right)\sin}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*x + 3*a)/b
^3 - 3/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)/b^3 - 1
/18*(b*d^2*x + b*c*d)*sin(3*b*x + 3*a)/b^3 + 3/2*(b*d^2*x + b*c*d)*sin(b*x
+ a)/b^3
```

3.19 $\int (c+dx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=75

$$\frac{d\sin^3(a+bx)}{9b^2}+\frac{2d\sin(a+bx)}{3b^2}-\frac{2(c+dx)\cos(a+bx)}{3b}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}$$

```
[Out] (-2*(c + d*x)*Cos[a + b*x])/(3*b) + (2*d*Sin[a + b*x])/(3*b^2) - ((c + d*x)
*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) + (d*Sin[a + b*x]^3)/(9*b^2)
```

Rubi [A] time = 0.0418427, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3310, 3296, 2637}

$$\frac{d\sin^3(a+bx)}{9b^2}+\frac{2d\sin(a+bx)}{3b^2}-\frac{2(c+dx)\cos(a+bx)}{3b}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]^3,x]
```

```
[Out] (-2*(c + d*x)*Cos[a + b*x])/(3*b) + (2*d*Sin[a + b*x])/(3*b^2) - ((c + d*x)
*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) + (d*Sin[a + b*x]^3)/(9*b^2)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)\sin^3(a+bx)\,dx = -\frac{(c+dx)\cos(a+bx)\sin^2(a+bx)}{3b}+\frac{d\sin^3(a+bx)}{9b^2}+\frac{2}{3}\int (c+dx)\sin(a+bx)\,dx$$
$$= -\frac{2(c+dx)\cos(a+bx)}{3b}-\frac{(c+dx)\cos(a+bx)\sin^2(a+bx)}{3b}+\frac{d\sin^3(a+bx)}{9b^2}+\frac{(2d)\int \cos(a}{3b}$$
$$= -\frac{2(c+dx)\cos(a+bx)}{3b}+\frac{2d\sin(a+bx)}{3b^2}-\frac{(c+dx)\cos(a+bx)\sin^2(a+bx)}{3b}+\frac{d\sin^3(a+bx}{9b^2}$$

Mathematica [A] time = 0.172696, size = 59, normalized size = 0.79

$$\frac{-27b(c+dx)\cos(a+bx)+3b(c+dx)\cos(3(a+bx))+d(27\sin(a+bx)-\sin(3(a+bx)))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x]^3,x]
```

```
[Out] (-27*b*(c + d*x)*Cos[a + b*x] + 3*b*(c + d*x)*Cos[3*(a + b*x)] + d*(27*Sin[
a + b*x] - Sin[3*(a + b*x)]))/(36*b^2)
```

Maple [A] time = 0.007, size = 95, normalized size = 1.3

$$\frac{1}{b}\left(\frac{d}{b}\left(-\frac{(bx+a)\left(2+(\sin(bx+a))^2\right)\cos(bx+a)}{3}+\frac{(\sin(bx+a))^3}{9}+\frac{2\sin(bx+a)}{3}\right)+\frac{da\left(2+(\sin(bx+a))^2\right)\cos(bx+}{3b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/9*sin(b*x+a)^3+2/3*s
in(b*x+a))+1/3/b*d*a*(2+sin(b*x+a)^2)*cos(b*x+a)-1/3*c*(2+sin(b*x+a)^2)*cos
(b*x+a))
```

Maxima [A] time = 1.05489, size = 140, normalized size = 1.87

$$\frac{12\left(\cos(bx+a)^3-3\cos(bx+a)\right)c-\frac{12\left(\cos(bx+a)^3-3\cos(bx+a)\right)ad}{b}+\frac{(3(bx+a)\cos(3bx+3a)-27(bx+a)\cos(bx+a)-\sin(3bx+3a)+27\sin(bx+}{b}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(12*(cos(b*x + a)^3 - 3*cos(b*x + a))*c - 12*(cos(b*x + a)^3 - 3*cos(b
*x + a))*a*d/b + (3*(b*x + a)*cos(3*b*x + 3*a) - 27*(b*x + a)*cos(b*x + a)
- sin(3*b*x + 3*a) + 27*sin(b*x + a))*d/b)/b
```

Fricas [A] time = 1.63869, size = 153, normalized size = 2.04

$$\frac{3\,(bdx+bc)\cos{(bx+a)}^{3}-9\,(bdx+bc)\cos{(bx+a)}-\left(d\cos{(bx+a)}^{2}-7\,d\right)\sin{(bx+a)}}{9\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b*d*x + b*c)*cos(b*x + a)^3 - 9*(b*d*x + b*c)*cos(b*x + a) - (d*cos
(b*x + a)^2 - 7*d)*sin(b*x + a))/b^2
```

Sympy [A] time = 1.11446, size = 126, normalized size = 1.68

$$\begin{cases} -\frac{c\sin^{2}{(a+bx)}\cos{(a+bx)}}{b}-\frac{2c\cos^{3}{(a+bx)}}{3b}-\frac{dx\sin^{2}{(a+bx)}\cos{(a+bx)}}{b}-\frac{2dx\cos^{3}{(a+bx)}}{3b}+\frac{7d\sin^{3}{(a+bx)}}{9b^{2}}+\frac{2d\sin{(a+bx)}\cos^{2}{(a+bx)}}{3b^{2}} & \text{for } b \neq \\ \left(cx+\frac{dx^{2}}{2}\right)\sin^{3}{(a)} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c*sin(a + b*x)**2*cos(a + b*x)/b - 2*c*cos(a + b*x)**3/(3*b) -
d*x*sin(a + b*x)**2*cos(a + b*x)/b - 2*d*x*cos(a + b*x)**3/(3*b) + 7*d*sin(
a + b*x)**3/(9*b**2) + 2*d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2), Ne(b, 0))
, ((c*x + d*x**2/2)*sin(a)**3, True))
```

Giac [A] time = 1.15119, size = 93, normalized size = 1.24

$$\frac{(bdx+bc)\cos{(3\,bx+3\,a)}}{12\,b^{2}}-\frac{3\,(bdx+bc)\cos{(bx+a)}}{4\,b^{2}}-\frac{d\sin{(3\,bx+3\,a)}}{36\,b^{2}}+\frac{3\,d\sin{(bx+a)}}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*(b*d*x + b*c)*cos(3*b*x + 3*a)/b^2 - 3/4*(b*d*x + b*c)*cos(b*x + a)/b^
2 - 1/36*d*sin(3*b*x + 3*a)/b^2 + 3/4*d*sin(b*x + a)/b^2
```

3.20 $\int \frac{\sin^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=121

$$-\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(\right.}{}$$

```
[Out] -(CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d) + (3*CosIntegr
al[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d) + (3*Cos[a - (b*c)/d]*SinIntegral
[(b*c)/d + b*x])/(4*d) - (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*
x])/(4*d)
```

Rubi [A] time = 0.245183, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3312, 3303, 3299, 3302}

$$-\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(c + d*x),x]
```

```
[Out] -(CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d) + (3*CosIntegr
al[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d) + (3*Cos[a - (b*c)/d]*SinIntegral
[(b*c)/d + b*x])/(4*d) - (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*
x])/(4*d)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{c+dx}\,dx = \int \left(\frac{3\sin(a+bx)}{4(c+dx)} - \frac{\sin(3a+3bx)}{4(c+dx)}\right)dx$$
$$= -\left(\frac{1}{4}\int \frac{\sin(3a+3bx)}{c+dx}\,dx\right) + \frac{3}{4}\int \frac{\sin(a+bx)}{c+dx}\,dx$$
$$= -\left(\frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx\right) + \frac{1}{4}\left(3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx - \frac{1}{4}\sin\left(3a-\right.$$
$$= -\frac{\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d} + \frac{3\mathrm{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d} + \frac{3\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{4d} - \frac{\cos\left(3a\right.}{}$$

Mathematica [A] time = 0.243051, size = 102, normalized size = 0.84

$$-\frac{\sin\left(3a-\frac{3bc}{d}\right)\mathrm{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) - 3\sin\left(a-\frac{bc}{d}\right)\mathrm{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right) - 3\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right) + \cos\left(3a\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(c + d*x),x]
```

```
[Out] -(CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] - 3*CosIntegral[b*(c/
d + x)]*Sin[a - (b*c)/d] - 3*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Co
s[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Maple [A] time = 0.009, size = 167, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{b}{12}\left(3\frac{1}{d}\mathrm{Si}\left(3\,bx+3\,a+3\,\frac{-da+cb}{d}\right)\cos\left(3\,\frac{-da+cb}{d}\right)-3\,\frac{1}{d}\mathrm{Ci}\left(3\,bx+3\,a+3\,\frac{-da+cb}{d}\right)\sin\left(3\,\frac{-da+cb}{d}\right)\right)+\frac{3\,b}{4}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/b*(-1/12*b*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b
*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)+3/4*b*(Si(b*x+a+(-a*d+b*c)/d)
*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d))
```

Maxima [C] time = 1.36547, size = 370, normalized size = 3.06

$$b\left(-3i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+3i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(i\,E_1\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)-i\,E_1\left(-\frac{3i\,bc+3i\,(bx+a)d-3i}{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/8*(b*(-3*I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 3*I*exp
_integral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b
*(I*exp_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_inte
gral_e(1, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d)
- 3*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(exp_inte
gral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1, -(3*
I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

Fricas [A] time = 1.6618, size = 406, normalized size = 3.36

$$\frac{3\left(\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)-\left(\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-2\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/8*(3*(cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*sin
(-(b*c - a*d)/d) - (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d*
x + b*c)/d))*sin(-3*(b*c - a*d)/d) - 2*cos(-3*(b*c - a*d)/d)*sin_integral(3
*(b*d*x + b*c)/d) + 6*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**3/(c + d*x), x)
```

Giac [C] time = 1.75939, size = 8500, normalized size = 70.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/8*(imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*imag_part(cos_integral(b*x + b*c/d))*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*imag_part(cos
_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*sin_integra
l((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 - 6*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*real_part(cos_integral(-b*x - b*c/d))*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_inte
gral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c
/d)^2 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*real_part(cos_integral(b*x + b*c/d))
*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*real_part(co
s_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*
```

```
b*c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*
b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_pa
rt(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^
2 + 3*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2 - 3*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a
)^2*tan(3/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)
^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 6*sin_integral((b*d*x + b*c)/d)*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 12*imag_part(cos_integral(b*x + b*
c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*imag_par
t(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d) - 24*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d) - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3
/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 3*imag_part(cos_integral(b*x + b*c/
d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*imag_part(cos_integral(-
b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_in
tegral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*si
n_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 -
6*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2
+ 4*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/
2*b*c/d)*tan(1/2*b*c/d)^2 - 4*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan
(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 8*sin_integral(3*(b*
d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + ima
g_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 + 3*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 - 3*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*
c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*
d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*sin_integr
al((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_p
art(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d)^2 - 3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)
^2*tan(1/2*b*c/d)^2 + 3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x
 + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*sin_integral(
(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*real_pa
rt(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)
+ 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d) - 6*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a
)*tan(3/2*b*c/d)^2 - 6*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*t
an(1/2*a)*tan(3/2*b*c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan
(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 6*real_part(cos_integra
```

```
l(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 6*real_part(cos_
integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*real_p
art(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
 + 6*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*ta
n(1/2*b*c/d) - 6*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d) - 6*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a
)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*real_part(cos_integral(b*x + b*c/d)
)*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 6*real_part(cos_integral(-b*x
- b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 2*real_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*real_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 +
 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan
(1/2*b*c/d)^2 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*ta
n(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*
tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-3*
b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(
cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d)^2 + 6*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*real_part(cos_integral(-b*x - b*c/d))*tan(1
/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(3*b*x + 3*
b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 + 3*imag_part(cos_integral(b*x + b*c/d))*
tan(3/2*a)^2*tan(1/2*a)^2 - 3*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2
*a)^2*tan(1/2*a)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2
*tan(1/2*a)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2
 + 6*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2 + 4*imag_part(
cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) - 4*i
mag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*
c/d) + 8*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*
c/d) + imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)
^2 - 3*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 +
 3*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - im
ag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 2*s
in_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - 6*sin_integr
al((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - imag_part(cos_integral(
3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 3*imag_part(cos_integral(
b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*imag_part(cos_integral(-b*x
 - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)
*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 6*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)
^2*tan(3/2*b*c/d)^2 - 12*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*
tan(1/2*a)*tan(1/2*b*c/d) + 12*imag_part(cos_integral(-b*x - b*c/d))*tan(3/
2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) - 24*sin_integral((b*d*x + b*c)/d)*tan(3/2
*a)^2*tan(1/2*a)*tan(1/2*b*c/d) - 12*imag_part(cos_integral(b*x + b*c/d))*t
an(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*imag_part(cos_integral(-b*x
```

```
- b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 24*sin_integral((b*d
*x + b*c)/d)*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - imag_part(cos_int
egral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + 3*imag_part(cos_int
egral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 3*imag_part(cos_integra
l(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(-3*
b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x + b
*c)/d)*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + 6*sin_integral((b*d*x + b*c)/d)*tan(
3/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/
2*a)^2*tan(1/2*b*c/d)^2 - 3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)
^2*tan(1/2*b*c/d)^2 + 3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*
tan(1/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*t
an(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*
c/d)^2 - 6*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*
imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*
b*c/d)^2 - 4*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b
*c/d)*tan(1/2*b*c/d)^2 + 8*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(3
/2*b*c/d)*tan(1/2*b*c/d)^2 - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*imag_part(cos_integral(b*x + b*c/d))*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*imag_part(cos_integral(-b*x - b*c/d))*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*c/d
)^2*tan(1/2*b*c/d)^2 - 6*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*
tan(1/2*a) - 6*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a
) + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2 + 2*
real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2 + 2*real_
part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d) + 2*real_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d) - 2*real_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d) - 2*real_part(
cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d) - 2*real_part(c
os_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2 - 2*real_part(cos
_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2 - 6*real_part(cos_
integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 - 6*real_part(cos_integr
al(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*real_part(cos_integral(b*
x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d) + 6*real_part(cos_integral(-b*x - b
*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d) - 6*real_part(cos_integral(b*x + b*c/d))
*tan(1/2*a)^2*tan(1/2*b*c/d) - 6*real_part(cos_integral(-b*x - b*c/d))*tan(
1/2*a)^2*tan(1/2*b*c/d) + 6*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*
c/d)^2*tan(1/2*b*c/d) + 6*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a
)*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)
*tan(1/2*b*c/d)^2 + 6*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1
/2*b*c/d)^2 + 6*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*
c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*
b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)*tan(1
```

```
/2*b*c/d)^2 - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2 - 3*ima
g_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2 + 3*imag_part(cos_integral(-
b*x - b*c/d))*tan(3/2*a)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(
3/2*a)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2 - 6*sin_integral(
(b*d*x + b*c)/d)*tan(3/2*a)^2 + imag_part(cos_integral(3*b*x + 3*b*c/d))*ta
n(1/2*a)^2 + 3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 - 3*imag_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - imag_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(1/2*a)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2
 + 6*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2 + 4*imag_part(cos_integral(
3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) - 4*imag_part(cos_integral(-3*b
*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) + 8*sin_integral(3*(b*d*x + b*c)/d
)*tan(3/2*a)*tan(3/2*b*c/d) - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(
3/2*b*c/d)^2 - 3*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*c/d)^2 + 3*
imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2 + imag_part(cos_inte
gral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x + b*c)/d
)*tan(3/2*b*c/d)^2 - 6*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*c/d)^2 - 12*
imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 12*imag_pa
rt(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - 24*sin_integral(
(b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d) + imag_part(cos_integral(3*b*x +
 3*b*c/d))*tan(1/2*b*c/d)^2 + 3*imag_part(cos_integral(b*x + b*c/d))*tan(1/
2*b*c/d)^2 - 3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - ima
g_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*c/d)^2 + 2*sin_integral(3*
(b*d*x + b*c)/d)*tan(1/2*b*c/d)^2 + 6*sin_integral((b*d*x + b*c)/d)*tan(1/2
*b*c/d)^2 + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a) + 2*real_
part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a) - 6*real_part(cos_integral(
b*x + b*c/d))*tan(1/2*a) - 6*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*
a) - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d) - 2*real_par
t(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d) + 6*real_part(cos_integral
(b*x + b*c/d))*tan(1/2*b*c/d) + 6*real_part(cos_integral(-b*x - b*c/d))*tan
(1/2*b*c/d) + imag_part(cos_integral(3*b*x + 3*b*c/d)) - 3*imag_part(cos_in
tegral(b*x + b*c/d)) + 3*imag_part(cos_integral(-b*x - b*c/d)) - imag_part(
cos_integral(-3*b*x - 3*b*c/d)) + 2*sin_integral(3*(b*d*x + b*c)/d) - 6*sin
_integral((b*d*x + b*c)/d))/(d*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + d*tan(3/2*
a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d)^2 + d*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a
)^2*tan(1/2*a)^2 + d*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + d*tan(1/2*a)^2*tan(3/2
*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)^2*tan(1/2*b*c/d)
^2 + d*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2 + d*tan(1/2*a)^2
+ d*tan(3/2*b*c/d)^2 + d*tan(1/2*b*c/d)^2 + d)
```

3.21 $\int \frac{\sin^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=145

$$\frac{3b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{3b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3}{-}$$

```
[Out] (3*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d^2) - (3*b*Cos[3*a -
(3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sin[a + b*x]^3/(d*(c +
 d*x)) - (3*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*S
in[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)
```

Rubi [A] time = 0.242383, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3313, 3303, 3299, 3302}

$$\frac{3b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{3b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3}{-}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(c + d*x)^2,x]
```


```
[Out] (3*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d^2) - (3*b*Cos[3*a -
(3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sin[a + b*x]^3/(d*(c +
 d*x)) - (3*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*S
in[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)
```


Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{(c+dx)^2}\,dx = -\frac{\sin^3(a+bx)}{d(c+dx)} + \frac{(3b)\int \left(\frac{\cos(a+bx)}{4(c+dx)} - \frac{\cos(3a+3bx)}{4(c+dx)}\right)dx}{d}$$

$$= -\frac{\sin^3(a+bx)}{d(c+dx)} + \frac{(3b)\int \frac{\cos(a+bx)}{c+dx}\,dx}{4d} - \frac{(3b)\int \frac{\cos(3a+3bx)}{c+dx}\,dx}{4d}$$

$$= -\frac{\sin^3(a+bx)}{d(c+dx)} - \frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d} + \frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{4d} + \frac{(3b\sin}{}$$

$$= \frac{3b\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{4d^2} - \frac{3b\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d^2} - \frac{\sin^3(a+bx)}{d(c+dx)} - \frac{3b\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}}{4d^2}$$

Mathematica [A] time = 1.05066, size = 175, normalized size = 1.21

$$\frac{3b(c+dx)\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right) - 3b(c+dx)\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) - 3b(c+dx)\sin\left(a-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] (3*b*(c + d*x)*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - 3*b*(c + d*x)*Co
s[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - 3*d*Cos[b*x]*Sin[a] + d
*Cos[3*b*x]*Sin[3*a] - 3*d*Cos[a]*Sin[b*x] + d*Cos[3*a]*Sin[3*b*x] - 3*b*(c
```

```
 + d*x)*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*b*(c + d*x)*Sin[3*a -
 (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d^2*(c + d*x))
```

Maple [A] time = 0.01, size = 240, normalized size = 1.7

$$\frac{1}{b}\left(-\frac{b^2}{12}\left(-3\frac{\sin(3bx+3a)}{((bx+a)d-da+cb)d}+3\frac{1}{d}\left(3\frac{1}{d}\mathrm{Si}\left(3bx+3a+3\frac{-da+cb}{d}\right)\sin\left(3\frac{-da+cb}{d}\right)+3\frac{1}{d}\mathrm{Ci}\left(3bx+3a+3\frac{-}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 1/b*(-1/12*b^2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d+3*(3*Si(3*b*x+3*a+3
*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(
-a*d+b*c)/d)/d)/d)+3/4*b^2*(-sin(b*x+a)/((b*x+a)*d-d*a+c*b)/d+(Si(b*x+a+(-a
*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/
d))
```

Maxima [C] time = 1.81778, size = 406, normalized size = 2.8

$$\frac{b^2\left(-3i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+3i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(i\,E_2\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)-i\,E_2\left(-\frac{3i\,bc+3i\,(bx+a)d-}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(b^2*(-3*I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 3*I*e
xp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) +
 b^2*(I*exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_
integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)
/d) - 3*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_int
egral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(
exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e
(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b*c
*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A] time = 1.99735, size = 595, normalized size = 4.1

$$\frac{6\,(bdx+bc)\sin\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-6\,(bdx+bc)\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)+3\left((bdx+bc)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\,\mathrm{C}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(6*(b*d*x + b*c)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d)
- 6*(b*d*x + b*c)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 3*((b
*d*x + b*c)*cos_integral((b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-(b*
d*x + b*c)/d))*cos(-(b*c - a*d)/d) - 3*((b*d*x + b*c)*cos_integral(3*(b*d*x
 + b*c)/d) + (b*d*x + b*c)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c -
a*d)/d) + 8*(d*cos(b*x + a)^2 - d)*sin(b*x + a))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**3/(c + d*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.22 $\int \frac{\sin^3(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=184

$$\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{3b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}+$$

```
[Out] (9*b^2*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8*d^3) - (3*b^
2*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^3) - (3*b*Cos[a + b*x]*
Sin[a + b*x]^2)/(2*d^2*(c + d*x)) - Sin[a + b*x]^3/(2*d*(c + d*x)^2) - (3*b
^2*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*Cos[3*a -
(3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rubi [A] time = 0.353643, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3314, 3303, 3299, 3302, 3312}

$$\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{3b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(c + d*x)^3,x]
```

```
[Out] (9*b^2*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8*d^3) - (3*b^
2*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^3) - (3*b*Cos[a + b*x]*
Sin[a + b*x]^2)/(2*d^2*(c + d*x)) - Sin[a + b*x]^3/(2*d*(c + d*x)^2) - (3*b
^2*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*Cos[3*a -
(3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{(c+dx)^3}\,dx = -\frac{3b\cos(a+bx)\sin^2(a+bx)}{2d^2(c+dx)} - \frac{\sin^3(a+bx)}{2d(c+dx)^2} + \frac{\left(3b^2\right)\int \frac{\sin(a+bx)}{c+dx}\,dx}{d^2} - \frac{\left(9b^2\right)\int \frac{\sin^3(a+bx)}{c+dx}\,dx}{2d^2}$$

$$= -\frac{3b\cos(a+bx)\sin^2(a+bx)}{2d^2(c+dx)} - \frac{\sin^3(a+bx)}{2d(c+dx)^2} - \frac{\left(9b^2\right)\int \left(\frac{3\sin(a+bx)}{4(c+dx)} - \frac{\sin(3a+3bx)}{4(c+dx)}\right)dx}{2d^2} + \frac{\left(3b^2\cos\left(a-\right.\right.}{}$$

$$= \frac{3b^2\operatorname{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3} - \frac{3b\cos(a+bx)\sin^2(a+bx)}{2d^2(c+dx)} - \frac{\sin^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}\right.}{d^3}$$

$$= \frac{3b^2\operatorname{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3} - \frac{3b\cos(a+bx)\sin^2(a+bx)}{2d^2(c+dx)} - \frac{\sin^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}\right.}{d^3}$$

$$= \frac{9b^2\operatorname{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{8d^3} - \frac{3b^2\operatorname{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{8d^3} - \frac{3b\cos(a+bx)\sin^2(a+bx)}{2d^2(c+dx)} - \frac{\sin}{2d}$$

Mathematica [A] time = 0.79574, size = 221, normalized size = 1.2

$$\frac{6b^2(c+dx)^2\left(3\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)-\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(c + d*x)^3,x]
```

```
[Out] (-6*d*Cos[b*x]*(b*(c + d*x)*Cos[a] + d*Sin[a]) + 2*d*Cos[3*b*x]*(3*b*(c + d
*x)*Cos[3*a] + d*Sin[3*a]) + 6*d*(-(d*Cos[a]) + b*(c + d*x)*Sin[a])*Sin[b*x
] + 2*d*(d*Cos[3*a] - 3*b*(c + d*x)*Sin[3*a])*Sin[3*b*x] + 6*b^2*(c + d*x)^
2*(3*CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] - CosIntegral[b*(c
/d + x)]*Sin[a - (b*c)/d] - Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*C
os[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]))/(16*d^3*(c + d*x)^2)
```

Maple [A] time = 0.01, size = 313, normalized size = 1.7

$$\frac{1}{b}\left(-\frac{b^3}{12}\left(-\frac{3\sin(3bx+3a)}{2\left((bx+a)d-da+cb\right)^2d}+\frac{3}{2d}\left(-3\frac{\cos(3bx+3a)}{((bx+a)d-da+cb)d}-3\frac{1}{d}\left(3\frac{1}{d}\text{Si}\left(3bx+3a+3\frac{-da+cb}{d}\right)\cos\left(3\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] 1/b*(-1/12*b^3*(-3/2*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)^2/d+3/2*(-3*cos(3*b
*x+3*a)/((b*x+a)*d-d*a+c*b)/d-3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d
+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d)+3/4*
b^3*(-1/2*sin(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-d*
a+c*b)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)
*sin((-a*d+b*c)/d)/d)/d)/d))
```

Maxima [C] time = 1.97265, size = 454, normalized size = 2.47

$$\frac{b^3\left(-3i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+3i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^3\left(i\,E_3\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)-i\,E_3\left(-\frac{3i\,bc+3i\,(bx+a)d-\right.\right.}{8\left(b^2c^2d-2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b^3*(-3*I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 3*I*e
xp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) +
 b^3*(I*exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_
integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)
/d) - 3*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_int
egral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*(
exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e
(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^2
*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
 + a))*b)
```

Fricas [B] time = 1.97958, size = 919, normalized size = 4.99

$$\frac{24\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3+18\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-6\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(24*(b*d^2*x + b*c*d)*cos(b*x + a)^3 + 18*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*cos(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 6*(b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*
c)/d) - 24*(b*d^2*x + b*c*d)*cos(b*x + a) + 8*(d^2*cos(b*x + a)^2 - d^2)*si
n(b*x + a) - 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x +
 b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x + b*c
)/d))*sin(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_in
tegral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integ
ral(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*
d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.23 $\int (c+dx)^3 \csc(a+bx)\,dx$

Optimal. Leaf size=185

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id(c+d$$

```
[Out] (-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b + ((3*I)*d*(c + d*x)^2*PolyLog[
2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))
])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*(c + d
*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x
))])/b^4 + ((6*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4
```

Rubi [A] time = 0.136988, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4183, 2531, 6609, 2282, 6589}

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x],x]
```


```
[Out] (-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b + ((3*I)*d*(c + d*x)^2*PolyLog[
2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))
])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*(c + d
*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x
))])/b^4 + ((6*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4
```


Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\csc(a+bx)\,dx = -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(3d)\int (c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b} + \frac{(3d)\int (c+dx)^2\log(\cdots}{b}$$

$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{3id(c+dx)^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2} - \cdots$$

$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{3id(c+dx)^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2} - \frac{6\cdots}{\cdots}$$

$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{3id(c+dx)^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2} - \frac{6\cdots}{\cdots}$$

$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{3id(c+dx)^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2} - \frac{6\cdots}{\cdots}$$

Mathematica [A] time = 0.457009, size = 221, normalized size = 1.19

$$\frac{3id\left(b^2(c+dx)^2\text{PolyLog}(2,-\cos(a+bx)-i\sin(a+bx))+2ibd(c+dx)\text{PolyLog}(3,-\cos(a+bx)-i\sin(a+bx))-2d^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x],x]
```

```
[Out] (-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] + (3*I)*d*(b^2*(
c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)
*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*PolyLog[4, -Cos[a + b*x
] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x] + I
*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*
x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]))/b^4
```

Maple [B] time = 0.095, size = 633, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a),x)
```

```
[Out] -6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+2/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)
))+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a))
)*x-6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x
^2+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-3/b*
c^2*d*ln(exp(I*(b*x+a))+1)*x-3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))-3/b^2*c^2
*d*ln(exp(I*(b*x+a))+1)*a+3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-3*I/b^2*
c^2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+3
*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+
3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))+1)-3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-1/b
^4*d^3*ln(exp(I*(b*x+a))+1)*a^3-6/b^3*c*d^2*a^2*arctanh(exp(I*(b*x+a)))+6/b
^2*c^2*d*a*arctanh(exp(I*(b*x+a)))+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d
^3*ln(1-exp(I*(b*x+a)))*a^3-2/b*c^3*arctanh(exp(I*(b*x+a)))+6*I/b^2*c*d^2*p
olylog(2,-exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+6*I*d
^3*polylog(4,exp(I*(b*x+a)))/b^4-6*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4
```

Maxima [B] time = 1.5116, size = 953, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^3*log(cot(b*x + a) + csc(b*x + a)) - 6*a*c^2*d*log(cot(b*x + a) +
 csc(b*x + a))/b + 6*a^2*c*d^2*log(cot(b*x + a) + csc(b*x + a))/b^2 - 2*a^3
*d^3*log(cot(b*x + a) + csc(b*x + a))/b^3 + (12*I*d^3*polylog(4, -e^(I*b*x
+ I*a)) - 12*I*d^3*polylog(4, e^(I*b*x + I*a)) + (2*I*(b*x + a)^3*d^3 + (6*
I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*
a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (2*I*(b*x + a
)^3*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b
*c*d^2 + 6*I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) +
 (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-1
2*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*dilog(-e^(I*b*x + I*a)) + (6*I*b^2*c^2
*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 1
2*I*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I*a)) + ((b*x + a)^3*d^3 + 3*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*l
og(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((b*x + a)^3*d^3
 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*
(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 12*(
b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -e^(I*b*x + I*a)) - 12*(b*c*d^2
 + (b*x + a)*d^3 - a*d^3)*polylog(3, e^(I*b*x + I*a)))/b^3)/b
```

Fricas [C] time = 2.1118, size = 2056, normalized size = 11.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4,
 cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(
b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^2*
d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x +
 a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(cos(b*x +
a) - I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)
*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x
```

```
 + 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a)
+ 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*
x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
+ a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
+ a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 6*(b*d^3*x + b*c*d^2)*
polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3
, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*
x + a) + I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) -
 I*sin(b*x + a)))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \csc (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*csc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \csc (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a), x)
```

3.24 $\int (c+dx)^2 \csc(a+bx)\,dx$

Optimal. Leaf size=123

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{PolyLog}\left(3,e^{i(\ldots}\right.}{b^3}$$

```
[Out] (-2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b + ((2*I)*d*(c + d*x)*PolyLog[2,
 -E^(I*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b
^2 - (2*d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, E^(I*(a +
 b*x))])/b^3
```

Rubi [A] time = 0.0882635, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4183, 2531, 2282, 6589}

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{PolyLog}\left(3,e^{i(\ldots}\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x],x]
```


```
[Out] (-2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b + ((2*I)*d*(c + d*x)*PolyLog[2,
 -E^(I*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b
^2 - (2*d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, E^(I*(a +
 b*x))])/b^3
```


Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \csc(a+bx)\,dx = -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(2d)\int (c+dx)\log\left(1-e^{i(a+bx)}\right)dx}{b} + \frac{(2d)\int (c+dx)\log\left(1+\right.}{b}$$

$$= -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\mathrm{Li}_2\left(e^{i(a+bx)}\right)}{b^2} - \frac{\left(2id^2\right.}{}$$

$$= -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\mathrm{Li}_2\left(e^{i(a+bx)}\right)}{b^2} - \frac{\left(2d^2\right.}{}$$

$$= -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\mathrm{Li}_2\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d^2\mathrm{L}}{}$$

Mathematica [A] time = 0.319925, size = 148, normalized size = 1.2

$$\frac{\frac{2id\left(b(c+dx)\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)+id\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)\right)}{b^2}+\frac{2d\left(d\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)-ib(c+dx)\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)\right)}{b^2}+(c+dx)^2\log\left(1-e^{i(a+bx)}\right)-}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x],x]
```

```
[Out] ((c + d*x)^2*Log[1 - E^(I*(a + b*x))] - (c + d*x)^2*Log[1 + E^(I*(a + b*x))
] + ((2*I)*d*(b*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + I*d*PolyLog[3, -E^
(I*(a + b*x))]))/b^2 + (2*d*((-I)*b*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] +
```

```
d*PolyLog[3, E^(I*(a + b*x))]))/b^2)/b
```

Maple [B] time = 0.048, size = 361, normalized size = 2.9

$$2\,\frac{d^2\mathrm{polylog}\left(3,\mathrm{e}^{i(bx+a)}\right)}{b^3}-2\,\frac{d^2\mathrm{polylog}\left(3,-\mathrm{e}^{i(bx+a)}\right)}{b^3}-2\,\frac{c^2\mathrm{Artanh}\left(\mathrm{e}^{i(bx+a)}\right)}{b}-2\,\frac{a^2d^2\mathrm{Artanh}\left(\mathrm{e}^{i(bx+a)}\right)}{b^3}-\frac{2\,icd\mathrm{polylog}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a),x)
```

```
[Out] 2*d^2*polylog(3,exp(I*(b*x+a)))/b^3-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3-2/
b*c^2*arctanh(exp(I*(b*x+a)))-2/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))-2*I/b^2
*c*d*polylog(2,exp(I*(b*x+a)))+4/b^2*c*d*a*arctanh(exp(I*(b*x+a)))+2*I/b^2*
d^2*polylog(2,-exp(I*(b*x+a)))*x+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2
*ln(1-exp(I*(b*x+a)))*a^2+2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-1/b*d^2*ln
(exp(I*(b*x+a))+1)*x^2+1/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2-2*I/b^2*d^2*polyl
og(2,exp(I*(b*x+a)))*x-2/b*c*d*ln(exp(I*(b*x+a))+1)*x-2/b^2*c*d*ln(exp(I*(b
*x+a))+1)*a+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a
```

Maxima [B] time = 1.32668, size = 529, normalized size = 4.3

$$-\frac{2\,c^2\log\left(\cot\left(bx+a\right)+\csc\left(bx+a\right)\right)-\frac{4\,acd\log(\cot(bx+a)+\csc(bx+a))}{b}+\frac{2\,a^2d^2\log(\cot(bx+a)+\csc(bx+a))}{b^2}+\frac{4\,d^2\mathrm{Li}_3(-e^{(i\,bx+i\,a)})-4\,d^2\mathrm{Li}_3(e}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^2*log(cot(b*x + a) + csc(b*x + a)) - 4*a*c*d*log(cot(b*x + a) + c
sc(b*x + a))/b + 2*a^2*d^2*log(cot(b*x + a) + csc(b*x + a))/b^2 + (4*d^2*po
lylog(3, -e^(I*b*x + I*a)) - 4*d^2*polylog(3, e^(I*b*x + I*a)) + (2*I*(b*x
+ a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), cos(b
*x + a) + 1) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*ar
ctan2(sin(b*x + a), -cos(b*x + a) + 1) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 +
4*I*a*d^2)*dilog(-e^(I*b*x + I*a)) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a
*d^2)*dilog(e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x +
a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((b*x + a)^
2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 -
```

```
2*cos(b*x + a) + 1))/b^2)/b
```

Fricas [C] time = 1.9378, size = 1330, normalized size = 10.81

$$\frac{2\,d^2\mathrm{polylog}\left(3,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+2\,d^2\mathrm{polylog}\left(3,\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)-2\,d^2\mathrm{polylog}\left(3,-\cos\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) - 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a)) - 2*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*
I*b*c*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*d
ilog(cos(b*x + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(-cos
(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-cos(b*x + a)
 - I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a)
 + I*sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x
+ a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b
*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-
1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x +
 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^
2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) +
 1))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2 \csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \csc(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a), x)
```

3.25 $\int (c + dx) \csc(a + bx) \, dx$

Optimal. Leaf size=67

$$\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

```
[Out] (-2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, -E^(I*(a + b*x)
)])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2
```

Rubi [A] time = 0.0394014, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4183, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x],x]
```

```
[Out] (-2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, -E^(I*(a + b*x)
)])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)\csc(a+bx)\,dx = -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\int \log\left(1-e^{i(a+bx)}\right)dx}{b} + \frac{d\int \log\left(1+e^{i(a+bx)}\right)dx}{b}$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(id)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}dx, x, e^{i(a+bx)}\right)}{b^2} - \frac{(id)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\right.}{b^2}$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{id\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{id\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}$$

Mathematica [A] time = 0.0751643, size = 134, normalized size = 2.

$$\frac{d\left(i\left(\text{PolyLog}\left(2,-e^{i(a+bx)}\right)-\text{PolyLog}\left(2,e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)-a\log\left(\tan\left(\frac{1}{2}(a+\right.\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x],x]
```

```
[Out] -((c*Log[Cos[a/2 + (b*x)/2]])/b) + (c*Log[Sin[a/2 + (b*x)/2]])/b + (d*((a +
 b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) - a*Log[Tan[(a
+ b*x)/2]] + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))])
))/b^2
```

Maple [B] time = 0.032, size = 151, normalized size = 2.3

$$-2\,\frac{c\text{Artanh}\left(e^{i(bx+a)}\right)}{b} + \frac{d\ln\left(1-e^{i(bx+a)}\right)x}{b} + \frac{d\ln\left(1-e^{i(bx+a)}\right)a}{b^2} - \frac{id\text{polylog}\left(2,e^{i(bx+a)}\right)}{b^2} - \frac{d\ln\left(e^{i(bx+a)}+1\right)x}{b} - \frac{d\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a),x)
```

```
[Out] -2/b*c*arctanh(exp(I*(b*x+a)))+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-ex
p(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b*d*ln(exp(I*(b*x+a))+1
)*x-1/b^2*d*ln(exp(I*(b*x+a))+1)*a+I*d*polylog(2,-exp(I*(b*x+a)))/b^2+2/b^2
*d*a*arctanh(exp(I*(b*x+a)))
```

Maxima [B] time = 1.32574, size = 235, normalized size = 3.51

$$-\frac{2i\,bdx\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right)-2i\,bc\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)-1\right)+\left(2i\,bdx+2i\,bc\right)\arctan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2(
sin(b*x + a), cos(b*x + a) - 1) + (2*I*b*d*x + 2*I*b*c)*arctan2(sin(b*x + a
), cos(b*x + a) + 1) - 2*I*d*dilog(-e^(I*b*x + I*a)) + 2*I*d*dilog(e^(I*b*x
 + I*a)) + (b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x +
a) + 1) - (b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a
) + 1))/b^2
```

Fricas [B] time = 1.86689, size = 721, normalized size = 10.76

$$\frac{-i\,d\mathrm{Li}_2\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+i\,d\mathrm{Li}_2\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)-i\,d\mathrm{Li}_2\left(-\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+i\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(cos(b*x + a) - I
*sin(b*x + a)) - I*d*dilog(-cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(-cos
(b*x + a) - I*sin(b*x + a)) - (b*d*x + b*c)*log(cos(b*x + a) + I*sin(b*x +
a) + 1) - (b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b*c - a*d
)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*log(-1/2*
cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x + a*d)*log(-cos(b*x + a)
+ I*sin(b*x + a) + 1) + (b*d*x + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) +
1))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c+dx\right)\csc\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a), x)
```

3.26 $\int \frac{\csc(a+bx)}{c+dx} dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\csc(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Csc[a + b*x]/(c + d*x), x]
```

Rubi [A] time = 0.0221008, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Csc[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 6.01612, size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[Csc[a + b*x]/(c + d*x), x]
```

Maple [A] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{\csc\left(bx+a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(bx+a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc\left(bx+a\right)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/(d*x + c), x)
```

3.27 $\int \frac{\csc(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Csc[a + b*x]/(c + d*x)^2, x]
```

Rubi [A] time = 0.0214692, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Csc[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)}{(c+dx)^2} dx = \int \frac{\csc(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 6.95177, size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[Csc[a + b*x]/(c + d*x)^2, x]
```

Maple [A] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\csc(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/(d*x + c)^2, x)
```

3.28 $\int (c+dx)^3 \csc^2(a+bx)\,dx$

Optimal. Leaf size=113

$$-\frac{3id^2(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{3d^3\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^4}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\cot(a+bx)}{b}$$

```
[Out] ((-I)*(c + d*x)^3)/b - ((c + d*x)^3*Cot[a + b*x])/b + (3*d*(c + d*x)^2*Log[
1 - E^((2*I)*(a + b*x))])/b^2 - ((3*I)*d^2*(c + d*x)*PolyLog[2, E^((2*I)*(a
 + b*x))])/b^3 + (3*d^3*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^4)
```

Rubi [A] time = 0.213008, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4184, 3717, 2190, 2531, 2282, 6589}

$$-\frac{3id^2(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{3d^3\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^4}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\cot(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^3)/b - ((c + d*x)^3*Cot[a + b*x])/b + (3*d*(c + d*x)^2*Log[
1 - E^((2*I)*(a + b*x))])/b^2 - ((3*I)*d^2*(c + d*x)*PolyLog[2, E^((2*I)*(a
 + b*x))])/b^3 + (3*d^3*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^4)
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\csc^2(a+bx)\,dx = -\frac{(c+dx)^3\cot(a+bx)}{b} + \frac{(3d)\int (c+dx)^2\cot(a+bx)\,dx}{b}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^3\cot(a+bx)}{b} - \frac{(6id)\int \frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^3\cot(a+bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{\left(6d^2\right)\int (c+dx)\log}{b^2}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^3\cot(a+bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\text{Li}_2\left(e^{2i(}\right.}{b^3}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^3\cot(a+bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\text{Li}_2\left(e^{2i(}\right.}{b^3}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^3\cot(a+bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\text{Li}_2\left(e^{2i(}\right.}{b^3}$$

Mathematica [B] time = 6.90861, size = 478, normalized size = 4.23

$$-\frac{3cd^2\csc(a)\sec(a)\left(\frac{\tan(a)\left(i\text{PolyLog}\left(2,e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)+ibx\left(2\tan^{-1}(\tan(a))-\pi\right)-2\left(\tan^{-1}(\tan(a))+bx\right)\log\left(1-e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)+2\tan^{-1}(\tan}{\sqrt{\tan^2(a)+1}}\right.}{b^3\sqrt{\sec^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2,x]
```

```
[Out] -(d^3*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a)
)*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1
+ E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a +
 b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*
I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a + b*x)
)]))/E^((2*I)*a)))/(2*b^4) + (3*c^2*d*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*
Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^2*(Cos[a]^2 + Sin[a]^2)) + (Csc[a]*Cs
c[a + b*x]*(c^3*Sin[b*x] + 3*c^2*d*x*Sin[b*x] + 3*c*d^2*x^2*Sin[b*x] + d^3*
x^3*Sin[b*x]))/b - (3*c*d^2*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + (
(I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + Arc
Tan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] +
 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b
*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^3*Sqrt[Sec[a]^2*(Co
```

```
s[a]^2 + Sin[a]^2)])
```

Maple [B] time = 0.085, size = 541, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2,x)
```

```
[Out] 3*d/b^2*c^2*ln(exp(I*(b*x+a))-1)+3*d/b^2*c^2*ln(exp(I*(b*x+a))+1)-6*d/b^2*c
^2*ln(exp(I*(b*x+a)))-6*d^3/b^4*a^2*ln(exp(I*(b*x+a)))+3*d^3/b^4*a^2*ln(exp
(I*(b*x+a))-1)+3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2+3*d^3/b^2*ln(1-exp(I*(b*x
+a)))*x^2-3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2-2*I*d^3/b*x^3+4*I*d^3/b^4*a^3-
2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a))-1)+6*d^3/b^4*po
lylog(3,exp(I*(b*x+a)))+6*d^3/b^4*polylog(3,-exp(I*(b*x+a)))-6*I*d^2/b^3*c*
polylog(2,-exp(I*(b*x+a)))-6*I*d^2/b*c*x^2-6*I*d^2/b^3*c*a^2-6*I*d^3/b^3*po
lylog(2,-exp(I*(b*x+a)))*x-6*I*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x-6*I*d^2/
b^3*c*polylog(2,exp(I*(b*x+a)))+6*I*d^3/b^3*a^2*x-6*d^2/b^3*c*a*ln(exp(I*(b
*x+a))-1)+6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x+6*d^2/b^2*c*ln(1-exp(I*(b*x+a)
))*x+6*d^2/b^3*c*ln(1-exp(I*(b*x+a)))*a-12*I*d^2/b^2*c*a*x+12*d^2/b^3*c*a*l
n(exp(I*(b*x+a)))
```

Maxima [B] time = 1.60182, size = 2228, normalized size = 19.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(3*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*
log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(2*b*x + 2*
a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + si
n(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*c^2*d/((
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) - 6*((
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b
*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + si
n(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a
```

```
)^2 - 2*cos(b*x + a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^2/((cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^2) + 3*((cos(2
*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x +
a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b
*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 -
 2*cos(b*x + a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^3/((cos(2*b*x +
2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^3) - 2*c^3/tan(b*x
+ a) + 6*a*c^2*d/(b*tan(b*x + a)) - 6*a^2*c*d^2/(b^2*tan(b*x + a)) + 2*a^3*
d^3/(b^3*tan(b*x + a)) - 2*((6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x
+ a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a)
 - (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(2*b*x
+ 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (6*(b*x + a)^2*d^3 + 12*(
b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
+ a))*cos(2*b*x + 2*a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^
3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) +
4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(2*b*x + 2*a) - (1
2*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d
^3)*cos(2*b*x + 2*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*si
n(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - (12*b*c*d^2 + 12*(b*x + a)*d^3 -
12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (-12*I*b
*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(2*b*x + 2*a))*dilog(e^(I*b*x
+ I*a)) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a) + (-3*
I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a)
+ 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log
(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (3*I*(b*x + a)^2*d
^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*
c*d^2 + 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*
c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 - 2*cos(b*x + a) + 1) - (-12*I*d^3*cos(2*b*x + 2*a) + 12*d^3*sin(2*b*x
 + 2*a) + 12*I*d^3)*polylog(3, -e^(I*b*x + I*a)) - (-12*I*d^3*cos(2*b*x + 2
*a) + 12*d^3*sin(2*b*x + 2*a) + 12*I*d^3)*polylog(3, e^(I*b*x + I*a)) - (-4
*I*(b*x + a)^3*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2)*sin(2*b*x +
2*a))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a) + 2*I*b^3))/b
```

Fricas [C] time = 2.04648, size = 1756, normalized size = 15.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*d^3*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3*p
olylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -c
os(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a)
 - I*sin(b*x + a))*sin(b*x + a) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(cos(b*
x + a) + I*sin(b*x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(c
os(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*di
log(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + (-6*I*b*d^3*x - 6*I*b*c*
d^2)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 +
2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x +
 a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) - I*sin(
b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2
*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a
*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b
*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(
b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*
x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x
+ a) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x
+ a))/(b^4*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^3 \csc^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*csc(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \csc(bx+a)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^2, x)
```

3.29 $\int (c+dx)^2 \csc^2(a+bx)\,dx$

Optimal. Leaf size=83

$$-\frac{id^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{i(c+dx)^2}{b}$$

```
[Out] ((-I)*(c + d*x)^2)/b - ((c + d*x)^2*Cot[a + b*x])/b + (2*d*(c + d*x)*Log[1
- E^((2*I)*(a + b*x))])/b^2 - (I*d^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^3
```

Rubi [A] time = 0.135961, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4184, 3717, 2190, 2279, 2391}

$$-\frac{id^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{i(c+dx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2,x]
```


```
[Out] ((-I)*(c + d*x)^2)/b - ((c + d*x)^2*Cot[a + b*x])/b + (2*d*(c + d*x)*Log[1
- E^((2*I)*(a + b*x))])/b^2 - (I*d^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^3
```


Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^2\csc^2(a+bx)\,dx = -\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{(2d)\int (c+dx)\cot(a+bx)\,dx}{b}$$
$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(4id)\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx}{b}$$
$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{\left(2d^2\right)\int \log\left(1-e^{2i(\cdots}\right.}{b^2}$$
$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}+\frac{\left(id^2\right)\text{Subst}\left(\int \frac{\log(\cdots}{x}\right.}{b^3}$$
$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{id^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}$$

Mathematica [B] time = 4.82682, size = 181, normalized size = 2.18

$$\frac{\csc(a)\left(d^2\left(-\sin(a)\left(i\text{PolyLog}\left(2,e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)-ibx\left(\pi-2\tan^{-1}(\tan(a))\right)-2\left(\tan^{-1}(\tan(a))+bx\right)\log\left(1-e^{2i(\cdots}\right.\right.\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2,x]
```

```
[Out] (Csc[a]*(-2*b*c*d*(b*x*Cos[a] - Log[Sin[a + b*x]]*Sin[a]) + d^2*(-(b^2*E^(I
*ArcTan[Tan[a]])*x^2*Cos[a]*Sqrt[Sec[a]^2]) - ((-I)*b*x*(Pi - 2*ArcTan[Tan[
a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*
```

```
I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b
*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Sin
[a]) + b^2*(c + d*x)^2*Csc[a + b*x]*Sin[b*x]))/b^3
```

Maple [B] time = 0.043, size = 276, normalized size = 3.3

$$\frac{-2\,i\left(d^2x^2+2\,cdx+c^2\right)}{b\left(e^{2\,i(bx+a)}-1\right)}-4\,\frac{cd\ln\left(e^{i(bx+a)}\right)}{b^2}+2\,\frac{cd\ln\left(e^{i(bx+a)}-1\right)}{b^2}+2\,\frac{cd\ln\left(e^{i(bx+a)}+1\right)}{b^2}-\frac{2\,id^2x^2}{b}-\frac{4\,id^2ax}{b^2}-\frac{2\,id^2a^2}{b^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2,x)
```

```
[Out] -2*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))-1)-4*d/b^2*c*ln(exp(I*(b*x+a
)))+2*d/b^2*c*ln(exp(I*(b*x+a))-1)+2*d/b^2*c*ln(exp(I*(b*x+a))+1)-2*I*d^2/b
*x^2-4*I*d^2/b^2*a*x-2*I*d^2/b^3*a^2+2*d^2/b^2*ln(1-exp(I*(b*x+a)))*x+2*d^2
/b^3*ln(1-exp(I*(b*x+a)))*a-2*I*d^2/b^3*polylog(2,exp(I*(b*x+a)))+2*d^2/b^2
*ln(exp(I*(b*x+a))+1)*x-2*I*d^2/b^3*polylog(2,-exp(I*(b*x+a)))+4*d^2/b^3*a*
ln(exp(I*(b*x+a)))-2*d^2/b^3*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] time = 1.46171, size = 749, normalized size = 9.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*b^2*c^2 + (2*b*d^2*x + 2*b*c*d - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a) -
 (2*I*b*d^2*x + 2*I*b*c*d)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x
+ a) + 1) - (2*b*c*d*cos(2*b*x + 2*a) + 2*I*b*c*d*sin(2*b*x + 2*a) - 2*b*c*
d)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d^2*x*cos(2*b*x + 2*a) +
2*I*b*d^2*x*sin(2*b*x + 2*a) - 2*b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x +
a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x)*cos(2*b*x + 2*a) + (2*d^2*cos(2*b*x
 + 2*a) + 2*I*d^2*sin(2*b*x + 2*a) - 2*d^2)*dilog(-e^(I*b*x + I*a)) + (2*d^
2*cos(2*b*x + 2*a) + 2*I*d^2*sin(2*b*x + 2*a) - 2*d^2)*dilog(e^(I*b*x + I*a
)) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^
2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(
b*x + a) + 1) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2
```

```
*a) + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)
^2 - 2*cos(b*x + a) + 1) - (-2*I*b^2*d^2*x^2 - 4*I*b^2*c*d*x)*sin(2*b*x + 2
*a))/(-I*b^3*cos(2*b*x + 2*a) + b^3*sin(2*b*x + 2*a) + I*b^3)
```

Fricas [B] time = 1.79532, size = 1026, normalized size = 12.36

$$\frac{-i\,d^2\mathrm{Li}_2\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\sin\left(bx+a\right)+i\,d^2\mathrm{Li}_2\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)\sin\left(bx+a\right)+i\,d^2\mathrm{Li}_2\left(-\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (-I*d^2*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + I*d^2*dilog(cos
(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + I*d^2*dilog(-cos(b*x + a) + I*si
n(b*x + a))*sin(b*x + a) - I*d^2*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(
b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x
 + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x +
a) + (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(
b*x + a) + (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2
)*sin(b*x + a) + (b*d^2*x + a*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*
sin(b*x + a) + (b*d^2*x + a*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*si
n(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a))/(b^3*sin(b
*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2\csc^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \csc (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2, x)
```

3.30 $\int (c+dx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=29

$$\frac{d\log(\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}$$

```
[Out] -(((c + d*x)*Cot[a + b*x])/b) + (d*Log[Sin[a + b*x]])/b^2
```

Rubi [A] time = 0.0276419, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4184, 3475}

$$\frac{d\log(\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2,x]
```

```
[Out] -(((c + d*x)*Cot[a + b*x])/b) + (d*Log[Sin[a + b*x]])/b^2
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\csc^2(a+bx)\,dx &= -\frac{(c+dx)\cot(a+bx)}{b}+\frac{d\int \cot(a+bx)\,dx}{b}\\ &= -\frac{(c+dx)\cot(a+bx)}{b}+\frac{d\log(\sin(a+bx))}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0835127, size = 52, normalized size = 1.79

$$\frac{d\log(\sin(a+bx))}{b^2}-\frac{c\cot(a+bx)}{b}-\frac{dx\cot(a)}{b}+\frac{dx\csc(a)\sin(bx)\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*x*Cot[a])/b) - (c*Cot[a + b*x])/b + (d*Log[Sin[a + b*x]])/b^2 + (d*x*C
sc[a]*Csc[a + b*x]*Sin[b*x])/b
```

Maple [A] time = 0.007, size = 39, normalized size = 1.3

$$-\frac{d\cot(bx+a)\,x}{b}+\frac{d\ln(\sin(bx+a))}{b^2}-\frac{c\cot(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2,x)
```

```
[Out] -1/b*d*cot(b*x+a)*x+d*ln(sin(b*x+a))/b^2-1/b*c*cot(b*x+a)
```

Maxima [B] time = 1.00505, size = 293, normalized size = 10.1

$$\frac{\frac{\left(\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2+2\cos(bx+a)+1\right)+\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1\right)\log\left(c\cdots\right.}{\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1\right)b}\cdots}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(
b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*d/((cos(2*
b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) - 2*c/tan(b*
x + a) + 2*a*d/(b*tan(b*x + a)))/b
```

Fricas [A] time = 1.70285, size = 119, normalized size = 4.1

$$\frac{d\log\left(\frac{1}{2}\sin\left(bx+a\right)\right)\sin\left(bx+a\right)-\left(bdx+bc\right)\cos\left(bx+a\right)}{b^2\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (d*log(1/2*sin(b*x + a))*sin(b*x + a) - (b*d*x + b*c)*cos(b*x + a))/(b^2*si
n(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c+dx\right)\csc^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)**2, x)
```

Giac [B] time = 2.02759, size = 1689, normalized size = 58.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 -
b*d*x*tan(1/2*b*x)^2 - 4*b*d*x*tan(1/2*b*x)*tan(1/2*a) + d*log(16*(tan(1/2*
a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*
tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) -
2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b
*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1
```

```
/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan
(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)
^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*
b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)^2*tan(1/2*a) - b*d*x*tan(1/2*
a)^2 + d*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2
*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan
(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*t
an(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*
x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*
a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*ta
n(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + ta
n(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)*tan(
1/2*a)^2 - b*c*tan(1/2*b*x)^2 - 4*b*c*tan(1/2*b*x)*tan(1/2*a) - b*c*tan(1/2
*a)^2 + b*d*x - d*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^
8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^
4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2
*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*t
an(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3
*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*
b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*
a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*
b*x) - d*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2
*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan
(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*t
an(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*
x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*
a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*ta
n(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + ta
n(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*a) + b*c)
/(b^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*tan(1/2*b*x)*tan(1/2*a)^2 - b^2*tan(1
/2*b*x) - b^2*tan(1/2*a))
```

3.31 $\int \frac{\csc^2(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\csc^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Csc[a + b*x]^2/(c + d*x), x]
```

Rubi [A] time = 0.0402203, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^2(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Csc[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)}{c+dx} \, dx = \int \frac{\csc^2(a+bx)}{c+dx} \, dx$$

Mathematica [A] time = 6.26745, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Csc[a + b*x]^2/(c + d*x), x]
```

Maple [A] time = 0.178, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^2/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\csc(bx+a)^2}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/(d*x + c), x)
```

3.32 $\int \frac{\csc^2(a+bx)}{(c+dx)^2} \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\csc^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Csc[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [A] time = 0.0374365, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[a + b*x]^2/(c + d*x)^2,x]

[Out] Defer[Int][Csc[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2} \, dx = \int \frac{\csc^2(a+bx)}{(c+dx)^2} \, dx$$

Mathematica [A] time = 6.35265, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[a + b*x]^2/(c + d*x)^2,x]

[Out] Integrate[Csc[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A] time = 0.339, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/(d*x + c)^2, x)
```

3.33 $\int (c+dx)^3 \csc^3(a+bx)\,dx$

Optimal. Leaf size=309

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-3id(c+$$

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^3 - ((c + d*x)^3*ArcTanh[E^(I
*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot
[a + b*x]*Csc[a + b*x])/(2*b) + ((3*I)*d^3*PolyLog[2, -E^(I*(a + b*x))])/b^
4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d^3
*PolyLog[2, E^(I*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^(
I*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (
3*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((3*I)*d^3*PolyLog[4, -E
^(I*(a + b*x))])/b^4 + ((3*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4
```

Rubi [A] time = 0.226242, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4186, 4183, 2279, 2391, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-3id(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^3,x]
```

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^3 - ((c + d*x)^3*ArcTanh[E^(I
*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot
[a + b*x]*Csc[a + b*x])/(2*b) + ((3*I)*d^3*PolyLog[2, -E^(I*(a + b*x))])/b^
4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d^3
*PolyLog[2, E^(I*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^(
I*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (
3*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((3*I)*d^3*PolyLog[4, -E
^(I*(a + b*x))])/b^4 + ((3*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
```

```
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \csc^3(a+bx)\,dx = -\frac{3d(c+dx)^2\csc(a+bx)}{2b^2} - \frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b} + \frac{1}{2}\int (c+dx)^3\csc(a+bx)$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\csc(a+bx)}{2b^2} -$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\csc(a+bx)}{2b^2} -$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\csc(a+bx)}{2b^2} -$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\csc(a+bx)}{2b^2} -$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\csc(a+bx)}{2b^2} -$$

Mathematica [A] time = 5.10294, size = 528, normalized size = 1.71

$$-\frac{-3id\left(b^2(c+dx)^2+2d^2\right)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)+3id\left(b^2(c+dx)^2+2d^2\right)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)+6bcd^2\operatorname{PolyLog}\left(3,-e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3,x]
```

```
[Out] -(b^2*(c + d*x)^2*(3*d + b*(c + d*x)*Cot[a + b*x])*Csc[a + b*x] - b^3*c^3*L
og[1 - E^(I*(a + b*x))] - 6*b*c*d^2*Log[1 - E^(I*(a + b*x))] - 3*b^3*c^2*d*
x*Log[1 - E^(I*(a + b*x))] - 6*b*d^3*x*Log[1 - E^(I*(a + b*x))] - 3*b^3*c*d
^2*x^2*Log[1 - E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 - E^(I*(a + b*x))] + b^
3*c^3*Log[1 + E^(I*(a + b*x))] + 6*b*c*d^2*Log[1 + E^(I*(a + b*x))] + 3*b^3
*c^2*d*x*Log[1 + E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 + E^(I*(a + b*x))] + 3*
b^3*c*d^2*x^2*Log[1 + E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 + E^(I*(a + b*x)
```

```
)] - (3*I)*d*(2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b*x))] + (3*I)
*d*(2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(I*(a + b*x))] + 6*b*c*d^2*PolyLo
g[3, -E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, -E^(I*(a + b*x))] - 6*b*c*d^2
*PolyLog[3, E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, E^(I*(a + b*x))] + (6*I
)*d^3*PolyLog[4, -E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, E^(I*(a + b*x))])
/(2*b^4)
```

Maple [B] time = 0.125, size = 1056, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^3,x)
```

```
[Out] -3/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+1/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)
))+3/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+3/b^3*d^3*polylog(3,exp(I*(b*x+a))
)*x-3/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+3/2/b*c*d^2*ln(1-exp(I*(b*x+a)))
*x^2+3/2/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/2/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*
a-3/2/b*c^2*d*ln(exp(I*(b*x+a))+1)*x-3/2/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))
-3/2/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a-1/2/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+3/
2/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))+1)-3/2/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-1
/2/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^3-3/b^3*c*d^2*a^2*arctanh(exp(I*(b*x+a)))
+3/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))+1/2/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1
/2/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^3*x^3*b
*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a
))+d^3*x^3*b*exp(I*(b*x+a))+c^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*exp(I*(b*x
+a))-3*I*d^3*x^2*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(I*(b*x+a))-6*I*c*d^2*x*ex
p(3*I*(b*x+a))+c^3*b*exp(I*(b*x+a))-3*I*c^2*d*exp(3*I*(b*x+a))+3*I*d^3*x^2*
exp(I*(b*x+a))+6*I*c*d^2*x*exp(I*(b*x+a))+3*I*c^2*d*exp(I*(b*x+a)))-3*I/b^2
*polylog(2,exp(I*(b*x+a)))*c*d^2*x+3*I/b^2*polylog(2,-exp(I*(b*x+a)))*c*d^2
*x+3/2*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3/2*I/b^2*d^3*polylog(2,exp
(I*(b*x+a)))*x^2-3/2*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+3/2*I/b^2*c^2*d*
polylog(2,-exp(I*(b*x+a)))-1/b*c^3*arctanh(exp(I*(b*x+a)))-3/b^3*d^3*ln(exp
(I*(b*x+a))+1)*x-3/b^4*d^3*ln(exp(I*(b*x+a))+1)*a+3/b^3*d^3*ln(1-exp(I*(b*x
+a)))*x+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a-6/b^3*c*d^2*arctanh(exp(I*(b*x+a))
)+6/b^4*d^3*a*arctanh(exp(I*(b*x+a)))+3*I*d^3*polylog(2,-exp(I*(b*x+a)))/b^
4+3*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b
^4-3*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4
```

Maxima [B] time = 6.51747, size = 5234, normalized size = 16.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(c^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(cos(b*x + a) + 1) + log
(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(
cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*cos(b*x + a)/
(cos(b*x + a)^2 - 1) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b^2 -
 a^3*d^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(cos(b*x + a) + 1) + log
(cos(b*x + a) - 1))/b^3 - 4*((2*(b*x + a)^3*d^3 + 12*b*c*d^2 - 12*a*d^3 + 6
*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3
)*(b*x + a) + 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos(
4*b*x + 4*a) - 4*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos
(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 - 12*I*a*d^3 + (6*I*b*c
*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6*I*a^2
+ 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^3*d^3 - 24*I*b*c
*d^2 + 24*I*a*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c
^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 - 24*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a)
)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (12*b*c*d^2 - 12*a*d^3 + 12*(b*
c*d^2 - a*d^3)*cos(4*b*x + 4*a) - 24*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) - (
-12*I*b*c*d^2 + 12*I*a*d^3)*sin(4*b*x + 4*a) - (24*I*b*c*d^2 - 24*I*a*d^3)*
sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*(b*x + a)^3*
d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 +
 2)*d^3)*(b*x + a) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) -
4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b
*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3
+ (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 +
 (6*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^3*d^3
+ (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*
d^2 + (-12*I*a^2 - 24*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x
+ a), -cos(b*x + a) + 1) + (4*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*a*b*c*d
^2 + 12*a^2*d^3 - 12*(-I*b*c*d^2 + (I*a - 1)*d^3)*(b*x + a)^2 + (12*I*b^2*c
^2*d - 24*(I*a - 1)*b*c*d^2 + (12*I*a^2 - 24*a)*d^3)*(b*x + a))*cos(3*b*x +
 3*a) + (4*I*(b*x + a)^3*d^3 - 12*b^2*c^2*d + 24*a*b*c*d^2 - 12*a^2*d^3 - 1
2*(-I*b*c*d^2 + (I*a + 1)*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d - 24*(I*a + 1)
*b*c*d^2 + (12*I*a^2 + 24*a)*d^3)*(b*x + a))*cos(b*x + a) - (6*b^2*c^2*d -
12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*(a^2 + 2)*d^3 + 12*(b*c*d^2 - a*d^3)*(
```

```
b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2
*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 12*(b^2*c^2*d - 2*a*b*c*d^
2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*
b*x + 2*a) - (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 + (-6*I
*a^2 - 12*I)*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a)
 - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + (12*I*a^2 + 24
*I)*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e
^(I*b*x + I*a)) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*(a^2
+ 2)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b
*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x +
4*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b
*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d
^2 + 6*I*(b*x + a)^2*d^3 + (6*I*a^2 + 12*I)*d^3 + (12*I*b*c*d^2 - 12*I*a*d^
3)*(b*x + a))*sin(4*b*x + 4*a) + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(
b*x + a)^2*d^3 + (-12*I*a^2 - 24*I)*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x
 + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x + a)^3*d^3 - 6*I
*b*c*d^2 + 6*I*a*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c
^2*d + 6*I*a*b*c*d^2 + (-3*I*a^2 - 6*I)*d^3)*(b*x + a) + (-I*(b*x + a)^3*d^
3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*
I*b^2*c^2*d + 6*I*a*b*c*d^2 + (-3*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(4*b*x +
4*a) + (2*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 - 12*I*a*d^3 + (6*I*b*c*d^2 - 6*
I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6*I*a^2 + 12*I)*d
^3)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 +
3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^
3)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 +
 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d
^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*co
s(b*x + a) + 1) + (I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (3*I*b*c*d
^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 + 6
*I)*d^3)*(b*x + a) + (I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (3*I*b*
c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2
+ 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^3*d^3 - 12*I*b*c*
d^2 + 12*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d
 + 12*I*a*b*c*d^2 + (-6*I*a^2 - 12*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((
b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*
(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(
(b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3
*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log
(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (12*d^3*cos(4*b*x
+ 4*a) - 24*d^3*cos(2*b*x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a) - 24*I*d^3*sin
(2*b*x + 2*a) + 12*d^3)*polylog(4, -e^(I*b*x + I*a)) - (12*d^3*cos(4*b*x +
4*a) - 24*d^3*cos(2*b*x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a) - 24*I*d^3*sin(2
*b*x + 2*a) + 12*d^3)*polylog(4, e^(I*b*x + I*a)) + (-12*I*b*c*d^2 - 12*I*(
b*x + a)*d^3 + 12*I*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^
3)*cos(4*b*x + 4*a) + (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*a*d^3)*cos(
```

```
2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 24*(
b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I
*a)) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*b*c*d^2 + 12
*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(4*b*x + 4*a) + (-24*I*b*c*d^2 - 24*I*(b*
x + a)*d^3 + 24*I*a*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a
*d^3)*sin(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2
*a))*polylog(3, e^(I*b*x + I*a)) - (4*(b*x + a)^3*d^3 - 12*I*b^2*c^2*d + 24
*I*a*b*c*d^2 - 12*I*a^2*d^3 + (12*b*c*d^2 - (12*a + 12*I)*d^3)*(b*x + a)^2
+ (12*b^2*c^2*d - (24*a + 24*I)*b*c*d^2 + 12*(a^2 + 2*I*a)*d^3)*(b*x + a))*
sin(3*b*x + 3*a) - (4*(b*x + a)^3*d^3 + 12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 1
2*I*a^2*d^3 + (12*b*c*d^2 - (12*a - 12*I)*d^3)*(b*x + a)^2 + (12*b^2*c^2*d
- (24*a - 24*I)*b*c*d^2 + 12*(a^2 - 2*I*a)*d^3)*(b*x + a))*sin(b*x + a))/(-
4*I*b^3*cos(4*b*x + 4*a) + 8*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(4*b*x + 4*a
) - 8*b^3*sin(2*b*x + 2*a) - 4*I*b^3))/b
```

Fricas [C] time = 2.68417, size = 4091, normalized size = 13.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x +
a) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 6*I*d^3 + (-3*I*b
^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^2)*dil
og(cos(b*x + a) + I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3
*I*b^2*c^2*d - 6*I*d^3 + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d
 + 6*I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (3*I*b^2
*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 6*I*d^3 + (-3*I*b^2*d^3*x^2 -
6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x +
 a) + I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d
 - 6*I*d^3 + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 6*I*d^3)*
cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^
3 + 6*b*c*d^2 + 3*(b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d +
2*b*d^3)*x)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 + b^3*c^3 + 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 +
 6*b*c*d^2 + 3*(b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d + 2*b
*d^3)*x)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*(a^2 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2
 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/
```

```
2*I*sin(b*x + a) + 1/2) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 2)*b*c*d^2 -
(a^3 + 6*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 2)*b*c*d^2 - (a^3 + 6
*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)
- (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 6
*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
(a^3 + 6*a)*d^3 + 3*(b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d
+ 2*b*d^3)*x)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 6*a)*d^3 - (b^3*d^3*x^
3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 6*a)*d^3 + 3*(
b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d + 2*b*d^3)*x)*log(-co
s(b*x + a) - I*sin(b*x + a) + 1) + (6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*polyl
og(4, cos(b*x + a) + I*sin(b*x + a)) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*
polylog(4, cos(b*x + a) - I*sin(b*x + a)) + (6*I*d^3*cos(b*x + a)^2 - 6*I*d
^3)*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + (-6*I*d^3*cos(b*x + a)^2 +
 6*I*d^3)*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2
 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, cos(b*x + a) + I*sin(b*x
+ a)) - 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(
3, cos(b*x + a) - I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d
^2)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x
 + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) -
 I*sin(b*x + a)) + 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(b*x + a)
)/(b^4*cos(b*x + a)^2 - b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \csc^3(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*csc(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \csc(bx + a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^3, x)
```

3.34 $\int (c+dx)^2 \csc^3(a+bx)\,dx$

Optimal. Leaf size=180

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{d^2\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}$$

```
[Out] -(((c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b) - (d^2*ArcTanh[Cos[a + b*x]])/b
^3 - (d*(c + d*x)*Csc[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]*Csc[a + b*x
])/(2*b) + (I*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*(c + d*x
)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3
 + (d^2*PolyLog[3, E^(I*(a + b*x))])/b^3
```

Rubi [A] time = 0.136007, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4186, 3770, 4183, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{d^2\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3,x]
```


```
[Out] -(((c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b) - (d^2*ArcTanh[Cos[a + b*x]])/b
^3 - (d*(c + d*x)*Csc[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]*Csc[a + b*x
])/(2*b) + (I*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*(c + d*x
)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3
 + (d^2*PolyLog[3, E^(I*(a + b*x))])/b^3
```


Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \csc^3(a+bx)\,dx = -\frac{d(c+dx)\csc(a+bx)}{b^2} - \frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b} + \frac{1}{2}\int (c+dx)^2\csc(a+bx)\,dx$$
$$= -\frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc(a+bx)}{b^2} - \frac{(c+dx)^2}{}$$
$$= -\frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc(a+bx)}{b^2} - \frac{(c+dx)^2}{}$$
$$= -\frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc(a+bx)}{b^2} - \frac{(c+dx)^2}{}$$
$$= -\frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc(a+bx)}{b^2} - \frac{(c+dx)^2}{}$$

Mathematica [B] time = 7.43912, size = 471, normalized size = 2.62

$$\frac{2ibd(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right) - 2ibd(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right) - 2d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right) + 2d^2\text{PolyLog}\left(3,e^{i(a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3,x]
```

```
[Out] -((d*(c + d*x)*Csc[a])/b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/2
]^2)/(8*b) + (b^2*c^2*Log[1 - E^(I*(a + b*x))] + 2*d^2*Log[1 - E^(I*(a + b*
x))] + 2*b^2*c*d*x*Log[1 - E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 - E^(I*(a +
 b*x))] - b^2*c^2*Log[1 + E^(I*(a + b*x))] - 2*d^2*Log[1 + E^(I*(a + b*x))]
 - 2*b^2*c*d*x*Log[1 + E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + E^(I*(a + b*x
))] + (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x
)*PolyLog[2, E^(I*(a + b*x))] - 2*d^2*PolyLog[3, -E^(I*(a + b*x))] + 2*d^2*
PolyLog[3, E^(I*(a + b*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a/2 +
 (b*x)/2]^2)/(8*b) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*Sin[(b*x)/2]) - d^
2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c*d*Sin[(b*x)/2]
 + d^2*x*Sin[(b*x)/2]))/(2*b^2)
```

Maple [B] time = 0.077, size = 548, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^3,x)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^2*x^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I*
(b*x+a))+c^2*b*exp(3*I*(b*x+a))+d^2*x^2*b*exp(I*(b*x+a))+2*c*d*x*b*exp(I*(b
*x+a))-2*I*d^2*x*exp(3*I*(b*x+a))+c^2*b*exp(I*(b*x+a))-2*I*d*c*exp(3*I*(b*x
+a))+2*I*d^2*x*exp(I*(b*x+a))+2*I*d*c*exp(I*(b*x+a)))+d^2*polylog(3,exp(I*(
b*x+a)))/b^3-d^2*polylog(3,-exp(I*(b*x+a)))/b^3-2/b^3*d^2*arctanh(exp(I*(b*
x+a)))-1/b*c^2*arctanh(exp(I*(b*x+a)))-1/b^3*d^2*a^2*arctanh(exp(I*(b*x+a))
)-1/2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/2/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2
+2/b^2*c*d*a*arctanh(exp(I*(b*x+a)))-I/b^2*polylog(2,exp(I*(b*x+a)))*d^2*x+
I/b^2*polylog(2,-exp(I*(b*x+a)))*d^2*x+1/2/b*d^2*ln(1-exp(I*(b*x+a)))*x^2+I
/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-1/2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2-I/b^
2*c*d*polylog(2,exp(I*(b*x+a)))+1/b*c*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*c*d*ln
(1-exp(I*(b*x+a)))*a-1/b*c*d*ln(exp(I*(b*x+a))+1)*x-1/b^2*c*d*ln(exp(I*(b*x
+a))+1)*a
```

Maxima [B] time = 2.34856, size = 2611, normalized size = 14.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(c^2*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(cos(b*x + a) + 1) + log
(cos(b*x + a) - 1)) - 2*a*c*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(co
s(b*x + a) + 1) + log(cos(b*x + a) - 1))/b + a^2*d^2*(2*cos(b*x + a)/(cos(b
*x + a)^2 - 1) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b^2 - 4*((2
*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + 4*d^2 + 2*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) - 4*((b*x + a)^2*d
^2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) + (2*I*(b*x + a)
^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(4*b*x + 4*a) + (-
4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) - 8*I*d^2)*sin(2*b
*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (4*d^2*cos(4*b*x + 4*a
) - 8*d^2*cos(2*b*x + 2*a) + 4*I*d^2*sin(4*b*x + 4*a) - 8*I*d^2*sin(2*b*x +
 2*a) + 4*d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*(b*x + a)^2*d^2
 + 4*(b*c*d - a*d^2)*(b*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*
x + a))*cos(4*b*x + 4*a) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)
)*cos(2*b*x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x +
a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*
x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (4*I*(
b*x + a)^2*d^2 + 8*b*c*d - 8*a*d^2 - 8*(-I*b*c*d + (I*a - 1)*d^2)*(b*x + a)
```

```
)*cos(3*b*x + 3*a) + (4*I*(b*x + a)^2*d^2 - 8*b*c*d + 8*a*d^2 - 8*(-I*b*c*d
 + (I*a + 1)*d^2)*(b*x + a))*cos(b*x + a) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*
a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 8*(b*c*d + (b*
x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*
I*a*d^2)*sin(4*b*x + 4*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin
(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d
^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 8*(b*c*d + (b*x +
 a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*
d^2)*sin(4*b*x + 4*a) + (-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*sin(2*
b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*
I*a*d^2)*(b*x + a) - 2*I*d^2 + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^
2)*(b*x + a) - 2*I*d^2)*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*
d - 4*I*a*d^2)*(b*x + a) + 4*I*d^2)*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2
*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 +
 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d -
2*I*a*d^2)*(b*x + a) + 2*I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^
2)*(b*x + a) + 2*I*d^2)*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*
c*d + 4*I*a*d^2)*(b*x + a) - 4*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 +
 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (-4*I*d^2*cos(4*b*x + 4*a) + 8*I
*d^2*cos(2*b*x + 2*a) + 4*d^2*sin(4*b*x + 4*a) - 8*d^2*sin(2*b*x + 2*a) - 4
*I*d^2)*polylog(3, -e^(I*b*x + I*a)) + (4*I*d^2*cos(4*b*x + 4*a) - 8*I*d^2*
cos(2*b*x + 2*a) - 4*d^2*sin(4*b*x + 4*a) + 8*d^2*sin(2*b*x + 2*a) + 4*I*d^
2)*polylog(3, e^(I*b*x + I*a)) - (4*(b*x + a)^2*d^2 - 8*I*b*c*d + 8*I*a*d^2
 + (8*b*c*d - (8*a + 8*I)*d^2)*(b*x + a))*sin(3*b*x + 3*a) - (4*(b*x + a)^2
*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (8*b*c*d - (8*a - 8*I)*d^2)*(b*x + a))*sin(b
*x + a))/(-4*I*b^2*cos(4*b*x + 4*a) + 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(
4*b*x + 4*a) - 8*b^2*sin(2*b*x + 2*a) - 4*I*b^2))/b
```

Fricas [C] time = 2.25953, size = 2379, normalized size = 13.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a) + (2*I*b*d^2*x +
2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) +
 I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d + (2*I*b*d^2*x + 2*I*b*c*d)*co
```

```
s(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c
*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin
(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x
+ a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*d^2)*cos(b*x + a)^2 +
2*d^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*d^2)*cos(b*x + a)^2 +
2*d^2)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2
 + 2)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2)*log(-1/2*
cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)
*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b
*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*
d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a
)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x +
 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*co
s(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 2*(d^2*cos(b*x + a)
^2 - d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*(d^2*cos(b*x + a)^2
 - d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 2*(d^2*cos(b*x + a)^2 -
 d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 2*(d^2*cos(b*x + a)^2 -
d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 4*(b*d^2*x + b*c*d)*sin(b
*x + a))/(b^3*cos(b*x + a)^2 - b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2 \csc^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^3, x)
```

3.35 $\int (c+dx)\csc^3(a+bx)\,dx$

Optimal. Leaf size=109

$$\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)\cot(a+bx)\,\text{cs}}{2b}$$

```
[Out] -(((c + d*x)*ArcTanh[E^(I*(a + b*x))])/b) - (d*Csc[a + b*x])/(2*b^2) - ((c
+ d*x)*Cot[a + b*x]*Csc[a + b*x])/(2*b) + ((I/2)*d*PolyLog[2, -E^(I*(a + b*
x))])/b^2 - ((I/2)*d*PolyLog[2, E^(I*(a + b*x))])/b^2
```

Rubi [A] time = 0.0666484, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4185, 4183, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)\cot(a+bx)\,\text{cs}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^3,x]
```

```
[Out] -(((c + d*x)*ArcTanh[E^(I*(a + b*x))])/b) - (d*Csc[a + b*x])/(2*b^2) - ((c
+ d*x)*Cot[a + b*x]*Csc[a + b*x])/(2*b) + ((I/2)*d*PolyLog[2, -E^(I*(a + b*
x))])/b^2 - ((I/2)*d*PolyLog[2, E^(I*(a + b*x))])/b^2
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)\csc^3(a+bx)\,dx = -\frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b} + \frac{1}{2}\int (c+dx)\csc(a+bx)\,dx$$
$$= -\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b} - \frac{d\int\log\left(1-\right.}{2}$$
$$= -\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b} + \frac{(id)\,\mathrm{Subst}}{}$$
$$= -\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b} + \frac{id\,\mathrm{Li}_2\left(-e^{i(a}\right.}{2b^2}$$

Mathematica [B] time = 1.82193, size = 292, normalized size = 2.68

$$\frac{d\left(i\left(\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)-\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)-a\log\left(\tan\left(\frac{1}{2}(a+b\right.\right.\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^3,x]
```

```
[Out] -(d*x*Csc[a/2 + (b*x)/2]^2)/(8*b) - (c*Csc[(a + b*x)/2]^2)/(8*b) - (c*Log[C
os[(a + b*x)/2]])/(2*b) + (c*Log[Sin[(a + b*x)/2]])/(2*b) + (d*((a + b*x)*(
Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) - a*Log[Tan[(a + b*x)/
2]] + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))])))/(2*b
^2) + (d*x*Sec[a/2 + (b*x)/2]^2)/(8*b) + (c*Sec[(a + b*x)/2]^2)/(8*b) + (d*
Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x)/2])/(4*b^2) - (d*Sec[a/2]*Sec[a/2 + (
b*x)/2]*Sin[(b*x)/2])/(4*b^2)
```

Maple [B] time = 0.054, size = 246, normalized size = 2.3

$$\frac{dxb\mathrm{e}^{3i(bx+a)}+cb\mathrm{e}^{3i(bx+a)}+dxb\mathrm{e}^{i(bx+a)}+cb\mathrm{e}^{i(bx+a)}-id\mathrm{e}^{3i(bx+a)}+id\mathrm{e}^{i(bx+a)}}{b^2\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}-\frac{c\mathrm{Artanh}\left(\mathrm{e}^{i(bx+a)}\right)}{b}+\frac{d\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x}{2b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3,x)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2*(d*x*b*exp(3*I*(b*x+a))+c*b*exp(3*I*(b*x+a))+d
*x*b*exp(I*(b*x+a))+c*b*exp(I*(b*x+a))-I*d*exp(3*I*(b*x+a))+I*d*exp(I*(b*x+
a)))-1/b*c*arctanh(exp(I*(b*x+a)))+1/2/b*d*ln(1-exp(I*(b*x+a)))*x+1/2/b^2*d
*ln(1-exp(I*(b*x+a)))*a-1/2*I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/2/b*d*ln(ex
p(I*(b*x+a))+1)*x-1/2/b^2*d*ln(exp(I*(b*x+a))+1)*a+1/2*I*d*polylog(2,-exp(I
*(b*x+a)))/b^2+1/b^2*d*a*arctanh(exp(I*(b*x+a)))
```

Maxima [B] time = 1.64639, size = 1044, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -((2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) - 4*(b*d*x + b*c)*cos
(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4*a) + (-4*I*b*d*x - 4*I*
b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*b*c*cos
(4*b*x + 4*a) - 4*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin(4*b*x + 4*a) - 4*I*b*c
*sin(2*b*x + 2*a) + 2*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d
*x*cos(4*b*x + 4*a) - 4*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*sin(4*b*x + 4*a)
 - 4*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a
) + 1) + (4*I*b*d*x + 4*I*b*c + 4*d)*cos(3*b*x + 3*a) + (4*I*b*d*x + 4*I*b*
c - 4*d)*cos(b*x + a) - (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I*
d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(-e^(I*b*x + I*a))
+ (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a) - 4
*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (
-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*cos(2*b*x + 2*a)
 + (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(c
os(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b*d*x + I*b*c + (
I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*cos(2*b*x + 2*a)
 - (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(c
```

```
os(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (4*b*d*x + 4*b*c - 4
*I*d)*sin(3*b*x + 3*a) - (4*b*d*x + 4*b*c + 4*I*d)*sin(b*x + a))/(-4*I*b^2*
cos(4*b*x + 4*a) + 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(4*b*x + 4*a) - 8*b^
2*sin(2*b*x + 2*a) - 4*I*b^2)
```

Fricas [B] time = 1.98429, size = 1191, normalized size = 10.93

$$\frac{2\,(bdx + bc)\cos{(bx + a)} + \left(-i\,d\cos{(bx + a)}^2 + i\,d\right)\mathrm{Li}_2\left(\cos{(bx + a)} + i\,\sin{(bx + a)}\right) + \left(i\,d\cos{(bx + a)}^2 - i\,d\right)\mathrm{Li}_2\left(\cos(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*d*x + b*c)*cos(b*x + a) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(cos(b
*x + a) + I*sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(cos(b*x + a) -
 I*sin(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(-cos(b*x + a) + I*sin(
b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)
) + (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) + I*sin(b
*x + a) + 1) + (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a
) - I*sin(b*x + a) + 1) + ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2
*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + ((b*c - a*d)*cos(b*x + a)^2 - b
*c + a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b*d*x - (b*d
*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (
b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) - I*sin(b*x +
 a) + 1) + 2*d*sin(b*x + a))/(b^2*cos(b*x + a)^2 - b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)\csc^3{(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \csc (bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)*csc(b*x + a)^3, x)
```

3.36 $\int \frac{\csc^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\csc^3(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Csc[a + b*x]^3/(c + d*x), x]
```

Rubi [A] time = 0.0390921, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[a + b*x]^3/(c + d*x),x]

[Out] Defer[Int][Csc[a + b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)}{c+dx}\,dx=\int \frac{\csc^3(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 31.3571, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[a + b*x]^3/(c + d*x),x]

[Out] Integrate[Csc[a + b*x]^3/(c + d*x), x]
```

Maple [A] time = 2.163, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx + a))^3}{dx + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^3/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] (((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - d*sin(3*b*x
 + 3*a) + d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x + b*c)
*cos(2*b*x + 2*a) - 2*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x + b*
c)*cos(b*x + a) + d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*cos(b*x
+ a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 -
4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) +
 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b
*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*d^2)*si
n(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) +
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x +
2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b
```

```
^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*
a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x +
2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*d^2)*sin(b*x +
 a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 - 2*(b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) + (d*cos(
3*b*x + 3*a) - d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) + (b*d*x + b
*c)*sin(b*x + a))*sin(4*b*x + 4*a) + (2*d*cos(2*b*x + 2*a) - 2*(b*d*x + b*c
)*sin(2*b*x + 2*a) - d)*sin(3*b*x + 3*a) + 2*(d*cos(b*x + a) - (b*d*x + b*c
)*sin(b*x + a))*sin(2*b*x + 2*a) + d*sin(b*x + a))/(b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^3}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^3/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**3/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/(d*x + c), x)
```

3.37 $\int \frac{\csc^3(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\csc^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Csc[a + b*x]^3/(c + d*x)^2, x]
```

Rubi [A] time = 0.0376024, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Csc[a + b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2} dx = \int \frac{\csc^3(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 34.7999, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] Integrate[Csc[a + b*x]^3/(c + d*x)^2, x]
```

Maple [A] time = 3.402, size = 0, normalized size = 0.

$$\int \frac{(\csc (bx + a))^3}{(dx + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - 2*d*sin(3*b
*x + 3*a) + 2*d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x +
b*c)*cos(2*b*x + 2*a) - 4*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x
+ b*c)*cos(b*x + a) + 2*d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*co
s(b*x + a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 +
 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*
a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x
+ 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 + 6*d^2)*sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x
^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d
^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*s
in(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^
2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
```

```
^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^
3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4
*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)
)*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 6*d^2)*sin(b*x + a)/
(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^
4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^
2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2
+ 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 - 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^
3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) + (2*d*c
os(3*b*x + 3*a) - 2*d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) + (b*d*
x + b*c)*sin(b*x + a))*sin(4*b*x + 4*a) + 2*(2*d*cos(2*b*x + 2*a) - (b*d*x
+ b*c)*sin(2*b*x + 2*a) - d)*sin(3*b*x + 3*a) + 2*(2*d*cos(b*x + a) - (b*d*
x + b*c)*sin(b*x + a))*sin(2*b*x + 2*a) + 2*d*sin(b*x + a))/(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a
) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x +
 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b
^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*c
os(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^
3)*cos(2*b*x + 2*a))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^3}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**3/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/(d*x + c)^2, x)
```

3.38 $\int (c+dx)^{5/2}\sin(a+bx)\,dx$

Optimal. Leaf size=195

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{4b^3}+5$$

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(4*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x]
)/b - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]])/(4*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqr
t[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(7/2)) + (5*
d*(c + d*x)^(3/2)*Sin[a + b*x])/(2*b^2)
```

Rubi [A] time = 0.434107, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{4b^3}+5$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(4*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x]
)/b - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]])/(4*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqr
t[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(7/2)) + (5*
d*(c + d*x)^(3/2)*Sin[a + b*x])/(2*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
```

```
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\sin(a+bx)\,dx = -\frac{(c+dx)^{5/2}\cos(a+bx)}{b} + \frac{(5d)\int (c+dx)^{3/2}\cos(a+bx)\,dx}{2b}$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{b} + \frac{5d(c+dx)^{3/2}\sin(a+bx)}{2b^2} - \frac{\left(15d^2\right)\int \sqrt{c+dx}\sin(a+bx)\,dx}{4b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{4b^3} - \frac{(c+dx)^{5/2}\cos(a+bx)}{b} + \frac{5d(c+dx)^{3/2}\sin(a+bx)}{2b^2} - \frac{\left(15d^3\right)\int \cdots}{8 \cdots}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{4b^3} - \frac{(c+dx)^{5/2}\cos(a+bx)}{b} + \frac{5d(c+dx)^{3/2}\sin(a+bx)}{2b^2} - \frac{\left(15d^3\cos \cdots\right.}{\cdots}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{4b^3} - \frac{(c+dx)^{5/2}\cos(a+bx)}{b} + \frac{5d(c+dx)^{3/2}\sin(a+bx)}{2b^2} - \frac{\left(15d^2\cos \cdots\right.}{\cdots}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{4b^3} - \frac{(c+dx)^{5/2}\cos(a+bx)}{b} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\cdots}{\sqrt{d}}\right.\cdots}{4b^{7/2}\cdots}$$

Mathematica [C] time = 0.111448, size = 124, normalized size = 0.64

$$\frac{d^2\sqrt{c+dx}e^{-\frac{i(ad+bc)}{d}}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{7}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}+\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{7}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Sin[a + b*x],x]
```

```
[Out] (d^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[7/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d] + (E^(((2*I)*b*c)/d)*Gamma[7/2, (I*b*(c + d*x))/d])/Sqrt
[(I*b*(c + d*x))/d]))/(2*b^3*E^((I*(b*c + a*d))/d))
```

Maple [A] time = 0.012, size = 233, normalized size = 1.2

$$2\,\frac{1}{d}\left(-1/2\,\frac{d\,(dx+c)^{5/2}}{b}\cos\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)+5/2\,\frac{d}{b}\left(1/2\,\frac{d\,(dx+c)^{3/2}}{b}\sin\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)-3/2\,\frac{d}{b}\left(-1/2\,\frac{d\sqrt{\cdots}}{\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/2/b*d*(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+5/2/b*d*(1/2/b*d
*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/
2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 1.88066, size = 891, normalized size = 4.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/32*(80*(d*x + c)^(3/2)*b*d^2*sqrt(abs(b)/abs(d))*sin(((d*x + c)*b - b*c +
 a*d)/d) - 8*(4*(d*x + c)^(5/2)*b^2*d*sqrt(abs(b)/abs(d)) - 15*sqrt(d*x + c
)*d^3*sqrt(abs(b)/abs(d)))*cos(((d*x + c)*b - b*c + a*d)/d) - ((15*sqrt(pi)
*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi
)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqr
t(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*
sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^
3*cos(-(b*c - a*d)/d) - (15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(I*b/d)) - ((15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))))*d^3*cos(-(b*c - a*d)/d) - (-15*I*sqrt(pi)*cos(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*co
s(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*
sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)
*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sin(-(
b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))/(b^3*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 1.77434, size = 467, normalized size = 2.39

$$-\frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+2\sqrt{dx+c}\left(\left(4b^3d\right.\right.}{8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresne
l_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 2*sqrt(d*
x + c)*((4*b^3*d^2*x^2 + 8*b^3*c*d*x + 4*b^3*c^2 - 15*b*d^2)*cos(b*x + a) -
 10*(b^2*d^2*x + b^2*c*d)*sin(b*x + a)))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.25858, size = 1376, normalized size = 7.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(4*(sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)
```

```
/d)/b + 2*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2 + d
^2*((I*sqrt(2)*sqrt(pi)*(-4*I*b^2*c^2*d + 12*b*c*d^2 + 15*I*d^3)*d*erf(-1/2
*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/
2)*b^2*d - 8*I*(d*x + c)^(3/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^2*d + 10*(
d*x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e
^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^2 + (I*sqrt(2)*sqrt(pi)*(-4*I*
b^2*c^2*d - 12*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*b^2*d - 8*I*(d*x + c)^(3
/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*s
qrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c +
I*a*d)/d)/b^3)/d^2) + 4*(I*sqrt(2)*sqrt(pi)*(2*I*b*c*d - 3*d^2)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*
a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + I*sqrt(2)*sqrt(pi)*(2*I
*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*
b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x
+ c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2 - 2*I*(2*I*(d*x + c)^(3
/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b
 + I*b*c - I*a*d)/d)/b^2)*c)/d
```

3.39 $\int (c+dx)^{3/2}\sin(a+bx)\,dx$

Optimal. Leaf size=170

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{2b^2}-(c+d$$

```
[Out] -(((c + d*x)^(3/2)*Cos[a + b*x])/b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d
]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*b^(5/2)) - (3*d^
(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
 - (b*c)/d])/(2*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[a + b*x])/(2*b^2)
```

Rubi [A] time = 0.242042, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{2b^2}-(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Sin[a + b*x],x]
```

```
[Out] -(((c + d*x)^(3/2)*Cos[a + b*x])/b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d
]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*b^(5/2)) - (3*d^
(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
 - (b*c)/d])/(2*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[a + b*x])/(2*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^{3/2}\sin(a+bx)\,dx &= -\frac{(c+dx)^{3/2}\cos(a+bx)}{b}+\frac{(3d)\int\sqrt{c+dx}\cos(a+bx)\,dx}{2b}\\
&= -\frac{(c+dx)^{3/2}\cos(a+bx)}{b}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{2b^2}-\frac{\left(3d^2\right)\int\frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{4b^2}\\
&= -\frac{(c+dx)^{3/2}\cos(a+bx)}{b}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{2b^2}-\frac{\left(3d^2\cos\left(a-\frac{bc}{d}\right)\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{4b^2}-\\
&= -\frac{(c+dx)^{3/2}\cos(a+bx)}{b}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{2b^2}-\frac{\left(3d\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int\sin\left(\frac{bx^2}{d}\right)\right.}{2b^2}\\
&= -\frac{(c+dx)^{3/2}\cos(a+bx)}{b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+d}}{\sqrt{d}}\right.}{2b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.100351, size = 125, normalized size = 0.74

$$-\frac{id\sqrt{c+dx}e^{-\frac{i(ad+bc)}{d}}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{5}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{5}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Sin[a + b*x],x]
```

```
[Out] ((-I/2)*d*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[5/2, ((-I)*b*(c + d*x))/d])/Sqr
t[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[5/2, (I*b*(c + d*x))/d])
/Sqrt[(I*b*(c + d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d))
```

Maple [A] time = 0.007, size = 188, normalized size = 1.1

$$2\frac{1}{d}\left(-1/2\frac{d(dx+c)^{3/2}}{b}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)+3/2\frac{d}{b}\left(1/2\frac{d\sqrt{dx+c}}{b}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)-1/4\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/2/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d
*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d))))
```

Maxima [C] time = 1.82981, size = 855, normalized size = 5.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/16*(16*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*cos(((d*x + c)*b - b*c +
a*d)/d) - 24*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*sin(((d*x + c)*b - b*c +
 a*d)/d) - ((-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))))*d^2*cos(-(b*c - a*d)/d) - (3*sqrt(pi)*cos(1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-(b*c - a*d)/d))
*erf(sqrt(d*x + c)*sqrt(I*b/d)) - ((3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-(b*c - a*d)/d) - (3*sqrt(
pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(
pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sq
rt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*
sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^
2*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))/(b^2*d*sqrt(abs(b)/
abs(d)))
```

Fricas [A] time = 1.79597, size = 394, normalized size = 2.32

$$-\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-2\left(3\,bd\sin\left(bx+a\right.\right.}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(
2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_
cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 2*(3*b*d*si
n(b*x + a) - 2*(b^2*d*x + b^2*c)*cos(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^{\frac{3}{2}}\sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x), x)
```

Giac [C] time = 1.25184, size = 764, normalized size = 4.49

$$-\frac{2\left(\frac{\sqrt{2}\sqrt{\pi}d^{2}\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)b}+\frac{\sqrt{2}\sqrt{\pi}d^{2}\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)b}+\frac{2\,\sqrt{dx+c}de^{\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{d}\right)}}{b}+\frac{2\,\sqrt{dx+c}de^{\left(-\cdots\right)}}{\cdots}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*(sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/
d)/b + 2*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c + I*sq
rt(2)*sqrt(pi)*(2*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) + I*sqrt(2)*sqrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/
2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b -
 I*b*c + I*a*d)/d)/b^2 - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b
*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d
```

3.40 $\int \sqrt{c+dx}\sin(a+bx)\,dx$

Optimal. Leaf size=142

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}$$

```
[Out] -((Sqrt[c + d*x]*Cos[a + b*x])/b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*Fr
esnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/b^(3/2) - (Sqrt[d]*Sqrt
[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d
])/b^(3/2)
```

Rubi [A] time = 0.175986, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sin[a + b*x],x]
```


```
[Out] -((Sqrt[c + d*x]*Cos[a + b*x])/b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*Fr
esnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/b^(3/2) - (Sqrt[d]*Sqrt
[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d
])/b^(3/2)
```


Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c+dx}\sin(a+bx)\,dx &= -\frac{\sqrt{c+dx}\cos(a+bx)}{b}+\frac{d\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{2b}\\ &= -\frac{\sqrt{c+dx}\cos(a+bx)}{b}+\frac{\left(d\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{2b}-\frac{\left(d\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{2b}\\ &= -\frac{\sqrt{c+dx}\cos(a+bx)}{b}+\frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)dx,x,\sqrt{c+dx}\right)}{b}-\frac{\sin\left(a-\frac{bc}{d}\right)\operatorname{Sub}}{}\\ &= -\frac{\sqrt{c+dx}\cos(a+bx)}{b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a\right.}{b^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.0938248, size = 123, normalized size = 0.87

$$\frac{\sqrt{c+dx}e^{-\frac{i(ad+bc)}{d}}\left(-\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sin[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[
(I*b*(c + d*x))/d]))/(2*b*E^((I*(b*c + a*d))/d))
```

Maple [A] time = 0.007, size = 145, normalized size = 1.

$$2\frac{1}{d}\left(-1/2\frac{d\sqrt{dx+c}}{b}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)+1/4\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\frac{da-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(\frac{da-}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/2/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 1.83438, size = 779, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*(8*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*cos((((d*x + c)*b - b*c + a*d)/d
) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
```

```
) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
))*d*cos(-(b*c - a*d)/d) - (I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))))*d*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/
d)) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))))*d*cos(-(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))))*d*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-
I*b/d)))/(b*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 1.76481, size = 327, normalized size = 2.3

$$\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-2\sqrt{dx+c}\,b\cos\left(bx+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sq
rt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(
2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 2*sqrt(d*x + c)*b*co
s(b*x + a))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c+dx}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*sin(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x), x)
```

Giac [C] time = 1.18217, size = 332, normalized size = 2.34

$$-\frac{\frac{\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i bd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{i bc-i ad}{d}\right)}}{\sqrt{bd}\left(\frac{i bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i bd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{-i bc+i ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{2\sqrt{dx+c}de^{\left(\frac{i(dx+c)b-i bc+i ad}{d}\right)}}{b}+\frac{2\sqrt{dx+c}de^{\left(\frac{-i(\cdots}{\cdots}\right)}}{b}\cdots}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
 + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/
b + 2*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

3.41 $\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] (Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sq
rt[d]])/(Sqrt[b]*Sqrt[d]) + (Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
 + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])
```

Rubi [A] time = 0.132519, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sqrt[c + d*x],x]
```


```
[Out] (Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sq
rt[d]])/(Sqrt[b]*Sqrt[d]) + (Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
 + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])
```


Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx &= \cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx+\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx\\ &= \frac{\left(2\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)dx,x,\sqrt{c+dx}\right)}{d}+\frac{\left(2\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)dx,x,\sqrt{c+dx}\right)}{d}\\ &= \frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{\sqrt{b}\sqrt{d}}\end{aligned}$$

Mathematica [C] time = 0.0538371, size = 121, normalized size = 1.03

$$-\frac{e^{-\frac{i(ad+bc)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\operatorname{Gamma}\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\operatorname{Gamma}\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)\right)}{2b\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] -(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-I)*b*(c + d*x))/d] +
 E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, (I*b*(c + d*x))/d])/(
```

```
2*b*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x])
```

Maple [A] time = 0.014, size = 99, normalized size = 0.9

$$\frac{\sqrt{2}\sqrt{\pi}}{d}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}b}{\sqrt{\pi}d}\sqrt{dx+c}\frac{1}{\sqrt{\frac{b}{d}}}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}b}{\sqrt{\pi}d}\sqrt{dx+c}\frac{1}{\sqrt{\frac{b}{d}}}\right)\right)\frac{1}{\sqrt{\frac{b}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x+c)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)
/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))
```

Maxima [C] time = 1.77746, size = 714, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(((-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))))*cos(-(b*c - a*d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d
)) + ((I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)))*cos(-(b*c - a*d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
```

```
tan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)
))/(d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 1.71886, size = 269, normalized size = 2.3

$$\frac{\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*pi*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + sqrt(2)*pi*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(
d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sin(a + b*x)/sqrt(c + d*x), x)
```

Giac [C] time = 1.12505, size = 227, normalized size = 1.94

$$-\frac{\frac{i\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{-ibc+iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
 + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^
2*d^2) + 1)))/d
```

3.42 $\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=139

$$\frac{2\sqrt{2\pi}\sqrt{b}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sqrt{2\pi}\sqrt{b}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}$$

```
[Out] (2*Sqrt[b]*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/d^(3/2) - (2*Sqrt[b]*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/d^(3/2) - (2*Sin[a + b*x])/(d
*Sqrt[c + d*x])
```

Rubi [A] time = 0.204376, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3297, 3306, 3305, 3351, 3304, 3352}

$$\frac{2\sqrt{2\pi}\sqrt{b}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sqrt{2\pi}\sqrt{b}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (2*Sqrt[b]*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/d^(3/2) - (2*Sqrt[b]*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/d^(3/2) - (2*Sin[a + b*x])/(d
*Sqrt[c + d*x])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
```

```
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\sin(a+bx)}{d\sqrt{c+dx}} + \frac{(2b)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2\sin(a+bx)}{d\sqrt{c+dx}} + \frac{\left(2b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{d} - \frac{\left(2b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2\sin(a+bx)}{d\sqrt{c+dx}} + \frac{\left(4b\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{d^2} - \frac{\left(4b\sin\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \right.}{d^2}$$

$$= \frac{2\sqrt{b}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sqrt{b}\sqrt{2\pi}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{3/2}} - \frac{2\sin(a+bx)}{d\sqrt{c+dx}}$$

Mathematica [C] time = 0.312954, size = 148, normalized size = 1.06

$$\frac{ie^{-\frac{i(ad+bc)}{d}}\left(-e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)+2ie^{\frac{i(ad+bc)}{d}}\sin(a+bx)\right)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (I*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-I)*b*(c + d*x))/
d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, (I*b*(c + d*x))/
d] + (2*I)*E^((I*(b*c + a*d))/d)*Sin[a + b*x]))/(d*E^((I*(b*c + a*d))/d)*Sq
rt[c + d*x])
```

Maple [A] time = 0.014, size = 140, normalized size = 1.

$$2\frac{1}{d}\left(-\frac{1}{\sqrt{dx+c}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)+\frac{b\sqrt{2}\sqrt{\pi}}{d}\left(\cos\left(\frac{da-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(\frac{da-cb}{d}\right)\text{Fre}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x+c)^(3/2),x)
```

```
[Out] 2/d*(-1/(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(
b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^
(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d)))
```

Maxima [C] time = 1.32221, size = 632, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(((I*gamma(-1/2, I*(d*x + c)*b/d) - I*gamma(-1/2, -I*(d*x + c)*b/d))*c
os(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (I*gamma(-1/
```

```
2, I*(d*x + c)*b/d) - I*gamma(-1/2, -I*(d*x + c)*b/d))*cos(-1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - (gamma(-1/2, I*(d*x + c)*b/d)
+ gamma(-1/2, -I*(d*x + c)*b/d))*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + (gamma(-1/2, I*(d*x + c)*b/d) + gamma(-1/2, -I*(d*x +
 c)*b/d))*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*c
os(-(b*c - a*d)/d) + ((gamma(-1/2, I*(d*x + c)*b/d) + gamma(-1/2, -I*(d*x +
 c)*b/d))*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (
gamma(-1/2, I*(d*x + c)*b/d) + gamma(-1/2, -I*(d*x + c)*b/d))*cos(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (I*gamma(-1/2, I*(d*x +
 c)*b/d) - I*gamma(-1/2, -I*(d*x + c)*b/d))*sin(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + (-I*gamma(-1/2, I*(d*x + c)*b/d) + I*gamma
(-1/2, -I*(d*x + c)*b/d))*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))))*sin(-(b*c - a*d)/d))*sqrt((d*x + c)*abs(b)/abs(d))/(sqrt(d*x
 + c)*d)
```

Fricas [A] time = 1.964, size = 362, normalized size = 2.6

$$\frac{2\left(\sqrt{2}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-\sqrt{dx+\right.}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(s
qrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d
))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) -
sqrt(d*x + c)*sin(b*x + a))/(d^2*x + c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)/(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/(d*x + c)^(3/2), x)
```

3.43 $\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=168

$$-\frac{4\sqrt{2\pi}b^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4\sqrt{2\pi}b^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4b\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}$$

```
[Out] (-4*b*Cos[a + b*x])/(3*d^2*Sqrt[c + d*x]) - (4*b^(3/2)*Sqrt[2*Pi]*Cos[a - (
b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) -
 (4*b^(3/2)*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]
*Sin[a - (b*c)/d])/(3*d^(5/2)) - (2*Sin[a + b*x])/(3*d*(c + d*x)^(3/2))
```

Rubi [A] time = 0.238313, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3297, 3306, 3305, 3351, 3304, 3352}

$$-\frac{4\sqrt{2\pi}b^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4\sqrt{2\pi}b^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4b\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x)^(5/2),x]
```



```
[Out] (-4*b*Cos[a + b*x])/(3*d^2*Sqrt[c + d*x]) - (4*b^(3/2)*Sqrt[2*Pi]*Cos[a - (
b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) -
 (4*b^(3/2)*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]
*Sin[a - (b*c)/d])/(3*d^(5/2)) - (2*Sin[a + b*x])/(3*d*(c + d*x)^(3/2))
```


Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
```

```
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}} + \frac{(2b)\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx}{3d}$$

$$= -\frac{4b\cos(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin(a+bx)}{3d(c+dx)^{3/2}} - \frac{\left(4b^2\right)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{4b\cos(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin(a+bx)}{3d(c+dx)^{3/2}} - \frac{\left(4b^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{3d^2} - \frac{\left(4b^2\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right.}{\sqrt{c+dx}}}{3d^2}$$

$$= -\frac{4b\cos(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin(a+bx)}{3d(c+dx)^{3/2}} - \frac{\left(8b^2\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{3d^3} - \frac{\left(8b^2\sin\right.}{}$$

$$= -\frac{4b\cos(a+bx)}{3d^2\sqrt{c+dx}} - \frac{4b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4b^{3/2}\sqrt{2\pi}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{3d^{5/2}} -$$

Mathematica [C] time = 0.629594, size = 162, normalized size = 0.96

$$\frac{2\left(-d\sin(a+bx)-b(c+dx)\left(e^{-i(a+bx)}\left(-e^{\frac{ib(c+dx)}{d}}\sqrt{\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)+e^{2i(a+bx)}+1\right)-e^{i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\text{Gamm}\right.\right.}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (2*(-(b*(c + d*x)*(-(E^(I*(a - (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1
/2, ((-I)*b*(c + d*x))/d]) + (1 + E^((2*I)*(a + b*x)) - E^((I*b*(c + d*x))/
d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, (I*b*(c + d*x))/d])/E^(I*(a + b*x)))
 - d*Sin[a + b*x]))/(3*d^2*(c + d*x)^(3/2))
```

Maple [A] time = 0.009, size = 180, normalized size = 1.1

$$2\,\frac{1}{d}\left(-1/3\,\frac{1}{(dx+c)^{3/2}}\sin\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)+2/3\,\frac{b}{d}\left(-\frac{1}{\sqrt{dx+c}}\cos\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)-\frac{b\sqrt{2}\sqrt{\pi}}{d}\left(\cos\left(\frac{da-cb}{d}\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x+c)^(5/2),x)
```

```
[Out] 2/d*(-1/3/(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)+2/3*b/d*(-1/(d*x+c)^
(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 1.31056, size = 632, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(((I*gamma(-3/2, I*(d*x + c)*b/d) - I*gamma(-3/2, -I*(d*x + c)*b/d))*c
os(3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (I*gamma(-3/
2, I*(d*x + c)*b/d) - I*gamma(-3/2, -I*(d*x + c)*b/d))*cos(-3/4*pi + 3/2*ar
ctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) - (gamma(-3/2, I*(d*x + c)*b/d)
+ gamma(-3/2, -I*(d*x + c)*b/d))*sin(3/4*pi + 3/2*arctan2(0, b) + 3/2*arcta
n2(0, d/sqrt(d^2))) + (gamma(-3/2, I*(d*x + c)*b/d) + gamma(-3/2, -I*(d*x +
 c)*b/d))*sin(-3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))))*c
os(-(b*c - a*d)/d) + ((gamma(-3/2, I*(d*x + c)*b/d) + gamma(-3/2, -I*(d*x +
 c)*b/d))*cos(3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (
gamma(-3/2, I*(d*x + c)*b/d) + gamma(-3/2, -I*(d*x + c)*b/d))*cos(-3/4*pi +
 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (I*gamma(-3/2, I*(d*x +
 c)*b/d) - I*gamma(-3/2, -I*(d*x + c)*b/d))*sin(3/4*pi + 3/2*arctan2(0, b)
+ 3/2*arctan2(0, d/sqrt(d^2))) + (-I*gamma(-3/2, I*(d*x + c)*b/d) + I*gamma
(-3/2, -I*(d*x + c)*b/d))*sin(-3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0,
d/sqrt(d^2))))*sin(-(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(3/2)/((d*x +
 c)^(3/2)*d)
```

Fricas [A] time = 2.26133, size = 510, normalized size = 3.04

$$-\frac{2\left(2\sqrt{2}\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+2\sqrt{2}\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\right.\right.}{3\left(d^4x^2+2cd^3x+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos
(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 2*sqrt
(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_cos(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(d*x + c)*(2*(
b*d*x + b*c)*cos(b*x + a) + d*sin(b*x + a)))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a + bx)}{(c + dx)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sin(a + b*x)/(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx + a)}{(dx + c)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/(d*x + c)^(5/2), x)
```

3.44 $\int \frac{\sin(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{8\sqrt{2\pi}b^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8\sqrt{2\pi}b^{5/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8b^2\sin(a+bx)}{15d^3\sqrt{c+dx}}-\frac{4b\cos(a+}{15d^2(c+}$$

```
[Out] (-4*b*Cos[a + b*x])/(15*d^2*(c + d*x)^(3/2)) - (8*b^(5/2)*Sqrt[2*Pi]*Cos[a
- (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2
)) + (8*b^(5/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt
[d]]*Sin[a - (b*c)/d])/(15*d^(7/2)) - (2*Sin[a + b*x])/(5*d*(c + d*x)^(5/2)
) + (8*b^2*Sin[a + b*x])/(15*d^3*Sqrt[c + d*x])
```

Rubi [A] time = 0.296832, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3297, 3306, 3305, 3351, 3304, 3352}

$$-\frac{8\sqrt{2\pi}b^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8\sqrt{2\pi}b^{5/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8b^2\sin(a+bx)}{15d^3\sqrt{c+dx}}-\frac{4b\cos(a+}{15d^2(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] (-4*b*Cos[a + b*x])/(15*d^2*(c + d*x)^(3/2)) - (8*b^(5/2)*Sqrt[2*Pi]*Cos[a
- (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2
)) + (8*b^(5/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt
[d]]*Sin[a - (b*c)/d])/(15*d^(7/2)) - (2*Sin[a + b*x])/(5*d*(c + d*x)^(5/2)
) + (8*b^2*Sin[a + b*x])/(15*d^3*Sqrt[c + d*x])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{2\sin(a+bx)}{5d(c+dx)^{5/2}} + \frac{(2b)\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx}{5d}$$

$$= -\frac{4b\cos(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin(a+bx)}{5d(c+dx)^{5/2}} - \frac{\left(4b^2\right)\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{4b\cos(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\sin(a+bx)}{15d^3\sqrt{c+dx}} - \frac{\left(8b^3\right)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{15d^3}$$

$$= -\frac{4b\cos(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\sin(a+bx)}{15d^3\sqrt{c+dx}} - \frac{\left(8b^3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{15d^3} + \frac{\left(8b^3 \sin\right.}{}$$

$$= -\frac{4b\cos(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\sin(a+bx)}{15d^3\sqrt{c+dx}} - \frac{\left(16b^3\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)dx, x\right.}{15d^4}$$

$$= -\frac{4b\cos(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{8b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8b^{5/2}\sqrt{2\pi}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{15d^{7/2}}$$

Mathematica [C] time = 0.458893, size = 208, normalized size = 1.08

$$-\frac{i\left(b(c+dx)\left(2e^{i\left(a-\frac{bc}{d}\right)}\left(e^{\frac{ib(c+dx)}{d}}(2b(c+dx)-id)-2id\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)\right)-ie^{-i(a+bx)}\left(4de^{\frac{ib(c+dx)}{d}}\left(\frac{ib(c+d}{d}\right.\right.\right.\right.}{15d^3(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] ((-I/15)*(b*(c + d*x)*(2*E^(I*(a - (b*c)/d))*(E^((I*b*(c + d*x))/d)*((-I)*d
 + 2*b*(c + d*x)) - (2*I)*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-I)*b
*(c + d*x))/d]) - (I*(2*d - (4*I)*b*(c + d*x) + 4*d*E^((I*b*(c + d*x))/d)*(
(I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, (I*b*(c + d*x))/d]))/E^(I*(a + b*x))) -
 (6*I)*d^2*Sin[a + b*x]))/(d^3*(c + d*x)^(5/2))
```

Maple [A] time = 0.007, size = 220, normalized size = 1.1

$$2\,\frac{1}{d}\left(-1/5\,\frac{1}{(dx+c)^{5/2}}\sin\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)+2/5\,\frac{b}{d}\left(-1/3\,\frac{1}{(dx+c)^{3/2}}\cos\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)-2/3\,\frac{b}{d}\left(-\frac{1}{\sqrt{dx+c}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x+c)^(7/2),x)
```

```
[Out] 2/d*(-1/5/(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)+2/5*b/d*(-1/3/(d*x+c
)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)-2/3*b/d*(-1/(d*x+c)^(1/2)*sin(1/d*(d
*x+c)*b+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*Fre
snelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*Fres
nelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 1.31408, size = 632, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/4*(((I*gamma(-5/2, I*(d*x + c)*b/d) - I*gamma(-5/2, -I*(d*x + c)*b/d))*c
os(5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (I*gamma(-5/
2, I*(d*x + c)*b/d) - I*gamma(-5/2, -I*(d*x + c)*b/d))*cos(-5/4*pi + 5/2*ar
ctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) - (gamma(-5/2, I*(d*x + c)*b/d)
+ gamma(-5/2, -I*(d*x + c)*b/d))*sin(5/4*pi + 5/2*arctan2(0, b) + 5/2*arcta
n2(0, d/sqrt(d^2))) + (gamma(-5/2, I*(d*x + c)*b/d) + gamma(-5/2, -I*(d*x +
 c)*b/d))*sin(-5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))))*c
os(-(b*c - a*d)/d) + ((gamma(-5/2, I*(d*x + c)*b/d) + gamma(-5/2, -I*(d*x +
 c)*b/d))*cos(5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (
gamma(-5/2, I*(d*x + c)*b/d) + gamma(-5/2, -I*(d*x + c)*b/d))*cos(-5/4*pi +
 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (I*gamma(-5/2, I*(d*x +
 c)*b/d) - I*gamma(-5/2, -I*(d*x + c)*b/d))*sin(5/4*pi + 5/2*arctan2(0, b)
+ 5/2*arctan2(0, d/sqrt(d^2))) + (-I*gamma(-5/2, I*(d*x + c)*b/d) + I*gamma
(-5/2, -I*(d*x + c)*b/d))*sin(-5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0,
d/sqrt(d^2))))*sin(-(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(5/2)/((d*x +
 c)^(5/2)*d)
```

Fricas [A] time = 2.36828, size = 682, normalized size = 3.53

$$-\frac{2\left(4\sqrt{2}\left(\pi b^{2}d^{3}x^{3}+3\,\pi b^{2}cd^{2}x^{2}+3\,\pi b^{2}c^{2}dx+\pi b^{2}c^{3}\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-4\sqrt{2}\left(\pi b^{2}d^{3}x^{3}+3\,\pi b^{2}cd\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(4*sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x +
pi*b^2*c^3)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) - 4*sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*
pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x +
c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(d*x + c)*(2*(b*d^2*x + b*c*d)
*cos(b*x + a) - (4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - 3*d^2)*sin(b*x +
 a)))/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)}{(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/(d*x + c)^(7/2), x)
```

3.45 $\int (c+dx)^{5/2}\sin^2(a+bx)\,dx$

Optimal. Leaf size=231

$$-\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sin(2a+2bx)}{64b^3}+$$

```
[Out] (-5*d*(c + d*x)^(3/2))/(16*b^2) + (c + d*x)^(7/2)/(7*d) - (15*d^(5/2)*Sqrt[
Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[P
i])])/(128*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x
])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(128*b^(7/2)) - ((c + d*x)^(5/
2)*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x]^2)/
(8*b^2) + (15*d^2*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(64*b^3)
```

Rubi [A] time = 0.442284, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3311, 32, 3312, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sin(2a+2bx)}{64b^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Sin[a + b*x]^2,x]
```


```
[Out] (-5*d*(c + d*x)^(3/2))/(16*b^2) + (c + d*x)^(7/2)/(7*d) - (15*d^(5/2)*Sqrt[
Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[P
i])])/(128*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x
])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(128*b^(7/2)) - ((c + d*x)^(5/
2)*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x]^2)/
(8*b^2) + (15*d^2*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(64*b^3)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\sin^2(a+bx)\,dx = -\frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{5d(c+dx)^{3/2}\sin^2(a+bx)}{8b^2} + \frac{1}{2}\int (c+dx)^{5/2}\,dx - $$

$$= \frac{(c+dx)^{7/2}}{7d} - \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{5d(c+dx)^{3/2}\sin^2(a+bx)}{8b^2} - \frac{\left(15d^2\right)\int}{}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{5d(c+dx)^{3/2}\sin^2(a}{8b^2}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{5d(c+dx)^{3/2}\sin^2(a}{8b^2}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{5d(c+dx)^{3/2}\sin^2(a}{8b^2}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{5d(c+dx)^{3/2}\sin^2(a}{8b^2}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}} - \frac{15d^{5/2}\sqrt{\pi}C\left(\frac{2\sqrt{b}\sqrt{}}{\sqrt{d}}\right.}{128}$$

Mathematica [A] time = 2.14366, size = 194, normalized size = 0.84

$$\frac{\sqrt{\frac{b}{d}}\left(2\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(-7d\sin(2(a+bx))\left(16b^2(c+dx)^2-15d^2\right)-140bd^2(c+dx)\cos(2(a+bx))+64b^3(c+dx)^3\right)-105\sqrt{\pi}\right.}{896b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(-105*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sq
rt[c + d*x])/Sqrt[Pi]] - 105*d^3*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*
x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(64*b^3*(c +
 d*x)^3 - 140*b*d^2*(c + d*x)*Cos[2*(a + b*x)] - 7*d*(-15*d^2 + 16*b^2*(c +
 d*x)^2)*Sin[2*(a + b*x)])))/(896*b^4)
```

Maple [A] time = 0.017, size = 242, normalized size = 1.1

$$2\,\frac{1}{d}\left(1/14\,(dx+c)^{7/2}-1/8\,\frac{d\,(dx+c)^{5/2}}{b}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{da-cb}{d}\right)+5/8\,\frac{d}{b}\left(-1/4\,\frac{d\,(dx+c)^{3/2}}{b}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{d}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/14*(d*x+c)^(7/2)-1/8/b*d*(d*x+c)^(5/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)
/d)+5/8/b*d*(-1/4/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/4/b*
d*(1/4/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/
(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/
d)))))
```

Maxima [C] time = 1.85487, size = 940, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/7168*sqrt(2)*(512*sqrt(2)*(d*x + c)^(7/2)*b^3*sqrt(abs(b)/abs(d)) - 1120*
sqrt(2)*(d*x + c)^(3/2)*b*d^2*sqrt(abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c
+ a*d)/d) - ((105*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + 105*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + 105*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) - 105*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))))*d^3*cos(-2*(b*c - a*d)/d) + (105*sqrt(pi)*cos(1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 105*sqrt(pi)*cos(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 105*I*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 105*I*sqrt
(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*si
n(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - ((-105*I*sqrt(pi)*c
os(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 105*I*sqrt(p
i)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 105*sqr
t(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 105*s
qrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3
```

```
*cos(-2*(b*c - a*d)/d) + (105*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))) + 105*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 105*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) - 105*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sin(-2*(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(-2*I*b/d)) - 56*(16*sqrt(2)*(d*x + c)^(5/2)*b^2*d*sqrt(abs(b)
/abs(d)) - 15*sqrt(2)*sqrt(d*x + c)*d^3*sqrt(abs(b)/abs(d)))*sin(2*((d*x +
c)*b - b*c + a*d)/d))/(b^3*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 2.29634, size = 614, normalized size = 2.66

$$-\frac{105\,\pi d^4\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+105\,\pi d^4\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\left(32\,b^4d^3x^3+96\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/896*(105*pi*d^4*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(
d*x + c)*sqrt(b/(pi*d))) + 105*pi*d^4*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x
 + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(32*b^4*d^3*x^3 + 96*b^4*c*
d^2*x^2 + 32*b^4*c^3 + 70*b^2*c*d^2 - 140*(b^2*d^3*x + b^2*c*d^2)*cos(b*x +
 a)^2 - 7*(16*b^3*d^3*x^2 + 32*b^3*c*d^2*x + 16*b^3*c^2*d - 15*b*d^3)*cos(b
*x + a)*sin(b*x + a) + 2*(48*b^4*c^2*d + 35*b^2*d^3)*x)*sqrt(d*x + c))/(b^4
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.34768, size = 1419, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/26880*(560*(3*I*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)*b) - 3*I*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
 + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b
) - 16*(d*x + c)^(3/2) - 6*I*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c
+ 2*I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*
a*d)/d)/b)*c^2 - d^2*(256*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(
d*x + c)^(3/2)*c^2)/d^2 + 105*(sqrt(pi)*(-16*I*b^2*c^2*d + 24*b*c*d^2 + 15*
I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*
b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(16*I*(d*x
+ c)^(5/2)*b^2*d - 32*I*(d*x + c)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*b^2*c^
2*d + 20*(d*x + c)^(3/2)*b*d^2 - 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x +
 c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3)/d^2 + 105*(sqrt(
pi)*(16*I*b^2*c^2*d + 24*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-16*I*(d*x + c)^(5/2)*b^2*d + 32*I*(d*x + c)
^(3/2)*b^2*c*d - 16*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 -
24*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((2*I*(d*x + c)*b - 2*
I*b*c + 2*I*a*d)/d)/b^3)/d^2) - 56*(192*(d*x + c)^(5/2) - 320*(d*x + c)^(3/
2)*c + 15*sqrt(pi)*(4*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b^2) + 15*sqrt(pi)*(-4*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)
*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 30*(-4*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(
d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a
*d)/d)/b^2 - 30*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt
(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)*c)/d
```

3.46 $\int (c+dx)^{3/2} \sin^2(a+bx)\,dx$

Optimal. Leaf size=203

$$\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\sin^2(a+bx)}{8b^2}-\frac{(c+dx}{}$$

```
[Out] (-3*d*Sqrt[c + d*x])/(16*b^2) + (c + d*x)^(5/2)/(5*d) + (3*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])
])/(32*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(S
qrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(32*b^(5/2)) - ((c + d*x)^(3/2)*Cos
[a + b*x]*Sin[a + b*x])/(2*b) + (3*d*Sqrt[c + d*x]*Sin[a + b*x]^2)/(8*b^2)
```

Rubi [A] time = 0.359744, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3311, 32, 3312, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\sin^2(a+bx)}{8b^2}-\frac{(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Sin[a + b*x]^2,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(16*b^2) + (c + d*x)^(5/2)/(5*d) + (3*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])
])/(32*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(S
qrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(32*b^(5/2)) - ((c + d*x)^(3/2)*Cos
[a + b*x]*Sin[a + b*x])/(2*b) + (3*d*Sqrt[c + d*x]*Sin[a + b*x]^2)/(8*b^2)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\sin^2(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{3d\sqrt{c+dx}\sin^2(a+bx)}{8b^2} + \frac{1}{2}\int (c+dx)^{3/2}\,dx - \frac{(3a}{}$$

$$= \frac{(c+dx)^{5/2}}{5d} - \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{3d\sqrt{c+dx}\sin^2(a+bx)}{8b^2} - \frac{(3d^2)\int\left(\frac{}{2\sqrt{}}\right.}{}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} - \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{3d\sqrt{c+dx}\sin^2(a+bx)}{8b^2}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} - \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{3d\sqrt{c+dx}\sin^2(a+bx)}{8b^2}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} - \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{3d\sqrt{c+dx}\sin^2(a+bx)}{8b^2}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} + \frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} - \frac{3d^{3/2}\sqrt{\pi}S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right.}{32b^{5/2}}$$

Mathematica [A] time = 1.69637, size = 175, normalized size = 0.86

$$\frac{\sqrt{\frac{b}{d}}\left(15\sqrt{\pi}d^2\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-15\sqrt{\pi}d^2\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+2\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(4b(c+dx)(4b(\right.\right.}{160b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(15*d^2*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt
[c + d*x])/Sqrt[Pi]] - 15*d^2*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])
/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(-15*d^2*Cos[2*
(a + b*x)] + 4*b*(c + d*x)*(4*b*(c + d*x) - 5*d*Sin[2*(a + b*x)]))))/(160*b
^3)
```

Maple [A] time = 0.013, size = 197, normalized size = 1.

$$2\frac{1}{d}\left(1/10\,(dx+c)^{5/2} - 1/8\,\frac{d\,(dx+c)^{3/2}}{b}\sin\left(2\,\frac{(dx+c)\,b}{d} + 2\,\frac{da-cb}{d}\right) + 3/8\,\frac{d}{b}\left(-1/4\,\frac{d\sqrt{dx+c}}{b}\cos\left(2\,\frac{(dx+c)\,b}{d} + 2\,\frac{da-c}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/10*(d*x+c)^(5/2)-1/8/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)
/d)+3/8/b*d*(-1/4/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*
d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*
(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+
c)^(1/2)*b/d))))
```

Maxima [C] time = 1.82384, size = 899, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1280*sqrt(2)*(128*sqrt(2)*(d*x + c)^(5/2)*b^2*sqrt(abs(b)/abs(d)) - 160*s
qrt(2)*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*sin(2*((d*x + c)*b - b*c + a
*d)/d) - 120*sqrt(2)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*cos(2*((d*x + c)
*b - b*c + a*d)/d) + ((15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-2*(b*c - a*d)/d) - (15*I*sqrt(pi)*
cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(p
i)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt
(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqr
t(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*s
in(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + ((15*sqrt(pi)*cos(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(
pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos
(-2*(b*c - a*d)/d) - (-15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(-2*I*b/d)))/(b^2*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 2.15745, size = 478, normalized size = 2.35

$$\frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+2\left(16\,b^3d^2x^2+32\,b^3cdx\right.}{160\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/160*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*
x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x
 + 16*b^3*c^2 - 30*b*d^2*cos(b*x + a)^2 + 15*b*d^2 - 40*(b^2*d^2*x + b^2*c*
d)*cos(b*x + a)*sin(b*x + a))*sqrt(d*x + c))/(b^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2, x)
```

Giac [C] time = 1.26014, size = 772, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/960*(192*(d*x + c)^(5/2) - 320*(d*x + c)^(3/2)*c - 20*(3*I*sqrt(pi)*d^2*e
rf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*
a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 3*I*sqrt(pi)*d^2*erf(-sqr
```

```
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*(d*x + c)^(3/2) - 6*I*sqrt
(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 6*I*sqrt(d*x +
c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)*c + 15*sqrt(pi)*(4*I*b
*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 15*sqr
t(pi)*(-4*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b^2) - 30*(-4*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d
*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^2 - 30*(4*I*(d*x
 + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(
d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)/d
```

3.47 $\int \sqrt{c+dx}\sin^2(a+bx)\,dx$

Optimal. Leaf size=158

$$\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}+\frac{(c+dx)^{3/2}}{3d}$$

```
[Out] (c + d*x)^(3/2)/(3*d) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*
Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]
*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d
])/(8*b^(3/2)) - (Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(4*b)
```

Rubi [A] time = 0.284661, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3312, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}+\frac{(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sin[a + b*x]^2,x]
```


```
[Out] (c + d*x)^(3/2)/(3*d) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*
Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]
*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d
])/(8*b^(3/2)) - (Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(4*b)
```


Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\sin^2(a+bx)\,dx = \int \left(\frac{1}{2}\sqrt{c+dx}-\frac{1}{2}\sqrt{c+dx}\cos(2a+2bx)\right)dx$$
$$= \frac{(c+dx)^{3/2}}{3d}-\frac{1}{2}\int \sqrt{c+dx}\cos(2a+2bx)\,dx$$
$$= \frac{(c+dx)^{3/2}}{3d}-\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}+\frac{d\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}\,dx}{8b}$$
$$= \frac{(c+dx)^{3/2}}{3d}-\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}+\frac{\left(d\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{8b}+\frac{\left(d\sin\left(2a-\frac{2\ldots}{\ldots}\right.\right.}{}$$
$$= \frac{(c+dx)^{3/2}}{3d}-\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)dx,x,\sqrt{c+dx}\right)}{4b}$$
$$= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}+\frac{\sqrt{d}\sqrt{\pi}C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{8b^{3/2}}-\frac{\sqrt{c}\ldots}{}$$

Mathematica [A] time = 0.531776, size = 149, normalized size = 0.94

$$\frac{3\sqrt{\pi}d\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+3\sqrt{\pi}d\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+2\sqrt{\frac{b}{d}}\sqrt{c+dx}(4b(c+dx)-3d\sin(2(a\ldots}{24d^2\left(\frac{b}{d}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqr
t[Pi]] + 3*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*
a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(4*b*(c + d*x) - 3*d*Sin[2*(a +
b*x)]))/(24*(b/d)^(3/2)*d^2)
```

Maple [A] time = 0.013, size = 150, normalized size = 1.

$$2\frac{1}{d}\left(1/6\,(dx+c)^{3/2}-1/8\,\frac{d\sqrt{dx+c}}{b}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{da-cb}{d}\right)+1/16\,\frac{d\sqrt{\pi}}{b}\left(\cos\left(2\,\frac{da-cb}{d}\right)\text{FresnelS}\left(2\,\frac{\sqrt{dx+c}\,b}{d\sqrt{\pi}}\,\frac{1}{\sqrt{\ldots}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/6*(d*x+c)^(3/2)-1/8/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/
d)+1/16/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b
/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 1.81975, size = 826, normalized size = 5.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/192*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b*sqrt(abs(b)/abs(d)) - 24*sqrt(2
)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*sin(2*((d*x + c)*b - b*c + a*d)/d) -
((-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))))*d*cos(-2*(b*c - a*d)/d) - (3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sin(-2*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(2*I*b/d)) - ((3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))))*d*cos(-2*(b*c - a*d)/d) - (3*sqrt(pi)*cos(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*si
n(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sin(-2*(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/(b*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 2.19164, size = 367, normalized size = 2.32

$$\frac{3\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+4\left(2\,b^2dx-3\,bd\cos\left(bx+\right.\right.}{24\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x
+ c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(2*b^2*d*x - 3*b*d*cos(b*x + a)*s
in(b*x + a) + 2*b^2*c)*sqrt(d*x + c))/(b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+dx}\sin^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2, x)
```

Giac [C] time = 1.20764, size = 331, normalized size = 2.09

$$-\frac{\frac{3i\,\sqrt{\pi}d^2\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2i\,bc-2i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{3i\,\sqrt{\pi}d^2\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2i\,bc+2i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-16\,(dx+c)^{\frac{3}{2}}-\frac{6i\,\sqrt{dx+c}de^{\left(\frac{2i\,(dx+c)b-2i\,bc+2i\,ad}{d}\right)}}{b}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/48*(3*I*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) -
```

```
3*I*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*(
d*x + c)^(3/2) - 6*I*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*
d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/
b)/d
```

3.48 $\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}$$

```
[Out] Sqrt[c + d*x]/d - (Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c
 + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2*Sqrt[b]*Sqrt[d]) + (Sqrt[Pi]*FresnelS[(2*S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2*Sqrt[b]*
Sqrt[d])
```

Rubi [A] time = 0.234267, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3312, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sqrt[c + d*x],x]
```


```
[Out] Sqrt[c + d*x]/d - (Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c
 + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2*Sqrt[b]*Sqrt[d]) + (Sqrt[Pi]*FresnelS[(2*S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2*Sqrt[b]*
Sqrt[d])
```


Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx &= \int \left(\frac{1}{2\sqrt{c+dx}} - \frac{\cos(2a+2bx)}{2\sqrt{c+dx}}\right) dx \\
&= \frac{\sqrt{c+dx}}{d} - \frac{1}{2}\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx \\
&= \frac{\sqrt{c+dx}}{d} - \frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx + \frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx \\
&= \frac{\sqrt{c+dx}}{d} - \frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{d} + \frac{\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right.\right.}{d} \\
&= \frac{\sqrt{c+dx}}{d} - \frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}} + \frac{\sqrt{\pi}S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{2\sqrt{b}\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.225902, size = 126, normalized size = 0.97

$$\frac{\sqrt{\frac{b}{d}}\left(-\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+2\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[b/d]*(2*Sqrt[b/d]*Sqrt[c + d*x] - Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresn
elC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*
Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d]))/(2*b)
```

Maple [A] time = 0.015, size = 108, normalized size = 0.8

$$2\frac{1}{d}\left(1/2\sqrt{dx+c}-1/4\sqrt{\pi}\left(\cos\left(2\frac{da-cb}{d}\right)\text{FresnelC}\left(2\frac{\sqrt{dx+cb}}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(2\frac{da-cb}{d}\right)\text{FresnelS}\left(2\frac{\sqrt{dx+cb}}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^(1/2),x)
```

```
[Out] 2/d*(1/2*(d*x+c)^(1/2)-1/4*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*Fresnel
C(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/P
i^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 1.85129, size = 747, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*(((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
```

```
sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))))*cos(-2*(b*c - a*d)/d) - (I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(2*I*b/d)) + ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))))*cos(-2*(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*
x + c)*sqrt(-2*I*b/d)) - 8*sqrt(2)*sqrt(d*x + c)*sqrt(abs(b)/abs(d)))/(d*sq
rt(abs(b)/abs(d)))
```

Fricas [A] time = 2.05816, size = 281, normalized size = 2.16

$$-\frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-2\sqrt{dx+c}b}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(
pi*d)))*sin(-2*(b*c - a*d)/d) - 2*sqrt(d*x + c)*b)/(b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/sqrt(c + d*x), x)
```

Giac [C] time = 1.18342, size = 220, normalized size = 1.69

$$\frac{\frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2i\,bc-2i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2i\,bc+2i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+4\sqrt{dx+c}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(pi)*d
*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c +
2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*sqrt(d*x + c))/d
```

3.49 $\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{2\sqrt{\pi}\sqrt{b}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{d^{3/2}}+\frac{2\sqrt{\pi}\sqrt{b}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{d^{3/2}}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}$$

```
[Out] (2*Sqrt[b]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])])/d^(3/2) + (2*Sqrt[b]*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqr
t[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/d^(3/2) - (2*Sin[a +
b*x]^2)/(d*Sqrt[c + d*x])
```

Rubi [A] time = 0.253951, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3313, 12, 3306, 3305, 3351, 3304, 3352}

$$\frac{2\sqrt{\pi}\sqrt{b}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{d^{3/2}}+\frac{2\sqrt{\pi}\sqrt{b}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{d^{3/2}}-\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (2*Sqrt[b]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])])/d^(3/2) + (2*Sqrt[b]*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqr
t[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/d^(3/2) - (2*Sin[a +
b*x]^2)/(d*Sqrt[c + d*x])
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}} + \frac{(4b)\int \frac{\sin(2a+2bx)}{2\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}} + \frac{(2b)\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}} + \frac{\left(2b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{d} + \frac{\left(2b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2\sin^2(a+bx)}{d\sqrt{c+dx}} + \frac{\left(4b\cos\left(2a-\frac{2bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{d^2} + \frac{\left(4b\sin\left(2a-\frac{2bc}{d}\right)\right)\text{S}}{}$$

$$= \frac{2\sqrt{b}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{d^{3/2}} + \frac{2\sqrt{b}\sqrt{\pi}C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^{3/2}} - \frac{2\sin^2(a+bx)}{d\sqrt{c+dx}}$$

Mathematica [A] time = 0.383539, size = 149, normalized size = 1.1

$$\frac{2\sqrt{\pi}\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + 2\sqrt{\pi}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \cos(2(a+bx))}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (-1 + Cos[2*(a + b*x)] + 2*Sqrt[b/d]*Sqrt[Pi]*Sqrt[c + d*x]*Cos[2*a - (2*b*
c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 2*Sqrt[b/d]*Sqrt[Pi]
*Sqrt[c + d*x]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*
b*c)/d])/(d*Sqrt[c + d*x])
```

Maple [A] time = 0.013, size = 145, normalized size = 1.1

$$2\frac{1}{d}\left(-1/2\frac{1}{\sqrt{dx+c}} + 1/2\frac{1}{\sqrt{dx+c}}\cos\left(2\frac{(dx+c)b}{d} + 2\frac{da-cb}{d}\right) + \frac{b\sqrt{\pi}}{d}\left(\cos\left(2\frac{da-cb}{d}\right)\text{FresnelS}\left(2\frac{\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right) + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^(3/2),x)
```

```
[Out] 2/d*(-1/2/(d*x+c)^(1/2)+1/2/(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+
b/d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*
x+c)^(1/2)*b/d)))
```

Maxima [C] time = 1.3232, size = 640, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(2)*(((gamma(-1/2, 2*I*(d*x + c)*b/d) + gamma(-1/2, -2*I*(d*x + c)
*b/d))*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (gam
ma(-1/2, 2*I*(d*x + c)*b/d) + gamma(-1/2, -2*I*(d*x + c)*b/d))*cos(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (I*gamma(-1/2, 2*I*(d*
x + c)*b/d) - I*gamma(-1/2, -2*I*(d*x + c)*b/d))*sin(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (-I*gamma(-1/2, 2*I*(d*x + c)*b/d) +
I*gamma(-1/2, -2*I*(d*x + c)*b/d))*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))))*cos(-2*(b*c - a*d)/d) + ((-I*gamma(-1/2, 2*I*(d*x +
 c)*b/d) + I*gamma(-1/2, -2*I*(d*x + c)*b/d))*cos(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) + (-I*gamma(-1/2, 2*I*(d*x + c)*b/d) + I*g
amma(-1/2, -2*I*(d*x + c)*b/d))*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + (gamma(-1/2, 2*I*(d*x + c)*b/d) + gamma(-1/2, -2*I*(d
*x + c)*b/d))*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 - (gamma(-1/2, 2*I*(d*x + c)*b/d) + gamma(-1/2, -2*I*(d*x + c)*b/d))*sin(-
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sin(-2*(b*c - a*
d)/d))*sqrt((d*x + c)*abs(b)/abs(d)) - 8)/(sqrt(d*x + c)*d)
```

Fricas [A] time = 2.12082, size = 340, normalized size = 2.52

$$\frac{2\left((\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+\sqrt{dx+c}(\mathrm{co}\right.}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(
d*x + c)*sqrt(b/(pi*d))) + (pi*d*x + pi*c)*sqrt(b/(pi*d))*fresnel_cos(2*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + sqrt(d*x + c)*(cos(b*x +
 a)^2 - 1))/(d^2*x + c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^2}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*x + c)^(3/2), x)
```

3.50 $\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=170

$$\frac{8\sqrt{\pi}b^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{3d^{5/2}}-\frac{8\sqrt{\pi}b^{3/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}}-\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sin}{3d(c}$$

```
[Out] (8*b^(3/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])])/(3*d^(5/2)) - (8*b^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]
*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(3*d^(5/2)) - (8*
b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2*Sqrt[c + d*x]) - (2*Sin[a + b*x]^2)/(3*
d*(c + d*x)^(3/2))
```

Rubi [A] time = 0.328234, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3314, 32, 3312, 3306, 3305, 3351, 3304, 3352}

$$\frac{8\sqrt{\pi}b^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{3d^{5/2}}-\frac{8\sqrt{\pi}b^{3/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}}-\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sin}{3d(c}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (8*b^(3/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])])/(3*d^(5/2)) - (8*b^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]
*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(3*d^(5/2)) - (8*
b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2*Sqrt[c + d*x]) - (2*Sin[a + b*x]^2)/(3*
d*(c + d*x)^(3/2))
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{(8b^2)\int \frac{1}{\sqrt{c+dx}}\,dx}{3d^2} - \frac{(16b^2)\int \frac{\sin^2(a+bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= \frac{16b^2\sqrt{c+dx}}{3d^3} - \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{(16b^2)\int \left(\frac{1}{2\sqrt{c+dx}} - \frac{\cos(2a+2bx)}{2\sqrt{c+dx}}\right)dx}{3d^2}$$

$$= -\frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{(8b^2)\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{\left(8b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{3d^2} - \frac{\left(8b^2\sin\left(2\right.\right.}{}$$

$$= -\frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{\left(16b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\text{Subst}\left(\int\cos\left(\frac{2bx^2}{d}\right)dx, x, \sqrt{c}\right.}{3d^3}$$

$$= \frac{8b^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}} - \frac{8b^{3/2}\sqrt{\pi}S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{3d^{5/2}} - \frac{8b\cos(a+bx)\sin(a}{3d^2\sqrt{c+dx}}$$

Mathematica [A] time = 1.39855, size = 158, normalized size = 0.93

$$\frac{2\left(4\sqrt{\pi}b\sqrt{\frac{b}{d}}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 4\sqrt{\pi}b\sqrt{\frac{b}{d}}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - \frac{\sin(a+bx)(4b(c+dx)\cos(a+bx)+d\sin(}{(c+dx)^{3/2}}\right.}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (2*(4*b*Sqrt[b/d]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[
c + d*x])/Sqrt[Pi]] - 4*b*Sqrt[b/d]*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c +
 d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] - (Sin[a + b*x]*(4*b*(c + d*x)*Cos[a
+ b*x] + d*Sin[a + b*x]))/(c + d*x)^(3/2)))/(3*d^2)
```

Maple [A] time = 0.014, size = 189, normalized size = 1.1

$$2\,\frac{1}{d}\left(-1/6\,(dx+c)^{-3/2} + 1/6\,\frac{1}{(dx+c)^{3/2}}\cos\left(2\,\frac{(dx+c)\,b}{d} + 2\,\frac{da-cb}{d}\right) + 2/3\,\frac{b}{d}\left(-\frac{1}{\sqrt{dx+c}}\sin\left(2\,\frac{(dx+c)\,b}{d} + 2\,\frac{da-cb}{d}\right) + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^(5/2),x)
```

```
[Out] 2/d*(-1/6/(d*x+c)^(3/2)+1/6/(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+
2/3*b/d*(-1/(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+2*b/d*Pi^(1/2)/(
b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d
))))
```

Maxima [C] time = 1.29074, size = 644, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sqrt(2)*((3*(gamma(-3/2, 2*I*(d*x + c)*b/d) + gamma(-3/2, -2*I*(d*x +
 c)*b/d))*cos(3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + 3
*(gamma(-3/2, 2*I*(d*x + c)*b/d) + gamma(-3/2, -2*I*(d*x + c)*b/d))*cos(-3/
4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (3*I*gamma(-3/2,
2*I*(d*x + c)*b/d) - 3*I*gamma(-3/2, -2*I*(d*x + c)*b/d))*sin(3/4*pi + 3/2*
arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (-3*I*gamma(-3/2, 2*I*(d*x +
 c)*b/d) + 3*I*gamma(-3/2, -2*I*(d*x + c)*b/d))*sin(-3/4*pi + 3/2*arctan2(0
, b) + 3/2*arctan2(0, d/sqrt(d^2))))*cos(-2*(b*c - a*d)/d) + ((-3*I*gamma(-
3/2, 2*I*(d*x + c)*b/d) + 3*I*gamma(-3/2, -2*I*(d*x + c)*b/d))*cos(3/4*pi +
 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (-3*I*gamma(-3/2, 2*I*(
d*x + c)*b/d) + 3*I*gamma(-3/2, -2*I*(d*x + c)*b/d))*cos(-3/4*pi + 3/2*arct
an2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + 3*(gamma(-3/2, 2*I*(d*x + c)*b/d
) + gamma(-3/2, -2*I*(d*x + c)*b/d))*sin(3/4*pi + 3/2*arctan2(0, b) + 3/2*a
rctan2(0, d/sqrt(d^2))) - 3*(gamma(-3/2, 2*I*(d*x + c)*b/d) + gamma(-3/2, -
2*I*(d*x + c)*b/d))*sin(-3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt
(d^2))))*sin(-2*(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(3/2) - 4)/((d*x
+ c)^(3/2)*d)
```

Fricas [A] time = 2.33142, size = 502, normalized size = 2.95

$$\frac{2\left(4\left(\pi bd^2x^2 + 2\,\pi bcdx + \pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 4\left(\pi bd^2x^2 + 2\,\pi bcdx + \pi bc^2\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx-\right.}\right.}{3\left(d^4x^2 + 2\,cd^3x + c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(4*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(-2*(b*c
- a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 4*(pi*b*d^2*x^2 + 2
*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(
pi*d)))*sin(-2*(b*c - a*d)/d) + (d*cos(b*x + a)^2 - 4*(b*d*x + b*c)*cos(b*x
 + a)*sin(b*x + a) - d)*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^2}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*x + c)^(5/2), x)
```

3.51 $\int \frac{\sin^2(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=216

$$-\frac{32\sqrt{\pi}b^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{15d^{7/2}}-\frac{32\sqrt{\pi}b^{5/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{15d^{7/2}}+\frac{32b^2\sin^2(a+bx)}{15d^3\sqrt{c+dx}}-\frac{8b\sin(a}{15}$$

```
[Out] (-16*b^2)/(15*d^3*Sqrt[c + d*x]) - (32*b^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d
]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(15*d^(7/2)) - (3
2*b^(5/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*S
in[2*a - (2*b*c)/d])/(15*d^(7/2)) - (8*b*Cos[a + b*x]*Sin[a + b*x])/(15*d^2
*(c + d*x)^(3/2)) - (2*Sin[a + b*x]^2)/(5*d*(c + d*x)^(5/2)) + (32*b^2*Sin[
a + b*x]^2)/(15*d^3*Sqrt[c + d*x])
```

Rubi [A] time = 0.335769, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3314, 32, 3313, 12, 3306, 3305, 3351, 3304, 3352}

$$-\frac{32\sqrt{\pi}b^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{15d^{7/2}}-\frac{32\sqrt{\pi}b^{5/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{15d^{7/2}}+\frac{32b^2\sin^2(a+bx)}{15d^3\sqrt{c+dx}}-\frac{8b\sin(a}{15}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] (-16*b^2)/(15*d^3*Sqrt[c + d*x]) - (32*b^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d
]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(15*d^(7/2)) - (3
2*b^(5/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*S
in[2*a - (2*b*c)/d])/(15*d^(7/2)) - (8*b*Cos[a + b*x]*Sin[a + b*x])/(15*d^2
*(c + d*x)^(3/2)) - (2*Sin[a + b*x]^2)/(5*d*(c + d*x)^(5/2)) + (32*b^2*Sin[
a + b*x]^2)/(15*d^3*Sqrt[c + d*x])
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
```

```
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{\left(8b^2\right)\int \frac{1}{(c+dx)^{3/2}}\,dx}{15d^2} - \frac{\left(16b^2\right)\int \frac{\sin^2(a+bx)}{(c+dx)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\sin^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{\left(64b^3\right)\int \frac{\sin(2}{2\sqrt{}}}{15d^3}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\sin^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{\left(32b^3\right)\int \frac{\sin(2}{\sqrt{}}}{15d^3}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\sin^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{\left(32b^3\cos\left(2a\right.\right.}{}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sin^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\sin^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{\left(64b^3\cos\left(2a\right.\right.}{}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{32b^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{15d^{7/2}} - \frac{32b^{5/2}\sqrt{\pi}C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{15d^{7/2}} -$$

Mathematica [A] time = 2.0029, size = 244, normalized size = 1.13

$$-\frac{16b^2c^2\cos(2(a+bx)) + 32b^2cdx\cos(2(a+bx)) + 16b^2d^2x^2\cos(2(a+bx)) + 32\sqrt{\pi}bd\left(\frac{b}{d}\right)^{3/2}(c+dx)^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\mathrm{I}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] -(3*d^2 + 16*b^2*c^2*Cos[2*(a + b*x)] - 3*d^2*Cos[2*(a + b*x)] + 32*b^2*c*d
*x*Cos[2*(a + b*x)] + 16*b^2*d^2*x^2*Cos[2*(a + b*x)] + 32*b*(b/d)^(3/2)*d*
Sqrt[Pi]*(c + d*x)^(5/2)*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
+ d*x])/Sqrt[Pi]] + 32*b*(b/d)^(3/2)*d*Sqrt[Pi]*(c + d*x)^(5/2)*FresnelC[(2
*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 4*b*c*d*Sin[2*(a
```

```
 + b*x)] + 4*b*d^2*x*Sin[2*(a + b*x)])/(15*d^3*(c + d*x)^(5/2))
```

Maple [A] time = 0.013, size = 230, normalized size = 1.1

$$2\,\frac{1}{d}\left(-1/10\,(dx+c)^{-5/2}+1/10\,\frac{1}{(dx+c)^{5/2}}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{da-cb}{d}\right)+2/5\,\frac{b}{d}\left(-1/3\,\frac{1}{(dx+c)^{3/2}}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{da}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^(7/2),x)
```

```
[Out] 2/d*(-1/10/(d*x+c)^(5/2)+1/10/(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d
)+2/5*b/d*(-1/3/(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+4/3*b/d*(-1/
(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-2*b/d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(2
*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 1.30236, size = 644, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/10*(sqrt(2)*((5*(gamma(-5/2, 2*I*(d*x + c)*b/d) + gamma(-5/2, -2*I*(d*x +
 c)*b/d))*cos(5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + 5
*(gamma(-5/2, 2*I*(d*x + c)*b/d) + gamma(-5/2, -2*I*(d*x + c)*b/d))*cos(-5/
4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (5*I*gamma(-5/2,
2*I*(d*x + c)*b/d) - 5*I*gamma(-5/2, -2*I*(d*x + c)*b/d))*sin(5/4*pi + 5/2*
arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (-5*I*gamma(-5/2, 2*I*(d*x +
 c)*b/d) + 5*I*gamma(-5/2, -2*I*(d*x + c)*b/d))*sin(-5/4*pi + 5/2*arctan2(0
, b) + 5/2*arctan2(0, d/sqrt(d^2))))*cos(-2*(b*c - a*d)/d) + ((-5*I*gamma(-
5/2, 2*I*(d*x + c)*b/d) + 5*I*gamma(-5/2, -2*I*(d*x + c)*b/d))*cos(5/4*pi +
 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (-5*I*gamma(-5/2, 2*I*(
d*x + c)*b/d) + 5*I*gamma(-5/2, -2*I*(d*x + c)*b/d))*cos(-5/4*pi + 5/2*arct
an2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + 5*(gamma(-5/2, 2*I*(d*x + c)*b/d
) + gamma(-5/2, -2*I*(d*x + c)*b/d))*sin(5/4*pi + 5/2*arctan2(0, b) + 5/2*a
```

```
rctan2(0, d/sqrt(d^2))) - 5*(gamma(-5/2, 2*I*(d*x + c)*b/d) + gamma(-5/2, -
2*I*(d*x + c)*b/d))*sin(-5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt
(d^2))))*sin(-2*(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(5/2) - 2)/((d*x
+ c)^(5/2)*d)
```

Fricas [A] time = 2.45532, size = 745, normalized size = 3.45

$$-\frac{2\left(16\left(\pi b^{2}d^{3}x^{3}+3\,\pi b^{2}cd^{2}x^{2}+3\,\pi b^{2}c^{2}dx+\pi b^{2}c^{3}\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+16\left(\pi b^{2}d^{3}x^{3}+3\,\pi b^{2}cd^{2}x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(16*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*
c^3)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(
b/(pi*d))) + 16*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + p
i*b^2*c^3)*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-
2*(b*c - a*d)/d) - (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - (16*b^2*d^2*
x^2 + 32*b^2*c*d*x + 16*b^2*c^2 - 3*d^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*
d)*cos(b*x + a)*sin(b*x + a) - 3*d^2)*sqrt(d*x + c))/(d^6*x^3 + 3*c*d^5*x^2
 + 3*c^2*d^4*x + c^3*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(bx+a\right)^{2}}{\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*x + c)^(7/2), x)
```

3.52 $\int \frac{\sin^2(a+bx)}{(c+dx)^{9/2}}\,dx$

Optimal. Leaf size=247

$$-\frac{128\sqrt{\pi}b^{7/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{105d^{9/2}}+\frac{128\sqrt{\pi}b^{7/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{105d^{9/2}}+\frac{32b^2\sin^2(a+bx)}{105d^3(c+dx)^{3/2}}+\frac{128b^3}{}$$

```
[Out] (-16*b^2)/(105*d^3*(c + d*x)^(3/2)) - (128*b^(7/2)*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(105*d^(9/2))
 + (128*b^(7/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[P
i])]*Sin[2*a - (2*b*c)/d])/(105*d^(9/2)) - (8*b*Cos[a + b*x]*Sin[a + b*x])/
(35*d^2*(c + d*x)^(5/2)) + (128*b^3*Cos[a + b*x]*Sin[a + b*x])/(105*d^4*Sqr
t[c + d*x]) - (2*Sin[a + b*x]^2)/(7*d*(c + d*x)^(7/2)) + (32*b^2*Sin[a + b*
x]^2)/(105*d^3*(c + d*x)^(3/2))
```

Rubi [A] time = 0.419004, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3314, 32, 3312, 3306, 3305, 3351, 3304, 3352}

$$-\frac{128\sqrt{\pi}b^{7/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{105d^{9/2}}+\frac{128\sqrt{\pi}b^{7/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{105d^{9/2}}+\frac{32b^2\sin^2(a+bx)}{105d^3(c+dx)^{3/2}}+\frac{128b^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(c + d*x)^(9/2),x]
```


```
[Out] (-16*b^2)/(105*d^3*(c + d*x)^(3/2)) - (128*b^(7/2)*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(105*d^(9/2))
 + (128*b^(7/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[P
i])]*Sin[2*a - (2*b*c)/d])/(105*d^(9/2)) - (8*b*Cos[a + b*x]*Sin[a + b*x])/
(35*d^2*(c + d*x)^(5/2)) + (128*b^3*Cos[a + b*x]*Sin[a + b*x])/(105*d^4*Sqr
t[c + d*x]) - (2*Sin[a + b*x]^2)/(7*d*(c + d*x)^(7/2)) + (32*b^2*Sin[a + b*
x]^2)/(105*d^3*(c + d*x)^(3/2))
```


Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
```

```
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(c+dx)^{9/2}}\,dx = -\frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{(8b^2)\int\frac{1}{(c+dx)^{5/2}}\,dx}{35d^2} - \frac{(16b^2)\int\frac{\sin^2(a+bx)}{(c+dx)^{5/2}}\,dx}{35d^2}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} + \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{256b^4\sqrt{c+dx}}{105d^5} - \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} + \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} + \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} + \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} + \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}} - \frac{2\sin^2(a+bx)}{7d(c+dx)^{7/2}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{128b^{7/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{105d^{9/2}} + \frac{128b^{7/2}\sqrt{\pi}S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\right.}{105d^{9/2}}$$

Mathematica [B] time = 4.63769, size = 661, normalized size = 2.68

$$\frac{\cos(2a)\left(2\cos\left(\frac{2bc}{d}\right)\left(15d^3\cos\left(\frac{2b(c+dx)}{d}\right) - 4b(c+dx)\left(3d^2\sin\left(\frac{2b(c+dx)}{d}\right) + 4b(c+dx)\left(8\sqrt{\pi}b\sqrt{\frac{b}{d}}(c+dx)^{3/2}\text{FresnelC}\left(\frac{2\right.\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(c + d*x)^(9/2),x]
```

```
[Out] (-30*d^3 + Cos[2*a]*(4*Cos[(b*c)/d]*Sin[(b*c)/d]*(15*d^3*Sin[(2*b*(c + d*x)
)/d] + 4*b*(c + d*x)*(3*d^2*Cos[(2*b*(c + d*x))/d] - 4*b*(c + d*x)*(4*b*(c
+ d*x)*Cos[(2*b*(c + d*x))/d] + 8*b*Sqrt[b/d]*Sqrt[Pi]*(c + d*x)^(3/2)*Fres
nelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + d*Sin[(2*b*(c + d*x))/d]))) +
2*Cos[(2*b*c)/d]*(15*d^3*Cos[(2*b*(c + d*x))/d] - 4*b*(c + d*x)*(3*d^2*Sin[
(2*b*(c + d*x))/d] + 4*b*(c + d*x)*(d*Cos[(2*b*(c + d*x))/d] + 8*b*Sqrt[b/d
]*Sqrt[Pi]*(c + d*x)^(3/2)*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] -
```

```
 4*b*(c + d*x)*Sin[(2*b*(c + d*x))/d])))) - 2*Cos[a]*Sin[a]*(2*(Cos[(b*c)/d
] - Sin[(b*c)/d])*(Cos[(b*c)/d] + Sin[(b*c)/d])*(15*d^3*Sin[(2*b*(c + d*x))
/d] + 4*b*(c + d*x)*(3*d^2*Cos[(2*b*(c + d*x))/d] - 4*b*(c + d*x)*(4*b*(c +
 d*x)*Cos[(2*b*(c + d*x))/d] + 8*b*Sqrt[b/d]*Sqrt[Pi]*(c + d*x)^(3/2)*Fresn
elS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + d*Sin[(2*b*(c + d*x))/d]))) - 2
*Sin[(2*b*c)/d]*(15*d^3*Cos[(2*b*(c + d*x))/d] - 4*b*(c + d*x)*(3*d^2*Sin[(
2*b*(c + d*x))/d] + 4*b*(c + d*x)*(d*Cos[(2*b*(c + d*x))/d] + 8*b*Sqrt[b/d]
*Sqrt[Pi]*(c + d*x)^(3/2)*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] -
4*b*(c + d*x)*Sin[(2*b*(c + d*x))/d])))))/(210*d^4*(c + d*x)^(7/2))
```

Maple [A] time = 0.015, size = 273, normalized size = 1.1

$$2\frac{1}{d}\left(-1/14\,(dx+c)^{-7/2}+1/14\,\frac{1}{(dx+c)^{7/2}}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{da-cb}{d}\right)+2/7\,\frac{b}{d}\left(-1/5\,\frac{1}{(dx+c)^{5/2}}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{da}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*x+c)^(9/2),x)
```

```
[Out] 2/d*(-1/14/(d*x+c)^(7/2)+1/14/(d*x+c)^(7/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d
)+2/7*b/d*(-1/5/(d*x+c)^(5/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+4/5*b/d*(-1/
3/(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-4/3*b/d*(-1/(d*x+c)^(1/2)*
sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+2*b/d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c
)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*
FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 1.29426, size = 644, normalized size = 2.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/7*(sqrt(2)*((7*(gamma(-7/2, 2*I*(d*x + c)*b/d) + gamma(-7/2, -2*I*(d*x +
c)*b/d))*cos(7/4*pi + 7/2*arctan2(0, b) + 7/2*arctan2(0, d/sqrt(d^2))) + 7*
(gamma(-7/2, 2*I*(d*x + c)*b/d) + gamma(-7/2, -2*I*(d*x + c)*b/d))*cos(-7/4
*pi + 7/2*arctan2(0, b) + 7/2*arctan2(0, d/sqrt(d^2))) + (7*I*gamma(-7/2, 2
```

```
*I*(d*x + c)*b/d) - 7*I*gamma(-7/2, -2*I*(d*x + c)*b/d))*sin(7/4*pi + 7/2*a
rctan2(0, b) + 7/2*arctan2(0, d/sqrt(d^2))) + (-7*I*gamma(-7/2, 2*I*(d*x +
c)*b/d) + 7*I*gamma(-7/2, -2*I*(d*x + c)*b/d))*sin(-7/4*pi + 7/2*arctan2(0,
 b) + 7/2*arctan2(0, d/sqrt(d^2)))))*cos(-2*(b*c - a*d)/d) + ((-7*I*gamma(-7
/2, 2*I*(d*x + c)*b/d) + 7*I*gamma(-7/2, -2*I*(d*x + c)*b/d))*cos(7/4*pi +
7/2*arctan2(0, b) + 7/2*arctan2(0, d/sqrt(d^2))) + (-7*I*gamma(-7/2, 2*I*(d
*x + c)*b/d) + 7*I*gamma(-7/2, -2*I*(d*x + c)*b/d))*cos(-7/4*pi + 7/2*arcta
n2(0, b) + 7/2*arctan2(0, d/sqrt(d^2))) + 7*(gamma(-7/2, 2*I*(d*x + c)*b/d)
 + gamma(-7/2, -2*I*(d*x + c)*b/d))*sin(7/4*pi + 7/2*arctan2(0, b) + 7/2*ar
ctan2(0, d/sqrt(d^2))) - 7*(gamma(-7/2, 2*I*(d*x + c)*b/d) + gamma(-7/2, -2
*I*(d*x + c)*b/d))*sin(-7/4*pi + 7/2*arctan2(0, b) + 7/2*arctan2(0, d/sqrt(
d^2))))*sin(-2*(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(7/2) - 1)/((d*x +
 c)^(7/2)*d)
```

Fricas [B] time = 2.89485, size = 941, normalized size = 3.81

$$-\frac{2\left(64\left(\pi b^3d^4x^4+4\,\pi b^3cd^3x^3+6\,\pi b^3c^2d^2x^2+4\,\pi b^3c^3dx+\pi b^3c^4\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-64\left(\pi b^3d^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(64*(pi*b^3*d^4*x^4 + 4*pi*b^3*c*d^3*x^3 + 6*pi*b^3*c^2*d^2*x^2 + 4*
pi*b^3*c^3*d*x + pi*b^3*c^4)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_c
os(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 64*(pi*b^3*d^4*x^4 + 4*pi*b^3*c*d^3*x^
3 + 6*pi*b^3*c^2*d^2*x^2 + 4*pi*b^3*c^3*d*x + pi*b^3*c^4)*sqrt(b/(pi*d))*fr
esnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - (8*b^2*d^
3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - 15*d^3 - (16*b^2*d^3*x^2 + 32*b^2*c*
d^2*x + 16*b^2*c^2*d - 15*d^3)*cos(b*x + a)^2 + 4*(16*b^3*d^3*x^3 + 48*b^3*
c*d^2*x^2 + 16*b^3*c^3 - 3*b*c*d^2 + 3*(16*b^3*c^2*d - b*d^3)*x)*cos(b*x +
a)*sin(b*x + a))*sqrt(d*x + c))/(d^8*x^4 + 4*c*d^7*x^3 + 6*c^2*d^6*x^2 + 4*
c^3*d^5*x + c^4*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^2}{(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*x + c)^(9/2), x)
```

3.53 $\int (c+dx)^{5/2}\sin^3(a+bx)\,dx$

Optimal. Leaf size=410

$$-\frac{45\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a\ \cdots}{14\cdots}$$

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - (2*(c + d*x)^(5/2)*Cos[a + b
*x])/(3*b) - (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - (45*d^(5/2)
*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sq
rt[d]])/(16*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[
(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(144*b^(7/2)) - (5*d^(5/2)*Sqr
t[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b
*c)/d])/(144*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]
*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3
/2)*Sin[a + b*x])/(3*b^2) - ((c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2)/(
3*b) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x]^3)/(18*b^2)
```

Rubi [A] time = 1.12658, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3311, 3296, 3306, 3305, 3351, 3304, 3352, 3312}

$$-\frac{45\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a\ \cdots}{14\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Sin[a + b*x]^3,x]
```

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - (2*(c + d*x)^(5/2)*Cos[a + b
*x])/(3*b) - (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - (45*d^(5/2)
*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sq
rt[d]])/(16*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[
(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(144*b^(7/2)) - (5*d^(5/2)*Sqr
t[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b
*c)/d])/(144*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]
*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3
/2)*Sin[a + b*x])/(3*b^2) - ((c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2)/(
3*b) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x]^3)/(18*b^2)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^{5/2}\sin^3(a+bx)\,dx = -\frac{(c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)}{3b} + \frac{5d(c+dx)^{3/2}\sin^3(a+bx)}{18b^2} + \frac{2}{3}\int (c+dx)^{5/2}\sin($$

$$= -\frac{2(c+dx)^{5/2}\cos(a+bx)}{3b} - \frac{(c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)}{3b} + \frac{5d(c+dx)^{3/2}\sin^3(a+}{18b^2}$$

$$= -\frac{2(c+dx)^{5/2}\cos(a+bx)}{3b} + \frac{5d(c+dx)^{3/2}\sin(a+bx)}{3b^2} - \frac{(c+dx)^{5/2}\cos(a+bx)\sin^2(a+}{3b}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cos(a+bx)}{3b} - \frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{144b^3} + \frac{5d(}{}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cos(a+bx)}{3b} - \frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{144b^3} + \frac{5d(}{}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cos(a+bx)}{3b} - \frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{144b^3} + \frac{5d(}{}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cos(a+bx)}{3b} - \frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{144b^3} - \frac{45d}{}$$

Mathematica [A] time = 3.1518, size = 542, normalized size = 1.32

$$\frac{-648b^3c^2\sqrt{c+dx}\cos(a+bx) + 72b^3c^2\sqrt{c+dx}\cos(3(a+bx)) - 648b^3d^2x^2\sqrt{c+dx}\cos(a+bx) + 72b^3d^2x^2\sqrt{c+dx}\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Sin[a + b*x]^3,x]
```

```
[Out] (-648*b^3*c^2*Sqrt[c + d*x]*Cos[a + b*x] + 2430*b*d^2*Sqrt[c + d*x]*Cos[a +
 b*x] - 1296*b^3*c*d*x*Sqrt[c + d*x]*Cos[a + b*x] - 648*b^3*d^2*x^2*Sqrt[c
+ d*x]*Cos[a + b*x] + 72*b^3*c^2*Sqrt[c + d*x]*Cos[3*(a + b*x)] - 30*b*d^2*
Sqrt[c + d*x]*Cos[3*(a + b*x)] + 144*b^3*c*d*x*Sqrt[c + d*x]*Cos[3*(a + b*x
)] + 72*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[3*(a + b*x)] - 1215*Sqrt[b/d]*d^3*Sqr
```

```
t[2*Pi]*Cos[a - (b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + 5*S
qrt[b/d]*d^3*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*
Sqrt[c + d*x]] - 5*Sqrt[b/d]*d^3*Sqrt[6*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*S
qrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 1215*Sqrt[b/d]*d^3*Sqrt[2*Pi]*FresnelS
[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[a - (b*c)/d] + 1620*b^2*c*d*Sqrt[c
 + d*x]*Sin[a + b*x] + 1620*b^2*d^2*x*Sqrt[c + d*x]*Sin[a + b*x] - 60*b^2*c
*d*Sqrt[c + d*x]*Sin[3*(a + b*x)] - 60*b^2*d^2*x*Sqrt[c + d*x]*Sin[3*(a + b
*x)])/(864*b^4)
```

Maple [A] time = 0.013, size = 476, normalized size = 1.2

$$2\,\frac{1}{d}\left(-3/8\,\frac{d\,(dx+c)^{5/2}}{b}\cos\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)+\frac{15\,d}{8\,b}\left(1/2\,\frac{d\,(dx+c)^{3/2}}{b}\sin\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)-3/2\,\frac{d}{b}\left(-1/2\,\frac{d\sqrt{dx}}{b}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/8/b*d*(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+15/8/b*d*(1/2/b*
d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1
/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin
((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1
/24/b*d*(d*x+c)^(5/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-5/24/b*d*(1/6/b*d*(d
*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/2
)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*
x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.20947, size = 1868, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3456*sqrt(3)*(80*sqrt(3)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(3*((d*x + c)*b
 - b*c + a*d)/d)/abs(d) - 2160*sqrt(3)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(((d
*x + c)*b - b*c + a*d)/d)/abs(d) - 8*(12*sqrt(3)*(d*x + c)^(5/2)*b^2*d*abs(
b)/abs(d) - 5*sqrt(3)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(3*((d*x + c)*b -
 b*c + a*d)/d) + 216*(4*sqrt(3)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sq
rt(3)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(((d*x + c)*b - b*c + a*d)/d) - (
(5*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 5*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
- 5*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) + 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))))*d^3*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (5*I*sqrt(pi)*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*cos(-
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)
/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(3)
*(405*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) + 405*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 405*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr
t(d^2))) + 405*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) + sqrt(3)*(-405*
I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
405*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - 405*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + 405*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqr
t(I*b/d)) + (sqrt(3)*(405*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) + 405*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) + 405*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) - 405*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)
/d) + sqrt(3)*(405*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + 405*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) - 405*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) + 405*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*er
f(sqrt(d*x + c)*sqrt(-I*b/d)) - ((5*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-3*(b*c
- a*d)/d) - (-5*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) - 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
```

```
(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(-3*I*b/d)))*abs(d)/(b^3*d*abs(b))
```

Fricas [A] time = 2.76394, size = 923, normalized size = 2.25

$$5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-1215\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+1215\sqrt{2}\pi d^3\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sq
rt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 1215*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 1215*
sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-(b*c - a*d)/d) - 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt
(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*((12*b^3*d^2*x
^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^3 - 3*(12*b^3*d^2*x^
2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2)*cos(b*x + a) + 10*(7*b^2*d^2*x +
7*b^2*c*d - (b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x +
c))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.62919, size = 2726, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/1728*(12*(sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b) - 27*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(
6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((3
*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^((I*(d*x +
c)*b - I*b*c + I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c
- I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)
/d)/b)*c^2 - d^2*((I*sqrt(6)*sqrt(pi)*(12*I*b^2*c^2*d - 12*b*c*d^2 - 5*I*d^
3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(
-12*I*(d*x + c)^(5/2)*b^2*d + 24*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x
+ c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)*b*c*d^2 + 5*I*
sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^2 +
27*(I*sqrt(2)*sqrt(pi)*(-12*I*b^2*c^2*d + 36*b*c*d^2 + 45*I*d^3)*d*erf(-1/2
*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(12*I*(d*x + c)^(5
/2)*b^2*d - 24*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d + 3
0*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*b*c*d^2 - 45*I*sqrt(d*x + c)*d^3
)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^2 + 27*(I*sqrt(2)*sqrt(pi)*
(-12*I*b^2*c^2*d - 36*b*c*d^2 + 45*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(d*
x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 30*(d*x + c)^(3/2)*b*
d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 45*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b
- I*b*c + I*a*d)/d)/b^3)/d^2 + (I*sqrt(6)*sqrt(pi)*(12*I*b^2*c^2*d + 12*b*c
*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b^3) - 6*I*(-12*I*(d*x + c)^(5/2)*b^2*d + 24*I*(d*x + c)^(3/2)*b^2*c*d
- 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c
)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)
/d)/b^3)/d^2) - 12*(I*sqrt(6)*sqrt(pi)*(-2*I*b*c*d + d^2)*d*erf(-1/2*sqrt(6
)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*I*sqrt(2)*sqrt(pi)*(6*I
*b*c*d - 9*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2
```

```
) + 9*I*sqrt(2)*sqrt(pi)*(6*I*b*c*d + 9*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + I*sqrt(6)*sqrt(pi)*(-2*I*b*c*d - d^2)*d
*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-
2*I*(d*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((
3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 - 18*I*(6*I*(d*x + c)^(3/2)*b*d
 - 6*I*sqrt(d*x + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c
 + I*a*d)/d)/b^2 - 18*I*(6*I*(d*x + c)^(3/2)*b*d - 6*I*sqrt(d*x + c)*b*c*d
+ 9*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 - 6*I*(-2
*I*(d*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((-
3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)*c)/d
```

3.54 $\int (c+dx)^{3/2}\sin^3(a+bx)\,dx$

Optimal. Leaf size=354

$$\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{9\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{9\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)}{8b^{5/2}}$$

```
[Out] (-2*(c + d*x)^(3/2)*Cos[a + b*x])/(3*b) - (9*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*
c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (
d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
 + d*x])/Sqrt[d]])/(24*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqr
t[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) - (9*d^(
3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(5/2)) + (d*Sqrt[c + d*x]*Sin[a + b*x])/b^2 - ((c + d*x)^(
3/2)*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) + (d*Sqrt[c + d*x]*Sin[a + b*x]^3)/
(6*b^2)
```

Rubi [A] time = 0.97218, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3311, 3296, 3306, 3305, 3351, 3304, 3352, 3312}

$$\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{9\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{9\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Sin[a + b*x]^3,x]
```


```
[Out] (-2*(c + d*x)^(3/2)*Cos[a + b*x])/(3*b) - (9*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*
c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (
d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
 + d*x])/Sqrt[d]])/(24*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqr
t[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) - (9*d^(
3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(5/2)) + (d*Sqrt[c + d*x]*Sin[a + b*x])/b^2 - ((c + d*x)^(
3/2)*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) + (d*Sqrt[c + d*x]*Sin[a + b*x]^3)/
(6*b^2)
```


<u>Rule 3311</u>

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^{3/2}\sin^3(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)}{3b}+\frac{d\sqrt{c+dx}\sin^3(a+bx)}{6b^2}+\frac{2}{3}\int (c+dx)^{3/2}\sin(a+$$

$$= -\frac{2(c+dx)^{3/2}\cos(a+bx)}{3b}-\frac{(c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)}{3b}+\frac{d\sqrt{c+dx}\sin^3(a+bx}{6b^2}$$

$$= -\frac{2(c+dx)^{3/2}\cos(a+bx)}{3b}+\frac{d\sqrt{c+dx}\sin(a+bx)}{b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)}{3b}$$

$$= -\frac{2(c+dx)^{3/2}\cos(a+bx)}{3b}+\frac{d\sqrt{c+dx}\sin(a+bx)}{b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)}{3b}$$

$$= -\frac{2(c+dx)^{3/2}\cos(a+bx)}{3b}+\frac{d\sqrt{c+dx}\sin(a+bx)}{b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)}{3b}$$

$$= -\frac{2(c+dx)^{3/2}\cos(a+bx)}{3b}-\frac{9d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3}{}\right.}{24b}$$

Mathematica [A] time = 1.63445, size = 389, normalized size = 1.1

$$\frac{\sqrt{6\pi}d\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)-81\sqrt{2\pi}d\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)-81\sqrt{2\pi}d\cos\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Sin[a + b*x]^3,x]
```

```
[Out] (-108*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[a + b*x] - 108*b*Sqrt[b/d]*d*x*Sqrt[c
 + d*x]*Cos[a + b*x] + 12*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] + 12
*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[3*(a + b*x)] - 81*d*Sqrt[2*Pi]*Cos[a - (
b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + d*Sqrt[6*Pi]*Cos[3*a
 - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + d*Sqrt[6*Pi]*F
resnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] - 81*d*Sqr
```

```
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[a - (b*c)/d] + 162
*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[a + b*x] - 6*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[3
*(a + b*x)])/(144*b^2*Sqrt[b/d])
```

Maple [A] time = 0.013, size = 384, normalized size = 1.1

$$2\frac{1}{d}\left(-3/8\frac{d\,(dx+c)^{3/2}}{b}\cos\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)+\frac{9\,d}{8\,b}\left(1/2\frac{d\sqrt{dx+c}}{b}\sin\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)-1/4\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\frac{da}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/8/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+9/8/b*d*(1/2/b*d
*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))+1/24/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/8/b*d*(1
/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1
/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 2.22983, size = 1790, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/576*sqrt(3)*(16*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*cos(3*((d*x + c)*b - b
*c + a*d)/d)/abs(d) - 144*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*cos(((d*x + c)
*b - b*c + a*d)/d)/abs(d) - 8*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*sin(3*((d*x
+ c)*b - b*c + a*d)/d)/abs(d) + 216*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*sin(((
d*x + c)*b - b*c + a*d)/d)/abs(d) - ((-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0
```

```
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a
*d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqr
t(3*I*b/d)) - (sqrt(3)*(27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) + 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d
) + sqrt(3)*(27*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) + 27*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) - 27*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + 27*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(I*b/d)) - (sqrt(3)*(-27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-(
b*c - a*d)/d) + sqrt(3)*(27*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))) + 27*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/
d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-3*(b*c -
a*d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(-3*I*b/d)))*abs(d)/(b^2*d*abs(b))
```

Fricas [A] time = 2.41929, size = 761, normalized size = 2.15

$$\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 81\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 81\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt
(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 81*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(
b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 81*sqrt(2
)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt
(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b^2*c)*c
os(b*x + a)^3 - 6*(b^2*d*x + b^2*c)*cos(b*x + a) - (b*d*cos(b*x + a)^2 - 7*
b*d)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.49296, size = 1513, normalized size = 4.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/288*(2*(sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) - 27*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a
```

```
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((3*I
*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^((I*(d*x + c)
*b - I*b*c + I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c -
I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d
)/b)*c - I*sqrt(6)*sqrt(pi)*(-2*I*b*c*d + d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 9*I*sqrt(2)*sqrt(pi)*(6*I*b*c*d - 9*
d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 9*I*sqr
t(2)*sqrt(pi)*(6*I*b*c*d + 9*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)*b^2) - I*sqrt(6)*sqrt(pi)*(-2*I*b*c*d - d^2)*d*erf(-1/2*s
qrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c +
 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*I*(-2*I*(d*x +
c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((3*I*(d*x +
c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 + 18*I*(6*I*(d*x + c)^(3/2)*b*d - 6*I*sqrt
(d*x + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d
)/b^2 + 18*I*(6*I*(d*x + c)^(3/2)*b*d - 6*I*sqrt(d*x + c)*b*c*d + 9*sqrt(d*
x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 + 6*I*(-2*I*(d*x + c
)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((-3*I*(d*x +
c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d
```

3.55 $\int \sqrt{c+dx}\sin^3(a+bx)\,dx$

Optimal. Leaf size=304

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)S}{12b^{3/2}}$$

```
[Out] (-3*Sqrt[c + d*x]*Cos[a + b*x])/(4*b) + (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(1
2*b) + (3*Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]])/(4*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c
)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin
[3*a - (3*b*c)/d])/(12*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*S
qrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Rubi [A] time = 0.497779, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3312, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)S}{12b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sin[a + b*x]^3,x]
```


```
[Out] (-3*Sqrt[c + d*x]*Cos[a + b*x])/(4*b) + (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(1
2*b) + (3*Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]])/(4*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c
)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin
[3*a - (3*b*c)/d])/(12*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*S
qrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```


Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\sin^3(a+bx)\,dx = \int \left(\frac{3}{4}\sqrt{c+dx}\sin(a+bx) - \frac{1}{4}\sqrt{c+dx}\sin(3a+3bx)\right)dx$$
$$= -\left(\frac{1}{4}\int \sqrt{c+dx}\sin(3a+3bx)\,dx\right) + \frac{3}{4}\int \sqrt{c+dx}\sin(a+bx)\,dx$$
$$= -\frac{3\sqrt{c+dx}\cos(a+bx)}{4b} + \frac{\sqrt{c+dx}\cos(3a+3bx)}{12b} - \frac{d\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}dx}{24b} + \frac{(3d)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}dx}{8b}$$
$$= -\frac{3\sqrt{c+dx}\cos(a+bx)}{4b} + \frac{\sqrt{c+dx}\cos(3a+3bx)}{12b} - \frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}dx}{24b} + \cdots$$
$$= -\frac{3\sqrt{c+dx}\cos(a+bx)}{4b} + \frac{\sqrt{c+dx}\cos(3a+3bx)}{12b} - \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Subst}\left(\int\cos\left(\frac{3bx^2}{d}\right)dx,\cdots\right.}{12b}$$
$$= -\frac{3\sqrt{c+dx}\cos(a+bx)}{4b} + \frac{\sqrt{c+dx}\cos(3a+3bx)}{12b} + \frac{3\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \cdots$$

Mathematica [A] time = 0.804446, size = 266, normalized size = 0.88

$$\frac{27\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) - \sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) + \sqrt{6\pi}\sin\left(3a-\frac{3bc}{d}\right)\cdots}{72b\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-54*Sqrt[b/d]*Sqrt[c + d*x]*Cos[a + b*x] + 6*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3
*(a + b*x)] + 27*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*
Sqrt[c + d*x]] - Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/
Pi]*Sqrt[c + d*x]] + Sqrt[6*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]
]*Sin[3*a - (3*b*c)/d] - 27*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c
 + d*x]]*Sin[a - (b*c)/d])/(72*b*Sqrt[b/d])
```

Maple [A] time = 0.01, size = 296, normalized size = 1.

$$2\frac{1}{d}\left(-3/8\frac{d\sqrt{dx+c}}{b}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right) + 3/16\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\frac{da-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right) - \sin\left(\frac{da-\cdots}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/8/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/16/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d))+1/24/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/
d)-1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)
/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.144, size = 1651, normalized size = 5.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/288*sqrt(3)*(8*sqrt(3)*sqrt(d*x + c)*d*abs(b)*cos(3*((d*x + c)*b - b*c +
a*d)/d)/abs(d) - 72*sqrt(3)*sqrt(d*x + c)*d*abs(b)*cos(((d*x + c)*b - b*c +
 a*d)/d)/abs(d) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (I*sqrt(pi)*
cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/
abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(3)*
(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
- 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) + 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))))*d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) + sqrt(3)*(-9*I*sqrt(pi)*co
s(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)
*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs
(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(3)*
```

```
(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) - 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))))*d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) + sqrt(3)*(9*I*sqrt(pi)*cos
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*c
os(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*
sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(
b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((sqrt(pi
)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)
*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs
(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)
/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*abs(d)/(b*d*abs(b))
```

Fricas [A] time = 2.31985, size = 645, normalized size = 2.12

$$-\frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-27\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\mathrm{S}\left(\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c
- a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi
*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b
*c - a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c
)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^3 - 3*b*cos(b*
x + a))*sqrt(d*x + c))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+dx}\sin^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*sin(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3, x)
```

Giac [C] time = 1.28787, size = 659, normalized size = 2.17

$$\frac{\sqrt{6}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{3ibc-3iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b} - \frac{27\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b} - \frac{27\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{-ib\cdots}{}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/144*(sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) - 27*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)*b) - 27*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((3*I*(d
*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^((I*(d*x + c)*b
- I*b*c + I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a
*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b
)/d
```

3.56 $\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=257

$$-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+}}{\sqrt{d}}\right.}{2\sqrt{b}\sqrt{d}}$$

```
[Out] (3*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/
Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(
Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/
6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(2*Sqrt[b]*Sqrt[d]) + (3*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(2*Sqrt[b]*Sqrt[d])
```

Rubi [A] time = 0.404988, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3312, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+}}{\sqrt{d}}\right.}{2\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] (3*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/
Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(
Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/
6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(2*Sqrt[b]*Sqrt[d]) + (3*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(2*Sqrt[b]*Sqrt[d])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \left(\frac{3\sin(a+bx)}{4\sqrt{c+dx}} - \frac{\sin(3a+3bx)}{4\sqrt{c+dx}}\right) dx$$
$$= -\left(\frac{1}{4}\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}\,dx\right) + \frac{3}{4}\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx$$
$$= -\left(\frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx\right) + \frac{1}{4}\left(3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx - \frac{1}{4}\sin\left(3a-\right.$$
$$= -\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Subst}\left(\int \sin\left(\frac{3bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{2d} + \frac{\left(3\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)dx, x, \sqrt{\right.}{}}{2d}$$
$$= \frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} - \frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} - \frac{\sqrt{\frac{\pi}{6}}C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a\right.}{2\sqrt{b}\sqrt{d}}$$

Mathematica [A] time = 0.559629, size = 202, normalized size = 0.79

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{\frac{b}{d}}\left(\sqrt{3}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) - 9\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) - 9\cos\left(a-\frac{bc}{d}\right)S\left(\right.\right.}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] -(Sqrt[b/d]*Sqrt[Pi/2]*(-9*Cos[a - (b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*S
qrt[c + d*x]] + Sqrt[3]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*
Sqrt[c + d*x]] + Sqrt[3]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3
*a - (3*b*c)/d] - 9*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[a - (b
*c)/d]))/(6*b)
```

Maple [A] time = 0.015, size = 210, normalized size = 0.8

$$2\frac{1}{d}\left(3/8\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right) + \sin\left(\frac{da-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)\right)\frac{1}{\sqrt{\frac{b}{d}}} - 1/24\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*x+c)^(1/2),x)
```

```
[Out] 2/d*(3/8*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^
(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/24*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d))
```

Maxima [C] time = 2.1193, size = 1527, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/48*sqrt(3)*(((-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))))*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (sqrt(pi)*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sqrt(abs(b)/abs(d))
*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(3)*(3*I*sq
rt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*
sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3
*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3
*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*s
qrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) + sqrt(3)*(3*sqrt(pi)*cos(1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sqrt(abs(b)/abs(d))*
sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(3)*(-3*I*sqrt(p
i)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt
(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqr
t(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqr
t(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sqrt(
abs(b)/abs(d))*cos(-(b*c - a*d)/d) + sqrt(3)*(3*sqrt(pi)*cos(1/4*pi + 1/2*a
```

```
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sqrt(abs(b)/abs(d))*sin(
-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I*sqrt(pi)*cos(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sqrt(abs(b)/abs(d))*cos(-
3*(b*c - a*d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))))*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x +
c)*sqrt(-3*I*b/d)))*abs(d)/(d*abs(b))
```

Fricas [A] time = 2.17241, size = 552, normalized size = 2.15

$$-\frac{\sqrt{6}\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\mathrm{C}\left(\sqrt{2}\sqrt{dx+\cdots}\right.}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(6)*pi*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*
sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*sqrt(b/(pi*d))*cos(-(b*c - a*d
)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*sqrt(
b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d
)/d) + sqrt(6)*pi*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(
pi*d)))*sin(-3*(b*c - a*d)/d))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sin(a + b*x)**3/sqrt(c + d*x), x)
```

Giac [C] time = 1.22059, size = 446, normalized size = 1.74

$$-\frac{-\frac{i\sqrt{6}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{3i\,bc-3i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{9i\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}-\frac{9i\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{-i}{}\right.}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(-I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) + 9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) - 9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))/d
```

3.57 $\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=270

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{b}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\frac{3\pi}{2}}\sqrt{b}\sin\left(3a-\frac{3bc}{d}\right)S}{d^{3/2}}$$

```
[Out] (3*Sqrt[b]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/d^(3/2) - (Sqrt[b]*Sqrt[(3*Pi)/2]*Cos[3*a - (3*b*c)/d]*Fr
esnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[b]*Sqrt
[(3*Pi)/2]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d])/d^(3/2) - (3*Sqrt[b]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqr
t[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/d^(3/2) - (2*Sin[a + b*x]^3)/(d*Sqrt
[c + d*x])
```

Rubi [A] time = 0.563834, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3313, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{b}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\frac{3\pi}{2}}\sqrt{b}\sin\left(3a-\frac{3bc}{d}\right)S}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (3*Sqrt[b]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/d^(3/2) - (Sqrt[b]*Sqrt[(3*Pi)/2]*Cos[3*a - (3*b*c)/d]*Fr
esnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[b]*Sqrt
[(3*Pi)/2]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d])/d^(3/2) - (3*Sqrt[b]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqr
t[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/d^(3/2) - (2*Sin[a + b*x]^3)/(d*Sqrt
[c + d*x])
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
```

```
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\sin^3(a+bx)}{d\sqrt{c+dx}} + \frac{(6b)\int\left(\frac{\cos(a+bx)}{4\sqrt{c+dx}} - \frac{\cos(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d}$$

$$= -\frac{2\sin^3(a+bx)}{d\sqrt{c+dx}} + \frac{(3b)\int\frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{2d} - \frac{(3b)\int\frac{\cos(3a+3bx)}{\sqrt{c+dx}}\,dx}{2d}$$

$$= -\frac{2\sin^3(a+bx)}{d\sqrt{c+dx}} - \frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int\frac{\cos\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx}{2d} + \frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{2d} + \frac{(3b\,\mathrm{si}}{}$$

$$= -\frac{2\sin^3(a+bx)}{d\sqrt{c+dx}} - \frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\mathrm{Subst}\left(\int\cos\left(\frac{3bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{d^2} + \frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\mathrm{Sub}}{}$$

$$= \frac{3\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{b}\sqrt{\frac{3\pi}{2}}S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c}}{\sqrt{d}}\right.}{d^{3}}$$

Mathematica [A] time = 1.00836, size = 300, normalized size = 1.11

$$\frac{3\sqrt{2\pi}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos\left(a-\frac{bc}{d}\right)\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) - \sqrt{6\pi}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos\left(3a-\frac{3bc}{d}\right)\mathrm{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (3*Sqrt[b/d]*Sqrt[2*Pi]*Sqrt[c + d*x]*Cos[a - (b*c)/d]*FresnelC[Sqrt[b/d]*S
qrt[2/Pi]*Sqrt[c + d*x]] - Sqrt[b/d]*Sqrt[6*Pi]*Sqrt[c + d*x]*Cos[3*a - (3*
b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[b/d]*Sqrt[6*Pi]
*Sqrt[c + d*x]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*
c)/d] - 3*Sqrt[b/d]*Sqrt[2*Pi]*Sqrt[c + d*x]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*
Sqrt[c + d*x]]*Sin[a - (b*c)/d] - 3*Sin[a + b*x] + Sin[3*(a + b*x)])/(2*d*S
qrt[c + d*x])
```

Maple [A] time = 0.013, size = 288, normalized size = 1.1

$$2\frac{1}{d}\left(-3/4\frac{1}{\sqrt{dx+c}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)+3/4\frac{b\sqrt{2}\sqrt{\pi}}{d}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(\frac{da-cb}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*x+c)^(3/2),x)
```

```
[Out] 2/d*(-3/4/(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/4*b/d*2^(1/2)*Pi^(
1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d
*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*
x+c)^(1/2)*b/d))+1/4/(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/4*b/d
*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS
(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))
```

Maxima [C] time = 1.49037, size = 1264, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(sqrt(3)*(((I*gamma(-1/2, 3*I*(d*x + c)*b/d) - I*gamma(-1/2, -3*I*(d*x
 + c)*b/d))*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 (I*gamma(-1/2, 3*I*(d*x + c)*b/d) - I*gamma(-1/2, -3*I*(d*x + c)*b/d))*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - (gamma(-1/2,
3*I*(d*x + c)*b/d) + gamma(-1/2, -3*I*(d*x + c)*b/d))*sin(1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (gamma(-1/2, 3*I*(d*x + c)*b/d)
+ gamma(-1/2, -3*I*(d*x + c)*b/d))*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))))*cos(-3*(b*c - a*d)/d) + ((gamma(-1/2, 3*I*(d*x + c)
*b/d) + gamma(-1/2, -3*I*(d*x + c)*b/d))*cos(1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) + (gamma(-1/2, 3*I*(d*x + c)*b/d) + gamma(-1/2,
 -3*I*(d*x + c)*b/d))*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + (I*gamma(-1/2, 3*I*(d*x + c)*b/d) - I*gamma(-1/2, -3*I*(d*x + c
)*b/d))*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (-I
*gamma(-1/2, 3*I*(d*x + c)*b/d) + I*gamma(-1/2, -3*I*(d*x + c)*b/d))*sin(-1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sin(-3*(b*c - a*d
)/d))*sqrt((d*x + c)*abs(b)/abs(d)) + (((-3*I*gamma(-1/2, I*(d*x + c)*b/d)
+ 3*I*gamma(-1/2, -I*(d*x + c)*b/d))*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) + (-3*I*gamma(-1/2, I*(d*x + c)*b/d) + 3*I*gamma(-1
/2, -I*(d*x + c)*b/d))*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) + 3*(gamma(-1/2, I*(d*x + c)*b/d) + gamma(-1/2, -I*(d*x + c)*b/d
))*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*(gamma
(-1/2, I*(d*x + c)*b/d) + gamma(-1/2, -I*(d*x + c)*b/d))*sin(-1/4*pi + 1/2*
```

```
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*cos(-(b*c - a*d)/d) - (3*(gam
ma(-1/2, I*(d*x + c)*b/d) + gamma(-1/2, -I*(d*x + c)*b/d))*cos(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*(gamma(-1/2, I*(d*x + c)*
b/d) + gamma(-1/2, -I*(d*x + c)*b/d))*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))) - (-3*I*gamma(-1/2, I*(d*x + c)*b/d) + 3*I*gamma(
-1/2, -I*(d*x + c)*b/d))*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) - (3*I*gamma(-1/2, I*(d*x + c)*b/d) - 3*I*gamma(-1/2, -I*(d*x +
 c)*b/d))*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*s
in(-(b*c - a*d)/d))*sqrt((d*x + c)*abs(b)/abs(d)))/(sqrt(d*x + c)*d)
```

Fricas [A] time = 2.45337, size = 707, normalized size = 2.62

$$-\frac{\sqrt{6}(\pi dx + \pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 3\sqrt{2}(\pi dx + \pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 3\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(6)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_
cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqrt(2)*(pi*d*x + pi*c)*sqrt(
b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*
d))) + 3*sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*
x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - sqrt(6)*(pi*d*x + pi*c)*sqrt(b
/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*
d)/d) - 4*sqrt(d*x + c)*(cos(b*x + a)^2 - 1)*sin(b*x + a))/(d^2*x + c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)**3/(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^3}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^3/(d*x + c)^(3/2), x)
```

3.58 $\int \frac{\sin^3(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=292

$$\frac{\sqrt{6\pi}b^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{2\pi}b^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{2\pi}b^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\right.}{d^{5/2}}$$

```
[Out] -((b^(3/2)*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/d^(5/2)) + (b^(3/2)*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*Fresn
elS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(5/2) + (b^(3/2)*Sqrt[6*
Pi]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/
d])/d^(5/2) - (b^(3/2)*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x
])/Sqrt[d]]*Sin[a - (b*c)/d])/d^(5/2) - (4*b*Cos[a + b*x]*Sin[a + b*x]^2)/(
d^2*Sqrt[c + d*x]) - (2*Sin[a + b*x]^3)/(3*d*(c + d*x)^(3/2))
```

Rubi [A] time = 0.710384, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3314, 3306, 3305, 3351, 3304, 3352, 3312}

$$\frac{\sqrt{6\pi}b^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{2\pi}b^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{2\pi}b^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\right.}{d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(c + d*x)^(5/2),x]
```

```
[Out] -((b^(3/2)*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/d^(5/2)) + (b^(3/2)*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*Fresn
elS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(5/2) + (b^(3/2)*Sqrt[6*
Pi]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/
d])/d^(5/2) - (b^(3/2)*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x
])/Sqrt[d]]*Sin[a - (b*c)/d])/d^(5/2) - (4*b*Cos[a + b*x]*Sin[a + b*x]^2)/(
d^2*Sqrt[c + d*x]) - (2*Sin[a + b*x]^3)/(3*d*(c + d*x)^(3/2))
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
```

```
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{4b\cos(a+bx)\sin^2(a+bx)}{d^2\sqrt{c+dx}} - \frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{(8b^2)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{d^2} - \frac{(12b^2)\int \frac{\sin^3(a+bx)}{\sqrt{c+dx}}\,dx}{d^2}$$

$$= -\frac{4b\cos(a+bx)\sin^2(a+bx)}{d^2\sqrt{c+dx}} - \frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{(12b^2)\int \left(\frac{3\sin(a+bx)}{4\sqrt{c+dx}} - \frac{\sin(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d^2} + \frac{\left(8b^2\cos\left(a\right.\right.}{}$$

$$= -\frac{4b\cos(a+bx)\sin^2(a+bx)}{d^2\sqrt{c+dx}} - \frac{2\sin^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{(3b^2)\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}\,dx}{d^2} - \frac{(9b^2)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{d^2} + \frac{(16b}{}$$

$$= \frac{8b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{8b^{3/2}\sqrt{2\pi}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{5/2}} - \frac{4b\cos(a+bx)\sin}{d^2\sqrt{c+dx}}$$

$$= \frac{8b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{8b^{3/2}\sqrt{2\pi}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{5/2}} - \frac{4b\cos(a+bx)\sin}{d^2\sqrt{c+dx}}$$

$$= -\frac{b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{b^{3/2}\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{b^{3/2}\sqrt{6\pi}C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}}{}\right.}{}$$

Mathematica [A] time = 2.42588, size = 496, normalized size = 1.7

$$\frac{6\sqrt{6\pi}bdx\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)+6\sqrt{6\pi}bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\frac{b}{d}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(c + d*x)^(5/2),x]
```

```
[Out] (-6*b*c*Cos[a + b*x] - 6*b*d*x*Cos[a + b*x] + 6*b*c*Cos[3*(a + b*x)] + 6*b*
d*x*Cos[3*(a + b*x)] - 6*b*Sqrt[b/d]*Sqrt[2*Pi]*(c + d*x)^(3/2)*Cos[a - (b*
c)/d]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + 6*b*Sqrt[b/d]*Sqrt[6*P
i]*(c + d*x)^(3/2)*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[
c + d*x]] + 6*b*c*Sqrt[b/d]*Sqrt[6*Pi]*Sqrt[c + d*x]*FresnelC[Sqrt[b/d]*Sqr
t[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 6*b*Sqrt[b/d]*d*Sqrt[6*Pi]*x*
Sqrt[c + d*x]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c
)/d] - 6*b*c*Sqrt[b/d]*Sqrt[2*Pi]*Sqrt[c + d*x]*FresnelC[Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]]*Sin[a - (b*c)/d] - 6*b*Sqrt[b/d]*d*Sqrt[2*Pi]*x*Sqrt[c +
d*x]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[a - (b*c)/d] - 3*d*Si
```

```
n[a + b*x] + d*Sin[3*(a + b*x)])/(6*d^2*(c + d*x)^(3/2))
```

Maple [A] time = 0.011, size = 368, normalized size = 1.3

$$2\,\frac{1}{d}\left(-1/4\,\frac{1}{(dx+c)^{3/2}}\sin\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)+1/2\,\frac{b}{d}\left(-\frac{1}{\sqrt{dx+c}}\cos\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)-\frac{b\sqrt{2}\sqrt{\pi}}{d}\left(\cos\left(\frac{da-cb}{d}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*x+c)^(5/2),x)
```

```
[Out] 2/d*(-1/4/(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/2*b/d*(-1/(d*x+c)^
(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))+1/12
/(d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/2*b/d*(-1/(d*x+c)^(1/2)*c
os(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(c
os(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d
*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 1.51922, size = 1265, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*(3*sqrt(3)*(((I*gamma(-3/2, 3*I*(d*x + c)*b/d) - I*gamma(-3/2, -3*I*(d
*x + c)*b/d))*cos(3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2)))
 + (I*gamma(-3/2, 3*I*(d*x + c)*b/d) - I*gamma(-3/2, -3*I*(d*x + c)*b/d))*c
os(-3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) - (gamma(-3/2
, 3*I*(d*x + c)*b/d) + gamma(-3/2, -3*I*(d*x + c)*b/d))*sin(3/4*pi + 3/2*ar
ctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (gamma(-3/2, 3*I*(d*x + c)*b/d
) + gamma(-3/2, -3*I*(d*x + c)*b/d))*sin(-3/4*pi + 3/2*arctan2(0, b) + 3/2*
arctan2(0, d/sqrt(d^2))))*cos(-3*(b*c - a*d)/d) + ((gamma(-3/2, 3*I*(d*x +
c)*b/d) + gamma(-3/2, -3*I*(d*x + c)*b/d))*cos(3/4*pi + 3/2*arctan2(0, b) +
```

```
3/2*arctan2(0, d/sqrt(d^2))) + (gamma(-3/2, 3*I*(d*x + c)*b/d) + gamma(-3/
2, -3*I*(d*x + c)*b/d))*cos(-3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/
sqrt(d^2))) + (I*gamma(-3/2, 3*I*(d*x + c)*b/d) - I*gamma(-3/2, -3*I*(d*x +
 c)*b/d))*sin(3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (
-I*gamma(-3/2, 3*I*(d*x + c)*b/d) + I*gamma(-3/2, -3*I*(d*x + c)*b/d))*sin(
-3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))))*sin(-3*(b*c - a
*d)/d))*((d*x + c)*abs(b)/abs(d))^(3/2) + (((-3*I*gamma(-3/2, I*(d*x + c)*b
/d) + 3*I*gamma(-3/2, -I*(d*x + c)*b/d))*cos(3/4*pi + 3/2*arctan2(0, b) + 3
/2*arctan2(0, d/sqrt(d^2))) + (-3*I*gamma(-3/2, I*(d*x + c)*b/d) + 3*I*gamm
a(-3/2, -I*(d*x + c)*b/d))*cos(-3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0,
 d/sqrt(d^2))) + 3*(gamma(-3/2, I*(d*x + c)*b/d) + gamma(-3/2, -I*(d*x + c)
*b/d))*sin(3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) - 3*(g
amma(-3/2, I*(d*x + c)*b/d) + gamma(-3/2, -I*(d*x + c)*b/d))*sin(-3/4*pi +
3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))))*cos(-(b*c - a*d)/d) - (3*
(gamma(-3/2, I*(d*x + c)*b/d) + gamma(-3/2, -I*(d*x + c)*b/d))*cos(3/4*pi +
 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + 3*(gamma(-3/2, I*(d*x +
 c)*b/d) + gamma(-3/2, -I*(d*x + c)*b/d))*cos(-3/4*pi + 3/2*arctan2(0, b) +
 3/2*arctan2(0, d/sqrt(d^2))) - (-3*I*gamma(-3/2, I*(d*x + c)*b/d) + 3*I*ga
mma(-3/2, -I*(d*x + c)*b/d))*sin(3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0
, d/sqrt(d^2))) - (3*I*gamma(-3/2, I*(d*x + c)*b/d) - 3*I*gamma(-3/2, -I*(d
*x + c)*b/d))*sin(-3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))
))*sin(-(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(3/2))/((d*x + c)^(3/2)*d
)
```

Fricas [A] time = 2.66328, size = 963, normalized size = 3.3

$$\frac{3\sqrt{6}\left(\pi bd^2x^2 + 2\pi bcdx + \pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 3\sqrt{2}\left(\pi bd^2x^2 + 2\pi bcdx + \pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{b}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*sqrt(6)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(
-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqr
t(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(-(b*c - a*
d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqrt(2)*(pi*b*d
^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d
*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 3*sqrt(6)*(pi*b*d^2*x^2 + 2*p
i*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 2*(6*(b*d*x + b*c)*cos(b*x + a)^3 - 6*(
```

```
b*d*x + b*c)*cos(b*x + a) + (d*cos(b*x + a)^2 - d)*sin(b*x + a))*sqrt(d*x +
 c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sin(a + b*x)**3/(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^3}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^3/(d*x + c)^(5/2), x)
```

3.59 $\int \frac{\sin^3(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=356

$$-\frac{2\sqrt{2\pi}b^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{6\sqrt{6\pi}b^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{6\sqrt{6\pi}b^{5/2}\sin\left(3a-\right.}{5}$$

```
[Out] (-2*b^(5/2)*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
 + d*x])/Sqrt[d]])/(5*d^(7/2)) + (6*b^(5/2)*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]
*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) - (6*b^(
5/2)*Sqrt[6*Pi]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*
a - (3*b*c)/d])/(5*d^(7/2)) + (2*b^(5/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(5*d^(7/2)) - (16*b^2*Sin[a
 + b*x])/(5*d^3*Sqrt[c + d*x]) - (4*b*Cos[a + b*x]*Sin[a + b*x]^2)/(5*d^2*(
c + d*x)^(3/2)) - (2*Sin[a + b*x]^3)/(5*d*(c + d*x)^(5/2)) + (24*b^2*Sin[a
+ b*x]^3)/(5*d^3*Sqrt[c + d*x])
```

Rubi [A] time = 0.796731, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3314, 3297, 3306, 3305, 3351, 3304, 3352, 3313}

$$-\frac{2\sqrt{2\pi}b^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{6\sqrt{6\pi}b^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{6\sqrt{6\pi}b^{5/2}\sin\left(3a-\right.}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(c + d*x)^(7/2),x]
```


```
[Out] (-2*b^(5/2)*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
 + d*x])/Sqrt[d]])/(5*d^(7/2)) + (6*b^(5/2)*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]
*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) - (6*b^(
5/2)*Sqrt[6*Pi]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*
a - (3*b*c)/d])/(5*d^(7/2)) + (2*b^(5/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(5*d^(7/2)) - (16*b^2*Sin[a
 + b*x])/(5*d^3*Sqrt[c + d*x]) - (4*b*Cos[a + b*x]*Sin[a + b*x]^2)/(5*d^2*(
c + d*x)^(3/2)) - (2*Sin[a + b*x]^3)/(5*d*(c + d*x)^(5/2)) + (24*b^2*Sin[a
+ b*x]^3)/(5*d^3*Sqrt[c + d*x])
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{4b\cos(a+bx)\sin^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{\left(8b^2\right)\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx}{5d^2} - \frac{\left(12b^2\right)\int \frac{\sin^3(a+bx)}{(c+dx)^{3/2}}\,dx}{5d^2}$$

$$= -\frac{16b^2\sin(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cos(a+bx)\sin^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\sin^3(a+bx)}{5d^3\sqrt{c+dx}} + \frac{\left(16b^3\right)\int \frac{\text{c}}{}}{5d}$$

$$= -\frac{16b^2\sin(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cos(a+bx)\sin^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\sin^3(a+bx)}{5d^3\sqrt{c+dx}} - \frac{\left(18b^3\right)\int \frac{\text{c}}{}}{5d}$$

$$= -\frac{16b^2\sin(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cos(a+bx)\sin^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sin^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\sin^3(a+bx)}{5d^3\sqrt{c+dx}} + \frac{\left(18b^3\cos\right.}{}$$

$$= \frac{16b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{16b^{5/2}\sqrt{2\pi}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{5d^{7/2}} - \frac{16b^2\sin(a+bx)}{5d^3\sqrt{c+dx}}$$

$$= -\frac{2b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{6b^{5/2}\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{6b^{5/2}\sqrt{6\pi}S\left(\right.}{}$$

Mathematica [B] time = 6.39826, size = 1429, normalized size = 4.01

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(c + d*x)^(7/2),x]
```

```
[Out] (3*(Cos[a]*((2*(b/d)^(5/2)*Sin[(b*c)/d]*(Cos[(b*(c + d*x))/d]/((b/d)^(5/2)*
(c + d*x)^(5/2)) - (2*(2*(Cos[(b*(c + d*x))/d]/(Sqrt[b/d]*Sqrt[c + d*x]) +
```

```
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]) + Sin[(b*(c + d*x)
)/d]/((b/d)^(3/2)*(c + d*x)^(3/2))))/3))/(5*d) - (2*(b/d)^(5/2)*Cos[(b*c)/d
]*(Sin[(b*(c + d*x))/d]/((b/d)^(5/2)*(c + d*x)^(5/2)) + (2*(Cos[(b*(c + d*x
))/d]/((b/d)^(3/2)*(c + d*x)^(3/2)) - 2*(-(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sq
rt[2/Pi]*Sqrt[c + d*x]]) + Sin[(b*(c + d*x))/d]/(Sqrt[b/d]*Sqrt[c + d*x])))
)/3))/(5*d)) + Sin[a]*((-2*(b/d)^(5/2)*Cos[(b*c)/d]*(Cos[(b*(c + d*x))/d]/(
(b/d)^(5/2)*(c + d*x)^(5/2)) - (2*(2*(Cos[(b*(c + d*x))/d]/(Sqrt[b/d]*Sqrt[
c + d*x]) + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]) + Sin[
(b*(c + d*x))/d]/((b/d)^(3/2)*(c + d*x)^(3/2))))/3))/(5*d) - (2*(b/d)^(5/2)
*Sin[(b*c)/d]*(Sin[(b*(c + d*x))/d]/((b/d)^(5/2)*(c + d*x)^(5/2)) + (2*(Cos
[(b*(c + d*x))/d]/((b/d)^(3/2)*(c + d*x)^(3/2)) - 2*(-(Sqrt[2*Pi]*FresnelC[
Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]) + Sin[(b*(c + d*x))/d]/(Sqrt[b/d]*Sqrt
[c + d*x]))))/3))/(5*d))))/4 + (-(Cos[3*a]*((18*Sqrt[3]*(b/d)^(5/2)*Sin[(3*
b*c)/d]*(Cos[(3*b*(c + d*x))/d]/(9*Sqrt[3]*(b/d)^(5/2)*(c + d*x)^(5/2)) - (
2*(2*(Cos[(3*b*(c + d*x))/d]/(Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]) + Sqrt[2*Pi]
*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) + Sin[(3*b*(c + d*x))/d]/(3*
Sqrt[3]*(b/d)^(3/2)*(c + d*x)^(3/2))))/3))/(5*d) - (18*Sqrt[3]*(b/d)^(5/2)*
Cos[(3*b*c)/d]*(Sin[(3*b*(c + d*x))/d]/(9*Sqrt[3]*(b/d)^(5/2)*(c + d*x)^(5/
2)) + (2*(Cos[(3*b*(c + d*x))/d]/(3*Sqrt[3]*(b/d)^(3/2)*(c + d*x)^(3/2)) -
2*(-(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) + Sin[(3*b*(c
 + d*x))/d]/(Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]))))/3))/(5*d))) - Sin[3*a]*((-
18*Sqrt[3]*(b/d)^(5/2)*Cos[(3*b*c)/d]*(Cos[(3*b*(c + d*x))/d]/(9*Sqrt[3]*(b
/d)^(5/2)*(c + d*x)^(5/2)) - (2*(2*(Cos[(3*b*(c + d*x))/d]/(Sqrt[3]*Sqrt[b/
d]*Sqrt[c + d*x]) + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]
) + Sin[(3*b*(c + d*x))/d]/(3*Sqrt[3]*(b/d)^(3/2)*(c + d*x)^(3/2))))/3))/(5
*d) - (18*Sqrt[3]*(b/d)^(5/2)*Sin[(3*b*c)/d]*(Sin[(3*b*(c + d*x))/d]/(9*Sqr
t[3]*(b/d)^(5/2)*(c + d*x)^(5/2)) + (2*(Cos[(3*b*(c + d*x))/d]/(3*Sqrt[3]*(
b/d)^(3/2)*(c + d*x)^(3/2)) - 2*(-(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]
*Sqrt[c + d*x]]) + Sin[(3*b*(c + d*x))/d]/(Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x])
)))/3))/(5*d)))/4
```

Maple [A] time = 0.012, size = 450, normalized size = 1.3

$$2\,\frac{1}{d}\left(-\frac{3}{20\,(dx+c)^{5/2}}\sin\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)+3/10\,\frac{b}{d}\left(-1/3\,\frac{1}{(dx+c)^{3/2}}\cos\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)-2/3\,\frac{b}{d}\left(-\frac{1}{\sqrt{dx+}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*x+c)^(7/2),x)
```

```
[Out] 2/d*(-3/20/(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/10*b/d*(-1/3/(d*x
+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)-2/3*b/d*(-1/(d*x+c)^(1/2)*sin(1/d*
```

```
(d*x+c)*b+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*F
resnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*Fr
esnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1/20/(d*x+c)^(5/2
)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-3/10*b/d*(-1/3/(d*x+c)^(3/2)*cos(3/d*(d*
x+c)*b+3*(a*d-b*c)/d)-2*b/d*(-1/(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)
/d)+b/d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2
^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*F
resnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 1.48198, size = 1265, normalized size = 3.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/16*(9*sqrt(3)*(((I*gamma(-5/2, 3*I*(d*x + c)*b/d) - I*gamma(-5/2, -3*I*(d
*x + c)*b/d))*cos(5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2)))
 + (I*gamma(-5/2, 3*I*(d*x + c)*b/d) - I*gamma(-5/2, -3*I*(d*x + c)*b/d))*c
os(-5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) - (gamma(-5/2
, 3*I*(d*x + c)*b/d) + gamma(-5/2, -3*I*(d*x + c)*b/d))*sin(5/4*pi + 5/2*ar
ctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (gamma(-5/2, 3*I*(d*x + c)*b/d
) + gamma(-5/2, -3*I*(d*x + c)*b/d))*sin(-5/4*pi + 5/2*arctan2(0, b) + 5/2*
arctan2(0, d/sqrt(d^2))))*cos(-3*(b*c - a*d)/d) + ((gamma(-5/2, 3*I*(d*x +
c)*b/d) + gamma(-5/2, -3*I*(d*x + c)*b/d))*cos(5/4*pi + 5/2*arctan2(0, b) +
 5/2*arctan2(0, d/sqrt(d^2))) + (gamma(-5/2, 3*I*(d*x + c)*b/d) + gamma(-5/
2, -3*I*(d*x + c)*b/d))*cos(-5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/
sqrt(d^2))) + (I*gamma(-5/2, 3*I*(d*x + c)*b/d) - I*gamma(-5/2, -3*I*(d*x +
 c)*b/d))*sin(5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (
-I*gamma(-5/2, 3*I*(d*x + c)*b/d) + I*gamma(-5/2, -3*I*(d*x + c)*b/d))*sin(
-5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))))*sin(-3*(b*c - a
*d)/d))*((d*x + c)*abs(b)/abs(d))^(5/2) + (((-3*I*gamma(-5/2, I*(d*x + c)*b
/d) + 3*I*gamma(-5/2, -I*(d*x + c)*b/d))*cos(5/4*pi + 5/2*arctan2(0, b) + 5
/2*arctan2(0, d/sqrt(d^2))) + (-3*I*gamma(-5/2, I*(d*x + c)*b/d) + 3*I*gamm
a(-5/2, -I*(d*x + c)*b/d))*cos(-5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0,
 d/sqrt(d^2))) + 3*(gamma(-5/2, I*(d*x + c)*b/d) + gamma(-5/2, -I*(d*x + c)
*b/d))*sin(5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) - 3*(g
amma(-5/2, I*(d*x + c)*b/d) + gamma(-5/2, -I*(d*x + c)*b/d))*sin(-5/4*pi +
5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))))*cos(-(b*c - a*d)/d) - (3*
(gamma(-5/2, I*(d*x + c)*b/d) + gamma(-5/2, -I*(d*x + c)*b/d))*cos(5/4*pi +
 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + 3*(gamma(-5/2, I*(d*x +
```

```
 c)*b/d) + gamma(-5/2, -I*(d*x + c)*b/d))*cos(-5/4*pi + 5/2*arctan2(0, b) +
 5/2*arctan2(0, d/sqrt(d^2))) - (-3*I*gamma(-5/2, I*(d*x + c)*b/d) + 3*I*ga
mma(-5/2, -I*(d*x + c)*b/d))*sin(5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0
, d/sqrt(d^2))) - (3*I*gamma(-5/2, I*(d*x + c)*b/d) - 3*I*gamma(-5/2, -I*(d
*x + c)*b/d))*sin(-5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))
))*sin(-(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(5/2))/((d*x + c)^(5/2)*d
)
```

Fricas [A] time = 3.42978, size = 1269, normalized size = 3.56

$$\frac{2\left(3\sqrt{6}\left(\pi b^2d^3x^3+3\,\pi b^2cd^2x^2+3\,\pi b^2c^2dx+\pi b^2c^3\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\left(\pi b^2d^3x^3+3\,\pi b^2c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*sqrt(6)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi
*b^2*c^3)*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) - sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi
*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(s
qrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c
*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_sin(sqrt(2
)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 3*sqrt(6)*(pi*b^2*d^3
*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*f
resnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + (2
*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 2*(b*d^2*x + b*c*d)*cos(b*x + a) + (4*b
^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - (12*b^2*d^2*x^2 + 24*b^2*c*d*x + 12*
b^2*c^2 - d^2)*cos(b*x + a)^2 - d^2)*sin(b*x + a))*sqrt(d*x + c))/(d^6*x^3
+ 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^3}{(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^3/(d*x + c)^(7/2), x)
```

3.60 $\int (dx)^{3/2}\sin(fx)\,dx$

Optimal. Leaf size=87

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{2f^{5/2}}+\frac{3d\sqrt{dx}\sin(fx)}{2f^2}-\frac{(dx)^{3/2}\cos(fx)}{f}$$

```
[Out] -(((d*x)^(3/2)*Cos[f*x])/f) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[f]*Sqrt[
2/Pi]*Sqrt[d*x])/Sqrt[d]])/(2*f^(5/2)) + (3*d*Sqrt[d*x]*Sin[f*x])/(2*f^2)
```

Rubi [A] time = 0.109295, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3296, 3305, 3351}

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{2f^{5/2}}+\frac{3d\sqrt{dx}\sin(fx)}{2f^2}-\frac{(dx)^{3/2}\cos(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*Sin[f*x],x]
```

```
[Out] -(((d*x)^(3/2)*Cos[f*x])/f) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[f]*Sqrt[
2/Pi]*Sqrt[d*x])/Sqrt[d]])/(2*f^(5/2)) + (3*d*Sqrt[d*x]*Sin[f*x])/(2*f^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (dx)^{3/2}\sin(fx)\,dx &= -\frac{(dx)^{3/2}\cos(fx)}{f}+\frac{(3d)\int \sqrt{dx}\cos(fx)\,dx}{2f}\\
&= -\frac{(dx)^{3/2}\cos(fx)}{f}+\frac{3d\sqrt{dx}\sin(fx)}{2f^2}-\frac{\left(3d^2\right)\int \frac{\sin(fx)}{\sqrt{dx}}\,dx}{4f^2}\\
&= -\frac{(dx)^{3/2}\cos(fx)}{f}+\frac{3d\sqrt{dx}\sin(fx)}{2f^2}-\frac{(3d)\,\mathrm{Subst}\left(\int \sin\left(\frac{fx^2}{d}\right)dx,x,\sqrt{dx}\right)}{2f^2}\\
&= -\frac{(dx)^{3/2}\cos(fx)}{f}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{2f^{5/2}}+\frac{3d\sqrt{dx}\sin(fx)}{2f^2}
\end{aligned}$$

Mathematica [C] time = 0.0131099, size = 60, normalized size = 0.69

$$\frac{d^2\left(\sqrt{-ifx}\,\mathrm{Gamma}\left(\frac{5}{2},-ifx\right)+\sqrt{ifx}\,\mathrm{Gamma}\left(\frac{5}{2},ifx\right)\right)}{2f^3\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*Sin[f*x],x]
```

```
[Out] (d^2*(Sqrt[(-I)*f*x]*Gamma[5/2, (-I)*f*x] + Sqrt[I*f*x]*Gamma[5/2, I*f*x]))
/(2*f^3*Sqrt[d*x])
```

Maple [A] time = 0.01, size = 87, normalized size = 1.

$$2\,\frac{1}{d}\left(-1/2\,\frac{d\,(dx)^{3/2}\cos\left(fx\right)}{f}+3/2\,\frac{d}{f}\left(1/2\,\frac{\sqrt{dx}\sin\left(fx\right)d}{f}-1/4\,\frac{d\sqrt{2}\sqrt{\pi}}{f}\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx}f}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{f}{d}}}\right)\frac{1}{\sqrt{\frac{f}{d}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*sin(f*x),x)
```

```
[Out] 2/d*(-1/2*d/f*(d*x)^(3/2)*cos(f*x)+3/2*d/f*(1/2*d/f*(d*x)^(1/2)*sin(f*x)-1/
4*d/f*2^(1/2)*Pi^(1/2)/(1/d*f)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(1/d*f)^(1/2
)*(d*x)^(1/2)/d*f)))
```

Maxima [C] time = 1.76174, size = 423, normalized size = 4.86

$$-\frac{16\,(dx)^{\frac{3}{2}}\,df\sqrt{\frac{|f|}{|d|}}\cos\left(fx\right)-\left(-3i\,\sqrt{\pi}\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,\arctan\left(0,f\right)+\frac{1}{2}\,\arctan\left(0,\frac{d}{\sqrt{d^2}}\right)\right)-3i\,\sqrt{\pi}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,\arctan\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*sin(f*x),x, algorithm="maxima")
```

```
[Out] -1/16*(16*(d*x)^(3/2)*d*f*sqrt(abs(f)/abs(d))*cos(f*x) - (-3*I*sqrt(pi)*cos
(1/4*pi + 1/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*c
os(-1/4*pi + 1/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*
sin(1/4*pi + 1/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*erf(sqr
t(d*x)*sqrt(I*f/d)) - (3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, f) + 1/2*ar
ctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, f) + 1/2
*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, f) + 1/2
*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, f) + 1/
2*arctan2(0, d/sqrt(d^2))))*d^2*erf(sqrt(d*x)*sqrt(-I*f/d)) - 24*sqrt(d*x)*
d^2*sqrt(abs(f)/abs(d))*sin(f*x))/(d*f^2*sqrt(abs(f)/abs(d)))
```

Fricas [A] time = 2.29349, size = 192, normalized size = 2.21

$$-\frac{3\,\sqrt{2}\pi d^2\sqrt{\frac{f}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx}\sqrt{\frac{f}{\pi d}}\right)+2\left(2\,df^2x\cos\left(fx\right)-3\,df\sin\left(fx\right)\right)\sqrt{dx}}{4\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*sin(f*x),x, algorithm="fricas")
```

```
[Out] -1/4*(3*sqrt(2)*pi*d^2*sqrt(f/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x)*sqrt(f/
(pi*d))) + 2*(2*d*f^2*x*cos(f*x) - 3*d*f*sin(f*x))*sqrt(d*x))/f^3
```

Sympy [A] time = 126.238, size = 117, normalized size = 1.34

$$-\frac{7d^{\frac{3}{2}}x^{\frac{3}{2}}\cos\left(fx\right)\Gamma\left(\frac{7}{4}\right)}{4f\Gamma\left(\frac{11}{4}\right)}+\frac{21d^{\frac{3}{2}}\sqrt{x}\sin\left(fx\right)\Gamma\left(\frac{7}{4}\right)}{8f^{2}\Gamma\left(\frac{11}{4}\right)}-\frac{21\sqrt{2}\sqrt{\pi}d^{\frac{3}{2}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{7}{4}\right)}{16f^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*sin(f*x),x)
```

```
[Out] -7*d**(3/2)*x**(3/2)*cos(f*x)*gamma(7/4)/(4*f*gamma(11/4)) + 21*d**(3/2)*sq
rt(x)*sin(f*x)*gamma(7/4)/(8*f**2*gamma(11/4)) - 21*sqrt(2)*sqrt(pi)*d**(3/
2)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)/sqrt(pi))*gamma(7/4)/(16*f**(5/2)*gamma
(11/4))
```

Giac [C] time = 1.15531, size = 286, normalized size = 3.29

$$-\frac{-\frac{3i\sqrt{2}\sqrt{\pi}d^{3}\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{df}\sqrt{dx}\left(\frac{idf}{\sqrt{d^{2}f^{2}}}+1\right)}{2d}\right)}{\sqrt{df}\left(\frac{idf}{\sqrt{d^{2}f^{2}}}+1\right)f^{2}}+\frac{3i\sqrt{2}\sqrt{\pi}d^{3}\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{df}\sqrt{dx}\left(-\frac{idf}{\sqrt{d^{2}f^{2}}}+1\right)}{2d}\right)}{\sqrt{df}\left(-\frac{idf}{\sqrt{d^{2}f^{2}}}+1\right)f^{2}}-\frac{2i\left(2i\sqrt{dx}d^{2}fx-3\sqrt{dx}d^{2}\right)e^{(ifx)}}{f^{2}}-\frac{2i\left(2i\sqrt{dx}d^{2}fx+3\sqrt{dx}d^{2}\right)e^{(-ifx)}}{f^{2}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*sin(f*x),x, algorithm="giac")
```

```
[Out] -1/8*(-3*I*sqrt(2)*sqrt(pi)*d^3*erf(-1/2*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(I*d*f
/sqrt(d^2*f^2) + 1)/d)/(sqrt(d*f)*(I*d*f/sqrt(d^2*f^2) + 1)*f^2) + 3*I*sqrt
(2)*sqrt(pi)*d^3*erf(-1/2*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(-I*d*f/sqrt(d^2*f^2)
 + 1)/d)/(sqrt(d*f)*(-I*d*f/sqrt(d^2*f^2) + 1)*f^2) - 2*I*(2*I*sqrt(d*x)*d^
2*f*x - 3*sqrt(d*x)*d^2)*e^(I*f*x)/f^2 - 2*I*(2*I*sqrt(d*x)*d^2*f*x + 3*sqr
t(d*x)*d^2)*e^(-I*f*x)/f^2)/d
```

3.61 $\int \sqrt{dx} \sin(fx)\, dx$

Optimal. Leaf size=65

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{f^{3/2}}-\frac{\sqrt{dx}\cos(fx)}{f}$$

```
[Out] -((Sqrt[d*x]*Cos[f*x])/f) + (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[f]*Sqrt[2/Pi
]*Sqrt[d*x])/Sqrt[d]])/f^(3/2)
```

Rubi [A] time = 0.0579075, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3296, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{f^{3/2}}-\frac{\sqrt{dx}\cos(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*Sin[f*x],x]
```

```
[Out] -((Sqrt[d*x]*Cos[f*x])/f) + (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[f]*Sqrt[2/Pi
]*Sqrt[d*x])/Sqrt[d]])/f^(3/2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{dx}\sin(fx)\,dx = -\frac{\sqrt{dx}\cos(fx)}{f} + \frac{d\int \frac{\cos(fx)}{\sqrt{dx}}\,dx}{2f}$$
$$= -\frac{\sqrt{dx}\cos(fx)}{f} + \frac{\text{Subst}\left(\int \cos\left(\frac{fx^2}{d}\right)dx, x, \sqrt{dx}\right)}{f}$$
$$= -\frac{\sqrt{dx}\cos(fx)}{f} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}C\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{f^{3/2}}$$

Mathematica [C] time = 0.0114526, size = 69, normalized size = 1.06

$$-\frac{\sqrt{dx}\text{Gamma}\left(\frac{3}{2}, -ifx\right)}{2f\sqrt{-ifx}} - \frac{\sqrt{dx}\text{Gamma}\left(\frac{3}{2}, ifx\right)}{2f\sqrt{ifx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*Sin[f*x],x]
```

```
[Out] -(Sqrt[d*x]*Gamma[3/2, (-I)*f*x])/(2*f*Sqrt[(-I)*f*x]) - (Sqrt[d*x]*Gamma[3
/2, I*f*x])/(2*f*Sqrt[I*f*x])
```

Maple [A] time = 0.009, size = 65, normalized size = 1.

$$2\frac{1}{d}\left(-1/2\frac{d\sqrt{dx}\cos(fx)}{f} + 1/4\frac{d\sqrt{2}\sqrt{\pi}}{f}\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx}f}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{f}{d}}}\right)\frac{1}{\sqrt{\frac{f}{d}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*sin(f*x),x)
```

```
[Out] 2/d*(-1/2*d/f*(d*x)^(1/2)*cos(f*x)+1/4*d/f*2^(1/2)*Pi^(1/2)/(1/d*f)^(1/2)*F
resnelC(2^(1/2)/Pi^(1/2)/(1/d*f)^(1/2)*(d*x)^(1/2)/d*f))
```

Maxima [C] time = 1.7341, size = 379, normalized size = 5.83

$$-\frac{8\sqrt{dx}d\sqrt{\frac{|f|}{|d|}}\cos\left(fx\right)-\left(\sqrt{\pi}\cos\left(\frac{1}{4}\pi+\frac{1}{2}\arctan\left(0,f\right)+\frac{1}{2}\arctan\left(0,\frac{d}{\sqrt{d^2}}\right)\right)+\sqrt{\pi}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\arctan\left(0,f\right)+\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*sin(f*x),x, algorithm="maxima")
```

```
[Out] -1/8*(8*sqrt(d*x)*d*sqrt(abs(f)/abs(d))*cos(f*x) - (sqrt(pi)*cos(1/4*pi + 1
/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/
2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/
2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1
/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2))))*d*erf(sqrt(d*x)*sqrt(I*f/d
)) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2))
) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2)))
 + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2)))
 - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2))
))*d*erf(sqrt(d*x)*sqrt(-I*f/d)))/(d*f*sqrt(abs(f)/abs(d)))
```

Fricas [A] time = 2.34448, size = 149, normalized size = 2.29

$$\frac{\sqrt{2}\pi d\sqrt{\frac{f}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx}\sqrt{\frac{f}{\pi d}}\right)-2\sqrt{dx}f\cos\left(fx\right)}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*sin(f*x),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*d*sqrt(f/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x)*sqrt(f/(pi*d
))) - 2*sqrt(d*x)*f*cos(f*x))/f^2
```

Sympy [A] time = 3.11813, size = 85, normalized size = 1.31

$$-\frac{5\sqrt{d}\sqrt{x}\cos\left(fx\right)\Gamma\left(\frac{5}{4}\right)}{4f\Gamma\left(\frac{9}{4}\right)}+\frac{5\sqrt{2}\sqrt{\pi}\sqrt{d}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{8f^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*sin(f*x),x)
```

```
[Out] -5*sqrt(d)*sqrt(x)*cos(f*x)*gamma(5/4)/(4*f*gamma(9/4)) + 5*sqrt(2)*sqrt(pi
)*sqrt(d)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)/sqrt(pi))*gamma(5/4)/(8*f**(3/2)
*gamma(9/4))
```

Giac [C] time = 1.15394, size = 238, normalized size = 3.66

$$-\frac{\frac{\sqrt{2}\sqrt{\pi}d^2\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{df}\sqrt{dx}\left(\frac{idf}{\sqrt{d^2f^2}}+1\right)}{2d}\right)}{\sqrt{df}\left(\frac{idf}{\sqrt{d^2f^2}}+1\right)f}+\frac{\sqrt{2}\sqrt{\pi}d^2\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{df}\sqrt{dx}\left(-\frac{idf}{\sqrt{d^2f^2}}+1\right)}{2d}\right)}{\sqrt{df}\left(-\frac{idf}{\sqrt{d^2f^2}}+1\right)f}+\frac{2\sqrt{dx}de^{(ifx)}}{f}+\frac{2\sqrt{dx}de^{(-ifx)}}{f}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*sin(f*x),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(I*d*f/sqrt
(d^2*f^2) + 1)/d)/(sqrt(d*f)*(I*d*f/sqrt(d^2*f^2) + 1)*f) + sqrt(2)*sqrt(pi
)*d^2*erf(-1/2*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(-I*d*f/sqrt(d^2*f^2) + 1)/d)/(s
qrt(d*f)*(-I*d*f/sqrt(d^2*f^2) + 1)*f) + 2*sqrt(d*x)*d*e^(I*f*x)/f + 2*sqrt
(d*x)*d*e^(-I*f*x)/f)/d
```

3.62 $\int \frac{\sin(fx)}{\sqrt{dx}} dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2\pi}S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{f}}$$

```
[Out] (Sqrt[2*Pi]*FresnelS[(Sqrt[f]*Sqrt[2/Pi]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt
[f])
```

Rubi [A] time = 0.0344381, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3305, 3351}

$$\frac{\sqrt{2\pi}S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[f*x]/Sqrt[d*x],x]
```

```
[Out] (Sqrt[2*Pi]*FresnelS[(Sqrt[f]*Sqrt[2/Pi]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt
[f])
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin(fx)}{\sqrt{dx}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \sin\left(\frac{fx^2}{d}\right)dx, x, \sqrt{dx}\right)}{d}$$
$$= \frac{\sqrt{2\pi}S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{f}}$$

Mathematica [C] time = 0.0076831, size = 59, normalized size = 1.28

$$\frac{-\sqrt{-ifx}\,\mathrm{Gamma}\left(\frac{1}{2}, -ifx\right) - \sqrt{ifx}\,\mathrm{Gamma}\left(\frac{1}{2}, ifx\right)}{2f\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[f*x]/Sqrt[d*x],x]
```

```
[Out] (-(Sqrt[(-I)*f*x]*Gamma[1/2, (-I)*f*x]) - Sqrt[I*f*x]*Gamma[1/2, I*f*x])/(2
*f*Sqrt[d*x])
```

Maple [A] time = 0.007, size = 42, normalized size = 0.9

$$\frac{\sqrt{2}\sqrt{\pi}}{d}\,\mathrm{FresnelS}\left(\frac{\sqrt{2}f}{\sqrt{\pi}d}\sqrt{dx}\frac{1}{\sqrt{\frac{f}{d}}}\right)\frac{1}{\sqrt{\frac{f}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x)/(d*x)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*Pi^(1/2)/(1/d*f)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(1/d*f)^(1/2)*
(d*x)^(1/2)/d*f)
```

Maxima [C] time = 1.66492, size = 344, normalized size = 7.48

$$\frac{\left(i\sqrt{\pi}\cos\left(\frac{1}{4}\pi+\frac{1}{2}\arctan\left(0,f\right)+\frac{1}{2}\arctan\left(0,\frac{d}{\sqrt{d^2}}\right)\right)+i\sqrt{\pi}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\arctan\left(0,f\right)+\frac{1}{2}\arctan\left(0,\frac{d}{\sqrt{d^2}}\right)\right)+\sqrt{\pi}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*((I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2
))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^
2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2)
)) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2))
))*erf(sqrt(d*x)*sqrt(I*f/d)) + (-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, f)
 + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, f
) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, f)
+ 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, f) +
 1/2*arctan2(0, d/sqrt(d^2))))*erf(sqrt(d*x)*sqrt(-I*f/d)))/(d*sqrt(abs(f)/
abs(d)))
```

Fricas [A] time = 2.04256, size = 101, normalized size = 2.2

$$\frac{\sqrt{2}\pi\sqrt{\frac{f}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx}\sqrt{\frac{f}{\pi d}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*pi*sqrt(f/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x)*sqrt(f/(pi*d)))/f
```

Sympy [A] time = 1.38134, size = 54, normalized size = 1.17

$$\frac{3\sqrt{2}\sqrt{\pi}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{4\sqrt{d}\sqrt{f}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)**(1/2),x)
```

```
[Out] 3*sqrt(2)*sqrt(pi)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)/sqrt(pi))*gamma(3/4)/(4
*sqrt(d)*sqrt(f)*gamma(7/4))
```

Giac [C] time = 1.15571, size = 184, normalized size = 4.

$$-\frac{\frac{i\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{df}\sqrt{dx}\left(\frac{idf}{\sqrt{d^2f^2}}+1\right)}{2d}\right)}{\sqrt{df}\left(\frac{idf}{\sqrt{d^2f^2}}+1\right)}-\frac{i\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{df}\sqrt{dx}\left(-\frac{idf}{\sqrt{d^2f^2}}+1\right)}{2d}\right)}{\sqrt{df}\left(-\frac{idf}{\sqrt{d^2f^2}}+1\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(I*d*f/sqrt
(d^2*f^2) + 1)/d)/(sqrt(d*f)*(I*d*f/sqrt(d^2*f^2) + 1)) - I*sqrt(2)*sqrt(pi
)*d*erf(-1/2*sqrt(2)*sqrt(d*f)*sqrt(d*x)*(-I*d*f/sqrt(d^2*f^2) + 1)/d)/(sqr
t(d*f)*(-I*d*f/sqrt(d^2*f^2) + 1)))/d
```

3.63 $\int \frac{\sin(fx)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{2\sqrt{2\pi}\sqrt{f}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sin(fx)}{d\sqrt{dx}}$$

```
[Out] (2*Sqrt[f]*Sqrt[2*Pi]*FresnelC[(Sqrt[f]*Sqrt[2/Pi]*Sqrt[d*x])/Sqrt[d]])/d^(
3/2) - (2*Sin[f*x])/(d*Sqrt[d*x])
```

Rubi [A] time = 0.0649724, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3297, 3304, 3352}

$$\frac{2\sqrt{2\pi}\sqrt{f}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sin(fx)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[f*x]/(d*x)^(3/2),x]
```

```
[Out] (2*Sqrt[f]*Sqrt[2*Pi]*FresnelC[(Sqrt[f]*Sqrt[2/Pi]*Sqrt[d*x])/Sqrt[d]])/d^(
3/2) - (2*Sin[f*x])/(d*Sqrt[d*x])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(fx)}{(dx)^{3/2}}\,dx &= -\frac{2\sin(fx)}{d\sqrt{dx}}+\frac{(2f)\int \frac{\cos(fx)}{\sqrt{dx}}\,dx}{d}\\ &= -\frac{2\sin(fx)}{d\sqrt{dx}}+\frac{(4f)\,\text{Subst}\left(\int \cos\left(\frac{fx^2}{d}\right)\,dx, x, \sqrt{dx}\right)}{d^2}\\ &= \frac{2\sqrt{f}\sqrt{2\pi}C\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sin(fx)}{d\sqrt{dx}}\end{aligned}$$

Mathematica [C] time = 0.0226545, size = 64, normalized size = 1.

$$\frac{x\left(-i\sqrt{-ifx}\text{Gamma}\left(\frac{1}{2},-ifx\right)+i\sqrt{ifx}\text{Gamma}\left(\frac{1}{2},ifx\right)-2\sin(fx)\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[f*x]/(d*x)^(3/2),x]
```

```
[Out] (x*((-I)*Sqrt[(-I)*f*x]*Gamma[1/2, (-I)*f*x] + I*Sqrt[I*f*x]*Gamma[1/2, I*f
*x] - 2*Sin[f*x]))/(d*x)^(3/2)
```

Maple [A] time = 0.007, size = 60, normalized size = 0.9

$$2\,\frac{1}{d}\left(-\frac{\sin\left(fx\right)}{\sqrt{dx}}+\frac{f\sqrt{2}\sqrt{\pi}}{d}\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx}f}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{f}{d}}}\right)\frac{1}{\sqrt{\frac{f}{d}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x)/(d*x)^(3/2),x)
```

```
[Out] 2/d*(-sin(f*x)/(d*x)^(1/2)+1/d*f*2^(1/2)*Pi^(1/2)/(1/d*f)^(1/2)*FresnelC(2^
(1/2)/Pi^(1/2)/(1/d*f)^(1/2)*(d*x)^(1/2)/d*f))
```

Maxima [C] time = 1.15665, size = 231, normalized size = 3.61

$$\frac{\sqrt{\frac{dx|f|}{|d|}}\left(\left(-i\,\Gamma\left(-\frac{1}{2},i\,fx\right)+i\,\Gamma\left(-\frac{1}{2},-i\,fx\right)\right)\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,\arctan\left(0,f\right)+\frac{1}{2}\,\arctan\left(0,\frac{d}{\sqrt{d^2}}\right)\right)+\left(-i\,\Gamma\left(-\frac{1}{2},i\,fx\right)+i\,\Gamma\left(-\frac{1}{2},\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(d*x*abs(f)/abs(d))*((-I*gamma(-1/2, I*f*x) + I*gamma(-1/2, -I*f*x)
)*cos(1/4*pi + 1/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2))) + (-I*gamma
(-1/2, I*f*x) + I*gamma(-1/2, -I*f*x))*cos(-1/4*pi + 1/2*arctan2(0, f) + 1/
2*arctan2(0, d/sqrt(d^2))) + (gamma(-1/2, I*f*x) + gamma(-1/2, -I*f*x))*sin
(1/4*pi + 1/2*arctan2(0, f) + 1/2*arctan2(0, d/sqrt(d^2))) - (gamma(-1/2, I
*f*x) + gamma(-1/2, -I*f*x))*sin(-1/4*pi + 1/2*arctan2(0, f) + 1/2*arctan2(
0, d/sqrt(d^2))))/(sqrt(d*x)*d)
```

Fricas [A] time = 2.22915, size = 149, normalized size = 2.33

$$\frac{2\left(\sqrt{2}\pi dx\sqrt{\frac{f}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx}\sqrt{\frac{f}{\pi d}}\right)-\sqrt{dx}\sin\left(fx\right)\right)}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*pi*d*x*sqrt(f/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x)*sqrt(f/(pi*d
))) - sqrt(d*x)*sin(f*x))/(d^2*x)
```

Sympy [A] time = 7.55038, size = 80, normalized size = 1.25

$$\frac{\sqrt{2}\sqrt{\pi}\sqrt{f}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}-\frac{\sin\left(fx\right)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)**(3/2),x)
```

```
[Out] sqrt(2)*sqrt(pi)*sqrt(f)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)/sqrt(pi))*gamma(1
/4)/(2*d**(3/2)*gamma(5/4)) - sin(f*x)*gamma(1/4)/(2*d**(3/2)*sqrt(x)*gamma
(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x)/(d*x)^(3/2), x)
```

3.64 $\int \frac{\sin(fx)}{(dx)^{5/2}} dx$

Optimal. Leaf size=87

$$-\frac{4\sqrt{2\pi}f^{3/2}S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4f\cos(fx)}{3d^2\sqrt{dx}}-\frac{2\sin(fx)}{3d(dx)^{3/2}}$$

```
[Out] (-4*f*Cos[f*x])/(3*d^2*Sqrt[d*x]) - (4*f^(3/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[f]
*Sqrt[2/Pi]*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2)) - (2*Sin[f*x])/(3*d*(d*x)^(3/2
))
```

Rubi [A] time = 0.0926116, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3297, 3305, 3351}

$$-\frac{4\sqrt{2\pi}f^{3/2}S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4f\cos(fx)}{3d^2\sqrt{dx}}-\frac{2\sin(fx)}{3d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[f*x]/(d*x)^(5/2),x]
```

```
[Out] (-4*f*Cos[f*x])/(3*d^2*Sqrt[d*x]) - (4*f^(3/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[f]
*Sqrt[2/Pi]*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2)) - (2*Sin[f*x])/(3*d*(d*x)^(3/2
))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(fx)}{(dx)^{5/2}}\,dx &= -\frac{2\sin(fx)}{3d(dx)^{3/2}}+\frac{(2f)\int \frac{\cos(fx)}{(dx)^{3/2}}\,dx}{3d}\\ &= -\frac{4f\cos(fx)}{3d^2\sqrt{dx}}-\frac{2\sin(fx)}{3d(dx)^{3/2}}-\frac{\left(4f^2\right)\int \frac{\sin(fx)}{\sqrt{dx}}\,dx}{3d^2}\\ &= -\frac{4f\cos(fx)}{3d^2\sqrt{dx}}-\frac{2\sin(fx)}{3d(dx)^{3/2}}-\frac{\left(8f^2\right)\text{Subst}\left(\int \sin\left(\frac{fx^2}{d}\right)\,dx,x,\sqrt{dx}\right)}{3d^3}\\ &= -\frac{4f\cos(fx)}{3d^2\sqrt{dx}}-\frac{4f^{3/2}\sqrt{2\pi}S\left(\frac{\sqrt{f}\sqrt{\frac{2}{\pi}}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{2\sin(fx)}{3d(dx)^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.0849214, size = 111, normalized size = 1.28

$$-\frac{2x\sin(fx)}{3(dx)^{5/2}}+\frac{2fx^{5/2}\left(\frac{\sqrt{ifx}\text{Gamma}\left(\frac{1}{2},ifx\right)-e^{-ifx}}{\sqrt{x}}-\frac{e^{ifx}-\sqrt{-ifx}\text{Gamma}\left(\frac{1}{2},-ifx\right)}{\sqrt{x}}\right)}{3(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[f*x]/(d*x)^(5/2),x]
```

```
[Out] (2*f*x^(5/2)*(-((E^(I*f*x) - Sqrt[(-I)*f*x]*Gamma[1/2, (-I)*f*x])/Sqrt[x])
+ (-E^((-I)*f*x) + Sqrt[I*f*x]*Gamma[1/2, I*f*x])/Sqrt[x]))/(3*(d*x)^(5/2))
 - (2*x*Sin[f*x])/(3*(d*x)^(5/2))
```

Maple [A] time = 0.006, size = 79, normalized size = 0.9

$$2\,\frac{1}{d}\left(-1/3\,\frac{\sin\left(fx\right)}{(dx)^{3/2}}+2/3\,\frac{f}{d}\left(-\frac{\cos\left(fx\right)}{\sqrt{dx}}-\frac{f\sqrt{2}\sqrt{\pi}}{d}\text{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx}f}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{f}{d}}}\right)\frac{1}{\sqrt{\frac{f}{d}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x)/(d*x)^(5/2),x)
```

```
[Out] 2/d*(-1/3*sin(f*x)/(d*x)^(3/2)+2/3/d*f*(-1/(d*x)^(1/2)*cos(f*x)-1/d*f*2^(1/
2)*Pi^(1/2)/(1/d*f)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(1/d*f)^(1/2)*(d*x)^(1/
2)/d*f)))
```

Maxima [C] time = 1.17412, size = 231, normalized size = 2.66

$$\left(\frac{dx|f|}{|d|}\right)^{\frac{3}{2}}\left(\left(\left(-i\,\Gamma\left(-\frac{3}{2},i\,fx\right)+i\,\Gamma\left(-\frac{3}{2},-i\,fx\right)\right)\cos\left(\frac{3}{4}\,\pi+\frac{3}{2}\,\arctan\left(0,f\right)+\frac{3}{2}\,\arctan\left(0,\frac{d}{\sqrt{d^2}}\right)\right)+\left(-i\,\Gamma\left(-\frac{3}{2},i\,fx\right)+i\,\Gamma\left(-\frac{3}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(d*x*abs(f)/abs(d))^(3/2)*((-I*gamma(-3/2, I*f*x) + I*gamma(-3/2, -I*f*
x))*cos(3/4*pi + 3/2*arctan2(0, f) + 3/2*arctan2(0, d/sqrt(d^2))) + (-I*gam
ma(-3/2, I*f*x) + I*gamma(-3/2, -I*f*x))*cos(-3/4*pi + 3/2*arctan2(0, f) +
3/2*arctan2(0, d/sqrt(d^2))) + (gamma(-3/2, I*f*x) + gamma(-3/2, -I*f*x))*s
in(3/4*pi + 3/2*arctan2(0, f) + 3/2*arctan2(0, d/sqrt(d^2))) - (gamma(-3/2,
 I*f*x) + gamma(-3/2, -I*f*x))*sin(-3/4*pi + 3/2*arctan2(0, f) + 3/2*arctan
2(0, d/sqrt(d^2)))))/((d*x)^(3/2)*d)
```

Fricas [A] time = 2.28062, size = 189, normalized size = 2.17

$$-\frac{2\left(2\sqrt{2}\pi dfx^2\sqrt{\frac{f}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx}\sqrt{\frac{f}{\pi d}}\right)+\left(2\,fx\cos\left(fx\right)+\sin\left(fx\right)\right)\sqrt{dx}\right)}{3\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*pi*d*f*x^2*sqrt(f/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x)*sqr
t(f/(pi*d))) + (2*f*x*cos(f*x) + sin(f*x))*sqrt(d*x))/(d^3*x^2)
```

Sympy [A] time = 145.307, size = 114, normalized size = 1.31

$$\frac{\sqrt{2}\sqrt{\pi}f^{\frac{3}{2}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{3}{4}\right)}+\frac{f\cos\left(fx\right)\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}+\frac{\sin\left(fx\right)\Gamma\left(-\frac{1}{4}\right)}{6d^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)**(5/2),x)
```

```
[Out] sqrt(2)*sqrt(pi)*f**(3/2)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)/sqrt(pi))*gamma(
-1/4)/(3*d**(5/2)*gamma(3/4)) + f*cos(f*x)*gamma(-1/4)/(3*d**(5/2)*sqrt(x)*
gamma(3/4)) + sin(f*x)*gamma(-1/4)/(6*d**(5/2)*x**(3/2)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(fx\right)}{\left(dx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x)/(d*x)^(5/2), x)
```

3.65 $\int \sqrt{c+dx}\csc(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\sqrt{c+dx}\csc(a+bx),x\right)$$

```
[Out] Unintegrable[Sqrt[c + d*x]*Csc[a + b*x], x]
```

Rubi [A] time = 0.0310791, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{c+dx}\csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + d*x]*Csc[a + b*x],x]
```

```
[Out] Defer[Int][Sqrt[c + d*x]*Csc[a + b*x], x]
```

Rubi steps

$$\int \sqrt{c+dx}\csc(a+bx)\,dx=\int \sqrt{c+dx}\csc(a+bx)\,dx$$

Mathematica [A] time = 15.3741, size = 0, normalized size = 0.

$$\int \sqrt{c+dx}\csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x]*Csc[a + b*x],x]
```

```
[Out] Integrate[Sqrt[c + d*x]*Csc[a + b*x], x]
```

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int \csc(bx+a)\sqrt{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*(d*x+c)^(1/2),x)
```

```
[Out] int(csc(b*x+a)*(d*x+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx+c}\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x + c)*csc(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{dx+c}\csc(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x + c)*csc(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+dx}\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*csc(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx+c}\,\csc\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x + c)*csc(b*x + a), x)
```

3.66 $\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\csc(a+bx)}{\sqrt{c+dx}},x\right)$$

```
[Out] Unintegrable[Csc[a + b*x]/Sqrt[c + d*x], x]
```

Rubi [A] time = 0.0310664, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] Defer[Int][Csc[a + b*x]/Sqrt[c + d*x], x]
```

Rubi steps

$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx$$

Mathematica [A] time = 14.6077, size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] Integrate[Csc[a + b*x]/Sqrt[c + d*x], x]
```

Maple [A] time = 0.05, size = 0, normalized size = 0.

$$\int \csc \left(b x +a \right) \frac{1}{\sqrt{d x +c}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*x+c)^(1/2),x)
```

```
[Out] int(csc(b*x+a)/(d*x+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc \left(bx + a\right)}{\sqrt{dx + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/sqrt(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\csc \left(bx + a\right)}{\sqrt{dx + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)/sqrt(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc \left(a + bx\right)}{\sqrt{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)/sqrt(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/sqrt(d*x + c), x)
```

3.67 $\int \left(\frac{x}{\sin^{\frac{3}{2}}(e+fx)}+x\sqrt{\sin(e+fx)}\right) dx$

Optimal. Leaf size=38

$$\frac{4\sqrt{\sin(e+fx)}}{f^2}-\frac{2x\cos(e+fx)}{f\sqrt{\sin(e+fx)}}$$

```
[Out] (-2*x*Cos[e + f*x])/(f*Sqrt[Sin[e + f*x]]) + (4*Sqrt[Sin[e + f*x]])/f^2
```

Rubi [A] time = 0.0613543, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {3315}

$$\frac{4\sqrt{\sin(e+fx)}}{f^2}-\frac{2x\cos(e+fx)}{f\sqrt{\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sin[e + f*x]^(3/2) + x*Sqrt[Sin[e + f*x]],x]
```

```
[Out] (-2*x*Cos[e + f*x])/(f*Sqrt[Sin[e + f*x]]) + (4*Sqrt[Sin[e + f*x]])/f^2
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\sin^{\frac{3}{2}}(e+fx)}+x\sqrt{\sin(e+fx)}\right) dx &= \int \frac{x}{\sin^{\frac{3}{2}}(e+fx)}\, dx+\int x\sqrt{\sin(e+fx)}\, dx\\ &= -\frac{2x\cos(e+fx)}{f\sqrt{\sin(e+fx)}}+\frac{4\sqrt{\sin(e+fx)}}{f^2}\end{aligned}$$

Mathematica [A] time = 0.408233, size = 33, normalized size = 0.87

$$\frac{4\sin(e+fx)-2fx\cos(e+fx)}{f^2\sqrt{\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sin[e + f*x]^(3/2) + x*Sqrt[Sin[e + f*x]],x]
```

```
[Out] (-2*f*x*Cos[e + f*x] + 4*Sin[e + f*x])/(f^2*Sqrt[Sin[e + f*x]])
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int x\left(\sin\left(fx+e\right)\right)^{-\frac{3}{2}}+x\sqrt{\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x)
```

```
[Out] int(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\sin\left(fx+e\right)}+\frac{x}{\sin\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(sin(f*x + e)) + x/sin(f*x + e)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\sin^2\left(e+fx\right)+1\right)}{\sin^{\frac{3}{2}}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)**(3/2)+x*sin(f*x+e)**(1/2),x)
```

```
[Out] Integral(x*(sin(e + f*x)**2 + 1)/sin(e + f*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\sin\left(fx+e\right)}+\frac{x}{\sin\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(3/2)+x*sin(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(sin(f*x + e)) + x/sin(f*x + e)^(3/2), x)
```

3.68 $\int \left(\frac{x^2}{\sin^{\frac{3}{2}}(e+fx)} + x^2\sqrt{\sin(e+fx)}\right) dx$

Optimal. Leaf size=62

$$\frac{8x\sqrt{\sin(e+fx)}}{f^2} - \frac{16E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f^3} - \frac{2x^2\cos(e+fx)}{f\sqrt{\sin(e+fx)}}$$

```
[Out] (-16*EllipticE[(e - Pi/2 + f*x)/2, 2])/f^3 - (2*x^2*Cos[e + f*x])/(f*Sqrt[S
in[e + f*x]]) + (8*x*Sqrt[Sin[e + f*x]])/f^2
```

Rubi [A] time = 0.106662, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {3316, 2639}

$$\frac{8x\sqrt{\sin(e+fx)}}{f^2} - \frac{16E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f^3} - \frac{2x^2\cos(e+fx)}{f\sqrt{\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sin[e + f*x]^(3/2) + x^2*Sqrt[Sin[e + f*x]],x]
```


```
[Out] (-16*EllipticE[(e - Pi/2 + f*x)/2, 2])/f^3 - (2*x^2*Cos[e + f*x])/(f*Sqrt[S
in[e + f*x]]) + (8*x*Sqrt[Sin[e + f*x]])/f^2
```


Rule 3316

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[((c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1
)), x] + (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n +
 2), x], x] + Dist[(d^2*m*(m - 1))/(b^2*f^2*(n + 1)*(n + 2)), Int[(c + d*x)
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] - Simp[(d*m*(c + d*x)^(m - 1)*(b*
Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; FreeQ[{b, c, d, e,
 f}, x] && LtQ[n, -1] && NeQ[n, -2] && GtQ[m, 1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(\frac{x^2}{\sin^{\frac{3}{2}}(e+fx)}+x^2\sqrt{\sin(e+fx)}\right)dx=\int \frac{x^2}{\sin^{\frac{3}{2}}(e+fx)}\,dx+\int x^2\sqrt{\sin(e+fx)}\,dx$$
$$=-\frac{2x^2\cos(e+fx)}{f\sqrt{\sin(e+fx)}}+\frac{8x\sqrt{\sin(e+fx)}}{f^2}-\frac{8\int \sqrt{\sin(e+fx)}\,dx}{f^2}$$
$$=-\frac{16E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f^3}-\frac{2x^2\cos(e+fx)}{f\sqrt{\sin(e+fx)}}+\frac{8x\sqrt{\sin(e+fx)}}{f^2}$$

Mathematica [C] time = 4.19399, size = 185, normalized size = 2.98

$$-\frac{\sec(e)\left(\left(f^2x^2-8\right)\cos(2e+fx)-8fx\cos(e)\sin(e+fx)+\left(f^2x^2+8\right)\cos(fx)\right)}{f^3\sqrt{\sin(e+fx)}}+\frac{8\sec(e)e^{-ifx}\sqrt{2-2e^{2i(e+fx)}}\left(3\,{}_2F_1\left(\cdots\right.\right.}{3f^3\sqrt{-ie^{-}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sin[e + f*x]^(3/2) + x^2*Sqrt[Sin[e + f*x]],x]
```

```
[Out] (8*Sqrt[2 - 2*E^((2*I)*(e + f*x))]*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, E^(
(2*I)*(e + f*x))] + E^((2*I)*f*x)*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)
*(e + f*x))])*Sec[e])/(3*E^(I*f*x)*Sqrt[((-I)*(-1 + E^((2*I)*(e + f*x))))/E
^(I*(e + f*x))]*f^3) - (Sec[e]*((8 + f^2*x^2)*Cos[f*x] + (-8 + f^2*x^2)*Cos
[2*e + f*x] - 8*f*x*Cos[e]*Sin[e + f*x]))/(f^3*Sqrt[Sin[e + f*x]])
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int x^2\left(\sin\left(fx+e\right)\right)^{-\frac{3}{2}}+x^2\sqrt{\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x)
```

```
[Out] int(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\sin\left(fx+e\right)}+\frac{x^2}{\sin\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(sin(f*x + e)) + x^2/sin(f*x + e)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\left(\sin^2\left(e+fx\right)+1\right)}{\sin^{\frac{3}{2}}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/sin(f*x+e)**(3/2)+x**2*sin(f*x+e)**(1/2),x)
```

```
[Out] Integral(x**2*(sin(e + f*x)**2 + 1)/sin(e + f*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\sin\left(fx+e\right)}+\frac{x^2}{\sin\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sin(f*x+e)^(3/2)+x^2*sin(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(sin(f*x + e)) + x^2/sin(f*x + e)^(3/2), x)
```

3.69 $\int \left(\frac{x}{\sin^{\frac{5}{2}}(e+fx)}-\frac{x}{3\sqrt{\sin(e+fx)}}\right) dx$

Optimal. Leaf size=42

$$-\frac{4}{3f^2\sqrt{\sin(e+fx)}}-\frac{2x\cos(e+fx)}{3f\sin^{\frac{3}{2}}(e+fx)}$$

```
[Out] (-2*x*Cos[e + f*x])/(3*f*Sin[e + f*x]^(3/2)) - 4/(3*f^2*Sqrt[Sin[e + f*x]])
```

Rubi [A] time = 0.0603062, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {3315}

$$-\frac{4}{3f^2\sqrt{\sin(e+fx)}}-\frac{2x\cos(e+fx)}{3f\sin^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sin[e + f*x]^(5/2) - x/(3*Sqrt[Sin[e + f*x]]),x]
```

```
[Out] (-2*x*Cos[e + f*x])/(3*f*Sin[e + f*x]^(3/2)) - 4/(3*f^2*Sqrt[Sin[e + f*x]])
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\sin^{\frac{5}{2}}(e+fx)}-\frac{x}{3\sqrt{\sin(e+fx)}}\right) dx &= -\left(\frac{1}{3}\int \frac{x}{\sqrt{\sin(e+fx)}}\, dx\right)+\int \frac{x}{\sin^{\frac{5}{2}}(e+fx)}\, dx\\ &= -\frac{2x\cos(e+fx)}{3f\sin^{\frac{3}{2}}(e+fx)}-\frac{4}{3f^2\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.404757, size = 35, normalized size = 0.83

$$-\frac{2(2\sin(e+fx)+fx\cos(e+fx))}{3f^2\sin^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sin[e + f*x]^(5/2) - x/(3*Sqrt[Sin[e + f*x]]),x]
```

```
[Out] (-2*(f*x*Cos[e + f*x] + 2*Sin[e + f*x]))/(3*f^2*Sin[e + f*x]^(3/2))
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int x\left(\sin\left(fx+e\right)\right)^{-\frac{5}{2}}-\frac{x}{3}\frac{1}{\sqrt{\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x)
```

```
[Out] int(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{3\sqrt{\sin\left(fx+e\right)}}+\frac{x}{\sin\left(fx+e\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x/sqrt(sin(f*x + e)) + x/sin(f*x + e)^(5/2), x)
```

Fricas [A] time = 1.72272, size = 117, normalized size = 2.79

$$\frac{2\left(fx\cos\left(fx+e\right)+2\,\sin\left(fx+e\right)\right)\sqrt{\sin\left(fx+e\right)}}{3\left(f^2\cos\left(fx+e\right)^2-f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(f*x*cos(f*x + e) + 2*sin(f*x + e))*sqrt(sin(f*x + e))/(f^2*cos(f*x + e
)^2 - f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{3x}{\sin^{\frac{5}{2}}(e+fx)}\,dx + \int \frac{x}{\sqrt{\sin(e+fx)}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)**(5/2)-1/3*x/sin(f*x+e)**(1/2),x)
```

```
[Out] -(Integral(-3*x/sin(e + f*x)**(5/2), x) + Integral(x/sqrt(sin(e + f*x)), x)
)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{3\sqrt{\sin(fx+e)}} + \frac{x}{\sin(fx+e)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(5/2)-1/3*x/sin(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x/sqrt(sin(f*x + e)) + x/sin(f*x + e)^(5/2), x)
```

3.70 $\int \left(\frac{x}{\sin^{\frac{7}{2}}(e+fx)}+\frac{3}{5}x\sqrt{\sin(e+fx)}\right)dx$

Optimal. Leaf size=83

$$-\frac{4}{15f^2\sin^{\frac{3}{2}}(e+fx)}+\frac{12\sqrt{\sin(e+fx)}}{5f^2}-\frac{2x\cos(e+fx)}{5f\sin^{\frac{5}{2}}(e+fx)}-\frac{6x\cos(e+fx)}{5f\sqrt{\sin(e+fx)}}$$

```
[Out] (-2*x*Cos[e + f*x])/(5*f*Sin[e + f*x]^(5/2)) - 4/(15*f^2*Sin[e + f*x]^(3/2)
) - (6*x*Cos[e + f*x])/(5*f*Sqrt[Sin[e + f*x]]) + (12*Sqrt[Sin[e + f*x]])/(
5*f^2)
```

Rubi [A] time = 0.0858376, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {3315}

$$-\frac{4}{15f^2\sin^{\frac{3}{2}}(e+fx)}+\frac{12\sqrt{\sin(e+fx)}}{5f^2}-\frac{2x\cos(e+fx)}{5f\sin^{\frac{5}{2}}(e+fx)}-\frac{6x\cos(e+fx)}{5f\sqrt{\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sin[e + f*x]^(7/2) + (3*x*Sqrt[Sin[e + f*x]])/5,x]
```

```
[Out] (-2*x*Cos[e + f*x])/(5*f*Sin[e + f*x]^(5/2)) - 4/(15*f^2*Sin[e + f*x]^(3/2)
) - (6*x*Cos[e + f*x])/(5*f*Sqrt[Sin[e + f*x]]) + (12*Sqrt[Sin[e + f*x]])/(
5*f^2)
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \left(\frac{x}{\sin^{\frac{7}{2}}(e+fx)} + \frac{3}{5}x\sqrt{\sin(e+fx)}\right)dx = \frac{3}{5}\int x\sqrt{\sin(e+fx)}\,dx + \int \frac{x}{\sin^{\frac{7}{2}}(e+fx)}\,dx$$
$$= -\frac{2x\cos(e+fx)}{5f\sin^{\frac{5}{2}}(e+fx)} - \frac{4}{15f^2\sin^{\frac{3}{2}}(e+fx)} + \frac{3}{5}\int \frac{x}{\sin^{\frac{3}{2}}(e+fx)}\,dx + \frac{3}{5}\int x\sqrt{\sin$$
$$= -\frac{2x\cos(e+fx)}{5f\sin^{\frac{5}{2}}(e+fx)} - \frac{4}{15f^2\sin^{\frac{3}{2}}(e+fx)} - \frac{6x\cos(e+fx)}{5f\sqrt{\sin(e+fx)}} + \frac{12\sqrt{\sin(e+fx)}}{5f^2}$$

Mathematica [A] time = 0.585283, size = 58, normalized size = 0.7

$$\frac{46\sin(e+fx) - 18\sin(3(e+fx)) - 21fx\cos(e+fx) + 9fx\cos(3(e+fx))}{30f^2\sin^{\frac{5}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sin[e + f*x]^(7/2) + (3*x*Sqrt[Sin[e + f*x]])/5,x]
```

```
[Out] (-21*f*x*Cos[e + f*x] + 9*f*x*Cos[3*(e + f*x)] + 46*Sin[e + f*x] - 18*Sin[3
*(e + f*x)])/(30*f^2*Sin[e + f*x]^(5/2))
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int x\left(\sin\left(fx+e\right)\right)^{-\frac{7}{2}} + \frac{3x}{5}\sqrt{\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x)
```

```
[Out] int(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3}{5}x\sqrt{\sin\left(fx+e\right)} + \frac{x}{\sin\left(fx+e\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(3/5*x*sqrt(sin(f*x + e)) + x/sin(f*x + e)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)**(7/2)+3/5*x*sin(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3}{5} x \sqrt{\sin\left(fx+e\right)} + \frac{x}{\sin\left(fx+e\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sin(f*x+e)^(7/2)+3/5*x*sin(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(3/5*x*sqrt(sin(f*x + e)) + x/sin(f*x + e)^(7/2), x)
```

3.71 $\int (c+dx)^m (b\sin(e+fx))^n\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((c+dx)^m (b\sin(e+fx))^n, x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*(b*Sin[e + f*x])^n, x]
```

Rubi [A] time = 0.0418718, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c+dx)^m (b\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*(b*Sin[e + f*x])^n,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(b*Sin[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m (b\sin(e+fx))^n\,dx = \int (c+dx)^m (b\sin(e+fx))^n\,dx$$

Mathematica [A] time = 0.721235, size = 0, normalized size = 0.

$$\int (c+dx)^m (b\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(b*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[(c + d*x)^m*(b*Sin[e + f*x])^n, x]
```

Maple [A] time = 0.299, size = 0, normalized size = 0.

$$\int (dx+c)^m \left(b \sin \left(fx+e\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(b*sin(f*x+e))^n,x)
```

```
[Out] int((d*x+c)^m*(b*sin(f*x+e))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \left(b \sin \left(fx+e\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*(b*sin(f*x + e))^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx+c)^m \left(b \sin \left(fx+e\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*(b*sin(f*x + e))^n, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(e+fx\right)\right)^n (c+dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(b*sin(f*x+e))**n,x)
```

```
[Out] Integral((b*sin(e + f*x))**n*(c + d*x)**m, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \left(b\sin\left(fx+e\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*(b*sin(f*x + e))^n, x)
```

3.72 $\int (c+dx)^m \sin^3(a+bx)\,dx$

Optimal. Leaf size=267

$$-\frac{3e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\right.}{8b}$$

```
[Out] (-3*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(8*
b*(((-I)*b*(c + d*x))/d)^m) - (3*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d
])/(8*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + (3^(-1 - m)*E^((3*I)*(
a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(8*b*(((-I)
*b*(c + d*x))/d)^m) + (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c + d*
x))/d])/(8*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.303063, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3312, 3308, 2181}

$$-\frac{3e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]^3,x]
```

```
[Out] (-3*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(8*
b*(((-I)*b*(c + d*x))/d)^m) - (3*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d
])/(8*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + (3^(-1 - m)*E^((3*I)*(
a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(8*b*(((-I)
*b*(c + d*x))/d)^m) + (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c + d*
x))/d])/(8*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m \sin^3(a+bx)\,dx = \int \left(\frac{3}{4}(c+dx)^m \sin(a+bx) - \frac{1}{4}(c+dx)^m \sin(3a+3bx)\right) dx$$

$$= -\left(\frac{1}{4}\int (c+dx)^m \sin(3a+3bx)\,dx\right) + \frac{3}{4}\int (c+dx)^m \sin(a+bx)\,dx$$

$$= -\left(\frac{1}{8}i\int e^{-i(3a+3bx)}(c+dx)^m\,dx\right) + \frac{1}{8}i\int e^{i(3a+3bx)}(c+dx)^m\,dx + \frac{3}{8}i\int e^{-i(a+bx)}(c+dx)^m\,dx$$

$$= -\frac{3e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b} - \frac{3e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1\right.}{8b}$$

Mathematica [A] time = 9.80897, size = 251, normalized size = 0.94

$$\frac{3^{-m-1}e^{-\frac{3i(ad+bc)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-3^{m+2}e^{2ia+\frac{4ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m \text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right) - 3^{m+2}e^{2i\left(2a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m \text{G}\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]^3,x]
```

```
[Out] (3^(-1 - m)*(c + d*x)^m*(-(3^(2 + m)*E^((2*I)*(2*a + (b*c)/d))*((I*b*(c + d
*x))/d)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d]) - 3^(2 + m)*E^((2*I)*a + ((4*
I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, (I*b*(c + d*x))/d] + E^((6
*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d] + E^(((6*
I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/d]))/(
8*b*E^(((3*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int (dx+c)^m (\sin(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sin(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*sin(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sin(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sin(b*x + a)^3, x)
```

Fricas [A] time = 1.90813, size = 459, normalized size = 1.72

$$\frac{e^{\left(-\frac{dm\log\left(\frac{3ib}{d}\right)-3ibc+3iad}{d}\right)}\Gamma\left(m+1,\frac{3ibdx+3ibc}{d}\right)-9e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}\Gamma\left(m+1,\frac{ibdx+ibc}{d}\right)-9e^{\left(-\frac{dm\log\left(-\frac{ib}{d}\right)+ibc-iad}{d}\right)}\Gamma\left(m+1,\frac{-ibdx\cdots}{d}\right.}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gamma(m + 1, (3*I*b*d*x
 + 3*I*b*c)/d) - 9*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*
b*d*x + I*b*c)/d) - 9*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m + 1,
 (-I*b*d*x - I*b*c)/d) + e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gam
ma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \sin^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sin(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**m*sin(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sin(bx+a)^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sin(b*x + a)^3, x)
```

3.73 $\int (c+dx)^m \sin^2(a+bx)\,dx$

Optimal. Leaf size=162

$$\frac{i2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{i2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m\right.}{b}$$

```
[Out] (c + d*x)^(1 + m)/(2*d*(1 + m)) + (I*2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c
+ d*x)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)
 - (I*2^(-3 - m)*(c + d*x)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*
I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.217265, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3312, 3307, 2181}

$$\frac{i2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{i2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(1 + m)/(2*d*(1 + m)) + (I*2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c
+ d*x)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)
 - (I*2^(-3 - m)*(c + d*x)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*
I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m \sin^2(a+bx)\,dx = \int \left(\frac{1}{2}(c+dx)^m - \frac{1}{2}(c+dx)^m\cos(2a+2bx)\right)dx$$
$$= \frac{(c+dx)^{1+m}}{2d(1+m)} - \frac{1}{2}\int (c+dx)^m\cos(2a+2bx)\,dx$$
$$= \frac{(c+dx)^{1+m}}{2d(1+m)} - \frac{1}{4}\int e^{-i(2a+2bx)}(c+dx)^m\,dx - \frac{1}{4}\int e^{i(2a+2bx)}(c+dx)^m\,dx$$
$$= \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{i2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} - \frac{i2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)} \cdots}{\cdots}$$

Mathematica [A] time = 0.612256, size = 211, normalized size = 1.3

$$\frac{2^{-m-3}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-id(m+1)\left(-\frac{ib(c+dx)}{d}\right)^m\left(\cos\left(2a-\frac{2bc}{d}\right)-i\sin\left(2a-\frac{2bc}{d}\right)\right)\text{Gamma}\left(m+1,\frac{2ib(c+dx)}{d}\right)+id(\cdots\right.}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]^2,x]
```

```
[Out] (2^(-3 - m)*(c + d*x)^m*(2^(2 + m)*b*(c + d*x)*((b^2*(c + d*x)^2)/d^2)^m -
I*d*(1 + m)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d]*(C
os[2*a - (2*b*c)/d] - I*Sin[2*a - (2*b*c)/d]) + I*d*(1 + m)*((I*b*(c + d*x)
)/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]*(Cos[2*a - (2*b*c)/d] + I*Sin[2
*a - (2*b*c)/d])))/(b*d*(1 + m)*((b^2*(c + d*x)^2)/d^2)^m)
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int (dx+c)^{m} (\sin (bx+a))^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sin(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*sin(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(dm+d)\int (dx+c)^{m}\cos (2\,bx+2\,a)\, dx - e^{(m\log(dx+c)+\log(dx+c))}}{2\,(dm+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((d*m + d)*integrate((d*x + c)^m*cos(2*b*x + 2*a), x) - e^(m*log(d*x +
 c) + log(d*x + c)))/(d*m + d)
```

Fricas [A] time = 1.81047, size = 340, normalized size = 2.1

$$\frac{(-i\,dm - i\,d)e^{\left(-\frac{dm\log\left(\frac{2i\,b}{d}\right)-2i\,bc+2i\,ad}{d}\right)}\Gamma\left(m+1,\frac{2i\,bdx+2i\,bc}{d}\right) + (i\,dm+i\,d)e^{\left(-\frac{dm\log\left(-\frac{2i\,b}{d}\right)+2i\,bc-2i\,ad}{d}\right)}\Gamma\left(m+1,\frac{-2i\,bdx-2i\,bc}{d}\right) + 4\,(bdx + \dots}{8\,(bdm+bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((-I*d*m - I*d)*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m +
 1, (2*I*b*d*x + 2*I*b*c)/d) + (I*d*m + I*d)*e^(-(d*m*log(-2*I*b/d) + 2*I*b
*c - 2*I*a*d)/d)*gamma(m + 1, (-2*I*b*d*x - 2*I*b*c)/d) + 4*(b*d*x + b*c)*(
d*x + c)^m)/(b*d*m + b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \sin^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*sin(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sin(b*x + a)^2, x)
```

3.74 $\int (c+dx)^m \sin(a+bx)\,dx$

Optimal. Leaf size=127

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

```
[Out] -(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*
(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(
2*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.0884659, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3308, 2181}

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Sin[a + b*x],x]
```

```
[Out] -(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*
(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(
2*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m \sin(a+bx)\,dx = \frac{1}{2}i\int e^{-i(a+bx)}(c+dx)^m\,dx - \frac{1}{2}i\int e^{i(a+bx)}(c+dx)^m\,dx$$
$$= -\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b} - \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+\right.}{2b}$$

Mathematica [A] time = 0.0493769, size = 121, normalized size = 0.95

$$\frac{e^{-\frac{i(ad+bc)}{d}}(c+dx)^m\left(-e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x],x]
```

```
[Out] ((c + d*x)^m*(-((E^((2*I)*a)*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(((-I)*b*(
c + d*x))/d)^m) - (E^(((2*I)*b*c)/d)*Gamma[1 + m, (I*b*(c + d*x))/d])/((I*b
*(c + d*x))/d)^m))/(2*b*E^((I*(b*c + a*d))/d))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int (dx+c)^m \sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sin(b*x+a),x)
```

```
[Out] int((d*x+c)^m*sin(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sin(b*x + a), x)
```

Fricas [A] time = 1.82587, size = 219, normalized size = 1.72

$$-\frac{e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}\Gamma\left(m+1,\frac{ibdx+ibc}{d}\right)+e^{\left(-\frac{dm\log\left(-\frac{ib}{d}\right)+ibc-iad}{d}\right)}\Gamma\left(m+1,\frac{-ibdx-ibc}{d}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I*b*c
)/d) + e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m + 1, (-I*b*d*x - I*
b*c)/d))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \sin(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*sin(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sin(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sin(b*x + a), x)
```

3.75 $\int (c+dx)^m \csc(a+bx)\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\csc(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*Csc[a + b*x], x]
```

Rubi [A] time = 0.0186464, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x],x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \csc(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\,dx$$

Mathematica [A] time = 5.71564, size = 0, normalized size = 0.

$$\int (c+dx)^m \csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Csc[a + b*x], x]
```

Maple [A] time = 0.04, size = 0, normalized size = 0.

$$\int (dx + c)^m \csc (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a),x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx + c)^m \csc (bx + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^m \csc (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*csc(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a), x)
```

3.76 $\int (c+dx)^m \csc^2(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\csc^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*Csc[a + b*x]^2, x]
```

Rubi [A] time = 0.0365495, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c+dx)^m \csc^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \csc^2(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\,dx$$

Mathematica [A] time = 1.1148, size = 0, normalized size = 0.

$$\int (c+dx)^m \csc^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2, x]
```

Maple [A] time = 0.048, size = 0, normalized size = 0.

$$\int (dx+c)^m (\csc (bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx+c)^m \csc (bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \csc^2 (a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*csc(a + b*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^2, x)
```

3.77 $\int x^{3+m}\sin(a+bx)\,dx$

Optimal. Leaf size=79

$$\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+4,-ibx)}{2b^4}-\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+4,ibx)}{2b^4}$$

```
[Out] ((I/2)*E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(b^4*((-I)*b*x)^m) - ((I/2)*x^m*
Gamma[4 + m, I*b*x])/(b^4*E^(I*a)*(I*b*x)^m)
```

Rubi [A] time = 0.0774144, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3308, 2181}

$$\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+4,-ibx)}{2b^4}-\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+4,ibx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Sin[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(b^4*((-I)*b*x)^m) - ((I/2)*x^m*
Gamma[4 + m, I*b*x])/(b^4*E^(I*a)*(I*b*x)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)]])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^{3+m}\sin(a+bx)\,dx = \frac{1}{2}i\int e^{-i(a+bx)}x^{3+m}\,dx - \frac{1}{2}i\int e^{i(a+bx)}x^{3+m}\,dx$$
$$= \frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(4+m,-ibx)}{2b^4} - \frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(4+m,ibx)}{2b^4}$$

Mathematica [A] time = 0.0191448, size = 79, normalized size = 1.

$$\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+4,-ibx)}{2b^4} - \frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+4,ibx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Sin[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(b^4*((-I)*b*x)^m) - ((I/2)*x^m*
Gamma[4 + m, I*b*x])/(b^4*E^(I*a)*(I*b*x)^m)
```

Maple [C] time = 0.119, size = 454, normalized size = 5.8

$$\frac{2^{3+m}\sqrt{\pi}\sin(a)}{b^4}\left(b^2\right)^{-\frac{m}{2}}\left(3\,\frac{2^{-4-m}x^{3+m}b^3\left(b^2\right)^{m/2}(8/3+2/3\,m)\sin(bx)}{\sqrt{\pi}\,(4+m)} - \frac{2^{-3-m}x^{1+m}b\left(-m^2-7\,m-12\right)(\cos(bx)\,xb-\sin(\right.}{\sqrt{\pi}\,(4+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3+m)*sin(b*x+a),x)
```

```
[Out] 2^(3+m)/b^4*(b^2)^(-1/2*m)*Pi^(1/2)*(3*2^(-4-m)/Pi^(1/2)/(4+m)*x^(3+m)*b^3*
(b^2)^(1/2*m)*(8/3+2/3*m)*sin(b*x)-2^(-3-m)/Pi^(1/2)/(4+m)*x^(1+m)*b*(b^2)^
(1/2*m)*(-m^2-7*m-12)*(cos(b*x)*x*b-sin(b*x))+2^(-3-m)/Pi^(1/2)/(4+m)*x^(2+
m)*b^2*(b^2)^(1/2*m)*(-m^3-8*m^2-19*m-12)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2
,b*x)*sin(b*x)-2^(-3-m)/Pi^(1/2)*x^(2+m)*b^2*(b^2)^(1/2*m)*(2+m)*(1+m)*(3+m
)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*sin(a)+2^
(3+m)*b^(-4-m)*Pi^(1/2)*(2^(-3-m)/Pi^(1/2)/(5+m)*x^(2+m)*b^(2+m)*(m^2+7*m+1
0)*sin(b*x)-2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(cos(b*x)*x*b-sin(b*x))-2^(-3
-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*m*(3+m)*(2+m)*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/
2,b*x)*sin(b*x)+2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(3+m)*(2+m)*(b*x)^(-5/2-m
)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*cos(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+3} \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 3)*sin(b*x + a), x)
```

Fricas [A] time = 1.8114, size = 149, normalized size = 1.89

$$-\frac{e^{(-(m+3)\log(i\,b)-i\,a)}\Gamma(m+4, i\,bx) + e^{(-(m+3)\log(-i\,b)+i\,a)}\Gamma(m+4, -i\,bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-(m + 3)*log(I*b) - I*a)*gamma(m + 4, I*b*x) + e^(-(m + 3)*log(-I*
b) + I*a)*gamma(m + 4, -I*b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+3} \sin(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*sin(b*x+a),x)
```

```
[Out] Integral(x**(m + 3)*sin(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+3} \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 3)*sin(b*x + a), x)
```

3.78 $\int x^{2+m}\sin(a+bx)\,dx$

Optimal. Leaf size=75

$$\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+3,-ibx)}{2b^3}+\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+3,ibx)}{2b^3}$$

```
[Out] (E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(2*b^3*((-I)*b*x)^m) + (x^m*Gamma[3 +
m, I*b*x])/(2*b^3*E^(I*a)*(I*b*x)^m)
```

Rubi [A] time = 0.0727324, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3308, 2181}

$$\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+3,-ibx)}{2b^3}+\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+3,ibx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Sin[a + b*x],x]
```

```
[Out] (E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(2*b^3*((-I)*b*x)^m) + (x^m*Gamma[3 +
m, I*b*x])/(2*b^3*E^(I*a)*(I*b*x)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^{2+m}\sin(a+bx)\,dx = \frac{1}{2}i\int e^{-i(a+bx)}x^{2+m}\,dx - \frac{1}{2}i\int e^{i(a+bx)}x^{2+m}\,dx$$
$$= \frac{e^{ia}x^m(-ibx)^{-m}\Gamma(3+m,-ibx)}{2b^3} + \frac{e^{-ia}x^m(ibx)^{-m}\Gamma(3+m,ibx)}{2b^3}$$

Mathematica [A] time = 0.0158561, size = 75, normalized size = 1.

$$\frac{e^{ia}x^m(-ibx)^{-m}\mathrm{Gamma}(m+3,-ibx)}{2b^3} + \frac{e^{-ia}x^m(ibx)^{-m}\mathrm{Gamma}(m+3,ibx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Sin[a + b*x],x]
```

```
[Out] (E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(2*b^3*((-I)*b*x)^m) + (x^m*Gamma[3 +
m, I*b*x])/(2*b^3*E^(I*a)*(I*b*x)^m)
```

Maple [C] time = 0.063, size = 353, normalized size = 4.7

$$\frac{2^{2+m}\sqrt{\pi}\sin(a)}{b^2}\left(b^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(3\,\frac{2^{-3-m}x^{2+m}\left(b^2\right)^{3/2+m/2}(2+2/3\,m)\sin(bx)}{\sqrt{\pi}\,(3+m)\,b} - \frac{2^{-2-m}x^{2+m}\,(2+m)\,m\sin(bx)}{\sqrt{\pi}b}\left(b^2\right)^{\frac{3}{2}+\frac{m}{2}}(bx)^{-\frac{3}{2}-m}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*sin(b*x+a),x)
```

```
[Out] 2^(2+m)/b^2*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(-3-m)/Pi^(1/2)/(3+m)*x^(2+m)*
(b^2)^(3/2+1/2*m)*(2+2/3*m)/b*sin(b*x)-2^(-2-m)/Pi^(1/2)*x^(2+m)*(b^2)^(3/2
+1/2*m)/b*(2+m)*m*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)+2^(-2-m)/
Pi^(1/2)*x^(2+m)*(b^2)^(3/2+1/2*m)/b*(2+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin
(b*x))*LommelS1(m+3/2,1/2,b*x))*sin(a)+2^(2+m)*b^(-3-m)*Pi^(1/2)*(-2^(-2-m)
/Pi^(1/2)*x^(1+m)*b^(1+m)*(cos(b*x)*x*b-sin(b*x))+2^(-2-m)/Pi^(1/2)/(4+m)*x
^(2+m)*b^(2+m)*(m^2+5*m+4)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)+
2^(-2-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(2+m)*(1+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-
sin(b*x))*LommelS1(m+1/2,1/2,b*x))*cos(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+2} \sin(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 2)*sin(b*x + a), x)
```

Fricas [A] time = 1.71217, size = 149, normalized size = 1.99

$$-\frac{e^{\left(-(m+2)\log(i\,b)-i\,a\right)}\Gamma\left(m+3, i\,bx\right)+e^{\left(-(m+2)\log(-i\,b)+i\,a\right)}\Gamma\left(m+3, -i\,bx\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-(m + 2)*log(I*b) - I*a)*gamma(m + 3, I*b*x) + e^(-(m + 2)*log(-I*
b) + I*a)*gamma(m + 3, -I*b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+2} \sin(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*sin(b*x+a),x)
```

```
[Out] Integral(x**(m + 2)*sin(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+2} \sin(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*sin(b*x + a), x)
```

3.79 $\int x^{1+m}\sin(a+bx)\,dx$

Optimal. Leaf size=79

$$\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+2,ibx)}{2b^2}-\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+2,-ibx)}{2b^2}$$

```
[Out] ((-I/2)*E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(b^2*((-I)*b*x)^m) + ((I/2)*x^m
*Gamma[2 + m, I*b*x])/(b^2*E^(I*a)*(I*b*x)^m)
```

Rubi [A] time = 0.0708763, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3308, 2181}

$$\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+2,ibx)}{2b^2}-\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+2,-ibx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Sin[a + b*x],x]
```

```
[Out] ((-I/2)*E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(b^2*((-I)*b*x)^m) + ((I/2)*x^m
*Gamma[2 + m, I*b*x])/(b^2*E^(I*a)*(I*b*x)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^{1+m}\sin(a+bx)\,dx = \frac{1}{2}i\int e^{-i(a+bx)}x^{1+m}\,dx - \frac{1}{2}i\int e^{i(a+bx)}x^{1+m}\,dx$$
$$= -\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(2+m,-ibx)}{2b^2} + \frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(2+m,ibx)}{2b^2}$$

Mathematica [A] time = 0.0161029, size = 79, normalized size = 1.

$$\frac{ie^{-ia}x^m(ibx)^{-m}\mathrm{Gamma}(m+2,ibx)}{2b^2} - \frac{ie^{ia}x^m(-ibx)^{-m}\mathrm{Gamma}(m+2,-ibx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Sin[a + b*x],x]
```

```
[Out] ((-I/2)*E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(b^2*((-I)*b*x)^m) + ((I/2)*x^m
*Gamma[2 + m, I*b*x])/(b^2*E^(I*a)*(I*b*x)^m)
```

Maple [C] time = 0.063, size = 290, normalized size = 3.7

$$\frac{2^{1+m}\sqrt{\pi}\sin(a)}{b^2}\left(b^2\right)^{-\frac{m}{2}}\left(\frac{2^{-1-m}x^{1+m}b\sin(bx)}{\sqrt{\pi}(2+m)}\left(b^2\right)^{\frac{m}{2}} + 3\,\frac{2^{-2-m}x^{2+m}b^2\left(b^2\right)^{m/2}(2/3+2/3\,m)(bx)^{-3/2-m}\,\mathrm{LommelS1}(m+3/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*sin(b*x+a),x)
```

```
[Out] 2^(1+m)/b^2*(b^2)^(-1/2*m)*Pi^(1/2)*(2^(-1-m)/Pi^(1/2)/(2+m)*x^(1+m)*b*(b^2
)^(1/2*m)*sin(b*x)+3*2^(-2-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^2*(b^2)^(1/2*m)*(2/3
+2/3*m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)+2^(-1-m)/Pi^(1/2)*x
^(2+m)*b^2*(b^2)^(1/2*m)*(1+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*Lomme
lS1(m+1/2,1/2,b*x))*sin(a)+2^(1+m)*b^(-2-m)*Pi^(1/2)*(2^(-1-m)/Pi^(1/2)*x^(
2+m)*b^(2+m)*m*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)-2^(-1-m)/Pi^
(1/2)*x^(2+m)*b^(2+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+3/2
,1/2,b*x))*cos(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1} \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 1)*sin(b*x + a), x)
```

Fricas [A] time = 1.71925, size = 149, normalized size = 1.89

$$-\frac{e^{\left(-(m+1)\log(i\,b)-i\,a\right)}\Gamma\left(m+2,i\,bx\right)+e^{\left(-(m+1)\log(-i\,b)+i\,a\right)}\Gamma\left(m+2,-i\,bx\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-(m + 1)*log(I*b) - I*a)*gamma(m + 2, I*b*x) + e^(-(m + 1)*log(-I*
b) + I*a)*gamma(m + 2, -I*b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1} \sin(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*sin(b*x+a),x)
```

```
[Out] Integral(x**(m + 1)*sin(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1} \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*sin(b*x + a), x)
```

3.80 $\int x^m \sin(a+bx)\,dx$

Optimal. Leaf size=75

$$-\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+1,-ibx)}{2b}-\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+1,ibx)}{2b}$$

```
[Out] -(E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(2*b*((-I)*b*x)^m) - (x^m*Gamma[1 + m
, I*b*x])/(2*b*E^(I*a)*(I*b*x)^m)
```

Rubi [A] time = 0.0657262, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3308, 2181}

$$-\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+1,-ibx)}{2b}-\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+1,ibx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + b*x],x]
```

```
[Out] -(E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(2*b*((-I)*b*x)^m) - (x^m*Gamma[1 + m
, I*b*x])/(2*b*E^(I*a)*(I*b*x)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^m \sin(a+bx)\,dx = \frac{1}{2}i\int e^{-i(a+bx)}x^m\,dx - \frac{1}{2}i\int e^{i(a+bx)}x^m\,dx$$
$$= -\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(1+m,-ibx)}{2b} - \frac{e^{-ia}x^m(ibx)^{-m}\Gamma(1+m,ibx)}{2b}$$

Mathematica [A] time = 0.013591, size = 75, normalized size = 1.

$$-\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+1,-ibx)}{2b} - \frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+1,ibx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sin[a + b*x],x]
```

```
[Out] -(E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(2*b*((-I)*b*x)^m) - (x^m*Gamma[1 + m
, I*b*x])/(2*b*E^(I*a)*(I*b*x)^m)
```

Maple [C] time = 0.062, size = 378, normalized size = 5.

$$2^m\left(b^2\right)^{-\frac{1}{2}-\frac{m}{2}}\sqrt{\pi}\left(3\,\frac{2^{-1-m}\left(b^2\right)^{1/2+m/2}x^m\,(6+2\,m)\sin(bx)}{\sqrt{\pi}\,(1+m)\,(9+3\,m)\,b} + \frac{x^m2^{-m}\,(\cos(bx)\,xb-\sin(bx))}{\sqrt{\pi}\,(1+m)\,b}\left(b^2\right)^{\frac{1}{2}+\frac{m}{2}} + \frac{2^{-m}x^{2+m}bm\sin(b}{\sqrt{\pi}\,(1+m)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(b*x+a),x)
```

```
[Out] 2^m*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(-1-m)/Pi^(1/2)/(1+m)*(b^2)^(1/2+1/2*m
)*x^m*(6+2*m)/(9+3*m)/b*sin(b*x)+1/Pi^(1/2)/(1+m)*(b^2)^(1/2+1/2*m)*x^m*2^(
-m)/b*(cos(b*x)*x*b-sin(b*x))+2^(-m)/Pi^(1/2)/(1+m)*x^(2+m)*(b^2)^(1/2+1/2*
m)*b*m*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)-2^(-m)/Pi^(1/2)/(1+m
)*x^(2+m)*(b^2)^(1/2+1/2*m)*b*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*Lommel
S1(m+3/2,1/2,b*x))*sin(a)+2^m*b^(-1-m)*Pi^(1/2)*(1/Pi^(1/2)/(2+m)*x^(1+m)*b
^(1+m)*2^(-m)*sin(b*x)-2^(-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^(2+m)*(b*x)^(-3/2-m)
*LommelS1(m+3/2,3/2,b*x)*sin(b*x)-3*2^(-1-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^(2+m)
*(4/3+2/3*m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x)
)*cos(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sin(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sin(b*x + a), x)
```

Fricas [A] time = 1.78999, size = 132, normalized size = 1.76

$$-\frac{e^{(-m\log(ib)-ia)}\Gamma(m+1, ibx) + e^{(-m\log(-ib)+ia)}\Gamma(m+1, -ibx)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-m*log(I*b) - I*a)*gamma(m + 1, I*b*x) + e^(-m*log(-I*b) + I*a)*ga
mma(m + 1, -I*b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sin(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(b*x+a),x)
```

```
[Out] Integral(x**m*sin(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sin(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*sin(b*x + a), x)
```

3.81 $\int x^{-1+m}\sin(a+bx)\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m,-ibx)-\frac{1}{2}ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m,ibx)$$

```
[Out] ((I/2)*E^(I*a)*x^m*Gamma[m, (-I)*b*x])/((-I)*b*x)^m - ((I/2)*x^m*Gamma[m, I
*b*x])/(E^(I*a)*(I*b*x)^m)
```

Rubi [A] time = 0.0679546, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3308, 2181}

$$\frac{1}{2}ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m,-ibx)-\frac{1}{2}ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m,ibx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Sin[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*a)*x^m*Gamma[m, (-I)*b*x])/((-I)*b*x)^m - ((I/2)*x^m*Gamma[m, I
*b*x])/(E^(I*a)*(I*b*x)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\int x^{-1+m}\sin(a+bx)\,dx = \frac{1}{2}i\int e^{-i(a+bx)}x^{-1+m}\,dx - \frac{1}{2}i\int e^{i(a+bx)}x^{-1+m}\,dx$$
$$= \frac{1}{2}ie^{ia}x^m(-ibx)^{-m}\Gamma(m,-ibx) - \frac{1}{2}ie^{-ia}x^m(ibx)^{-m}\Gamma(m,ibx)$$

Mathematica [A] time = 0.020435, size = 63, normalized size = 0.91

$$\frac{1}{2}ie^{-ia}x^m\left(e^{2ia}(-ibx)^{-m}\text{Gamma}(m,-ibx) - (ibx)^{-m}\text{Gamma}(m,ibx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Sin[a + b*x],x]
```

```
[Out] ((I/2)*x^m*((E^((2*I)*a)*Gamma[m, (-I)*b*x])/((-I)*b*x)^m - Gamma[m, I*b*x]
/(I*b*x)^m))/E^(I*a)
```

Maple [C] time = 0.065, size = 426, normalized size = 6.2

$$2^{-1+m}\left(b^2\right)^{-\frac{m}{2}}\sqrt{\pi}\left(3\frac{x^{-1+m}2^{-m}\left(b^2\right)^{m/2}\left(2x^2b^2+2m+4\right)\sin(bx)}{\sqrt{\pi}m(6+3m)b} + \frac{2^{1-m}x^{-1+m}(\cos(bx)xb-\sin(bx))}{\sqrt{\pi}mb}\left(b^2\right)^{\frac{m}{2}} - 3\frac{x^{2+m}2^{1-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*sin(b*x+a),x)
```

```
[Out] 2^(-1+m)*(b^2)^(-1/2*m)*Pi^(1/2)*(3/Pi^(1/2)/m*x^(-1+m)*2^(-m)*(b^2)^(1/2*m
)*(2*b^2*x^2+2*m+4)/(6+3*m)/b*sin(b*x)+2^(1-m)/Pi^(1/2)/m*x^(-1+m)*(b^2)^(1
/2*m)/b*(cos(b*x)*x*b-sin(b*x))-3/Pi^(1/2)/m*x^(2+m)*2^(1-m)*(b^2)^(1/2*m)*
b^2/(6+3*m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)-1/Pi^(1/2)/m*x^
(2+m)*2^(1-m)*(b^2)^(1/2*m)*b^2*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*Lomm
elS1(m+1/2,1/2,b*x))*sin(a)+2^(-1+m)*b^(-m)*Pi^(1/2)*(2^(1-m)/Pi^(1/2)/(1+m
)*x^m*b^m*sin(b*x)-2^(1-m)/Pi^(1/2)/(1+m)*x^m*b^m/m*(cos(b*x)*x*b-sin(b*x))
-1/Pi^(1/2)/(1+m)*x^(2+m)*b^(2+m)*2^(1-m)*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2
,b*x)*sin(b*x)+1/Pi^(1/2)/(1+m)*x^(2+m)*b^(2+m)*2^(1-m)/m*(b*x)^(-5/2-m)*(c
os(b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*cos(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1} \sin (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 1)*sin(b*x + a), x)
```

Fricas [A] time = 1.69296, size = 138, normalized size = 2.

$$-\frac{e^{\left(-(m-1)\log(i\,b)-i\,a\right)}\Gamma\left(m, i\,bx\right) + e^{\left(-(m-1)\log(-i\,b)+i\,a\right)}\Gamma\left(m, -i\,bx\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-(m - 1)*log(I*b) - I*a)*gamma(m, I*b*x) + e^(-(m - 1)*log(-I*b) +
 I*a)*gamma(m, -I*b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1} \sin (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*sin(b*x+a),x)
```

```
[Out] Integral(x**(m - 1)*sin(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1} \sin (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*sin(b*x + a), x)
```

3.82 $\int x^{-2+m}\sin(a+bx)\,dx$

Optimal. Leaf size=71

$$\frac{1}{2}e^{ia}bx^m(-ibx)^{-m}\text{Gamma}(m-1,-ibx)+\frac{1}{2}e^{-ia}bx^m(ibx)^{-m}\text{Gamma}(m-1,ibx)$$

```
[Out] (b*E^(I*a)*x^m*Gamma[-1 + m, (-I)*b*x])/(2*((-I)*b*x)^m) + (b*x^m*Gamma[-1
+ m, I*b*x])/(2*E^(I*a)*(I*b*x)^m)
```

Rubi [A] time = 0.0706639, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3308, 2181}

$$\frac{1}{2}e^{ia}bx^m(-ibx)^{-m}\text{Gamma}(m-1,-ibx)+\frac{1}{2}e^{-ia}bx^m(ibx)^{-m}\text{Gamma}(m-1,ibx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Sin[a + b*x],x]
```

```
[Out] (b*E^(I*a)*x^m*Gamma[-1 + m, (-I)*b*x])/(2*((-I)*b*x)^m) + (b*x^m*Gamma[-1
+ m, I*b*x])/(2*E^(I*a)*(I*b*x)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^{-2+m}\sin(a+bx)\,dx = \frac{1}{2}i\int e^{-i(a+bx)}x^{-2+m}\,dx - \frac{1}{2}i\int e^{i(a+bx)}x^{-2+m}\,dx$$
$$= \frac{1}{2}be^{ia}x^m(-ibx)^{-m}\Gamma(-1+m,-ibx) + \frac{1}{2}be^{-ia}x^m(ibx)^{-m}\Gamma(-1+m,ibx)$$

Mathematica [A] time = 0.0186229, size = 65, normalized size = 0.92

$$\frac{1}{2}e^{-ia}bx^m\left(e^{2ia}(-ibx)^{-m}\text{Gamma}(m-1,-ibx)+(ibx)^{-m}\text{Gamma}(m-1,ibx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Sin[a + b*x],x]
```

```
[Out] (b*x^m*((E^((2*I)*a)*Gamma[-1 + m, (-I)*b*x])/((-I)*b*x)^m + Gamma[-1 + m,
I*b*x]/(I*b*x)^m))/(2*E^(I*a))
```

Maple [C] time = 0.072, size = 529, normalized size = 7.5

$$2^{m-2}\left(b^2\right)^{-\frac{1}{2}-\frac{m}{2}}b^2\sqrt{\pi}\left(3\,\frac{2^{1-m}x^{m-2}\left(b^2\right)^{-1/2+m/2}\left(2\,x^2b^2+2\,m+2\right)\sin(bx)}{\sqrt{\pi}\,(-1+m)\,(3+3\,m)\,b}-\frac{2^{2-m}x^{m-2}\left(x^2b^2-m^2-m\right)(\cos(bx)\,xb-\sin}{\sqrt{\pi}\,(-1+m)\,b\,(1+m)\,m}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-2)*sin(b*x+a),x)
```

```
[Out] 2^(m-2)*(b^2)^(-1/2-1/2*m)*b^2*Pi^(1/2)*(3*2^(1-m)/Pi^(1/2)/(-1+m)*x^(m-2)*
(b^2)^(-1/2+1/2*m)*(2*b^2*x^2+2*m+2)/(3+3*m)/b*sin(b*x)-2^(2-m)/Pi^(1/2)/(-
1+m)*x^(m-2)*(b^2)^(-1/2+1/2*m)/b*(b^2*x^2-m^2-m)/(1+m)/m*(cos(b*x)*x*b-sin
(b*x))-3*2^(2-m)/Pi^(1/2)/(-1+m)*x^(2+m)*(b^2)^(-1/2+1/2*m)*b^3/(3+3*m)*(b*
x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)+2^(2-m)/Pi^(1/2)/(-1+m)*x^(2+m
)*(b^2)^(-1/2+1/2*m)*b^3/(1+m)/m*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*Lom
melS1(m+3/2,1/2,b*x))*sin(a)+2^(m-2)*b^(1-m)*Pi^(1/2)*(2^(1-m)/Pi^(1/2)/m*x
^(-1+m)*b^(-1+m)*(-2*b^2*x^2+2*m^2+2*m-4)/(2+m)/(-1+m)*sin(b*x)-3*2^(2-m)/P
i^(1/2)/m*x^(-1+m)*b^(-1+m)/(-3+3*m)*(cos(b*x)*x*b-sin(b*x))+2^(2-m)/Pi^(1/
2)/m*x^(2+m)*b^(2+m)/(2+m)/(-1+m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*si
n(b*x)+3*2^(2-m)/Pi^(1/2)/m*x^(2+m)*b^(2+m)/(-3+3*m)*(b*x)^(-5/2-m)*(cos(b*
x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*cos(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-2}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 2)*sin(b*x + a), x)
```

Fricas [A] time = 1.82031, size = 149, normalized size = 2.1

$$-\frac{e^{(-(m-2)\log(i\,b)-i\,a)}\Gamma\left(m-1,i\,bx\right)+e^{(-(m-2)\log(-i\,b)+i\,a)}\Gamma\left(m-1,-i\,bx\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-(m - 2)*log(I*b) - I*a)*gamma(m - 1, I*b*x) + e^(-(m - 2)*log(-I*
b) + I*a)*gamma(m - 1, -I*b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-2}\sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)*sin(b*x+a),x)
```

```
[Out] Integral(x**(m - 2)*sin(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-2}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*sin(b*x + a), x)
```

3.83 $\int x^{-3+m}\sin(a+bx)\,dx$

Optimal. Leaf size=79

$$\frac{1}{2}ie^{-ia}b^2x^m(ibx)^{-m}\text{Gamma}(m-2,ibx)-\frac{1}{2}ie^{ia}b^2x^m(-ibx)^{-m}\text{Gamma}(m-2,-ibx)$$

```
[Out] ((-I/2)*b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/((-I)*b*x)^m + ((I/2)*b^2*
x^m*Gamma[-2 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

Rubi [A] time = 0.0717161, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3308, 2181}

$$\frac{1}{2}ie^{-ia}b^2x^m(ibx)^{-m}\text{Gamma}(m-2,ibx)-\frac{1}{2}ie^{ia}b^2x^m(-ibx)^{-m}\text{Gamma}(m-2,-ibx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Sin[a + b*x],x]
```

```
[Out] ((-I/2)*b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/((-I)*b*x)^m + ((I/2)*b^2*
x^m*Gamma[-2 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)]])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^{-3+m}\sin(a+bx)\,dx = \frac{1}{2}i\int e^{-i(a+bx)}x^{-3+m}\,dx - \frac{1}{2}i\int e^{i(a+bx)}x^{-3+m}\,dx$$
$$= -\frac{1}{2}ib^2e^{ia}x^m(-ibx)^{-m}\Gamma(-2+m,-ibx) + \frac{1}{2}ib^2e^{-ia}x^m(ibx)^{-m}\Gamma(-2+m,ibx)$$

Mathematica [A] time = 0.0156524, size = 79, normalized size = 1.

$$\frac{1}{2}ie^{-ia}b^2x^m(ibx)^{-m}\text{Gamma}(m-2,ibx) - \frac{1}{2}ie^{ia}b^2x^m(-ibx)^{-m}\text{Gamma}(m-2,-ibx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Sin[a + b*x],x]
```

```
[Out] ((-I/2)*b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/((-I)*b*x)^m + ((I/2)*b^2*
x^m*Gamma[-2 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

Maple [C] time = 0.079, size = 599, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-3)*sin(b*x+a),x)
```

```
[Out] 2^(m-3)*(b^2)^(-1/2*m)*b^2*Pi^(1/2)*(2^(2-m)/Pi^(1/2)/(m-2)*x^(m-3)/b^3*(b^
2)^(1/2*m)*(-2*b^4*x^4+2*b^2*m^2*x^2+2*b^2*m*x^2-4*b^2*x^2+2*m^3+2*m^2-4*m)
/m/(2+m)/(-1+m)*sin(b*x)-2^(-m+3)/Pi^(1/2)/(m-2)*x^(m-3)/b^3*(b^2)^(1/2*m)*
(b^2*x^2-m^2+m)/m/(-1+m)*(cos(b*x)*x*b-sin(b*x))+2^(-m+3)/Pi^(1/2)/(m-2)*x^
(2+m)*b^2*(b^2)^(1/2*m)/m/(2+m)/(-1+m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*
x)*sin(b*x)+2^(-m+3)/Pi^(1/2)/(m-2)*x^(2+m)*b^2*(b^2)^(1/2*m)/m/(-1+m)*(b*x
)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*sin(a)+2^(m-3)*
b^(2-m)*Pi^(1/2)*(2^(2-m)/Pi^(1/2)/(-1+m)*x^(m-2)*b^(m-2)*(-2*b^2*x^2+2*m^2
-2*m-4)/(1+m)/(m-2)*sin(b*x)+2^(-m+3)/Pi^(1/2)/(-1+m)*x^(m-2)*b^(m-2)*(b^2*
x^2-m^2-m)/(1+m)/(m-2)/m*(cos(b*x)*x*b-sin(b*x))+2^(-m+3)/Pi^(1/2)/(-1+m)*x
^(2+m)*b^(2+m)/(1+m)/(m-2)*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)-
2^(-m+3)/Pi^(1/2)/(-1+m)*x^(2+m)*b^(2+m)/(1+m)/(m-2)/m*(b*x)^(-5/2-m)*(cos(
b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*cos(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-3}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 3)*sin(b*x + a), x)
```

Fricas [A] time = 1.7661, size = 149, normalized size = 1.89

$$-\frac{e^{\left(-(m-3)\log(i\,b)-i\,a\right)}\Gamma\left(m-2,i\,bx\right)+e^{\left(-(m-3)\log(-i\,b)+i\,a\right)}\Gamma\left(m-2,-i\,bx\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-(m - 3)*log(I*b) - I*a)*gamma(m - 2, I*b*x) + e^(-(m - 3)*log(-I*
b) + I*a)*gamma(m - 2, -I*b*x))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-3}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 3)*sin(b*x + a), x)
```

3.84 $\int x^{3+m} \sin^2(a+bx)\,dx$

Optimal. Leaf size=97

$$\frac{e^{2ia}2^{-m-6}x^m(-ibx)^{-m}\text{Gamma}(m+4,-2ibx)}{b^4}+\frac{e^{-2ia}2^{-m-6}x^m(ibx)^{-m}\text{Gamma}(m+4,2ibx)}{b^4}+\frac{x^{m+4}}{2(m+4)}$$

```
[Out] x^(4 + m)/(2*(4 + m)) + (2^(-6 - m)*E^((2*I)*a)*x^m*Gamma[4 + m, (-2*I)*b*x
])/(b^4*((-I)*b*x)^m) + (2^(-6 - m)*x^m*Gamma[4 + m, (2*I)*b*x])/(b^4*E^((2
*I)*a)*(I*b*x)^m)
```

Rubi [A] time = 0.161679, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$\frac{e^{2ia}2^{-m-6}x^m(-ibx)^{-m}\text{Gamma}(m+4,-2ibx)}{b^4}+\frac{e^{-2ia}2^{-m-6}x^m(ibx)^{-m}\text{Gamma}(m+4,2ibx)}{b^4}+\frac{x^{m+4}}{2(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Sin[a + b*x]^2,x]
```

```
[Out] x^(4 + m)/(2*(4 + m)) + (2^(-6 - m)*E^((2*I)*a)*x^m*Gamma[4 + m, (-2*I)*b*x
])/(b^4*((-I)*b*x)^m) + (2^(-6 - m)*x^m*Gamma[4 + m, (2*I)*b*x])/(b^4*E^((2
*I)*a)*(I*b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
```

```
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{3+m}\sin^2(a+bx)\,dx &= \int\left(\frac{x^{3+m}}{2}-\frac{1}{2}x^{3+m}\cos(2a+2bx)\right)dx\\ &= \frac{x^{4+m}}{2(4+m)}-\frac{1}{2}\int x^{3+m}\cos(2a+2bx)\,dx\\ &= \frac{x^{4+m}}{2(4+m)}-\frac{1}{4}\int e^{-i(2a+2bx)}x^{3+m}\,dx-\frac{1}{4}\int e^{i(2a+2bx)}x^{3+m}\,dx\\ &= \frac{x^{4+m}}{2(4+m)}+\frac{2^{-6-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(4+m,-2ibx)}{b^4}+\frac{2^{-6-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(4+m,2ibx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.328603, size = 118, normalized size = 1.22

$$\frac{2^{-m-6}x^m\left(b^2x^2\right)^{-m}\left((m+4)(\cos(a)-i\sin(a))^2(-ibx)^m\text{Gamma}(m+4,2ibx)+(m+4)(\cos(a)+i\sin(a))^2(ibx)^m\text{Gamma}\right.}{b^4(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Sin[a + b*x]^2,x]
```

```
[Out] (2^(-6 - m)*x^m*(2^(5 + m)*b^4*x^4*(b^2*x^2)^m + (4 + m)*((-I)*b*x)^m*Gamma
[4 + m, (2*I)*b*x]*(Cos[a] - I*Sin[a])^2 + (4 + m)*(I*b*x)^m*Gamma[4 + m, (
-2*I)*b*x]*(Cos[a] + I*Sin[a])^2))/(b^4*(4 + m)*(b^2*x^2)^m)
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int x^{3+m}\left(\sin\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3+m)*sin(b*x+a)^2,x)
```

```
[Out] int(x^(3+m)*sin(b*x+a)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.83243, size = 235, normalized size = 2.42

$$\frac{4\,bxx^{m+3} + (-i\,m - 4i)e^{(-(m+3)\log(2i\,b) - 2i\,a)}\Gamma\left(m+4, 2i\,bx\right) + (i\,m + 4i)e^{(-(m+3)\log(-2i\,b) + 2i\,a)}\Gamma\left(m+4, -2i\,bx\right)}{8\,(bm + 4\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m + 3) + (-I*m - 4*I)*e^(-(m + 3)*log(2*I*b) - 2*I*a)*gamma(m
 + 4, 2*I*b*x) + (I*m + 4*I)*e^(-(m + 3)*log(-2*I*b) + 2*I*a)*gamma(m + 4,
-2*I*b*x))/(b*m + 4*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+3} \sin(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 3)*sin(b*x + a)^2, x)
```

3.85 $\int x^{2+m}\sin^2(a+bx)\,dx$

Optimal. Leaf size=103

$$-\frac{ie^{2ia}2^{-m-5}x^m(-ibx)^{-m}\text{Gamma}(m+3,-2ibx)}{b^3}+\frac{ie^{-2ia}2^{-m-5}x^m(ibx)^{-m}\text{Gamma}(m+3,2ibx)}{b^3}+\frac{x^{m+3}}{2(m+3)}$$

```
[Out] x^(3 + m)/(2*(3 + m)) - (I*2^(-5 - m)*E^((2*I)*a)*x^m*Gamma[3 + m, (-2*I)*b
*x])/(b^3*((-I)*b*x)^m) + (I*2^(-5 - m)*x^m*Gamma[3 + m, (2*I)*b*x])/(b^3*E
^((2*I)*a)*(I*b*x)^m)
```

Rubi [A] time = 0.142807, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$-\frac{ie^{2ia}2^{-m-5}x^m(-ibx)^{-m}\text{Gamma}(m+3,-2ibx)}{b^3}+\frac{ie^{-2ia}2^{-m-5}x^m(ibx)^{-m}\text{Gamma}(m+3,2ibx)}{b^3}+\frac{x^{m+3}}{2(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Sin[a + b*x]^2,x]
```

```
[Out] x^(3 + m)/(2*(3 + m)) - (I*2^(-5 - m)*E^((2*I)*a)*x^m*Gamma[3 + m, (-2*I)*b
*x])/(b^3*((-I)*b*x)^m) + (I*2^(-5 - m)*x^m*Gamma[3 + m, (2*I)*b*x])/(b^3*E
^((2*I)*a)*(I*b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
```

```
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int x^{2+m}\sin^2(a+bx)\,dx &= \int\left(\frac{x^{2+m}}{2}-\frac{1}{2}x^{2+m}\cos(2a+2bx)\right)dx\\
&= \frac{x^{3+m}}{2(3+m)}-\frac{1}{2}\int x^{2+m}\cos(2a+2bx)\,dx\\
&= \frac{x^{3+m}}{2(3+m)}-\frac{1}{4}\int e^{-i(2a+2bx)}x^{2+m}\,dx-\frac{1}{4}\int e^{i(2a+2bx)}x^{2+m}\,dx\\
&= \frac{x^{3+m}}{2(3+m)}-\frac{i2^{-5-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(3+m,-2ibx)}{b^3}+\frac{i2^{-5-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(3+m,2ibx)}{b^3}
\end{aligned}
$$

Mathematica [A] time = 0.310719, size = 120, normalized size = 1.17

$$
\frac{2^{-m-5}x^m\left(b^2x^2\right)^{-m}\left((m+3)(\sin(2a)+i\cos(2a))(-ibx)^m\text{Gamma}(m+3,2ibx)+(m+3)(\sin(2a)-i\cos(2a))(ibx)^m\text{Gamm}\right.}{b^3(m+3)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Sin[a + b*x]^2,x]
```

```
[Out] (2^(-5 - m)*x^m*(2^(4 + m)*b*x*(b^2*x^2)^(1 + m) + (3 + m)*(I*b*x)^m*Gamma[
3 + m, (-2*I)*b*x]*((-I)*Cos[2*a] + Sin[2*a]) + (3 + m)*((-I)*b*x)^m*Gamma[
3 + m, (2*I)*b*x]*(I*Cos[2*a] + Sin[2*a])))/(b^3*(3 + m)*(b^2*x^2)^m)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int x^{2+m}\left(\sin(bx+a)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*sin(b*x+a)^2,x)
```

```
[Out] int(x^(2+m)*sin(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(m+3)\int x^2x^m\cos(2\,bx+2\,a)\,dx - e^{(m\log(x)+3\,\log(x))}}{2\,(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((m + 3)*integrate(x^2*x^m*cos(2*b*x + 2*a), x) - e^(m*log(x) + 3*log(
x)))/(m + 3)
```

Fricas [A] time = 1.77347, size = 235, normalized size = 2.28

$$\frac{4\,bxx^{m+2} + (-i\,m-3i)e^{(-(m+2)\log(2i\,b)-2i\,a)}\Gamma\left(m+3,2i\,bx\right) + (i\,m+3i)e^{(-(m+2)\log(-2i\,b)+2i\,a)}\Gamma\left(m+3,-2i\,bx\right)}{8\,(bm+3\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m + 2) + (-I*m - 3*I)*e^(-(m + 2)*log(2*I*b) - 2*I*a)*gamma(m
 + 3, 2*I*b*x) + (I*m + 3*I)*e^(-(m + 2)*log(-2*I*b) + 2*I*a)*gamma(m + 3,
-2*I*b*x))/(b*m + 3*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+2} \sin(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*sin(b*x + a)^2, x)
```

3.86 $\int x^{1+m}\sin^2(a+bx)\,dx$

Optimal. Leaf size=99

$$-\frac{e^{2ia}2^{-m-4}x^m(-ibx)^{-m}\text{Gamma}(m+2,-2ibx)}{b^2}-\frac{e^{-2ia}2^{-m-4}x^m(ibx)^{-m}\text{Gamma}(m+2,2ibx)}{b^2}+\frac{x^{m+2}}{2(m+2)}$$

```
[Out] x^(2 + m)/(2*(2 + m)) - (2^(-4 - m)*E^((2*I)*a)*x^m*Gamma[2 + m, (-2*I)*b*x
])/(b^2*((-I)*b*x)^m) - (2^(-4 - m)*x^m*Gamma[2 + m, (2*I)*b*x])/(b^2*E^((2
*I)*a)*(I*b*x)^m)
```

Rubi [A] time = 0.142854, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$-\frac{e^{2ia}2^{-m-4}x^m(-ibx)^{-m}\text{Gamma}(m+2,-2ibx)}{b^2}-\frac{e^{-2ia}2^{-m-4}x^m(ibx)^{-m}\text{Gamma}(m+2,2ibx)}{b^2}+\frac{x^{m+2}}{2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Sin[a + b*x]^2,x]
```

```
[Out] x^(2 + m)/(2*(2 + m)) - (2^(-4 - m)*E^((2*I)*a)*x^m*Gamma[2 + m, (-2*I)*b*x
])/(b^2*((-I)*b*x)^m) - (2^(-4 - m)*x^m*Gamma[2 + m, (2*I)*b*x])/(b^2*E^((2
*I)*a)*(I*b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
```

```
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int x^{1+m}\sin^2(a+bx)\,dx &= \int \left(\frac{x^{1+m}}{2}-\frac{1}{2}x^{1+m}\cos(2a+2bx)\right)dx\\
&= \frac{x^{2+m}}{2(2+m)}-\frac{1}{2}\int x^{1+m}\cos(2a+2bx)\,dx\\
&= \frac{x^{2+m}}{2(2+m)}-\frac{1}{4}\int e^{-i(2a+2bx)}x^{1+m}\,dx-\frac{1}{4}\int e^{i(2a+2bx)}x^{1+m}\,dx\\
&= \frac{x^{2+m}}{2(2+m)}-\frac{2^{-4-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(2+m,-2ibx)}{b^2}-\frac{2^{-4-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(2+m,2ibx)}{b^2}
\end{aligned}
$$

Mathematica [A] time = 0.305028, size = 116, normalized size = 1.17

$$
\frac{2^{-m-4}x^m\left(b^2x^2\right)^{-m}\left(-(m+2)(\cos(a)-i\sin(a))^2(-ibx)^m\text{Gamma}(m+2,2ibx)-(m+2)(\cos(a)+i\sin(a))^2(ibx)^m\text{Gamm}\right.}{b^2(m+2)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Sin[a + b*x]^2,x]
```

```
[Out] (2^(-4 - m)*x^m*(2^(3 + m)*(b^2*x^2)^(1 + m) - (2 + m)*((-I)*b*x)^m*Gamma[2
 + m, (2*I)*b*x]*(Cos[a] - I*Sin[a])^2 - (2 + m)*(I*b*x)^m*Gamma[2 + m, (-2
*I)*b*x]*(Cos[a] + I*Sin[a])^2))/(b^2*(2 + m)*(b^2*x^2)^m)
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int x^{1+m}\left(\sin\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*sin(b*x+a)^2,x)
```

```
[Out] int(x^(1+m)*sin(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(m+2)\int xx^m\cos(2\,bx+2\,a)\,dx - e^{(m\log(x)+2\,\log(x))}}{2\,(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((m + 2)*integrate(x*x^m*cos(2*b*x + 2*a), x) - e^(m*log(x) + 2*log(x)
))/(m + 2)
```

Fricas [A] time = 1.74335, size = 235, normalized size = 2.37

$$\frac{4\,bxx^{m+1} + (-i\,m-2i)e^{(-(m+1)\log(2i\,b)-2i\,a)}\Gamma\left(m+2,2i\,bx\right) + (i\,m+2i)e^{(-(m+1)\log(-2i\,b)+2i\,a)}\Gamma\left(m+2,-2i\,bx\right)}{8\,(bm+2\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m + 1) + (-I*m - 2*I)*e^(-(m + 1)*log(2*I*b) - 2*I*a)*gamma(m
 + 2, 2*I*b*x) + (I*m + 2*I)*e^(-(m + 1)*log(-2*I*b) + 2*I*a)*gamma(m + 2,
-2*I*b*x))/(b*m + 2*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1}\sin^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*sin(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 1)*sin(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*sin(b*x + a)^2, x)
```

3.87 $\int x^m \sin^2(a+bx)\,dx$

Optimal. Leaf size=103

$$\frac{ie^{2ia}2^{-m-3}x^m(-ibx)^{-m}\text{Gamma}(m+1,-2ibx)}{b}-\frac{ie^{-2ia}2^{-m-3}x^m(ibx)^{-m}\text{Gamma}(m+1,2ibx)}{b}+\frac{x^{m+1}}{2(m+1)}$$

```
[Out] x^(1 + m)/(2*(1 + m)) + (I*2^(-3 - m)*E^((2*I)*a)*x^m*Gamma[1 + m, (-2*I)*b
*x])/(b*((-I)*b*x)^m) - (I*2^(-3 - m)*x^m*Gamma[1 + m, (2*I)*b*x])/(b*E^((2
*I)*a)*(I*b*x)^m)
```

Rubi [A] time = 0.134033, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3312, 3307, 2181}

$$\frac{ie^{2ia}2^{-m-3}x^m(-ibx)^{-m}\text{Gamma}(m+1,-2ibx)}{b}-\frac{ie^{-2ia}2^{-m-3}x^m(ibx)^{-m}\text{Gamma}(m+1,2ibx)}{b}+\frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + b*x]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) + (I*2^(-3 - m)*E^((2*I)*a)*x^m*Gamma[1 + m, (-2*I)*b
*x])/(b*((-I)*b*x)^m) - (I*2^(-3 - m)*x^m*Gamma[1 + m, (2*I)*b*x])/(b*E^((2
*I)*a)*(I*b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
```

```
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^m \sin^2(a+bx)\,dx &= \int \left(\frac{x^m}{2}-\frac{1}{2}x^m\cos(2a+2bx)\right)dx\\ &= \frac{x^{1+m}}{2(1+m)}-\frac{1}{2}\int x^m\cos(2a+2bx)\,dx\\ &= \frac{x^{1+m}}{2(1+m)}-\frac{1}{4}\int e^{-i(2a+2bx)}x^m\,dx-\frac{1}{4}\int e^{i(2a+2bx)}x^m\,dx\\ &= \frac{x^{1+m}}{2(1+m)}+\frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(1+m,-2ibx)}{b}-\frac{i2^{-3-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(1+m,2ibx)}{b}\end{aligned}$$

Mathematica [A] time = 0.268765, size = 120, normalized size = 1.17

$$\frac{2^{-m-3}x^m\left(b^2x^2\right)^{-m}\left(-i(m+1)(\cos(a)-i\sin(a))^2(-ibx)^m\text{Gamma}(m+1,2ibx)+i(m+1)(\cos(a)+i\sin(a))^2(ibx)^m\text{Gamm}\right.}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sin[a + b*x]^2,x]
```

```
[Out] (2^(-3 - m)*x^m*(2^(2 + m)*b*x*(b^2*x^2)^m - I*(1 + m)*((-I)*b*x)^m*Gamma[1
 + m, (2*I)*b*x]*(Cos[a] - I*Sin[a])^2 + I*(1 + m)*(I*b*x)^m*Gamma[1 + m, (
-2*I)*b*x]*(Cos[a] + I*Sin[a])^2))/(b*(1 + m)*(b^2*x^2)^m)
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int x^m \left(\sin\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(b*x+a)^2,x)
```

```
[Out] int(x^m*sin(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(m+1)\int x^m \cos(2bx+2a)\ dx - e^{(m\log(x)+\log(x))}}{2(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((m + 1)*integrate(x^m*cos(2*b*x + 2*a), x) - e^(m*log(x) + log(x)))/(
m + 1)
```

Fricas [A] time = 1.75863, size = 203, normalized size = 1.97

$$\frac{4bxx^m + (-im-i)e^{(-m\log(2ib)-2ia)}\Gamma(m+1, 2ibx) + (im+i)e^{(-m\log(-2ib)+2ia)}\Gamma(m+1, -2ibx)}{8(bm+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^m + (-I*m - I)*e^(-m*log(2*I*b) - 2*I*a)*gamma(m + 1, 2*I*b*x)
 + (I*m + I)*e^(-m*log(-2*I*b) + 2*I*a)*gamma(m + 1, -2*I*b*x))/(b*m + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sin^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(b*x+a)**2,x)
```

```
[Out] Integral(x**m*sin(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sin(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*sin(b*x + a)^2, x)
```

3.88 $\int x^{-1+m}\sin^2(a+bx)\,dx$

Optimal. Leaf size=83

$$e^{2ia}2^{-m-2}x^m(-ibx)^{-m}\text{Gamma}(m,-2ibx)+e^{-2ia}2^{-m-2}x^m(ibx)^{-m}\text{Gamma}(m,2ibx)+\frac{x^m}{2m}$$

```
[Out] x^m/(2*m) + (2^(-2 - m)*E^((2*I)*a)*x^m*Gamma[m, (-2*I)*b*x])/((-I)*b*x)^m
+ (2^(-2 - m)*x^m*Gamma[m, (2*I)*b*x])/(E^((2*I)*a)*(I*b*x)^m)
```

Rubi [A] time = 0.130349, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$e^{2ia}2^{-m-2}x^m(-ibx)^{-m}\text{Gamma}(m,-2ibx)+e^{-2ia}2^{-m-2}x^m(ibx)^{-m}\text{Gamma}(m,2ibx)+\frac{x^m}{2m}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Sin[a + b*x]^2,x]
```

```
[Out] x^m/(2*m) + (2^(-2 - m)*E^((2*I)*a)*x^m*Gamma[m, (-2*I)*b*x])/((-I)*b*x)^m
+ (2^(-2 - m)*x^m*Gamma[m, (2*I)*b*x])/(E^((2*I)*a)*(I*b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
```

```
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^{-1+m}\sin^2(a+bx)\,dx &= \int\left(\frac{x^{-1+m}}{2}-\frac{1}{2}x^{-1+m}\cos(2a+2bx)\right)dx\\
&= \frac{x^m}{2m}-\frac{1}{2}\int x^{-1+m}\cos(2a+2bx)\,dx\\
&= \frac{x^m}{2m}-\frac{1}{4}\int e^{-i(2a+2bx)}x^{-1+m}\,dx-\frac{1}{4}\int e^{i(2a+2bx)}x^{-1+m}\,dx\\
&= \frac{x^m}{2m}+2^{-2-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(m,-2ibx)+2^{-2-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(m,2ibx)
\end{aligned}$$

Mathematica [A] time = 0.230728, size = 99, normalized size = 1.19

$$\frac{2^{-m-2}x^m\left(b^2x^2\right)^{-m}\left(m(\cos(a)-i\sin(a))^2(-ibx)^m\text{Gamma}(m,2ibx)+m(\cos(a)+i\sin(a))^2(ibx)^m\text{Gamma}(m,-2ibx)+2^{m}\right.}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Sin[a + b*x]^2,x]
```

```
[Out] (2^(-2 - m)*x^m*(2^(1 + m)*(b^2*x^2)^m + m*((-I)*b*x)^m*Gamma[m, (2*I)*b*x]
*(Cos[a] - I*Sin[a])^2 + m*(I*b*x)^m*Gamma[m, (-2*I)*b*x]*(Cos[a] + I*Sin[a
])^2))/(m*(b^2*x^2)^m)
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int x^{-1+m}\left(\sin(bx+a)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*sin(b*x+a)^2,x)
```

```
[Out] int(x^(-1+m)*sin(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{m\int\frac{x^m\cos(2\,bx+2\,a)}{x}\,dx - x^m}{2\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(m*integrate(x^m*cos(2*b*x + 2*a)/x, x) - x^m)/m
```

Fricas [A] time = 1.74727, size = 193, normalized size = 2.33

$$\frac{4\,bxx^{m-1} - i\,me^{(-(m-1)\log(2i\,b)-2i\,a)}\Gamma\left(m,2i\,bx\right) + i\,me^{(-(m-1)\log(-2i\,b)+2i\,a)}\Gamma\left(m,-2i\,bx\right)}{8\,bm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m - 1) - I*m*e^(-(m - 1)*log(2*I*b) - 2*I*a)*gamma(m, 2*I*b*x
) + I*m*e^(-(m - 1)*log(-2*I*b) + 2*I*a)*gamma(m, -2*I*b*x))/(b*m)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1}\sin^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*sin(b*x+a)**2,x)
```

```
[Out] Integral(x**(m - 1)*sin(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1}\sin\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*sin(b*x + a)^2, x)
```

3.89 $\int x^{-2+m} \sin^2(a+bx)\,dx$

Optimal. Leaf size=101

$$-ie^{2ia}b2^{-m-1}x^m(-ibx)^{-m}\text{Gamma}(m-1,-2ibx)+ie^{-2ia}b2^{-m-1}x^m(ibx)^{-m}\text{Gamma}(m-1,2ibx)-\frac{x^{m-1}}{2(1-m)}$$

```
[Out] -x^(-1 + m)/(2*(1 - m)) - (I*2^(-1 - m)*b*E^((2*I)*a)*x^m*Gamma[-1 + m, (-2
*I)*b*x])/((-I)*b*x)^m + (I*2^(-1 - m)*b*x^m*Gamma[-1 + m, (2*I)*b*x])/(E^(
(2*I)*a)*(I*b*x)^m)
```

Rubi [A] time = 0.137095, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$-ie^{2ia}b2^{-m-1}x^m(-ibx)^{-m}\text{Gamma}(m-1,-2ibx)+ie^{-2ia}b2^{-m-1}x^m(ibx)^{-m}\text{Gamma}(m-1,2ibx)-\frac{x^{m-1}}{2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Sin[a + b*x]^2,x]
```

```
[Out] -x^(-1 + m)/(2*(1 - m)) - (I*2^(-1 - m)*b*E^((2*I)*a)*x^m*Gamma[-1 + m, (-2
*I)*b*x])/((-I)*b*x)^m + (I*2^(-1 - m)*b*x^m*Gamma[-1 + m, (2*I)*b*x])/(E^(
(2*I)*a)*(I*b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
```

```
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{-2+m}\sin^2(a+bx)\,dx &= \int\left(\frac{x^{-2+m}}{2}-\frac{1}{2}x^{-2+m}\cos(2a+2bx)\right)dx\\ &= -\frac{x^{-1+m}}{2(1-m)}-\frac{1}{2}\int x^{-2+m}\cos(2a+2bx)\,dx\\ &= -\frac{x^{-1+m}}{2(1-m)}-\frac{1}{4}\int e^{-i(2a+2bx)}x^{-2+m}\,dx-\frac{1}{4}\int e^{i(2a+2bx)}x^{-2+m}\,dx\\ &= -\frac{x^{-1+m}}{2(1-m)}-i2^{-1-m}be^{2ia}x^m(-ibx)^{-m}\Gamma(-1+m,-2ibx)+i2^{-1-m}be^{-2ia}x^m(ibx)^{-m}\Gamma(-1+m,2ibx\end{aligned}$$

Mathematica [A] time = 0.31142, size = 117, normalized size = 1.16

$$\frac{2^{-m-1}x^{m-1}\left(b^2x^2\right)^{-m}\left(b(m-1)x(\sin(2a)+i\cos(2a))(-ibx)^m\text{Gamma}(m-1,2ibx)+b(m-1)x(\sin(2a)-i\cos(2a))(ibx)^m\right.}{m-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Sin[a + b*x]^2,x]
```

```
[Out] (2^(-1 - m)*x^(-1 + m)*(2^m*(b^2*x^2)^m + b*(-1 + m)*x*(I*b*x)^m*Gamma[-1 +
 m, (-2*I)*b*x]*((-I)*Cos[2*a] + Sin[2*a]) + b*(-1 + m)*x*((-I)*b*x)^m*Gamm
a[-1 + m, (2*I)*b*x]*(I*Cos[2*a] + Sin[2*a])))/((-1 + m)*(b^2*x^2)^m)
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int x^{m-2}\left(\sin\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-2)*sin(b*x+a)^2,x)
```

```
[Out] int(x^(m-2)*sin(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(m-1)x\int\frac{x^m\cos(2\,bx+2\,a)}{x^2}\,dx-x^m}{2\,(m-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((m - 1)*x*integrate(x^m*cos(2*b*x + 2*a)/x^2, x) - x^m)/((m - 1)*x)
```

Fricas [A] time = 1.73722, size = 227, normalized size = 2.25

$$\frac{4\,bxx^{m-2}+(-i\,m+i)e^{(-(m-2)\log(2i\,b)-2i\,a)}\Gamma\left(m-1,2i\,bx\right)+(i\,m-i)e^{(-(m-2)\log(-2i\,b)+2i\,a)}\Gamma\left(m-1,-2i\,bx\right)}{8\,(bm-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m - 2) + (-I*m + I)*e^(-(m - 2)*log(2*I*b) - 2*I*a)*gamma(m -
 1, 2*I*b*x) + (I*m - I)*e^(-(m - 2)*log(-2*I*b) + 2*I*a)*gamma(m - 1, -2*I
*b*x))/(b*m - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-2} \sin (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*sin(b*x + a)^2, x)
```

3.90 $\int x^{-3+m} \sin^2(a+bx)\,dx$

Optimal. Leaf size=97

$$-e^{2ia}b^2 2^{-m}x^m(-ibx)^{-m}\text{Gamma}(m-2,-2ibx)-e^{-2ia}b^2 2^{-m}x^m(ibx)^{-m}\text{Gamma}(m-2,2ibx)-\frac{x^{m-2}}{2(2-m)}$$

```
[Out] -x^(-2 + m)/(2*(2 - m)) - (b^2*E^((2*I)*a)*x^m*Gamma[-2 + m, (-2*I)*b*x])/(
2^m*((-I)*b*x)^m) - (b^2*x^m*Gamma[-2 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*
b*x)^m)
```

Rubi [A] time = 0.173118, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$-e^{2ia}b^2 2^{-m}x^m(-ibx)^{-m}\text{Gamma}(m-2,-2ibx)-e^{-2ia}b^2 2^{-m}x^m(ibx)^{-m}\text{Gamma}(m-2,2ibx)-\frac{x^{m-2}}{2(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Sin[a + b*x]^2,x]
```

```
[Out] -x^(-2 + m)/(2*(2 - m)) - (b^2*E^((2*I)*a)*x^m*Gamma[-2 + m, (-2*I)*b*x])/(
2^m*((-I)*b*x)^m) - (b^2*x^m*Gamma[-2 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*
b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
```

```
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int x^{-3+m}\sin^2(a+bx)\,dx &= \int\left(\frac{x^{-3+m}}{2}-\frac{1}{2}x^{-3+m}\cos(2a+2bx)\right)dx\\
&= -\frac{x^{-2+m}}{2(2-m)}-\frac{1}{2}\int x^{-3+m}\cos(2a+2bx)\,dx\\
&= -\frac{x^{-2+m}}{2(2-m)}-\frac{1}{4}\int e^{-i(2a+2bx)}x^{-3+m}\,dx-\frac{1}{4}\int e^{i(2a+2bx)}x^{-3+m}\,dx\\
&= -\frac{x^{-2+m}}{2(2-m)}-2^{-m}b^2e^{2ia}x^m(-ibx)^{-m}\Gamma(-2+m,-2ibx)-2^{-m}b^2e^{-2ia}x^m(ibx)^{-m}\Gamma(-2+m,2ibx)
\end{aligned}
$$

Mathematica [A] time = 0.364202, size = 121, normalized size = 1.25

$$
\frac{2^{-m-1}x^{m-2}\left(b^2x^2\right)^{-m}\left(-2b^2(m-2)x^2(\cos(a)-i\sin(a))^2(-ibx)^m\text{Gamma}(m-2,2ibx)+2(m-2)(\cos(2a)+i\sin(2a))(ibx\right.}{m-2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Sin[a + b*x]^2,x]
```

```
[Out] (2^(-1 - m)*x^(-2 + m)*(2^m*(b^2*x^2)^m - 2*b^2*(-2 + m)*x^2*((-I)*b*x)^m*G
amma[-2 + m, (2*I)*b*x]*(Cos[a] - I*Sin[a])^2 + 2*(-2 + m)*(I*b*x)^(2 + m)*
Gamma[-2 + m, (-2*I)*b*x]*(Cos[2*a] + I*Sin[2*a])))/((-2 + m)*(b^2*x^2)^m)
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$
\int x^{m-3}(\sin(bx+a))^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-3)*sin(b*x+a)^2,x)
```

```
[Out] int(x^(m-3)*sin(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(m-2)x^2\int\frac{x^m\cos(2\,bx+2\,a)}{x^3}\,dx-x^m}{2\,(m-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((m - 2)*x^2*integrate(x^m*cos(2*b*x + 2*a)/x^3, x) - x^m)/((m - 2)*x^
2)
```

Fricas [A] time = 1.81052, size = 235, normalized size = 2.42

$$\frac{4\,bxx^{m-3}+(-i\,m+2i)e^{(-(m-3)\log(2i\,b)-2i\,a)}\Gamma\left(m-2,2i\,bx\right)+(i\,m-2i)e^{(-(m-3)\log(-2i\,b)+2i\,a)}\Gamma\left(m-2,-2i\,bx\right)}{8\,(bm-2\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m - 3) + (-I*m + 2*I)*e^(-(m - 3)*log(2*I*b) - 2*I*a)*gamma(m
 - 2, 2*I*b*x) + (I*m - 2*I)*e^(-(m - 3)*log(-2*I*b) + 2*I*a)*gamma(m - 2,
-2*I*b*x))/(b*m - 2*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-3} \sin(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 3)*sin(b*x + a)^2, x)
```

3.91 $\int \left(\frac{x}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x\sqrt{\csc(e+fx)}\right)dx$

Optimal. Leaf size=42

$$\frac{4}{9f^2\csc^{\frac{3}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}$$

```
[Out] 4/(9*f^2*Csc[e + f*x]^(3/2)) - (2*x*Cos[e + f*x])/(3*f*Sqrt[Csc[e + f*x]])
```

Rubi [A] time = 0.123271, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {4187, 4189}

$$\frac{4}{9f^2\csc^{\frac{3}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Csc[e + f*x]^(3/2) - (x*Sqrt[Csc[e + f*x]])/3,x]
```

```
[Out] 4/(9*f^2*Csc[e + f*x]^(3/2)) - (2*x*Cos[e + f*x])/(3*f*Sqrt[Csc[e + f*x]])
```

Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \left(\frac{x}{\csc^{\frac{3}{2}}(e+fx)} - \frac{1}{3}x\sqrt{\csc(e+fx)}\right) dx = -\left(\frac{1}{3}\int x\sqrt{\csc(e+fx)}\,dx\right) + \int \frac{x}{\csc^{\frac{3}{2}}(e+fx)}\,dx$$
$$= \frac{4}{9f^2\csc^{\frac{3}{2}}(e+fx)} - \frac{2x\cos(e+fx)}{3f\sqrt{\csc(e+fx)}} + \frac{1}{3}\int x\sqrt{\csc(e+fx)}\,dx - \frac{1}{3}\left(\sqrt{\csc(e+\right.$$
$$= \frac{4}{9f^2\csc^{\frac{3}{2}}(e+fx)} - \frac{2x\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}$$

Mathematica [A] time = 0.487129, size = 29, normalized size = 0.69

$$-\frac{2(3fx\cot(e+fx)-2)}{9f^2\csc^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Csc[e + f*x]^(3/2) - (x*Sqrt[Csc[e + f*x]])/3,x]
```

```
[Out] (-2*(-2 + 3*f*x*Cot[e + f*x]))/(9*f^2*Csc[e + f*x]^(3/2))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int x\left(\csc\left(fx+e\right)\right)^{-\frac{3}{2}} - \frac{x}{3}\sqrt{\csc\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x)
```

```
[Out] int(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{3}x\sqrt{\csc\left(fx+e\right)} + \frac{x}{\csc\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x*sqrt(csc(f*x + e)) + x/csc(f*x + e)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{3x}{\csc^{\frac{3}{2}}(e+fx)}\,dx + \int x\sqrt{\csc\left(e+fx\right)}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)**(3/2)-1/3*x*csc(f*x+e)**(1/2),x)
```

```
[Out] -(Integral(-3*x/csc(e + f*x)**(3/2), x) + Integral(x*sqrt(csc(e + f*x)), x)
)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{3}\,x\sqrt{\csc\left(fx+e\right)} + \frac{x}{\csc\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(3/2)-1/3*x*csc(f*x+e)^(1/2),x, algorithm="giac")

[Out] integrate(-1/3*x*sqrt(csc(f*x + e)) + x/csc(f*x + e)^(3/2), x)
```

3.92 $\int \left(\frac{x^2}{\csc^{\frac{3}{2}}(e+fx)} - \frac{1}{3}x^2\sqrt{\csc(e+fx)}\right) dx$

Optimal. Leaf size=111

$$\frac{8x}{9f^2\csc^{\frac{3}{2}}(e+fx)} + \frac{16\cos(e+fx)}{27f^3\sqrt{\csc(e+fx)}} - \frac{16\sqrt{\sin(e+fx)}\sqrt{\csc(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{27f^3} - \frac{2x^2\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}$$

```
[Out] (8*x)/(9*f^2*Csc[e + f*x]^(3/2)) + (16*Cos[e + f*x])/(27*f^3*Sqrt[Csc[e + f
*x]]) - (2*x^2*Cos[e + f*x])/(3*f*Sqrt[Csc[e + f*x]]) - (16*Sqrt[Csc[e + f*
x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(27*f^3)
```

Rubi [A] time = 0.209395, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4188, 4189, 3769, 3771, 2641}

$$\frac{8x}{9f^2\csc^{\frac{3}{2}}(e+fx)} + \frac{16\cos(e+fx)}{27f^3\sqrt{\csc(e+fx)}} - \frac{16\sqrt{\sin(e+fx)}\sqrt{\csc(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{27f^3} - \frac{2x^2\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Csc[e + f*x]^(3/2) - (x^2*Sqrt[Csc[e + f*x]])/3,x]
```

```
[Out] (8*x)/(9*f^2*Csc[e + f*x]^(3/2)) + (16*Cos[e + f*x])/(27*f^3*Sqrt[Csc[e + f
*x]]) - (2*x^2*Cos[e + f*x])/(3*f*Sqrt[Csc[e + f*x]]) - (16*Sqrt[Csc[e + f*
x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(27*f^3)
```

Rule 4188

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(n + 1)/(b^2*n), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n + 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^n, x], x]
+ Simp[((c + d*x)^m*Cos[e + f*x]*(b*Csc[e + f*x])^(n + 1))/(b*f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(\frac{x^2}{\csc^{\frac{3}{2}}(e+fx)}-\frac{1}{3}x^2\sqrt{\csc(e+fx)}\right)dx=-\left(\frac{1}{3}\int x^2\sqrt{\csc(e+fx)}\,dx\right)+\int \frac{x^2}{\csc^{\frac{3}{2}}(e+fx)}\,dx$$

$$=\frac{8x}{9f^2\csc^{\frac{3}{2}}(e+fx)}-\frac{2x^2\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}+\frac{1}{3}\int x^2\sqrt{\csc(e+fx)}\,dx-\frac{8\int \frac{1}{\csc^{\frac{3}{2}}(e+}}{9f^2}$$

$$=\frac{8x}{9f^2\csc^{\frac{3}{2}}(e+fx)}+\frac{16\cos(e+fx)}{27f^3\sqrt{\csc(e+fx)}}-\frac{2x^2\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}-\frac{8\int \sqrt{\csc(e+f}}{27f^2}$$

$$=\frac{8x}{9f^2\csc^{\frac{3}{2}}(e+fx)}+\frac{16\cos(e+fx)}{27f^3\sqrt{\csc(e+fx)}}-\frac{2x^2\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}-\frac{\left(8\sqrt{\csc(e+fx}\right.}{}$$

$$=\frac{8x}{9f^2\csc^{\frac{3}{2}}(e+fx)}+\frac{16\cos(e+fx)}{27f^3\sqrt{\csc(e+fx)}}-\frac{2x^2\cos(e+fx)}{3f\sqrt{\csc(e+fx)}}-\frac{16\sqrt{\csc(e+fx}}{}$$

Mathematica [A] time = 0.556044, size = 87, normalized size = 0.78

$$-\frac{\sqrt{\csc(e+fx)}\left(9f^2x^2\sin(2(e+fx))-8\sin(2(e+fx))+12fx\cos(2(e+fx))-16\sqrt{\sin(e+fx)}F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right.}{27f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Csc[e + f*x]^(3/2) - (x^2*Sqrt[Csc[e + f*x]])/3,x]
```

```
[Out] -(Sqrt[Csc[e + f*x]]*(-12*f*x + 12*f*x*Cos[2*(e + f*x)] - 16*EllipticF[(-2*
e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f*x]] - 8*Sin[2*(e + f*x)] + 9*f^2*x^2*S
in[2*(e + f*x)]))/(27*f^3)
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int x^2\left(\csc\left(fx+e\right)\right)^{-\frac{3}{2}}-\frac{x^2}{3}\sqrt{\csc\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x)
```

```
[Out] int(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{3}\,x^2\sqrt{\csc\left(fx+e\right)}+\frac{x^2}{\csc\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(-1/3*x^2*sqrt(csc(f*x + e)) + x^2/csc(f*x + e)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{3x^2}{\csc^{\frac{3}{2}}(e+fx)}\,dx + \int x^2\sqrt{\csc\left(e+fx\right)}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/csc(f*x+e)**(3/2)-1/3*x**2*csc(f*x+e)**(1/2),x)
```

```
[Out] -(Integral(-3*x**2/csc(e + f*x)**(3/2), x) + Integral(x**2*sqrt(csc(e + f*x
)), x))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{3}\,x^2\sqrt{\csc\left(fx+e\right)} + \frac{x^2}{\csc\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csc(f*x+e)^(3/2)-1/3*x^2*csc(f*x+e)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(-1/3*x^2*sqrt(csc(f*x + e)) + x^2/csc(f*x + e)^(3/2), x)
```

3.93 $\int \left(\frac{x}{\csc^{\frac{5}{2}}(e+fx)}-\frac{3x}{5\sqrt{\csc(e+fx)}}\right)dx$

Optimal. Leaf size=42

$$\frac{4}{25f^2\csc^{\frac{5}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{5f\csc^{\frac{3}{2}}(e+fx)}$$

```
[Out] 4/(25*f^2*Csc[e + f*x]^(5/2)) - (2*x*Cos[e + f*x])/(5*f*Csc[e + f*x]^(3/2))
```

Rubi [A] time = 0.106179, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {4187, 4189}

$$\frac{4}{25f^2\csc^{\frac{5}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{5f\csc^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Csc[e + f*x]^(5/2) - (3*x)/(5*Sqrt[Csc[e + f*x]]),x]
```

```
[Out] 4/(25*f^2*Csc[e + f*x]^(5/2)) - (2*x*Cos[e + f*x])/(5*f*Csc[e + f*x]^(3/2))
```

Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \left(\frac{x}{\csc^{\frac{5}{2}}(e+fx)} - \frac{3x}{5\sqrt{\csc(e+fx)}}\right) dx &= -\left(\frac{3}{5}\int \frac{x}{\sqrt{\csc(e+fx)}}\, dx\right) + \int \frac{x}{\csc^{\frac{5}{2}}(e+fx)}\, dx \\
&= \frac{4}{25f^2\csc^{\frac{5}{2}}(e+fx)} - \frac{2x\cos(e+fx)}{5f\csc^{\frac{3}{2}}(e+fx)} + \frac{3}{5}\int \frac{x}{\sqrt{\csc(e+fx)}}\, dx - \frac{1}{5}\left(3\sqrt{\csc(e+\right. \\
&= \frac{4}{25f^2\csc^{\frac{5}{2}}(e+fx)} - \frac{2x\cos(e+fx)}{5f\csc^{\frac{3}{2}}(e+fx)}
\end{aligned}
$$

Mathematica [A] time = 0.447213, size = 29, normalized size = 0.69

$$
-\frac{2(5fx\cot(e+fx)-2)}{25f^2\csc^{\frac{5}{2}}(e+fx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Csc[e + f*x]^(5/2) - (3*x)/(5*Sqrt[Csc[e + f*x]]),x]
```

```
[Out] (-2*(-2 + 5*f*x*Cot[e + f*x]))/(25*f^2*Csc[e + f*x]^(5/2))
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$
\int x\left(\csc\left(fx+e\right)\right)^{-\frac{5}{2}} - \frac{3x}{5}\frac{1}{\sqrt{\csc\left(fx+e\right)}}\, dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x)
```

```
[Out] int(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int -\frac{3x}{5\sqrt{\csc\left(fx+e\right)}} + \frac{x}{\csc\left(fx+e\right)^{\frac{5}{2}}}\, dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-3/5*x/sqrt(csc(f*x + e)) + x/csc(f*x + e)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{5x}{\csc^{\frac{5}{2}}(e+fx)}\,dx + \int \frac{3x}{\sqrt{\csc(e+fx)}}\,dx}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)**(5/2)-3/5*x/csc(f*x+e)**(1/2),x)
```

```
[Out] -(Integral(-5*x/csc(e + f*x)**(5/2), x) + Integral(3*x/sqrt(csc(e + f*x)),
x))/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{3\,x}{5\,\sqrt{\csc\left(fx+e\right)}} + \frac{x}{\csc\left(fx+e\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(5/2)-3/5*x/csc(f*x+e)^(1/2),x, algorithm="giac")

[Out] integrate(-3/5*x/sqrt(csc(f*x + e)) + x/csc(f*x + e)^(5/2), x)
```

3.94 $\int \left(\frac{x}{\csc^{\frac{7}{2}}(e+fx)} - \frac{5}{21}x\sqrt{\csc(e+fx)}\right) dx$

Optimal. Leaf size=83

$$\frac{20}{63f^2\csc^{\frac{3}{2}}(e+fx)} + \frac{4}{49f^2\csc^{\frac{7}{2}}(e+fx)} - \frac{2x\cos(e+fx)}{7f\csc^{\frac{5}{2}}(e+fx)} - \frac{10x\cos(e+fx)}{21f\sqrt{\csc(e+fx)}}$$

```
[Out] 4/(49*f^2*Csc[e + f*x]^(7/2)) - (2*x*Cos[e + f*x])/(7*f*Csc[e + f*x]^(5/2))
 + 20/(63*f^2*Csc[e + f*x]^(3/2)) - (10*x*Cos[e + f*x])/(21*f*Sqrt[Csc[e +
f*x]])
```

Rubi [A] time = 0.133373, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {4187, 4189}

$$\frac{20}{63f^2\csc^{\frac{3}{2}}(e+fx)} + \frac{4}{49f^2\csc^{\frac{7}{2}}(e+fx)} - \frac{2x\cos(e+fx)}{7f\csc^{\frac{5}{2}}(e+fx)} - \frac{10x\cos(e+fx)}{21f\sqrt{\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Csc[e + f*x]^(7/2) - (5*x*Sqrt[Csc[e + f*x]])/21,x]
```

```
[Out] 4/(49*f^2*Csc[e + f*x]^(7/2)) - (2*x*Cos[e + f*x])/(7*f*Csc[e + f*x]^(5/2))
 + 20/(63*f^2*Csc[e + f*x]^(3/2)) - (10*x*Cos[e + f*x])/(21*f*Sqrt[Csc[e +
f*x]])
```

Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
```

```
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \left(\frac{x}{\csc^{\frac{7}{2}}(e+fx)}-\frac{5}{21}x\sqrt{\csc(e+fx)}\right)dx=-\left(\frac{5}{21}\int x\sqrt{\csc(e+fx)}\,dx\right)+\int \frac{x}{\csc^{\frac{7}{2}}(e+fx)}\,dx$$
$$=\frac{4}{49f^2\csc^{\frac{7}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{7f\csc^{\frac{5}{2}}(e+fx)}+\frac{5}{7}\int \frac{x}{\csc^{\frac{3}{2}}(e+fx)}\,dx-\frac{1}{21}\left(5\sqrt{\csc(\right.$$
$$=\frac{4}{49f^2\csc^{\frac{7}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{7f\csc^{\frac{5}{2}}(e+fx)}+\frac{20}{63f^2\csc^{\frac{3}{2}}(e+fx)}-\frac{10x\cos(e+f}{21f\sqrt{\csc(e+}}$$
$$=\frac{4}{49f^2\csc^{\frac{7}{2}}(e+fx)}-\frac{2x\cos(e+fx)}{7f\csc^{\frac{5}{2}}(e+fx)}+\frac{20}{63f^2\csc^{\frac{3}{2}}(e+fx)}-\frac{10x\cos(e+f}{21f\sqrt{\csc(e+}}$$

Mathematica [A] time = 2.22757, size = 57, normalized size = 0.69

$$\frac{-36\cos(2(e+fx))-483fx\cot(e+fx)+63fx\cos(3(e+fx))\csc(e+fx)+316}{882f^2\csc^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Csc[e + f*x]^(7/2) - (5*x*Sqrt[Csc[e + f*x]])/21,x]
```

```
[Out] (316 - 36*Cos[2*(e + f*x)] - 483*f*x*Cot[e + f*x] + 63*f*x*Cos[3*(e + f*x)]
*Csc[e + f*x])/(882*f^2*Csc[e + f*x]^(3/2))
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int x\left(\csc\left(fx+e\right)\right)^{-\frac{7}{2}}-\frac{5x}{21}\sqrt{\csc\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x)
```

```
[Out] int(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{5}{21}\,x\sqrt{\csc\left(fx+e\right)}+\frac{x}{\csc\left(fx+e\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-5/21*x*sqrt(csc(f*x + e)) + x/csc(f*x + e)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)**(7/2)-5/21*x*csc(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{5}{21}\,x\sqrt{\csc\left(fx+e\right)}+\frac{x}{\csc\left(fx+e\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csc(f*x+e)^(7/2)-5/21*x*csc(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-5/21*x*sqrt(csc(f*x + e)) + x/csc(f*x + e)^(7/2), x)
```

3.95 $\int (c+dx)^3(a+a\sin(e+fx))\,dx$

Optimal. Leaf size=90

$$\frac{6ad^2(c+dx)\cos(e+fx)}{f^3}+\frac{3ad(c+dx)^2\sin(e+fx)}{f^2}-\frac{a(c+dx)^3\cos(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6ad^3\sin(e+fx)}{f^4}$$

```
[Out] (a*(c + d*x)^4)/(4*d) + (6*a*d^2*(c + d*x)*Cos[e + f*x])/f^3 - (a*(c + d*x)
^3*Cos[e + f*x])/f - (6*a*d^3*Sin[e + f*x])/f^4 + (3*a*d*(c + d*x)^2*Sin[e
+ f*x])/f^2
```

Rubi [A] time = 0.118222, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3296, 2637}

$$\frac{6ad^2(c+dx)\cos(e+fx)}{f^3}+\frac{3ad(c+dx)^2\sin(e+fx)}{f^2}-\frac{a(c+dx)^3\cos(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6ad^3\sin(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + a*Sin[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) + (6*a*d^2*(c + d*x)*Cos[e + f*x])/f^3 - (a*(c + d*x)
^3*Cos[e + f*x])/f - (6*a*d^3*Sin[e + f*x])/f^4 + (3*a*d*(c + d*x)^2*Sin[e
+ f*x])/f^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^3(a+a\sin(e+fx))\,dx = \int \left(a(c+dx)^3 + a(c+dx)^3\sin(e+fx)\right)\,dx$$
$$= \frac{a(c+dx)^4}{4d} + a\int (c+dx)^3\sin(e+fx)\,dx$$
$$= \frac{a(c+dx)^4}{4d} - \frac{a(c+dx)^3\cos(e+fx)}{f} + \frac{(3ad)\int (c+dx)^2\cos(e+fx)\,dx}{f}$$
$$= \frac{a(c+dx)^4}{4d} - \frac{a(c+dx)^3\cos(e+fx)}{f} + \frac{3ad(c+dx)^2\sin(e+fx)}{f^2} - \frac{\left(6ad^2\right)\int (c+dx)\,\mathrm{s}}{f^2}$$
$$= \frac{a(c+dx)^4}{4d} + \frac{6ad^2(c+dx)\cos(e+fx)}{f^3} - \frac{a(c+dx)^3\cos(e+fx)}{f} + \frac{3ad(c+dx)^2\sin(e}{f^2}$$
$$= \frac{a(c+dx)^4}{4d} + \frac{6ad^2(c+dx)\cos(e+fx)}{f^3} - \frac{a(c+dx)^3\cos(e+fx)}{f} - \frac{6ad^3\sin(e+fx)}{f^4} +$$

Mathematica [A] time = 0.843487, size = 123, normalized size = 1.37

$$a\left(\frac{3d\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2-2\right)\right)\sin(e+fx)}{f^4} - \frac{(c+dx)\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2-6\right)\right)\cos(e+fx)}{f^3} + \frac{1}{4}x\left(6c^2dx+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + a*Sin[e + f*x]),x]
```

```
[Out] a*((x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - ((c + d*x)*(c^2*f^2
+ 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Cos[e + f*x])/f^3 + (3*d*(c^2*f^2 + 2*c
*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x])/f^4)
```

Maple [B] time = 0.016, size = 482, normalized size = 5.4

$$\frac{1}{f}\left(\frac{ad^3\left(-\left(fx+e\right)^3\cos\left(fx+e\right)+3\left(fx+e\right)^2\sin\left(fx+e\right)-6\sin\left(fx+e\right)+6\left(fx+e\right)\cos\left(fx+e\right)\right)}{f^3}+3\frac{acd^2\left(-\left(fx-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+a*sin(f*x+e)),x)
```

```
[Out] 1/f*(a/f^3*d^3*(-(f*x+e)^3*cos(f*x+e)+3*(f*x+e)^2*sin(f*x+e)-6*sin(f*x+e)+6
*(f*x+e)*cos(f*x+e))+3*a/f^2*c*d^2*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f
*x+e)*sin(f*x+e))-3*a/f^3*d^3*e*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f*x+
e)*sin(f*x+e))+3*a/f*c^2*d*(sin(f*x+e)-(f*x+e)*cos(f*x+e))-6*a/f^2*c*d^2*e*
(sin(f*x+e)-(f*x+e)*cos(f*x+e))+3*a/f^3*d^3*e^2*(sin(f*x+e)-(f*x+e)*cos(f*x
+e))-a*c^3*cos(f*x+e)+3*a/f*c^2*d*e*cos(f*x+e)-3*a/f^2*c*d^2*e^2*cos(f*x+e)
+a/f^3*d^3*e^3*cos(f*x+e)+1/4*a/f^3*d^3*(f*x+e)^4+a/f^2*c*d^2*(f*x+e)^3-a/f
^3*d^3*e*(f*x+e)^3+3/2*a/f*c^2*d*(f*x+e)^2-3*a/f^2*c*d^2*e*(f*x+e)^2+3/2*a/
f^3*d^3*e^2*(f*x+e)^2+a*c^3*(f*x+e)-3*a/f*c^2*d*e*(f*x+e)+3*a/f^2*c*d^2*e^2
*(f*x+e)-a/f^3*d^3*e^3*(f*x+e))
```

Maxima [B] time = 1.03568, size = 624, normalized size = 6.93

$$\frac{4\left(fx+e\right)ac^3 + \frac{\left(fx+e\right)^4 ad^3}{f^3} - \frac{4\left(fx+e\right)^3 ad^3 e}{f^3} + \frac{6\left(fx+e\right)^2 ad^3 e^2}{f^3} - \frac{4\left(fx+e\right)ad^3 e^3}{f^3} + \frac{4\left(fx+e\right)^3 acd^2}{f^2} - \frac{12\left(fx+e\right)^2 acd^2 e}{f^2} + \frac{12\left(fx+e\right)acd^2 e^2}{f^2} + \frac{6\left(f\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c^3 + (f*x + e)^4*a*d^3/f^3 - 4*(f*x + e)^3*a*d^3*e/f^3
+ 6*(f*x + e)^2*a*d^3*e^2/f^3 - 4*(f*x + e)*a*d^3*e^3/f^3 + 4*(f*x + e)^3*a
*c*d^2/f^2 - 12*(f*x + e)^2*a*c*d^2*e/f^2 + 12*(f*x + e)*a*c*d^2*e^2/f^2 +
6*(f*x + e)^2*a*c^2*d/f - 12*(f*x + e)*a*c^2*d*e/f - 4*a*c^3*cos(f*x + e) +
 4*a*d^3*e^3*cos(f*x + e)/f^3 - 12*a*c*d^2*e^2*cos(f*x + e)/f^2 + 12*a*c^2*
d*e*cos(f*x + e)/f - 12*((f*x + e)*cos(f*x + e) - sin(f*x + e))*a*d^3*e^2/f
^3 + 24*((f*x + e)*cos(f*x + e) - sin(f*x + e))*a*c*d^2*e/f^2 - 12*((f*x +
e)*cos(f*x + e) - sin(f*x + e))*a*c^2*d/f + 12*(((f*x + e)^2 - 2)*cos(f*x +
 e) - 2*(f*x + e)*sin(f*x + e))*a*d^3*e/f^3 - 12*(((f*x + e)^2 - 2)*cos(f*x
 + e) - 2*(f*x + e)*sin(f*x + e))*a*c*d^2/f^2 - 4*(((f*x + e)^3 - 6*f*x - 6
*e)*cos(f*x + e) - 3*((f*x + e)^2 - 2)*sin(f*x + e))*a*d^3/f^3)/f
```

Fricas [A] time = 1.79098, size = 362, normalized size = 4.02

$$\frac{ad^3f^4x^4 + 4\,acd^2f^4x^3 + 6\,ac^2df^4x^2 + 4\,ac^3f^4x - 4\left(ad^3f^3x^3 + 3\,acd^2f^3x^2 + ac^3f^3 - 6\,acd^2f + 3\left(ac^2df^3 - 2\,ad^3f\right)x\right)\mathrm{co}}{4\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
- 4*(a*d^3*f^3*x^3 + 3*a*c*d^2*f^3*x^2 + a*c^3*f^3 - 6*a*c*d^2*f + 3*(a*c^2
*d*f^3 - 2*a*d^3*f)*x)*cos(f*x + e) + 12*(a*d^3*f^2*x^2 + 2*a*c*d^2*f^2*x +
 a*c^2*d*f^2 - 2*a*d^3)*sin(f*x + e))/f^4
```

Sympy [A] time = 1.86536, size = 264, normalized size = 2.93

$$\begin{cases} ac^3x - \frac{ac^3\cos(e+fx)}{f} + \frac{3ac^2dx^2}{2} - \frac{3ac^2dx\cos(e+fx)}{f} + \frac{3ac^2d\sin(e+fx)}{f^2} + acd^2x^3 - \frac{3acd^2x^2\cos(e+fx)}{f} + \frac{6acd^2x\sin(e+fx)}{f^2} + \frac{6acd^2\cos(e+f\ldots}{f^3} \\ (a\sin(e)+a)\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c**3*x - a*c**3*cos(e + f*x)/f + 3*a*c**2*d*x**2/2 - 3*a*c**2*
d*x*cos(e + f*x)/f + 3*a*c**2*d*sin(e + f*x)/f**2 + a*c*d**2*x**3 - 3*a*c*d
**2*x**2*cos(e + f*x)/f + 6*a*c*d**2*x*sin(e + f*x)/f**2 + 6*a*c*d**2*cos(e
 + f*x)/f**3 + a*d**3*x**4/4 - a*d**3*x**3*cos(e + f*x)/f + 3*a*d**3*x**2*s
in(e + f*x)/f**2 + 6*a*d**3*x*cos(e + f*x)/f**3 - 6*a*d**3*sin(e + f*x)/f**
4, Ne(f, 0)), ((a*sin(e) + a)*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**
3*x**4/4), True))
```

Giac [A] time = 1.16655, size = 212, normalized size = 2.36

$$\frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x - \frac{\left(ad^3f^3x^3 + 3\,acd^2f^3x^2 + 3\,ac^2df^3x + ac^3f^3 - 6\,ad^3fx - 6\,acd^2f\right)\cos\left(fx+e\right)}{f^4} + \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x - (a*d^3*f^3*x^3 +
3*a*c*d^2*f^3*x^2 + 3*a*c^2*d*f^3*x + a*c^3*f^3 - 6*a*d^3*f*x - 6*a*c*d^2*f
)*cos(f*x + e)/f^4 + 3*(a*d^3*f^2*x^2 + 2*a*c*d^2*f^2*x + a*c^2*d*f^2 - 2*a
```

```
*d^3)*sin(f*x + e)/f^4
```

3.96 $\int (c+dx)^2(a+a\sin(e+fx))\,dx$

Optimal. Leaf size=68

$$\frac{2ad(c+dx)\sin(e+fx)}{f^2}-\frac{a(c+dx)^2\cos(e+fx)}{f}+\frac{a(c+dx)^3}{3d}+\frac{2ad^2\cos(e+fx)}{f^3}$$

```
[Out] (a*(c + d*x)^3)/(3*d) + (2*a*d^2*Cos[e + f*x])/f^3 - (a*(c + d*x)^2*Cos[e +
 f*x])/f + (2*a*d*(c + d*x)*Sin[e + f*x])/f^2
```

Rubi [A] time = 0.0883813, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3296, 2638}

$$\frac{2ad(c+dx)\sin(e+fx)}{f^2}-\frac{a(c+dx)^2\cos(e+fx)}{f}+\frac{a(c+dx)^3}{3d}+\frac{2ad^2\cos(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + a*Sin[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) + (2*a*d^2*Cos[e + f*x])/f^3 - (a*(c + d*x)^2*Cos[e +
 f*x])/f + (2*a*d*(c + d*x)*Sin[e + f*x])/f^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^2(a+a\sin(e+fx))\,dx = \int \left(a(c+dx)^2 + a(c+dx)^2\sin(e+fx)\right)\,dx$$
$$= \frac{a(c+dx)^3}{3d} + a\int (c+dx)^2\sin(e+fx)\,dx$$
$$= \frac{a(c+dx)^3}{3d} - \frac{a(c+dx)^2\cos(e+fx)}{f} + \frac{(2ad)\int (c+dx)\cos(e+fx)\,dx}{f}$$
$$= \frac{a(c+dx)^3}{3d} - \frac{a(c+dx)^2\cos(e+fx)}{f} + \frac{2ad(c+dx)\sin(e+fx)}{f^2} - \frac{\left(2ad^2\right)\int \sin(e+f}{f^2}$$
$$= \frac{a(c+dx)^3}{3d} + \frac{2ad^2\cos(e+fx)}{f^3} - \frac{a(c+dx)^2\cos(e+fx)}{f} + \frac{2ad(c+dx)\sin(e+fx)}{f^2}$$

Mathematica [A] time = 0.499129, size = 81, normalized size = 1.19

$$a\left(-\frac{\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2-2\right)\right)\cos(e+fx)}{f^3}+c^2x+\frac{2d(c+dx)\sin(e+fx)}{f^2}+cdx^2+\frac{d^2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + a*Sin[e + f*x]),x]
```

```
[Out] a*(c^2*x + c*d*x^2 + (d^2*x^3)/3 - ((c^2*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2*
x^2))*Cos[e + f*x])/f^3 + (2*d*(c + d*x)*Sin[e + f*x])/f^2)
```

Maple [B] time = 0.012, size = 241, normalized size = 3.5

$$\frac{1}{f}\left(\frac{ad^2\left(-\left(fx+e\right)^2\cos\left(fx+e\right)+2\cos\left(fx+e\right)+2\left(fx+e\right)\sin\left(fx+e\right)\right)}{f^2}+2\frac{acd\left(\sin\left(fx+e\right)-\left(fx+e\right)\cos\left(fx+\right.\right.}{f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+a*sin(f*x+e)),x)
```

```
[Out] 1/f*(a/f^2*d^2*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f*x+e)*sin(f*x+e))+2*
a/f*c*d*(sin(f*x+e)-(f*x+e)*cos(f*x+e))-2*a/f^2*d^2*e*(sin(f*x+e)-(f*x+e)*c
```

```
os(f*x+e))-a*c^2*cos(f*x+e)+2*a/f*c*d*e*cos(f*x+e)-a/f^2*d^2*e^2*cos(f*x+e)
+1/3*a/f^2*d^2*(f*x+e)^3+a/f*c*d*(f*x+e)^2-a/f^2*d^2*e*(f*x+e)^2+a*c^2*(f*x
+e)-2*a/f*c*d*e*(f*x+e)+a/f^2*d^2*e^2*(f*x+e))
```

Maxima [B] time = 1.01276, size = 323, normalized size = 4.75

$$\frac{3\left(fx+e\right)ac^2 + \frac{\left(fx+e\right)^3ad^2}{f^2} - \frac{3\left(fx+e\right)^2ad^2e}{f^2} + \frac{3\left(fx+e\right)ad^2e^2}{f^2} + \frac{3\left(fx+e\right)^2acd}{f} - \frac{6\left(fx+e\right)acde}{f} - 3\,ac^2\cos\left(fx+e\right) - \frac{3\,ad^2e^2\cos\left(fx+e\right)}{f^2} + \frac{6}{}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a*c^2 + (f*x + e)^3*a*d^2/f^2 - 3*(f*x + e)^2*a*d^2*e/f^2
+ 3*(f*x + e)*a*d^2*e^2/f^2 + 3*(f*x + e)^2*a*c*d/f - 6*(f*x + e)*a*c*d*e/f
 - 3*a*c^2*cos(f*x + e) - 3*a*d^2*e^2*cos(f*x + e)/f^2 + 6*a*c*d*e*cos(f*x
+ e)/f + 6*((f*x + e)*cos(f*x + e) - sin(f*x + e))*a*d^2*e/f^2 - 6*((f*x +
e)*cos(f*x + e) - sin(f*x + e))*a*c*d/f - 3*(((f*x + e)^2 - 2)*cos(f*x + e)
 - 2*(f*x + e)*sin(f*x + e))*a*d^2/f^2)/f
```

Fricas [A] time = 1.74285, size = 228, normalized size = 3.35

$$\frac{ad^2f^3x^3 + 3\,acdf^3x^2 + 3\,ac^2f^3x - 3\left(ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2 - 2\,ad^2\right)\cos\left(fx+e\right) + 6\left(ad^2fx + acdf\right)\sin\left(fx+e\right)}{3\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x - 3*(a*d^2*f^2*x^2 + 2
*a*c*d*f^2*x + a*c^2*f^2 - 2*a*d^2)*cos(f*x + e) + 6*(a*d^2*f*x + a*c*d*f)*
sin(f*x + e))/f^3
```

Sympy [A] time = 0.864488, size = 151, normalized size = 2.22

$$\begin{cases} ac^2x - \frac{ac^2\cos\left(e+fx\right)}{f} + acdx^2 - \frac{2acdx\cos\left(e+fx\right)}{f} + \frac{2acd\sin\left(e+fx\right)}{f^2} + \frac{ad^2x^3}{3} - \frac{ad^2x^2\cos\left(e+fx\right)}{f} + \frac{2ad^2x\sin\left(e+fx\right)}{f^2} + \frac{2ad^2\cos\left(e+fx\right)}{f^3} & \text{for} \\ \left(a\sin\left(e\right)+a\right)\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c**2*x - a*c**2*cos(e + f*x)/f + a*c*d*x**2 - 2*a*c*d*x*cos(e
+ f*x)/f + 2*a*c*d*sin(e + f*x)/f**2 + a*d**2*x**3/3 - a*d**2*x**2*cos(e +
f*x)/f + 2*a*d**2*x*sin(e + f*x)/f**2 + 2*a*d**2*cos(e + f*x)/f**3, Ne(f, 0
)), ((a*sin(e) + a)*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Giac [A] time = 1.12443, size = 128, normalized size = 1.88

$$\frac{1}{3}ad^2x^3 + acdx^2 + ac^2x - \frac{\left(ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2 - 2\,ad^2\right)\cos\left(fx + e\right)}{f^3} + \frac{2\left(ad^2fx + acdf\right)\sin\left(fx + e\right)}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x - (a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^
2*f^2 - 2*a*d^2)*cos(f*x + e)/f^3 + 2*(a*d^2*f*x + a*c*d*f)*sin(f*x + e)/f^
3
```

3.97 $\int (c+dx)(a+a\sin(e+fx))\,dx$

Optimal. Leaf size=45

$$-\frac{a(c+dx)\cos(e+fx)}{f}+\frac{a(c+dx)^2}{2d}+\frac{ad\sin(e+fx)}{f^2}$$

```
[Out] (a*(c + d*x)^2)/(2*d) - (a*(c + d*x)*Cos[e + f*x])/f + (a*d*Sin[e + f*x])/f
^2
```

Rubi [A] time = 0.0422202, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3317, 3296, 2637}

$$-\frac{a(c+dx)\cos(e+fx)}{f}+\frac{a(c+dx)^2}{2d}+\frac{ad\sin(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + a*Sin[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) - (a*(c + d*x)*Cos[e + f*x])/f + (a*d*Sin[e + f*x])/f
^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)(a+a\sin(e+fx))\,dx &= \int (a(c+dx)+a(c+dx)\sin(e+fx))\,dx \\
&= \frac{a(c+dx)^2}{2d} + a\int (c+dx)\sin(e+fx)\,dx \\
&= \frac{a(c+dx)^2}{2d} - \frac{a(c+dx)\cos(e+fx)}{f} + \frac{(ad)\int \cos(e+fx)\,dx}{f} \\
&= \frac{a(c+dx)^2}{2d} - \frac{a(c+dx)\cos(e+fx)}{f} + \frac{ad\sin(e+fx)}{f^2}
\end{aligned}
$$

Mathematica [A] time = 0.356213, size = 51, normalized size = 1.13

$$
-\frac{a((e+fx)(-2cf+de-dfx)+2f(c+dx)\cos(e+fx)-2d\sin(e+fx))}{2f^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + a*Sin[e + f*x]),x]
```

```
[Out] -(a*((e + f*x)*(d*e - 2*c*f - d*f*x) + 2*f*(c + d*x)*Cos[e + f*x] - 2*d*Sin
[e + f*x]))/(2*f^2)
```

Maple [B] time = 0.011, size = 90, normalized size = 2.

$$
\frac{1}{f}\left(\frac{da\left(\sin\left(fx+e\right)-\left(fx+e\right)\cos\left(fx+e\right)\right)}{f} - ac\cos\left(fx+e\right) + \frac{ade\cos\left(fx+e\right)}{f} + \frac{da\left(fx+e\right)^2}{2f} + ac\left(fx+e\right) - \frac{ade\left(\right.}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+a*sin(f*x+e)),x)
```

```
[Out] 1/f*(a/f*d*(sin(f*x+e)-(f*x+e)*cos(f*x+e))-a*c*cos(f*x+e)+a/f*d*e*cos(f*x+e
)+1/2*a/f*d*(f*x+e)^2+a*c*(f*x+e)-a/f*d*e*(f*x+e))
```

Maxima [B] time = 0.969237, size = 126, normalized size = 2.8

$$\frac{2\left(fx+e\right)ac+\frac{\left(fx+e\right)^{2}ad}{f}-\frac{2\left(fx+e\right)ade}{f}-2\,ac\cos\left(fx+e\right)+\frac{2\,ade\cos\left(fx+e\right)}{f}-\frac{2\left(\left(fx+e\right)\cos\left(fx+e\right)-\sin\left(fx+e\right)\right)ad}{f}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(f*x + e)*a*c + (f*x + e)^2*a*d/f - 2*(f*x + e)*a*d*e/f - 2*a*c*cos(
f*x + e) + 2*a*d*e*cos(f*x + e)/f - 2*((f*x + e)*cos(f*x + e) - sin(f*x + e
))*a*d/f)/f
```

Fricas [A] time = 1.76993, size = 126, normalized size = 2.8

$$\frac{adf^{2}x^{2}+2\,acf^{2}x+2\,ad\sin\left(fx+e\right)-2\left(adfx+acf\right)\cos\left(fx+e\right)}{2\,f^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x + 2*a*d*sin(f*x + e) - 2*(a*d*f*x + a*c*f)*c
os(f*x + e))/f^2
```

Sympy [A] time = 0.343821, size = 68, normalized size = 1.51

$$\begin{cases} acx-\frac{ac\cos\left(e+fx\right)}{f}+\frac{adx^{2}}{2}-\frac{adx\cos\left(e+fx\right)}{f}+\frac{ad\sin\left(e+fx\right)}{f^{2}} & \text{for}\ f\neq 0\\ \left(a\sin\left(e\right)+a\right)\left(cx+\frac{dx^{2}}{2}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x - a*c*cos(e + f*x)/f + a*d*x**2/2 - a*d*x*cos(e + f*x)/f +
 a*d*sin(e + f*x)/f**2, Ne(f, 0)), ((a*sin(e) + a)*(c*x + d*x**2/2), True))
```

Giac [A] time = 1.09574, size = 63, normalized size = 1.4

$$\frac{1}{2}adx^2 + acx + \frac{ad\sin(fx+e)}{f^2} - \frac{(adfx + acf)\cos(fx+e)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + a*d*sin(f*x + e)/f^2 - (a*d*f*x + a*c*f)*cos(f*x + e)
/f^2
```

3.98 $\int \frac{a+a\sin(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=64

$$\frac{a\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{a\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

```
[Out] (a*Log[c + d*x])/d + (a*CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d + (a
*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d
```

Rubi [A] time = 0.150089, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3317, 3303, 3299, 3302}

$$\frac{a\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{a\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*x),x]
```

```
[Out] (a*Log[c + d*x])/d + (a*CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d + (a
*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{c+dx}\,dx &= \int \left(\frac{a}{c+dx}+\frac{a\sin(e+fx)}{c+dx}\right)dx\\ &= \frac{a\log(c+dx)}{d}+a\int \frac{\sin(e+fx)}{c+dx}\,dx\\ &= \frac{a\log(c+dx)}{d}+\left(a\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx+\left(a\sin\left(e-\frac{cf}{d}\right)\right)\int \frac{\cos\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx\\ &= \frac{a\log(c+dx)}{d}+\frac{a\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{a\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.294371, size = 54, normalized size = 0.84

$$\frac{a\left(\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)+\log(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*x),x]
```

```
[Out] (a*(Log[c + d*x] + CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + Cos[e - (c*f
)/d]*SinIntegral[f*(c/d + x)]))/d
```

Maple [A] time = 0.014, size = 96, normalized size = 1.5

$$\frac{a}{d}\text{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)-\frac{a}{d}\text{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)+\frac{a\ln\left(\left(fx+e\right)d+cf-de\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(d*x+c),x)
```

```
[Out] a*Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-a*Ci(f*x+e+(c*f-d*e)/d)*sin((c*f
-d*e)/d)/d+a*ln((f*x+e)*d+c*f-d*e)/d
```

Maxima [C] time = 1.20674, size = 231, normalized size = 3.61

$$\frac{\frac{2af\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d}+\frac{\left(f\left(-iE_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)+iE_1\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f\left(E_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)+E_1\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\sin\left(-\frac{de-cf}{d}\right)\right)a}{d}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*f*log(c + (f*x + e)*d/f - d*e/f)/d + (f*(-I*exp_integral_e(1, (I*(
f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(1, -(I*(f*x + e)*d - I*d*
e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f*(exp_integral_e(1, (I*(f*x + e)*d -
I*d*e + I*c*f)/d) + exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*
sin(-(d*e - c*f)/d))*a/d)/f
```

Fricas [A] time = 1.66127, size = 234, normalized size = 3.66

$$\frac{2\,a\cos\left(-\frac{de-cf}{d}\right)\operatorname{Si}\left(\frac{dfx+cf}{d}\right)+2\,a\log\left(dx+c\right)-\left(a\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)+a\operatorname{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\sin\left(-\frac{de-cf}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*cos(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/d) + 2*a*log(d*x +
c) - (a*cos_integral((d*f*x + c*f)/d) + a*cos_integral(-(d*f*x + c*f)/d))*s
in(-(d*e - c*f)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{\sin\left(e+fx\right)}{c+dx}\,dx+\int \frac{1}{c+dx}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c),x)
```

```
[Out] a*(Integral(sin(e + f*x)/(c + d*x), x) + Integral(1/(c + d*x), x))
```

Giac [C] time = 1.25651, size = 961, normalized size = 15.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 -
 a*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*
a*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*sin_integral((d*f*x
 + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*a*real_part(cos_integral(f*x +
 c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*a*real_part(cos_integral(-f*x - c*
f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) + 2*a*real_part(cos_integral(f*x + c*f/d)
)*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*a*real_part(cos_integral(-f*x - c*f/d))*t
an(1/2*c*f/d)*tan(1/2*e)^2 - a*imag_part(cos_integral(f*x + c*f/d))*tan(1/2
*c*f/d)^2 + a*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2 + 2*a*
log(abs(d*x + c))*tan(1/2*c*f/d)^2 - 2*a*sin_integral((d*f*x + c*f)/d)*tan(
1/2*c*f/d)^2 + 4*a*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(
1/2*e) - 4*a*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e
) + 8*a*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e) - a*imag_pa
rt(cos_integral(f*x + c*f/d))*tan(1/2*e)^2 + a*imag_part(cos_integral(-f*x
- c*f/d))*tan(1/2*e)^2 + 2*a*log(abs(d*x + c))*tan(1/2*e)^2 - 2*a*sin_integ
ral((d*f*x + c*f)/d)*tan(1/2*e)^2 - 2*a*real_part(cos_integral(f*x + c*f/d)
)*tan(1/2*c*f/d) - 2*a*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)
 + 2*a*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e) + 2*a*real_part(cos_
integral(-f*x - c*f/d))*tan(1/2*e) + a*imag_part(cos_integral(f*x + c*f/d))
 - a*imag_part(cos_integral(-f*x - c*f/d)) + 2*a*log(abs(d*x + c)) + 2*a*si
n_integral((d*f*x + c*f)/d))/(d*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + d*tan(1/2*c
*f/d)^2 + d*tan(1/2*e)^2 + d)
```

3.99 $\int \frac{a+a\sin(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=88

$$\frac{af\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d^2}-\frac{af\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a\sin(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

```
[Out] -(a/(d*(c + d*x))) + (a*f*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d^2
- (a*Sin[e + f*x])/(d*(c + d*x)) - (a*f*Sin[e - (c*f)/d]*SinIntegral[(c*f)/
d + f*x])/d^2
```

Rubi [A] time = 0.21372, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3317, 3297, 3303, 3299, 3302}

$$\frac{af\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d^2}-\frac{af\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a\sin(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) + (a*f*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d^2
- (a*Sin[e + f*x])/(d*(c + d*x)) - (a*f*Sin[e - (c*f)/d]*SinIntegral[(c*f)/
d + f*x])/d^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sin(e+fx)}{(c+dx)^2}\,dx &= \int \left(\frac{a}{(c+dx)^2}+\frac{a\sin(e+fx)}{(c+dx)^2}\right)dx \\
&= -\frac{a}{d(c+dx)}+a\int \frac{\sin(e+fx)}{(c+dx)^2}\,dx \\
&= -\frac{a}{d(c+dx)}-\frac{a\sin(e+fx)}{d(c+dx)}+\frac{(af)\int \frac{\cos(e+fx)}{c+dx}\,dx}{d} \\
&= -\frac{a}{d(c+dx)}-\frac{a\sin(e+fx)}{d(c+dx)}+\frac{\left(af\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\cos\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d}-\frac{\left(af\sin\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+f\right.}{c+dx}}{d} \\
&= -\frac{a}{d(c+dx)}+\frac{af\cos\left(e-\frac{cf}{d}\right)\text{Ci}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{a\sin(e+fx)}{d(c+dx)}-\frac{af\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.493286, size = 110, normalized size = 1.25

$$\frac{a(\sin(e+fx)+1)\left(f(c+dx)\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\cos\left(e-\frac{cf}{d}\right)-f(c+dx)\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)-d(\sin(e+f\right.}{d^2(c+dx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*x)^2,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(f*(c + d*x)*Cos[e - (c*f)/d]*CosIntegral[f*(c/d + x)
] - d*(1 + Sin[e + f*x]) - f*(c + d*x)*Sin[e - (c*f)/d]*SinIntegral[f*(c/d
+ x)]))/(d^2*(c + d*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

Maple [A] time = 0.014, size = 141, normalized size = 1.6

$$\frac{1}{f}\left(af^{2}\left(-\frac{\sin\left(fx+e\right)}{\left(\left(fx+e\right)d+cf-de\right)d}+\frac{1}{d}\left(\frac{1}{d}\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)+\frac{1}{d}\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(d*x+c)^2,x)
```

```
[Out] 1/f*(a*f^2*(-sin(f*x+e)/((f*x+e)*d+c*f-d*e)/d+(Si(f*x+e+(c*f-d*e)/d)*sin((c
*f-d*e)/d)/d+Ci(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d)/d)-a*f^2/((f*x+e)*d+
c*f-d*e)/d)
```

Maxima [C] time = 1.30553, size = 265, normalized size = 3.01

$$-\frac{\frac{2\,af^{2}}{\left(fx+e\right)d^{2}-d^{2}e+cdf}-\frac{\left(f^{2}\left(-i\,E_{2}\left(\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)+i\,E_{2}\left(-\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f^{2}\left(E_{2}\left(\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)+E_{2}\left(-\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)\right)\sin\left(-\frac{de-cf}{d}\right)\right)a}{\left(fx+e\right)d^{2}-d^{2}e+cdf}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*f^2/((f*x + e)*d^2 - d^2*e + c*d*f) - (f^2*(-I*exp_integral_e(2,
(I*(f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(2, -(I*(f*x + e)*d -
I*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f^2*(exp_integral_e(2, (I*(f*x + e
)*d - I*d*e + I*c*f)/d) + exp_integral_e(2, -(I*(f*x + e)*d - I*d*e + I*c*f
)/d))*sin(-(d*e - c*f)/d))*a/((f*x + e)*d^2 - d^2*e + c*d*f))/f
```

Fricas [A] time = 1.82915, size = 332, normalized size = 3.77

$$-\frac{2\,ad\sin\left(fx+e\right)-2\left(adfx+acf\right)\sin\left(-\frac{de-cf}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)+2\,ad-\left(\left(adfx+acf\right)\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)+\left(adfx+acf\right)\mathrm{Ci}\left(-\right.\right.}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*sin(f*x + e) - 2*(a*d*f*x + a*c*f)*sin(-(d*e - c*f)/d)*sin_inte
gral((d*f*x + c*f)/d) + 2*a*d - ((a*d*f*x + a*c*f)*cos_integral((d*f*x + c*
f)/d) + (a*d*f*x + a*c*f)*cos_integral(-(d*f*x + c*f)/d))*cos(-(d*e - c*f)/
d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{\sin\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx+\int\frac{1}{c^2+2cdx+d^2x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)**2,x)
```

```
[Out] a*(Integral(sin(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**
2 + 2*c*d*x + d**2*x**2), x))
```

Giac [C] time = 1.34078, size = 4251, normalized size = 48.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(d*f*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/
d)^2*tan(1/2*e)^2 + d*f*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x
)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*d*f*x*imag_part(cos_integral(f*x + c*
f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*d*f*x*imag_part(cos_in
```

```
tegral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) + 4*d*f*x*
sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) -
2*d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*
tan(1/2*e)^2 + 2*d*f*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2
*tan(1/2*c*f/d)*tan(1/2*e)^2 - 4*d*f*x*sin_integral((d*f*x + c*f)/d)*tan(1/
2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + c*f*real_part(cos_integral(f*x + c*f
/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + c*f*real_part(cos_integ
ral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - d*f*x*rea
l_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 - d*f*x*r
eal_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + 4*d*
f*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(
1/2*e) + 4*d*f*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1
/2*c*f/d)*tan(1/2*e) + 2*c*f*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f
*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*c*f*imag_part(cos_integral(-f*x - c*f
/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) + 4*c*f*sin_integral((d*f*x
 + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - d*f*x*real_part(cos
_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 - d*f*x*real_part(cos_i
ntegral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*c*f*imag_part(cos_in
tegral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*c*f*ima
g_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)
^2 - 4*c*f*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(
1/2*e)^2 + d*f*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(
1/2*e)^2 + d*f*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan
(1/2*e)^2 + 2*d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan
(1/2*c*f/d) - 2*d*f*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*
tan(1/2*c*f/d) + 4*d*f*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1
/2*c*f/d) - c*f*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2
*c*f/d)^2 - c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/
2*c*f/d)^2 - 2*d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*ta
n(1/2*e) + 2*d*f*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan
(1/2*e) - 4*d*f*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e) +
 4*c*f*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*t
an(1/2*e) + 4*c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(
1/2*c*f/d)*tan(1/2*e) + 2*d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/
2*c*f/d)^2*tan(1/2*e) - 2*d*f*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1
/2*c*f/d)^2*tan(1/2*e) + 4*d*f*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/
d)^2*tan(1/2*e) - c*f*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*t
an(1/2*e)^2 - c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(
1/2*e)^2 - 2*d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(
1/2*e)^2 + 2*d*f*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan
(1/2*e)^2 - 4*d*f*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e)
^2 + c*f*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2
 + c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2
+ d*f*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2 + d*f*x*real_pa
rt(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2 + 2*c*f*imag_part(cos_integra
```

```
l(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*c*f*imag_part(cos_integra
l(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) + 4*c*f*sin_integral((d*f*x
+ c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d) - d*f*x*real_part(cos_integral(f*x
+ c*f/d))*tan(1/2*c*f/d)^2 - d*f*x*real_part(cos_integral(-f*x - c*f/d))*ta
n(1/2*c*f/d)^2 - 2*c*f*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*
tan(1/2*e) + 2*c*f*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan
(1/2*e) - 4*c*f*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e) + 4
*d*f*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) + 4*d
*f*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) + 2*c*
f*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*c*f*
imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) + 4*c*f*s
in_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e) + 4*d*tan(1/2*f*x)
^2*tan(1/2*c*f/d)^2*tan(1/2*e) - d*f*x*real_part(cos_integral(f*x + c*f/d))
*tan(1/2*e)^2 - d*f*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2 -
2*c*f*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*
c*f*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 - 4*c
*f*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e)^2 + 4*d*tan(1/2*
f*x)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + c*f*real_part(cos_integral(f*x + c*f/d
))*tan(1/2*f*x)^2 + c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^
2 + 2*d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 2*d*f*x*i
mag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) + 4*d*f*x*sin_integral(
(d*f*x + c*f)/d)*tan(1/2*c*f/d) - c*f*real_part(cos_integral(f*x + c*f/d))*
tan(1/2*c*f/d)^2 - c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)
^2 - 2*d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e) + 2*d*f*x*imag
_part(cos_integral(-f*x - c*f/d))*tan(1/2*e) - 4*d*f*x*sin_integral((d*f*x
+ c*f)/d)*tan(1/2*e) + 4*c*f*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c
*f/d)*tan(1/2*e) + 4*c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/
d)*tan(1/2*e) - c*f*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2 - c*f
*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2 + d*f*x*real_part(cos_i
ntegral(f*x + c*f/d)) + d*f*x*real_part(cos_integral(-f*x - c*f/d)) + 2*c*f
*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 2*c*f*imag_part(cos_
integral(-f*x - c*f/d))*tan(1/2*c*f/d) + 4*c*f*sin_integral((d*f*x + c*f)/d
)*tan(1/2*c*f/d) - 4*d*tan(1/2*f*x)*tan(1/2*c*f/d)^2 - 2*c*f*imag_part(cos_
integral(f*x + c*f/d))*tan(1/2*e) + 2*c*f*imag_part(cos_integral(-f*x - c*f
/d))*tan(1/2*e) - 4*c*f*sin_integral((d*f*x + c*f)/d)*tan(1/2*e) + 4*d*tan(
1/2*f*x)^2*tan(1/2*e) - 4*d*tan(1/2*c*f/d)^2*tan(1/2*e) + 4*d*tan(1/2*f*x)*
tan(1/2*e)^2 + c*f*real_part(cos_integral(f*x + c*f/d)) + c*f*real_part(cos
_integral(-f*x - c*f/d)) - 4*d*tan(1/2*f*x) - 4*d*tan(1/2*e))*a/(d^3*x*tan(
1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + c*d^2*tan(1/2*f*x)^2*tan(1/2*c*f
/d)^2*tan(1/2*e)^2 + d^3*x*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + d^3*x*tan(1/2*
f*x)^2*tan(1/2*e)^2 + d^3*x*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + c*d^2*tan(1/2*f
*x)^2*tan(1/2*c*f/d)^2 + c*d^2*tan(1/2*f*x)^2*tan(1/2*e)^2 + c*d^2*tan(1/2*
c*f/d)^2*tan(1/2*e)^2 + d^3*x*tan(1/2*f*x)^2 + d^3*x*tan(1/2*c*f/d)^2 + d^3
*x*tan(1/2*e)^2 + c*d^2*tan(1/2*f*x)^2 + c*d^2*tan(1/2*c*f/d)^2 + c*d^2*tan
(1/2*e)^2 + d^3*x + c*d^2) - a/((d*x + c)*d)
```

3.100 $\int \frac{a+a\sin(e+fx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=123

$$-\frac{af^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{af^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{af\cos(e+fx)}{2d^2(c+dx)}-\frac{a\sin(e+fx)}{2d(c+dx)^2}-\frac{a}{2d(c+dx)^2}$$

```
[Out] -a/(2*d*(c + d*x)^2) - (a*f*Cos[e + f*x])/(2*d^2*(c + d*x)) - (a*f^2*CosInt
egral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/(2*d^3) - (a*Sin[e + f*x])/(2*d*(c +
 d*x)^2) - (a*f^2*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/(2*d^3)
```

Rubi [A] time = 0.256565, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3317, 3297, 3303, 3299, 3302}

$$-\frac{af^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{af^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{af\cos(e+fx)}{2d^2(c+dx)}-\frac{a\sin(e+fx)}{2d(c+dx)^2}-\frac{a}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*x)^3,x]
```



```
[Out] -a/(2*d*(c + d*x)^2) - (a*f*Cos[e + f*x])/(2*d^2*(c + d*x)) - (a*f^2*CosInt
egral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/(2*d^3) - (a*Sin[e + f*x])/(2*d*(c +
 d*x)^2) - (a*f^2*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/(2*d^3)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sin(e+fx)}{(c+dx)^3}\,dx &= \int \left(\frac{a}{(c+dx)^3}+\frac{a\sin(e+fx)}{(c+dx)^3}\right)dx\\
&= -\frac{a}{2d(c+dx)^2}+a\int \frac{\sin(e+fx)}{(c+dx)^3}\,dx\\
&= -\frac{a}{2d(c+dx)^2}-\frac{a\sin(e+fx)}{2d(c+dx)^2}+\frac{(af)\int \frac{\cos(e+fx)}{(c+dx)^2}\,dx}{2d}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{af\cos(e+fx)}{2d^2(c+dx)}-\frac{a\sin(e+fx)}{2d(c+dx)^2}-\frac{\left(af^2\right)\int \frac{\sin(e+fx)}{c+dx}\,dx}{2d^2}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{af\cos(e+fx)}{2d^2(c+dx)}-\frac{a\sin(e+fx)}{2d(c+dx)^2}-\frac{\left(af^2\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{2d^2}-\frac{\left(af^2\sin\right.}{}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{af\cos(e+fx)}{2d^2(c+dx)}-\frac{af^2\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{a\sin(e+fx)}{2d(c+dx)^2}-\frac{af^2\cos\left(e-\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.672044, size = 104, normalized size = 0.85

$$-\frac{a\left(f^2(c+dx)^2\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+f^2(c+dx)^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)+d(f(c+dx)\cos(e+fx)\right.}{2d^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*x)^3,x]
```

```
[Out] -(a*(f^2*(c + d*x)^2*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + d*(f*(c +
d*x)*Cos[e + f*x] + d*(1 + Sin[e + f*x])) + f^2*(c + d*x)^2*Cos[e - (c*f)/d
]*SinIntegral[f*(c/d + x)]))/(2*d^3*(c + d*x)^2)
```

Maple [A] time = 0.017, size = 177, normalized size = 1.4

$$\frac{1}{f}\left(af^3\left(-\frac{\sin\left(fx+e\right)}{2\left(\left(fx+e\right)d+cf-de\right)^2 d}+\frac{1}{2d}\left(-\frac{\cos\left(fx+e\right)}{\left(\left(fx+e\right)d+cf-de\right)d}-\frac{1}{d}\left(\frac{1}{d}\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)-\frac{1}{d}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(d*x+c)^3,x)
```

```
[Out] 1/f*(a*f^3*(-1/2*sin(f*x+e)/((f*x+e)*d+c*f-d*e)^2/d+1/2*(-cos(f*x+e)/((f*x+
e)*d+c*f-d*e)/d-(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*e
)/d)*sin((c*f-d*e)/d)/d)/d)/d)-1/2*a*f^3/((f*x+e)*d+c*f-d*e)^2/d)
```

Maxima [C] time = 1.44191, size = 358, normalized size = 2.91

$$-\frac{\frac{af^3}{\left(fx+e\right)^2 d^3+d^3e^2-2\,cd^2ef+c^2df^2-2\left(d^3e-cd^2f\right)\left(fx+e\right)}-\frac{\left(f^3\left(-i\,E_3\left(\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)+i\,E_3\left(-\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f^3\left(E_3\left(\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)+E_3\right.\right.}{\left(fx+e\right)^2 d^3+d^3e^2-2\,cd^2ef+c^2df^2-2\left(d^3e-cd^2f\right)\left(fx+e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*f^3/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*(d^3*e
 - c*d^2*f)*(f*x + e)) - (f^3*(-I*exp_integral_e(3, (I*(f*x + e)*d - I*d*e
+ I*c*f)/d) + I*exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*cos(
-(d*e - c*f)/d) + f^3*(exp_integral_e(3, (I*(f*x + e)*d - I*d*e + I*c*f)/d)
 + exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*sin(-(d*e - c*f)/
d))*a/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*(d^3*e - c*d
^2*f)*(f*x + e)))/f
```

Fricas [A] time = 1.75286, size = 517, normalized size = 4.2

$$-\frac{2\,ad^2\sin\left(fx+e\right)+2\,ad^2+2\left(ad^2f^2x^2+2\,acdf^2x+ac^2f^2\right)\cos\left(-\frac{de-cf}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)+2\left(ad^2fx+acdf\right)\cos\left(fx+e\right)-}{4\left(d^5x^2+2\,cd^4x+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d^2*sin(f*x + e) + 2*a*d^2 + 2*(a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a
*c^2*f^2)*cos(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/d) + 2*(a*d^2*f*x
+ a*c*d*f)*cos(f*x + e) - ((a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2)*cos_
integral((d*f*x + c*f)/d) + (a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2)*cos
_integral(-(d*f*x + c*f)/d))*sin(-(d*e - c*f)/d))/(d^5*x^2 + 2*c*d^4*x + c^
2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{\sin\left(e+fx\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int\frac{1}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)**3,x)
```

```
[Out] a*(Integral(sin(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x
) + Integral(1/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))
```

Giac [C] time = 1.51854, size = 8312, normalized size = 67.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan
(1/2*c*f/d)^2*tan(1/2*e)^2 - a*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*
f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*d^2*f^2*x^2*sin_in
tegral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*a*
d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f
/d)^2*tan(1/2*e) - 2*a*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*ta
n(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) + 2*a*d^2*f^2*x^2*real_part(cos_in
tegral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*a*d^2*f
^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*
tan(1/2*e)^2 + 2*a*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f
*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*a*c*d*f^2*x*imag_part(cos_integral(
-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*a*c*d*f^2*x
*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2
 - a*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/
2*c*f/d)^2 + a*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*
x)^2*tan(1/2*c*f/d)^2 - 2*a*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1
/2*f*x)^2*tan(1/2*c*f/d)^2 + 4*a*d^2*f^2*x^2*imag_part(cos_integral(f*x + c
*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) - 4*a*d^2*f^2*x^2*imag_part
(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) + 8*a
*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)*ta
n(1/2*e) - 4*a*c*d*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^
2*tan(1/2*c*f/d)^2*tan(1/2*e) - 4*a*c*d*f^2*x*real_part(cos_integral(-f*x -
 c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - a*d^2*f^2*x^2*imag_pa
rt(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 + a*d^2*f^2*x^2*i
mag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*a*d^2*
f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e)^2 + 4*a*c*d
*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*t
an(1/2*e)^2 + 4*a*c*d*f^2*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f
*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + a*d^2*f^2*x^2*imag_part(cos_integral(f*
x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - a*d^2*f^2*x^2*imag_part(cos_int
egral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*d^2*f^2*x^2*sin_in
tegral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a*c^2*f^2*imag_part
(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 -
a*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/
d)^2*tan(1/2*e)^2 + 2*a*c^2*f^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^
2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*a*d^2*f^2*x^2*real_part(cos_integral(f*
x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*a*d^2*f^2*x^2*real_part(cos_i
ntegral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*a*c*d*f^2*x*imag_p
art(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + 2*a*c*d*f^
2*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 -
 4*a*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^
2 + 2*a*d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan
(1/2*e) + 2*a*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x
)^2*tan(1/2*e) + 8*a*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2
*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) - 8*a*c*d*f^2*x*imag_part(cos_integral(-f
```

```
*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) + 16*a*c*d*f^2*x*sin_
integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) - 2*a*d^
2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)
- 2*a*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*ta
n(1/2*e) - 2*a*c^2*f^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*
tan(1/2*c*f/d)^2*tan(1/2*e) - 2*a*c^2*f^2*real_part(cos_integral(-f*x - c*f
/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*a*c*d*f^2*x*imag_part(c
os_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*a*c*d*f^2*x*imag_
part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 - 4*a*c*d*f^2*
x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*a*d^2*f^2*x
^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*a*d
^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^
2 + 2*a*c^2*f^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2
*c*f/d)*tan(1/2*e)^2 + 2*a*c^2*f^2*real_part(cos_integral(-f*x - c*f/d))*ta
n(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*a*c*d*f^2*x*imag_part(cos_inte
gral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*a*c*d*f^2*x*imag_part(
cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*a*c*d*f^2*x*s
in_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*d^2*f*x*ta
n(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a*d^2*f^2*x^2*imag_part(cos_in
tegral(f*x + c*f/d))*tan(1/2*f*x)^2 - a*d^2*f^2*x^2*imag_part(cos_integral(
-f*x - c*f/d))*tan(1/2*f*x)^2 + 2*a*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/
d)*tan(1/2*f*x)^2 - 4*a*c*d*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(
1/2*f*x)^2*tan(1/2*c*f/d) - 4*a*c*d*f^2*x*real_part(cos_integral(-f*x - c*f
/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - a*d^2*f^2*x^2*imag_part(cos_integral(f
*x + c*f/d))*tan(1/2*c*f/d)^2 + a*d^2*f^2*x^2*imag_part(cos_integral(-f*x -
 c*f/d))*tan(1/2*c*f/d)^2 - 2*a*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*t
an(1/2*c*f/d)^2 - a*c^2*f^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*
x)^2*tan(1/2*c*f/d)^2 + a*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan
(1/2*f*x)^2*tan(1/2*c*f/d)^2 - 2*a*c^2*f^2*sin_integral((d*f*x + c*f)/d)*ta
n(1/2*f*x)^2*tan(1/2*c*f/d)^2 + 4*a*c*d*f^2*x*real_part(cos_integral(f*x +
c*f/d))*tan(1/2*f*x)^2*tan(1/2*e) + 4*a*c*d*f^2*x*real_part(cos_integral(-f
*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e) + 4*a*d^2*f^2*x^2*imag_part(cos_inte
gral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) - 4*a*d^2*f^2*x^2*imag_part(co
s_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) + 8*a*d^2*f^2*x^2*sin_i
ntegral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e) + 4*a*c^2*f^2*imag_part(
cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) - 4*a*c
^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*
tan(1/2*e) + 8*a*c^2*f^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1
/2*c*f/d)*tan(1/2*e) - 4*a*c*d*f^2*x*real_part(cos_integral(f*x + c*f/d))*t
an(1/2*c*f/d)^2*tan(1/2*e) - 4*a*c*d*f^2*x*real_part(cos_integral(-f*x - c*
f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - a*d^2*f^2*x^2*imag_part(cos_integral(f*
x + c*f/d))*tan(1/2*e)^2 + a*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*f/
d))*tan(1/2*e)^2 - 2*a*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)
^2 - a*c^2*f^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*
e)^2 + a*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1
```

/2*e)^2 - 2*a*c^2*f^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*
e)^2 + 4*a*c*d*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*ta
n(1/2*e)^2 + 4*a*c*d*f^2*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*
f/d)*tan(1/2*e)^2 + a*c^2*f^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*
c*f/d)^2*tan(1/2*e)^2 - a*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan
(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*c^2*f^2*sin_integral((d*f*x + c*f)/d)*tan(
1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*c*d*f*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1
/2*e)^2 + 2*a*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2
 - 2*a*c*d*f^2*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2 + 4*a
*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2 - 2*a*d^2*f^2*x^2*r
eal_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 2*a*d^2*f^2*x^2*real_p
art(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) - 2*a*c^2*f^2*real_part(cos_
integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*a*c^2*f^2*real_par
t(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*a*c*d*f^2*x
*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2 + 2*a*c*d*f^2*x*imag
_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2 - 4*a*c*d*f^2*x*sin_inte
gral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2 - 2*a*d^2*f*x*tan(1/2*f*x)^2*tan(1/2
*c*f/d)^2 + 2*a*d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e)
 + 2*a*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e) + 2*a*c
^2*f^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e) + 2*a
*c^2*f^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e) +
8*a*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e
) - 8*a*c*d*f^2*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(
1/2*e) + 16*a*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/
2*e) - 2*a*c^2*f^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*ta
n(1/2*e) - 2*a*c^2*f^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)
^2*tan(1/2*e) - 8*a*d^2*f*x*tan(1/2*f*x)*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*a*
c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2 + 2*a*c*d*f^2*x
*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2 - 4*a*c*d*f^2*x*sin_int
egral((d*f*x + c*f)/d)*tan(1/2*e)^2 + 2*a*d^2*f*x*tan(1/2*f*x)^2*tan(1/2*e)
^2 + 2*a*c^2*f^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/
2*e)^2 + 2*a*c^2*f^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*t
an(1/2*e)^2 - 2*a*d^2*f*x*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*d^2*tan(1/2*f
*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a*d^2*f^2*x^2*imag_part(cos_integral(
f*x + c*f/d)) - a*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*f/d)) + 2*a*d
^2*f^2*x^2*sin_integral((d*f*x + c*f)/d) + a*c^2*f^2*imag_part(cos_integral
(f*x + c*f/d))*tan(1/2*f*x)^2 - a*c^2*f^2*imag_part(cos_integral(-f*x - c*f
/d))*tan(1/2*f*x)^2 + 2*a*c^2*f^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x
)^2 - 4*a*c*d*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 4
*a*c*d*f^2*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) - a*c^2*f
^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2 + a*c^2*f^2*imag_p
art(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2 - 2*a*c^2*f^2*sin_integral
((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2 - 2*a*c*d*f*tan(1/2*f*x)^2*tan(1/2*c*f/d
)^2 + 4*a*c*d*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e) + 4*a*c
*d*f^2*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e) + 4*a*c^2*f^2*ima

```
g_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) - 4*a*c^2*f^2*i
mag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) + 8*a*c^2*f^
2*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e) - 8*a*c*d*f*tan(1
/2*f*x)*tan(1/2*c*f/d)^2*tan(1/2*e) - 4*a*d^2*tan(1/2*f*x)^2*tan(1/2*c*f/d)
^2*tan(1/2*e) - a*c^2*f^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2
 + a*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2 - 2*a*c^2*f
^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)^2 + 2*a*c*d*f*tan(1/2*f*x)^2*ta
n(1/2*e)^2 - 2*a*c*d*f*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 4*a*d^2*tan(1/2*f*x)
*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*c*d*f^2*x*imag_part(cos_integral(f*x +
 c*f/d)) - 2*a*c*d*f^2*x*imag_part(cos_integral(-f*x - c*f/d)) + 4*a*c*d*f^
2*x*sin_integral((d*f*x + c*f)/d) - 2*a*d^2*f*x*tan(1/2*f*x)^2 - 2*a*c^2*f^
2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 2*a*c^2*f^2*real_pa
rt(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) + 2*a*d^2*f*x*tan(1/2*c*f/d)^
2 + 2*a*d^2*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + 2*a*c^2*f^2*real_part(cos_int
egral(f*x + c*f/d))*tan(1/2*e) + 2*a*c^2*f^2*real_part(cos_integral(-f*x -
c*f/d))*tan(1/2*e) - 8*a*d^2*f*x*tan(1/2*f*x)*tan(1/2*e) - 2*a*d^2*f*x*tan(
1/2*e)^2 + 2*a*d^2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*a*d^2*tan(1/2*c*f/d)^2*t
an(1/2*e)^2 + a*c^2*f^2*imag_part(cos_integral(f*x + c*f/d)) - a*c^2*f^2*im
ag_part(cos_integral(-f*x - c*f/d)) + 2*a*c^2*f^2*sin_integral((d*f*x + c*f
)/d) - 2*a*c*d*f*tan(1/2*f*x)^2 + 2*a*c*d*f*tan(1/2*c*f/d)^2 + 4*a*d^2*tan(
1/2*f*x)*tan(1/2*c*f/d)^2 - 8*a*c*d*f*tan(1/2*f*x)*tan(1/2*e) - 4*a*d^2*tan
(1/2*f*x)^2*tan(1/2*e) + 4*a*d^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*a*c*d*f*ta
n(1/2*e)^2 - 4*a*d^2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*a*d^2*f*x + 2*a*d^2*tan(
1/2*f*x)^2 + 2*a*d^2*tan(1/2*c*f/d)^2 + 2*a*d^2*tan(1/2*e)^2 + 2*a*c*d*f +
4*a*d^2*tan(1/2*f*x) + 4*a*d^2*tan(1/2*e) + 2*a*d^2)/(d^5*x^2*tan(1/2*f*x)^
2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*c*d^4*x*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2
*tan(1/2*e)^2 + d^5*x^2*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + d^5*x^2*tan(1/2*f
*x)^2*tan(1/2*e)^2 + d^5*x^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + c^2*d^3*tan(1/
2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*c*d^4*x*tan(1/2*f*x)^2*tan(1/2*c
*f/d)^2 + 2*c*d^4*x*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*c*d^4*x*tan(1/2*c*f/d)^
2*tan(1/2*e)^2 + d^5*x^2*tan(1/2*f*x)^2 + d^5*x^2*tan(1/2*c*f/d)^2 + c^2*d^
3*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + d^5*x^2*tan(1/2*e)^2 + c^2*d^3*tan(1/2*
f*x)^2*tan(1/2*e)^2 + c^2*d^3*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*c*d^4*x*tan
(1/2*f*x)^2 + 2*c*d^4*x*tan(1/2*c*f/d)^2 + 2*c*d^4*x*tan(1/2*e)^2 + d^5*x^2
 + c^2*d^3*tan(1/2*f*x)^2 + c^2*d^3*tan(1/2*c*f/d)^2 + c^2*d^3*tan(1/2*e)^2
 + 2*c*d^4*x + c^2*d^3)
```

3.101 $\int (c+dx)^3 (a+a\sin(e+fx))^2 \, dx$

Optimal. Leaf size=237

$$\frac{12a^2d^2(c+dx)\cos(e+fx)}{f^3}+\frac{3a^2d^2(c+dx)\sin(e+fx)\cos(e+fx)}{4f^3}-\frac{3a^2cd^2x}{4f^2}+\frac{3a^2d(c+dx)^2\sin^2(e+fx)}{4f^2}+\frac{6a^2d(c\cdots}{}$$

```
[Out] (-3*a^2*c*d^2*x)/(4*f^2) - (3*a^2*d^3*x^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8
*d) + (12*a^2*d^2*(c + d*x)*Cos[e + f*x])/f^3 - (2*a^2*(c + d*x)^3*Cos[e +
f*x])/f - (12*a^2*d^3*Sin[e + f*x])/f^4 + (6*a^2*d*(c + d*x)^2*Sin[e + f*x]
)/f^2 + (3*a^2*d^2*(c + d*x)*Cos[e + f*x]*Sin[e + f*x])/(4*f^3) - (a^2*(c +
 d*x)^3*Cos[e + f*x]*Sin[e + f*x])/(2*f) - (3*a^2*d^3*Sin[e + f*x]^2)/(8*f^
4) + (3*a^2*d*(c + d*x)^2*Sin[e + f*x]^2)/(4*f^2)
```

Rubi [A] time = 0.295401, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3317, 3296, 2637, 3311, 32, 3310}

$$\frac{12a^2d^2(c+dx)\cos(e+fx)}{f^3}+\frac{3a^2d^2(c+dx)\sin(e+fx)\cos(e+fx)}{4f^3}-\frac{3a^2cd^2x}{4f^2}+\frac{3a^2d(c+dx)^2\sin^2(e+fx)}{4f^2}+\frac{6a^2d(c\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + a*Sin[e + f*x])^2,x]
```


```
[Out] (-3*a^2*c*d^2*x)/(4*f^2) - (3*a^2*d^3*x^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8
*d) + (12*a^2*d^2*(c + d*x)*Cos[e + f*x])/f^3 - (2*a^2*(c + d*x)^3*Cos[e +
f*x])/f - (12*a^2*d^3*Sin[e + f*x])/f^4 + (6*a^2*d*(c + d*x)^2*Sin[e + f*x]
)/f^2 + (3*a^2*d^2*(c + d*x)*Cos[e + f*x]*Sin[e + f*x])/(4*f^3) - (a^2*(c +
 d*x)^3*Cos[e + f*x]*Sin[e + f*x])/(2*f) - (3*a^2*d^3*Sin[e + f*x]^2)/(8*f^
4) + (3*a^2*d*(c + d*x)^2*Sin[e + f*x]^2)/(4*f^2)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)^3(a+a\sin(e+fx))^2\,dx = \int \left(a^2(c+dx)^3 + 2a^2(c+dx)^3\sin(e+fx) + a^2(c+dx)^3\sin^2(e+fx)\right)dx$$
$$= \frac{a^2(c+dx)^4}{4d} + a^2\int (c+dx)^3\sin^2(e+fx)\,dx + \left(2a^2\right)\int (c+dx)^3\sin(e+fx)\,dx$$
$$= \frac{a^2(c+dx)^4}{4d} - \frac{2a^2(c+dx)^3\cos(e+fx)}{f} - \frac{a^2(c+dx)^3\cos(e+fx)\sin(e+fx)}{2f} + 3a$$
$$= \frac{3a^2(c+dx)^4}{8d} - \frac{2a^2(c+dx)^3\cos(e+fx)}{f} + \frac{6a^2d(c+dx)^2\sin(e+fx)}{f^2} + \frac{3a^2d^2(c+}{}$$
$$= -\frac{3a^2cd^2x}{4f^2} - \frac{3a^2d^3x^2}{8f^2} + \frac{3a^2(c+dx)^4}{8d} + \frac{12a^2d^2(c+dx)\cos(e+fx)}{f^3} - \frac{2a^2(c+dx)^3}{f}$$
$$= -\frac{3a^2cd^2x}{4f^2} - \frac{3a^2d^3x^2}{8f^2} + \frac{3a^2(c+dx)^4}{8d} + \frac{12a^2d^2(c+dx)\cos(e+fx)}{f^3} - \frac{2a^2(c+dx)^3}{f}$$

Mathematica [A] time = 1.34344, size = 216, normalized size = 0.91

$$\frac{a^2\left(-2f(c+dx)\left(2c^2f^2+4cdf^2x+d^2\left(2f^2x^2-3\right)\right)\sin(2(e+fx))+96d\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2-2\right)\right)\sin(e+fx)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(6*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 32*f*(c + d*x)*
(c^2*f^2 + 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Cos[e + f*x] - 3*d*(2*c^2*f^2
+ 4*c*d*f^2*x + d^2*(-1 + 2*f^2*x^2))*Cos[2*(e + f*x)] + 96*d*(c^2*f^2 + 2*
c*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x] - 2*f*(c + d*x)*(2*c^2*f^2 + 4
*c*d*f^2*x + d^2*(-3 + 2*f^2*x^2))*Sin[2*(e + f*x)]))/(16*f^4)
```

Maple [B] time = 0.022, size = 1135, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+a*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(a^2*c^3*(f*x+e)+a^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a
^2*c^3*cos(f*x+e)+1/4*a^2/f^3*d^3*(f*x+e)^4+2*a^2/f^3*d^3*(-(f*x+e)^3*cos(f
```

```
*x+e)+3*(f*x+e)^2*sin(f*x+e)-6*sin(f*x+e)+6*(f*x+e)*cos(f*x+e))+a^2/f^3*d^3
*((f*x+e)^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-3/4*(f*x+e)^2*cos(f*
x+e)^2+3/2*(f*x+e)*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-3/8*(f*x+e)^2-
3/8*sin(f*x+e)^2-3/8*(f*x+e)^4)-a^2/f^3*d^3*e^3*(-1/2*sin(f*x+e)*cos(f*x+e)
+1/2*f*x+1/2*e)-a^2/f^3*d^3*e*(f*x+e)^3+2*a^2/f^3*d^3*e^3*cos(f*x+e)-a^2/f^
3*d^3*e^3*(f*x+e)+a^2/f^2*c*d^2*(f*x+e)^3+6*a^2/f*c^2*d*(sin(f*x+e)-(f*x+e)
*cos(f*x+e))+3/2*a^2/f^3*d^3*e^2*(f*x+e)^2+3/2*a^2/f*c^2*d*(f*x+e)^2+3*a^2/
f*c^2*d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1
/4*sin(f*x+e)^2)+6*a^2/f^2*c*d^2*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f*x
+e)*sin(f*x+e))+3*a^2/f^3*d^3*e^2*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*
f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)-6*a^2/f^3*d^3*e*(-(f*x+e)^2*cos(
f*x+e)+2*cos(f*x+e)+2*(f*x+e)*sin(f*x+e))+6*a^2/f^3*d^3*e^2*(sin(f*x+e)-(f*
x+e)*cos(f*x+e))+3*a^2/f^2*c*d^2*((f*x+e)^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2
*f*x+1/2*e)-1/2*(f*x+e)*cos(f*x+e)^2+1/4*sin(f*x+e)*cos(f*x+e)+1/4*f*x+1/4*
e-1/3*(f*x+e)^3)-3*a^2/f^3*d^3*e*((f*x+e)^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2
*f*x+1/2*e)-1/2*(f*x+e)*cos(f*x+e)^2+1/4*sin(f*x+e)*cos(f*x+e)+1/4*f*x+1/4*
e-1/3*(f*x+e)^3)-6*a^2/f^2*c*d^2*e*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2
*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)-12*a^2/f^2*c*d^2*e*(sin(f*x+e)-
(f*x+e)*cos(f*x+e))+6*a^2/f*c^2*d*e*cos(f*x+e)-3*a^2/f*c^2*d*e*(-1/2*sin(f*
x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-3*a^2/f^2*c*d^2*e*(f*x+e)^2-6*a^2/f^2*c*d^2*
e^2*cos(f*x+e)+3*a^2/f^2*c*d^2*e^2*(f*x+e)-3*a^2/f*c^2*d*e*(f*x+e)+3*a^2/f^
2*c*d^2*e^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [B] time = 1.10759, size = 1308, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^3 + 16*(f*x + e)*a^2*c^3 + 4
*(f*x + e)^4*a^2*d^3/f^3 - 16*(f*x + e)^3*a^2*d^3*e/f^3 + 24*(f*x + e)^2*a^
2*d^3*e^2/f^3 - 4*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*d^3*e^3/f^3 - 16*(f*
x + e)*a^2*d^3*e^3/f^3 + 16*(f*x + e)^3*a^2*c*d^2/f^2 - 48*(f*x + e)^2*a^2*
c*d^2*e/f^2 + 12*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c*d^2*e^2/f^2 + 48*(f
*x + e)*a^2*c*d^2*e^2/f^2 + 24*(f*x + e)^2*a^2*c^2*d/f - 12*(2*f*x + 2*e -
sin(2*f*x + 2*e))*a^2*c^2*d*e/f - 48*(f*x + e)*a^2*c^2*d*e/f - 32*a^2*c^3*c
os(f*x + e) + 32*a^2*d^3*e^3*cos(f*x + e)/f^3 - 96*a^2*c*d^2*e^2*cos(f*x +
e)/f^2 + 96*a^2*c^2*d*e*cos(f*x + e)/f + 6*(2*(f*x + e)^2 - 2*(f*x + e)*sin
(2*f*x + 2*e) - cos(2*f*x + 2*e))*a^2*d^3*e^2/f^3 - 96*((f*x + e)*cos(f*x +
 e) - sin(f*x + e))*a^2*d^3*e^2/f^3 - 12*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2
```

```
*f*x + 2*e) - cos(2*f*x + 2*e))*a^2*c*d^2*e/f^2 + 192*((f*x + e)*cos(f*x +
e) - sin(f*x + e))*a^2*c*d^2*e/f^2 + 6*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*f
*x + 2*e) - cos(2*f*x + 2*e))*a^2*c^2*d/f - 96*((f*x + e)*cos(f*x + e) - si
n(f*x + e))*a^2*c^2*d/f - 2*(4*(f*x + e)^3 - 6*(f*x + e)*cos(2*f*x + 2*e) -
 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))*a^2*d^3*e/f^3 + 96*(((f*x + e)^2 -
 2)*cos(f*x + e) - 2*(f*x + e)*sin(f*x + e))*a^2*d^3*e/f^3 + 2*(4*(f*x + e)
^3 - 6*(f*x + e)*cos(2*f*x + 2*e) - 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))
*a^2*c*d^2/f^2 - 96*(((f*x + e)^2 - 2)*cos(f*x + e) - 2*(f*x + e)*sin(f*x +
 e))*a^2*c*d^2/f^2 + (2*(f*x + e)^4 - 3*(2*(f*x + e)^2 - 1)*cos(2*f*x + 2*e
) - 2*(2*(f*x + e)^3 - 3*f*x - 3*e)*sin(2*f*x + 2*e))*a^2*d^3/f^3 - 32*(((f
*x + e)^3 - 6*f*x - 6*e)*cos(f*x + e) - 3*((f*x + e)^2 - 2)*sin(f*x + e))*a
^2*d^3/f^3)/f
```

Fricas [A] time = 1.88544, size = 752, normalized size = 3.17

$$\frac{3\,a^2d^3f^4x^4+12\,a^2cd^2f^4x^3+3\left(6\,a^2c^2df^4+a^2d^3f^2\right)x^2-3\left(2\,a^2d^3f^2x^2+4\,a^2cd^2f^2x+2\,a^2c^2df^2-a^2d^3\right)\cos\left(fx+e\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*a^2*d^3*f^4*x^4 + 12*a^2*c*d^2*f^4*x^3 + 3*(6*a^2*c^2*d*f^4 + a^2*d^
3*f^2)*x^2 - 3*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2 - a
^2*d^3)*cos(f*x + e)^2 + 6*(2*a^2*c^3*f^4 + a^2*c*d^2*f^2)*x - 16*(a^2*d^3*
f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + a^2*c^3*f^3 - 6*a^2*c*d^2*f + 3*(a^2*c^2*d*
f^3 - 2*a^2*d^3*f)*x)*cos(f*x + e) + 2*(24*a^2*d^3*f^2*x^2 + 48*a^2*c*d^2*f
^2*x + 24*a^2*c^2*d*f^2 - 48*a^2*d^3 - (2*a^2*d^3*f^3*x^3 + 6*a^2*c*d^2*f^3
*x^2 + 2*a^2*c^3*f^3 - 3*a^2*c*d^2*f + 3*(2*a^2*c^2*d*f^3 - a^2*d^3*f)*x)*c
os(f*x + e))*sin(f*x + e))/f^4
```

Sympy [A] time = 4.78442, size = 779, normalized size = 3.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**3*x*sin(e + f*x)**2/2 + a**2*c**3*x*cos(e + f*x)**2/2 +
a**2*c**3*x - a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c**3*cos(e
 + f*x)/f + 3*a**2*c**2*d*x**2*sin(e + f*x)**2/4 + 3*a**2*c**2*d*x**2*cos(e
 + f*x)**2/4 + 3*a**2*c**2*d*x**2/2 - 3*a**2*c**2*d*x*sin(e + f*x)*cos(e +
f*x)/(2*f) - 6*a**2*c**2*d*x*cos(e + f*x)/f + 6*a**2*c**2*d*sin(e + f*x)/f*
*2 - 3*a**2*c**2*d*cos(e + f*x)**2/(4*f**2) + a**2*c*d**2*x**3*sin(e + f*x)
**2/2 + a**2*c*d**2*x**3*cos(e + f*x)**2/2 + a**2*c*d**2*x**3 - 3*a**2*c*d*
*2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 6*a**2*c*d**2*x**2*cos(e + f*x)/f
 + 3*a**2*c*d**2*x*sin(e + f*x)**2/(4*f**2) + 12*a**2*c*d**2*x*sin(e + f*x)
/f**2 - 3*a**2*c*d**2*x*cos(e + f*x)**2/(4*f**2) + 3*a**2*c*d**2*sin(e + f*
x)*cos(e + f*x)/(4*f**3) + 12*a**2*c*d**2*cos(e + f*x)/f**3 + a**2*d**3*x**
4*sin(e + f*x)**2/8 + a**2*d**3*x**4*cos(e + f*x)**2/8 + a**2*d**3*x**4/4 -
 a**2*d**3*x**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*d**3*x**3*cos(e +
f*x)/f + 3*a**2*d**3*x**2*sin(e + f*x)**2/(8*f**2) + 6*a**2*d**3*x**2*sin(e
 + f*x)/f**2 - 3*a**2*d**3*x**2*cos(e + f*x)**2/(8*f**2) + 3*a**2*d**3*x*si
n(e + f*x)*cos(e + f*x)/(4*f**3) + 12*a**2*d**3*x*cos(e + f*x)/f**3 - 12*a*
*2*d**3*sin(e + f*x)/f**4 + 3*a**2*d**3*cos(e + f*x)**2/(8*f**4), Ne(f, 0))
, ((a*sin(e) + a)**2*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)
, True))
```

Giac [A] time = 1.15698, size = 458, normalized size = 1.93

$$\frac{3}{8}\,a^2d^3x^4+\frac{3}{2}\,a^2cd^2x^3+\frac{9}{4}\,a^2c^2dx^2+\frac{3}{2}\,a^2c^3x-\frac{3\left(2\,a^2d^3f^2x^2+4\,a^2cd^2f^2x+2\,a^2c^2df^2-a^2d^3\right)\cos\left(2\,fx+2\,e\right)}{16\,f^4}-\frac{2\left(a^2d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/8*a^2*d^3*x^4 + 3/2*a^2*c*d^2*x^3 + 9/4*a^2*c^2*d*x^2 + 3/2*a^2*c^3*x - 3
/16*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2 - a^2*d^3)*cos
(2*f*x + 2*e)/f^4 - 2*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*
f^3*x + a^2*c^3*f^3 - 6*a^2*d^3*f*x - 6*a^2*c*d^2*f)*cos(f*x + e)/f^4 - 1/8
*(2*a^2*d^3*f^3*x^3 + 6*a^2*c*d^2*f^3*x^2 + 6*a^2*c^2*d*f^3*x + 2*a^2*c^3*f
^3 - 3*a^2*d^3*f*x - 3*a^2*c*d^2*f)*sin(2*f*x + 2*e)/f^4 + 6*(a^2*d^3*f^2*x
^2 + 2*a^2*c*d^2*f^2*x + a^2*c^2*d*f^2 - 2*a^2*d^3)*sin(f*x + e)/f^4
```

3.102 $\int (c+dx)^2 (a+a\sin(e+fx))^2 \, dx$

Optimal. Leaf size=168

$$\frac{a^2 d(c+dx)\sin^2(e+fx)}{2f^2}+\frac{4a^2 d(c+dx)\sin(e+fx)}{f^2}-\frac{2a^2(c+dx)^2\cos(e+fx)}{f}-\frac{a^2(c+dx)^2\sin(e+fx)\cos(e+fx)}{2f}+$$

```
[Out] -(a^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(2*d) + (4*a^2*d^2*Cos[e + f*x])/f
^3 - (2*a^2*(c + d*x)^2*Cos[e + f*x])/f + (4*a^2*d*(c + d*x)*Sin[e + f*x])/
f^2 + (a^2*d^2*Cos[e + f*x]*Sin[e + f*x])/(4*f^3) - (a^2*(c + d*x)^2*Cos[e
+ f*x]*Sin[e + f*x])/(2*f) + (a^2*d*(c + d*x)*Sin[e + f*x]^2)/(2*f^2)
```

Rubi [A] time = 0.19208, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3317, 3296, 2638, 3311, 32, 2635, 8}

$$\frac{a^2 d(c+dx)\sin^2(e+fx)}{2f^2}+\frac{4a^2 d(c+dx)\sin(e+fx)}{f^2}-\frac{2a^2(c+dx)^2\cos(e+fx)}{f}-\frac{a^2(c+dx)^2\sin(e+fx)\cos(e+fx)}{2f}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + a*Sin[e + f*x])^2,x]
```


```
[Out] -(a^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(2*d) + (4*a^2*d^2*Cos[e + f*x])/f
^3 - (2*a^2*(c + d*x)^2*Cos[e + f*x])/f + (4*a^2*d*(c + d*x)*Sin[e + f*x])/
f^2 + (a^2*d^2*Cos[e + f*x]*Sin[e + f*x])/(4*f^3) - (a^2*(c + d*x)^2*Cos[e
+ f*x]*Sin[e + f*x])/(2*f) + (a^2*d*(c + d*x)*Sin[e + f*x]^2)/(2*f^2)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^2(a+a\sin(e+fx))^2\,dx = \int \left(a^2(c+dx)^2+2a^2(c+dx)^2\sin(e+fx)+a^2(c+dx)^2\sin^2(e+fx)\right)dx$$
$$= \frac{a^2(c+dx)^3}{3d}+a^2\int (c+dx)^2\sin^2(e+fx)\,dx+\left(2a^2\right)\int (c+dx)^2\sin(e+fx)\,dx$$
$$= \frac{a^2(c+dx)^3}{3d}-\frac{2a^2(c+dx)^2\cos(e+fx)}{f}-\frac{a^2(c+dx)^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{a^2d$$
$$= \frac{a^2(c+dx)^3}{2d}-\frac{2a^2(c+dx)^2\cos(e+fx)}{f}+\frac{4a^2d(c+dx)\sin(e+fx)}{f^2}+\frac{a^2d^2\cos(e+f}{4}$$
$$= -\frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}+\frac{4a^2d^2\cos(e+fx)}{f^3}-\frac{2a^2(c+dx)^2\cos(e+fx)}{f}+\frac{4a^2d(c+a}{}$$

Mathematica [A] time = 0.613693, size = 182, normalized size = 1.08

$$a^2 \left(-16 \left(c^2 f^2+2 c d f^2 x+d^2 \left(f^2 x^2-2\right)\right) \cos(e+fx)-2c^2 f^2 \sin(2(e+fx))+12c^2 f^3 x-4cdf^2 x \sin(2(e+fx))+32cdf$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2*f^3*x + 12*c*d*f^3*x^2 + 4*d^2*f^3*x^3 - 16*(c^2*f^2 + 2*c*d*f
^2*x + d^2*(-2 + f^2*x^2))*Cos[e + f*x] - 2*d*f*(c + d*x)*Cos[2*(e + f*x)]
+ 32*c*d*f*Sin[e + f*x] + 32*d^2*f*x*Sin[e + f*x] + d^2*Sin[2*(e + f*x)] -
2*c^2*f^2*Sin[2*(e + f*x)] - 4*c*d*f^2*x*Sin[2*(e + f*x)] - 2*d^2*f^2*x^2*S
in[2*(e + f*x)]))/(8*f^3)
```

Maple [B] time = 0.02, size = 567, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+a*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(a^2/f^2*d^2*((f*x+e)^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/2*
(f*x+e)*cos(f*x+e)^2+1/4*sin(f*x+e)*cos(f*x+e)+1/4*f*x+1/4*e-1/3*(f*x+e)^3)
+2*a^2/f*c*d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e
)^2+1/4*sin(f*x+e)^2)-2*a^2/f^2*d^2*e*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+
1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)+a^2*c^2*(-1/2*sin(f*x+e)*cos
(f*x+e)+1/2*f*x+1/2*e)-2*a^2/f*c*d*e*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/
2*e)+a^2/f^2*d^2*e^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a^2/f^2*d
^2*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f*x+e)*sin(f*x+e))+4*a^2/f*c*d*(s
in(f*x+e)-(f*x+e)*cos(f*x+e))-4*a^2/f^2*d^2*e*(sin(f*x+e)-(f*x+e)*cos(f*x+e
))-2*a^2*c^2*cos(f*x+e)+4*a^2/f*c*d*e*cos(f*x+e)-2*a^2/f^2*d^2*e^2*cos(f*x+
e)+1/3*a^2/f^2*d^2*(f*x+e)^3+a^2/f*c*d*(f*x+e)^2-a^2/f^2*d^2*e*(f*x+e)^2+a^
2*c^2*(f*x+e)-2*a^2/f*c*d*e*(f*x+e)+a^2/f^2*d^2*e^2*(f*x+e))
```

Maxima [B] time = 1.02159, size = 686, normalized size = 4.08

$$6\left(2fx+2e-\sin\left(2fx+2e\right)\right)a^2c^2+24\left(fx+e\right)a^2c^2+\frac{8\left(fx+e\right)^3a^2d^2}{f^2}-\frac{24\left(fx+e\right)^2a^2d^2e}{f^2}+\frac{6\left(2fx+2e-\sin\left(2fx+2e\right)\right)a^2d^2e^2}{f^2}+\frac{24\left(f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(6*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^2 + 24*(f*x + e)*a^2*c^2 + 8
*(f*x + e)^3*a^2*d^2/f^2 - 24*(f*x + e)^2*a^2*d^2*e/f^2 + 6*(2*f*x + 2*e -
sin(2*f*x + 2*e))*a^2*d^2*e^2/f^2 + 24*(f*x + e)*a^2*d^2*e^2/f^2 + 24*(f*x
+ e)^2*a^2*c*d/f - 12*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c*d*e/f - 48*(f*
x + e)*a^2*c*d*e/f - 48*a^2*c^2*cos(f*x + e) - 48*a^2*d^2*e^2*cos(f*x + e)/
f^2 + 96*a^2*c*d*e*cos(f*x + e)/f - 6*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*f*
x + 2*e) - cos(2*f*x + 2*e))*a^2*d^2*e/f^2 + 96*((f*x + e)*cos(f*x + e) - s
in(f*x + e))*a^2*d^2*e/f^2 + 6*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e
) - cos(2*f*x + 2*e))*a^2*c*d/f - 96*((f*x + e)*cos(f*x + e) - sin(f*x + e)
)*a^2*c*d/f + (4*(f*x + e)^3 - 6*(f*x + e)*cos(2*f*x + 2*e) - 3*(2*(f*x + e
)^2 - 1)*sin(2*f*x + 2*e))*a^2*d^2/f^2 - 48*(((f*x + e)^2 - 2)*cos(f*x + e)
 - 2*(f*x + e)*sin(f*x + e))*a^2*d^2/f^2)/f
```

Fricas [A] time = 1.78484, size = 447, normalized size = 2.66

$$\frac{2\,a^2d^2f^3x^3+6\,a^2cdf^3x^2-2\left(a^2d^2fx+a^2cdf\right)\cos\left(fx+e\right)^2+\left(6\,a^2c^2f^3+a^2d^2f\right)x-8\left(a^2d^2f^2x^2+2\,a^2cdf^2x+a^2c^2f^2\cdots\right.}{4\,f\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*d^2*f^3*x^3 + 6*a^2*c*d*f^3*x^2 - 2*(a^2*d^2*f*x + a^2*c*d*f)*co
s(f*x + e)^2 + (6*a^2*c^2*f^3 + a^2*d^2*f)*x - 8*(a^2*d^2*f^2*x^2 + 2*a^2*c
*d*f^2*x + a^2*c^2*f^2 - 2*a^2*d^2)*cos(f*x + e) + (16*a^2*d^2*f*x + 16*a^2
*c*d*f - (2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - a^2*d^2)*co
s(f*x + e))*sin(f*x + e))/f^3
```

Sympy [A] time = 2.15149, size = 456, normalized size = 2.71

$$\begin{cases} \frac{a^2c^2x\sin^2\left(e+fx\right)}{2}+\frac{a^2c^2x\cos^2\left(e+fx\right)}{2}+a^2c^2x-\frac{a^2c^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2a^2c^2\cos\left(e+fx\right)}{f}+\frac{a^2cdx^2\sin^2\left(e+fx\right)}{2}+\frac{a^2cdx^2\cos^2\left(e+fx\right)}{2}+a^2c\cdots \\ \left(a\sin\left(e\right)+a\right)^2\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x*sin(e + f*x)**2/2 + a**2*c**2*x*cos(e + f*x)**2/2 +
a**2*c**2*x - a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c**2*cos(e
 + f*x)/f + a**2*c*d*x**2*sin(e + f*x)**2/2 + a**2*c*d*x**2*cos(e + f*x)**2
/2 + a**2*c*d*x**2 - a**2*c*d*x*sin(e + f*x)*cos(e + f*x)/f - 4*a**2*c*d*x*
cos(e + f*x)/f + 4*a**2*c*d*sin(e + f*x)/f**2 - a**2*c*d*cos(e + f*x)**2/(2
*f**2) + a**2*d**2*x**3*sin(e + f*x)**2/6 + a**2*d**2*x**3*cos(e + f*x)**2/
6 + a**2*d**2*x**3/3 - a**2*d**2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a
**2*d**2*x**2*cos(e + f*x)/f + a**2*d**2*x*sin(e + f*x)**2/(4*f**2) + 4*a**
2*d**2*x*sin(e + f*x)/f**2 - a**2*d**2*x*cos(e + f*x)**2/(4*f**2) + a**2*d*
*2*sin(e + f*x)*cos(e + f*x)/(4*f**3) + 4*a**2*d**2*cos(e + f*x)/f**3, Ne(f
, 0)), ((a*sin(e) + a)**2*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Giac [A] time = 1.14398, size = 279, normalized size = 1.66

$$\frac{1}{2}a^2d^2x^3 + \frac{3}{2}a^2cdx^2 + \frac{3}{2}a^2c^2x - \frac{\left(a^2d^2fx + a^2cdf\right)\cos\left(2fx + 2e\right)}{4f^3} - \frac{2\left(a^2d^2f^2x^2 + 2a^2cdf^2x + a^2c^2f^2 - 2a^2d^2\right)\cos\left(\right.}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*d^2*x^3 + 3/2*a^2*c*d*x^2 + 3/2*a^2*c^2*x - 1/4*(a^2*d^2*f*x + a^2*
c*d*f)*cos(2*f*x + 2*e)/f^3 - 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^
2*f^2 - 2*a^2*d^2)*cos(f*x + e)/f^3 - 1/8*(2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^
2*x + 2*a^2*c^2*f^2 - a^2*d^2)*sin(2*f*x + 2*e)/f^3 + 4*(a^2*d^2*f*x + a^2*
c*d*f)*sin(f*x + e)/f^3
```

3.103 $\int (c+dx)(a+a\sin(e+fx))^2\,dx$

Optimal. Leaf size=118

$$-\frac{2a^2(c+dx)\cos(e+fx)}{f}-\frac{a^2(c+dx)\sin(e+fx)\cos(e+fx)}{2f}+\frac{a^2(c+dx)^2}{2d}+\frac{1}{2}a^2cx+\frac{a^2d\sin^2(e+fx)}{4f^2}+\frac{2a^2d\sin(e+$$

```
[Out] (a^2*c*x)/2 + (a^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) - (2*a^2*(c + d*x)*Co
s[e + f*x])/f + (2*a^2*d*Sin[e + f*x])/f^2 - (a^2*(c + d*x)*Cos[e + f*x]*Si
n[e + f*x])/(2*f) + (a^2*d*Sin[e + f*x]^2)/(4*f^2)
```

Rubi [A] time = 0.10374, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3317, 3296, 2637, 3310}

$$-\frac{2a^2(c+dx)\cos(e+fx)}{f}-\frac{a^2(c+dx)\sin(e+fx)\cos(e+fx)}{2f}+\frac{a^2(c+dx)^2}{2d}+\frac{1}{2}a^2cx+\frac{a^2d\sin^2(e+fx)}{4f^2}+\frac{2a^2d\sin(e+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*c*x)/2 + (a^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) - (2*a^2*(c + d*x)*Co
s[e + f*x])/f + (2*a^2*d*Sin[e + f*x])/f^2 - (a^2*(c + d*x)*Cos[e + f*x]*Si
n[e + f*x])/(2*f) + (a^2*d*Sin[e + f*x]^2)/(4*f^2)
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)(a+a\sin(e+fx))^2\,dx = \int \left(a^2(c+dx)+2a^2(c+dx)\sin(e+fx)+a^2(c+dx)\sin^2(e+fx)\right)dx$$

$$= \frac{a^2(c+dx)^2}{2d}+a^2\int (c+dx)\sin^2(e+fx)\,dx+\left(2a^2\right)\int (c+dx)\sin(e+fx)\,dx$$

$$= \frac{a^2(c+dx)^2}{2d}-\frac{2a^2(c+dx)\cos(e+fx)}{f}-\frac{a^2(c+dx)\cos(e+fx)\sin(e+fx)}{2f}+\frac{a^2d\,\mathrm{s}}{}$$

$$= \frac{1}{2}a^2cx+\frac{1}{4}a^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{2a^2(c+dx)\cos(e+fx)}{f}+\frac{2a^2d\sin(e+fx)}{f^2}-\frac{a^2(}{}$$

Mathematica [A] time = 1.04758, size = 80, normalized size = 0.68

$$-\frac{a^2(6(e+fx)(d(e-fx)-2cf)+2f(c+dx)\sin(2(e+fx))+16f(c+dx)\cos(e+fx)-16d\sin(e+fx)+d\cos(2(e+f}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -(a^2*(6*(e + f*x)*(-2*c*f + d*(e - f*x)) + 16*f*(c + d*x)*Cos[e + f*x] + d
*Cos[2*(e + f*x)] - 16*d*Sin[e + f*x] + 2*f*(c + d*x)*Sin[2*(e + f*x)]))/(8
*f^2)
```

Maple [B] time = 0.02, size = 219, normalized size = 1.9

$$\frac{1}{f}\left(\frac{a^2d}{f}\left(\left(fx+e\right)\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{\left(fx+e\right)^2}{4}+\frac{\left(\sin\left(fx+e\right)\right)^2}{4}\right)+a^2c\left(-\frac{\sin\left(fx+e\right)\cos\left(fx\right.}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+a*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(a^2/f*d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e
)^2+1/4*sin(f*x+e)^2)+a^2*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a^2/
f*d*e*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a^2/f*d*(sin(f*x+e)-(f*x
+e)*cos(f*x+e))-2*a^2*c*cos(f*x+e)+2*a^2/f*d*e*cos(f*x+e)+1/2*a^2/f*d*(f*x+
e)^2+a^2*c*(f*x+e)-a^2/f*d*e*(f*x+e))
```

Maxima [A] time = 0.99571, size = 277, normalized size = 2.35

$$\frac{2\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^2c+8\left(fx+e\right)a^2c+\frac{4\left(fx+e\right)^2a^2d}{f}-\frac{2\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^2de}{f}-\frac{8\left(fx+e\right)a^2de}{f}-16\,a^2c\cos\left(fx\right.}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c + 8*(f*x + e)*a^2*c + 4*(f*x
+ e)^2*a^2*d/f - 2*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*d*e/f - 8*(f*x + e)
*a^2*d*e/f - 16*a^2*c*cos(f*x + e) + 16*a^2*d*e*cos(f*x + e)/f + (2*(f*x +
e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e))*a^2*d/f - 16*((f*x
+ e)*cos(f*x + e) - sin(f*x + e))*a^2*d/f)/f
```

Fricas [A] time = 1.73688, size = 228, normalized size = 1.93

$$\frac{3\,a^2df^2x^2+6\,a^2cf^2x-a^2d\cos\left(fx+e\right)^2-8\left(a^2dfx+a^2cf\right)\cos\left(fx+e\right)+2\left(4\,a^2d-\left(a^2dfx+a^2cf\right)\cos\left(fx+e\right)\right)\sin\left(\right.}{4\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*d*f^2*x^2 + 6*a^2*c*f^2*x - a^2*d*cos(f*x + e)^2 - 8*(a^2*d*f*x
+ a^2*c*f)*cos(f*x + e) + 2*(4*a^2*d - (a^2*d*f*x + a^2*c*f)*cos(f*x + e))*
sin(f*x + e))/f^2
```

Sympy [A] time = 0.844491, size = 219, normalized size = 1.86

$$\begin{cases} \frac{a^2cx\sin^2(e+fx)}{2} + \frac{a^2cx\cos^2(e+fx)}{2} + a^2cx - \frac{a^2c\sin(e+fx)\cos(e+fx)}{2f} - \frac{2a^2c\cos(e+fx)}{f} + \frac{a^2dx^2\sin^2(e+fx)}{4} + \frac{a^2dx^2\cos^2(e+fx)}{4} + \frac{a^2dx^2}{2} - \\ (a\sin(e)+a)^2\left(cx+\frac{dx^2}{2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c*x*sin(e + f*x)**2/2 + a**2*c*x*cos(e + f*x)**2/2 + a**2*c
*x - a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c*cos(e + f*x)/f + a**
2*d*x**2*sin(e + f*x)**2/4 + a**2*d*x**2*cos(e + f*x)**2/4 + a**2*d*x**2/2
- a**2*d*x*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*d*x*cos(e + f*x)/f + 2*
a**2*d*sin(e + f*x)/f**2 - a**2*d*cos(e + f*x)**2/(4*f**2), Ne(f, 0)), ((a*
sin(e) + a)**2*(c*x + d*x**2/2), True))
```

Giac [A] time = 1.10989, size = 144, normalized size = 1.22

$$\frac{3}{4}a^2dx^2 + \frac{3}{2}a^2cx - \frac{a^2d\cos\left(2fx+2e\right)}{8f^2} + \frac{2a^2d\sin\left(fx+e\right)}{f^2} - \frac{2\left(a^2dfx+a^2cf\right)\cos\left(fx+e\right)}{f^2} - \frac{\left(a^2dfx+a^2cf\right)\sin\left(2\right.}{4f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/4*a^2*d*x^2 + 3/2*a^2*c*x - 1/8*a^2*d*cos(2*f*x + 2*e)/f^2 + 2*a^2*d*sin(
f*x + e)/f^2 - 2*(a^2*d*f*x + a^2*c*f)*cos(f*x + e)/f^2 - 1/4*(a^2*d*f*x +
a^2*c*f)*sin(2*f*x + 2*e)/f^2
```

3.104 $\int \frac{(a+a\sin(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=145

$$\frac{2a^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}-\frac{a^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{2d}+\frac{a^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{2d}$$

```
[Out] -(a^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (3*a^2*L
og[c + d*x])/(2*d) + (2*a^2*CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d
+ (2*a^2*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d + (a^2*Sin[2*e - (2
*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rubi [A] time = 0.370685, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3318, 3312, 3303, 3299, 3302}

$$\frac{2a^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}-\frac{a^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{2d}+\frac{a^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*x),x]
```


```
[Out] -(a^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (3*a^2*L
og[c + d*x])/(2*d) + (2*a^2*CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d
+ (2*a^2*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d + (a^2*Sin[2*e - (2
*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2}{c+dx}\,dx &= \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)}{c+dx}\,dx\\ &= \left(4a^2\right)\int \left(\frac{3}{8(c+dx)}-\frac{\cos(2e+2fx)}{8(c+dx)}+\frac{\sin(e+fx)}{2(c+dx)}\right)dx\\ &= \frac{3a^2\log(c+dx)}{2d}-\frac{1}{2}a^2\int \frac{\cos(2e+2fx)}{c+dx}\,dx+\left(2a^2\right)\int \frac{\sin(e+fx)}{c+dx}\,dx\\ &= \frac{3a^2\log(c+dx)}{2d}-\frac{1}{2}\left(a^2\cos\left(2e-\frac{2cf}{d}\right)\right)\int \frac{\cos\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx+\left(2a^2\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin}{}\\ &= -\frac{a^2\cos\left(2e-\frac{2cf}{d}\right)\text{Ci}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{3a^2\log(c+dx)}{2d}+\frac{2a^2\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{2a^2}{}\end{aligned}$$

Mathematica [A] time = 0.27671, size = 114, normalized size = 0.79

$$\frac{a^2\left(4\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)\left(-\cos\left(2e-\frac{2cf}{d}\right)\right)+\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2f(c+dx)}{d}\right)+4\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*x),x]
```

```
[Out] (a^2*(-(Cos[2*e - (2*c*f)/d]*CosIntegral[(2*f*(c + d*x))/d]) + 3*Log[c + d*
x] + 4*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + 4*Cos[e - (c*f)/d]*SinIn
tegral[f*(c/d + x)] + Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d]))
/(2*d)
```

Maple [A] time = 0.021, size = 192, normalized size = 1.3

$$\frac{3\,a^2\ln\left(\left(fx+e\right)d+cf-de\right)}{2\,d}-\frac{a^2}{2\,d}\mathrm{Si}\left(2\,fx+2\,e+2\,\frac{cf-de}{d}\right)\sin\left(2\,\frac{cf-de}{d}\right)-\frac{a^2}{2\,d}\mathrm{Ci}\left(2\,fx+2\,e+2\,\frac{cf-de}{d}\right)\cos\left(2\,\frac{cf}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(d*x+c),x)
```

```
[Out] 3/2*a^2*ln((f*x+e)*d+c*f-d*e)/d-1/2*a^2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(
c*f-d*e)/d)/d-1/2*a^2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d+2*a^
2*Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-2*a^2*Ci(f*x+e+(c*f-d*e)/d)*sin(
(c*f-d*e)/d)/d
```

Maxima [C] time = 1.31331, size = 452, normalized size = 3.12

$$\frac{\frac{4\,a^2f\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d}+\frac{4\left(f\left(-iE_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)+iE_1\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f\left(E_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)+E_1\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\sin\left(-\frac{de-cf}{d}\right.}{d}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^2*f*log(c + (f*x + e)*d/f - d*e/f)/d + 4*(f*(-I*exp_integral_e(1,
(I*(f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(1, -(I*(f*x + e)*d -
I*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f*(exp_integral_e(1, (I*(f*x + e)*
d - I*d*e + I*c*f)/d) + exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I*c*f)/
d))*sin(-(d*e - c*f)/d))*a^2/d + (f*(exp_integral_e(1, (2*I*(f*x + e)*d - 2
*I*d*e + 2*I*c*f)/d) + exp_integral_e(1, -(2*I*(f*x + e)*d - 2*I*d*e + 2*I*
c*f)/d))*cos(-2*(d*e - c*f)/d) + f*(I*exp_integral_e(1, (2*I*(f*x + e)*d -
2*I*d*e + 2*I*c*f)/d) - I*exp_integral_e(1, -(2*I*(f*x + e)*d - 2*I*d*e + 2
*I*c*f)/d))*sin(-2*(d*e - c*f)/d) + 2*f*log((f*x + e)*d - d*e + c*f))*a^2/d
```

```
)/f
```

Fricas [A] time = 1.75776, size = 468, normalized size = 3.23

$$-\frac{2\,a^2\sin\left(-\frac{2\,(de-cf)}{d}\right)\mathrm{Si}\left(\frac{2\,(dfx+cf)}{d}\right)-8\,a^2\cos\left(-\frac{de-cf}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)-6\,a^2\log\left(dx+c\right)+\left(a^2\,\mathrm{Ci}\left(\frac{2\,(dfx+cf)}{d}\right)+a^2\,\mathrm{Ci}\left(-\frac{2\,(\cdots}{}\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*sin(-2*(d*e - c*f)/d)*sin_integral(2*(d*f*x + c*f)/d) - 8*a^2*c
os(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/d) - 6*a^2*log(d*x + c) + (a^
2*cos_integral(2*(d*f*x + c*f)/d) + a^2*cos_integral(-2*(d*f*x + c*f)/d))*c
os(-2*(d*e - c*f)/d) + 4*(a^2*cos_integral((d*f*x + c*f)/d) + a^2*cos_integ
ral(-(d*f*x + c*f)/d))*sin(-(d*e - c*f)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{2\sin\left(e+fx\right)}{c+dx}\,dx+\int\frac{\sin^2\left(e+fx\right)}{c+dx}\,dx+\int\frac{1}{c+dx}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(d*x+c),x)
```

```
[Out] a**2*(Integral(2*sin(e + f*x)/(c + d*x), x) + Integral(sin(e + f*x)**2/(c +
 d*x), x) + Integral(1/(c + d*x), x))
```

Giac [C] time = 1.62394, size = 9516, normalized size = 65.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(4*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)
^2*tan(1/2*e)^2*tan(e)^2 - 4*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(
c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 6*a^2*log(abs(d*x + c))*t
an(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - a^2*real_part(cos_inte
gral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2
- a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)
^2*tan(1/2*e)^2*tan(e)^2 + 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2
*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 2*a^2*imag_part(cos_integral(2*f*
x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) + 2*a^2*ima
g_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/
2*e)^2*tan(e) - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*
c*f/d)^2*tan(1/2*e)^2*tan(e) - 8*a^2*real_part(cos_integral(f*x + c*f/d))*t
an(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 - 8*a^2*real_part(cos_inte
gral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 + 8*a
^2*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2
*e)^2*tan(e)^2 + 8*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*t
an(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 + 2*a^2*imag_part(cos_integral(2*f*x +
2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 2*a^2*imag_pa
rt(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2
*tan(e)^2 + 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(1/2*c*f/d)
^2*tan(1/2*e)^2*tan(e)^2 + 4*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c
*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 4*a^2*imag_part(cos_integral(-f*x -
 c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 6*a^2*log(abs(d*x + c
))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a^2*real_part(cos_integral(
2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a^2*real_par
t(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^
2 + 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1
/2*e)^2 - 4*a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2
*c*f/d)^2*tan(1/2*e)^2*tan(e) - 4*a^2*real_part(cos_integral(-2*f*x - 2*c*f
/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 4*a^2*imag_part(cos_
integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + 4*a^2*imag_
part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + 6
*a^2*log(abs(d*x + c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - a^2*real_pa
rt(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 -
a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2
*tan(e)^2 - 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)
^2*tan(e)^2 + 16*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(
1/2*c*f/d)*tan(1/2*e)*tan(e)^2 - 16*a^2*imag_part(cos_integral(-f*x - c*f/d
))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 + 32*a^2*sin_integral((d
*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 - 4*a^2*imag
_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a^2
*imag_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 +
 6*a^2*log(abs(d*x + c))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - a^2*real_part
(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - a^2*re
al_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2
```

```
- 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 +
4*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*ta
n(e)^2 - 4*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1
/2*e)^2*tan(e)^2 + 6*a^2*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*ta
n(e)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(
1/2*e)^2*tan(e)^2 + a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*c
*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2
*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 8*a^2*real_part(cos_integral(f*x + c*f/d)
)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 8*a^2*real_part(cos_integral(-
f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e) + 8*a^2*real_part(co
s_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 8*a^2*r
eal_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)
^2 - 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/
d)^2*tan(1/2*e)^2 + 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f
/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*t
an(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*a^2*imag_part(cos_integral(2*f*
x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e) + 2*a^2*imag_part(cos_in
tegral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e) - 4*a^2*sin_
integral(2*(d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e) - 2*a^2*im
ag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e) + 2
*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*ta
n(e) - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*e)^2*tan(
e) + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/
2*e)^2*tan(e) - 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*c*f
/d)^2*tan(1/2*e)^2*tan(e) + 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(1/2*c
*f/d)^2*tan(1/2*e)^2*tan(e) - 8*a^2*real_part(cos_integral(f*x + c*f/d))*ta
n(c*f/d)^2*tan(1/2*c*f/d)*tan(e)^2 - 8*a^2*real_part(cos_integral(-f*x - c*
f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(e)^2 + 2*a^2*imag_part(cos_integral(2
*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(e)^2 - 2*a^2*imag_part(cos
_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(e)^2 + 4*a^2*s
in_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(e)^2 + 8*a^2
*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*e)*tan(e)^2 + 8*
a^2*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*e)*tan(e)^2
- 8*a^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)*ta
n(e)^2 - 8*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1
/2*e)*tan(e)^2 + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*
tan(1/2*e)^2*tan(e)^2 - 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan
(c*f/d)*tan(1/2*e)^2*tan(e)^2 + 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c
*f/d)*tan(1/2*e)^2*tan(e)^2 + 8*a^2*real_part(cos_integral(f*x + c*f/d))*ta
n(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 + 8*a^2*real_part(cos_integral(-f*x - c*
f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 - 4*a^2*imag_part(cos_integral(f
*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 4*a^2*imag_part(cos_integral(-
f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 6*a^2*log(abs(d*x + c))*tan(c
*f/d)^2*tan(1/2*c*f/d)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan
(c*f/d)^2*tan(1/2*c*f/d)^2 + a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*
```

```
tan(c*f/d)^2*tan(1/2*c*f/d)^2 - 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f
/d)^2*tan(1/2*c*f/d)^2 + 16*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c*
f/d)^2*tan(1/2*c*f/d)*tan(1/2*e) - 16*a^2*imag_part(cos_integral(-f*x - c*f
/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e) + 32*a^2*sin_integral((d*f*x +
c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e) - 4*a^2*imag_part(cos_integr
al(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2 + 4*a^2*imag_part(cos_integral(-
f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2 + 6*a^2*log(abs(d*x + c))*tan(c*f/d
)^2*tan(1/2*e)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^
2*tan(1/2*e)^2 + a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2
*tan(1/2*e)^2 - 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*e)
^2 + 4*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)
^2 - 4*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e
)^2 + 6*a^2*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - a^2*real_part
(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - a^2*real_pa
rt(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 8*a^2*si
n_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 4*a^2*real_part
(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(e) - 4*a^2*
real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(e
) - 4*a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*e)^2*
tan(e) - 4*a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2
*e)^2*tan(e) + 4*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(
e)^2 - 4*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(e)^2 +
6*a^2*log(abs(d*x + c))*tan(c*f/d)^2*tan(e)^2 - a^2*real_part(cos_integral(
2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(e)^2 - a^2*real_part(cos_integral(-2*f*x
 - 2*c*f/d))*tan(c*f/d)^2*tan(e)^2 + 8*a^2*sin_integral((d*f*x + c*f)/d)*ta
n(c*f/d)^2*tan(e)^2 - 4*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*
f/d)^2*tan(e)^2 + 4*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d
)^2*tan(e)^2 + 6*a^2*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(e)^2 + a^2*real
_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(e)^2 + a^2*real_p
art(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)^2*tan(e)^2 - 8*a^2*sin_i
ntegral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(e)^2 + 16*a^2*imag_part(cos_i
ntegral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 - 16*a^2*imag_part
(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 + 32*a^2*si
n_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 - 4*a^2*imag
_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2*tan(e)^2 + 4*a^2*imag_part(co
s_integral(-f*x - c*f/d))*tan(1/2*e)^2*tan(e)^2 + 6*a^2*log(abs(d*x + c))*t
an(1/2*e)^2*tan(e)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2
*e)^2*tan(e)^2 + a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2
*tan(e)^2 - 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e)^2 - 8*a
^2*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d) - 8*a^2
*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d) - 2*a^2*
imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2 + 2*a^
2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2 - 4
*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(1/2*c*f/d)^2 + 8*a^2*re
al_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*e) + 8*a^2*real_par
```

```
t(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*e) - 8*a^2*real_part(cos
_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - 8*a^2*real_part(cos_i
ntegral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*a^2*imag_part(cos_in
tegral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*e)^2 + 2*a^2*imag_part(cos_inte
gral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*e)^2 - 4*a^2*sin_integral(2*(d*f
*x + c*f)/d)*tan(c*f/d)*tan(1/2*e)^2 + 8*a^2*real_part(cos_integral(f*x + c
*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 + 8*a^2*real_part(cos_integral(-f*x - c*
f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 - 2*a^2*imag_part(cos_integral(2*f*x + 2*
c*f/d))*tan(c*f/d)^2*tan(e) + 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d
))*tan(c*f/d)^2*tan(e) - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)^2
*tan(e) + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*t
an(e) - 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)^2*ta
n(e) + 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(e) + 2*a^
2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(e) - 2*a^2*imag
_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(e) + 4*a^2*sin_integ
ral(2*(d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e) + 2*a^2*imag_part(cos_integral(2
*f*x + 2*c*f/d))*tan(c*f/d)*tan(e)^2 - 2*a^2*imag_part(cos_integral(-2*f*x
- 2*c*f/d))*tan(c*f/d)*tan(e)^2 + 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan
(c*f/d)*tan(e)^2 - 8*a^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d
)*tan(e)^2 - 8*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan
(e)^2 + 8*a^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e)*tan(e)^2 + 8*
a^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)*tan(e)^2 + 4*a^2*imag_
part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2 - 4*a^2*imag_part(cos_integral
(-f*x - c*f/d))*tan(c*f/d)^2 + 6*a^2*log(abs(d*x + c))*tan(c*f/d)^2 + a^2*r
eal_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2 + a^2*real_part(cos_in
tegral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2 + 8*a^2*sin_integral((d*f*x + c*f)/d
)*tan(c*f/d)^2 - 4*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^
2 + 4*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2 + 6*a^2*lo
g(abs(d*x + c))*tan(1/2*c*f/d)^2 - a^2*real_part(cos_integral(2*f*x + 2*c*f
/d))*tan(1/2*c*f/d)^2 - a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1
/2*c*f/d)^2 - 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2 + 16*a^2
*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) - 16*a^2*im
ag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) + 32*a^2*sin_
integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e) - 4*a^2*imag_part(cos_i
ntegral(f*x + c*f/d))*tan(1/2*e)^2 + 4*a^2*imag_part(cos_integral(-f*x - c*
f/d))*tan(1/2*e)^2 + 6*a^2*log(abs(d*x + c))*tan(1/2*e)^2 - a^2*real_part(c
os_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2 - a^2*real_part(cos_integral(-2*
f*x - 2*c*f/d))*tan(1/2*e)^2 - 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*
e)^2 - 4*a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(e) - 4
*a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(e) + 4*a^2*im
ag_part(cos_integral(f*x + c*f/d))*tan(e)^2 - 4*a^2*imag_part(cos_integral(
-f*x - c*f/d))*tan(e)^2 + 6*a^2*log(abs(d*x + c))*tan(e)^2 + a^2*real_part(
cos_integral(2*f*x + 2*c*f/d))*tan(e)^2 + a^2*real_part(cos_integral(-2*f*x
 - 2*c*f/d))*tan(e)^2 + 8*a^2*sin_integral((d*f*x + c*f)/d)*tan(e)^2 - 2*a^
2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d) + 2*a^2*imag_part(cos
```

```
_integral(-2*f*x - 2*c*f/d))*tan(c*f/d) - 4*a^2*sin_integral(2*(d*f*x + c*f
)/d)*tan(c*f/d) - 8*a^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)
 - 8*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) + 8*a^2*real_
part(cos_integral(f*x + c*f/d))*tan(1/2*e) + 8*a^2*real_part(cos_integral(-
f*x - c*f/d))*tan(1/2*e) + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*t
an(e) - 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(e) + 4*a^2*sin_
integral(2*(d*f*x + c*f)/d)*tan(e) + 4*a^2*imag_part(cos_integral(f*x + c*f
/d)) - 4*a^2*imag_part(cos_integral(-f*x - c*f/d)) + 6*a^2*log(abs(d*x + c)
) - a^2*real_part(cos_integral(2*f*x + 2*c*f/d)) - a^2*real_part(cos_integr
al(-2*f*x - 2*c*f/d)) + 8*a^2*sin_integral((d*f*x + c*f)/d))/(d*tan(c*f/d)^
2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + d*tan(c*f/d)^2*tan(1/2*c*f/d)^2*
tan(1/2*e)^2 + d*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + d*tan(c*f/d)^2*ta
n(1/2*e)^2*tan(e)^2 + d*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + d*tan(c*f/
d)^2*tan(1/2*c*f/d)^2 + d*tan(c*f/d)^2*tan(1/2*e)^2 + d*tan(1/2*c*f/d)^2*ta
n(1/2*e)^2 + d*tan(c*f/d)^2*tan(e)^2 + d*tan(1/2*c*f/d)^2*tan(e)^2 + d*tan(
1/2*e)^2*tan(e)^2 + d*tan(c*f/d)^2 + d*tan(1/2*c*f/d)^2 + d*tan(1/2*e)^2 +
d*tan(e)^2 + d)
```

3.105 $\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=162

$$\frac{a^2 f\,\mathrm{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{2a^2 f\,\mathrm{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d^2}-\frac{2a^2 f\sin\left(e-\frac{cf}{d}\right)\mathrm{Si}\left(xf+\frac{cf}{d}\right.}{d^2}$$

```
[Out] (2*a^2*f*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d^2 + (a^2*f*CosInteg
ral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/d^2 - (4*a^2*Sin[e/2 + Pi/4 +
(f*x)/2]^4)/(d*(c + d*x)) - (2*a^2*f*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d +
 f*x])/d^2 + (a^2*f*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/d^
2
```

Rubi [A] time = 0.332581, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3318, 3313, 3303, 3299, 3302}

$$\frac{a^2 f\,\mathrm{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{2a^2 f\,\mathrm{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d^2}-\frac{2a^2 f\sin\left(e-\frac{cf}{d}\right)\mathrm{Si}\left(xf+\frac{cf}{d}\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (2*a^2*f*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d^2 + (a^2*f*CosInteg
ral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/d^2 - (4*a^2*Sin[e/2 + Pi/4 +
(f*x)/2]^4)/(d*(c + d*x)) - (2*a^2*f*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d +
 f*x])/d^2 + (a^2*f*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/d^
2
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^2}\,dx = \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)}{(c+dx)^2}\,dx$$

$$= -\frac{4a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)} + \frac{\left(8a^2f\right)\int\left(\frac{\cos(e+fx)}{4(c+dx)}+\frac{\sin(2e+2fx)}{8(c+dx)}\right)dx}{d}$$

$$= -\frac{4a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)} + \frac{\left(a^2f\right)\int\frac{\sin(2e+2fx)}{c+dx}\,dx}{d} + \frac{\left(2a^2f\right)\int\frac{\cos(e+fx)}{c+dx}\,dx}{d}$$

$$= -\frac{4a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)} + \frac{\left(a^2f\cos\left(2e-\frac{2cf}{d}\right)\right)\int\frac{\sin\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx}{d} + \frac{\left(2a^2f\cos\left(e-\frac{cf}{d}\right)\right)\int\frac{\cos}{}}{d}$$

$$= \frac{2a^2f\cos\left(e-\frac{cf}{d}\right)\text{Ci}\left(\frac{cf}{d}+fx\right)}{d^2} + \frac{a^2f\text{Ci}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2} - \frac{4a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)}$$

Mathematica [A] time = 0.587249, size = 206, normalized size = 1.27

$$\frac{a^2\left(2f(c+dx)\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)+4f(c+dx)\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\cos\left(e-\frac{cf}{d}\right)-4dfx\sin\left(e-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (a^2*(-3*d + d*Cos[2*(e + f*x)] + 4*f*(c + d*x)*Cos[e - (c*f)/d]*CosIntegra
l[f*(c/d + x)] + 2*f*(c + d*x)*CosIntegral[(2*f*(c + d*x))/d]*Sin[2*e - (2*
c*f)/d] - 4*d*Sin[e + f*x] - 4*c*f*Sin[e - (c*f)/d]*SinIntegral[f*(c/d + x)
] - 4*d*f*x*Sin[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + 2*c*f*Cos[2*e - (2*
c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 2*d*f*x*Cos[2*e - (2*c*f)/d]*SinIn
tegral[(2*f*(c + d*x))/d]))/(2*d^2*(c + d*x))
```

Maple [A] time = 0.026, size = 274, normalized size = 1.7

$$\frac{1}{f}\left(-\frac{3a^2f^2}{\left(2\left(fx+e\right)d+2cf-2de\right)d}-\frac{a^2f^2}{4}\left(-2\frac{\cos\left(2fx+2e\right)}{\left(\left(fx+e\right)d+cf-de\right)d}-2\frac{1}{d}\left(2\frac{1}{d}\text{Si}\left(2fx+2e+2\frac{cf-de}{d}\right)\cos\left(2\frac{cf}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(d*x+c)^2,x)
```

```
[Out] 1/f*(-3/2*a^2*f^2/((f*x+e)*d+c*f-d*e)/d-1/4*a^2*f^2*(-2*cos(2*f*x+2*e)/((f*
x+e)*d+c*f-d*e)/d-2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2*C
i(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)/d)+2*a^2*f^2*(-sin(f*x+e)/
((f*x+e)*d+c*f-d*e)/d+(Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+Ci(f*x+e+(c
*f-d*e)/d)*cos((c*f-d*e)/d)/d)/d))
```

Maxima [C] time = 1.51391, size = 500, normalized size = 3.09

$$-\frac{\frac{64a^2f^2}{\left(fx+e\right)d^2-d^2e+cdf}-\frac{64\left(f^2\left(-iE_2\left(\frac{i\left(fx+e\right)d-ide+icf}{d}\right)+iE_2\left(-\frac{i\left(fx+e\right)d-ide+icf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f^2\left(E_2\left(\frac{i\left(fx+e\right)d-ide+icf}{d}\right)+E_2\left(-\frac{i\left(fx+e\right)d-ide+icf}{d}\right)\right)\sin\left(-\frac{de-c}{d}\right.\right.}{\left(fx+e\right)d^2-d^2e+cdf}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/64*(64*a^2*f^2/((f*x + e)*d^2 - d^2*e + c*d*f) - 64*(f^2*(-I*exp_integra
l_e(2, (I*(f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(2, -(I*(f*x +
e)*d - I*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f^2*(exp_integral_e(2, (I*(
f*x + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(2, -(I*(f*x + e)*d - I*d*e
+ I*c*f)/d))*sin(-(d*e - c*f)/d))*a^2/((f*x + e)*d^2 - d^2*e + c*d*f) - (16
*f^2*(exp_integral_e(2, (2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d) + exp_inte
gral_e(2, -(2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d))*cos(-2*(d*e - c*f)/d)
+ f^2*(16*I*exp_integral_e(2, (2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d) - 16
*I*exp_integral_e(2, -(2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d))*sin(-2*(d*e
 - c*f)/d) - 32*f^2)*a^2/((f*x + e)*d^2 - d^2*e + c*d*f))/f
```

Fricas [A] time = 2.01274, size = 682, normalized size = 4.21

$$\frac{2\,a^2 d\cos\left(fx+e\right)^2 - 4\,a^2 d\sin\left(fx+e\right) - 4\,a^2 d + 2\left(a^2 dfx + a^2 cf\right)\cos\left(-\frac{2\left(de-cf\right)}{d}\right)\operatorname{Si}\left(\frac{2\left(dfx+cf\right)}{d}\right) + 4\left(a^2 dfx + a^2 cf\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*d*cos(f*x + e)^2 - 4*a^2*d*sin(f*x + e) - 4*a^2*d + 2*(a^2*d*f*x
 + a^2*c*f)*cos(-2*(d*e - c*f)/d)*sin_integral(2*(d*f*x + c*f)/d) + 4*(a^2*
d*f*x + a^2*c*f)*sin(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/d) + 2*((a^
2*d*f*x + a^2*c*f)*cos_integral((d*f*x + c*f)/d) + (a^2*d*f*x + a^2*c*f)*co
s_integral(-(d*f*x + c*f)/d))*cos(-(d*e - c*f)/d) - ((a^2*d*f*x + a^2*c*f)*
cos_integral(2*(d*f*x + c*f)/d) + (a^2*d*f*x + a^2*c*f)*cos_integral(-2*(d*
f*x + c*f)/d))*sin(-2*(d*e - c*f)/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{2\sin\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{\sin^2\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{1}{c^2+2cdx+d^2x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*sin(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(s
in(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**2 + 2*c*d*
x + d**2*x**2), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.106 $\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{a^2f^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3}+\frac{a^2f^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{d^3}-\frac{a^2f^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf\right.}{d^3}$$

```
[Out] (a^2*f^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/d^3 - (a^2*f^
2*CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d^3 - (4*a^2*f*Cos[e/2 + Pi/
4 + (f*x)/2]*Sin[e/2 + Pi/4 + (f*x)/2]^3)/(d^2*(c + d*x)) - (2*a^2*Sin[e/2
+ Pi/4 + (f*x)/2]^4)/(d*(c + d*x)^2) - (a^2*f^2*Cos[e - (c*f)/d]*SinIntegra
l[(c*f)/d + f*x])/d^3 - (a^2*f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d
 + 2*f*x])/d^3
```

Rubi [A] time = 0.506181, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3318, 3314, 3309, 31, 3303, 3299, 3302, 3312}

$$-\frac{a^2f^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3}+\frac{a^2f^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{d^3}-\frac{a^2f^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf\right.}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*x)^3,x]
```



```
[Out] (a^2*f^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/d^3 - (a^2*f^
2*CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d^3 - (4*a^2*f*Cos[e/2 + Pi/
4 + (f*x)/2]*Sin[e/2 + Pi/4 + (f*x)/2]^3)/(d^2*(c + d*x)) - (2*a^2*Sin[e/2
+ Pi/4 + (f*x)/2]^4)/(d*(c + d*x)^2) - (a^2*f^2*Cos[e - (c*f)/d]*SinIntegra
l[(c*f)/d + f*x])/d^3 - (a^2*f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d
 + 2*f*x])/d^3
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+dx)^3}\,dx = \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)}{(c+dx)^3}\,dx$$

$$= -\frac{4a^2f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{2a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} + \frac{\left(6a^2f^2\right)\int \frac{\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f}{}\right.}{c+dx}}{d^2}$$

$$= -\frac{4a^2f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{2a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} + \frac{\left(3a^2f^2\right)\int \frac{1}{c+dx}\,dx}{d^2} -$$

$$= -\frac{4a^2f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{2a^2\sin^4\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} + \frac{\left(a^2f^2\right)\int \frac{\cos(2e+2fx)}{c+dx}\,d}{d^2}$$

$$= \frac{3a^2f^2\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3} - \frac{4a^2f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{2a^2\sin^4\left(\frac{e}{2}+\right.}{d(c+dx}$$

$$= \frac{a^2f^2\cos\left(2e-\frac{2cf}{d}\right)\text{Ci}\left(\frac{2cf}{d}+2fx\right)}{d^3} - \frac{a^2f^2\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3} - \frac{4a^2f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right.}{d^2(c+}$$

Mathematica [A] time = 0.900699, size = 353, normalized size = 1.57

$$-\frac{a^2\left(4c^2f^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2f(c+dx)}{d}\right)+4c^2f^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)+4f^2(c+dx)^2\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] -(a^2*(3*d^2 + 4*c*d*f*Cos[e + f*x] + 4*d^2*f*x*Cos[e + f*x] - d^2*Cos[2*(e
 + f*x)] - 4*f^2*(c + d*x)^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*f*(c + d*x
))/d] + 4*f^2*(c + d*x)^2*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + 4*d^2
*Sin[e + f*x] + 2*c*d*f*Sin[2*(e + f*x)] + 2*d^2*f*x*Sin[2*(e + f*x)] + 4*c
^2*f^2*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + 8*c*d*f^2*x*Cos[e - (c*f
)/d]*SinIntegral[f*(c/d + x)] + 4*d^2*f^2*x^2*Cos[e - (c*f)/d]*SinIntegral[
f*(c/d + x)] + 4*c^2*f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d
] + 8*c*d*f^2*x*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 4*d^2
*f^2*x^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d]))/(4*d^3*(c +
d*x)^2)
```

Maple [A] time = 0.023, size = 347, normalized size = 1.5

$$\frac{1}{f}\left(-\frac{3\,a^2f^3}{4\,\left(\left(fx+e\right)d+cf-de\right)^2d}-\frac{a^2f^3}{4}\left(-\frac{\cos\left(2\,fx+2\,e\right)}{\left(\left(fx+e\right)d+cf-de\right)^2d}-\frac{1}{d}\left(-2\,\frac{\sin\left(2\,fx+2\,e\right)}{\left(\left(fx+e\right)d+cf-de\right)d}+2\,\frac{1}{d}\left(2\,\frac{1}{d}\mathrm{Si}\left(2\,f\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(d*x+c)^3,x)
```

```
[Out] 1/f*(-3/4*a^2*f^3/((f*x+e)*d+c*f-d*e)^2/d-1/4*a^2*f^3*(-cos(2*f*x+2*e)/((f*
x+e)*d+c*f-d*e)^2/d-(-2*sin(2*f*x+2*e)/((f*x+e)*d+c*f-d*e)/d+2*(2*Si(2*f*x+
2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2
*(c*f-d*e)/d)/d)/d)/d)+2*a^2*f^3*(-1/2*sin(f*x+e)/((f*x+e)*d+c*f-d*e)^2/d+1
/2*(-cos(f*x+e)/((f*x+e)*d+c*f-d*e)/d-(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/
d)/d-Ci(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)/d)/d))
```

Maxima [C] time = 1.95089, size = 641, normalized size = 2.85

$$-\frac{\frac{32\,a^2f^3}{\left(fx+e\right)^2d^3+d^3e^2-2\,cd^2ef+c^2df^2-2\,\left(d^3e-cd^2f\right)\left(fx+e\right)}-\frac{64\left(f^3\left(-i\,E_3\left(\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)+i\,E_3\left(-\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f^3\left(E_3\left(\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)+\right.\right.}{\left(fx+e\right)^2d^3+d^3e^2-2\,cd^2ef+c^2df^2-2\,\left(d^3e-cd^2f\right)\left(fx+e\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(32*a^2*f^3/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*
(d^3*e - c*d^2*f)*(f*x + e)) - 64*(f^3*(-I*exp_integral_e(3, (I*(f*x + e)*d
 - I*d*e + I*c*f)/d) + I*exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)
/d))*cos(-(d*e - c*f)/d) + f^3*(exp_integral_e(3, (I*(f*x + e)*d - I*d*e +
I*c*f)/d) + exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*sin(-(d*
e - c*f)/d))*a^2/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*(
d^3*e - c*d^2*f)*(f*x + e)) - (16*f^3*(exp_integral_e(3, (2*I*(f*x + e)*d -
 2*I*d*e + 2*I*c*f)/d) + exp_integral_e(3, -(2*I*(f*x + e)*d - 2*I*d*e + 2*
I*c*f)/d))*cos(-2*(d*e - c*f)/d) + f^3*(16*I*exp_integral_e(3, (2*I*(f*x +
e)*d - 2*I*d*e + 2*I*c*f)/d) - 16*I*exp_integral_e(3, -(2*I*(f*x + e)*d - 2
*I*d*e + 2*I*c*f)/d))*sin(-2*(d*e - c*f)/d) - 16*f^3)*a^2/((f*x + e)^2*d^3
+ d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*(d^3*e - c*d^2*f)*(f*x + e)))/f
```

Fricas [B] time = 2.07093, size = 1049, normalized size = 4.66

$$\frac{a^2d^2\cos\left(fx+e\right)^2 - 2\,a^2d^2 + 2\left(a^2d^2f^2x^2 + 2\,a^2cdf^2x + a^2c^2f^2\right)\sin\left(-\frac{2\left(de-cf\right)}{d}\right)\mathrm{Si}\left(\frac{2\left(dfx+cf\right)}{d}\right) - 2\left(a^2d^2f^2x^2 + 2\,a^2cdf^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*d^2*cos(f*x + e)^2 - 2*a^2*d^2 + 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^
2*x + a^2*c^2*f^2)*sin(-2*(d*e - c*f)/d)*sin_integral(2*(d*f*x + c*f)/d) -
2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*cos(-(d*e - c*f)/d)*sin
_integral((d*f*x + c*f)/d) - 2*(a^2*d^2*f*x + a^2*c*d*f)*cos(f*x + e) + ((a
^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*cos_integral(2*(d*f*x + c*f
)/d) + (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*cos_integral(-2*(d
*f*x + c*f)/d))*cos(-2*(d*e - c*f)/d) - 2*(a^2*d^2 + (a^2*d^2*f*x + a^2*c*d
*f)*cos(f*x + e))*sin(f*x + e) + ((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*
c^2*f^2)*cos_integral((d*f*x + c*f)/d) + (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x
 + a^2*c^2*f^2)*cos_integral(-(d*f*x + c*f)/d))*sin(-(d*e - c*f)/d))/(d^5*x
^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{2\sin\left(e+fx\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int\frac{\sin^2\left(e+fx\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int\frac{1}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(d*x+c)**3,x)
```

```
[Out] a**2*(Integral(2*sin(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**
3), x) + Integral(sin(e + f*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3), x) + Integral(1/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.107 $\int \frac{(c+dx)^3}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=148

$$-\frac{12id^2(c+dx)\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{af^3}+\frac{12d^3\text{PolyLog}\left(3,ie^{i(e+fx)}\right)}{af^4}+\frac{6d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)}{af^2}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{f}{2}\cdots\right.}{af}$$

```
[Out] ((-I)*(c + d*x)^3)/(a*f) - ((c + d*x)^3*Cot[e/2 + Pi/4 + (f*x)/2])/(a*f) +
(6*d*(c + d*x)^2*Log[1 - I*E^(I*(e + f*x))])/(a*f^2) - ((12*I)*d^2*(c + d*x
)*PolyLog[2, I*E^(I*(e + f*x))])/(a*f^3) + (12*d^3*PolyLog[3, I*E^(I*(e + f
*x))])/(a*f^4)
```

Rubi [A] time = 0.306161, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3318, 4184, 3717, 2190, 2531, 2282, 6589}

$$-\frac{12id^2(c+dx)\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{af^3}+\frac{12d^3\text{PolyLog}\left(3,ie^{i(e+fx)}\right)}{af^4}+\frac{6d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)}{af^2}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{f}{2}\cdots\right.}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^3)/(a*f) - ((c + d*x)^3*Cot[e/2 + Pi/4 + (f*x)/2])/(a*f) +
(6*d*(c + d*x)^2*Log[1 - I*E^(I*(e + f*x))])/(a*f^2) - ((12*I)*d^2*(c + d*x
)*PolyLog[2, I*E^(I*(e + f*x))])/(a*f^3) + (12*d^3*PolyLog[3, I*E^(I*(e + f
*x))])/(a*f^4)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
```

```
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+a\sin(e+fx)}\,dx = \frac{\int (c+dx)^3\csc^2\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)dx}{2a}$$

$$= -\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{(3d)\int (c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)dx}{af}$$

$$= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{(6d)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)^2}{1-ie^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}dx}{af}$$

$$= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)}{af^2} - \frac{\left(12d^2\right)\int (c+dx)\,\mathrm{l}}{}$$

$$= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\mathrm{Li}_2\left(\right.}{af^3}$$

$$= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\mathrm{Li}_2\left(\right.}{af^3}$$

$$= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\mathrm{Li}_2\left(\right.}{af^3}$$

Mathematica [A] time = 1.03841, size = 126, normalized size = 0.85

$$\frac{-12id^2f(c+dx)\mathrm{PolyLog}\left(2,ie^{i(e+fx)}\right)+12d^3\mathrm{PolyLog}\left(3,ie^{i(e+fx)}\right)+f^2(c+dx)^2\left(f(c+dx)\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)-if(\right.}{af^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((-12*I)*d^2*f*(c + d*x)*PolyLog[2, I*E^(I*(e + f*x))] + 12*d^3*PolyLog[3,
I*E^(I*(e + f*x))] + f^2*(c + d*x)^2*((-I)*f*(c + d*x) + 6*d*Log[1 - I*E^(I
*(e + f*x))] + f*(c + d*x)*Tan[(2*e - Pi + 2*f*x)/4]))/(a*f^4)
```

Maple [B] time = 0.141, size = 484, normalized size = 3.3

$$-2\frac{d^3x^3+3cd^2x^2+3c^2dx+c^3}{af\left(e^{i(fx+e)}+i\right)}+6\frac{d^3\ln\left(1-ie^{i(fx+e)}\right)x^2}{af^2}-6\frac{d^3\ln\left(1-ie^{i(fx+e)}\right)e^2}{f^4a}+6\frac{\ln\left(e^{i(fx+e)}+i\right)c^2d}{af^2}+6\frac{d^3e^2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+a*sin(f*x+e)),x)
```

```
[Out] -2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(I*(f*x+e))+I)+6/f^2/a*d^3*l
n(1-I*exp(I*(f*x+e)))*x^2-6/f^4/a*d^3*ln(1-I*exp(I*(f*x+e)))*e^2+6/f^2/a*ln
(exp(I*(f*x+e))+I)*c^2*d+6/f^4/a*d^3*e^2*ln(exp(I*(f*x+e))+I)+6*I/f^3/a*d^3
*e^2*x-6*I/f^3/a*c*d^2*e^2-12*I/f^3/a*d^3*polylog(2,I*exp(I*(f*x+e)))*x-6/f
^4/a*d^3*e^2*ln(exp(I*(f*x+e)))+12*d^3*polylog(3,I*exp(I*(f*x+e)))/a/f^4-12
*I/f^2/a*c*d^2*e*x-6*I/f/a*c*d^2*x^2-6/f^2/a*ln(exp(I*(f*x+e)))*c^2*d+12/f^
3/a*c*d^2*e*ln(exp(I*(f*x+e)))-12/f^3/a*c*d^2*e*ln(exp(I*(f*x+e))+I)-2*I/f/
a*d^3*x^3+4*I/f^4/a*d^3*e^3-12*I/f^3/a*c*d^2*polylog(2,I*exp(I*(f*x+e)))+12
/f^2/a*c*d^2*ln(1-I*exp(I*(f*x+e)))*x+12/f^3/a*c*d^2*ln(1-I*exp(I*(f*x+e)))
*e
```

Maxima [B] time = 1.47734, size = 1315, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] (6*(2*(f*x + e)*cos(f*x + e) - (cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x
 + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1))*c*d^2
*e/(a*f^2*cos(f*x + e)^2 + a*f^2*sin(f*x + e)^2 + 2*a*f^2*sin(f*x + e) + a*
f^2) - 6*c*d^2*e^2/(a*f^2 + a*f^2*sin(f*x + e)/(cos(f*x + e) + 1)) - 3*(2*(
f*x + e)*cos(f*x + e) - (cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) +
 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1))*c^2*d/(a*f*c
os(f*x + e)^2 + a*f*sin(f*x + e)^2 + 2*a*f*sin(f*x + e) + a*f) + 6*c^2*d*e/
(a*f + a*f*sin(f*x + e)/(cos(f*x + e) + 1)) - 2*c^3/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1)) + (-2*I*d^3*e^3 + (6*d^3*e^2*cos(f*x + e) + 6*I*d^3*e^2*si
n(f*x + e) + 6*I*d^3*e^2)*arctan2(sin(f*x + e) + 1, cos(f*x + e)) + (-6*I*(
f*x + e)^2*d^3 + (12*I*d^3*e - 12*I*c*d^2*f)*(f*x + e) - 6*((f*x + e)^2*d^3
 - 2*(d^3*e - c*d^2*f)*(f*x + e))*cos(f*x + e) + (-6*I*(f*x + e)^2*d^3 + (1
```

```
2*I*d^3*e - 12*I*c*d^2*f)*(f*x + e))*sin(f*x + e))*arctan2(cos(f*x + e), si
n(f*x + e) + 1) - 2*((f*x + e)^3*d^3 + 3*(f*x + e)*d^3*e^2 - 3*(d^3*e - c*d
^2*f)*(f*x + e)^2)*cos(f*x + e) + (-12*I*(f*x + e)*d^3 + 12*I*d^3*e - 12*I*
c*d^2*f - 12*((f*x + e)*d^3 - d^3*e + c*d^2*f)*cos(f*x + e) + (-12*I*(f*x +
 e)*d^3 + 12*I*d^3*e - 12*I*c*d^2*f)*sin(f*x + e))*dilog(I*e^(I*f*x + I*e))
 + (3*(f*x + e)^2*d^3 + 3*d^3*e^2 - 6*(d^3*e - c*d^2*f)*(f*x + e) + (-3*I*(
f*x + e)^2*d^3 - 3*I*d^3*e^2 + (6*I*d^3*e - 6*I*c*d^2*f)*(f*x + e))*cos(f*x
 + e) + 3*((f*x + e)^2*d^3 + d^3*e^2 - 2*(d^3*e - c*d^2*f)*(f*x + e))*sin(f
*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) + (-12*I
*d^3*cos(f*x + e) + 12*d^3*sin(f*x + e) + 12*d^3)*polylog(3, I*e^(I*f*x + I
*e)) + (-2*I*(f*x + e)^3*d^3 - 6*I*(f*x + e)*d^3*e^2 + (6*I*d^3*e - 6*I*c*d
^2*f)*(f*x + e)^2)*sin(f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e
) + a*f^3))/f
```

Fricas [C] time = 2.02191, size = 2134, normalized size = 14.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3 + (d^3*f^3*x^3 +
3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3)*cos(f*x + e) - (-6*I*d^3*f*x - 6
*I*c*d^2*f + (-6*I*d^3*f*x - 6*I*c*d^2*f)*cos(f*x + e) + (-6*I*d^3*f*x - 6*
I*c*d^2*f)*sin(f*x + e))*dilog(I*cos(f*x + e) - sin(f*x + e)) - (6*I*d^3*f*
x + 6*I*c*d^2*f + (6*I*d^3*f*x + 6*I*c*d^2*f)*cos(f*x + e) + (6*I*d^3*f*x +
 6*I*c*d^2*f)*sin(f*x + e))*dilog(-I*cos(f*x + e) - sin(f*x + e)) - 3*(d^3*
e^2 - 2*c*d^2*e*f + c^2*d*f^2 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*cos(f*x
 + e) + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*sin(f*x + e))*log(cos(f*x + e)
+ I*sin(f*x + e) + I) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*
e*f + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*cos(f*x + e) +
(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*sin(f*x + e))*log(I*c
os(f*x + e) + sin(f*x + e) + 1) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2
+ 2*c*d^2*e*f + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*cos(f
*x + e) + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*sin(f*x + e
))*log(-I*cos(f*x + e) + sin(f*x + e) + 1) - 3*(d^3*e^2 - 2*c*d^2*e*f + c^2
*d*f^2 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*cos(f*x + e) + (d^3*e^2 - 2*c*
d^2*e*f + c^2*d*f^2)*sin(f*x + e))*log(-cos(f*x + e) + I*sin(f*x + e) + I)
- 6*(d^3*cos(f*x + e) + d^3*sin(f*x + e) + d^3)*polylog(3, I*cos(f*x + e) -
 sin(f*x + e)) - 6*(d^3*cos(f*x + e) + d^3*sin(f*x + e) + d^3)*polylog(3, -
I*cos(f*x + e) - sin(f*x + e)) - (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f
```

```
^3*x + c^3*f^3)*sin(f*x + e))/(a*f^4*cos(f*x + e) + a*f^4*sin(f*x + e) + a*
f^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^3}{\sin(e+fx)+1}\,dx + \int \frac{d^3x^3}{\sin(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\sin(e+fx)+1}\,dx + \int \frac{3c^2dx}{\sin(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(c**3/(sin(e + f*x) + 1), x) + Integral(d**3*x**3/(sin(e + f*x) +
1), x) + Integral(3*c*d**2*x**2/(sin(e + f*x) + 1), x) + Integral(3*c**2*d*
x/(sin(e + f*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{a\sin(fx+e)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(a*sin(f*x + e) + a), x)
```

3.108 $\int \frac{(c+dx)^2}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=113

$$-\frac{4id^2\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{af^3}+\frac{4d(c+dx)\log\left(1-ie^{i(e+fx)}\right)}{af^2}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{af}-\frac{i(c+dx)^2}{af}$$

```
[Out] ((-I)*(c + d*x)^2)/(a*f) - ((c + d*x)^2*Cot[e/2 + Pi/4 + (f*x)/2])/(a*f) +
(4*d*(c + d*x)*Log[1 - I*E^(I*(e + f*x))])/(a*f^2) - ((4*I)*d^2*PolyLog[2,
I*E^(I*(e + f*x))])/(a*f^3)
```

Rubi [A] time = 0.21833, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3318, 4184, 3717, 2190, 2279, 2391}

$$-\frac{4id^2\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{af^3}+\frac{4d(c+dx)\log\left(1-ie^{i(e+fx)}\right)}{af^2}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{af}-\frac{i(c+dx)^2}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^2)/(a*f) - ((c + d*x)^2*Cot[e/2 + Pi/4 + (f*x)/2])/(a*f) +
(4*d*(c + d*x)*Log[1 - I*E^(I*(e + f*x))])/(a*f^2) - ((4*I)*d^2*PolyLog[2,
I*E^(I*(e + f*x))])/(a*f^3)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+a\sin(e+fx)}\,dx = \frac{\int (c+dx)^2\csc^2\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)dx}{2a}$$
$$= -\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{(2d)\int (c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)dx}{af}$$
$$= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{(4d)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)}{1-ie^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}dx}{af}$$
$$= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{4d(c+dx)\log\left(1-ie^{i(e+fx)}\right)}{af^2}-\frac{\left(4d^2\right)\int \log\left(1-ie^{2}\right.}{af^2}$$
$$= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{4d(c+dx)\log\left(1-ie^{i(e+fx)}\right)}{af^2}+\frac{\left(4id^2\right)\text{Subst}\left(\int \frac{\log}{}\right.}{a}$$
$$= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{4d(c+dx)\log\left(1-ie^{i(e+fx)}\right)}{af^2}-\frac{4id^2\text{Li}_2\left(ie^{i(e+fx)}\right)}{af^3}$$

Mathematica [A] time = 0.654299, size = 94, normalized size = 0.83

$$\frac{f(c+dx)\left(f(c+dx)\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)-if(c+dx)+4d\log\left(1-ie^{i(e+fx)}\right)\right)-4id^2\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{af^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((-4*I)*d^2*PolyLog[2, I*E^(I*(e + f*x))] + f*(c + d*x)*((-I)*f*(c + d*x) +
 4*d*Log[1 - I*E^(I*(e + f*x))] + f*(c + d*x)*Tan[(2*e - Pi + 2*f*x)/4]))/(
a*f^3)
```

Maple [B] time = 0.07, size = 254, normalized size = 2.3

$$-2\,\frac{d^2x^2+2\,cdx+c^2}{af\left(\mathrm{e}^{i(fx+e)}+i\right)}+4\,\frac{\ln\left(\mathrm{e}^{i(fx+e)}+i\right)cd}{af^2}-4\,\frac{\ln\left(\mathrm{e}^{i(fx+e)}\right)cd}{af^2}-\frac{2\,id^2x^2}{af}-\frac{4\,id^2ex}{af^2}-\frac{2\,id^2e^2}{f^3a}+4\,\frac{d^2\ln\left(1-i\mathrm{e}^{i(fx+e)}\right)x}{af^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+a*sin(f*x+e)),x)
```

```
[Out] -2*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(I*(f*x+e))+I)+4/f^2/a*ln(exp(I*(f*x+e))+I
)*c*d-4/f^2/a*ln(exp(I*(f*x+e)))*c*d-2*I/f/a*d^2*x^2-4*I/f^2/a*d^2*e*x-2*I/
f^3/a*d^2*e^2+4/f^2/a*d^2*ln(1-I*exp(I*(f*x+e)))*x+4/f^3/a*d^2*ln(1-I*exp(I
*(f*x+e)))*e-4*I*d^2*polylog(2,I*exp(I*(f*x+e)))/a/f^3-4/f^3/a*d^2*e*ln(exp
(I*(f*x+e))+I)+4/f^3/a*d^2*e*ln(exp(I*(f*x+e)))
```

Maxima [B] time = 1.35275, size = 421, normalized size = 3.73

$$\frac{2i\,c^2f^2 + \left(4\,cdf\cos\left(fx+e\right) + 4i\,cdf\sin\left(fx+e\right) + 4i\,cdf\right)\arctan\left(\sin\left(fx+e\right)+1,\cos\left(fx+e\right)\right) - \left(4\,d^2fx\cos\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] (2*I*c^2*f^2 + (4*c*d*f*cos(f*x + e) + 4*I*c*d*f*sin(f*x + e) + 4*I*c*d*f)*
arctan2(sin(f*x + e) + 1, cos(f*x + e)) - (4*d^2*f*x*cos(f*x + e) + 4*I*d^2
*f*x*sin(f*x + e) + 4*I*d^2*f*x)*arctan2(cos(f*x + e), sin(f*x + e) + 1) -
2*(d^2*f^2*x^2 + 2*c*d*f^2*x)*cos(f*x + e) - (4*d^2*cos(f*x + e) + 4*I*d^2*
sin(f*x + e) + 4*I*d^2)*dilog(I*e^(I*f*x + I*e)) + (2*d^2*f*x + 2*c*d*f + (
-2*I*d^2*f*x - 2*I*c*d*f)*cos(f*x + e) + 2*(d^2*f*x + c*d*f)*sin(f*x + e))*
log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) + (-2*I*d^2*f^2*x
^2 - 4*I*c*d*f^2*x)*sin(f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x +
e) + a*f^3)
```

Fricas [B] time = 1.81435, size = 1196, normalized size = 10.58

$$-\frac{d^2f^2x^2 + 2\,cdf^2x + c^2f^2 + \left(d^2f^2x^2 + 2\,cdf^2x + c^2f^2\right)\cos\left(fx+e\right) - \left(-2i\,d^2\cos\left(fx+e\right) - 2i\,d^2\sin\left(fx+e\right) - 2i\,d^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^
2)*cos(f*x + e) - (-2*I*d^2*cos(f*x + e) - 2*I*d^2*sin(f*x + e) - 2*I*d^2)*
dilog(I*cos(f*x + e) - sin(f*x + e)) - (2*I*d^2*cos(f*x + e) + 2*I*d^2*sin(
f*x + e) + 2*I*d^2)*dilog(-I*cos(f*x + e) - sin(f*x + e)) + 2*(d^2*e - c*d*
f + (d^2*e - c*d*f)*cos(f*x + e) + (d^2*e - c*d*f)*sin(f*x + e))*log(cos(f*
x + e) + I*sin(f*x + e) + I) - 2*(d^2*f*x + d^2*e + (d^2*f*x + d^2*e)*cos(f
*x + e) + (d^2*f*x + d^2*e)*sin(f*x + e))*log(I*cos(f*x + e) + sin(f*x + e)
 + 1) - 2*(d^2*f*x + d^2*e + (d^2*f*x + d^2*e)*cos(f*x + e) + (d^2*f*x + d^
2*e)*sin(f*x + e))*log(-I*cos(f*x + e) + sin(f*x + e) + 1) + 2*(d^2*e - c*d
*f + (d^2*e - c*d*f)*cos(f*x + e) + (d^2*e - c*d*f)*sin(f*x + e))*log(-cos(
f*x + e) + I*sin(f*x + e) + I) - (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*sin(
f*x + e))/(a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) + a*f^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^2}{\sin(e+fx)+1}\,dx + \int \frac{d^2x^2}{\sin(e+fx)+1}\,dx + \int \frac{2cdx}{\sin(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(c**2/(sin(e + f*x) + 1), x) + Integral(d**2*x**2/(sin(e + f*x) +
1), x) + Integral(2*c*d*x/(sin(e + f*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(a*sin(f*x + e) + a), x)
```

3.109 $\int \frac{c+dx}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=60

$$\frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\right)}{af^2}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{af}$$

```
[Out] -(((c + d*x)*Cot[e/2 + Pi/4 + (f*x)/2])/(a*f)) + (2*d*Log[Sin[e/2 + Pi/4 +
(f*x)/2]])/(a*f^2)
```

Rubi [A] time = 0.0636577, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3318, 4184, 3475}

$$\frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\right)}{af^2}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(((c + d*x)*Cot[e/2 + Pi/4 + (f*x)/2])/(a*f)) + (2*d*Log[Sin[e/2 + Pi/4 +
(f*x)/2]])/(a*f^2)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{a+a\sin(e+fx)}\,dx &= \frac{\int (c+dx)\csc^2\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)dx}{2a}\\ &= -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{d\int \cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)dx}{af}\\ &= -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\right)}{af^2}\end{aligned}$$

Mathematica [A] time = 0.149789, size = 51, normalized size = 0.85

$$\frac{f(c+dx)\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)+2d\log\left(\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)}{af^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (2*d*Log[Cos[(2*e - Pi + 2*f*x)/4]] + f*(c + d*x)*Tan[(2*e - Pi + 2*f*x)/4]
)/(a*f^2)
```

Maple [B] time = 0.043, size = 122, normalized size = 2.

$$-2\,\frac{c}{af\left(\tan\left(1/2\,fx+e/2\right)+1\right)}+\frac{dx}{af}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1}-\frac{dx}{af}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1}+2\,\frac{d\ln\left(\tan\left(1/2\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+a*sin(f*x+e)),x)
```

```
[Out] -2/a*c/f/(tan(1/2*f*x+1/2*e)+1)+1/a*d/(tan(1/2*f*x+1/2*e)+1)*x/f*tan(1/2*f*
x+1/2*e)-1/a*d/(tan(1/2*f*x+1/2*e)+1)*x/f+2/a*d/f^2*ln(tan(1/2*f*x+1/2*e)+1
```

```
)-1/a*d/f^2*ln(1+tan(1/2*f*x+1/2*e)^2)
```

Maxima [B] time = 0.985481, size = 228, normalized size = 3.8

$$-\frac{\frac{\left(2\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}+2\sin\left(fx+e\right)+1\right)\log\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}+2\sin\left(fx+e\right)+1\right)\right)d}{af\cos\left(fx+e\right)^{2}+af\sin\left(fx+e\right)^{2}+2af\sin\left(fx+e\right)+af}-\frac{2de}{af+\frac{af\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}+\frac{2c}{a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -((2*(f*x + e)*cos(f*x + e) - (cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x
+ e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1))*d/(a*f
*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 + 2*a*f*sin(f*x + e) + a*f) - 2*d*e/(a
*f + a*f*sin(f*x + e)/(cos(f*x + e) + 1)) + 2*c/(a + a*sin(f*x + e)/(cos(f*
x + e) + 1)))/f
```

Fricas [B] time = 1.77034, size = 251, normalized size = 4.18

$$-\frac{dfx+cf+\left(dfx+cf\right)\cos\left(fx+e\right)-\left(d\cos\left(fx+e\right)+d\sin\left(fx+e\right)+d\right)\log\left(\sin\left(fx+e\right)+1\right)-\left(dfx+cf\right)\sin\left(f\right.}{af^{2}\cos\left(fx+e\right)+af^{2}\sin\left(fx+e\right)+af^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d*f*x + c*f + (d*f*x + c*f)*cos(f*x + e) - (d*cos(f*x + e) + d*sin(f*x +
e) + d)*log(sin(f*x + e) + 1) - (d*f*x + c*f)*sin(f*x + e))/(a*f^2*cos(f*x
+ e) + a*f^2*sin(f*x + e) + a*f^2)
```

Sympy [A] time = 1.12534, size = 272, normalized size = 4.53

$$\begin{cases}-\frac{2cf}{af^{2}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^{2}}+\frac{dfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af^{2}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^{2}}-\frac{dfx}{af^{2}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^{2}}+\frac{2d\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af^{2}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^{2}}+\frac{2d\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af^{2}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af^{2}}-\frac{d\log\left(\right.}{} \\ \frac{cx+\frac{dx^{2}}{2}}{a\sin\left(e\right)+a} & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*c*f/(a*f**2*tan(e/2 + f*x/2) + a*f**2) + d*f*x*tan(e/2 + f*x/
2)/(a*f**2*tan(e/2 + f*x/2) + a*f**2) - d*f*x/(a*f**2*tan(e/2 + f*x/2) + a*
f**2) + 2*d*log(tan(e/2 + f*x/2) + 1)*tan(e/2 + f*x/2)/(a*f**2*tan(e/2 + f*
x/2) + a*f**2) + 2*d*log(tan(e/2 + f*x/2) + 1)/(a*f**2*tan(e/2 + f*x/2) + a
*f**2) - d*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/2)/(a*f**2*tan(e/2 +
f*x/2) + a*f**2) - d*log(tan(e/2 + f*x/2)**2 + 1)/(a*f**2*tan(e/2 + f*x/2)
+ a*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*sin(e) + a), True))
```

Giac [B] time = 1.24122, size = 940, normalized size = 15.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -(d*f*x*tan(1/2*f*x)*tan(1/2*e) + d*f*x*tan(1/2*f*x) + d*f*x*tan(1/2*e) + c
*f*tan(1/2*f*x)*tan(1/2*e) - d*log(2*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan
(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + t
an(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/
2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*
tan(1/2*e) + 1))*tan(1/2*f*x)*tan(1/2*e) - d*f*x + c*f*tan(1/2*f*x) + d*log
(2*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1
/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*t
an(1/2*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*
x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1))*tan(1/2*f*x) + c*
f*tan(1/2*e) + d*log(2*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*
tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4
+ 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2
*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1
))*tan(1/2*e) - c*f + d*log(2*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)
^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*
f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*
tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2
*e) + 1)))/(a*f^2*tan(1/2*f*x)*tan(1/2*e) - a*f^2*tan(1/2*f*x) - a*f^2*tan(
1/2*e) - a*f^2)
```

3.110 $\int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a\sin(e+fx)+a)},x\right)$$

[Out] Unintegrable[1/((c + d*x)*(a + a*Sin[e + f*x])), x]

Rubi [A] time = 0.0606529, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d*x)*(a + a*Sin[e + f*x])),x]

[Out] Defer[Int][1/((c + d*x)*(a + a*Sin[e + f*x])), x]

Rubi steps

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx$$

Mathematica [A] time = 4.76525, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d*x)*(a + a*Sin[e + f*x])),x]

[Out] Integrate[1/((c + d*x)*(a + a*Sin[e + f*x])), x]

Maple [A] time = 0.133, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a+a\sin\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+a*sin(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a+a*sin(f*x+e)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{adx+ac+(adx+ac)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c + (a*d*x + a*c)*sin(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c\sin\left(e+fx\right)+c+dx\sin\left(e+fx\right)+dx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral(1/(c*sin(e + f*x) + c + d*x*sin(e + f*x) + d*x), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(a*sin(f*x + e) + a)), x)
```

3.111 $\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a\sin(e+fx)+a)},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a + a*Sin[e + f*x])), x]
```

Rubi [A] time = 0.0651236, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + a*Sin[e + f*x])),x]

[Out] Defer[Int][1/((c + d*x)^2*(a + a*Sin[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx$$

Mathematica [A] time = 4.61851, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + a*Sin[e + f*x])),x]

[Out] Integrate[1/((c + d*x)^2*(a + a*Sin[e + f*x])), x]
```

Maple [A] time = 0.325, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2 \left(a + a\sin\left(fx+e\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+a*sin(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a+a*sin(f*x+e)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{ad^2x^2 + 2\,acdx + ac^2 + \left(ad^2x^2 + 2\,acdx + ac^2\right)\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (a*d^2*x^2 + 2*a*c*d*x + a*c^2)
*sin(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c^2\sin(e+fx)+c^2+2cdx\sin(e+fx)+2cdx+d^2x^2\sin(e+fx)+d^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral(1/(c**2*sin(e + f*x) + c**2 + 2*c*d*x*sin(e + f*x) + 2*c*d*x + d**
2*x**2*sin(e + f*x) + d**2*x**2), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(a*sin(f*x + e) + a)), x)
```

3.112 $\int \frac{(c+dx)^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=309

$$-\frac{4id^2(c+dx)\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{a^2f^3}+\frac{4d^3\text{PolyLog}\left(3,ie^{i(e+fx)}\right)}{a^2f^4}-\frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{a^2f^3}+\frac{2d(c+dx)^2\log\left(1-\right.}{a^2f^2}$$

```
[Out] ((-I/3)*(c + d*x)^3)/(a^2*f) - (2*d^2*(c + d*x)*Cot[e/2 + Pi/4 + (f*x)/2])/
(a^2*f^3) - ((c + d*x)^3*Cot[e/2 + Pi/4 + (f*x)/2])/(3*a^2*f) - (d*(c + d*x
)^2*Csc[e/2 + Pi/4 + (f*x)/2]^2)/(2*a^2*f^2) - ((c + d*x)^3*Cot[e/2 + Pi/4
+ (f*x)/2]*Csc[e/2 + Pi/4 + (f*x)/2]^2)/(6*a^2*f) + (2*d*(c + d*x)^2*Log[1
- I*E^(I*(e + f*x))])/(a^2*f^2) + (4*d^3*Log[Sin[e/2 + Pi/4 + (f*x)/2]])/(a
^2*f^4) - ((4*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(e + f*x))])/(a^2*f^3) + (
4*d^3*PolyLog[3, I*E^(I*(e + f*x))])/(a^2*f^4)
```

Rubi [A] time = 0.376733, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {3318, 4186, 4184, 3475, 3717, 2190, 2531, 2282, 6589}

$$-\frac{4id^2(c+dx)\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{a^2f^3}+\frac{4d^3\text{PolyLog}\left(3,ie^{i(e+fx)}\right)}{a^2f^4}-\frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{a^2f^3}+\frac{2d(c+dx)^2\log\left(1-\right.}{a^2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + a*Sin[e + f*x])^2,x]
```


```
[Out] ((-I/3)*(c + d*x)^3)/(a^2*f) - (2*d^2*(c + d*x)*Cot[e/2 + Pi/4 + (f*x)/2])/
(a^2*f^3) - ((c + d*x)^3*Cot[e/2 + Pi/4 + (f*x)/2])/(3*a^2*f) - (d*(c + d*x
)^2*Csc[e/2 + Pi/4 + (f*x)/2]^2)/(2*a^2*f^2) - ((c + d*x)^3*Cot[e/2 + Pi/4
+ (f*x)/2]*Csc[e/2 + Pi/4 + (f*x)/2]^2)/(6*a^2*f) + (2*d*(c + d*x)^2*Log[1
- I*E^(I*(e + f*x))])/(a^2*f^2) + (4*d^3*Log[Sin[e/2 + Pi/4 + (f*x)/2]])/(a
^2*f^4) - ((4*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(e + f*x))])/(a^2*f^3) + (
4*d^3*PolyLog[3, I*E^(I*(e + f*x))])/(a^2*f^4)
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
```

```
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+a\sin(e+fx))^2}\,dx = \frac{\int (c+dx)^3 \csc^4\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)dx}{4a^2}$$

$$= -\frac{d(c+dx)^2\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2a^2f^2} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f} + \frac{\int (c+dx)}{}$$

$$= -\frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{a^2f^3} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx)^2\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2a^2f^2}$$

$$= -\frac{i(c+dx)^3}{3a^2f} - \frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{a^2f^3} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx)^2\csc^2}{2a^2}$$

$$= -\frac{i(c+dx)^3}{3a^2f} - \frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{a^2f^3} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx)^2\csc^2}{2a^2}$$

$$= -\frac{i(c+dx)^3}{3a^2f} - \frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{a^2f^3} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx)^2\csc^2}{2a^2}$$

$$= -\frac{i(c+dx)^3}{3a^2f} - \frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{a^2f^3} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx)^2\csc^2}{2a^2}$$

$$= -\frac{i(c+dx)^3}{3a^2f} - \frac{2d^2(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{a^2f^3} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx)^2\csc^2}{2a^2}$$

Mathematica [A] time = 1.90441, size = 257, normalized size = 0.83

$$\frac{\frac{24d^2\left(d\text{PolyLog}\left(3,ie^{i(e+fx)}\right)-if(c+dx)\text{PolyLog}\left(2,ie^{i(e+fx)}\right)\right)}{f^2} + \frac{12d^2(c+dx)\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)}{f} + 12d(c+dx)^2\log\left(1-ie^{i(e+fx)}\right) + 2f(c+dx)^3\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((-2*I)*f*(c + d*x)^3 + 12*d*(c + d*x)^2*Log[1 - I*E^(I*(e + f*x))] + (24*d
^3*Log[Cos[(2*e - Pi + 2*f*x)/4]])/f^2 + (24*d^2*((-I)*f*(c + d*x)*PolyLog[
2, I*E^(I*(e + f*x))] + d*PolyLog[3, I*E^(I*(e + f*x))]))/f^2 - 3*d*(c + d*
x)^2*Sec[(2*e - Pi + 2*f*x)/4]^2 + (12*d^2*(c + d*x)*Tan[(2*e - Pi + 2*f*x)
```

```
/4])/f + 2*f*(c + d*x)^3*Tan[(2*e - Pi + 2*f*x)/4] + f*(c + d*x)^3*Sec[(2*e
 - Pi + 2*f*x)/4]^2*Tan[(2*e - Pi + 2*f*x)/4])/(6*a^2*f^2)
```

Maple [B] time = 0.641, size = 807, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -4*I/f^3/a^2*polylog(2,I*exp(I*(f*x+e)))*d^3*x-4*I/f^2/a^2*c*d^2*e*x+2/f^4/
a^2*ln(exp(I*(f*x+e))+I)*d^3*e^2-2/f^4/a^2*ln(1-I*exp(I*(f*x+e)))*d^3*e^2-2
/f^4/a^2*ln(exp(I*(f*x+e)))*d^3*e^2+2/f^2/a^2*ln(exp(I*(f*x+e))+I)*c^2*d-2/
f^2/a^2*ln(exp(I*(f*x+e)))*c^2*d+2*I/f^3/a^2*d^3*e^2*x-2*I/f^3/a^2*c*d^2*e^
2-4/f^3/a^2*ln(exp(I*(f*x+e))+I)*c*d^2*e+4/f^3/a^2*ln(exp(I*(f*x+e)))*c*d^2
*e+4/f^2/a^2*ln(1-I*exp(I*(f*x+e)))*c*d^2*x+4/f^3/a^2*ln(1-I*exp(I*(f*x+e))
)*c*d^2*e+2/f^2/a^2*ln(1-I*exp(I*(f*x+e)))*d^3*x^2-2/3*I/f/a^2*d^3*x^3-2*I/
f/a^2*c*d^2*x^2+4/3*I/f^4/a^2*d^3*e^3-2/3*I*(6*I*c*d^2+3*d^3*f^2*x^3*exp(I*
(f*x+e))+I*c^3*f^2+3*I*c^2*d*f^2*x+9*c*d^2*f^2*x^2*exp(I*(f*x+e))+3*f*d^3*x
^2*exp(2*I*(f*x+e))+3*I*c*d^2*f^2*x^2-6*I*d^3*x*exp(2*I*(f*x+e))+I*d^3*f^2*
x^3+9*c^2*d*f^2*x*exp(I*(f*x+e))+6*f*c*d^2*x*exp(2*I*(f*x+e))+3*I*f*c^2*d*e
xp(I*(f*x+e))+6*I*d^3*x+3*I*f*d^3*x^2*exp(I*(f*x+e))+3*c^3*f^2*exp(I*(f*x+e
))+3*f*c^2*d*exp(2*I*(f*x+e))+6*I*f*c*d^2*x*exp(I*(f*x+e))+12*d^3*x*exp(I*(
f*x+e))-6*I*c*d^2*exp(2*I*(f*x+e))+12*c*d^2*exp(I*(f*x+e)))/(exp(I*(f*x+e))
+I)^3/f^3/a^2-4*I/f^3/a^2*c*d^2*polylog(2,I*exp(I*(f*x+e)))+4*d^3*polylog(3
,I*exp(I*(f*x+e)))/a^2/f^4+4/f^4/a^2*ln(exp(I*(f*x+e))+I)*d^3-4/f^4/a^2*ln(
exp(I*(f*x+e)))*d^3
```

Maxima [B] time = 4.04143, size = 4834, normalized size = 15.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(6*c*d^2*e^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 2)/(a^2*f^2 + 3*a^2*f^2*sin(f*x + e)/(cos(f*x + e) + 1)
```

```
 + 3*a^2*f^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*f^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3) + 6*(2*(f*x + 3*(f*x + e)*sin(f*x + e) + e + cos(f*x +
 e) + sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - 2*(9*(f*x + e)*cos(f*x + e) - 6*
sin(f*x + e) - 1)*cos(2*f*x + 2*e) - 6*cos(2*f*x + 2*e)^2 - 6*cos(f*x + e)^
2 - (6*(cos(f*x + e) + sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - cos(3*f*x + 3*e
)^2 + 6*(3*sin(f*x + e) + 1)*cos(2*f*x + 2*e) - 9*cos(2*f*x + 2*e)^2 - 9*co
s(f*x + e)^2 - 2*(3*cos(2*f*x + 2*e) - 3*sin(f*x + e) - 1)*sin(3*f*x + 3*e)
 - sin(3*f*x + 3*e)^2 - 18*cos(f*x + e)*sin(2*f*x + 2*e) - 9*sin(2*f*x + 2*
e)^2 - 9*sin(f*x + e)^2 - 6*sin(f*x + e) - 1)*log(cos(f*x + e)^2 + sin(f*x
+ e)^2 + 2*sin(f*x + e) + 1) - 2*(3*(f*x + e)*cos(f*x + e) + cos(2*f*x + 2*
e) - sin(f*x + e))*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*sin(f*x + e) + e
 + 2*cos(f*x + e))*sin(2*f*x + 2*e) - 6*sin(2*f*x + 2*e)^2 - 6*sin(f*x + e)
^2 - 2*sin(f*x + e))*c*d^2*e/(a^2*f^2*cos(3*f*x + 3*e)^2 + 9*a^2*f^2*cos(2*
f*x + 2*e)^2 + 9*a^2*f^2*cos(f*x + e)^2 + a^2*f^2*sin(3*f*x + 3*e)^2 + 18*a
^2*f^2*cos(f*x + e)*sin(2*f*x + 2*e) + 9*a^2*f^2*sin(2*f*x + 2*e)^2 + 9*a^2
*f^2*sin(f*x + e)^2 + 6*a^2*f^2*sin(f*x + e) + a^2*f^2 - 6*(a^2*f^2*cos(f*x
 + e) + a^2*f^2*sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - 6*(3*a^2*f^2*sin(f*x +
 e) + a^2*f^2)*cos(2*f*x + 2*e) + 2*(3*a^2*f^2*cos(2*f*x + 2*e) - 3*a^2*f^2
*sin(f*x + e) - a^2*f^2)*sin(3*f*x + 3*e)) - 6*c^2*d*e*(3*sin(f*x + e)/(cos
(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2*f + 3*a^2*
f*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*f*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + a^2*f*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 3*(2*(f*x + 3*(f*x + e)
*sin(f*x + e) + e + cos(f*x + e) + sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - 2*(
9*(f*x + e)*cos(f*x + e) - 6*sin(f*x + e) - 1)*cos(2*f*x + 2*e) - 6*cos(2*f
*x + 2*e)^2 - 6*cos(f*x + e)^2 - (6*(cos(f*x + e) + sin(2*f*x + 2*e))*cos(3
*f*x + 3*e) - cos(3*f*x + 3*e)^2 + 6*(3*sin(f*x + e) + 1)*cos(2*f*x + 2*e)
- 9*cos(2*f*x + 2*e)^2 - 9*cos(f*x + e)^2 - 2*(3*cos(2*f*x + 2*e) - 3*sin(f
*x + e) - 1)*sin(3*f*x + 3*e) - sin(3*f*x + 3*e)^2 - 18*cos(f*x + e)*sin(2*
f*x + 2*e) - 9*sin(2*f*x + 2*e)^2 - 9*sin(f*x + e)^2 - 6*sin(f*x + e) - 1)*
log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - 2*(3*(f*x + e)*
cos(f*x + e) + cos(2*f*x + 2*e) - sin(f*x + e))*sin(3*f*x + 3*e) - 6*(f*x +
 3*(f*x + e)*sin(f*x + e) + e + 2*cos(f*x + e))*sin(2*f*x + 2*e) - 6*sin(2*
f*x + 2*e)^2 - 6*sin(f*x + e)^2 - 2*sin(f*x + e))*c^2*d/(a^2*f*cos(3*f*x +
3*e)^2 + 9*a^2*f*cos(2*f*x + 2*e)^2 + 9*a^2*f*cos(f*x + e)^2 + a^2*f*sin(3*
f*x + 3*e)^2 + 18*a^2*f*cos(f*x + e)*sin(2*f*x + 2*e) + 9*a^2*f*sin(2*f*x +
 2*e)^2 + 9*a^2*f*sin(f*x + e)^2 + 6*a^2*f*sin(f*x + e) + a^2*f - 6*(a^2*f*
cos(f*x + e) + a^2*f*sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - 6*(3*a^2*f*sin(f*
x + e) + a^2*f)*cos(2*f*x + 2*e) + 2*(3*a^2*f*cos(2*f*x + 2*e) - 3*a^2*f*si
n(f*x + e) - a^2*f)*sin(3*f*x + 3*e)) + 2*c^3*(3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e
)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3) - 3*(2*I*d^3*e^3 + 12*I*d^3*e - 12*I*c*d^2
*f + (-6*I*d^3*e^2 - 12*I*d^3 + 6*(d^3*e^2 + 2*d^3)*cos(3*f*x + 3*e) + (18*
I*d^3*e^2 + 36*I*d^3)*cos(2*f*x + 2*e) - 18*(d^3*e^2 + 2*d^3)*cos(f*x + e)
+ (6*I*d^3*e^2 + 12*I*d^3)*sin(3*f*x + 3*e) - 18*(d^3*e^2 + 2*d^3)*sin(2*f*
```

```
x + 2*e) + (-18*I*d^3*e^2 - 36*I*d^3)*sin(f*x + e))*arctan2(sin(f*x + e) +
1, cos(f*x + e)) + (6*I*(f*x + e)^2*d^3 + (-12*I*d^3*e + 12*I*c*d^2*f)*(f*x
 + e) - 6*((f*x + e)^2*d^3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*cos(3*f*x + 3*e
) + (-18*I*(f*x + e)^2*d^3 + (36*I*d^3*e - 36*I*c*d^2*f)*(f*x + e))*cos(2*f
*x + 2*e) + 18*((f*x + e)^2*d^3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*cos(f*x +
e) + (-6*I*(f*x + e)^2*d^3 + (12*I*d^3*e - 12*I*c*d^2*f)*(f*x + e))*sin(3*f
*x + 3*e) + 18*((f*x + e)^2*d^3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*sin(2*f*x
+ 2*e) + (18*I*(f*x + e)^2*d^3 + (-36*I*d^3*e + 36*I*c*d^2*f)*(f*x + e))*si
n(f*x + e))*arctan2(cos(f*x + e), sin(f*x + e) + 1) - 2*((f*x + e)^3*d^3 -
3*(d^3*e - c*d^2*f)*(f*x + e)^2 + 3*(d^3*e^2 + 2*d^3)*(f*x + e))*cos(3*f*x
+ 3*e) + (-6*I*(f*x + e)^3*d^3 - 6*d^3*e^2 - 12*I*d^3*e + 12*I*c*d^2*f + (1
8*I*d^3*e - 18*I*c*d^2*f - 6*d^3)*(f*x + e)^2 + (-18*I*d^3*e^2 + 12*d^3*e -
 12*c*d^2*f - 24*I*d^3)*(f*x + e))*cos(2*f*x + 2*e) + (6*d^3*e^3 - 6*I*(f*x
 + e)^2*d^3 - 6*I*d^3*e^2 + 24*d^3*e - 24*c*d^2*f + (12*I*d^3*e - 12*I*c*d^
2*f + 12*d^3)*(f*x + e))*cos(f*x + e) + (12*I*(f*x + e)*d^3 - 12*I*d^3*e +
12*I*c*d^2*f - 12*((f*x + e)*d^3 - d^3*e + c*d^2*f)*cos(3*f*x + 3*e) + (-36
*I*(f*x + e)*d^3 + 36*I*d^3*e - 36*I*c*d^2*f)*cos(2*f*x + 2*e) + 36*((f*x +
 e)*d^3 - d^3*e + c*d^2*f)*cos(f*x + e) + (-12*I*(f*x + e)*d^3 + 12*I*d^3*e
 - 12*I*c*d^2*f)*sin(3*f*x + 3*e) + 36*((f*x + e)*d^3 - d^3*e + c*d^2*f)*si
n(2*f*x + 2*e) + (36*I*(f*x + e)*d^3 - 36*I*d^3*e + 36*I*c*d^2*f)*sin(f*x +
 e))*dilog(I*e^(I*f*x + I*e)) - (3*(f*x + e)^2*d^3 + 3*d^3*e^2 + 6*d^3 - 6*
(d^3*e - c*d^2*f)*(f*x + e) - (-3*I*(f*x + e)^2*d^3 - 3*I*d^3*e^2 - 6*I*d^3
 + (6*I*d^3*e - 6*I*c*d^2*f)*(f*x + e))*cos(3*f*x + 3*e) - 9*((f*x + e)^2*d
^3 + d^3*e^2 + 2*d^3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*cos(2*f*x + 2*e) - (9
*I*(f*x + e)^2*d^3 + 9*I*d^3*e^2 + 18*I*d^3 + (-18*I*d^3*e + 18*I*c*d^2*f)*
(f*x + e))*cos(f*x + e) - 3*((f*x + e)^2*d^3 + d^3*e^2 + 2*d^3 - 2*(d^3*e -
 c*d^2*f)*(f*x + e))*sin(3*f*x + 3*e) - (9*I*(f*x + e)^2*d^3 + 9*I*d^3*e^2
+ 18*I*d^3 + (-18*I*d^3*e + 18*I*c*d^2*f)*(f*x + e))*sin(2*f*x + 2*e) + 9*(
(f*x + e)^2*d^3 + d^3*e^2 + 2*d^3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*sin(f*x
+ e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) + (-12*I*d^
3*cos(3*f*x + 3*e) + 36*d^3*cos(2*f*x + 2*e) + 36*I*d^3*cos(f*x + e) + 12*d
^3*sin(3*f*x + 3*e) + 36*I*d^3*sin(2*f*x + 2*e) - 36*d^3*sin(f*x + e) - 12*
d^3)*polylog(3, I*e^(I*f*x + I*e)) + (-2*I*(f*x + e)^3*d^3 + (6*I*d^3*e - 6
*I*c*d^2*f)*(f*x + e)^2 + (-6*I*d^3*e^2 - 12*I*d^3)*(f*x + e))*sin(3*f*x +
3*e) + (6*(f*x + e)^3*d^3 - 6*I*d^3*e^2 + 12*d^3*e - 12*c*d^2*f - (18*d^3*e
 - 18*c*d^2*f + 6*I*d^3)*(f*x + e)^2 + (18*d^3*e^2 + 12*I*d^3*e - 12*I*c*d^
2*f + 24*d^3)*(f*x + e))*sin(2*f*x + 2*e) + (6*I*d^3*e^3 + 6*(f*x + e)^2*d^
3 + 6*d^3*e^2 + 24*I*d^3*e - 24*I*c*d^2*f - (12*d^3*e - 12*c*d^2*f - 12*I*d
^3)*(f*x + e))*sin(f*x + e))/(-3*I*a^2*f^3*cos(3*f*x + 3*e) + 9*a^2*f^3*cos
(2*f*x + 2*e) + 9*I*a^2*f^3*cos(f*x + e) + 3*a^2*f^3*sin(3*f*x + 3*e) + 9*I
*a^2*f^3*sin(2*f*x + 2*e) - 9*a^2*f^3*sin(f*x + e) - 3*a^2*f^3))/f
```

Fricas [C] time = 2.69471, size = 3872, normalized size = 12.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(d^3*f^3*x^3 + c^3*f^3 + 3*c^2*d*f^2 + 3*(c*d^2*f^3 + d^3*f^2)*x^2 + (d
^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + c^3*f^3 + 6*c*d^2*f + 3*(c^2*d*f^3 + 2*d^3*f
)*x)*cos(f*x + e)^2 + 3*(c^2*d*f^3 + 2*c*d^2*f^2)*x + (2*d^3*f^3*x^3 + 2*c^
3*f^3 + 3*c^2*d*f^2 + 6*c*d^2*f + 3*(2*c*d^2*f^3 + d^3*f^2)*x^2 + 6*(c^2*d*
f^3 + c*d^2*f^2 + d^3*f)*x)*cos(f*x + e) - (-12*I*d^3*f*x - 12*I*c*d^2*f +
(6*I*d^3*f*x + 6*I*c*d^2*f)*cos(f*x + e)^2 + (-6*I*d^3*f*x - 6*I*c*d^2*f)*c
os(f*x + e) + (-12*I*d^3*f*x - 12*I*c*d^2*f + (-6*I*d^3*f*x - 6*I*c*d^2*f)*
cos(f*x + e))*sin(f*x + e))*dilog(I*cos(f*x + e) - sin(f*x + e)) - (12*I*d^
3*f*x + 12*I*c*d^2*f + (-6*I*d^3*f*x - 6*I*c*d^2*f)*cos(f*x + e)^2 + (6*I*d
^3*f*x + 6*I*c*d^2*f)*cos(f*x + e) + (12*I*d^3*f*x + 12*I*c*d^2*f + (6*I*d^
3*f*x + 6*I*c*d^2*f)*cos(f*x + e))*sin(f*x + e))*dilog(-I*cos(f*x + e) - si
n(f*x + e)) - 3*(2*d^3*e^2 - 4*c*d^2*e*f + 2*c^2*d*f^2 + 4*d^3 - (d^3*e^2 -
 2*c*d^2*e*f + c^2*d*f^2 + 2*d^3)*cos(f*x + e)^2 + (d^3*e^2 - 2*c*d^2*e*f +
 c^2*d*f^2 + 2*d^3)*cos(f*x + e) + (2*d^3*e^2 - 4*c*d^2*e*f + 2*c^2*d*f^2 +
 4*d^3 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 + 2*d^3)*cos(f*x + e))*sin(f*x
+ e))*log(cos(f*x + e) + I*sin(f*x + e) + I) - 3*(2*d^3*f^2*x^2 + 4*c*d^2*f
^2*x - 2*d^3*e^2 + 4*c*d^2*e*f - (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2
*c*d^2*e*f)*cos(f*x + e)^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d
^2*e*f)*cos(f*x + e) + (2*d^3*f^2*x^2 + 4*c*d^2*f^2*x - 2*d^3*e^2 + 4*c*d^2
*e*f + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*cos(f*x + e))*
sin(f*x + e))*log(I*cos(f*x + e) + sin(f*x + e) + 1) - 3*(2*d^3*f^2*x^2 + 4
*c*d^2*f^2*x - 2*d^3*e^2 + 4*c*d^2*e*f - (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3
*e^2 + 2*c*d^2*e*f)*cos(f*x + e)^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2
 + 2*c*d^2*e*f)*cos(f*x + e) + (2*d^3*f^2*x^2 + 4*c*d^2*f^2*x - 2*d^3*e^2 +
 4*c*d^2*e*f + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*cos(f*
x + e))*sin(f*x + e))*log(-I*cos(f*x + e) + sin(f*x + e) + 1) - 3*(2*d^3*e^
2 - 4*c*d^2*e*f + 2*c^2*d*f^2 + 4*d^3 - (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2
+ 2*d^3)*cos(f*x + e)^2 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 + 2*d^3)*cos(f
*x + e) + (2*d^3*e^2 - 4*c*d^2*e*f + 2*c^2*d*f^2 + 4*d^3 + (d^3*e^2 - 2*c*d
^2*e*f + c^2*d*f^2 + 2*d^3)*cos(f*x + e))*sin(f*x + e))*log(-cos(f*x + e) +
 I*sin(f*x + e) + I) + 6*(d^3*cos(f*x + e)^2 - d^3*cos(f*x + e) - 2*d^3 - (
d^3*cos(f*x + e) + 2*d^3)*sin(f*x + e))*polylog(3, I*cos(f*x + e) - sin(f*x
 + e)) + 6*(d^3*cos(f*x + e)^2 - d^3*cos(f*x + e) - 2*d^3 - (d^3*cos(f*x +
e) + 2*d^3)*sin(f*x + e))*polylog(3, -I*cos(f*x + e) - sin(f*x + e)) - (d^3
*f^3*x^3 + c^3*f^3 - 3*c^2*d*f^2 + 3*(c*d^2*f^3 - d^3*f^2)*x^2 + 3*(c^2*d*f
^3 - 2*c*d^2*f^2)*x - (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + c^3*f^3 + 6*c*d^2*f
```

```
+ 3*(c^2*d*f^3 + 2*d^3*f)*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f^4*cos(f*x +
 e)^2 - a^2*f^4*cos(f*x + e) - 2*a^2*f^4 - (a^2*f^4*cos(f*x + e) + 2*a^2*f^
4)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^3}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx + \int \frac{d^3x^3}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx + \int \frac{3c^2dx}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*sin(f*x+e))**2,x)
```

```
[Out] (Integral(c**3/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(d**3*x
**3/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(3*c*d**2*x**2/(si
n(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(3*c**2*d*x/(sin(e + f*x)
**2 + 2*sin(e + f*x) + 1), x))/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{\left(a\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(a*sin(f*x + e) + a)^2, x)
```

3.113 $\int \frac{(c+dx)^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=243

$$-\frac{4id^2\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{3a^2f^3}+\frac{4d(c+dx)\log\left(1-ie^{i(e+fx)}\right)}{3a^2f^2}-\frac{d(c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3a^2f^2}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3a^2f}-$$

```
[Out] ((-I/3)*(c + d*x)^2)/(a^2*f) - (2*d^2*Cot[e/2 + Pi/4 + (f*x)/2])/(3*a^2*f^3
) - ((c + d*x)^2*Cot[e/2 + Pi/4 + (f*x)/2])/(3*a^2*f) - (d*(c + d*x)*Csc[e/
2 + Pi/4 + (f*x)/2]^2)/(3*a^2*f^2) - ((c + d*x)^2*Cot[e/2 + Pi/4 + (f*x)/2]
*Csc[e/2 + Pi/4 + (f*x)/2]^2)/(6*a^2*f) + (4*d*(c + d*x)*Log[1 - I*E^(I*(e
+ f*x))])/(3*a^2*f^2) - (((4*I)/3)*d^2*PolyLog[2, I*E^(I*(e + f*x))])/(a^2*
f^3)
```

Rubi [A] time = 0.287215, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {3318, 4186, 3767, 8, 4184, 3717, 2190, 2279, 2391}

$$-\frac{4id^2\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{3a^2f^3}+\frac{4d(c+dx)\log\left(1-ie^{i(e+fx)}\right)}{3a^2f^2}-\frac{d(c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3a^2f^2}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3a^2f}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + a*Sin[e + f*x])^2,x]
```


```
[Out] ((-I/3)*(c + d*x)^2)/(a^2*f) - (2*d^2*Cot[e/2 + Pi/4 + (f*x)/2])/(3*a^2*f^3
) - ((c + d*x)^2*Cot[e/2 + Pi/4 + (f*x)/2])/(3*a^2*f) - (d*(c + d*x)*Csc[e/
2 + Pi/4 + (f*x)/2]^2)/(3*a^2*f^2) - ((c + d*x)^2*Cot[e/2 + Pi/4 + (f*x)/2]
*Csc[e/2 + Pi/4 + (f*x)/2]^2)/(6*a^2*f) + (4*d*(c + d*x)*Log[1 - I*E^(I*(e
+ f*x))])/(3*a^2*f^2) - (((4*I)/3)*d^2*PolyLog[2, I*E^(I*(e + f*x))])/(a^2*
f^3)
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+a\sin(e+fx))^2}\,dx = \frac{\int (c+dx)^2\csc^4\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)dx}{4a^2}$$
$$= -\frac{d(c+dx)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f} + \frac{\int (c+dx)^2 \cdots}{\cdots}$$
$$= -\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\csc \cdots}{6a^2f}$$
$$= -\frac{i(c+dx)^2}{3a^2f} - \frac{2d^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^3} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\cdots\right)}{3a^2f^2}$$
$$= -\frac{i(c+dx)^2}{3a^2f} - \frac{2d^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^3} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\cdots\right)}{3a^2f^2}$$
$$= -\frac{i(c+dx)^2}{3a^2f} - \frac{2d^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^3} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\cdots\right)}{3a^2f^2}$$
$$= -\frac{i(c+dx)^2}{3a^2f} - \frac{2d^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f^3} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f} - \frac{d(c+dx)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\cdots\right)}{3a^2f^2}$$

Mathematica [A] time = 2.20246, size = 175, normalized size = 0.72

$$\frac{-8id^2\text{PolyLog}\left(2,ie^{i(e+fx)}\right)+2\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+2\right)\right)\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)-2if(c+dx)\left(f(c+dx)+4id\log\cdots\right.}{6a^2f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((-2*I)*f*(c + d*x)*(f*(c + d*x) + (4*I)*d*Log[1 - I*E^(I*(e + f*x))]) - (8
*I)*d^2*PolyLog[2, I*E^(I*(e + f*x))] + 2*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 +
 f^2*x^2))*Tan[(2*e - Pi + 2*f*x)/4] + f*(c + d*x)*Sec[(2*e - Pi + 2*f*x)/4
]^2*(-2*d + f*(c + d*x)*Tan[(2*e - Pi + 2*f*x)/4]))/(6*a^2*f^3)
```

Maple [B] time = 0.495, size = 421, normalized size = 1.7

$$-\frac{\frac{2i}{3}\left(id^2f^2x^2+3d^2f^2x^2\mathrm{e}^{i(fx+e)}+2icdf^2x+2ifd^2x\mathrm{e}^{i(fx+e)}+6cdf^2x\mathrm{e}^{i(fx+e)}+2fd^2x\mathrm{e}^{2i(fx+e)}+ic^2f^2+2ifcd\mathrm{e}^{i(fx+e)}\right.}{\left(\mathrm{e}^{i(fx+e)}+i\right)^3f^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/3*I*(I*d^2*f^2*x^2+3*d^2*f^2*x^2*exp(I*(f*x+e))+2*I*c*d*f^2*x+2*I*f*d^2*
x*exp(I*(f*x+e))+6*c*d*f^2*x*exp(I*(f*x+e))+2*f*d^2*x*exp(2*I*(f*x+e))+I*c^
2*f^2+2*I*f*c*d*exp(I*(f*x+e))-2*I*d^2*exp(2*I*(f*x+e))+3*c^2*f^2*exp(I*(f*
x+e))+2*f*c*d*exp(2*I*(f*x+e))+2*I*d^2+4*d^2*exp(I*(f*x+e)))/(exp(I*(f*x+e)
)+I)^3/f^3/a^2+4/3/f^2/a^2*ln(exp(I*(f*x+e))+I)*c*d-4/3/f^2/a^2*ln(exp(I*(f
*x+e)))*c*d-2/3*I/f/a^2*d^2*x^2-4/3*I/f^2/a^2*d^2*e*x-2/3*I/f^3/a^2*d^2*e^2
+4/3/f^2/a^2*d^2*ln(1-I*exp(I*(f*x+e)))*x+4/3/f^3/a^2*d^2*ln(1-I*exp(I*(f*x
+e)))*e-4/3*I*d^2*polylog(2,I*exp(I*(f*x+e)))/a^2/f^3-4/3/f^3/a^2*d^2*e*ln(
exp(I*(f*x+e))+I)+4/3/f^3/a^2*d^2*e*ln(exp(I*(f*x+e)))
```

Maxima [B] time = 2.21389, size = 1123, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (-2*I*c^2*f^2 - 4*I*d^2 + (4*c*d*f*cos(3*f*x + 3*e) + 12*I*c*d*f*cos(2*f*x
+ 2*e) - 12*c*d*f*cos(f*x + e) + 4*I*c*d*f*sin(3*f*x + 3*e) - 12*c*d*f*sin(
2*f*x + 2*e) - 12*I*c*d*f*sin(f*x + e) - 4*I*c*d*f)*arctan2(sin(f*x + e) +
1, cos(f*x + e)) - (4*d^2*f*x*cos(3*f*x + 3*e) + 12*I*d^2*f*x*cos(2*f*x + 2
*e) - 12*d^2*f*x*cos(f*x + e) + 4*I*d^2*f*x*sin(3*f*x + 3*e) - 12*d^2*f*x*s
in(2*f*x + 2*e) - 12*I*d^2*f*x*sin(f*x + e) - 4*I*d^2*f*x)*arctan2(cos(f*x
```

```
+ e), sin(f*x + e) + 1) - 2*(d^2*f^2*x^2 + 2*c*d*f^2*x)*cos(3*f*x + 3*e) +
(-6*I*d^2*f^2*x^2 - 4*c*d*f + 4*I*d^2 - 4*(3*I*c*d*f^2 + d^2*f)*x)*cos(2*f*
x + 2*e) - (6*c^2*f^2 + 4*I*d^2*f*x + 4*I*c*d*f + 8*d^2)*cos(f*x + e) - (4*
d^2*cos(3*f*x + 3*e) + 12*I*d^2*cos(2*f*x + 2*e) - 12*d^2*cos(f*x + e) + 4*
I*d^2*sin(3*f*x + 3*e) - 12*d^2*sin(2*f*x + 2*e) - 12*I*d^2*sin(f*x + e) -
4*I*d^2)*dilog(I*e^(I*f*x + I*e)) - (2*d^2*f*x + 2*c*d*f - (-2*I*d^2*f*x -
2*I*c*d*f)*cos(3*f*x + 3*e) - 6*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e) - (6*I*d
^2*f*x + 6*I*c*d*f)*cos(f*x + e) - 2*(d^2*f*x + c*d*f)*sin(3*f*x + 3*e) - (
6*I*d^2*f*x + 6*I*c*d*f)*sin(2*f*x + 2*e) + 6*(d^2*f*x + c*d*f)*sin(f*x + e
))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) + (-2*I*d^2*f^
2*x^2 - 4*I*c*d*f^2*x)*sin(3*f*x + 3*e) + (6*d^2*f^2*x^2 - 4*I*c*d*f - 4*d^
2 + (12*c*d*f^2 - 4*I*d^2*f)*x)*sin(2*f*x + 2*e) + (-6*I*c^2*f^2 + 4*d^2*f*
x + 4*c*d*f - 8*I*d^2)*sin(f*x + e))/(-3*I*a^2*f^3*cos(3*f*x + 3*e) + 9*a^2
*f^3*cos(2*f*x + 2*e) + 9*I*a^2*f^3*cos(f*x + e) + 3*a^2*f^3*sin(3*f*x + 3*
e) + 9*I*a^2*f^3*sin(2*f*x + 2*e) - 9*a^2*f^3*sin(f*x + e) - 3*a^2*f^3)
```

Fricas [B] time = 2.06389, size = 2074, normalized size = 8.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(d^2*f^2*x^2 + c^2*f^2 + 2*c*d*f + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2
 + 2*d^2)*cos(f*x + e)^2 + 2*(c*d*f^2 + d^2*f)*x + 2*(d^2*f^2*x^2 + c^2*f^2
 + c*d*f + d^2 + (2*c*d*f^2 + d^2*f)*x)*cos(f*x + e) - (2*I*d^2*cos(f*x + e
)^2 - 2*I*d^2*cos(f*x + e) - 4*I*d^2 + (-2*I*d^2*cos(f*x + e) - 4*I*d^2)*si
n(f*x + e))*dilog(I*cos(f*x + e) - sin(f*x + e)) - (-2*I*d^2*cos(f*x + e)^2
 + 2*I*d^2*cos(f*x + e) + 4*I*d^2 + (2*I*d^2*cos(f*x + e) + 4*I*d^2)*sin(f*
x + e))*dilog(-I*cos(f*x + e) - sin(f*x + e)) + 2*(2*d^2*e - 2*c*d*f - (d^2
*e - c*d*f)*cos(f*x + e)^2 + (d^2*e - c*d*f)*cos(f*x + e) + (2*d^2*e - 2*c*
d*f + (d^2*e - c*d*f)*cos(f*x + e))*sin(f*x + e))*log(cos(f*x + e) + I*sin(
f*x + e) + I) - 2*(2*d^2*f*x + 2*d^2*e - (d^2*f*x + d^2*e)*cos(f*x + e)^2 +
 (d^2*f*x + d^2*e)*cos(f*x + e) + (2*d^2*f*x + 2*d^2*e + (d^2*f*x + d^2*e)*
cos(f*x + e))*sin(f*x + e))*log(I*cos(f*x + e) + sin(f*x + e) + 1) - 2*(2*d
^2*f*x + 2*d^2*e - (d^2*f*x + d^2*e)*cos(f*x + e)^2 + (d^2*f*x + d^2*e)*cos
(f*x + e) + (2*d^2*f*x + 2*d^2*e + (d^2*f*x + d^2*e)*cos(f*x + e))*sin(f*x
+ e))*log(-I*cos(f*x + e) + sin(f*x + e) + 1) + 2*(2*d^2*e - 2*c*d*f - (d^2
*e - c*d*f)*cos(f*x + e)^2 + (d^2*e - c*d*f)*cos(f*x + e) + (2*d^2*e - 2*c*
d*f + (d^2*e - c*d*f)*cos(f*x + e))*sin(f*x + e))*log(-cos(f*x + e) + I*sin
(f*x + e) + I) - (d^2*f^2*x^2 + c^2*f^2 - 2*c*d*f + 2*(c*d*f^2 - d^2*f)*x -
```

```
 (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*d^2)*cos(f*x + e))*sin(f*x + e))/
(a^2*f^3*cos(f*x + e)^2 - a^2*f^3*cos(f*x + e) - 2*a^2*f^3 - (a^2*f^3*cos(f
*x + e) + 2*a^2*f^3)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^2}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx + \int \frac{d^2x^2}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx + \int \frac{2cdx}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] (Integral(c**2/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(d**2*x
**2/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(2*c*d*x/(sin(e +
f*x)**2 + 2*sin(e + f*x) + 1), x))/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{(a\sin(fx+e)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(a*sin(f*x + e) + a)^2, x)
```

3.114 $\int \frac{c+dx}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=148

$$-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3a^2f}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{6a^2f}-\frac{d\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{6a^2f^2}+\frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right.\right.}{3a^2f^2}$$

```
[Out] -((c + d*x)*Cot[e/2 + Pi/4 + (f*x)/2])/(3*a^2*f) - (d*Csc[e/2 + Pi/4 + (f*x
)/2]^2)/(6*a^2*f^2) - ((c + d*x)*Cot[e/2 + Pi/4 + (f*x)/2]*Csc[e/2 + Pi/4 +
 (f*x)/2]^2)/(6*a^2*f) + (2*d*Log[Sin[e/2 + Pi/4 + (f*x)/2]])/(3*a^2*f^2)
```

Rubi [A] time = 0.0891686, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3318, 4185, 4184, 3475}

$$-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{3a^2f}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{6a^2f}-\frac{d\csc^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{6a^2f^2}+\frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right.\right.}{3a^2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((c + d*x)*Cot[e/2 + Pi/4 + (f*x)/2])/(3*a^2*f) - (d*Csc[e/2 + Pi/4 + (f*x
)/2]^2)/(6*a^2*f^2) - ((c + d*x)*Cot[e/2 + Pi/4 + (f*x)/2]*Csc[e/2 + Pi/4 +
 (f*x)/2]^2)/(6*a^2*f) + (2*d*Log[Sin[e/2 + Pi/4 + (f*x)/2]])/(3*a^2*f^2)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+a\sin(e+fx))^2}\,dx &= \frac{\int (c+dx)\csc^4\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)dx}{4a^2}\\ &= -\frac{d\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f}+\frac{\int (c+dx)\csc^2\left(\frac{e}{2}\right.}{6a^2}\\ &= -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f}-\frac{d\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\csc^2\left(\frac{e}{2}+\right.}{6a^2f}\\ &= -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3a^2f}-\frac{d\csc^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\csc^2\left(\frac{e}{2}+\right.}{6a^2f}\end{aligned}$$

Mathematica [A] time = 1.08169, size = 225, normalized size = 1.52

$$-\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{3}{2}(e+fx)\right)\left(2cf+2d\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)-de+dfx\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(d*Cos[(e + f*x)/2]*(2 + 3*e + 3*f*
x - 6*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + Cos[(3*(e + f*x))/2]*(-(d
*e) + 2*c*f + d*f*x + 2*d*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2*(d
+ 2*d*e - 3*c*f - d*f*x + d*Cos[e + f*x]*(e + f*x - 2*Log[Cos[(e + f*x)/2]
```

```
+ Sin[(e + f*x)/2]]) - 4*d*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[(e
 + f*x)/2]))/(6*a^2*f^2*(1 + Sin[e + f*x])^2)
```

Maple [B] time = 0.197, size = 233, normalized size = 1.6

$$-2\,\frac{c}{a^2f\left(\tan\left(1/2\,fx+e/2\right)+1\right)}-\frac{4\,c}{3\,a^2f}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-3}+2\,\frac{c}{a^2f\left(\tan\left(1/2\,fx+e/2\right)+1\right)^2}-\frac{2\,dx}{3\,a^2f}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/a^2*c/f/(tan(1/2*f*x+1/2*e)+1)-4/3/a^2*c/f/(tan(1/2*f*x+1/2*e)+1)^3+2/a^
2*c/f/(tan(1/2*f*x+1/2*e)+1)^2-2/3/a^2/(tan(1/2*f*x+1/2*e)+1)^3/f*x*d+2/3/a
^2/(tan(1/2*f*x+1/2*e)+1)^3*d/f^2*tan(1/2*f*x+1/2*e)+2/3/a^2/(tan(1/2*f*x+1
/2*e)+1)^3*d/f^2*tan(1/2*f*x+1/2*e)^2+2/3/a^2/(tan(1/2*f*x+1/2*e)+1)^3/f*x*
d*tan(1/2*f*x+1/2*e)^3+2/3/a^2*d/f^2*ln(tan(1/2*f*x+1/2*e)+1)-1/3/a^2*d/f^2
*ln(1+tan(1/2*f*x+1/2*e)^2)
```

Maxima [B] time = 1.05719, size = 1229, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(2*d*e*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 2)/(a^2*f + 3*a^2*f*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*f*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*f*sin(f*x + e)^3/(cos(f*x + e) +
1)^3) + (2*(f*x + 3*(f*x + e)*sin(f*x + e) + e + cos(f*x + e) + sin(2*f*x +
 2*e))*cos(3*f*x + 3*e) - 2*(9*(f*x + e)*cos(f*x + e) - 6*sin(f*x + e) - 1)
*cos(2*f*x + 2*e) - 6*cos(2*f*x + 2*e)^2 - 6*cos(f*x + e)^2 - (6*(cos(f*x +
 e) + sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - cos(3*f*x + 3*e)^2 + 6*(3*sin(f*
x + e) + 1)*cos(2*f*x + 2*e) - 9*cos(2*f*x + 2*e)^2 - 9*cos(f*x + e)^2 - 2*
(3*cos(2*f*x + 2*e) - 3*sin(f*x + e) - 1)*sin(3*f*x + 3*e) - sin(3*f*x + 3*
e)^2 - 18*cos(f*x + e)*sin(2*f*x + 2*e) - 9*sin(2*f*x + 2*e)^2 - 9*sin(f*x
+ e)^2 - 6*sin(f*x + e) - 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*
x + e) + 1) - 2*(3*(f*x + e)*cos(f*x + e) + cos(2*f*x + 2*e) - sin(f*x + e)
```

```
)*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*sin(f*x + e) + e + 2*cos(f*x + e)
)*sin(2*f*x + 2*e) - 6*sin(2*f*x + 2*e)^2 - 6*sin(f*x + e)^2 - 2*sin(f*x +
e))*d/(a^2*f*cos(3*f*x + 3*e)^2 + 9*a^2*f*cos(2*f*x + 2*e)^2 + 9*a^2*f*cos(
f*x + e)^2 + a^2*f*sin(3*f*x + 3*e)^2 + 18*a^2*f*cos(f*x + e)*sin(2*f*x + 2
*e) + 9*a^2*f*sin(2*f*x + 2*e)^2 + 9*a^2*f*sin(f*x + e)^2 + 6*a^2*f*sin(f*x
 + e) + a^2*f - 6*(a^2*f*cos(f*x + e) + a^2*f*sin(2*f*x + 2*e))*cos(3*f*x +
 3*e) - 6*(3*a^2*f*sin(f*x + e) + a^2*f)*cos(2*f*x + 2*e) + 2*(3*a^2*f*cos(
2*f*x + 2*e) - 3*a^2*f*sin(f*x + e) - a^2*f)*sin(3*f*x + 3*e)) - 2*c*(3*sin
(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(
a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [A] time = 1.68312, size = 495, normalized size = 3.34

$$\frac{dfx + \left(dfx + cf\right)\cos\left(fx+e\right)^2 + cf + \left(2\,dfx + 2\,cf + d\right)\cos\left(fx+e\right) + \left(d\cos\left(fx+e\right)^2 - d\cos\left(fx+e\right) - \left(d\cos\left(fx\right.\right.\right.}{3\left(a^2f^2\cos\left(fx+e\right)^2 - a^2f^2\cos\left(fx+e\right) - 2\,a^2f^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(d*f*x + (d*f*x + c*f)*cos(f*x + e)^2 + c*f + (2*d*f*x + 2*c*f + d)*cos
(f*x + e) + (d*cos(f*x + e)^2 - d*cos(f*x + e) - (d*cos(f*x + e) + 2*d)*sin
(f*x + e) - 2*d)*log(sin(f*x + e) + 1) - (d*f*x + c*f - (d*f*x + c*f)*cos(f
*x + e) - d)*sin(f*x + e) + d)/(a^2*f^2*cos(f*x + e)^2 - a^2*f^2*cos(f*x +
e) - 2*a^2*f^2 - (a^2*f^2*cos(f*x + e) + 2*a^2*f^2)*sin(f*x + e))
```

Sympy [A] time = 2.56087, size = 1246, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((6*c*f*tan(e/2 + f*x/2)**3/(9*a**2*f**2*tan(e/2 + f*x/2)**3 + 27*
a**2*f**2*tan(e/2 + f*x/2)**2 + 27*a**2*f**2*tan(e/2 + f*x/2) + 9*a**2*f**2
) - 6*c*f/(9*a**2*f**2*tan(e/2 + f*x/2)**3 + 27*a**2*f**2*tan(e/2 + f*x/2)*
```

```
*2 + 27*a**2*f**2*tan(e/2 + f*x/2) + 9*a**2*f**2) + 6*d*f*x*tan(e/2 + f*x/2
)**3/(9*a**2*f**2*tan(e/2 + f*x/2)**3 + 27*a**2*f**2*tan(e/2 + f*x/2)**2 +
27*a**2*f**2*tan(e/2 + f*x/2) + 9*a**2*f**2) - 6*d*f*x/(9*a**2*f**2*tan(e/2
 + f*x/2)**3 + 27*a**2*f**2*tan(e/2 + f*x/2)**2 + 27*a**2*f**2*tan(e/2 + f*
x/2) + 9*a**2*f**2) + 6*d*log(tan(e/2 + f*x/2) + 1)*tan(e/2 + f*x/2)**3/(9*
a**2*f**2*tan(e/2 + f*x/2)**3 + 27*a**2*f**2*tan(e/2 + f*x/2)**2 + 27*a**2*
f**2*tan(e/2 + f*x/2) + 9*a**2*f**2) + 18*d*log(tan(e/2 + f*x/2) + 1)*tan(e
/2 + f*x/2)**2/(9*a**2*f**2*tan(e/2 + f*x/2)**3 + 27*a**2*f**2*tan(e/2 + f*
x/2)**2 + 27*a**2*f**2*tan(e/2 + f*x/2) + 9*a**2*f**2) + 18*d*log(tan(e/2 +
 f*x/2) + 1)*tan(e/2 + f*x/2)/(9*a**2*f**2*tan(e/2 + f*x/2)**3 + 27*a**2*f*
*2*tan(e/2 + f*x/2)**2 + 27*a**2*f**2*tan(e/2 + f*x/2) + 9*a**2*f**2) + 6*d
*log(tan(e/2 + f*x/2) + 1)/(9*a**2*f**2*tan(e/2 + f*x/2)**3 + 27*a**2*f**2*
tan(e/2 + f*x/2)**2 + 27*a**2*f**2*tan(e/2 + f*x/2) + 9*a**2*f**2) - 3*d*lo
g(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/2)**3/(9*a**2*f**2*tan(e/2 + f*x/2
)**3 + 27*a**2*f**2*tan(e/2 + f*x/2)**2 + 27*a**2*f**2*tan(e/2 + f*x/2) + 9
*a**2*f**2) - 9*d*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/2)**2/(9*a**2*
f**2*tan(e/2 + f*x/2)**3 + 27*a**2*f**2*tan(e/2 + f*x/2)**2 + 27*a**2*f**2*
tan(e/2 + f*x/2) + 9*a**2*f**2) - 9*d*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2
+ f*x/2)/(9*a**2*f**2*tan(e/2 + f*x/2)**3 + 27*a**2*f**2*tan(e/2 + f*x/2)**
2 + 27*a**2*f**2*tan(e/2 + f*x/2) + 9*a**2*f**2) - 3*d*log(tan(e/2 + f*x/2)
**2 + 1)/(9*a**2*f**2*tan(e/2 + f*x/2)**3 + 27*a**2*f**2*tan(e/2 + f*x/2)**
2 + 27*a**2*f**2*tan(e/2 + f*x/2) + 9*a**2*f**2) - 2*d*tan(e/2 + f*x/2)**3/
(9*a**2*f**2*tan(e/2 + f*x/2)**3 + 27*a**2*f**2*tan(e/2 + f*x/2)**2 + 27*a*
*2*f**2*tan(e/2 + f*x/2) + 9*a**2*f**2) - 2*d/(9*a**2*f**2*tan(e/2 + f*x/2)
**3 + 27*a**2*f**2*tan(e/2 + f*x/2)**2 + 27*a**2*f**2*tan(e/2 + f*x/2) + 9*
a**2*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*sin(e) + a)**2, True))
```

Giac [B] time = 1.83268, size = 4177, normalized size = 28.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/3*(2*d*f*x*tan(1/2*f*x)^3*tan(1/2*e)^3 + 2*c*f*tan(1/2*f*x)^3*tan(1/2*e)
^3 - d*log(2*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*
x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/
2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*
tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1))*tan(1/2
*f*x)^3*tan(1/2*e)^3 - 6*d*f*x*tan(1/2*f*x)^2*tan(1/2*e)^2 + 3*d*log(2*(tan
(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) -
```

```
2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*
e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 +
tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1))*tan(1/2*f*x)^3*tan(1/2*e
)^2 + 3*d*log(2*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2
*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan
(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 +
 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1))*tan(
1/2*f*x)^2*tan(1/2*e)^3 + d*tan(1/2*f*x)^3*tan(1/2*e)^3 + 2*d*f*x*tan(1/2*f
*x)^3 + 6*d*f*x*tan(1/2*f*x)^2*tan(1/2*e) - 3*d*log(2*(tan(1/2*e)^2 + 1)/(t
an(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3
*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*
f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*
tan(1/2*f*x) + 2*tan(1/2*e) + 1))*tan(1/2*f*x)^3*tan(1/2*e) + 6*d*f*x*tan(1
/2*f*x)*tan(1/2*e)^2 - 6*c*f*tan(1/2*f*x)^2*tan(1/2*e)^2 - 3*d*log(2*(tan(1
/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2
*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)
^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + ta
n(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1))*tan(1/2*f*x)^2*tan(1/2*e)^
2 - d*tan(1/2*f*x)^3*tan(1/2*e)^2 + 2*d*f*x*tan(1/2*e)^3 - 3*d*log(2*(tan(1
/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2
*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)
^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + ta
n(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1))*tan(1/2*f*x)*tan(1/2*e)^3
- d*tan(1/2*f*x)^2*tan(1/2*e)^3 + 2*c*f*tan(1/2*f*x)^3 + d*log(2*(tan(1/2*e
)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan
(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 +
 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/
2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1))*tan(1/2*f*x)^3 + 6*d*f*x*tan(1
/2*f*x)*tan(1/2*e) + 6*c*f*tan(1/2*f*x)^2*tan(1/2*e) - 3*d*log(2*(tan(1/2*e
)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan
(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 +
 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/
2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1))*tan(1/2*f*x)^2*tan(1/2*e) + d*
tan(1/2*f*x)^3*tan(1/2*e) + 6*c*f*tan(1/2*f*x)*tan(1/2*e)^2 - 3*d*log(2*(ta
n(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e)
- 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2
*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 +
 tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1))*tan(1/2*f*x)*tan(1/2*e)
^2 - d*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*c*f*tan(1/2*e)^3 + d*log(2*(tan(1/2*
e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*ta
n(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2
+ 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1
/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1))*tan(1/2*e)^3 + d*tan(1/2*f*x)
*tan(1/2*e)^3 + 3*d*log(2*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 -
 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)
```

```
^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(
1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e)
+ 1))*tan(1/2*f*x)^2 - d*tan(1/2*f*x)^3 + 6*c*f*tan(1/2*f*x)*tan(1/2*e) + 3
*d*log(2*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4
*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*
x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(
1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1))*tan(1/2*f*x
)*tan(1/2*e) - d*tan(1/2*f*x)^2*tan(1/2*e) + 3*d*log(2*(tan(1/2*e)^2 + 1)/(
tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^
3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2
*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2
*tan(1/2*f*x) + 2*tan(1/2*e) + 1))*tan(1/2*e)^2 - d*tan(1/2*f*x)*tan(1/2*e)
^2 - d*tan(1/2*e)^3 - 2*d*f*x + 3*d*log(2*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^
4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*e)^
2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 - 2*t
an(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x)
 + 2*tan(1/2*e) + 1))*tan(1/2*f*x) - d*tan(1/2*f*x)^2 + 3*d*log(2*(tan(1/2*
e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*ta
n(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2
+ 2*tan(1/2*f*x)^3 - 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1
/2*e)^2 + 2*tan(1/2*f*x) + 2*tan(1/2*e) + 1))*tan(1/2*e) + d*tan(1/2*f*x)*t
an(1/2*e) - d*tan(1/2*e)^2 - 2*c*f + d*log(2*(tan(1/2*e)^2 + 1)/(tan(1/2*f*
x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^4*tan(1/2*e) - 2*tan(1/2*f*x)^3*tan(1/2*
e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^3 -
2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f
*x) + 2*tan(1/2*e) + 1)) - d*tan(1/2*f*x) - d*tan(1/2*e) - d)/(a^2*f^2*tan(
1/2*f*x)^3*tan(1/2*e)^3 - 3*a^2*f^2*tan(1/2*f*x)^3*tan(1/2*e)^2 - 3*a^2*f^2
*tan(1/2*f*x)^2*tan(1/2*e)^3 + 3*a^2*f^2*tan(1/2*f*x)^3*tan(1/2*e) + 3*a^2*
f^2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 3*a^2*f^2*tan(1/2*f*x)*tan(1/2*e)^3 - a^2
*f^2*tan(1/2*f*x)^3 + 3*a^2*f^2*tan(1/2*f*x)^2*tan(1/2*e) + 3*a^2*f^2*tan(1
/2*f*x)*tan(1/2*e)^2 - a^2*f^2*tan(1/2*e)^3 - 3*a^2*f^2*tan(1/2*f*x)^2 - 3*
a^2*f^2*tan(1/2*f*x)*tan(1/2*e) - 3*a^2*f^2*tan(1/2*e)^2 - 3*a^2*f^2*tan(1/
2*f*x) - 3*a^2*f^2*tan(1/2*e) - a^2*f^2)
```

3.115 $\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a\sin(e+fx)+a)^2},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)*(a + a*Sin[e + f*x])^2), x]
```

Rubi [A] time = 0.0557266, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + a*Sin[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + a*Sin[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx=\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx$$

Mathematica [A] time = 13.9464, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + a*Sin[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + a*Sin[e + f*x])^2), x]
```

Maple [A] time = 3.104, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a+a\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)/(a+a*sin(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(6*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 - 4*d^2*cos(f*x + e) + 6*(d^2*f
*x + c*d*f)*cos(f*x + e)^2 + 6*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 + 6*(d^
2*f*x + c*d*f)*sin(f*x + e)^2 + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 - 2*
d^2*cos(2*f*x + 2*e) + 2*d^2 - (d^2*f*x + c*d*f)*cos(f*x + e) - (d^2*f*x +
c*d*f)*sin(2*f*x + 2*e) + (3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 + 4*d^2)
*sin(f*x + e))*cos(3*f*x + 3*e) - 2*(d^2*f*x + c*d*f + 3*(3*d^2*f^2*x^2 + 6
*c*d*f^2*x + 3*c^2*f^2 + 2*d^2)*cos(f*x + e) + 6*(d^2*f*x + c*d*f)*sin(f*x
+ e))*cos(2*f*x + 2*e) - 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c
^2*d*f^3*x + a^2*c^3*f^3 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c
^2*d*f^3*x + a^2*c^3*f^3)*cos(3*f*x + 3*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c
*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2*e)^2 + 9*(a^2
*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f
*x + e)^2 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^
2*c^3*f^3)*sin(3*f*x + 3*e)^2 + 18*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 +
 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e)*sin(2*f*x + 2*e) + 9*(a^2*d^
3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2*f*
x + 2*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x +
 a^2*c^3*f^3)*sin(f*x + e)^2 - 6*((a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 +
3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e) + (a^2*d^3*f^3*x^3 + 3*a^2*c*
d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e))*cos(3*f*x
+ 3*e) - 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2
```

```
*c^3*f^3 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a
^2*c^3*f^3)*sin(f*x + e))*cos(2*f*x + 2*e) - 2*(a^2*d^3*f^3*x^3 + 3*a^2*c*d
^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 - 3*(a^2*d^3*f^3*x^3 + 3*a^2*c
*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2*e) + 3*(a^2*d
^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x
 + e))*sin(3*f*x + 3*e) + 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*
c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e))*integrate(2/3*(d^3*f^2*x^2 + 2*c*d
^2*f^2*x + c^2*d*f^2 + 6*d^3)*cos(f*x + e)/(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f
^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4
*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x
+ a^2*c^4*f^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*
a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e)^2 + 2*(
a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d
*f^3*x + a^2*c^4*f^3)*sin(f*x + e)), x) - 2*(2*d^2*sin(2*f*x + 2*e) - (d^2*
f*x + c*d*f)*cos(2*f*x + 2*e) + (3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 +
4*d^2)*cos(f*x + e) + (d^2*f*x + c*d*f)*sin(f*x + e))*sin(3*f*x + 3*e) - 2*
(3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 + 4*d^2 - 6*(d^2*f*x + c*d*f)*cos(
f*x + e) + 3*(3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 + 2*d^2)*sin(f*x + e)
)*sin(2*f*x + 2*e) + 2*(d^2*f*x + c*d*f)*sin(f*x + e))/(a^2*d^3*f^3*x^3 + 3
*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + (a^2*d^3*f^3*x^3 + 3
*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(3*f*x + 3*e)^2 +
9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)
*cos(2*f*x + 2*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*
d*f^3*x + a^2*c^3*f^3)*cos(f*x + e)^2 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*
x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(3*f*x + 3*e)^2 + 18*(a^2*d^3*f^3
*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e)*
sin(2*f*x + 2*e) + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f
^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f
^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e)^2 - 6*((a^2*d^3*f^3*
x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e) +
 (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*
sin(2*f*x + 2*e))*cos(3*f*x + 3*e) - 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x
^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3
*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e))*cos(2*f*x + 2*e) - 2*
(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 -
3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)
*cos(2*f*x + 2*e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*
f^3*x + a^2*c^3*f^3)*sin(f*x + e))*sin(3*f*x + 3*e) + 6*(a^2*d^3*f^3*x^3 +
3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{2\,a^2dx+2\,a^2c-\left(a^2dx+a^2c\right)\cos\left(fx+e\right)^2+2\left(a^2dx+a^2c\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(2*a^2*d*x + 2*a^2*c - (a^2*d*x + a^2*c)*cos(f*x + e)^2 + 2*(a^2
*d*x + a^2*c)*sin(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c\sin^2\left(e+fx\right)+2c\sin\left(e+fx\right)+c+dx\sin^2\left(e+fx\right)+2dx\sin\left(e+fx\right)+dx}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral(1/(c*sin(e + f*x)**2 + 2*c*sin(e + f*x) + c + d*x*sin(e + f*x)**2
+ 2*d*x*sin(e + f*x) + d*x), x)/a**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(a*sin(f*x + e) + a)^2), x)
```

3.116 $\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a\sin(e+fx)+a)^2},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a + a*Sin[e + f*x])^2), x]
```

Rubi [A] time = 0.0518527, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + a*Sin[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + a*Sin[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx$$

Mathematica [A] time = 14.8007, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + a*Sin[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + a*Sin[e + f*x])^2), x]
```

Maple [A] time = 4.749, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx+c\right)^{2} \left(a+a\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x)

[Out] int(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] 1/3*(12*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 - 12*d^2*cos(f*x + e) + 12*(d^
2*f*x + c*d*f)*cos(f*x + e)^2 + 12*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 + 1
2*(d^2*f*x + c*d*f)*sin(f*x + e)^2 + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2
 - 6*d^2*cos(2*f*x + 2*e) + 6*d^2 - 2*(d^2*f*x + c*d*f)*cos(f*x + e) - 2*(d
^2*f*x + c*d*f)*sin(2*f*x + 2*e) + 3*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 +
 4*d^2)*sin(f*x + e))*cos(3*f*x + 3*e) - 2*(2*d^2*f*x + 2*c*d*f + 9*(d^2*f^
2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*d^2)*cos(f*x + e) + 12*(d^2*f*x + c*d*f)*
sin(f*x + e))*cos(2*f*x + 2*e) - 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 +
 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4
 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^
4*f^3)*cos(3*f*x + 3*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^
2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e)^2 + 9
*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3
*d*f^3*x + a^2*c^4*f^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3
*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(3*f*x +
 3*e)^2 + 18*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2
 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(f*x + e)*sin(2*f*x + 2*e) + 9*(a^2*
d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3
*x + a^2*c^4*f^3)*sin(2*f*x + 2*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3
*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e
)^2 - 6*((a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4
```

```
*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(f*x + e) + (a^2*d^4*f^3*x^4 + 4*a^2*c*d
^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(2
*f*x + 2*e))*cos(3*f*x + 3*e) - 6*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 +
6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^
4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c
^4*f^3)*sin(f*x + e))*cos(2*f*x + 2*e) - 2*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f
^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 - 3*(a^2*d
^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*
x + a^2*c^4*f^3)*cos(2*f*x + 2*e) + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^
3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e))*
sin(3*f*x + 3*e) + 6*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2
*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e))*integrate(4/3*(d^
3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 12*d^3)*cos(f*x + e)/(a^2*d^5*f^3*x
^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2
+ 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3 + (a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4
+ 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2
*c^5*f^3)*cos(f*x + e)^2 + (a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*
c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)
*sin(f*x + e)^2 + 2*(a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3
*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*sin(f*
x + e)), x) - 2*(6*d^2*sin(2*f*x + 2*e) - 2*(d^2*f*x + c*d*f)*cos(2*f*x + 2
*e) + 3*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 4*d^2)*cos(f*x + e) + 2*(d^2
*f*x + c*d*f)*sin(f*x + e))*sin(3*f*x + 3*e) - 6*(d^2*f^2*x^2 + 2*c*d*f^2*x
 + c^2*f^2 + 4*d^2 - 4*(d^2*f*x + c*d*f)*cos(f*x + e) + 3*(d^2*f^2*x^2 + 2*
c*d*f^2*x + c^2*f^2 + 2*d^2)*sin(f*x + e))*sin(2*f*x + 2*e) + 4*(d^2*f*x +
c*d*f)*sin(f*x + e))/(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2
*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3
*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(3*f
*x + 3*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*
x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e)^2 + 9*(a^2*d^4*f^3*
x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2
*c^4*f^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c
^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(3*f*x + 3*e)^2 + 18*(
a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d
*f^3*x + a^2*c^4*f^3)*cos(f*x + e)*sin(2*f*x + 2*e) + 9*(a^2*d^4*f^3*x^4 +
4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f
^3)*sin(2*f*x + 2*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c
^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e)^2 - 6*((a^2*
d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3
*x + a^2*c^4*f^3)*cos(f*x + e) + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6
*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e))*c
os(3*f*x + 3*e) - 6*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*
f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^
3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*
x + e))*cos(2*f*x + 2*e) - 2*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2
```

```
*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 - 3*(a^2*d^4*f^3*x^4 + 4
*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^
3)*cos(2*f*x + 2*e) + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*
d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e))*sin(3*f*x + 3*
e) + 6*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a
^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{2\,a^2d^2x^2+4\,a^2cdx+2\,a^2c^2-\left(a^2d^2x^2+2\,a^2cdx+a^2c^2\right)\cos\left(fx+e\right)^2+2\left(a^2d^2x^2+2\,a^2cdx+a^2c^2\right)\sin\left(fx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(2*a^2*d^2*x^2 + 4*a^2*c*d*x + 2*a^2*c^2 - (a^2*d^2*x^2 + 2*a^2*
c*d*x + a^2*c^2)*cos(f*x + e)^2 + 2*(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2)*s
in(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c^2\sin^2\left(e+fx\right)+2c^2\sin\left(e+fx\right)+c^2+2cdx\sin^2\left(e+fx\right)+4cdx\sin\left(e+fx\right)+2cdx+d^2x^2\sin^2\left(e+fx\right)+2d^2x^2\sin\left(e+fx\right)+d^2x^2}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral(1/(c**2*sin(e + f*x)**2 + 2*c**2*sin(e + f*x) + c**2 + 2*c*d*x*sin
(e + f*x)**2 + 4*c*d*x*sin(e + f*x) + 2*c*d*x + d**2*x**2*sin(e + f*x)**2 +
 2*d**2*x**2*sin(e + f*x) + d**2*x**2), x)/a**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(a*sin(f*x + e) + a)^2), x)
```

3.117 $\int \frac{(c+dx)^3}{a-a\sin(e+fx)}\,dx$

Optimal. Leaf size=147

$$-\frac{12id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{af^3}+\frac{12d^3\text{PolyLog}\left(3,-ie^{i(e+fx)}\right)}{af^4}+\frac{6d(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{af^2}+\frac{(c+dx)^3\tan\left(\frac{e}{2}\right.}{af}$$

```
[Out] ((-I)*(c + d*x)^3)/(a*f) + (6*d*(c + d*x)^2*Log[1 + I*E^(I*(e + f*x))])/(a*
f^2) - ((12*I)*d^2*(c + d*x)*PolyLog[2, (-I)*E^(I*(e + f*x))])/(a*f^3) + (1
2*d^3*PolyLog[3, (-I)*E^(I*(e + f*x))])/(a*f^4) + ((c + d*x)^3*Tan[e/2 + Pi
/4 + (f*x)/2])/(a*f)
```

Rubi [A] time = 0.29514, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3318, 4184, 3717, 2190, 2531, 2282, 6589}

$$-\frac{12id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{af^3}+\frac{12d^3\text{PolyLog}\left(3,-ie^{i(e+fx)}\right)}{af^4}+\frac{6d(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{af^2}+\frac{(c+dx)^3\tan\left(\frac{e}{2}\right.}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a - a*Sin[e + f*x]),x]
```


```
[Out] ((-I)*(c + d*x)^3)/(a*f) + (6*d*(c + d*x)^2*Log[1 + I*E^(I*(e + f*x))])/(a*
f^2) - ((12*I)*d^2*(c + d*x)*PolyLog[2, (-I)*E^(I*(e + f*x))])/(a*f^3) + (1
2*d^3*PolyLog[3, (-I)*E^(I*(e + f*x))])/(a*f^4) + ((c + d*x)^3*Tan[e/2 + Pi
/4 + (f*x)/2])/(a*f)
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
```

```
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a-a\sin(e+fx)}\,dx = \frac{\int (c+dx)^3\csc^2\left(\frac{1}{2}\left(e-\frac{\pi}{2}\right)+\frac{fx}{2}\right)dx}{2a}$$

$$= \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{(3d)\int (c+dx)^2\cot\left(\frac{e}{2}-\frac{\pi}{4}+\frac{fx}{2}\right)dx}{af}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} - \frac{(6d)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)^2}{1+ie^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}dx}{af}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{af^2} + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} - \frac{\left(12d^2\right)\int (c+dx)\,\mathrm{l}}{}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(e+fx)}\right)}{af^3} + \frac{(c+dx)^3\tan\left(\frac{e}{2}\right.}{af}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(e+fx)}\right)}{af^3} + \frac{(c+dx)^3\tan\left(\frac{e}{2}\right.}{af}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+ie^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(e+fx)}\right)}{af^3} + \frac{12d^3\mathrm{Li}_3\left(-ie^{i(e+f}\right.}{af^4}$$

<u>Mathematica</u> [A] time = 1.12939, size = 124, normalized size = 0.84

$$\frac{-12id^2f(c+dx)\mathrm{PolyLog}\left(2,-ie^{i(e+fx)}\right)+12d^3\mathrm{PolyLog}\left(3,-ie^{i(e+fx)}\right)+f^2(c+dx)^2\left(f(c+dx)\tan\left(\frac{1}{4}(2e+2fx+\pi)\right)-i\right.}{af^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a - a*Sin[e + f*x]),x]
```

```
[Out] ((-12*I)*d^2*f*(c + d*x)*PolyLog[2, (-I)*E^(I*(e + f*x))] + 12*d^3*PolyLog[
3, (-I)*E^(I*(e + f*x))] + f^2*(c + d*x)^2*((-I)*f*(c + d*x) + 6*d*Log[1 +
I*E^(I*(e + f*x))] + f*(c + d*x)*Tan[(2*e + Pi + 2*f*x)/4]))/(a*f^4)
```

Maple [B] time = 0.132, size = 484, normalized size = 3.3

$$2\frac{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}{af\left(\mathrm{e}^{i(fx+e)}-i\right)}+6\frac{\ln\left(\mathrm{e}^{i(fx+e)}-i\right)c^2d}{af^2}+6\frac{d^3e^2\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{f^4a}+12\frac{cd^2e\ln\left(\mathrm{e}^{i(fx+e)}\right)}{af^3}+12\frac{d^3\mathrm{polylog}(}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a-a*sin(f*x+e)),x)
```

```
[Out] 2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(I*(f*x+e))-I)+6/f^2/a*ln(exp
(I*(f*x+e))-I)*c^2*d+6/f^4/a*d^3*e^2*ln(exp(I*(f*x+e))-I)+12/f^3/a*c*d^2*e*
ln(exp(I*(f*x+e)))+12*d^3*polylog(3,-I*exp(I*(f*x+e)))/a/f^4+6/f^2/a*d^3*ln
(1+I*exp(I*(f*x+e)))*x^2-6/f^4/a*d^3*ln(1+I*exp(I*(f*x+e)))*e^2-6/f^2/a*ln(
exp(I*(f*x+e)))*c^2*d+6*I/f^3/a*d^3*e^2*x-12*I/f^3/a*d^3*polylog(2,-I*exp(I
*(f*x+e)))*x-12/f^3/a*c*d^2*e*ln(exp(I*(f*x+e))-I)-6/f^4/a*d^3*e^2*ln(exp(I
*(f*x+e)))-6*I/f^3/a*c*d^2*e^2-12*I/f^2/a*c*d^2*e*x-6*I/f/a*c*d^2*x^2+4*I/f
^4/a*d^3*e^3-12*I/f^3/a*c*d^2*polylog(2,-I*exp(I*(f*x+e)))-2*I/f/a*d^3*x^3+
12/f^2/a*c*d^2*ln(1+I*exp(I*(f*x+e)))*x+12/f^3/a*c*d^2*ln(1+I*exp(I*(f*x+e)
))*e
```

Maxima [B] time = 1.49563, size = 1326, normalized size = 9.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a-a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(6*(2*(f*x + e)*cos(f*x + e) + (cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*
x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1))*c*d^
2*e/(a*f^2*cos(f*x + e)^2 + a*f^2*sin(f*x + e)^2 - 2*a*f^2*sin(f*x + e) + a
*f^2) - 6*c*d^2*e^2/(a*f^2 - a*f^2*sin(f*x + e)/(cos(f*x + e) + 1)) - 3*(2*
(f*x + e)*cos(f*x + e) + (cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e)
+ 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1))*c^2*d/(a*f*
cos(f*x + e)^2 + a*f*sin(f*x + e)^2 - 2*a*f*sin(f*x + e) + a*f) + 6*c^2*d*e
/(a*f - a*f*sin(f*x + e)/(cos(f*x + e) + 1)) - 2*c^3/(a - a*sin(f*x + e)/(c
os(f*x + e) + 1)) - (2*I*d^3*e^3 + (6*d^3*e^2*cos(f*x + e) + 6*I*d^3*e^2*si
n(f*x + e) - 6*I*d^3*e^2)*arctan2(sin(f*x + e) - 1, cos(f*x + e)) + (-6*I*(
f*x + e)^2*d^3 + (12*I*d^3*e - 12*I*c*d^2*f)*(f*x + e) + 6*((f*x + e)^2*d^3
 - 2*(d^3*e - c*d^2*f)*(f*x + e))*cos(f*x + e) + (6*I*(f*x + e)^2*d^3 + (-1
```

```
2*I*d^3*e + 12*I*c*d^2*f)*(f*x + e))*sin(f*x + e))*arctan2(cos(f*x + e), -s
in(f*x + e) + 1) - 2*((f*x + e)^3*d^3 + 3*(f*x + e)*d^3*e^2 - 3*(d^3*e - c*
d^2*f)*(f*x + e)^2)*cos(f*x + e) + (12*I*(f*x + e)*d^3 - 12*I*d^3*e + 12*I*
c*d^2*f - 12*((f*x + e)*d^3 - d^3*e + c*d^2*f)*cos(f*x + e) + (-12*I*(f*x +
 e)*d^3 + 12*I*d^3*e - 12*I*c*d^2*f)*sin(f*x + e))*dilog(-I*e^(I*f*x + I*e)
) - (3*(f*x + e)^2*d^3 + 3*d^3*e^2 - 6*(d^3*e - c*d^2*f)*(f*x + e) - (-3*I*
(f*x + e)^2*d^3 - 3*I*d^3*e^2 + (6*I*d^3*e - 6*I*c*d^2*f)*(f*x + e))*cos(f*
x + e) - 3*((f*x + e)^2*d^3 + d^3*e^2 - 2*(d^3*e - c*d^2*f)*(f*x + e))*sin(
f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) + (-12*
I*d^3*cos(f*x + e) + 12*d^3*sin(f*x + e) - 12*d^3)*polylog(3, -I*e^(I*f*x +
 I*e)) + (-2*I*(f*x + e)^3*d^3 - 6*I*(f*x + e)*d^3*e^2 + (6*I*d^3*e - 6*I*c
*d^2*f)*(f*x + e)^2)*sin(f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x +
 e) - a*f^3))/f
```

Fricas [C] time = 1.9757, size = 2133, normalized size = 14.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a-a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3 + (d^3*f^3*x^3 + 3
*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3)*cos(f*x + e) - (-6*I*d^3*f*x - 6*
I*c*d^2*f + (-6*I*d^3*f*x - 6*I*c*d^2*f)*cos(f*x + e) + (6*I*d^3*f*x + 6*I*
c*d^2*f)*sin(f*x + e))*dilog(I*cos(f*x + e) + sin(f*x + e)) - (6*I*d^3*f*x
+ 6*I*c*d^2*f + (6*I*d^3*f*x + 6*I*c*d^2*f)*cos(f*x + e) + (-6*I*d^3*f*x -
6*I*c*d^2*f)*sin(f*x + e))*dilog(-I*cos(f*x + e) + sin(f*x + e)) + 3*(d^3*e
^2 - 2*c*d^2*e*f + c^2*d*f^2 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*cos(f*x
+ e) - (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*sin(f*x + e))*log(cos(f*x + e) -
 I*sin(f*x + e) + I) + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e
*f + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*cos(f*x + e) - (
d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*sin(f*x + e))*log(I*co
s(f*x + e) - sin(f*x + e) + 1) + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 +
 2*c*d^2*e*f + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*cos(f*
x + e) - (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*sin(f*x + e)
)*log(-I*cos(f*x + e) - sin(f*x + e) + 1) + 3*(d^3*e^2 - 2*c*d^2*e*f + c^2*
d*f^2 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*cos(f*x + e) - (d^3*e^2 - 2*c*d
^2*e*f + c^2*d*f^2)*sin(f*x + e))*log(-cos(f*x + e) - I*sin(f*x + e) + I) +
 6*(d^3*cos(f*x + e) - d^3*sin(f*x + e) + d^3)*polylog(3, I*cos(f*x + e) +
sin(f*x + e)) + 6*(d^3*cos(f*x + e) - d^3*sin(f*x + e) + d^3)*polylog(3, -I
*cos(f*x + e) + sin(f*x + e)) + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^
```

```
3*x + c^3*f^3)*sin(f*x + e))/(a*f^4*cos(f*x + e) - a*f^4*sin(f*x + e) + a*f
^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{c^3}{\sin(e+fx)-1}\,dx + \int \frac{d^3x^3}{\sin(e+fx)-1}\,dx + \int \frac{3cd^2x^2}{\sin(e+fx)-1}\,dx + \int \frac{3c^2dx}{\sin(e+fx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a-a*sin(f*x+e)),x)
```

```
[Out] -(Integral(c**3/(sin(e + f*x) - 1), x) + Integral(d**3*x**3/(sin(e + f*x) -
 1), x) + Integral(3*c*d**2*x**2/(sin(e + f*x) - 1), x) + Integral(3*c**2*d
*x/(sin(e + f*x) - 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(dx+c)^3}{a\sin\left(fx+e\right)-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a-a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-(d*x + c)^3/(a*sin(f*x + e) - a), x)
```

3.118 $\int \frac{(c+dx)^2}{a-a\sin(e+fx)}\,dx$

Optimal. Leaf size=112

$$-\frac{4id^2\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{af^3}+\frac{4d(c+dx)\log\left(1+ie^{i(e+fx)}\right)}{af^2}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{af}-\frac{i(c+dx)^2}{af}$$

```
[Out] ((-I)*(c + d*x)^2)/(a*f) + (4*d*(c + d*x)*Log[1 + I*E^(I*(e + f*x))])/(a*f^
2) - ((4*I)*d^2*PolyLog[2, (-I)*E^(I*(e + f*x))])/(a*f^3) + ((c + d*x)^2*Ta
n[e/2 + Pi/4 + (f*x)/2])/(a*f)
```

Rubi [A] time = 0.212183, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3318, 4184, 3717, 2190, 2279, 2391}

$$-\frac{4id^2\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{af^3}+\frac{4d(c+dx)\log\left(1+ie^{i(e+fx)}\right)}{af^2}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{af}-\frac{i(c+dx)^2}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a - a*Sin[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^2)/(a*f) + (4*d*(c + d*x)*Log[1 + I*E^(I*(e + f*x))])/(a*f^
2) - ((4*I)*d^2*PolyLog[2, (-I)*E^(I*(e + f*x))])/(a*f^3) + ((c + d*x)^2*Ta
n[e/2 + Pi/4 + (f*x)/2])/(a*f)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a-a\sin(e+fx)}\,dx = \frac{\int (c+dx)^2\csc^2\left(\frac{1}{2}\left(e-\frac{\pi}{2}\right)+\frac{fx}{2}\right)dx}{2a}$$

$$= \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{(2d)\int (c+dx)\cot\left(\frac{e}{2}-\frac{\pi}{4}+\frac{fx}{2}\right)dx}{af}$$

$$= -\frac{i(c+dx)^2}{af} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} - \frac{(4d)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)}{1+ie^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}dx}{af}$$

$$= -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+ie^{i(e+fx)}\right)}{af^2} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} - \frac{(4d^2)\int \log\left(1+ie^{2\ldots}\right.}{af^2}$$

$$= -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+ie^{i(e+fx)}\right)}{af^2} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{(4id^2)\,\mathrm{Subst}\left(\int \frac{\log\ldots}{}\right.}{}$$

$$= -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+ie^{i(e+fx)}\right)}{af^2} - \frac{4id^2\operatorname{Li}_2\left(-ie^{i(e+fx)}\right)}{af^3} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}$$

Mathematica [A] time = 0.722677, size = 92, normalized size = 0.82

$$\frac{f(c+dx)\left(f(c+dx)\tan\left(\frac{1}{4}(2e+2fx+\pi)\right)-if(c+dx)+4d\log\left(1+ie^{i(e+fx)}\right)\right)-4id^2\mathrm{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{af^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a - a*Sin[e + f*x]),x]
```

```
[Out] ((-4*I)*d^2*PolyLog[2, (-I)*E^(I*(e + f*x))] + f*(c + d*x)*((-I)*f*(c + d*x
) + 4*d*Log[1 + I*E^(I*(e + f*x))] + f*(c + d*x)*Tan[(2*e + Pi + 2*f*x)/4])
)/(a*f^3)
```

Maple [B] time = 0.087, size = 254, normalized size = 2.3

$$2\,\frac{d^2x^2+2\,cdx+c^2}{af\left(e^{i(fx+e)}-i\right)} + 4\,\frac{\ln\left(e^{i(fx+e)}-i\right)cd}{af^2} - 4\,\frac{\ln\left(e^{i(fx+e)}\right)cd}{af^2} - \frac{2\,id^2x^2}{af} - \frac{4\,id^2ex}{af^2} - \frac{2\,id^2e^2}{f^3a} + 4\,\frac{d^2\ln\left(1+ie^{i(fx+e)}\right)x}{af^2} + 4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a-a*sin(f*x+e)),x)
```

```
[Out] 2*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(I*(f*x+e))-I)+4/f^2/a*ln(exp(I*(f*x+e))-I)
*c*d-4/f^2/a*ln(exp(I*(f*x+e)))*c*d-2*I/f/a*d^2*x^2-4*I/f^2/a*d^2*e*x-2*I/f
^3/a*d^2*e^2+4/f^2/a*d^2*ln(1+I*exp(I*(f*x+e)))*x+4/f^3/a*d^2*ln(1+I*exp(I*
(f*x+e)))*e-4*I*d^2*polylog(2,-I*exp(I*(f*x+e)))/a/f^3-4/f^3/a*d^2*e*ln(exp
(I*(f*x+e))-I)+4/f^3/a*d^2*e*ln(exp(I*(f*x+e)))
```

Maxima [B] time = 1.35062, size = 427, normalized size = 3.81

$$-2i c^2 f^2 + \left(4 cdf \cos\left(fx+e\right) + 4i cdf \sin\left(fx+e\right) - 4i cdf\right) \arctan\left(\sin\left(fx+e\right) - 1, \cos\left(fx+e\right)\right) + \left(4 d^2 fx \cos\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] (-2*I*c^2*f^2 + (4*c*d*f*cos(f*x + e) + 4*I*c*d*f*sin(f*x + e) - 4*I*c*d*f)
*arctan2(sin(f*x + e) - 1, cos(f*x + e)) + (4*d^2*f*x*cos(f*x + e) + 4*I*d^
2*f*x*sin(f*x + e) - 4*I*d^2*f*x)*arctan2(cos(f*x + e), -sin(f*x + e) + 1)
- 2*(d^2*f^2*x^2 + 2*c*d*f^2*x)*cos(f*x + e) - (4*d^2*cos(f*x + e) + 4*I*d^
2*sin(f*x + e) - 4*I*d^2)*dilog(-I*e^(I*f*x + I*e)) - (2*d^2*f*x + 2*c*d*f
- (-2*I*d^2*f*x - 2*I*c*d*f)*cos(f*x + e) - 2*(d^2*f*x + c*d*f)*sin(f*x + e
))*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) + (-2*I*d^2*f^
2*x^2 - 4*I*c*d*f^2*x)*sin(f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x
 + e) - a*f^3)
```

Fricas [B] time = 1.87766, size = 1195, normalized size = 10.67

$$d^2 f^2 x^2 + 2 cdf^2 x + c^2 f^2 + \left(d^2 f^2 x^2 + 2 cdf^2 x + c^2 f^2\right) \cos\left(fx+e\right) - \left(-2i d^2 \cos\left(fx+e\right) + 2i d^2 \sin\left(fx+e\right) - 2i d^2\right) \mathrm{L}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2
)*cos(f*x + e) - (-2*I*d^2*cos(f*x + e) + 2*I*d^2*sin(f*x + e) - 2*I*d^2)*d
ilog(I*cos(f*x + e) + sin(f*x + e)) - (2*I*d^2*cos(f*x + e) - 2*I*d^2*sin(f
*x + e) + 2*I*d^2)*dilog(-I*cos(f*x + e) + sin(f*x + e)) - 2*(d^2*e - c*d*f
 + (d^2*e - c*d*f)*cos(f*x + e) - (d^2*e - c*d*f)*sin(f*x + e))*log(cos(f*x
 + e) - I*sin(f*x + e) + I) + 2*(d^2*f*x + d^2*e + (d^2*f*x + d^2*e)*cos(f*
x + e) - (d^2*f*x + d^2*e)*sin(f*x + e))*log(I*cos(f*x + e) - sin(f*x + e)
+ 1) + 2*(d^2*f*x + d^2*e + (d^2*f*x + d^2*e)*cos(f*x + e) - (d^2*f*x + d^2
*e)*sin(f*x + e))*log(-I*cos(f*x + e) - sin(f*x + e) + 1) - 2*(d^2*e - c*d*
f + (d^2*e - c*d*f)*cos(f*x + e) - (d^2*e - c*d*f)*sin(f*x + e))*log(-cos(f
*x + e) - I*sin(f*x + e) + I) + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*sin(f
*x + e))/(a*f^3*cos(f*x + e) - a*f^3*sin(f*x + e) + a*f^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{c^2}{\sin(e+fx)-1}\,dx + \int \frac{d^2x^2}{\sin(e+fx)-1}\,dx + \int \frac{2cdx}{\sin(e+fx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a-a*sin(f*x+e)),x)
```

```
[Out] -(Integral(c**2/(sin(e + f*x) - 1), x) + Integral(d**2*x**2/(sin(e + f*x) -
 1), x) + Integral(2*c*d*x/(sin(e + f*x) - 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(dx+c)^2}{a\sin\left(fx+e\right)-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-(d*x + c)^2/(a*sin(f*x + e) - a), x)
```

3.119 $\int \frac{c+dx}{a-a\sin(e+fx)}\,dx$

Optimal. Leaf size=59

$$\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{af}+\frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\right)}{af^2}$$

```
[Out] (2*d*Log[Cos[e/2 + Pi/4 + (f*x)/2]])/(a*f^2) + ((c + d*x)*Tan[e/2 + Pi/4 +
(f*x)/2])/(a*f)
```

Rubi [A] time = 0.06638, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3318, 4184, 3475}

$$\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)}{af}+\frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\right)}{af^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - a*Sin[e + f*x]),x]
```

```
[Out] (2*d*Log[Cos[e/2 + Pi/4 + (f*x)/2]])/(a*f^2) + ((c + d*x)*Tan[e/2 + Pi/4 +
(f*x)/2])/(a*f)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{a-a\sin(e+fx)}\,dx &= \frac{\int (c+dx)\csc^2\left(\frac{1}{2}\left(e-\frac{\pi}{2}\right)+\frac{fx}{2}\right)dx}{2a}\\ &= \frac{(c+dx)\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{d\int \cot\left(\frac{e}{2}-\frac{\pi}{4}+\frac{fx}{2}\right)dx}{af}\\ &= \frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\right)}{af^2}+\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{af}\end{aligned}$$

Mathematica [A] time = 0.150707, size = 47, normalized size = 0.8

$$\frac{f(c+dx)\tan\left(\frac{1}{4}(2e+2fx+\pi)\right)+2d\log\left(\cos\left(\frac{1}{4}(2e+2fx+\pi)\right)\right)}{af^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - a*Sin[e + f*x]),x]
```

```
[Out] (2*d*Log[Cos[(2*e + Pi + 2*f*x)/4]] + f*(c + d*x)*Tan[(2*e + Pi + 2*f*x)/4]
)/(a*f^2)
```

Maple [B] time = 0.06, size = 123, normalized size = 2.1

$$-2\,\frac{c}{af\left(\tan\left(1/2\,fx+e/2\right)-1\right)}-\frac{dx}{af}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{-1}-\frac{dx}{af}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{-1}-\frac{d}{af^2}\ln\left(1+\left(\tan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a-a*sin(f*x+e)),x)
```

```
[Out] -2/a*c/f/(tan(1/2*f*x+1/2*e)-1)-1/a*d/(tan(1/2*f*x+1/2*e)-1)*x/f-1/a*d/(tan
(1/2*f*x+1/2*e)-1)*x/f*tan(1/2*f*x+1/2*e)-1/a*d/f^2*ln(1+tan(1/2*f*x+1/2*e)
```

```
^2)+2/a*d/f^2*ln(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.00975, size = 228, normalized size = 3.86

$$\frac{\frac{\left(2\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}-2\sin\left(fx+e\right)+1\right)\log\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}-2\sin\left(fx+e\right)+1\right)\right)d}{af\cos\left(fx+e\right)^{2}+af\sin\left(fx+e\right)^{2}-2af\sin\left(fx+e\right)+af}-\frac{2de}{af-\frac{af\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}+\frac{2c}{a-\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] ((2*(f*x + e)*cos(f*x + e) + (cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x +
 e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1))*d/(a*f*
cos(f*x + e)^2 + a*f*sin(f*x + e)^2 - 2*a*f*sin(f*x + e) + a*f) - 2*d*e/(a*
f - a*f*sin(f*x + e)/(cos(f*x + e) + 1)) + 2*c/(a - a*sin(f*x + e)/(cos(f*x
 + e) + 1)))/f
```

Fricas [B] time = 1.632, size = 251, normalized size = 4.25

$$\frac{dfx+cf+\left(dfx+cf\right)\cos\left(fx+e\right)+\left(d\cos\left(fx+e\right)-d\sin\left(fx+e\right)+d\right)\log\left(-\sin\left(fx+e\right)+1\right)+\left(dfx+cf\right)\sin\left(f\right.}{af^{2}\cos\left(fx+e\right)-af^{2}\sin\left(fx+e\right)+af^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] (d*f*x + c*f + (d*f*x + c*f)*cos(f*x + e) + (d*cos(f*x + e) - d*sin(f*x + e
) + d)*log(-sin(f*x + e) + 1) + (d*f*x + c*f)*sin(f*x + e))/(a*f^2*cos(f*x
+ e) - a*f^2*sin(f*x + e) + a*f^2)
```

Sympy [A] time = 1.10533, size = 272, normalized size = 4.61

$$\begin{cases}-\frac{2cf}{af^{2}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^{2}}-\frac{dfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af^{2}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^{2}}-\frac{dfx}{af^{2}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^{2}}+\frac{2d\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af^{2}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^{2}}-\frac{2d\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{af^{2}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-af^{2}}-\frac{d\log\left(t\right.}{} \\ \frac{cx+\frac{dx^{2}}{2}}{-a\sin\left(e\right)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*c*f/(a*f**2*tan(e/2 + f*x/2) - a*f**2) - d*f*x*tan(e/2 + f*x/
2)/(a*f**2*tan(e/2 + f*x/2) - a*f**2) - d*f*x/(a*f**2*tan(e/2 + f*x/2) - a*
f**2) + 2*d*log(tan(e/2 + f*x/2) - 1)*tan(e/2 + f*x/2)/(a*f**2*tan(e/2 + f*
x/2) - a*f**2) - 2*d*log(tan(e/2 + f*x/2) - 1)/(a*f**2*tan(e/2 + f*x/2) - a
*f**2) - d*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/2)/(a*f**2*tan(e/2 +
f*x/2) - a*f**2) + d*log(tan(e/2 + f*x/2)**2 + 1)/(a*f**2*tan(e/2 + f*x/2)
- a*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(-a*sin(e) + a), True))
```

Giac [B] time = 1.28914, size = 941, normalized size = 15.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] (d*f*x*tan(1/2*f*x)*tan(1/2*e) - d*f*x*tan(1/2*f*x) - d*f*x*tan(1/2*e) + c*
f*tan(1/2*f*x)*tan(1/2*e) + d*log(2*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(
1/2*e)^2 + 2*tan(1/2*f*x)^4*tan(1/2*e) + 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + ta
n(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^3 + 2*tan(1/2
*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 2*t
an(1/2*e) + 1))*tan(1/2*f*x)*tan(1/2*e) - d*f*x - c*f*tan(1/2*f*x) + d*log(
2*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 + 2*tan(1/2*f*x)^4*tan(1/
2*e) + 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^2*ta
n(1/2*e)^2 - 2*tan(1/2*f*x)^3 + 2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*tan(1/2*f*x
)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 2*tan(1/2*e) + 1))*tan(1/2*f*x) - c*f
*tan(1/2*e) + d*log(2*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 + 2*t
an(1/2*f*x)^4*tan(1/2*e) + 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f*x)^4 +
 2*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^3 + 2*tan(1/2*f*x)*tan(1/2*
e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 2*tan(1/2*e) + 1)
)*tan(1/2*e) - c*f - d*log(2*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^
2 + 2*tan(1/2*f*x)^4*tan(1/2*e) + 2*tan(1/2*f*x)^3*tan(1/2*e)^2 + tan(1/2*f
*x)^4 + 2*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^3 + 2*tan(1/2*f*x)*t
an(1/2*e)^2 + 2*tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 2*tan(1/2*
e) + 1)))/(a*f^2*tan(1/2*f*x)*tan(1/2*e) + a*f^2*tan(1/2*f*x) + a*f^2*tan(1
/2*e) - a*f^2)
```

3.120 $\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a-a\sin(e+fx))},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)*(a - a*Sin[e + f*x])), x]
```

Rubi [A] time = 0.0748442, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a - a*Sin[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a - a*Sin[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx = \int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx$$

Mathematica [A] time = 4.90179, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a-a\sin(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a - a*Sin[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)*(a - a*Sin[e + f*x])), x]
```

Maple [A] time = 0.136, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a-a\sin\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a-a*sin(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a-a*sin(f*x+e)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{adx+ac-(adx+ac)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c - (a*d*x + a*c)*sin(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{c\sin\left(e+fx\right)-c+dx\sin\left(e+fx\right)-dx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*sin(f*x+e)),x)
```

```
[Out] -Integral(1/(c*sin(e + f*x) - c + d*x*sin(e + f*x) - d*x), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(dx+c)\left(a\sin\left(fx+e\right)-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x + c)*(a*sin(f*x + e) - a)), x)
```

3.121 $\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a-a\sin(e+fx))},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a - a*Sin[e + f*x])), x]
```

Rubi [A] time = 0.0642, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a - a*Sin[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a - a*Sin[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx=\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx$$

Mathematica [A] time = 4.71838, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a-a\sin(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a - a*Sin[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a - a*Sin[e + f*x])), x]
```

Maple [A] time = 0.322, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx+c\right)^{2}\left(a-a\sin\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a-a*sin(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a-a*sin(f*x+e)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{ad^{2}x^{2}+2\,acdx+ac^{2}-\left(ad^{2}x^{2}+2\,acdx+ac^{2}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - (a*d^2*x^2 + 2*a*c*d*x + a*c^2)
*sin(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{c^2\sin\left(e+fx\right)-c^2+2cdx\sin\left(e+fx\right)-2cdx+d^2x^2\sin\left(e+fx\right)-d^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a-a*sin(f*x+e)),x)
```

```
[Out] -Integral(1/(c**2*sin(e + f*x) - c**2 + 2*c*d*x*sin(e + f*x) - 2*c*d*x + d*
*2*x**2*sin(e + f*x) - d**2*x**2), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(dx+c)^2\left(a\sin\left(fx+e\right)-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a-a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x + c)^2*(a*sin(f*x + e) - a)), x)
```

3.122 $\int x^3\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{12x^2\sqrt{a\sin(c+dx)+a}}{d^2}-\frac{96\sqrt{a\sin(c+dx)+a}}{d^4}+\frac{48x\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}}{d^3}-\frac{2x^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a}}{d}$$

```
[Out] (-96*Sqrt[a + a*Sin[c + d*x]])/d^4 + (12*x^2*Sqrt[a + a*Sin[c + d*x]])/d^2
+ (48*x*Cot[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin[c + d*x]])/d^3 - (2*x^3*Co
t[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin[c + d*x]])/d
```

Rubi [A] time = 0.140527, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3319, 3296, 2638}

$$\frac{12x^2\sqrt{a\sin(c+dx)+a}}{d^2}-\frac{96\sqrt{a\sin(c+dx)+a}}{d^4}+\frac{48x\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}}{d^3}-\frac{2x^3\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-96*Sqrt[a + a*Sin[c + d*x]])/d^4 + (12*x^2*Sqrt[a + a*Sin[c + d*x]])/d^2
+ (48*x*Cot[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin[c + d*x]])/d^3 - (2*x^3*Co
t[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin[c + d*x]])/d
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^3\sqrt{a+a\sin(c+dx)}\,dx = \left(\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int x^3\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)dx$$

$$= -\frac{2x^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d}+\frac{\left(6\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int x^2 c}{d}$$

$$= \frac{12x^2\sqrt{a+a\sin(c+dx)}}{d^2}-\frac{2x^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d}-\frac{\left(24\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right.}{}$$

$$= \frac{12x^2\sqrt{a+a\sin(c+dx)}}{d^2}+\frac{48x\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d^3}-\frac{2x^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d}$$

$$= -\frac{96\sqrt{a+a\sin(c+dx)}}{d^4}+\frac{12x^2\sqrt{a+a\sin(c+dx)}}{d^2}+\frac{48x\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+d}}{d^3}$$

Mathematica [A] time = 0.287665, size = 108, normalized size = 0.9

$$-\frac{2\sqrt{a(\sin(c+dx)+1)}\left(\left(-d^3x^3-6d^2x^2+24dx+48\right)\sin\left(\frac{1}{2}(c+dx)\right)+\left(d^3x^3-6d^2x^2-24dx+48\right)\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d^4\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*((48 - 24*d*x - 6*d^2*x^2 + d^3*x^3)*Cos[(c + d*x)/2] + (48 + 24*d*x -
6*d^2*x^2 - d^3*x^3)*Sin[(c + d*x)/2])*Sqrt[a*(1 + Sin[c + d*x])])/(d^4*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [C] time = 0.103, size = 145, normalized size = 1.2

$$\frac{-i\sqrt{2}\left(-ix^3d^3+d^3x^3e^{i(dx+c)}+6\,id^2x^2e^{i(dx+c)}-6\,d^2x^2+24\,idx-24\,dxe^{i(dx+c)}-48\,ie^{i(dx+c)}+48\right)\left(e^{i(dx+c)}+i\right)}{\left(e^{2\,i(dx+c)}-1+2\,ie^{i(dx+c)}\right)d^4}\sqrt{-a\,(-2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -I*2^(1/2)*(-a*(-2-2*sin(d*x+c)))^(1/2)/(exp(2*I*(d*x+c))-1+2*I*exp(I*(d*x+
c)))*(-I*x^3*d^3+d^3*x^3*exp(I*(d*x+c))+6*I*d^2*x^2*exp(I*(d*x+c))-6*d^2*x^
2+24*I*d*x-24*d*x*exp(I*(d*x+c))-48*I*exp(I*(d*x+c))+48)*(exp(I*(d*x+c))+I)
/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin(dx+c) + a}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{a(\sin(c+dx)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin (dx + c) + a} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*x^3, x)
```

3.123 $\int x^2\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{8x\sqrt{a\sin(c+dx)+a}}{d^2}+\frac{16\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}}{d^3}-\frac{2x^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}}{d}$$

```
[Out] (8*x*Sqrt[a + a*Sin[c + d*x]])/d^2 + (16*Cot[c/2 + Pi/4 + (d*x)/2]*Sqrt[a +
 a*Sin[c + d*x]])/d^3 - (2*x^2*Cot[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin[c +
 d*x]])/d
```

Rubi [A] time = 0.10307, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3319, 3296, 2638}

$$\frac{8x\sqrt{a\sin(c+dx)+a}}{d^2}+\frac{16\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}}{d^3}-\frac{2x^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (8*x*Sqrt[a + a*Sin[c + d*x]])/d^2 + (16*Cot[c/2 + Pi/4 + (d*x)/2]*Sqrt[a +
 a*Sin[c + d*x]])/d^3 - (2*x^2*Cot[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin[c +
 d*x]])/d
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^2\sqrt{a+a\sin(c+dx)}\,dx = \left(\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int x^2\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)dx$$

$$= -\frac{2x^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d} + \frac{\left(4\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int x\,\mathrm{co}}{d}$$

$$= \frac{8x\sqrt{a+a\sin(c+dx)}}{d^2} - \frac{2x^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d} - \frac{\left(8\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a}\right.}{}$$

$$= \frac{8x\sqrt{a+a\sin(c+dx)}}{d^2} + \frac{16\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d^3} - \frac{2x^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a}}{d}$$

Mathematica [A] time = 0.209777, size = 92, normalized size = 0.94

$$-\frac{2\sqrt{a(\sin(c+dx)+1)}\left(\left(d^2x^2-4dx-8\right)\cos\left(\frac{1}{2}(c+dx)\right)-\left(d^2x^2+4dx-8\right)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*((-8 - 4*d*x + d^2*x^2)*Cos[(c + d*x)/2] - (-8 + 4*d*x + d^2*x^2)*Sin[(
c + d*x)/2])*Sqrt[a*(1 + Sin[c + d*x])])/(d^3*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]))
```

Maple [C] time = 0.054, size = 119, normalized size = 1.2

$$\frac{-i\sqrt{2}\left(-id^2x^2+d^2x^2e^{i(dx+c)}+4\,idxe^{i(dx+c)}-4\,dx+8\,i-8\,e^{i(dx+c)}\right)\left(e^{i(dx+c)}+i\right)}{\left(e^{2\,i(dx+c)}-1+2\,ie^{i(dx+c)}\right)d^3}\sqrt{-a\,(-2-2\,\sin\,(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -I*2^(1/2)*(-a*(-2-2*sin(d*x+c)))^(1/2)/(exp(2*I*(d*x+c))-1+2*I*exp(I*(d*x+
c)))*(-I*d^2*x^2+d^2*x^2*exp(I*(d*x+c))+4*I*d*x*exp(I*(d*x+c))-4*d*x+8*I-8*
exp(I*(d*x+c)))*(exp(I*(d*x+c))+I)/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin(dx + c) + a} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{a (\sin(c + dx) + 1)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin(dx + c) + a}x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*x^2, x)
```

3.124 $\int x\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{4\sqrt{a\sin(c+dx)+a}}{d^2}-\frac{2x\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}}{d}$$

```
[Out] (4*Sqrt[a + a*Sin[c + d*x]])/d^2 - (2*x*Cot[c/2 + Pi/4 + (d*x)/2]*Sqrt[a +
a*Sin[c + d*x]])/d
```

Rubi [A] time = 0.0681968, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3319, 3296, 2638}

$$\frac{4\sqrt{a\sin(c+dx)+a}}{d^2}-\frac{2x\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + a*Sin[c + d*x]])/d^2 - (2*x*Cot[c/2 + Pi/4 + (d*x)/2]*Sqrt[a +
a*Sin[c + d*x]])/d
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x\sqrt{a+a\sin(c+dx)}\,dx &= \left(\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int x\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)dx\\
&= -\frac{2x\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d}+\frac{\left(2\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int\cos\left(\frac{c}{2}\right.}{d}\\
&= \frac{4\sqrt{a+a\sin(c+dx)}}{d^2}-\frac{2x\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{d}
\end{aligned}
$$

Mathematica [A] time = 0.154155, size = 76, normalized size = 1.31

$$
-\frac{2\sqrt{a(\sin(c+dx)+1)}\left((dx-2)\cos\left(\frac{1}{2}(c+dx)\right)-(dx+2)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*((-2 + d*x)*Cos[(c + d*x)/2] - (2 + d*x)*Sin[(c + d*x)/2])*Sqrt[a*(1 +
Sin[c + d*x])])/(d^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [C] time = 0.053, size = 93, normalized size = 1.6

$$
\frac{-i\sqrt{2}\left(-idx+dxe^{i(dx+c)}+2\,ie^{i(dx+c)}-2\right)\left(e^{i(dx+c)}+i\right)}{\left(e^{2\,i(dx+c)}-1+2\,ie^{i(dx+c)}\right)d^2}\sqrt{-a\,(-2-2\,\sin\,(dx+c))}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -I*2^(1/2)*(-a*(-2-2*sin(d*x+c)))^(1/2)/(exp(2*I*(d*x+c))-1+2*I*exp(I*(d*x+
c)))*(-I*d*x+d*x*exp(I*(d*x+c))+2*I*exp(I*(d*x+c))-2)*(exp(I*(d*x+c))+I)/d^
```

2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a(\sin(c+dx)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*x, x)
```

3.125 $\int \frac{\sqrt{a+a\sin(c+dx)}}{x}\,dx$

Optimal. Leaf size=101

$$\sin\left(\frac{1}{4}(2c+\pi)\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}+\cos\left(\frac{1}{4}(2c+\pi)\right)\text{Si}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)$$

```
[Out] CosIntegral[(d*x)/2]*Csc[c/2 + Pi/4 + (d*x)/2]*Sin[(2*c + Pi)/4]*Sqrt[a + a
*Sin[c + d*x]] + Cos[(2*c + Pi)/4]*Csc[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin
[c + d*x]]*SinIntegral[(d*x)/2]
```

Rubi [A] time = 0.139043, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3319, 3303, 3299, 3302}

$$\sin\left(\frac{1}{4}(2c+\pi)\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}+\cos\left(\frac{1}{4}(2c+\pi)\right)\text{Si}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/x,x]
```

```
[Out] CosIntegral[(d*x)/2]*Csc[c/2 + Pi/4 + (d*x)/2]*Sin[(2*c + Pi)/4]*Sqrt[a + a
*Sin[c + d*x]] + Cos[(2*c + Pi)/4]*Csc[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin
[c + d*x]]*SinIntegral[(d*x)/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x}\,dx = \left(\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{x}\,dx$$

$$= \left(\cos\left(\frac{1}{4}(2c+\pi)\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sin\left(\frac{dx}{2}\right)}{x}\,dx + \left(\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right.\right.$$

$$= \operatorname{Ci}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sin\left(\frac{1}{4}(2c+\pi)\right)\sqrt{a+a\sin(c+dx)} + \cos\left(\frac{1}{4}(2c+\pi)\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}\right.$$

Mathematica [A] time = 0.161715, size = 83, normalized size = 0.82

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\operatorname{CosIntegral}\left(\frac{dx}{2}\right)+\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\operatorname{Si}\left(\frac{dx}{2}\right)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/x,x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(CosIntegral[(d*x)/2]*(Cos[c/2] + Sin[c/2]) + (
Cos[c/2] - Sin[c/2])*SinIntegral[(d*x)/2]))/(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])
```

Maple [F] time = 0.229, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/x,x)
```

```
[Out] int((a+a*sin(d*x+c))^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a \sin (dx+c)+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a (\sin (c+dx)+1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin(dx+c)+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/x, x)
```

3.126 $\int \frac{\sqrt{a+a\sin(c+dx)}}{x^2}\,dx$

Optimal. Leaf size=130

$$-\frac{1}{2}d\sin\left(\frac{1}{4}(2c-\pi)\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}-\frac{1}{2}d\sin\left(\frac{1}{4}(2c+\pi)\right)\text{Si}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\right.$$

```
[Out] -(Sqrt[a + a*Sin[c + d*x]]/x) - (d*CosIntegral[(d*x)/2]*Csc[c/2 + Pi/4 + (d
*x)/2]*Sin[(2*c - Pi)/4]*Sqrt[a + a*Sin[c + d*x]])/2 - (d*Csc[c/2 + Pi/4 +
(d*x)/2]*Sin[(2*c + Pi)/4]*Sqrt[a + a*Sin[c + d*x]]*SinIntegral[(d*x)/2])/2
```

Rubi [A] time = 0.15319, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3319, 3297, 3303, 3299, 3302}

$$-\frac{1}{2}d\sin\left(\frac{1}{4}(2c-\pi)\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}-\frac{1}{2}d\sin\left(\frac{1}{4}(2c+\pi)\right)\text{Si}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/x^2,x]
```


```
[Out] -(Sqrt[a + a*Sin[c + d*x]]/x) - (d*CosIntegral[(d*x)/2]*Csc[c/2 + Pi/4 + (d
*x)/2]*Sin[(2*c - Pi)/4]*Sqrt[a + a*Sin[c + d*x]])/2 - (d*Csc[c/2 + Pi/4 +
(d*x)/2]*Sin[(2*c + Pi)/4]*Sqrt[a + a*Sin[c + d*x]]*SinIntegral[(d*x)/2])/2
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x^2}\,dx = \left(\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{x^2}\,dx$$
$$= -\frac{\sqrt{a+a\sin(c+dx)}}{x}+\frac{1}{2}\left(d\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int \frac{\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{x}\,dx$$
$$= -\frac{\sqrt{a+a\sin(c+dx)}}{x}-\frac{1}{2}\left(d\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sin\left(\frac{1}{4}(2c-\pi)\right)\sqrt{a+a\sin(c+dx)}\right)\int \frac{\cos\left(\frac{d}{2}\right.}{x}$$
$$= -\frac{\sqrt{a+a\sin(c+dx)}}{x}-\frac{1}{2}d\text{Ci}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sin\left(\frac{1}{4}(2c-\pi)\right)\sqrt{a+a\sin(c+dx)}-\frac{1}{2}$$

Mathematica [A] time = 0.300279, size = 117, normalized size = 0.9

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(dx\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\text{CosIntegral}\left(\frac{dx}{2}\right)-dx\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\text{Si}\left(\frac{dx}{2}\right)-2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\right.\right.}{2x\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/x^2,x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(d*x*CosIntegral[(d*x)/2]*(Cos[c/2] - Sin[c/2])
 - 2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - d*x*(Cos[c/2] + Sin[c/2])*SinI
ntegral[(d*x)/2]))/(2*x*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{a + a\sin(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/x^2,x)
```

```
[Out] int((a+a*sin(d*x+c))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin(dx + c) + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(dx+c\right)+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/x^2, x)
```

3.127 $\int \frac{\sqrt{a+a\sin(c+dx)}}{x^3}\,dx$

Optimal. Leaf size=174

$$-\frac{1}{8}d^2\sin\left(\frac{1}{4}(2c+\pi)\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}-\frac{1}{8}d^2\cos\left(\frac{1}{4}(2c+\pi)\right)\text{Si}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\right.$$

```
[Out] -Sqrt[a + a*Sin[c + d*x]]/(2*x^2) - (d*Cot[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a
*Sin[c + d*x]])/(4*x) - (d^2*CosIntegral[(d*x)/2]*Csc[c/2 + Pi/4 + (d*x)/2]
*Sin[(2*c + Pi)/4]*Sqrt[a + a*Sin[c + d*x]])/8 - (d^2*Cos[(2*c + Pi)/4]*Csc
[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin[c + d*x]]*SinIntegral[(d*x)/2])/8
```

Rubi [A] time = 0.193083, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3319, 3297, 3303, 3299, 3302}

$$-\frac{1}{8}d^2\sin\left(\frac{1}{4}(2c+\pi)\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(c+dx)+a}-\frac{1}{8}d^2\cos\left(\frac{1}{4}(2c+\pi)\right)\text{Si}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/x^3,x]
```

```
[Out] -Sqrt[a + a*Sin[c + d*x]]/(2*x^2) - (d*Cot[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a
*Sin[c + d*x]])/(4*x) - (d^2*CosIntegral[(d*x)/2]*Csc[c/2 + Pi/4 + (d*x)/2]
*Sin[(2*c + Pi)/4]*Sqrt[a + a*Sin[c + d*x]])/8 - (d^2*Cos[(2*c + Pi)/4]*Csc
[c/2 + Pi/4 + (d*x)/2]*Sqrt[a + a*Sin[c + d*x]]*SinIntegral[(d*x)/2])/8
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{x^3}\,dx = \left(\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{x^3}\,dx$$

$$= -\frac{\sqrt{a+a\sin(c+dx)}}{2x^2}+\frac{1}{4}\left(d\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}\right)\int \frac{\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{x^2}\,dx$$

$$= -\frac{\sqrt{a+a\sin(c+dx)}}{2x^2}-\frac{d\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{4x}-\frac{1}{8}\left(d^2\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a\right.$$

$$= -\frac{\sqrt{a+a\sin(c+dx)}}{2x^2}-\frac{d\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{4x}-\frac{1}{8}\left(d^2\cos\left(\frac{1}{4}(2c+\pi)\right)\csc\left(\frac{c}{2}\right.\right.$$

$$= -\frac{\sqrt{a+a\sin(c+dx)}}{2x^2}-\frac{d\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\sqrt{a+a\sin(c+dx)}}{4x}-\frac{1}{8}d^2\text{Ci}\left(\frac{dx}{2}\right)\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d}{2}\right.$$

Mathematica [A] time = 0.327245, size = 153, normalized size = 0.88

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(d^2x^2\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\text{CosIntegral}\left(\frac{dx}{2}\right)+d^2x^2\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\text{Si}\left(\frac{dx}{2}\right)-2dx\sin\left(\frac{1}{2}(c+d\right.\right.}{8x^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/x^3,x]
```

```
[Out] -(Sqrt[a*(1 + Sin[c + d*x])]*(4*Cos[(c + d*x)/2] + 2*d*x*Cos[(c + d*x)/2] +
 d^2*x^2*CosIntegral[(d*x)/2]*(Cos[c/2] + Sin[c/2]) + 4*Sin[(c + d*x)/2] -
2*d*x*Sin[(c + d*x)/2] + d^2*x^2*(Cos[c/2] - Sin[c/2])*SinIntegral[(d*x)/2]
))/(8*x^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/x^3,x)
```

```
[Out] int((a+a*sin(d*x+c))^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin(dx+c)+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(dx+c\right)+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/x^3, x)
```

3.128 $\int x^3(a+a\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=337

$$\frac{8ax^2\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{3f^2}+\frac{16ax^2\sqrt{a\sin(e+fx)+a}}{f^2}-\frac{64a\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{27f^4}-1$$

```
[Out] (-1280*a*Sqrt[a + a*Sin[e + f*x]])/(9*f^4) + (16*a*x^2*Sqrt[a + a*Sin[e + f
*x]])/f^2 + (640*a*x*Cot[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(9
*f^3) - (8*a*x^3*Cot[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
+ (32*a*x*Cos[e/2 + Pi/4 + (f*x)/2]*Sin[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Si
n[e + f*x]])/(9*f^3) - (4*a*x^3*Cos[e/2 + Pi/4 + (f*x)/2]*Sin[e/2 + Pi/4 +
(f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(3*f) - (64*a*Sin[e/2 + Pi/4 + (f*x)/2]^
2*Sqrt[a + a*Sin[e + f*x]])/(27*f^4) + (8*a*x^2*Sin[e/2 + Pi/4 + (f*x)/2]^2
*Sqrt[a + a*Sin[e + f*x]])/(3*f^2)
```

Rubi [A] time = 0.230083, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3319, 3311, 3296, 2638, 3310}

$$\frac{8ax^2\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{3f^2}+\frac{16ax^2\sqrt{a\sin(e+fx)+a}}{f^2}-\frac{64a\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{27f^4}-1$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-1280*a*Sqrt[a + a*Sin[e + f*x]])/(9*f^4) + (16*a*x^2*Sqrt[a + a*Sin[e + f
*x]])/f^2 + (640*a*x*Cot[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(9
*f^3) - (8*a*x^3*Cot[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
+ (32*a*x*Cos[e/2 + Pi/4 + (f*x)/2]*Sin[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Si
n[e + f*x]])/(9*f^3) - (4*a*x^3*Cos[e/2 + Pi/4 + (f*x)/2]*Sin[e/2 + Pi/4 +
(f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(3*f) - (64*a*Sin[e/2 + Pi/4 + (f*x)/2]^
2*Sqrt[a + a*Sin[e + f*x]])/(27*f^4) + (8*a*x^2*Sin[e/2 + Pi/4 + (f*x)/2]^2
*Sqrt[a + a*Sin[e + f*x]])/(3*f^2)
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
```

```
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int x^3(a+a\sin(e+fx))^{3/2}\,dx = \left(2a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int x^3\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)dx$$

$$= -\frac{4ax^3\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f} + \frac{8ax^2\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3f^2}$$

$$= -\frac{8ax^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f} + \frac{32ax\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{9f^3}$$

$$= \frac{16ax^2\sqrt{a+a\sin(e+fx)}}{f^2} + \frac{64ax\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{9f^3} - \frac{8ax^3\cot\left(\frac{e}{2}+\right.}{}$$

$$= -\frac{128a\sqrt{a+a\sin(e+fx)}}{9f^4} + \frac{16ax^2\sqrt{a+a\sin(e+fx)}}{f^2} + \frac{640ax\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a}}{9f^3}$$

$$= -\frac{1280a\sqrt{a+a\sin(e+fx)}}{9f^4} + \frac{16ax^2\sqrt{a+a\sin(e+fx)}}{f^2} + \frac{640ax\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+}}{9f^3}$$

Mathematica [A] time = 1.11269, size = 231, normalized size = 0.69

$$\frac{2a\sqrt{a(\sin(e+fx)+1)}\left(-\frac{2\left(\sin\left(\frac{e}{2}\right)\left(-18f^3x^3-117f^2x^2+480fx+968\right)+\cos\left(\frac{e}{2}\right)\left(18f^3x^3-117f^2x^2-480fx+968\right)\right)}{\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)}-\cos(fx)\left(2\sin(e)\left(8-9f^2x\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*a*((-2*((968 - 480*f*x - 117*f^2*x^2 + 18*f^3*x^3)*Cos[e/2] + (968 + 480
*f*x - 117*f^2*x^2 - 18*f^3*x^3)*Sin[e/2]))/(Cos[e/2] + Sin[e/2]) - Cos[f*x
]*(3*f*x*(-8 + 3*f^2*x^2)*Cos[e] + 2*(8 - 9*f^2*x^2)*Sin[e]) + (2*(-8 + 9*f
^2*x^2)*Cos[e] + 3*f*x*(-8 + 3*f^2*x^2)*Sin[e])*Sin[f*x] + (24*f*x*(-80 + 3
*f^2*x^2)*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])))*Sqrt[a*(1 + Sin[e + f*x])])/(27*f^4)
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int x^3\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x^3*(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*x^3, x)
```

3.129 $\int x^2(a+a\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=271

$$\frac{16ax\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{9f^2}+\frac{32ax\sqrt{a\sin(e+fx)+a}}{3f^2}+\frac{224a\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{9f^3}-\frac{32a}{}$$

```
[Out] (32*a*x*Sqrt[a + a*Sin[e + f*x]])/(3*f^2) + (224*a*Cot[e/2 + Pi/4 + (f*x)/2
]*Sqrt[a + a*Sin[e + f*x]])/(9*f^3) - (8*a*x^2*Cot[e/2 + Pi/4 + (f*x)/2]*Sq
rt[a + a*Sin[e + f*x]])/(3*f) - (32*a*Cos[e/2 + Pi/4 + (f*x)/2]^2*Cot[e/2 +
 Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(27*f^3) - (4*a*x^2*Cos[e/2 + Pi
/4 + (f*x)/2]*Sin[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(3*f) + (
16*a*x*Sin[e/2 + Pi/4 + (f*x)/2]^2*Sqrt[a + a*Sin[e + f*x]])/(9*f^2)
```

Rubi [A] time = 0.179307, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3319, 3311, 3296, 2638, 2633}

$$\frac{16ax\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{9f^2}+\frac{32ax\sqrt{a\sin(e+fx)+a}}{3f^2}+\frac{224a\cot\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{9f^3}-\frac{32a}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + a*Sin[e + f*x])^(3/2),x]
```


```
[Out] (32*a*x*Sqrt[a + a*Sin[e + f*x]])/(3*f^2) + (224*a*Cot[e/2 + Pi/4 + (f*x)/2
]*Sqrt[a + a*Sin[e + f*x]])/(9*f^3) - (8*a*x^2*Cot[e/2 + Pi/4 + (f*x)/2]*Sq
rt[a + a*Sin[e + f*x]])/(3*f) - (32*a*Cos[e/2 + Pi/4 + (f*x)/2]^2*Cot[e/2 +
 Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(27*f^3) - (4*a*x^2*Cos[e/2 + Pi
/4 + (f*x)/2]*Sin[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(3*f) + (
16*a*x*Sin[e/2 + Pi/4 + (f*x)/2]^2*Sqrt[a + a*Sin[e + f*x]])/(9*f^2)
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int x^2(a+a\sin(e+fx))^{3/2}\,dx = \left(2a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int x^2\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)dx$$

$$= -\frac{4ax^2\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f}+\frac{16ax\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{9f^2}$$

$$= -\frac{8ax^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{3f}-\frac{4ax^2\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{3f}$$

$$= \frac{32ax\sqrt{a+a\sin(e+fx)}}{3f^2}+\frac{32a\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{9f^3}-\frac{8ax^2\cot\left(\frac{e}{2}+\frac{\pi}{4}\right.}{}$$

$$= \frac{32ax\sqrt{a+a\sin(e+fx)}}{3f^2}+\frac{224a\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{9f^3}-\frac{8ax^2\cot\left(\frac{e}{2}+\right.}{}$$

Mathematica [A] time = 0.822052, size = 191, normalized size = 0.7

$$\frac{2a\sqrt{a(\sin(e+fx)+1)}\left(-\frac{4\left(\sin\left(\frac{e}{2}\right)\left(-9f^2x^2-39fx+80\right)+\cos\left(\frac{e}{2}\right)\left(9f^2x^2-39fx-80\right)\right)}{\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)}-\cos(fx)\left(\cos(e)\left(9f^2x^2-8\right)-12fx\sin(e)\right)+\sin(\right.}{27f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*a*((-4*((-80 - 39*f*x + 9*f^2*x^2)*Cos[e/2] + (80 - 39*f*x - 9*f^2*x^2)*
Sin[e/2]))/(Cos[e/2] + Sin[e/2]) - Cos[f*x]*((-8 + 9*f^2*x^2)*Cos[e] - 12*f
*x*Sin[e]) + (12*f*x*Cos[e] + (-8 + 9*f^2*x^2)*Sin[e])*Sin[f*x] + (8*(-80 +
 9*f^2*x^2)*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])))*Sqrt[a*(1 + Sin[e + f*x])])/(27*f^3)
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int x^2\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x^2*(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2}\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**2*(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*x^2, x)
```

3.130 $\int x(a + a\sin(e + fx))^{3/2}\,dx$

Optimal. Leaf size=165

$$\frac{8a\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{9f^2}+\frac{16a\sqrt{a\sin(e+fx)+a}}{3f^2}-\frac{4ax\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+f}}{3f}$$

```
[Out] (16*a*Sqrt[a + a*Sin[e + f*x]])/(3*f^2) - (8*a*x*Cot[e/2 + Pi/4 + (f*x)/2]*
Sqrt[a + a*Sin[e + f*x]])/(3*f) - (4*a*x*Cos[e/2 + Pi/4 + (f*x)/2]*Sin[e/2
+ Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(3*f) + (8*a*Sin[e/2 + Pi/4 + (
f*x)/2]^2*Sqrt[a + a*Sin[e + f*x]])/(9*f^2)
```

Rubi [A] time = 0.0911621, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3319, 3310, 3296, 2638}

$$\frac{8a\sin^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}}{9f^2}+\frac{16a\sqrt{a\sin(e+fx)+a}}{3f^2}-\frac{4ax\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\cos\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+f}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (16*a*Sqrt[a + a*Sin[e + f*x]])/(3*f^2) - (8*a*x*Cot[e/2 + Pi/4 + (f*x)/2]*
Sqrt[a + a*Sin[e + f*x]])/(3*f) - (4*a*x*Cos[e/2 + Pi/4 + (f*x)/2]*Sin[e/2
+ Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]])/(3*f) + (8*a*Sin[e/2 + Pi/4 + (
f*x)/2]^2*Sqrt[a + a*Sin[e + f*x]])/(9*f^2)
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
```

```
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x(a + a\sin(e + fx))^{3/2}\,dx = \left(2a\csc\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a + a\sin(e + fx)}\right)\int x\sin^3\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)dx$$
$$= -\frac{4ax\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a + a\sin(e + fx)}}{3f} + \frac{8a\sin^2\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a}}{9f^2}$$
$$= -\frac{8ax\cot\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a + a\sin(e + fx)}}{3f} - \frac{4ax\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a}}{3f}$$
$$= \frac{16a\sqrt{a + a\sin(e + fx)}}{3f^2} - \frac{8ax\cot\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a + a\sin(e + fx)}}{3f} - \frac{4ax\cos\left(\frac{e}{2} + \frac{\pi}{4} + \right.}{}$$

Mathematica [A] time = 0.680619, size = 113, normalized size = 0.68

$$-\frac{(a(\sin(e + fx) + 1))^{3/2}\left(27(fx - 2)\cos\left(\frac{1}{2}(e + fx)\right) + (3fx + 2)\cos\left(\frac{3}{2}(e + fx)\right) + 2\sin\left(\frac{1}{2}(e + fx)\right)((3fx - 2)\cos(e + \right.}{9f^2\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((27*(-2 + f*x)*Cos[(e + f*x)/2] + (2 + 3*f*x)*Cos[(3*(e + f*x))/2] + 2*(-
4*(7 + 3*f*x) + (-2 + 3*f*x)*Cos[e + f*x])*Sin[(e + f*x)/2])*(a*(1 + Sin[e
```

```
+ f*x]))^(3/2))/(9*f^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int x\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x*(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x*(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*x, x)
```

3.131 $\int \frac{(a+a\sin(e+fx))^{3/2}}{x}\,dx$

Optimal. Leaf size=221

$$\frac{3}{2}a\sin\left(\frac{1}{4}(2e+\pi)\right)\text{CosIntegral}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}+\frac{1}{2}a\cos\left(\frac{3}{4}(2e-\pi)\right)\text{CosIntegral}\left(\frac{3fx}{2}\right)$$

```
[Out] (a*Cos[(3*(2*e - Pi))/4]*CosIntegral[(3*f*x)/2]*Csc[e/2 + Pi/4 + (f*x)/2]*S
qrt[a + a*Sin[e + f*x]])/2 + (3*a*CosIntegral[(f*x)/2]*Csc[e/2 + Pi/4 + (f*
x)/2]*Sin[(2*e + Pi)/4]*Sqrt[a + a*Sin[e + f*x]])/2 + (3*a*Cos[(2*e + Pi)/4
]*Csc[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]]*SinIntegral[(f*x)/2])/
2 - (a*Csc[e/2 + Pi/4 + (f*x)/2]*Sin[(3*(2*e - Pi))/4]*Sqrt[a + a*Sin[e + f
*x]]*SinIntegral[(3*f*x)/2])/2
```

Rubi [A] time = 0.275993, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3319, 3312, 3303, 3299, 3302}

$$\frac{3}{2}a\sin\left(\frac{1}{4}(2e+\pi)\right)\text{CosIntegral}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}+\frac{1}{2}a\cos\left(\frac{3}{4}(2e-\pi)\right)\text{CosIntegral}\left(\frac{3fx}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/x,x]
```

```
[Out] (a*Cos[(3*(2*e - Pi))/4]*CosIntegral[(3*f*x)/2]*Csc[e/2 + Pi/4 + (f*x)/2]*S
qrt[a + a*Sin[e + f*x]])/2 + (3*a*CosIntegral[(f*x)/2]*Csc[e/2 + Pi/4 + (f*
x)/2]*Sin[(2*e + Pi)/4]*Sqrt[a + a*Sin[e + f*x]])/2 + (3*a*Cos[(2*e + Pi)/4
]*Csc[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]]*SinIntegral[(f*x)/2])/
2 - (a*Csc[e/2 + Pi/4 + (f*x)/2]*Sin[(3*(2*e - Pi))/4]*Sqrt[a + a*Sin[e + f
*x]]*SinIntegral[(3*f*x)/2])/2
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x}\,dx = \left(2a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int \frac{\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{x}\,dx$$

$$= \left(2a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int \left(\frac{3\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{4x}+\frac{\sin\left(\frac{3e}{2}-\frac{\pi}{4}+\frac{3fx}{2}\right)}{4x}\right.$$

$$= \frac{1}{2}\left(a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int \frac{\sin\left(\frac{3e}{2}-\frac{\pi}{4}+\frac{3fx}{2}\right)}{x}\,dx+\frac{1}{2}\left(3a\csc\left(\frac{e}{2}+\frac{\pi}{4}\right.\right.$$

$$= \frac{1}{2}\left(a\cos\left(\frac{3}{4}(2e-\pi)\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int \frac{\cos\left(\frac{3fx}{2}\right)}{x}\,dx+\frac{1}{2}\left(a\cos\left(\right.\right.$$

$$= \frac{1}{2}a\cos\left(\frac{3}{4}(2e-\pi)\right)\mathrm{Ci}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}+\frac{3}{2}a\mathrm{Ci}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}\right.$$

Mathematica [A] time = 0.662187, size = 127, normalized size = 0.57

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(3\text{CosIntegral}\left(\frac{fx}{2}\right)\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)+\text{CosIntegral}\left(\frac{3fx}{2}\right)\left(\sin\left(\frac{3e}{2}\right)-\cos\left(\frac{3e}{2}\right)\right)+\left(\cos\left(\frac{e}{2}\right)-\text{s}\right.\right.}{2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/x,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(3*CosIntegral[(f*x)/2]*(Cos[e/2] + Sin[e/2])
 + CosIntegral[(3*f*x)/2]*(-Cos[(3*e)/2] + Sin[(3*e)/2]) + (Cos[e/2] - Sin[
e/2])*(3*SinIntegral[(f*x)/2] + (1 + 2*Sin[e])*SinIntegral[(3*f*x)/2])))/(2
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/x,x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/x,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/x, x)
```

3.132 $\int \frac{(a+a\sin(e+fx))^{3/2}}{x^2}\,dx$

Optimal. Leaf size=263

$$-\frac{3}{4}af\sin\left(\frac{1}{4}(2e-\pi)\right)\text{CosIntegral}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}+\frac{3}{4}af\sin\left(\frac{1}{4}(6e+\pi)\right)\text{CosIntegral}\left(\frac{3f}{2}\right.$$

```
[Out] (-3*a*f*CosIntegral[(f*x)/2]*Csc[e/2 + Pi/4 + (f*x)/2]*Sin[(2*e - Pi)/4]*Sq
rt[a + a*Sin[e + f*x]])/4 + (3*a*f*CosIntegral[(3*f*x)/2]*Csc[e/2 + Pi/4 +
(f*x)/2]*Sin[(6*e + Pi)/4]*Sqrt[a + a*Sin[e + f*x]])/4 - (2*a*Sin[e/2 + Pi/
4 + (f*x)/2]^2*Sqrt[a + a*Sin[e + f*x]])/x - (3*a*f*Csc[e/2 + Pi/4 + (f*x)/
2]*Sin[(2*e + Pi)/4]*Sqrt[a + a*Sin[e + f*x]]*SinIntegral[(f*x)/2])/4 + (3*
a*f*Cos[(6*e + Pi)/4]*Csc[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]]*Si
nIntegral[(3*f*x)/2])/4
```

Rubi [A] time = 0.300412, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3319, 3313, 3303, 3299, 3302}

$$-\frac{3}{4}af\sin\left(\frac{1}{4}(2e-\pi)\right)\text{CosIntegral}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}+\frac{3}{4}af\sin\left(\frac{1}{4}(6e+\pi)\right)\text{CosIntegral}\left(\frac{3f}{2}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/x^2,x]
```



```
[Out] (-3*a*f*CosIntegral[(f*x)/2]*Csc[e/2 + Pi/4 + (f*x)/2]*Sin[(2*e - Pi)/4]*Sq
rt[a + a*Sin[e + f*x]])/4 + (3*a*f*CosIntegral[(3*f*x)/2]*Csc[e/2 + Pi/4 +
(f*x)/2]*Sin[(6*e + Pi)/4]*Sqrt[a + a*Sin[e + f*x]])/4 - (2*a*Sin[e/2 + Pi/
4 + (f*x)/2]^2*Sqrt[a + a*Sin[e + f*x]])/x - (3*a*f*Csc[e/2 + Pi/4 + (f*x)/
2]*Sin[(2*e + Pi)/4]*Sqrt[a + a*Sin[e + f*x]]*SinIntegral[(f*x)/2])/4 + (3*
a*f*Cos[(6*e + Pi)/4]*Csc[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]]*Si
nIntegral[(3*f*x)/2])/4
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
```

```
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^2}\,dx = \left(2a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int \frac{\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{x^2}\,dx$$

$$= -\frac{2a\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{x} + \left(3af\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)$$

$$= -\frac{2a\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{x} + \frac{1}{4}\left(3af\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right.$$

$$= -\frac{2a\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{x} + \frac{1}{4}\left(3af\cos\left(\frac{1}{4}(6e+\pi)\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a}\right.$$

$$= -\frac{3}{4}af\text{Ci}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{1}{4}(2e-\pi)\right)\sqrt{a+a\sin(e+fx)} + \frac{3}{4}af\text{Ci}\left(\frac{3fx}{2}\right)\csc\left(\right.$$

Mathematica [C] time = 0.916471, size = 226, normalized size = 0.86

$$\frac{i\left(-iae^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2\right)^{3/2}\left(3fxe^{ie+\frac{3ifx}{2}}\text{Ei}\left(-\frac{1}{2}ifx\right)+3ifxe^{2ie+\frac{3ifx}{2}}\text{Ei}\left(\frac{ifx}{2}\right)+3fxe^{\frac{3}{2}i(2e+fx)}\text{Ei}\left(\frac{3ifx}{2}\right)-6ie^{i(e+fx)}-6e^{2i(e}\right.}{4\sqrt{2}x\left(e^{i(e+fx)}+i\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/x^2,x]
```

```
[Out] ((I/4)*(((-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x)))^(3/2)*(2 - (6*I)*
E^(I*(e + f*x)) - 6*E^((2*I)*(e + f*x)) + (2*I)*E^((3*I)*(e + f*x)) + 3*E^(
I*e + ((3*I)/2)*f*x)*f*x*ExpIntegralEi[(-I/2)*f*x] + (3*I)*E^((2*I)*e + ((3
*I)/2)*f*x)*f*x*ExpIntegralEi[(I/2)*f*x] + (3*I)*E^(((3*I)/2)*f*x)*f*x*ExpI
ntegralEi[((-3*I)/2)*f*x] + 3*E^(((3*I)/2)*(2*e + f*x))*f*x*ExpIntegralEi[(
(3*I)/2)*f*x]))/(Sqrt[2]*(I + E^(I*(e + f*x)))^3*x)
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/x^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/x**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sin \left(fx+e\right)+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/x^2, x)
```

3.133 $\int \frac{(a+a\sin(e+fx))^{3/2}}{x^3}\,dx$

Optimal. Leaf size=332

$$-\frac{3}{16}af^2\sin\left(\frac{1}{4}(2e+\pi)\right)\text{CosIntegral}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}-\frac{9}{16}af^2\cos\left(\frac{3}{4}(2e-\pi)\right)\text{CosIntegr}$$

```
[Out] (-9*a*f^2*Cos[(3*(2*e - Pi))/4]*CosIntegral[(3*f*x)/2]*Csc[e/2 + Pi/4 + (f*
x)/2]*Sqrt[a + a*Sin[e + f*x]])/16 - (3*a*f^2*CosIntegral[(f*x)/2]*Csc[e/2
+ Pi/4 + (f*x)/2]*Sin[(2*e + Pi)/4]*Sqrt[a + a*Sin[e + f*x]])/16 - (3*a*f*C
os[e/2 + Pi/4 + (f*x)/2]*Sin[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]]
)/(2*x) - (a*Sin[e/2 + Pi/4 + (f*x)/2]^2*Sqrt[a + a*Sin[e + f*x]])/x^2 - (3
*a*f^2*Cos[(2*e + Pi)/4]*Csc[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]]
*SinIntegral[(f*x)/2])/16 + (9*a*f^2*Csc[e/2 + Pi/4 + (f*x)/2]*Sin[(3*(2*e
- Pi))/4]*Sqrt[a + a*Sin[e + f*x]]*SinIntegral[(3*f*x)/2])/16
```

Rubi [A] time = 0.375586, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3319, 3314, 3303, 3299, 3302, 3312}

$$-\frac{3}{16}af^2\sin\left(\frac{1}{4}(2e+\pi)\right)\text{CosIntegral}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\sqrt{a\sin(e+fx)+a}-\frac{9}{16}af^2\cos\left(\frac{3}{4}(2e-\pi)\right)\text{CosIntegr}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/x^3,x]
```


```
[Out] (-9*a*f^2*Cos[(3*(2*e - Pi))/4]*CosIntegral[(3*f*x)/2]*Csc[e/2 + Pi/4 + (f*
x)/2]*Sqrt[a + a*Sin[e + f*x]])/16 - (3*a*f^2*CosIntegral[(f*x)/2]*Csc[e/2
+ Pi/4 + (f*x)/2]*Sin[(2*e + Pi)/4]*Sqrt[a + a*Sin[e + f*x]])/16 - (3*a*f*C
os[e/2 + Pi/4 + (f*x)/2]*Sin[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]]
)/(2*x) - (a*Sin[e/2 + Pi/4 + (f*x)/2]^2*Sqrt[a + a*Sin[e + f*x]])/x^2 - (3
*a*f^2*Cos[(2*e + Pi)/4]*Csc[e/2 + Pi/4 + (f*x)/2]*Sqrt[a + a*Sin[e + f*x]]
*SinIntegral[(f*x)/2])/16 + (9*a*f^2*Csc[e/2 + Pi/4 + (f*x)/2]*Sin[(3*(2*e
- Pi))/4]*Sqrt[a + a*Sin[e + f*x]]*SinIntegral[(3*f*x)/2])/16
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
```

```
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{x^3}\,dx = \left(2a\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}\right)\int \frac{\sin^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{x^3}\,dx$$

$$= -\frac{3af\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{2x} - \frac{a\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+}}{x^2}$$

$$= -\frac{3af\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)}}{2x} - \frac{a\sin^2\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+}}{x^2}$$

$$= \frac{3}{2}af^2\text{Ci}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{1}{4}(2e+\pi)\right)\sqrt{a+a\sin(e+fx)} - \frac{3af\cos\left(\frac{e}{2}+\frac{\pi}{4}+\right.}{}$$

$$= \frac{3}{2}af^2\text{Ci}\left(\frac{fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sin\left(\frac{1}{4}(2e+\pi)\right)\sqrt{a+a\sin(e+fx)} - \frac{3af\cos\left(\frac{e}{2}+\frac{\pi}{4}+\right.}{}$$

$$= -\frac{9}{16}af^2\cos\left(\frac{3}{4}(2e-\pi)\right)\text{Ci}\left(\frac{3fx}{2}\right)\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\sqrt{a+a\sin(e+fx)} - \frac{3}{16}af^2\text{Ci}\left(\frac{fx}{2}\right.$$

Mathematica [C] time = 0.875693, size = 295, normalized size = 0.89

$$-\frac{i\left(-iae^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2\right)^{3/2}\left(3if^2x^2e^{ie+\frac{3ifx}{2}}\text{Ei}\left(-\frac{1}{2}ifx\right)+3f^2x^2e^{2ie+\frac{3ifx}{2}}\text{Ei}\left(\frac{ifx}{2}\right)-9if^2x^2e^{\frac{3}{2}i(2e+fx)}\text{Ei}\left(\frac{3ifx}{2}\right)+6fxe^{i(e}\right.}{16\sqrt{2}x^2\left(e^{i(e+fx)}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/x^3,x]
```

```
[Out] ((-I/16)*(((-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x)))^(3/2)*(-4 + (12
*I)*E^(I*(e + f*x)) + 12*E^((2*I)*(e + f*x)) - (4*I)*E^((3*I)*(e + f*x)) +
(6*I)*f*x + 6*E^(I*(e + f*x))*f*x + (6*I)*E^((2*I)*(e + f*x))*f*x + 6*E^((3
*I)*(e + f*x))*f*x + (3*I)*E^(I*e + ((3*I)/2)*f*x)*f^2*x^2*ExpIntegralEi[(-
I/2)*f*x] + 3*E^((2*I)*e + ((3*I)/2)*f*x)*f^2*x^2*ExpIntegralEi[(I/2)*f*x]
- 9*E^(((3*I)/2)*f*x)*f^2*x^2*ExpIntegralEi[((-3*I)/2)*f*x] - (9*I)*E^(((3*
I)/2)*(2*e + f*x))*f^2*x^2*ExpIntegralEi[((3*I)/2)*f*x]))/(Sqrt[2]*(I + E^(
I*(e + f*x)))^3*x^2)
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/x^3,x)

[Out] int((a+a*sin(f*x+e))^(3/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x^3,x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^(3/2)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x^3,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/x**3,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/x^3, x)
```

3.134 $\int \frac{x^3}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=417

$$\frac{12ix^2\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\mathrm{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{12ix^2\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\mathrm{PolyLog}\left(2,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{48x\sin\left(\frac{c}{2}+\ldots\right.}{\ldots}$$

```
[Out] (-4*x^3*ArcTanh[E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d
*Sqrt[a + a*Sin[c + d*x]]) + ((12*I)*x^2*PolyLog[2, -E^((I/4)*(2*c + Pi + 2
*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^2*Sqrt[a + a*Sin[c + d*x]]) - ((12*I)
*x^2*PolyLog[2, E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d
^2*Sqrt[a + a*Sin[c + d*x]]) - (48*x*PolyLog[3, -E^((I/4)*(2*c + Pi + 2*d*x
))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^3*Sqrt[a + a*Sin[c + d*x]]) + (48*x*PolyL
og[3, E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^3*Sqrt[a
+ a*Sin[c + d*x]]) - ((96*I)*PolyLog[4, -E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[
c/2 + Pi/4 + (d*x)/2])/(d^4*Sqrt[a + a*Sin[c + d*x]]) + ((96*I)*PolyLog[4,
E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^4*Sqrt[a + a*Si
n[c + d*x]])
```

Rubi [A] time = 0.249256, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3319, 4183, 2531, 6609, 2282, 6589}

$$\frac{12ix^2\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\mathrm{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{12ix^2\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\mathrm{PolyLog}\left(2,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{48x\sin\left(\frac{c}{2}+\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-4*x^3*ArcTanh[E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d
*Sqrt[a + a*Sin[c + d*x]]) + ((12*I)*x^2*PolyLog[2, -E^((I/4)*(2*c + Pi + 2
*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^2*Sqrt[a + a*Sin[c + d*x]]) - ((12*I)
*x^2*PolyLog[2, E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d
^2*Sqrt[a + a*Sin[c + d*x]]) - (48*x*PolyLog[3, -E^((I/4)*(2*c + Pi + 2*d*x
))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^3*Sqrt[a + a*Sin[c + d*x]]) + (48*x*PolyL
og[3, E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^3*Sqrt[a
+ a*Sin[c + d*x]]) - ((96*I)*PolyLog[4, -E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[
c/2 + Pi/4 + (d*x)/2])/(d^4*Sqrt[a + a*Sin[c + d*x]]) + ((96*I)*PolyLog[4,
```

```
E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^4*Sqrt[a + a*Si
n[c + d*x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\int x^3\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)dx}{\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\left(6\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x^2\log\left(1-e^{i\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}\right.}{d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{12ix^2\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{12ix^2\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{12ix^2\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{12ix^2\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}} -$$

Mathematica [A] time = 0.765819, size = 306, normalized size = 0.73

$$\frac{\sqrt[4]{-1}\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\left(e^{i(c+dx)}+i\right)\left(6d^2x^2\text{PolyLog}\left(2,-\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)-6d^2x^2\text{PolyLog}\left(2,\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)+24idx\text{PolyLog}\left(3,\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-1)^(1/4)*Sqrt[2]*(I + E^(I*(c + d*x)))*((-I)*d^3*x^3*Log[1 - (-1)^(1/4)*
E^((I/2)*(c + d*x))] + I*d^3*x^3*Log[1 + (-1)^(1/4)*E^((I/2)*(c + d*x))] +
6*d^2*x^2*PolyLog[2, -((-1)^(1/4)*E^((I/2)*(c + d*x)))] - 6*d^2*x^2*PolyLog
[2, (-1)^(1/4)*E^((I/2)*(c + d*x))] + (24*I)*d*x*PolyLog[3, -((-1)^(1/4)*E^
```

```
((I/2)*(c + d*x)))] - (24*I)*d*x*PolyLog[3, (-1)^(1/4)*E^((I/2)*(c + d*x))]
 - 48*PolyLog[4, -((-1)^(1/4)*E^((I/2)*(c + d*x)))] + 48*PolyLog[4, (-1)^(1
/4)*E^((I/2)*(c + d*x))]))/(d^4*E^((I/2)*(c + d*x))*Sqrt[((-I)*a*(I + E^(I*
(c + d*x)))^2)/E^(I*(c + d*x))])
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int x^3 \frac{1}{\sqrt{a + a\sin(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int(x^3/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a\sin(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^3}{\sqrt{a\sin(dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(a*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(a*sin(d*x + c) + a), x)
```

3.135 $\int \frac{x^2}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=293

$$\frac{8ix\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{8ix\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{16\sin\left(\frac{c}{2}+\frac{dx}{2}+\right.}{d^3\sqrt{}}$$

```
[Out] (-4*x^2*ArcTanh[E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d
*Sqrt[a + a*Sin[c + d*x]]) + ((8*I)*x*PolyLog[2, -E^((I/4)*(2*c + Pi + 2*d*
x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^2*Sqrt[a + a*Sin[c + d*x]]) - ((8*I)*x*P
olyLog[2, E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^2*Sqr
t[a + a*Sin[c + d*x]]) - (16*PolyLog[3, -E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[
c/2 + Pi/4 + (d*x)/2])/(d^3*Sqrt[a + a*Sin[c + d*x]]) + (16*PolyLog[3, E^((
I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^3*Sqrt[a + a*Sin[c
+ d*x]])
```

Rubi [A] time = 0.183318, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3319, 4183, 2531, 2282, 6589}

$$\frac{8ix\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{8ix\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{16\sin\left(\frac{c}{2}+\frac{dx}{2}+\right.}{d^3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-4*x^2*ArcTanh[E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d
*Sqrt[a + a*Sin[c + d*x]]) + ((8*I)*x*PolyLog[2, -E^((I/4)*(2*c + Pi + 2*d*
x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^2*Sqrt[a + a*Sin[c + d*x]]) - ((8*I)*x*P
olyLog[2, E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^2*Sqr
t[a + a*Sin[c + d*x]]) - (16*PolyLog[3, -E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[
c/2 + Pi/4 + (d*x)/2])/(d^3*Sqrt[a + a*Sin[c + d*x]]) + (16*PolyLog[3, E^((
I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^3*Sqrt[a + a*Sin[c
+ d*x]])
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
```

```
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\int x^2\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)dx}{\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{4x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\left(4\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x\log\left(1-e^{i\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right.}\right.}{d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{4x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{8ix\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{4x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{8ix\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{4x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{8ix\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}} -$$

Mathematica [A] time = 0.594169, size = 245, normalized size = 0.84

$$\frac{\sqrt[4]{-1}\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\left(e^{i(c+dx)}+i\right)\left(4dx\text{PolyLog}\left(2,-\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)-i\left(-4idx\text{PolyLog}\left(2,\sqrt[4]{-1}e^{\frac{1}{2}i(c+dx)}\right)-8\text{PolyLog}\left(3,-\right.\right.\right.}{d^3\sqrt{-iae^{-i(c+dx)}\left(e^{i(c+\right.}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-1)^(1/4)*Sqrt[2]*(I + E^(I*(c + d*x)))*(4*d*x*PolyLog[2, -((-1)^(1/4)*E^
((I/2)*(c + d*x)))] - I*(d^2*x^2*Log[1 - (-1)^(1/4)*E^((I/2)*(c + d*x))] -
d^2*x^2*Log[1 + (-1)^(1/4)*E^((I/2)*(c + d*x))] - (4*I)*d*x*PolyLog[2, (-1)
^(1/4)*E^((I/2)*(c + d*x))] - 8*PolyLog[3, -((-1)^(1/4)*E^((I/2)*(c + d*x))
)] + 8*PolyLog[3, (-1)^(1/4)*E^((I/2)*(c + d*x))])))/(d^3*E^((I/2)*(c + d*x
))*Sqrt[((-I)*a*(I + E^(I*(c + d*x)))^2)/E^(I*(c + d*x))])
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int x^2\frac{1}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int(x^2/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{a\sin(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(a*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(a*sin(d*x + c) + a), x)
```

3.136 $\int \frac{x}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=175

$$\frac{4i\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{4i\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{4x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{t}}{d\sqrt{a\sin(c}}$$

```
[Out] (-4*x*ArcTanh[E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d*S
qrt[a + a*Sin[c + d*x]]) + ((4*I)*PolyLog[2, -E^((I/4)*(2*c + Pi + 2*d*x))]
*Sin[c/2 + Pi/4 + (d*x)/2])/(d^2*Sqrt[a + a*Sin[c + d*x]]) - ((4*I)*PolyLog
[2, E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^2*Sqrt[a +
a*Sin[c + d*x]])
```

Rubi [A] time = 0.092206, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3319, 4183, 2279, 2391}

$$\frac{4i\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{4i\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)}{d^2\sqrt{a\sin(c+dx)+a}}-\frac{4x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\text{t}}{d\sqrt{a\sin(c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-4*x*ArcTanh[E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d*S
qrt[a + a*Sin[c + d*x]]) + ((4*I)*PolyLog[2, -E^((I/4)*(2*c + Pi + 2*d*x))]
*Sin[c/2 + Pi/4 + (d*x)/2])/(d^2*Sqrt[a + a*Sin[c + d*x]]) - ((4*I)*PolyLog
[2, E^((I/4)*(2*c + Pi + 2*d*x))]*Sin[c/2 + Pi/4 + (d*x)/2])/(d^2*Sqrt[a +
a*Sin[c + d*x]])
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\int x\csc\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)dx}{\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{4x\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\left(2\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int\log\left(1-e^{i\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}\right)}{d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{4x\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\left(4i\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x\right.}{d^2\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{4x\tanh^{-1}\left(e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d\sqrt{a+a\sin(c+dx)}} + \frac{4i\text{Li}_2\left(-e^{\frac{1}{4}i(2c+\pi+2dx)}\right)\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{d^2\sqrt{a+a\sin(c+dx)}} - \frac{4i}{}$$

Mathematica [A] time = 1.52358, size = 231, normalized size = 1.32

$$\frac{2\left(\frac{c\sin\left(\frac{1}{4}(2c+2dx-\pi)\right)\sin^{-1}\left(\csc\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)}{\sqrt{\frac{\sin(c+dx)-1}{\sin(c+dx)+1}}} + \frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(2i\left(\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2c+2dx+\pi)}\right)-\text{PolyLog}\left(2,e^{\frac{1}{4}i(2c+2dx+\pi)}\right)\right)+\frac{1}{2}(2c+2d\right.}{\sqrt{2}}\right.}{d^2\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*(((-(Pi*ArcTanh[(-1 + Tan[(c + d*x)/4])/Sqrt[2]]) + ((2*c + Pi + 2*d*x)*
(Log[1 - E^((I/4)*(2*c + Pi + 2*d*x))] - Log[1 + E^((I/4)*(2*c + Pi + 2*d*x
))]))/2 + (2*I)*(PolyLog[2, -E^((I/4)*(2*c + Pi + 2*d*x))] - PolyLog[2, E^(
(I/4)*(2*c + Pi + 2*d*x))]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/Sqrt[2]
 + (c*ArcSin[Csc[(2*c + Pi + 2*d*x)/4]]*Sin[(2*c - Pi + 2*d*x)/4])/Sqrt[(-1
 + Sin[c + d*x])/(1 + Sin[c + d*x])]))/(d^2*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int x \frac{1}{\sqrt{a + a \sin(dx + c)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int(x/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a \sin(dx + c) + a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{\sqrt{a \sin(dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(a*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(a*sin(d*x + c) + a), x)
```

3.137 $\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{a\sin(c+dx)+a}},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[a + a*Sin[c + d*x]]), x]
```

Rubi [A] time = 0.0736267, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[a + a*Sin[c + d*x]]),x]

[Out] Defer[Int][1/(x*Sqrt[a + a*Sin[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx$$

Mathematica [A] time = 3.03961, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+a\sin(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[a + a*Sin[c + d*x]]),x]

[Out] Integrate[1/(x*Sqrt[a + a*Sin[c + d*x]]), x]
```

Maple [A] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{a+a\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int(1/x/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(dx+c\right)+a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*sin(d*x + c) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sin\left(dx+c\right)+a}}{ax\sin\left(dx+c\right)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(d*x + c) + a)/(a*x*sin(d*x + c) + a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*(sin(c + d*x) + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin(dx+c)+a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sin(d*x + c) + a)*x), x)
```

3.138 $\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a\sin(c+dx)+a}},x\right)$$

```
[Out] Unintegrable[1/(x^2*Sqrt[a + a*Sin[c + d*x]]), x]
```

Rubi [A] time = 0.0735011, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[a + a*Sin[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx$$

Mathematica [A] time = 0.747242, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a+a\sin(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[a + a*Sin[c + d*x]]), x]
```

Maple [A] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt{a+a\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int(1/x^2/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(dx+c\right)+a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*sin(d*x + c) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sin\left(dx+c\right)+a}}{ax^2\sin\left(dx+c\right)+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(d*x + c) + a)/(a*x^2*sin(d*x + c) + a*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*(sin(c + d*x) + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin(dx+c)+a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sin(d*x + c) + a)*x^2), x)
```

3.139 $\int \frac{x^3}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=691

$$\frac{3ix^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{3ix^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{12x\sin\left(\frac{e}{2}+\frac{fx}{2}\right.}{af}$$

```
[Out] (-3*x^2)/(a*f^2*Sqrt[a + a*Sin[e + f*x]]) - (x^3*Cot[e/2 + Pi/4 + (f*x)/2])
/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - (24*x*ArcTanh[E^((I/4)*(2*e + Pi + 2*f*
x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^3*Sqrt[a + a*Sin[e + f*x]]) - (x^3*Arc
Tanh[E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f*Sqrt[a +
 a*Sin[e + f*x]]) + ((24*I)*PolyLog[2, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e
/2 + Pi/4 + (f*x)/2])/(a*f^4*Sqrt[a + a*Sin[e + f*x]]) + ((3*I)*x^2*PolyLog
[2, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^2*Sqrt[a
 + a*Sin[e + f*x]]) - ((24*I)*PolyLog[2, E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[
e/2 + Pi/4 + (f*x)/2])/(a*f^4*Sqrt[a + a*Sin[e + f*x]]) - ((3*I)*x^2*PolyLo
g[2, E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^2*Sqrt[a
 + a*Sin[e + f*x]]) - (12*x*PolyLog[3, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e
/2 + Pi/4 + (f*x)/2])/(a*f^3*Sqrt[a + a*Sin[e + f*x]]) + (12*x*PolyLog[3, E
^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^3*Sqrt[a + a*S
in[e + f*x]]) - ((24*I)*PolyLog[4, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 +
 Pi/4 + (f*x)/2])/(a*f^4*Sqrt[a + a*Sin[e + f*x]]) + ((24*I)*PolyLog[4, E^(
(I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^4*Sqrt[a + a*Sin
[e + f*x]])
```

Rubi [A] time = 0.353678, antiderivative size = 691, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3319, 4186, 4183, 2279, 2391, 2531, 6609, 2282, 6589}

$$\frac{3ix^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{3ix^2\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{12x\sin\left(\frac{e}{2}+\frac{fx}{2}\right.}{af}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*x^2)/(a*f^2*Sqrt[a + a*Sin[e + f*x]]) - (x^3*Cot[e/2 + Pi/4 + (f*x)/2])
/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - (24*x*ArcTanh[E^((I/4)*(2*e + Pi + 2*f*
x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^3*Sqrt[a + a*Sin[e + f*x]]) - (x^3*Arc
```

```
Tanh[E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f*Sqrt[a +
 a*Sin[e + f*x]]) + ((24*I)*PolyLog[2, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e
/2 + Pi/4 + (f*x)/2])/(a*f^4*Sqrt[a + a*Sin[e + f*x]]) + ((3*I)*x^2*PolyLog
[2, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^2*Sqrt[a
 + a*Sin[e + f*x]]) - ((24*I)*PolyLog[2, E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[
e/2 + Pi/4 + (f*x)/2])/(a*f^4*Sqrt[a + a*Sin[e + f*x]]) - ((3*I)*x^2*PolyLo
g[2, E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^2*Sqrt[a
 + a*Sin[e + f*x]]) - (12*x*PolyLog[3, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e
/2 + Pi/4 + (f*x)/2])/(a*f^3*Sqrt[a + a*Sin[e + f*x]]) + (12*x*PolyLog[3, E
^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^3*Sqrt[a + a*S
in[e + f*x]]) - ((24*I)*PolyLog[4, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 +
 Pi/4 + (f*x)/2])/(a*f^4*Sqrt[a + a*Sin[e + f*x]]) + ((24*I)*PolyLog[4, E^(
(I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^4*Sqrt[a + a*Sin
[e + f*x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\int x^3\csc^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)dx}{2a\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{3x^2}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} + \frac{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\int x^3\csc\left(\frac{e}{2}+\frac{\pi}{4}+\right.}{4a\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{3x^2}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{24x\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\right.}{af^3\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{3x^2}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{24x\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\right.}{af^3\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{3x^2}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{24x\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\right.}{af^3\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{3x^2}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{24x\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\right.}{af^3\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{3x^2}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^3\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{24x\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\right.}{af^3\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 2.80731, size = 455, normalized size = 0.66

$$-\frac{x^2\sqrt{a(\sin(e+fx)+1)}\left((6-fx)\sin\left(\frac{1}{2}(e+fx)\right)+(fx+6)\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2a^2f^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3} - \frac{(-1)^{3/4}e^{-\frac{3}{2}i(e+fx)}\left(e^{i(e+fx)}+i\right)^3\left(6\left(f^2x^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((-1)^(3/4)*(I + E^(I*(e + f*x)))^3*(6*(8 + f^2*x^2)*PolyLog[2, -((-1)^(1/
4)*E^((I/2)*(e + f*x)))] - 6*(8 + f^2*x^2)*PolyLog[2, (-1)^(1/4)*E^((I/2)*(
e + f*x))] - I*(24*f*x*Log[1 - (-1)^(1/4)*E^((I/2)*(e + f*x))] + f^3*x^3*Lo
g[1 - (-1)^(1/4)*E^((I/2)*(e + f*x))] - 24*f*x*Log[1 + (-1)^(1/4)*E^((I/2)*
(e + f*x))] - f^3*x^3*Log[1 + (-1)^(1/4)*E^((I/2)*(e + f*x))] - 24*f*x*Poly
Log[3, -((-1)^(1/4)*E^((I/2)*(e + f*x)))] + 24*f*x*PolyLog[3, (-1)^(1/4)*E^
```

```
((I/2)*(e + f*x))] - (48*I)*PolyLog[4, -((-1)^(1/4)*E^((I/2)*(e + f*x)))] +
 (48*I)*PolyLog[4, (-1)^(1/4)*E^((I/2)*(e + f*x))])))/(2*Sqrt[2]*E^(((3*I)/
2)*(e + f*x))*(((-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x)))^(3/2)*f^4)
 - (x^2*((6 + f*x)*Cos[(e + f*x)/2] + (6 - f*x)*Sin[(e + f*x)/2])*Sqrt[a*(1
 + Sin[e + f*x]])])/(2*a^2*f^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int x^{3} \left(a + a \sin\left(f x + e\right)\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x^3/(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\left(a \sin\left(f x + e\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{a \sin\left(f x + e\right) + a} x^{3}}{a^{2} \cos\left(f x + e\right)^{2} - 2 \, a^{2} \sin\left(f x + e\right) - 2 \, a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*x^3/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x
+ e) - 2*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**3/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(a*sin(f*x + e) + a)^(3/2), x)
```

3.140 $\int \frac{x^2}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=435

$$\frac{2ix\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{2ix\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{4\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right.}{af^3\sqrt{}}$$

```
[Out] (-2*x)/(a*f^2*Sqrt[a + a*Sin[e + f*x]]) - (x^2*Cot[e/2 + Pi/4 + (f*x)/2])/(
2*a*f*Sqrt[a + a*Sin[e + f*x]]) - (x^2*ArcTanh[E^((I/4)*(2*e + Pi + 2*f*x))
]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f*Sqrt[a + a*Sin[e + f*x]]) - (4*ArcTanh[Co
s[e/2 + Pi/4 + (f*x)/2]]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^3*Sqrt[a + a*Sin[e
 + f*x]]) + ((2*I)*x*PolyLog[2, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi
/4 + (f*x)/2])/(a*f^2*Sqrt[a + a*Sin[e + f*x]]) - ((2*I)*x*PolyLog[2, E^((I
/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^2*Sqrt[a + a*Sin[e
 + f*x]]) - (4*PolyLog[3, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (
f*x)/2])/(a*f^3*Sqrt[a + a*Sin[e + f*x]]) + (4*PolyLog[3, E^((I/4)*(2*e + P
i + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^3*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.235191, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3319, 4186, 3770, 4183, 2531, 2282, 6589}

$$\frac{2ix\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{2ix\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{4\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right.}{af^3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*x)/(a*f^2*Sqrt[a + a*Sin[e + f*x]]) - (x^2*Cot[e/2 + Pi/4 + (f*x)/2])/(
2*a*f*Sqrt[a + a*Sin[e + f*x]]) - (x^2*ArcTanh[E^((I/4)*(2*e + Pi + 2*f*x))
]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f*Sqrt[a + a*Sin[e + f*x]]) - (4*ArcTanh[Co
s[e/2 + Pi/4 + (f*x)/2]]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^3*Sqrt[a + a*Sin[e
 + f*x]]) + ((2*I)*x*PolyLog[2, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi
/4 + (f*x)/2])/(a*f^2*Sqrt[a + a*Sin[e + f*x]]) - ((2*I)*x*PolyLog[2, E^((I
/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^2*Sqrt[a + a*Sin[e
 + f*x]]) - (4*PolyLog[3, -E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (
f*x)/2])/(a*f^3*Sqrt[a + a*Sin[e + f*x]]) + (4*PolyLog[3, E^((I/4)*(2*e + P
i + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^3*Sqrt[a + a*Sin[e + f*x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\int x^2\csc^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)dx}{2a\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2x}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} + \frac{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\int x^2\csc\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f}{2}\right.}{4a\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2x}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\right.}{af\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2x}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\right.}{af\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2x}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\right.}{af\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2x}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x^2\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{x^2\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}+\right.}{af\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 2.03136, size = 352, normalized size = 0.81

$$-\frac{x\sqrt{a(\sin(e+fx)+1)}\left((4-fx)\sin\left(\frac{1}{2}(e+fx)\right)+(fx+4)\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2a^2f^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3} + \frac{\sqrt[4]{-1}e^{-\frac{3}{2}i(e+fx)}\left(e^{i(e+fx)}+i\right)^3\left(-4ifx\text{PolyL}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((-1)^(1/4)*(I + E^(I*(e + f*x)))^3*(16*ArcTanh[(-1)^(1/4)*E^((I/2)*(e + f*
x))] - f^2*x^2*Log[1 - (-1)^(1/4)*E^((I/2)*(e + f*x))] + f^2*x^2*Log[1 + (-
1)^(1/4)*E^((I/2)*(e + f*x))] - (4*I)*f*x*PolyLog[2, -((-1)^(1/4)*E^((I/2)*
(e + f*x)))] + (4*I)*f*x*PolyLog[2, (-1)^(1/4)*E^((I/2)*(e + f*x))] + 8*Pol
yLog[3, -((-1)^(1/4)*E^((I/2)*(e + f*x)))] - 8*PolyLog[3, (-1)^(1/4)*E^((I/
2)*(e + f*x))]))/(2*Sqrt[2]*E^(((3*I)/2)*(e + f*x))*(((-I)*a*(I + E^(I*(e +
 f*x)))^2)/E^(I*(e + f*x)))^(3/2)*f^3) - (x*((4 + f*x)*Cos[(e + f*x)/2] + (
4 - f*x)*Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])])/(2*a^2*f^2*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int x^{2} \left(a + a \sin \left(f x + e\right)\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x^2/(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(a \sin \left(f x + e\right) + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{a \sin \left(f x + e\right) + a} x^{2}}{a^{2} \cos \left(f x + e\right)^{2} - 2 \, a^{2} \sin \left(f x + e\right) - 2 \, a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*x^2/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x
+ e) - 2*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**2/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(a*sin(f*x + e) + a)^(3/2), x)
```

3.141 $\int \frac{x}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=249

$$\frac{i\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{i\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{1}{af^2\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(1/(a*f^2*Sqrt[a + a*Sin[e + f*x]])) - (x*Cot[e/2 + Pi/4 + (f*x)/2])/(2*a*
f*Sqrt[a + a*Sin[e + f*x]]) - (x*ArcTanh[E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[
e/2 + Pi/4 + (f*x)/2])/(a*f*Sqrt[a + a*Sin[e + f*x]]) + (I*PolyLog[2, -E^((
I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^2*Sqrt[a + a*Sin[
e + f*x]]) - (I*PolyLog[2, E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (
f*x)/2])/(a*f^2*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.127779, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3319, 4185, 4183, 2279, 2391}

$$\frac{i\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{i\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)}{af^2\sqrt{a\sin(e+fx)+a}}-\frac{1}{af^2\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(1/(a*f^2*Sqrt[a + a*Sin[e + f*x]])) - (x*Cot[e/2 + Pi/4 + (f*x)/2])/(2*a*
f*Sqrt[a + a*Sin[e + f*x]]) - (x*ArcTanh[E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[
e/2 + Pi/4 + (f*x)/2])/(a*f*Sqrt[a + a*Sin[e + f*x]]) + (I*PolyLog[2, -E^((
I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (f*x)/2])/(a*f^2*Sqrt[a + a*Sin[
e + f*x]]) - (I*PolyLog[2, E^((I/4)*(2*e + Pi + 2*f*x))]*Sin[e/2 + Pi/4 + (
f*x)/2])/(a*f^2*Sqrt[a + a*Sin[e + f*x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\int x\csc^3\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)dx}{2a\sqrt{a+a\sin(e+fx)}}$$
$$= -\frac{1}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} + \frac{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)\int x\csc\left(\frac{e}{2}+\frac{\pi}{4}+\right.}{4a\sqrt{a+a\sin(e+fx)}}$$
$$= -\frac{1}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{x\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}\right.}{af\sqrt{a+a\sin(e+fx)}}$$
$$= -\frac{1}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{x\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}\right.}{af\sqrt{a+a\sin(e+fx)}}$$
$$= -\frac{1}{af^2\sqrt{a+a\sin(e+fx)}} - \frac{x\cot\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{x\tanh^{-1}\left(e^{\frac{1}{4}i(2e+\pi+2fx)}\right)\sin\left(\frac{e}{2}+\frac{\pi}{4}\right.}{af\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 2.54506, size = 308, normalized size = 1.24

$$\frac{\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(2i\left(\text{PolyLog}\left(2,-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)-\text{PolyLog}\left(2,e^{\frac{1}{4}i(2e+2fx+\pi)}\right)\right)+\frac{1}{2}(2e+2fx+\pi)\left(\log\left(1-e^{\frac{1}{4}i(2e+2fx+\pi)}\right)-\log\left(1+e^{\frac{1}{4}i(2e+2fx+\pi)}\right)\right)-\right.}{\sqrt{2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*f*x*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - (2 + f*x)*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + ((-(Pi*ArcTanh[(-1 + Tan[(e + f*x)
/4])/Sqrt[2]]) + ((2*e + Pi + 2*f*x)*(Log[1 - E^((I/4)*(2*e + Pi + 2*f*x))]
 - Log[1 + E^((I/4)*(2*e + Pi + 2*f*x))]))/2 + (2*I)*(PolyLog[2, -E^((I/4)*
(2*e + Pi + 2*f*x))] - PolyLog[2, E^((I/4)*(2*e + Pi + 2*f*x))]))*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[2] + (e*ArcSin[Csc[(2*e + Pi + 2*f*x)/
4]]*(1 + Sin[e + f*x])*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[(-1 + Sin[e + f*x])/
(1 + Sin[e + f*x])])/(2*f^2*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int x\left(a+a\sin\left(fx+e\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x/(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\,x}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*x/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x +
e) - 2*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(a*sin(f*x + e) + a)^(3/2), x)
```

3.142 $\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x(a\sin(e+fx)+a)^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(x*(a + a*Sin[e + f*x])^(3/2)), x]
```

Rubi [A] time = 0.0833333, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + a*Sin[e + f*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/(x*(a + a*Sin[e + f*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx$$

Mathematica [A] time = 32.8651, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+a\sin(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + a*Sin[e + f*x])^(3/2)),x]
```

```
[Out] Integrate[1/(x*(a + a*Sin[e + f*x])^(3/2)), x]
```

Maple [A] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+a\sin\left(fx+e\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(1/x/(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}}{a^2x\cos\left(fx+e\right)^2-2\,a^2x\sin\left(fx+e\right)-2\,a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)/(a^2*x*cos(f*x + e)^2 - 2*a^2*x*sin(f*x
+ e) - 2*a^2*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(x*(a*(sin(e + f*x) + 1))**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*x), x)
```

3.143 $\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^2(a\sin(e+fx)+a)^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(x^2*(a + a*Sin[e + f*x])^(3/2)), x]
```

Rubi [A] time = 0.08029, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(a + a*Sin[e + f*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/(x^2*(a + a*Sin[e + f*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx$$

Mathematica [A] time = 17.4709, size = 0, normalized size = 0.

$$\int \frac{1}{x^2(a+a\sin(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(a + a*Sin[e + f*x])^(3/2)),x]
```

```
[Out] Integrate[1/(x^2*(a + a*Sin[e + f*x])^(3/2)), x]
```

Maple [A] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \left(a + a\sin\left(fx + e\right)\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(1/x^2/(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx + e\right) + a\right)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx + e\right) + a}}{a^2x^2\cos\left(fx + e\right)^2 - 2\,a^2x^2\sin\left(fx + e\right) - 2\,a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)/(a^2*x^2*cos(f*x + e)^2 - 2*a^2*x^2*sin(
f*x + e) - 2*a^2*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a \left(\sin \left(e + fx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a*(sin(e + f*x) + 1))**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \sin \left(fx + e\right) + a\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*x^2), x)
```

3.144 $\int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{\sqrt[3]{a\sin(c+dx)+a}}{x},x\right)$$

```
[Out] Unintegrable[(a + a*Sin[c + d*x])^(1/3)/x, x]
```

Rubi [A] time = 0.0689206, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + a*Sin[c + d*x])^(1/3)/x,x]

[Out] Defer[Int][(a + a*Sin[c + d*x])^(1/3)/x, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx=\int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx$$

Mathematica [A] time = 2.97031, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+a\sin(c+dx)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[c + d*x])^(1/3)/x,x]

[Out] Integrate[(a + a*Sin[c + d*x])^(1/3)/x, x]
```

Maple [A] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt[3]{a+a\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/3)/x,x)
```

```
[Out] int((a+a*sin(d*x+c))^(1/3)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(1/3)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a\left(\sin\left(c+dx\right)+1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/3)/x,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(1/3)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(1/3)/x, x)
```

3.145 $\int (c+dx)^m (a+a\sin(e+fx))^n\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left((c+dx)^m (a\sin(e+fx)+a)^n, x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*(a + a*Sin[e + f*x])^n, x]
```

Rubi [A] time = 0.0477629, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m (a+a\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + a*Sin[e + f*x])^n,x]

[Out] Defer[Int][(c + d*x)^m*(a + a*Sin[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m (a+a\sin(e+fx))^n\,dx = \int (c+dx)^m (a+a\sin(e+fx))^n\,dx$$

Mathematica [A] time = 1.09577, size = 0, normalized size = 0.

$$\int (c+dx)^m (a+a\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + a*Sin[e + f*x])^n,x]

[Out] Integrate[(c + d*x)^m*(a + a*Sin[e + f*x])^n, x]
```

Maple [A] time = 0.329, size = 0, normalized size = 0.

$$\int (dx+c)^m \left(a+a\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*sin(f*x+e))^n,x)
```

```
[Out] int((d*x+c)^m*(a+a*sin(f*x+e))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \left(a\sin\left(fx+e\right)+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*(a*sin(f*x + e) + a)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx+c)^m\left(a\sin\left(fx+e\right)+a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*(a*sin(f*x + e) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a\sin\left(fx+e\right)+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*(a*sin(f*x + e) + a)^n, x)
```

3.146 $\int (c+dx)^m (a+a\sin(e+fx))^3\,dx$

Optimal. Leaf size=449

$$-\frac{15a^3e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{8f}+\frac{3ia^32^{-m-3}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m\right.}{f}$$

```
[Out] (5*a^3*(c + d*x)^(1 + m))/(2*d*(1 + m)) - (15*a^3*E^(I*(e - (c*f)/d))*(c +
d*x)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d])/(8*f*(((-I)*f*(c + d*x))/d)^m) -
 (15*a^3*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(8*E^(I*(e - (c*f)/d)
)*f*((I*f*(c + d*x))/d)^m) + ((3*I)*2^(-3 - m)*a^3*E^((2*I)*(e - (c*f)/d))*
(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)
^m) - ((3*I)*2^(-3 - m)*a^3*(c + d*x)^m*Gamma[1 + m, ((2*I)*f*(c + d*x))/d]
)/(E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + (3^(-1 - m)*a^3*E^((3
*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*f*(c + d*x))/d])/(8*f*(
((-I)*f*(c + d*x))/d)^m) + (3^(-1 - m)*a^3*(c + d*x)^m*Gamma[1 + m, ((3*I)*
f*(c + d*x))/d])/(8*E^((3*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)
```

Rubi [A] time = 0.605078, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3318, 3312, 3307, 2181, 3308}

$$-\frac{15a^3e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{8f}+\frac{3ia^32^{-m-3}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + a*Sin[e + f*x])^3,x]
```


```
[Out] (5*a^3*(c + d*x)^(1 + m))/(2*d*(1 + m)) - (15*a^3*E^(I*(e - (c*f)/d))*(c +
d*x)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d])/(8*f*(((-I)*f*(c + d*x))/d)^m) -
 (15*a^3*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(8*E^(I*(e - (c*f)/d)
)*f*((I*f*(c + d*x))/d)^m) + ((3*I)*2^(-3 - m)*a^3*E^((2*I)*(e - (c*f)/d))*
(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)
^m) - ((3*I)*2^(-3 - m)*a^3*(c + d*x)^m*Gamma[1 + m, ((2*I)*f*(c + d*x))/d]
)/(E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + (3^(-1 - m)*a^3*E^((3
*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*f*(c + d*x))/d])/(8*f*(
((-I)*f*(c + d*x))/d)^m) + (3^(-1 - m)*a^3*(c + d*x)^m*Gamma[1 + m, ((3*I)*
f*(c + d*x))/d])/(8*E^((3*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\int (c+dx)^m(a+a\sin(e+fx))^3\,dx = \left(8a^3\right)\int (c+dx)^m \sin^6\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)dx$$

$$= \left(8a^3\right)\int \left(\frac{5}{16}(c+dx)^m - \frac{3}{16}(c+dx)^m\cos(2e+2fx) + \frac{15}{32}(c+dx)^m\sin(e+fx) - \frac{1}{32}\right.$$

$$= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)} - \frac{1}{4}a^3\int (c+dx)^m\sin(3e+3fx)\,dx - \frac{1}{2}\left(3a^3\right)\int (c+dx)^m\cos(2e+$$

$$= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)} - \frac{1}{8}\left(ia^3\right)\int e^{-i(3e+3fx)}(c+dx)^m\,dx + \frac{1}{8}\left(ia^3\right)\int e^{i(3e+3fx)}(c+dx)^m\,d$$

$$= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)} - \frac{15a^3e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{8f} - \frac{15a^3e^{-i}}{}$$

Mathematica [A] time = 0.844823, size = 376, normalized size = 0.84

$$\frac{1}{24}a^3(c+dx)^m\left(-\frac{45e^{i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{f} + \frac{9i2^{-m}e^{2i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2if}{}\right.}{f}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(c + d*x)^m*((60*(c + d*x))/(d*(1 + m)) - (45*E^(I*(e - (c*f)/d))*Gamm
a[1 + m, ((-I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (45*Gamma[1
+ m, (I*f*(c + d*x))/d])/(E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + ((
9*I)*E^((2*I)*(e - (c*f)/d))*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(2^m*f*(
((-I)*f*(c + d*x))/d)^m) - ((9*I)*Gamma[1 + m, ((2*I)*f*(c + d*x))/d])/(2^m
*E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + (E^((3*I)*(e - (c*f)/d)
)*Gamma[1 + m, ((-3*I)*f*(c + d*x))/d])/(3^m*f*(((-I)*f*(c + d*x))/d)^m) +
Gamma[1 + m, ((3*I)*f*(c + d*x))/d]/(3^m*E^((3*I)*(e - (c*f)/d))*f*((I*f*(c
 + d*x))/d)^m)))/24
```

Maple [F] time = 0.279, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+a\sin\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((d*x+c)^m*(a+a*sin(f*x+e))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.1351, size = 934, normalized size = 2.08

$$\left(a^3dm + a^3d\right)e^{\left(-\frac{dm\log\left(\frac{3if}{d}\right)+3ide-3icf}{d}\right)}\Gamma\left(m+1, \frac{3idfx+3icf}{d}\right) + \left(-9ia^3dm - 9ia^3d\right)e^{\left(-\frac{dm\log\left(\frac{2if}{d}\right)+2ide-2icf}{d}\right)}\Gamma\left(m+1, \frac{2idfx+2icf}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*((a^3*d*m + a^3*d)*e^(-(d*m*log(3*I*f/d) + 3*I*d*e - 3*I*c*f)/d)*gamma
(m + 1, (3*I*d*f*x + 3*I*c*f)/d) + (-9*I*a^3*d*m - 9*I*a^3*d)*e^(-(d*m*log(
2*I*f/d) + 2*I*d*e - 2*I*c*f)/d)*gamma(m + 1, (2*I*d*f*x + 2*I*c*f)/d) - 45
*(a^3*d*m + a^3*d)*e^(-(d*m*log(I*f/d) + I*d*e - I*c*f)/d)*gamma(m + 1, (I*
d*f*x + I*c*f)/d) - 45*(a^3*d*m + a^3*d)*e^(-(d*m*log(-I*f/d) - I*d*e + I*c
*f)/d)*gamma(m + 1, (-I*d*f*x - I*c*f)/d) + (9*I*a^3*d*m + 9*I*a^3*d)*e^(-(
d*m*log(-2*I*f/d) - 2*I*d*e + 2*I*c*f)/d)*gamma(m + 1, (-2*I*d*f*x - 2*I*c*
f)/d) + (a^3*d*m + a^3*d)*e^(-(d*m*log(-3*I*f/d) - 3*I*d*e + 3*I*c*f)/d)*ga
mma(m + 1, (-3*I*d*f*x - 3*I*c*f)/d) + 60*(a^3*d*f*x + a^3*c*f)*(d*x + c)^m
)/(d*f*m + d*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^3 (dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*x + c)^m, x)
```

3.147 $\int (c+dx)^m(a+a\sin(e+fx))^2\,dx$

Optimal. Leaf size=299

$$-\frac{a^2e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{f}+\frac{ia^22^{-m-3}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+\right.}{f}$$

```
[Out] (3*a^2*(c + d*x)^(1 + m))/(2*d*(1 + m)) - (a^2*E^(I*(e - (c*f)/d))*(c + d*x
)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (a^2
*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(E^(I*(e - (c*f)/d))*f*((I*f*
(c + d*x))/d)^m) + (I*2^(-3 - m)*a^2*E^((2*I)*(e - (c*f)/d))*(c + d*x)^m*Ga
mma[1 + m, ((-2*I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (I*2^(-3
 - m)*a^2*(c + d*x)^m*Gamma[1 + m, ((2*I)*f*(c + d*x))/d])/(E^((2*I)*(e - (
c*f)/d))*f*((I*f*(c + d*x))/d)^m)
```

Rubi [A] time = 0.369239, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3318, 3312, 3307, 2181, 3308}

$$-\frac{a^2e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{f}+\frac{ia^22^{-m-3}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + a*Sin[e + f*x])^2,x]
```



```
[Out] (3*a^2*(c + d*x)^(1 + m))/(2*d*(1 + m)) - (a^2*E^(I*(e - (c*f)/d))*(c + d*x
)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (a^2
*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(E^(I*(e - (c*f)/d))*f*((I*f*
(c + d*x))/d)^m) + (I*2^(-3 - m)*a^2*E^((2*I)*(e - (c*f)/d))*(c + d*x)^m*Ga
mma[1 + m, ((-2*I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (I*2^(-3
 - m)*a^2*(c + d*x)^m*Gamma[1 + m, ((2*I)*f*(c + d*x))/d])/(E^((2*I)*(e - (
c*f)/d))*f*((I*f*(c + d*x))/d)^m)
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
```

```
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\int (c+dx)^m(a+a\sin(e+fx))^2\,dx = \left(4a^2\right)\int (c+dx)^m \sin^4\left(\frac{1}{2}\left(e+\frac{\pi}{2}\right)+\frac{fx}{2}\right)dx$$
$$= \left(4a^2\right)\int \left(\frac{3}{8}(c+dx)^m - \frac{1}{8}(c+dx)^m\cos(2e+2fx)+\frac{1}{2}(c+dx)^m\sin(e+fx)\right)dx$$
$$= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)} - \frac{1}{2}a^2\int (c+dx)^m\cos(2e+2fx)\,dx + \left(2a^2\right)\int (c+dx)^m\sin(e+f$$
$$= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)} + \left(ia^2\right)\int e^{-i(e+fx)}(c+dx)^m\,dx - \left(ia^2\right)\int e^{i(e+fx)}(c+dx)^m\,dx - \frac{1}{4}a$$
$$= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)} - \frac{a^2e^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{f} - \frac{a^2e^{-i\left(e-\right.}}{}$$

Mathematica [A] time = 0.284949, size = 260, normalized size = 0.87

$$\frac{1}{8}a^2(c+dx)^m\left(-\frac{8e^{i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{f}+\frac{i2^{-m}e^{2i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2if(c\right.}{d}}{f}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(c + d*x)^m*((12*(c + d*x))/(d*(1 + m)) - (8*E^(I*(e - (c*f)/d))*Gamma
[1 + m, ((-I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (8*Gamma[1 +
m, (I*f*(c + d*x))/d])/(E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + (I*E
^((2*I)*(e - (c*f)/d))*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(2^m*f*(((-I)*
f*(c + d*x))/d)^m) - (I*Gamma[1 + m, ((2*I)*f*(c + d*x))/d])/(2^m*E^((2*I)*
(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)))/8
```

Maple [F] time = 0.193, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+a\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+a*sin(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97924, size = 636, normalized size = 2.13

$$\frac{\left(-i\,a^2dm - i\,a^2d\right)e^{\left(-\frac{dm\log\left(\frac{2i\,f}{d}\right)+2i\,de-2i\,cf}{d}\right)}\Gamma\left(m+1, \frac{2i\,dfx+2i\,cf}{d}\right) - 8\left(a^2dm + a^2d\right)e^{\left(-\frac{dm\log\left(\frac{i\,f}{d}\right)+i\,de-i\,cf}{d}\right)}\Gamma\left(m+1, \frac{i\,dfx+i\,cf}{d}\right) - 8\left(a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((-I*a^2*d*m - I*a^2*d)*e^(-(d*m*log(2*I*f/d) + 2*I*d*e - 2*I*c*f)/d)*g
amma(m + 1, (2*I*d*f*x + 2*I*c*f)/d) - 8*(a^2*d*m + a^2*d)*e^(-(d*m*log(I*f
/d) + I*d*e - I*c*f)/d)*gamma(m + 1, (I*d*f*x + I*c*f)/d) - 8*(a^2*d*m + a^
2*d)*e^(-(d*m*log(-I*f/d) - I*d*e + I*c*f)/d)*gamma(m + 1, (-I*d*f*x - I*c*
f)/d) + (I*a^2*d*m + I*a^2*d)*e^(-(d*m*log(-2*I*f/d) - 2*I*d*e + 2*I*c*f)/d
)*gamma(m + 1, (-2*I*d*f*x - 2*I*c*f)/d) + 12*(a^2*d*f*x + a^2*c*f)*(d*x +
c)^m)/(d*f*m + d*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\,(c+dx)^m\sin\left(e+fx\right)dx + \int (c+dx)^m\sin^2\left(e+fx\right)dx + \int (c+dx)^m\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*sin(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(2*(c + d*x)**m*sin(e + f*x), x) + Integral((c + d*x)**m*sin(
e + f*x)**2, x) + Integral((c + d*x)**m, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^2 (dx+c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*x + c)^m, x)
```

3.148 $\int (c+dx)^m (a+a\sin(e+fx))\,dx$

Optimal. Leaf size=148

$$-\frac{ae^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{2f}-\frac{ae^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if(c+dx)}{d}\right)}{2f}$$

```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) - (a*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gamm
a[1 + m, ((-I)*f*(c + d*x))/d])/(2*f*(((-I)*f*(c + d*x))/d)^m) - (a*(c + d*
x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(2*E^(I*(e - (c*f)/d))*f*((I*f*(c + d
*x))/d)^m)
```

Rubi [A] time = 0.144218, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3308, 2181}

$$-\frac{ae^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{2f}-\frac{ae^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if(c+dx)}{d}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + a*Sin[e + f*x]),x]
```


```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) - (a*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gamm
a[1 + m, ((-I)*f*(c + d*x))/d])/(2*f*(((-I)*f*(c + d*x))/d)^m) - (a*(c + d*
x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(2*E^(I*(e - (c*f)/d))*f*((I*f*(c + d
*x))/d)^m)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m(a+a\sin(e+fx))\,dx &= \int \left(a(c+dx)^m + a(c+dx)^m\sin(e+fx)\right)\,dx\\ &= \frac{a(c+dx)^{1+m}}{d(1+m)} + a\int (c+dx)^m\sin(e+fx)\,dx\\ &= \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{1}{2}(ia)\int e^{-i(e+fx)}(c+dx)^m\,dx - \frac{1}{2}(ia)\int e^{i(e+fx)}(c+dx)^m\,dx\\ &= \frac{a(c+dx)^{1+m}}{d(1+m)} - \frac{ae^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{2f} - \frac{ae^{-i\left(e-\frac{cf}{d}\right)}(c - \cdots}{\cdots}\end{aligned}$$

Mathematica [A] time = 2.71381, size = 199, normalized size = 1.34

$$-\frac{a(c+dx)^m(\sin(e+fx)+1)\left(d(m+1)\left(-\frac{if(c+dx)}{d}\right)^{-m}\left(\cos\left(e-\frac{cf}{d}\right)+i\sin\left(e-\frac{cf}{d}\right)\right)\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)+d(m+\cdots}{2df(m+1)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + a*Sin[e + f*x]),x]
```

```
[Out] -(a*(c + d*x)^m*(2*d*e - 2*c*f - 2*d*(e + f*x) + (d*(1 + m)*Gamma[1 + m, (I
*f*(c + d*x))/d]*(Cos[e - (c*f)/d] - I*Sin[e - (c*f)/d]))/((I*f*(c + d*x))/
d)^m + (d*(1 + m)*Gamma[1 + m, ((-I)*f*(c + d*x))/d]*(Cos[e - (c*f)/d] + I*
Sin[e - (c*f)/d]))/(((-I)*f*(c + d*x))/d)^m)*(1 + Sin[e + f*x]))/(2*d*f*(1
+ m)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int (dx+c)^m \left(a+a\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*sin(f*x+e)),x)
```

```
[Out] int((d*x+c)^m*(a+a*sin(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89878, size = 319, normalized size = 2.16

$$-\frac{(adm+ad)e^{\left(-\frac{dm\log\left(\frac{if}{d}\right)+ide-icf}{d}\right)}\Gamma\left(m+1,\frac{idfx+icf}{d}\right)+(adm+ad)e^{\left(-\frac{dm\log\left(-\frac{if}{d}\right)-ide+icf}{d}\right)}\Gamma\left(m+1,\frac{-idfx-icf}{d}\right)-2\left(adfx+acf\right)(}{2\left(dfm+df\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*((a*d*m + a*d)*e^(-(d*m*log(I*f/d) + I*d*e - I*c*f)/d)*gamma(m + 1, (I
*d*f*x + I*c*f)/d) + (a*d*m + a*d)*e^(-(d*m*log(-I*f/d) - I*d*e + I*c*f)/d)
*gamma(m + 1, (-I*d*f*x - I*c*f)/d) - 2*(a*d*f*x + a*c*f)*(d*x + c)^m)/(d*f
*m + d*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int (c+dx)^m \sin\left(e+fx\right)dx+\int (c+dx)^m\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*sin(f*x+e)),x)
```

```
[Out] a*(Integral((c + d*x)**m*sin(e + f*x), x) + Integral((c + d*x)**m, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*x + c)^m, x)
```

3.149 $\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(c+dx)^m}{a\sin(e+fx)+a},x\right)$$

```
[Out] Unintegrable[(c + d*x)^m/(a + a*Sin[e + f*x]), x]
```

Rubi [A] time = 0.0550764, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + a*Sin[e + f*x]),x]

[Out] Defer[Int][(c + d*x)^m/(a + a*Sin[e + f*x]), x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx$$

Mathematica [A] time = 0.880904, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{a+a\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + a*Sin[e + f*x]),x]

[Out] Integrate[(c + d*x)^m/(a + a*Sin[e + f*x]), x]
```

Maple [A] time = 0.1, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+a*sin(f*x+e)),x)
```

```
[Out] int((d*x+c)^m/(a+a*sin(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(a*sin(f*x + e) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^m}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m/(a*sin(f*x + e) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(c+dx)^m}{\sin(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**m/(sin(e + f*x) + 1), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{a\sin(fx+e)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(a*sin(f*x + e) + a), x)
```

3.150 $\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(c+dx)^m}{(a\sin(e+fx)+a)^2},x\right)$$

```
[Out] Unintegrable[(c + d*x)^m/(a + a*Sin[e + f*x])^2, x]
```

Rubi [A] time = 0.0530645, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + a*Sin[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx=\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx$$

Mathematica [A] time = 9.12508, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*x)^m/(a + a*Sin[e + f*x])^2, x]
```

Maple [A] time = 0.137, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(a+a\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m/(a+a*sin(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(a\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(a*sin(f*x + e) + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(dx+c)^m}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*x + c)^m/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(c+dx)^m}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**m/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x)/a**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(a\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(a*sin(f*x + e) + a)^2, x)
```

3.151 $\int (c+dx)^3(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=90

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cos(e+fx)}{f^3}+\frac{3bd(c+dx)^2\sin(e+fx)}{f^2}-\frac{b(c+dx)^3\cos(e+fx)}{f}-\frac{6bd^3\sin(e+fx)}{f^4}$$

```
[Out] (a*(c + d*x)^4)/(4*d) + (6*b*d^2*(c + d*x)*Cos[e + f*x])/f^3 - (b*(c + d*x)
^3*Cos[e + f*x])/f - (6*b*d^3*Sin[e + f*x])/f^4 + (3*b*d*(c + d*x)^2*Sin[e
+ f*x])/f^2
```

Rubi [A] time = 0.122539, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3296, 2637}

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cos(e+fx)}{f^3}+\frac{3bd(c+dx)^2\sin(e+fx)}{f^2}-\frac{b(c+dx)^3\cos(e+fx)}{f}-\frac{6bd^3\sin(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) + (6*b*d^2*(c + d*x)*Cos[e + f*x])/f^3 - (b*(c + d*x)
^3*Cos[e + f*x])/f - (6*b*d^3*Sin[e + f*x])/f^4 + (3*b*d*(c + d*x)^2*Sin[e
+ f*x])/f^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^3(a+b\sin(e+fx))\,dx = \int \left(a(c+dx)^3+b(c+dx)^3\sin(e+fx)\right)\,dx$$
$$= \frac{a(c+dx)^4}{4d}+b\int (c+dx)^3\sin(e+fx)\,dx$$
$$= \frac{a(c+dx)^4}{4d}-\frac{b(c+dx)^3\cos(e+fx)}{f}+\frac{(3bd)\int (c+dx)^2\cos(e+fx)\,dx}{f}$$
$$= \frac{a(c+dx)^4}{4d}-\frac{b(c+dx)^3\cos(e+fx)}{f}+\frac{3bd(c+dx)^2\sin(e+fx)}{f^2}-\frac{\left(6bd^2\right)\int (c+dx)}{f^2}$$
$$= \frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cos(e+fx)}{f^3}-\frac{b(c+dx)^3\cos(e+fx)}{f}+\frac{3bd(c+dx)^2\sin}{f^2}$$
$$= \frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cos(e+fx)}{f^3}-\frac{b(c+dx)^3\cos(e+fx)}{f}-\frac{6bd^3\sin(e+fx)}{f^4}$$

Mathematica [A] time = 0.432017, size = 124, normalized size = 1.38

$$\frac{1}{4}ax\left(6c^2dx+4c^3+4cd^2x^2+d^3x^3\right)+\frac{3bd\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2-2\right)\right)\sin(e+fx)}{f^4}-\frac{b(c+dx)\left(c^2f^2+2cdf^2x+d^2\right.}{f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - (b*(c + d*x)*(c^2*f^2
 + 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Cos[e + f*x])/f^3 + (3*b*d*(c^2*f^2 +
2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x])/f^4
```

Maple [B] time = 0.011, size = 482, normalized size = 5.4

$$\frac{1}{f}\left(\frac{ad^3\left(fx+e\right)^4}{4f^3}+\frac{acd^2\left(fx+e\right)^3}{f^2}-\frac{ad^3e\left(fx+e\right)^3}{f^3}+\frac{3ac^2d\left(fx+e\right)^2}{2f}-3\frac{acd^2e\left(fx+e\right)^2}{f^2}+\frac{3ad^3e^2\left(fx+e\right)^2}{2f^3}+ac^3\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*(1/4*a/f^3*d^3*(f*x+e)^4+a/f^2*c*d^2*(f*x+e)^3-a/f^3*d^3*e*(f*x+e)^3+3/
2*a/f*c^2*d*(f*x+e)^2-3*a/f^2*c*d^2*e*(f*x+e)^2+3/2*a/f^3*d^3*e^2*(f*x+e)^2
+a*c^3*(f*x+e)-3*a/f*c^2*d*e*(f*x+e)+3*a/f^2*c*d^2*e^2*(f*x+e)-a/f^3*d^3*e^
3*(f*x+e)+1/f^3*b*d^3*(-(f*x+e)^3*cos(f*x+e)+3*(f*x+e)^2*sin(f*x+e)-6*sin(f
*x+e)+6*(f*x+e)*cos(f*x+e))+3/f^2*b*c*d^2*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+
e)+2*(f*x+e)*sin(f*x+e))-3/f^3*b*d^3*e*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+
2*(f*x+e)*sin(f*x+e))+3/f*b*c^2*d*(sin(f*x+e)-(f*x+e)*cos(f*x+e))-6/f^2*b*c
*d^2*e*(sin(f*x+e)-(f*x+e)*cos(f*x+e))+3/f^3*b*d^3*e^2*(sin(f*x+e)-(f*x+e)*
cos(f*x+e))-c^3*b*cos(f*x+e)+3/f*b*c^2*d*e*cos(f*x+e)-3/f^2*b*c*d^2*e^2*cos
(f*x+e)+1/f^3*b*d^3*e^3*cos(f*x+e))
```

Maxima [B] time = 1.05, size = 624, normalized size = 6.93

$$\frac{4\left(fx+e\right)ac^3 + \frac{\left(fx+e\right)^4ad^3}{f^3} - \frac{4\left(fx+e\right)^3ad^3e}{f^3} + \frac{6\left(fx+e\right)^2ad^3e^2}{f^3} - \frac{4\left(fx+e\right)ad^3e^3}{f^3} + \frac{4\left(fx+e\right)^3acd^2}{f^2} - \frac{12\left(fx+e\right)^2acd^2e}{f^2} + \frac{12\left(fx+e\right)acd^2e^2}{f^2} + 6\left(fx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c^3 + (f*x + e)^4*a*d^3/f^3 - 4*(f*x + e)^3*a*d^3*e/f^3
+ 6*(f*x + e)^2*a*d^3*e^2/f^3 - 4*(f*x + e)*a*d^3*e^3/f^3 + 4*(f*x + e)^3*a
*c*d^2/f^2 - 12*(f*x + e)^2*a*c*d^2*e/f^2 + 12*(f*x + e)*a*c*d^2*e^2/f^2 +
6*(f*x + e)^2*a*c^2*d/f - 12*(f*x + e)*a*c^2*d*e/f - 4*b*c^3*cos(f*x + e) +
 4*b*d^3*e^3*cos(f*x + e)/f^3 - 12*b*c*d^2*e^2*cos(f*x + e)/f^2 + 12*b*c^2*
d*e*cos(f*x + e)/f - 12*((f*x + e)*cos(f*x + e) - sin(f*x + e))*b*d^3*e^2/f
^3 + 24*((f*x + e)*cos(f*x + e) - sin(f*x + e))*b*c*d^2*e/f^2 - 12*((f*x +
e)*cos(f*x + e) - sin(f*x + e))*b*c^2*d/f + 12*(((f*x + e)^2 - 2)*cos(f*x +
 e) - 2*(f*x + e)*sin(f*x + e))*b*d^3*e/f^3 - 12*(((f*x + e)^2 - 2)*cos(f*x
 + e) - 2*(f*x + e)*sin(f*x + e))*b*c*d^2/f^2 - 4*(((f*x + e)^3 - 6*f*x - 6
*e)*cos(f*x + e) - 3*((f*x + e)^2 - 2)*sin(f*x + e))*b*d^3/f^3)/f
```

Fricas [A] time = 1.69688, size = 362, normalized size = 4.02

$$\frac{ad^3f^4x^4 + 4\,acd^2f^4x^3 + 6\,ac^2df^4x^2 + 4\,ac^3f^4x - 4\left(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + bc^3f^3 - 6\,bcd^2f + 3\left(bc^2df^3 - 2\,bd^3f\right)x\right)\cos}{4\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
- 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + b*c^3*f^3 - 6*b*c*d^2*f + 3*(b*c^2
*d*f^3 - 2*b*d^3*f)*x)*cos(f*x + e) + 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x +
 b*c^2*d*f^2 - 2*b*d^3)*sin(f*x + e))/f^4
```

Sympy [A] time = 1.75372, size = 264, normalized size = 2.93

$$\begin{cases} ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4} - \frac{bc^3\cos\left(e+fx\right)}{f} - \frac{3bc^2dx\cos\left(e+fx\right)}{f} + \frac{3bc^2d\sin\left(e+fx\right)}{f^2} - \frac{3bcd^2x^2\cos\left(e+fx\right)}{f} + \frac{6bcd^2x\sin\left(e+fx\right)}{f^2} + \frac{6bc \\ (a+b\sin(e))\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 - b
*c**3*cos(e + f*x)/f - 3*b*c**2*d*x*cos(e + f*x)/f + 3*b*c**2*d*sin(e + f*x
)/f**2 - 3*b*c*d**2*x**2*cos(e + f*x)/f + 6*b*c*d**2*x*sin(e + f*x)/f**2 +
6*b*c*d**2*cos(e + f*x)/f**3 - b*d**3*x**3*cos(e + f*x)/f + 3*b*d**3*x**2*s
in(e + f*x)/f**2 + 6*b*d**3*x*cos(e + f*x)/f**3 - 6*b*d**3*sin(e + f*x)/f**
4, Ne(f, 0)), ((a + b*sin(e))*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**
3*x**4/4), True))
```

Giac [A] time = 1.46284, size = 212, normalized size = 2.36

$$\frac{1}{4}ad^3x^4 + acd^2x^3 + \frac{3}{2}ac^2dx^2 + ac^3x - \frac{\left(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + 3\,bc^2df^3x + bc^3f^3 - 6\,bd^3fx - 6\,bcd^2f\right)\cos\left(fx+e\right)}{f^4} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x - (b*d^3*f^3*x^3 +
3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*c^3*f^3 - 6*b*d^3*f*x - 6*b*c*d^2*f
)*cos(f*x + e)/f^4 + 3*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2 - 2*b
```

```
*d^3)*sin(f*x + e)/f^4
```

3.152 $\int (c+dx)^2(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=68

$$\frac{a(c+dx)^3}{3d}+\frac{2bd(c+dx)\sin(e+fx)}{f^2}-\frac{b(c+dx)^2\cos(e+fx)}{f}+\frac{2bd^2\cos(e+fx)}{f^3}$$

```
[Out] (a*(c + d*x)^3)/(3*d) + (2*b*d^2*Cos[e + f*x])/f^3 - (b*(c + d*x)^2*Cos[e +
 f*x])/f + (2*b*d*(c + d*x)*Sin[e + f*x])/f^2
```

Rubi [A] time = 0.0856717, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3296, 2638}

$$\frac{a(c+dx)^3}{3d}+\frac{2bd(c+dx)\sin(e+fx)}{f^2}-\frac{b(c+dx)^2\cos(e+fx)}{f}+\frac{2bd^2\cos(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) + (2*b*d^2*Cos[e + f*x])/f^3 - (b*(c + d*x)^2*Cos[e +
 f*x])/f + (2*b*d*(c + d*x)*Sin[e + f*x])/f^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^2(a+b\sin(e+fx))\,dx &= \int \left(a(c+dx)^2+b(c+dx)^2\sin(e+fx)\right)\,dx\\
&= \frac{a(c+dx)^3}{3d}+b\int (c+dx)^2\sin(e+fx)\,dx\\
&= \frac{a(c+dx)^3}{3d}-\frac{b(c+dx)^2\cos(e+fx)}{f}+\frac{(2bd)\int (c+dx)\cos(e+fx)\,dx}{f}\\
&= \frac{a(c+dx)^3}{3d}-\frac{b(c+dx)^2\cos(e+fx)}{f}+\frac{2bd(c+dx)\sin(e+fx)}{f^2}-\frac{\left(2bd^2\right)\int \sin(e+fx
\end{aligned}
$$
$$
= \frac{a(c+dx)^3}{3d}+\frac{2bd^2\cos(e+fx)}{f^3}-\frac{b(c+dx)^2\cos(e+fx)}{f}+\frac{2bd(c+dx)\sin(e+fx)}{f^2}
$$

Mathematica [A] time = 0.313986, size = 84, normalized size = 1.24

$$
\frac{1}{3}ax\left(3c^2+3cdx+d^2x^2\right)-\frac{b\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2-2\right)\right)\cos(e+fx)}{f^3}+\frac{2bd(c+dx)\sin(e+fx)}{f^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*x*(3*c^2 + 3*c*d*x + d^2*x^2))/3 - (b*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-2 +
 f^2*x^2))*Cos[e + f*x])/f^3 + (2*b*d*(c + d*x)*Sin[e + f*x])/f^2
```

Maple [B] time = 0.008, size = 241, normalized size = 3.5

$$
\frac{1}{f}\left(\frac{ad^2\left(fx+e\right)^3}{3f^2}+\frac{acd\left(fx+e\right)^2}{f}-\frac{ad^2e\left(fx+e\right)^2}{f^2}+ac^2\left(fx+e\right)-2\frac{acde\left(fx+e\right)}{f}+\frac{ad^2e^2\left(fx+e\right)}{f^2}+\frac{bd^2\left(-\left(fx+e\right)^2\right.}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*(1/3*a/f^2*d^2*(f*x+e)^3+a/f*c*d*(f*x+e)^2-a/f^2*d^2*e*(f*x+e)^2+a*c^2*
(f*x+e)-2*a/f*c*d*e*(f*x+e)+a/f^2*d^2*e^2*(f*x+e)+1/f^2*b*d^2*(-(f*x+e)^2*c
os(f*x+e)+2*cos(f*x+e)+2*(f*x+e)*sin(f*x+e))+2/f*b*c*d*(sin(f*x+e)-(f*x+e)*
```

```
cos(f*x+e))-2/f^2*b*d^2*e*(sin(f*x+e)-(f*x+e)*cos(f*x+e))-c^2*b*cos(f*x+e)+
2/f*b*c*d*e*cos(f*x+e)-1/f^2*b*d^2*e^2*cos(f*x+e))
```

Maxima [B] time = 0.994145, size = 323, normalized size = 4.75

$$\frac{3\left(fx+e\right)ac^2+\frac{\left(fx+e\right)^3ad^2}{f^2}-\frac{3\left(fx+e\right)^2ad^2e}{f^2}+\frac{3\left(fx+e\right)ad^2e^2}{f^2}+\frac{3\left(fx+e\right)^2acd}{f}-\frac{6\left(fx+e\right)acde}{f}-3\,bc^2\cos\left(fx+e\right)-\frac{3\,bd^2e^2\cos\left(fx+e\right)}{f^2}+}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a*c^2 + (f*x + e)^3*a*d^2/f^2 - 3*(f*x + e)^2*a*d^2*e/f^2
+ 3*(f*x + e)*a*d^2*e^2/f^2 + 3*(f*x + e)^2*a*c*d/f - 6*(f*x + e)*a*c*d*e/f
 - 3*b*c^2*cos(f*x + e) - 3*b*d^2*e^2*cos(f*x + e)/f^2 + 6*b*c*d*e*cos(f*x
+ e)/f + 6*((f*x + e)*cos(f*x + e) - sin(f*x + e))*b*d^2*e/f^2 - 6*((f*x +
e)*cos(f*x + e) - sin(f*x + e))*b*c*d/f - 3*(((f*x + e)^2 - 2)*cos(f*x + e)
 - 2*(f*x + e)*sin(f*x + e))*b*d^2/f^2)/f
```

Fricas [A] time = 1.68959, size = 228, normalized size = 3.35

$$\frac{ad^2f^3x^3+3\,acdf^3x^2+3\,ac^2f^3x-3\left(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2-2\,bd^2\right)\cos\left(fx+e\right)+6\left(bd^2fx+bcdf\right)\sin\left(fx+e\right)}{3\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x - 3*(b*d^2*f^2*x^2 + 2
*b*c*d*f^2*x + b*c^2*f^2 - 2*b*d^2)*cos(f*x + e) + 6*(b*d^2*f*x + b*c*d*f)*
sin(f*x + e))/f^3
```

Sympy [A] time = 0.813716, size = 151, normalized size = 2.22

$$\begin{cases} ac^2x+acdx^2+\frac{ad^2x^3}{3}-\frac{bc^2\cos\left(e+fx\right)}{f}-\frac{2bcdx\cos\left(e+fx\right)}{f}+\frac{2bcd\sin\left(e+fx\right)}{f^2}-\frac{bd^2x^2\cos\left(e+fx\right)}{f}+\frac{2bd^2x\sin\left(e+fx\right)}{f^2}+\frac{2bd^2\cos\left(e+fx\right)}{f^3} & \text{fo} \\ \left(a+b\sin\left(e\right)\right)\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) & \text{o} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 - b*c**2*cos(e + f*x)/f -
2*b*c*d*x*cos(e + f*x)/f + 2*b*c*d*sin(e + f*x)/f**2 - b*d**2*x**2*cos(e +
f*x)/f + 2*b*d**2*x*sin(e + f*x)/f**2 + 2*b*d**2*cos(e + f*x)/f**3, Ne(f, 0
)), ((a + b*sin(e))*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Giac [A] time = 1.49884, size = 128, normalized size = 1.88

$$\frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x - \frac{\left(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 - 2\,bd^2\right)\cos\left(fx + e\right)}{f^3} + \frac{2\left(bd^2fx + bcdf\right)\sin\left(fx + e\right)}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x - (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^
2*f^2 - 2*b*d^2)*cos(f*x + e)/f^3 + 2*(b*d^2*f*x + b*c*d*f)*sin(f*x + e)/f^
3
```

3.153 $\int (c+dx)(a+b\sin (e+fx))\,dx$

Optimal. Leaf size=45

$$\frac{a(c+dx)^2}{2d}-\frac{b(c+dx)\cos (e+fx)}{f}+\frac{bd\sin (e+fx)}{f^2}$$

```
[Out] (a*(c + d*x)^2)/(2*d) - (b*(c + d*x)*Cos[e + f*x])/f + (b*d*Sin[e + f*x])/f
^2
```

Rubi [A] time = 0.0423611, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3317, 3296, 2637}

$$\frac{a(c+dx)^2}{2d}-\frac{b(c+dx)\cos (e+fx)}{f}+\frac{bd\sin (e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) - (b*(c + d*x)*Cos[e + f*x])/f + (b*d*Sin[e + f*x])/f
^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)(a+b\sin(e+fx))\,dx = \int (a(c+dx)+b(c+dx)\sin(e+fx))\,dx$$
$$= \frac{a(c+dx)^2}{2d} + b\int (c+dx)\sin(e+fx)\,dx$$
$$= \frac{a(c+dx)^2}{2d} - \frac{b(c+dx)\cos(e+fx)}{f} + \frac{(bd)\int \cos(e+fx)\,dx}{f}$$
$$= \frac{a(c+dx)^2}{2d} - \frac{b(c+dx)\cos(e+fx)}{f} + \frac{bd\sin(e+fx)}{f^2}$$

Mathematica [A] time = 0.109109, size = 43, normalized size = 0.96

$$\frac{1}{2}ax(2c+dx) - \frac{b(c+dx)\cos(e+fx)}{f} + \frac{bd\sin(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*x*(2*c + d*x))/2 - (b*(c + d*x)*Cos[e + f*x])/f + (b*d*Sin[e + f*x])/f^2
```

Maple [B] time = 0.006, size = 90, normalized size = 2.

$$\frac{1}{f}\left(\frac{da\left(fx+e\right)^2}{2f} + ac\left(fx+e\right) - \frac{ade\left(fx+e\right)}{f} + \frac{bd\left(\sin\left(fx+e\right) - \left(fx+e\right)\cos\left(fx+e\right)\right)}{f} - cb\cos\left(fx+e\right) + \frac{bde\cos\left(fx\right.}{f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*(1/2*a/f*d*(f*x+e)^2+a*c*(f*x+e)-a/f*d*e*(f*x+e)+1/f*b*d*(sin(f*x+e)-(f
*x+e)*cos(f*x+e))-c*b*cos(f*x+e)+1/f*b*d*e*cos(f*x+e))
```

Maxima [B] time = 0.972822, size = 126, normalized size = 2.8

$$\frac{2\left(fx+e\right)ac+\frac{\left(fx+e\right)^{2}ad}{f}-\frac{2\left(fx+e\right)ade}{f}-2\,bc\cos\left(fx+e\right)+\frac{2\,bde\cos\left(fx+e\right)}{f}-\frac{2\left(\left(fx+e\right)\cos\left(fx+e\right)-\sin\left(fx+e\right)\right)bd}{f}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(f*x + e)*a*c + (f*x + e)^2*a*d/f - 2*(f*x + e)*a*d*e/f - 2*b*c*cos(
f*x + e) + 2*b*d*e*cos(f*x + e)/f - 2*((f*x + e)*cos(f*x + e) - sin(f*x + e
))*b*d/f)/f
```

Fricas [A] time = 1.62692, size = 126, normalized size = 2.8

$$\frac{adf^{2}x^{2}+2\,acf^{2}x+2\,bd\sin\left(fx+e\right)-2\left(bdfx+bcf\right)\cos\left(fx+e\right)}{2\,f^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x + 2*b*d*sin(f*x + e) - 2*(b*d*f*x + b*c*f)*c
os(f*x + e))/f^2
```

Sympy [A] time = 0.342548, size = 68, normalized size = 1.51

$$\begin{cases} acx+\frac{adx^{2}}{2}-\frac{bc\cos\left(e+fx\right)}{f}-\frac{bdx\cos\left(e+fx\right)}{f}+\frac{bd\sin\left(e+fx\right)}{f^{2}} & \text{for}\: f\neq 0\\ \left(a+b\sin\left(e\right)\right)\left(cx+\frac{dx^{2}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x + a*d*x**2/2 - b*c*cos(e + f*x)/f - b*d*x*cos(e + f*x)/f +
 b*d*sin(e + f*x)/f**2, Ne(f, 0)), ((a + b*sin(e))*(c*x + d*x**2/2), True))
```

Giac [A] time = 1.75076, size = 63, normalized size = 1.4

$$\frac{1}{2}\,adx^2 + acx + \frac{bd\sin\left(fx+e\right)}{f^2} - \frac{\left(bdfx+bcf\right)\cos\left(fx+e\right)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + b*d*sin(f*x + e)/f^2 - (b*d*f*x + b*c*f)*cos(f*x + e)
/f^2
```

3.154 $\int \frac{a+b\sin(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=64

$$\frac{a\log(c+dx)}{d}+\frac{b\operatorname{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cos\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(xf+\frac{cf}{d}\right)}{d}$$

```
[Out] (a*Log[c + d*x])/d + (b*CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d + (b
*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d
```

Rubi [A] time = 0.123518, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3317, 3303, 3299, 3302}

$$\frac{a\log(c+dx)}{d}+\frac{b\operatorname{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cos\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(xf+\frac{cf}{d}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*x),x]
```

```
[Out] (a*Log[c + d*x])/d + (b*CosIntegral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d + (b
*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin(e+fx)}{c+dx}\,dx &= \int \left(\frac{a}{c+dx}+\frac{b\sin(e+fx)}{c+dx}\right)dx\\
&= \frac{a\log(c+dx)}{d}+b\int \frac{\sin(e+fx)}{c+dx}\,dx\\
&= \frac{a\log(c+dx)}{d}+\left(b\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx+\left(b\sin\left(e-\frac{cf}{d}\right)\right)\int \frac{\cos\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx\\
&= \frac{a\log(c+dx)}{d}+\frac{b\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.150375, size = 57, normalized size = 0.89

$$\frac{a\log(c+dx)+b\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+b\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*x),x]
```

```
[Out] (a*Log[c + d*x] + b*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + b*Cos[e - (
c*f)/d]*SinIntegral[f*(c/d + x)])/d
```

Maple [A] time = 0.01, size = 96, normalized size = 1.5

$$\frac{a\ln\left(\left(fx+e\right)d+cf-de\right)}{d}+\frac{b}{d}\text{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)-\frac{b}{d}\text{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(d*x+c),x)
```

```
[Out] a*ln((f*x+e)*d+c*f-d*e)/d+b*Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-b*Ci(f
*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d
```

Maxima [C] time = 1.23759, size = 231, normalized size = 3.61

$$\frac{\frac{2af\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d}+\frac{\left(f\left(-iE_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)+iE_1\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f\left(E_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)+E_1\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\sin\left(-\frac{de-cf}{d}\right)\right)}{d}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*f*log(c + (f*x + e)*d/f - d*e/f)/d + (f*(-I*exp_integral_e(1, (I*(
f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(1, -(I*(f*x + e)*d - I*d*
e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f*(exp_integral_e(1, (I*(f*x + e)*d -
I*d*e + I*c*f)/d) + exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*
sin(-(d*e - c*f)/d))*b/d)/f
```

Fricas [A] time = 1.76993, size = 234, normalized size = 3.66

$$\frac{2\,b\cos\left(-\frac{de-cf}{d}\right)\operatorname{Si}\left(\frac{dfx+cf}{d}\right)+2\,a\log\left(dx+c\right)-\left(b\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)+b\operatorname{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\sin\left(-\frac{de-cf}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*cos(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/d) + 2*a*log(d*x +
c) - (b*cos_integral((d*f*x + c*f)/d) + b*cos_integral(-(d*f*x + c*f)/d))*s
in(-(d*e - c*f)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\sin\left(e + fx\right)}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c),x)
```

```
[Out] Integral((a + b*sin(e + f*x))/(c + d*x), x)
```

Giac [C] time = 1.92385, size = 961, normalized size = 15.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 -
 b*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*
a*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b*sin_integral((d*f*x
 + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*b*real_part(cos_integral(f*x +
 c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*b*real_part(cos_integral(-f*x - c*
f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) + 2*b*real_part(cos_integral(f*x + c*f/d)
)*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*b*real_part(cos_integral(-f*x - c*f/d))*t
an(1/2*c*f/d)*tan(1/2*e)^2 - b*imag_part(cos_integral(f*x + c*f/d))*tan(1/2
*c*f/d)^2 + b*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2 + 2*a*
log(abs(d*x + c))*tan(1/2*c*f/d)^2 - 2*b*sin_integral((d*f*x + c*f)/d)*tan(
1/2*c*f/d)^2 + 4*b*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(
1/2*e) - 4*b*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e
) + 8*b*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e) - b*imag_pa
rt(cos_integral(f*x + c*f/d))*tan(1/2*e)^2 + b*imag_part(cos_integral(-f*x
- c*f/d))*tan(1/2*e)^2 + 2*a*log(abs(d*x + c))*tan(1/2*e)^2 - 2*b*sin_integ
ral((d*f*x + c*f)/d)*tan(1/2*e)^2 - 2*b*real_part(cos_integral(f*x + c*f/d)
)*tan(1/2*c*f/d) - 2*b*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)
 + 2*b*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e) + 2*b*real_part(cos_
integral(-f*x - c*f/d))*tan(1/2*e) + b*imag_part(cos_integral(f*x + c*f/d))
 - b*imag_part(cos_integral(-f*x - c*f/d)) + 2*a*log(abs(d*x + c)) + 2*b*si
n_integral((d*f*x + c*f)/d))/(d*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + d*tan(1/2*c
*f/d)^2 + d*tan(1/2*e)^2 + d)
```

3.155 $\int \frac{a+b\sin(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=88

$$-\frac{a}{d(c+dx)}+\frac{bf\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d^2}-\frac{bf\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{b\sin(e+fx)}{d(c+dx)}$$

```
[Out] -(a/(d*(c + d*x))) + (b*f*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d^2
- (b*Sin[e + f*x])/(d*(c + d*x)) - (b*f*Sin[e - (c*f)/d]*SinIntegral[(c*f)/
d + f*x])/d^2
```

Rubi [A] time = 0.155229, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3317, 3297, 3303, 3299, 3302}

$$-\frac{a}{d(c+dx)}+\frac{bf\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d^2}-\frac{bf\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{b\sin(e+fx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) + (b*f*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d^2
- (b*Sin[e + f*x])/(d*(c + d*x)) - (b*f*Sin[e - (c*f)/d]*SinIntegral[(c*f)/
d + f*x])/d^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin(e+fx)}{(c+dx)^2}\,dx &= \int \left(\frac{a}{(c+dx)^2}+\frac{b\sin(e+fx)}{(c+dx)^2}\right)dx \\
&= -\frac{a}{d(c+dx)}+b\int \frac{\sin(e+fx)}{(c+dx)^2}\,dx \\
&= -\frac{a}{d(c+dx)}-\frac{b\sin(e+fx)}{d(c+dx)}+\frac{(bf)\int \frac{\cos(e+fx)}{c+dx}\,dx}{d} \\
&= -\frac{a}{d(c+dx)}-\frac{b\sin(e+fx)}{d(c+dx)}+\frac{\left(bf\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\cos\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d}-\frac{\left(bf\sin\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+fx\right)}{c+dx}}{d} \\
&= -\frac{a}{d(c+dx)}+\frac{bf\cos\left(e-\frac{cf}{d}\right)\mathrm{Ci}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{b\sin(e+fx)}{d(c+dx)}-\frac{bf\sin\left(e-\frac{cf}{d}\right)\mathrm{Si}\left(\frac{cf}{d}+fx\right)}{d^2}
\end{aligned}
$$

Mathematica [A] time = 0.34858, size = 72, normalized size = 0.82

$$
\frac{-\frac{d(a+b\sin(e+fx))}{c+dx}+bf\,\mathrm{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\cos\left(e-\frac{cf}{d}\right)-bf\sin\left(e-\frac{cf}{d}\right)\mathrm{Si}\left(f\left(\frac{c}{d}+x\right)\right)}{d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*x)^2,x]
```

```
[Out] (b*f*Cos[e - (c*f)/d]*CosIntegral[f*(c/d + x)] - (d*(a + b*Sin[e + f*x]))/(
c + d*x) - b*f*Sin[e - (c*f)/d]*SinIntegral[f*(c/d + x)])/d^2
```

Maple [A] time = 0.012, size = 141, normalized size = 1.6

$$\frac{1}{f}\left(-\frac{f^2 a}{\left(\left(fx+e\right)d+cf-de\right)d}+f^2 b\left(-\frac{\sin\left(fx+e\right)}{\left(\left(fx+e\right)d+cf-de\right)d}+\frac{1}{d}\left(\frac{1}{d}\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)+\frac{1}{d}\mathrm{Ci}\left(fx\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(d*x+c)^2,x)
```

```
[Out] 1/f*(-a*f^2/((f*x+e)*d+c*f-d*e)/d+f^2*b*(-sin(f*x+e)/((f*x+e)*d+c*f-d*e)/d+
(Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+Ci(f*x+e+(c*f-d*e)/d)*cos((c*f-d*
e)/d)/d)/d))
```

Maxima [C] time = 1.27957, size = 265, normalized size = 3.01

$$-\frac{\frac{2af^2}{\left(fx+e\right)d^2-d^2e+cdf}-\frac{\left(f^2\left(-iE_2\left(\frac{i\left(fx+e\right)d-ide+icf}{d}\right)+iE_2\left(-\frac{i\left(fx+e\right)d-ide+icf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f^2\left(E_2\left(\frac{i\left(fx+e\right)d-ide+icf}{d}\right)+E_2\left(-\frac{i\left(fx+e\right)d-ide+icf}{d}\right)\right)\sin\left(-\frac{de-cf}{d}\right)\right.}{\left(fx+e\right)d^2-d^2e+cdf}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*f^2/((f*x + e)*d^2 - d^2*e + c*d*f) - (f^2*(-I*exp_integral_e(2,
(I*(f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(2, -(I*(f*x + e)*d -
I*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f^2*(exp_integral_e(2, (I*(f*x + e
)*d - I*d*e + I*c*f)/d) + exp_integral_e(2, -(I*(f*x + e)*d - I*d*e + I*c*f
)/d))*sin(-(d*e - c*f)/d))*b/((f*x + e)*d^2 - d^2*e + c*d*f))/f
```

Fricas [A] time = 1.8318, size = 332, normalized size = 3.77

$$-\frac{2\,bd\sin\left(fx+e\right)-2\left(bdfx+bcf\right)\sin\left(-\frac{de-cf}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)+2\,ad-\left(\left(bdfx+bcf\right)\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)+\left(bdfx+bcf\right)\mathrm{Ci}\left(-\frac{d\ldots}{}\right.\right.}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*sin(f*x + e) - 2*(b*d*f*x + b*c*f)*sin(-(d*e - c*f)/d)*sin_inte
gral((d*f*x + c*f)/d) + 2*a*d - ((b*d*f*x + b*c*f)*cos_integral((d*f*x + c*
f)/d) + (b*d*f*x + b*c*f)*cos_integral(-(d*f*x + c*f)/d))*cos(-(d*e - c*f)/
d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\sin\left(e + fx\right)}{(c + dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))/(c + d*x)**2, x)
```

Giac [C] time = 1.47167, size = 4251, normalized size = 48.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(d*f*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/
d)^2*tan(1/2*e)^2 + d*f*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x
)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*d*f*x*imag_part(cos_integral(f*x + c*
f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*d*f*x*imag_part(cos_in
tegral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) + 4*d*f*x*
sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) -
2*d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*
tan(1/2*e)^2 + 2*d*f*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2
*tan(1/2*c*f/d)*tan(1/2*e)^2 - 4*d*f*x*sin_integral((d*f*x + c*f)/d)*tan(1/
2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + c*f*real_part(cos_integral(f*x + c*f
/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + c*f*real_part(cos_integ
```

ral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - d*f*x*rea
l_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 - d*f*x*r
eal_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + 4*d*
f*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(
1/2*e) + 4*d*f*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1
/2*c*f/d)*tan(1/2*e) + 2*c*f*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f
*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*c*f*imag_part(cos_integral(-f*x - c*f
/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) + 4*c*f*sin_integral((d*f*x
 + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - d*f*x*real_part(cos
_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 - d*f*x*real_part(cos_i
ntegral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*c*f*imag_part(cos_in
tegral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*c*f*ima
g_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)
^2 - 4*c*f*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(
1/2*e)^2 + d*f*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(
1/2*e)^2 + d*f*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan
(1/2*e)^2 + 2*d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan
(1/2*c*f/d) - 2*d*f*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*
tan(1/2*c*f/d) + 4*d*f*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1
/2*c*f/d) - c*f*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2
*c*f/d)^2 - c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/
2*c*f/d)^2 - 2*d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*ta
n(1/2*e) + 2*d*f*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan
(1/2*e) - 4*d*f*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e) +
 4*c*f*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*t
an(1/2*e) + 4*c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(
1/2*c*f/d)*tan(1/2*e) + 2*d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/
2*c*f/d)^2*tan(1/2*e) - 2*d*f*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1
/2*c*f/d)^2*tan(1/2*e) + 4*d*f*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/
d)^2*tan(1/2*e) - c*f*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*t
an(1/2*e)^2 - c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(
1/2*e)^2 - 2*d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(
1/2*e)^2 + 2*d*f*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan
(1/2*e)^2 - 4*d*f*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e)
^2 + c*f*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2
 + c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2
+ d*f*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2 + d*f*x*real_pa
rt(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2 + 2*c*f*imag_part(cos_integra
l(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*c*f*imag_part(cos_integra
l(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) + 4*c*f*sin_integral((d*f*x
+ c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d) - d*f*x*real_part(cos_integral(f*x
+ c*f/d))*tan(1/2*c*f/d)^2 - d*f*x*real_part(cos_integral(-f*x - c*f/d))*ta
n(1/2*c*f/d)^2 - 2*c*f*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*
tan(1/2*e) + 2*c*f*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan
(1/2*e) - 4*c*f*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e) + 4

```
*d*f*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) + 4*d
*f*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) + 2*c*
f*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*c*f*
imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) + 4*c*f*s
in_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e) + 4*d*tan(1/2*f*x)
^2*tan(1/2*c*f/d)^2*tan(1/2*e) - d*f*x*real_part(cos_integral(f*x + c*f/d))
*tan(1/2*e)^2 - d*f*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2 -
2*c*f*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*
c*f*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 - 4*c
*f*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e)^2 + 4*d*tan(1/2*
f*x)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + c*f*real_part(cos_integral(f*x + c*f/d
))*tan(1/2*f*x)^2 + c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^
2 + 2*d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 2*d*f*x*i
mag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) + 4*d*f*x*sin_integral(
(d*f*x + c*f)/d)*tan(1/2*c*f/d) - c*f*real_part(cos_integral(f*x + c*f/d))*
tan(1/2*c*f/d)^2 - c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)
^2 - 2*d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e) + 2*d*f*x*imag
_part(cos_integral(-f*x - c*f/d))*tan(1/2*e) - 4*d*f*x*sin_integral((d*f*x
+ c*f)/d)*tan(1/2*e) + 4*c*f*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c
*f/d)*tan(1/2*e) + 4*c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/
d)*tan(1/2*e) - c*f*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2 - c*f
*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2 + d*f*x*real_part(cos_i
ntegral(f*x + c*f/d)) + d*f*x*real_part(cos_integral(-f*x - c*f/d)) + 2*c*f
*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 2*c*f*imag_part(cos_
integral(-f*x - c*f/d))*tan(1/2*c*f/d) + 4*c*f*sin_integral((d*f*x + c*f)/d
)*tan(1/2*c*f/d) - 4*d*tan(1/2*f*x)*tan(1/2*c*f/d)^2 - 2*c*f*imag_part(cos_
integral(f*x + c*f/d))*tan(1/2*e) + 2*c*f*imag_part(cos_integral(-f*x - c*f
/d))*tan(1/2*e) - 4*c*f*sin_integral((d*f*x + c*f)/d)*tan(1/2*e) + 4*d*tan(
1/2*f*x)^2*tan(1/2*e) - 4*d*tan(1/2*c*f/d)^2*tan(1/2*e) + 4*d*tan(1/2*f*x)*
tan(1/2*e)^2 + c*f*real_part(cos_integral(f*x + c*f/d)) + c*f*real_part(cos
_integral(-f*x - c*f/d)) - 4*d*tan(1/2*f*x) - 4*d*tan(1/2*e))*b/(d^3*x*tan(
1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + c*d^2*tan(1/2*f*x)^2*tan(1/2*c*f
/d)^2*tan(1/2*e)^2 + d^3*x*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + d^3*x*tan(1/2*
f*x)^2*tan(1/2*e)^2 + d^3*x*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + c*d^2*tan(1/2*f
*x)^2*tan(1/2*c*f/d)^2 + c*d^2*tan(1/2*f*x)^2*tan(1/2*e)^2 + c*d^2*tan(1/2*
c*f/d)^2*tan(1/2*e)^2 + d^3*x*tan(1/2*f*x)^2 + d^3*x*tan(1/2*c*f/d)^2 + d^3
*x*tan(1/2*e)^2 + c*d^2*tan(1/2*f*x)^2 + c*d^2*tan(1/2*c*f/d)^2 + c*d^2*tan
(1/2*e)^2 + d^3*x + c*d^2) - a/((d*x + c)*d)
```

3.156 $\int \frac{a+b\sin(e+fx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=123

$$-\frac{a}{2d(c+dx)^2}-\frac{bf^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{bf^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{bf\cos(e+fx)}{2d^2(c+dx)}-\frac{b\sin(e+f}{2d(c+dx)}$$

```
[Out] -a/(2*d*(c + d*x)^2) - (b*f*Cos[e + f*x])/(2*d^2*(c + d*x)) - (b*f^2*CosInt
egral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/(2*d^3) - (b*Sin[e + f*x])/(2*d*(c +
 d*x)^2) - (b*f^2*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/(2*d^3)
```

Rubi [A] time = 0.189929, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3317, 3297, 3303, 3299, 3302}

$$-\frac{a}{2d(c+dx)^2}-\frac{bf^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{bf^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{bf\cos(e+fx)}{2d^2(c+dx)}-\frac{b\sin(e+f}{2d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*x)^3,x]
```


```
[Out] -a/(2*d*(c + d*x)^2) - (b*f*Cos[e + f*x])/(2*d^2*(c + d*x)) - (b*f^2*CosInt
egral[(c*f)/d + f*x]*Sin[e - (c*f)/d])/(2*d^3) - (b*Sin[e + f*x])/(2*d*(c +
 d*x)^2) - (b*f^2*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/(2*d^3)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin(e+fx)}{(c+dx)^3}\,dx &= \int \left(\frac{a}{(c+dx)^3}+\frac{b\sin(e+fx)}{(c+dx)^3}\right)dx\\
&= -\frac{a}{2d(c+dx)^2}+b\int \frac{\sin(e+fx)}{(c+dx)^3}\,dx\\
&= -\frac{a}{2d(c+dx)^2}-\frac{b\sin(e+fx)}{2d(c+dx)^2}+\frac{(bf)\int \frac{\cos(e+fx)}{(c+dx)^2}\,dx}{2d}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{bf\cos(e+fx)}{2d^2(c+dx)}-\frac{b\sin(e+fx)}{2d(c+dx)^2}-\frac{\left(bf^2\right)\int \frac{\sin(e+fx)}{c+dx}\,dx}{2d^2}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{bf\cos(e+fx)}{2d^2(c+dx)}-\frac{b\sin(e+fx)}{2d(c+dx)^2}-\frac{\left(bf^2\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{2d^2}-\frac{\left(bf^2\sin\right.}{}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{bf\cos(e+fx)}{2d^2(c+dx)}-\frac{bf^2\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{b\sin(e+fx)}{2d(c+dx)^2}-\frac{bf^2\cos\left(e-\right.}{}\end{aligned}$$

Mathematica [A] time = 0.814259, size = 94, normalized size = 0.76

$$-\frac{\frac{d(d(a+b\sin(e+fx))+bf(c+dx)\cos(e+fx))}{(c+dx)^2}+bf^2\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+bf^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*x)^3,x]
```

```
[Out] -(b*f^2*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d] + (d*(b*f*(c + d*x)*Cos[e
 + f*x] + d*(a + b*Sin[e + f*x])))/(c + d*x)^2 + b*f^2*Cos[e - (c*f)/d]*Sin
Integral[f*(c/d + x)])/(2*d^3)
```

Maple [A] time = 0.01, size = 177, normalized size = 1.4

$$\frac{1}{f}\left(-\frac{f^3 a}{2\left(\left(fx+e\right)d+cf-de\right)^2 d}+f^3 b\left(-\frac{\sin\left(fx+e\right)}{2\left(\left(fx+e\right)d+cf-de\right)^2 d}+\frac{1}{2d}\left(-\frac{\cos\left(fx+e\right)}{\left(\left(fx+e\right)d+cf-de\right)d}-\frac{1}{d}\left(\frac{1}{d}\mathrm{Si}\left(fx+\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(d*x+c)^3,x)
```

```
[Out] 1/f*(-1/2*a*f^3/((f*x+e)*d+c*f-d*e)^2/d+f^3*b*(-1/2*sin(f*x+e)/((f*x+e)*d+c
*f-d*e)^2/d+1/2*(-cos(f*x+e)/((f*x+e)*d+c*f-d*e)/d-(Si(f*x+e+(c*f-d*e)/d)*c
os((c*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)/d)/d))
```

Maxima [C] time = 1.46564, size = 358, normalized size = 2.91

$$-\frac{\frac{af^3}{\left(fx+e\right)^2 d^3+d^3e^2-2\,cd^2ef+c^2df^2-2\left(d^3e-cd^2f\right)\left(fx+e\right)}-\frac{\left(f^3\left(-i\,E_3\left(\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)+i\,E_3\left(-\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f^3\left(E_3\left(\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)+E_3 \cdots\right.\right.}{\left(fx+e\right)^2 d^3+d^3e^2-2\,cd^2ef+c^2df^2-2\left(d^3e-cd^2f\right)\left(fx+e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*f^3/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*(d^3*e
 - c*d^2*f)*(f*x + e)) - (f^3*(-I*exp_integral_e(3, (I*(f*x + e)*d - I*d*e
+ I*c*f)/d) + I*exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*cos(
-(d*e - c*f)/d) + f^3*(exp_integral_e(3, (I*(f*x + e)*d - I*d*e + I*c*f)/d)
 + exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*sin(-(d*e - c*f)/
d))*b/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*(d^3*e - c*d
^2*f)*(f*x + e)))/f
```

Fricas [A] time = 2.16258, size = 517, normalized size = 4.2

$$-\frac{2\,bd^2\sin\left(fx+e\right)+2\,ad^2+2\left(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2\right)\cos\left(-\frac{de-cf}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)+2\left(bd^2fx+bcdf\right)\cos\left(fx+e\right)-}{4\left(d^5x^2+2\,cd^4x+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*d^2*sin(f*x + e) + 2*a*d^2 + 2*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b
*c^2*f^2)*cos(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/d) + 2*(b*d^2*f*x
+ b*c*d*f)*cos(f*x + e) - ((b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*cos_
integral((d*f*x + c*f)/d) + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*cos
_integral(-(d*f*x + c*f)/d))*sin(-(d*e - c*f)/d))/(d^5*x^2 + 2*c*d^4*x + c^
2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\sin\left(e+fx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sin(e + f*x))/(c + d*x)**3, x)
```

Giac [C] time = 1.4657, size = 8312, normalized size = 67.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan
(1/2*c*f/d)^2*tan(1/2*e)^2 - b*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*
```

f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b*d^2*f^2*x^2*sin_in
tegral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*b*
d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f
/d)^2*tan(1/2*e) - 2*b*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*ta
n(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) + 2*b*d^2*f^2*x^2*real_part(cos_in
tegral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*b*d^2*f
^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*
tan(1/2*e)^2 + 2*b*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f
*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*b*c*d*f^2*x*imag_part(cos_integral(
-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*b*c*d*f^2*x
*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2
 - b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/
2*c*f/d)^2 + b*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*
x)^2*tan(1/2*c*f/d)^2 - 2*b*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1
/2*f*x)^2*tan(1/2*c*f/d)^2 + 4*b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c
*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) - 4*b*d^2*f^2*x^2*imag_part
(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) + 8*b
*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)*ta
n(1/2*e) - 4*b*c*d*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^
2*tan(1/2*c*f/d)^2*tan(1/2*e) - 4*b*c*d*f^2*x*real_part(cos_integral(-f*x -
 c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - b*d^2*f^2*x^2*imag_pa
rt(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 + b*d^2*f^2*x^2*i
mag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*b*d^2*
f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e)^2 + 4*b*c*d
*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*t
an(1/2*e)^2 + 4*b*c*d*f^2*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f
*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + b*d^2*f^2*x^2*imag_part(cos_integral(f*
x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - b*d^2*f^2*x^2*imag_part(cos_int
egral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b*d^2*f^2*x^2*sin_in
tegral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + b*c^2*f^2*imag_part
(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 -
b*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/
d)^2*tan(1/2*e)^2 + 2*b*c^2*f^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^
2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*b*d^2*f^2*x^2*real_part(cos_integral(f*
x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*b*d^2*f^2*x^2*real_part(cos_i
ntegral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*b*c*d*f^2*x*imag_p
art(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + 2*b*c*d*f^
2*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 -
 4*b*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^
2 + 2*b*d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan
(1/2*e) + 2*b*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x
)^2*tan(1/2*e) + 8*b*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2
*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) - 8*b*c*d*f^2*x*imag_part(cos_integral(-f
*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) + 16*b*c*d*f^2*x*sin_
integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) - 2*b*d^

2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*b*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*b*c^2*f^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*b*c^2*f^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*b*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*b*c*d*f^2*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 - 4*b*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*b*d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*b*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*b*c^2*f^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*b*c^2*f^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*b*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*b*c*d*f^2*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*b*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b*d^2*f*x*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2 - b*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2 + 2*b*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2 - 4*b*c*d*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 4*b*c*d*f^2*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2 + b*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2 - 2*b*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2 - b*c^2*f^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + b*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 - 2*b*c^2*f^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + 4*b*c*d*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e) + 4*b*c*d*f^2*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e) + 4*b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) - 4*b*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) + 8*b*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e) + 4*b*c^2*f^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) - 4*b*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) + 8*b*c^2*f^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e) - 4*b*c*d*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - 4*b*c*d*f^2*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2 + b*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2 - 2*b*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)^2 - b*c^2*f^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 + b*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*b*c^2*f^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e)^2 + 4*b*c*d*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*ta n(1/2*e)^2 + 4*b*c*d*f^2*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 + b*c^2*f^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - b*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b*c^2*f^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b*c*d*f*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2 - 2*b*c*d*f^2*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2 + 4*b*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2 - 2*b*d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 2*b*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) - 2*b*c^2*f^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*b*c^2*f^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*b*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2 + 2*b*c*d*f^2*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2 - 4*b*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2 - 2*b*d^2*f*x*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + 2*b*d^2*f^2*x^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e) + 2*b*d^2*f^2*x^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e) + 2*b*c^2*f^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e) + 2*b*c^2*f^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e) + 8*b*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) - 8*b*c*d*f^2*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) + 16*b*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e) - 2*b*c^2*f^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*b*c^2*f^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - 8*b*d^2*f*x*tan(1/2*f*x)*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*b*c*d*f^2*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2 + 2*b*c*d*f^2*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2 - 4*b*c*d*f^2*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)^2 + 2*b*d^2*f*x*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*b*c^2*f^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*b*c^2*f^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 - 2*b*d^2*f*x*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*d^2*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + b*d^2*f^2*x^2*imag_part(cos_integral(f*x + c*f/d)) - b*d^2*f^2*x^2*imag_part(cos_integral(-f*x - c*f/d)) + 2*b*d^2*f^2*x^2*sin_integral((d*f*x + c*f)/d) + b*c^2*f^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2 - b*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2 + 2*b*c^2*f^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2 - 4*b*c*d*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 4*b*c*d*f^2*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) - b*c^2*f^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2 + b*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2 - 2*b*c^2*f^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2 - 2*b*c*d*f*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + 4*b*c*d*f^2*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e) + 4*b*c*d*f^2*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e) + 4*b*c^2*f^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) - 4*b*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) + 8*b*c^2*f^

```
2*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e) - 8*b*c*d*f*tan(1
/2*f*x)*tan(1/2*c*f/d)^2*tan(1/2*e) - 4*b*d^2*tan(1/2*f*x)^2*tan(1/2*c*f/d)
^2*tan(1/2*e) - b*c^2*f^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2
 + b*c^2*f^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2 - 2*b*c^2*f
^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)^2 + 2*b*c*d*f*tan(1/2*f*x)^2*ta
n(1/2*e)^2 - 2*b*c*d*f*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 4*b*d^2*tan(1/2*f*x)
*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b*c*d*f^2*x*imag_part(cos_integral(f*x +
 c*f/d)) - 2*b*c*d*f^2*x*imag_part(cos_integral(-f*x - c*f/d)) + 4*b*c*d*f^
2*x*sin_integral((d*f*x + c*f)/d) - 2*b*d^2*f*x*tan(1/2*f*x)^2 - 2*b*c^2*f^
2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 2*b*c^2*f^2*real_pa
rt(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) + 2*b*d^2*f*x*tan(1/2*c*f/d)^
2 + 2*a*d^2*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + 2*b*c^2*f^2*real_part(cos_int
egral(f*x + c*f/d))*tan(1/2*e) + 2*b*c^2*f^2*real_part(cos_integral(-f*x -
c*f/d))*tan(1/2*e) - 8*b*d^2*f*x*tan(1/2*f*x)*tan(1/2*e) - 2*b*d^2*f*x*tan(
1/2*e)^2 + 2*a*d^2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*a*d^2*tan(1/2*c*f/d)^2*t
an(1/2*e)^2 + b*c^2*f^2*imag_part(cos_integral(f*x + c*f/d)) - b*c^2*f^2*im
ag_part(cos_integral(-f*x - c*f/d)) + 2*b*c^2*f^2*sin_integral((d*f*x + c*f
)/d) - 2*b*c*d*f*tan(1/2*f*x)^2 + 2*b*c*d*f*tan(1/2*c*f/d)^2 + 4*b*d^2*tan(
1/2*f*x)*tan(1/2*c*f/d)^2 - 8*b*c*d*f*tan(1/2*f*x)*tan(1/2*e) - 4*b*d^2*tan
(1/2*f*x)^2*tan(1/2*e) + 4*b*d^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*b*c*d*f*ta
n(1/2*e)^2 - 4*b*d^2*tan(1/2*f*x)*tan(1/2*e)^2 + 2*b*d^2*f*x + 2*a*d^2*tan(
1/2*f*x)^2 + 2*a*d^2*tan(1/2*c*f/d)^2 + 2*a*d^2*tan(1/2*e)^2 + 2*b*c*d*f +
4*b*d^2*tan(1/2*f*x) + 4*b*d^2*tan(1/2*e) + 2*a*d^2)/(d^5*x^2*tan(1/2*f*x)^
2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*c*d^4*x*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2
*tan(1/2*e)^2 + d^5*x^2*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + d^5*x^2*tan(1/2*f
*x)^2*tan(1/2*e)^2 + d^5*x^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + c^2*d^3*tan(1/
2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*c*d^4*x*tan(1/2*f*x)^2*tan(1/2*c
*f/d)^2 + 2*c*d^4*x*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*c*d^4*x*tan(1/2*c*f/d)^
2*tan(1/2*e)^2 + d^5*x^2*tan(1/2*f*x)^2 + d^5*x^2*tan(1/2*c*f/d)^2 + c^2*d^
3*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + d^5*x^2*tan(1/2*e)^2 + c^2*d^3*tan(1/2*
f*x)^2*tan(1/2*e)^2 + c^2*d^3*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*c*d^4*x*tan
(1/2*f*x)^2 + 2*c*d^4*x*tan(1/2*c*f/d)^2 + 2*c*d^4*x*tan(1/2*e)^2 + d^5*x^2
 + c^2*d^3*tan(1/2*f*x)^2 + c^2*d^3*tan(1/2*c*f/d)^2 + c^2*d^3*tan(1/2*e)^2
 + 2*c*d^4*x + c^2*d^3)
```

3.157 $\int (c+dx)^3(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=250

$$\frac{a^2(c+dx)^4}{4d}+\frac{12abd^2(c+dx)\cos(e+fx)}{f^3}+\frac{6abd(c+dx)^2\sin(e+fx)}{f^2}-\frac{2ab(c+dx)^3\cos(e+fx)}{f}-\frac{12abd^3\sin(e+fx}{f^4}$$

```
[Out] (-3*b^2*c*d^2*x)/(4*f^2) - (3*b^2*d^3*x^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d
) + (b^2*(c + d*x)^4)/(8*d) + (12*a*b*d^2*(c + d*x)*Cos[e + f*x])/f^3 - (2*
a*b*(c + d*x)^3*Cos[e + f*x])/f - (12*a*b*d^3*Sin[e + f*x])/f^4 + (6*a*b*d*
(c + d*x)^2*Sin[e + f*x])/f^2 + (3*b^2*d^2*(c + d*x)*Cos[e + f*x]*Sin[e + f
*x])/(4*f^3) - (b^2*(c + d*x)^3*Cos[e + f*x]*Sin[e + f*x])/(2*f) - (3*b^2*d
^3*Sin[e + f*x]^2)/(8*f^4) + (3*b^2*d*(c + d*x)^2*Sin[e + f*x]^2)/(4*f^2)
```

Rubi [A] time = 0.267118, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3317, 3296, 2637, 3311, 32, 3310}

$$\frac{a^2(c+dx)^4}{4d}+\frac{12abd^2(c+dx)\cos(e+fx)}{f^3}+\frac{6abd(c+dx)^2\sin(e+fx)}{f^2}-\frac{2ab(c+dx)^3\cos(e+fx)}{f}-\frac{12abd^3\sin(e+fx}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-3*b^2*c*d^2*x)/(4*f^2) - (3*b^2*d^3*x^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d
) + (b^2*(c + d*x)^4)/(8*d) + (12*a*b*d^2*(c + d*x)*Cos[e + f*x])/f^3 - (2*
a*b*(c + d*x)^3*Cos[e + f*x])/f - (12*a*b*d^3*Sin[e + f*x])/f^4 + (6*a*b*d*
(c + d*x)^2*Sin[e + f*x])/f^2 + (3*b^2*d^2*(c + d*x)*Cos[e + f*x]*Sin[e + f
*x])/(4*f^3) - (b^2*(c + d*x)^3*Cos[e + f*x]*Sin[e + f*x])/(2*f) - (3*b^2*d
^3*Sin[e + f*x]^2)/(8*f^4) + (3*b^2*d*(c + d*x)^2*Sin[e + f*x]^2)/(4*f^2)
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)^3(a+b\sin(e+fx))^2\,dx = \int \left(a^2(c+dx)^3 + 2ab(c+dx)^3\sin(e+fx) + b^2(c+dx)^3\sin^2(e+fx)\right)dx$$
$$= \frac{a^2(c+dx)^4}{4d} + (2ab)\int (c+dx)^3\sin(e+fx)\,dx + b^2\int (c+dx)^3\sin^2(e+fx)\,dx$$
$$= \frac{a^2(c+dx)^4}{4d} - \frac{2ab(c+dx)^3\cos(e+fx)}{f} - \frac{b^2(c+dx)^3\cos(e+fx)\sin(e+fx)}{2f} + 3b$$
$$= \frac{a^2(c+dx)^4}{4d} + \frac{b^2(c+dx)^4}{8d} - \frac{2ab(c+dx)^3\cos(e+fx)}{f} + \frac{6abd(c+dx)^2\sin(e+fx)}{f^2}$$
$$= -\frac{3b^2cd^2x}{4f^2} - \frac{3b^2d^3x^2}{8f^2} + \frac{a^2(c+dx)^4}{4d} + \frac{b^2(c+dx)^4}{8d} + \frac{12abd^2(c+dx)\cos(e+fx)}{f^3} -$$
$$= -\frac{3b^2cd^2x}{4f^2} - \frac{3b^2d^3x^2}{8f^2} + \frac{a^2(c+dx)^4}{4d} + \frac{b^2(c+dx)^4}{8d} + \frac{12abd^2(c+dx)\cos(e+fx)}{f^3} -$$

Mathematica [A] time = 1.298, size = 232, normalized size = 0.93

$$\frac{2f^4x\left(2a^2+b^2\right)\left(6c^2dx+4c^3+4cd^2x^2+d^3x^3\right)+96abd\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2-2\right)\right)\sin(e+fx)-32abf(c+dx)\left(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (2*(2*a^2 + b^2)*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 32*a*b
*f*(c + d*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Cos[e + f*x] - 3*
b^2*d*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(-1 + 2*f^2*x^2))*Cos[2*(e + f*x)] + 9
6*a*b*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x] - 2*b^2*f
*(c + d*x)*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(-3 + 2*f^2*x^2))*Sin[2*(e + f*x)
])/(16*f^4)
```

Maple [B] time = 0.016, size = 1125, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(a^2*c^3*(f*x+e)+b^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-6/f
^2*a*b*c*d^2*e^2*cos(f*x+e)+6/f*a*b*c^2*d*e*cos(f*x+e)-12/f^2*a*b*c*d^2*e*(
sin(f*x+e)-(f*x+e)*cos(f*x+e))+1/f^3*b^2*d^3*((f*x+e)^3*(-1/2*sin(f*x+e)*co
s(f*x+e)+1/2*f*x+1/2*e)-3/4*(f*x+e)^2*cos(f*x+e)^2+3/2*(f*x+e)*(1/2*sin(f*x
+e)*cos(f*x+e)+1/2*f*x+1/2*e)-3/8*(f*x+e)^2-3/8*sin(f*x+e)^2-3/8*(f*x+e)^4)
-2*a*b*c^3*cos(f*x+e)+1/4*a^2/f^3*d^3*(f*x+e)^4-a^2/f^3*d^3*e*(f*x+e)^3-a^2
/f^3*d^3*e^3*(f*x+e)+a^2/f^2*c*d^2*(f*x+e)^3+3/2*a^2/f^3*d^3*e^2*(f*x+e)^2+
3/2*a^2/f*c^2*d*(f*x+e)^2+3/f^3*b^2*d^3*e^2*((f*x+e)*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)+3/f^2*b^2*c*d^2*((f*x+
e)^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/2*(f*x+e)*cos(f*x+e)^2+1/
4*sin(f*x+e)*cos(f*x+e)+1/4*f*x+1/4*e-1/3*(f*x+e)^3)+3/f^2*b^2*c*d^2*e^2*(-
1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-6/f^2*b^2*c*d^2*e*((f*x+e)*(-1/2*s
in(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)-6/f^3*a
*b*d^3*e*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f*x+e)*sin(f*x+e))+6/f^3*a*
b*d^3*e^2*(sin(f*x+e)-(f*x+e)*cos(f*x+e))+6/f^2*a*b*c*d^2*(-(f*x+e)^2*cos(f
*x+e)+2*cos(f*x+e)+2*(f*x+e)*sin(f*x+e))-3/f*b^2*c^2*d*e*(-1/2*sin(f*x+e)*c
os(f*x+e)+1/2*f*x+1/2*e)+2/f^3*a*b*d^3*e^3*cos(f*x+e)+6/f*a*b*c^2*d*(sin(f*
x+e)-(f*x+e)*cos(f*x+e))-3*a^2/f^2*c*d^2*e*(f*x+e)^2+3*a^2/f^2*c*d^2*e^2*(f
*x+e)-3*a^2/f*c^2*d*e*(f*x+e)-1/f^3*b^2*d^3*e^3*(-1/2*sin(f*x+e)*cos(f*x+e)
+1/2*f*x+1/2*e)+3/f*b^2*c^2*d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+
1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)+2/f^3*a*b*d^3*(-(f*x+e)^3*cos(f*x+e)
+3*(f*x+e)^2*sin(f*x+e)-6*sin(f*x+e)+6*(f*x+e)*cos(f*x+e))-3/f^3*b^2*d^3*e*
((f*x+e)^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/2*(f*x+e)*cos(f*x+e
)^2+1/4*sin(f*x+e)*cos(f*x+e)+1/4*f*x+1/4*e-1/3*(f*x+e)^3))
```

Maxima [B] time = 1.13452, size = 1295, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(f*x + e)*a^2*c^3 + 4*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c^3 + 4
*(f*x + e)^4*a^2*d^3/f^3 - 16*(f*x + e)^3*a^2*d^3*e/f^3 + 24*(f*x + e)^2*a^
2*d^3*e^2/f^3 - 16*(f*x + e)*a^2*d^3*e^3/f^3 - 4*(2*f*x + 2*e - sin(2*f*x +
 2*e))*b^2*d^3*e^3/f^3 + 16*(f*x + e)^3*a^2*c*d^2/f^2 - 48*(f*x + e)^2*a^2*
c*d^2*e/f^2 + 48*(f*x + e)*a^2*c*d^2*e^2/f^2 + 12*(2*f*x + 2*e - sin(2*f*x
+ 2*e))*b^2*c*d^2*e^2/f^2 + 24*(f*x + e)^2*a^2*c^2*d/f - 48*(f*x + e)*a^2*c
^2*d*e/f - 12*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c^2*d*e/f - 32*a*b*c^3*c
os(f*x + e) + 32*a*b*d^3*e^3*cos(f*x + e)/f^3 - 96*a*b*c*d^2*e^2*cos(f*x +
e)/f^2 + 96*a*b*c^2*d*e*cos(f*x + e)/f - 96*((f*x + e)*cos(f*x + e) - sin(f
```

```
*x + e))*a*b*d^3*e^2/f^3 + 6*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e)
- cos(2*f*x + 2*e))*b^2*d^3*e^2/f^3 + 192*((f*x + e)*cos(f*x + e) - sin(f*x
 + e))*a*b*c*d^2*e/f^2 - 12*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e) -
 cos(2*f*x + 2*e))*b^2*c*d^2*e/f^2 - 96*((f*x + e)*cos(f*x + e) - sin(f*x +
 e))*a*b*c^2*d/f + 6*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e) - cos(2*
f*x + 2*e))*b^2*c^2*d/f + 96*(((f*x + e)^2 - 2)*cos(f*x + e) - 2*(f*x + e)*
sin(f*x + e))*a*b*d^3*e/f^3 - 2*(4*(f*x + e)^3 - 6*(f*x + e)*cos(2*f*x + 2*
e) - 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))*b^2*d^3*e/f^3 - 96*(((f*x + e)
^2 - 2)*cos(f*x + e) - 2*(f*x + e)*sin(f*x + e))*a*b*c*d^2/f^2 + 2*(4*(f*x
+ e)^3 - 6*(f*x + e)*cos(2*f*x + 2*e) - 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2
*e))*b^2*c*d^2/f^2 - 32*(((f*x + e)^3 - 6*f*x - 6*e)*cos(f*x + e) - 3*((f*x
 + e)^2 - 2)*sin(f*x + e))*a*b*d^3/f^3 + (2*(f*x + e)^4 - 3*(2*(f*x + e)^2
- 1)*cos(2*f*x + 2*e) - 2*(2*(f*x + e)^3 - 3*f*x - 3*e)*sin(2*f*x + 2*e))*b
^2*d^3/f^3)/f
```

Fricas [A] time = 2.17466, size = 805, normalized size = 3.22

$$\frac{\left(2\,a^2+b^2\right)d^3f^4x^4+4\left(2\,a^2+b^2\right)cd^2f^4x^3+3\left(2\left(2\,a^2+b^2\right)c^2df^4+b^2d^3f^2\right)x^2-3\left(2\,b^2d^3f^2x^2+4\,b^2cd^2f^2x+2\,b^2c^2df^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((2*a^2 + b^2)*d^3*f^4*x^4 + 4*(2*a^2 + b^2)*c*d^2*f^4*x^3 + 3*(2*(2*a^
2 + b^2)*c^2*d*f^4 + b^2*d^3*f^2)*x^2 - 3*(2*b^2*d^3*f^2*x^2 + 4*b^2*c*d^2*
f^2*x + 2*b^2*c^2*d*f^2 - b^2*d^3)*cos(f*x + e)^2 + 2*(2*(2*a^2 + b^2)*c^3*
f^4 + 3*b^2*c*d^2*f^2)*x - 16*(a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + a*b*
c^3*f^3 - 6*a*b*c*d^2*f + 3*(a*b*c^2*d*f^3 - 2*a*b*d^3*f)*x)*cos(f*x + e) +
 2*(24*a*b*d^3*f^2*x^2 + 48*a*b*c*d^2*f^2*x + 24*a*b*c^2*d*f^2 - 48*a*b*d^3
 - (2*b^2*d^3*f^3*x^3 + 6*b^2*c*d^2*f^3*x^2 + 2*b^2*c^3*f^3 - 3*b^2*c*d^2*f
 + 3*(2*b^2*c^2*d*f^3 - b^2*d^3*f)*x)*cos(f*x + e))*sin(f*x + e))/f^4
```

Sympy [A] time = 4.59935, size = 779, normalized size = 3.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**3*x + 3*a**2*c**2*d*x**2/2 + a**2*c*d**2*x**3 + a**2*d**
3*x**4/4 - 2*a*b*c**3*cos(e + f*x)/f - 6*a*b*c**2*d*x*cos(e + f*x)/f + 6*a*
b*c**2*d*sin(e + f*x)/f**2 - 6*a*b*c*d**2*x**2*cos(e + f*x)/f + 12*a*b*c*d*
*2*x*sin(e + f*x)/f**2 + 12*a*b*c*d**2*cos(e + f*x)/f**3 - 2*a*b*d**3*x**3*
cos(e + f*x)/f + 6*a*b*d**3*x**2*sin(e + f*x)/f**2 + 12*a*b*d**3*x*cos(e +
f*x)/f**3 - 12*a*b*d**3*sin(e + f*x)/f**4 + b**2*c**3*x*sin(e + f*x)**2/2 +
 b**2*c**3*x*cos(e + f*x)**2/2 - b**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f)
+ 3*b**2*c**2*d*x**2*sin(e + f*x)**2/4 + 3*b**2*c**2*d*x**2*cos(e + f*x)**2
/4 - 3*b**2*c**2*d*x*sin(e + f*x)*cos(e + f*x)/(2*f) - 3*b**2*c**2*d*cos(e
+ f*x)**2/(4*f**2) + b**2*c*d**2*x**3*sin(e + f*x)**2/2 + b**2*c*d**2*x**3*
cos(e + f*x)**2/2 - 3*b**2*c*d**2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 3*
b**2*c*d**2*x*sin(e + f*x)**2/(4*f**2) - 3*b**2*c*d**2*x*cos(e + f*x)**2/(4
*f**2) + 3*b**2*c*d**2*sin(e + f*x)*cos(e + f*x)/(4*f**3) + b**2*d**3*x**4*
sin(e + f*x)**2/8 + b**2*d**3*x**4*cos(e + f*x)**2/8 - b**2*d**3*x**3*sin(e
 + f*x)*cos(e + f*x)/(2*f) + 3*b**2*d**3*x**2*sin(e + f*x)**2/(8*f**2) - 3*
b**2*d**3*x**2*cos(e + f*x)**2/(8*f**2) + 3*b**2*d**3*x*sin(e + f*x)*cos(e
+ f*x)/(4*f**3) + 3*b**2*d**3*cos(e + f*x)**2/(8*f**4), Ne(f, 0)), ((a + b*
sin(e))**2*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), True))
```

Giac [A] time = 1.12116, size = 501, normalized size = 2.

$$\frac{1}{4}a^2d^3x^4 + \frac{1}{8}b^2d^3x^4 + a^2cd^2x^3 + \frac{1}{2}b^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 + \frac{3}{4}b^2c^2dx^2 + a^2c^3x + \frac{1}{2}b^2c^3x - \frac{3\left(2b^2d^3f^2x^2 + 4b^2cd^2f^2x + \cdots\right.}{1\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/4*a^2*d^3*x^4 + 1/8*b^2*d^3*x^4 + a^2*c*d^2*x^3 + 1/2*b^2*c*d^2*x^3 + 3/2
*a^2*c^2*d*x^2 + 3/4*b^2*c^2*d*x^2 + a^2*c^3*x + 1/2*b^2*c^3*x - 3/16*(2*b^
2*d^3*f^2*x^2 + 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 - b^2*d^3)*cos(2*f*x +
2*e)/f^4 - 2*(a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3*x + a
*b*c^3*f^3 - 6*a*b*d^3*f*x - 6*a*b*c*d^2*f)*cos(f*x + e)/f^4 - 1/8*(2*b^2*d
^3*f^3*x^3 + 6*b^2*c*d^2*f^3*x^2 + 6*b^2*c^2*d*f^3*x + 2*b^2*c^3*f^3 - 3*b^
2*d^3*f*x - 3*b^2*c*d^2*f)*sin(2*f*x + 2*e)/f^4 + 6*(a*b*d^3*f^2*x^2 + 2*a*
b*c*d^2*f^2*x + a*b*c^2*d*f^2 - 2*a*b*d^3)*sin(f*x + e)/f^4
```

3.158 $\int (c+dx)^2(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=182

$$\frac{a^2(c+dx)^3}{3d}+\frac{4abd(c+dx)\sin(e+fx)}{f^2}-\frac{2ab(c+dx)^2\cos(e+fx)}{f}+\frac{4abd^2\cos(e+fx)}{f^3}+\frac{b^2d(c+dx)\sin^2(e+fx)}{2f^2}-$$

```
[Out] -(b^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(3*d) + (b^2*(c + d*x)^3)/(6*d) +
(4*a*b*d^2*Cos[e + f*x])/f^3 - (2*a*b*(c + d*x)^2*Cos[e + f*x])/f + (4*a*b*
d*(c + d*x)*Sin[e + f*x])/f^2 + (b^2*d^2*Cos[e + f*x]*Sin[e + f*x])/(4*f^3)
 - (b^2*(c + d*x)^2*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b^2*d*(c + d*x)*Sin
[e + f*x]^2)/(2*f^2)
```

Rubi [A] time = 0.191988, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3317, 3296, 2638, 3311, 32, 2635, 8}

$$\frac{a^2(c+dx)^3}{3d}+\frac{4abd(c+dx)\sin(e+fx)}{f^2}-\frac{2ab(c+dx)^2\cos(e+fx)}{f}+\frac{4abd^2\cos(e+fx)}{f^3}+\frac{b^2d(c+dx)\sin^2(e+fx)}{2f^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Sin[e + f*x])^2,x]
```


```
[Out] -(b^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(3*d) + (b^2*(c + d*x)^3)/(6*d) +
(4*a*b*d^2*Cos[e + f*x])/f^3 - (2*a*b*(c + d*x)^2*Cos[e + f*x])/f + (4*a*b*
d*(c + d*x)*Sin[e + f*x])/f^2 + (b^2*d^2*Cos[e + f*x]*Sin[e + f*x])/(4*f^3)
 - (b^2*(c + d*x)^2*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b^2*d*(c + d*x)*Sin
[e + f*x]^2)/(2*f^2)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^2(a+b\sin(e+fx))^2\,dx = \int \left(a^2(c+dx)^2 + 2ab(c+dx)^2\sin(e+fx) + b^2(c+dx)^2\sin^2(e+fx)\right)dx$$

$$= \frac{a^2(c+dx)^3}{3d} + (2ab)\int (c+dx)^2\sin(e+fx)\,dx + b^2\int (c+dx)^2\sin^2(e+fx)\,dx$$

$$= \frac{a^2(c+dx)^3}{3d} - \frac{2ab(c+dx)^2\cos(e+fx)}{f} - \frac{b^2(c+dx)^2\cos(e+fx)\sin(e+fx)}{2f} + b^2 \cdots$$

$$= \frac{a^2(c+dx)^3}{3d} + \frac{b^2(c+dx)^3}{6d} - \frac{2ab(c+dx)^2\cos(e+fx)}{f} + \frac{4abd(c+dx)\sin(e+fx)}{f^2} \cdots$$

$$= -\frac{b^2d^2x}{4f^2} + \frac{a^2(c+dx)^3}{3d} + \frac{b^2(c+dx)^3}{6d} + \frac{4abd^2\cos(e+fx)}{f^3} - \frac{2ab(c+dx)^2\cos(e+ \cdots}{f}$$

Mathematica [A] time = 0.726044, size = 249, normalized size = 1.37

$$\frac{24a^2c^2f^3x + 24a^2cdf^3x^2 + 8a^2d^2f^3x^3 - 48ab\left(c^2f^2 + 2cdf^2x + d^2\left(f^2x^2-2\right)\right)\cos(e+fx) + 96abcdf\sin(e+fx) + 96a \cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (24*a^2*c^2*f^3*x + 12*b^2*c^2*f^3*x + 24*a^2*c*d*f^3*x^2 + 12*b^2*c*d*f^3*
x^2 + 8*a^2*d^2*f^3*x^3 + 4*b^2*d^2*f^3*x^3 - 48*a*b*(c^2*f^2 + 2*c*d*f^2*x
 + d^2*(-2 + f^2*x^2))*Cos[e + f*x] - 6*b^2*d*f*(c + d*x)*Cos[2*(e + f*x)]
+ 96*a*b*c*d*f*Sin[e + f*x] + 96*a*b*d^2*f*x*Sin[e + f*x] + 3*b^2*d^2*Sin[2
*(e + f*x)] - 6*b^2*c^2*f^2*Sin[2*(e + f*x)] - 12*b^2*c*d*f^2*x*Sin[2*(e +
f*x)] - 6*b^2*d^2*f^2*x^2*Sin[2*(e + f*x)])/(24*f^3)
```

Maple [B] time = 0.013, size = 561, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(1/3*a^2/f^2*d^2*(f*x+e)^3+a^2/f*c*d*(f*x+e)^2-a^2/f^2*d^2*e*(f*x+e)^2+
a^2*c^2*(f*x+e)-2*a^2/f*c*d*e*(f*x+e)+a^2/f^2*d^2*e^2*(f*x+e)+2/f^2*a*b*d^2
```

```
*(-(f*x+e)^2*cos(f*x+e)+2*cos(f*x+e)+2*(f*x+e)*sin(f*x+e))+4/f*a*b*c*d*(sin
(f*x+e)-(f*x+e)*cos(f*x+e))-4/f^2*a*b*d^2*e*(sin(f*x+e)-(f*x+e)*cos(f*x+e))
-2*a*b*c^2*cos(f*x+e)+4/f*a*b*c*d*e*cos(f*x+e)-2/f^2*a*b*d^2*e^2*cos(f*x+e)
+1/f^2*b^2*d^2*((f*x+e)^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/2*(f
*x+e)*cos(f*x+e)^2+1/4*sin(f*x+e)*cos(f*x+e)+1/4*f*x+1/4*e-1/3*(f*x+e)^3)+2
/f*b^2*c*d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^
2+1/4*sin(f*x+e)^2)-2/f^2*b^2*d^2*e*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/
2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)+b^2*c^2*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)-2/f*b^2*c*d*e*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*
e)+1/f^2*b^2*d^2*e^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [B] time = 1.04999, size = 678, normalized size = 3.73

$$\frac{24\left(fx+e\right)a^2c^2+6\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)b^2c^2+\frac{8\left(fx+e\right)^3a^2d^2}{f^2}-\frac{24\left(fx+e\right)^2a^2d^2e}{f^2}+\frac{24\left(fx+e\right)a^2d^2e^2}{f^2}+\frac{6\left(2\,fx+2\,e-\sin\left(2\,fx+2\right.\right.}{f^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(24*(f*x + e)*a^2*c^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c^2 + 8
*(f*x + e)^3*a^2*d^2/f^2 - 24*(f*x + e)^2*a^2*d^2*e/f^2 + 24*(f*x + e)*a^2*
d^2*e^2/f^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*d^2*e^2/f^2 + 24*(f*x
+ e)^2*a^2*c*d/f - 48*(f*x + e)*a^2*c*d*e/f - 12*(2*f*x + 2*e - sin(2*f*x +
 2*e))*b^2*c*d*e/f - 48*a*b*c^2*cos(f*x + e) - 48*a*b*d^2*e^2*cos(f*x + e)/
f^2 + 96*a*b*c*d*e*cos(f*x + e)/f + 96*((f*x + e)*cos(f*x + e) - sin(f*x +
e))*a*b*d^2*e/f^2 - 6*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e) - cos(2
*f*x + 2*e))*b^2*d^2*e/f^2 - 96*((f*x + e)*cos(f*x + e) - sin(f*x + e))*a*b
*c*d/f + 6*(2*(f*x + e)^2 - 2*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e)
)*b^2*c*d/f - 48*(((f*x + e)^2 - 2)*cos(f*x + e) - 2*(f*x + e)*sin(f*x + e)
)*a*b*d^2/f^2 + (4*(f*x + e)^3 - 6*(f*x + e)*cos(2*f*x + 2*e) - 3*(2*(f*x +
 e)^2 - 1)*sin(2*f*x + 2*e))*b^2*d^2/f^2)/f
```

Fricas [A] time = 2.08423, size = 495, normalized size = 2.72

$$\frac{2\left(2\,a^2+b^2\right)d^2f^3x^3+6\left(2\,a^2+b^2\right)cdf^3x^2-6\left(b^2d^2fx+b^2cdf\right)\cos\left(fx+e\right)^2+3\left(2\left(2\,a^2+b^2\right)c^2f^3+b^2d^2f\right)x-24\left(abd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(2*a^2 + b^2)*d^2*f^3*x^3 + 6*(2*a^2 + b^2)*c*d*f^3*x^2 - 6*(b^2*d^
2*f*x + b^2*c*d*f)*cos(f*x + e)^2 + 3*(2*(2*a^2 + b^2)*c^2*f^3 + b^2*d^2*f)
*x - 24*(a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 - 2*a*b*d^2)*cos(f
*x + e) + 3*(16*a*b*d^2*f*x + 16*a*b*c*d*f - (2*b^2*d^2*f^2*x^2 + 4*b^2*c*d
*f^2*x + 2*b^2*c^2*f^2 - b^2*d^2)*cos(f*x + e))*sin(f*x + e))/f^3
```

Sympy [A] time = 2.06795, size = 456, normalized size = 2.51

$$\begin{cases} a^2c^2x + a^2cdx^2 + \frac{a^2d^2x^3}{3} - \frac{2abc^2\cos\left(e+fx\right)}{f} - \frac{4abcdx\cos\left(e+fx\right)}{f} + \frac{4abcd\sin\left(e+fx\right)}{f^2} - \frac{2abd^2x^2\cos\left(e+fx\right)}{f} + \frac{4abd^2x\sin\left(e+fx\right)}{f^2} + \frac{4abd^2\cos}{f^3} \\ (a+b\sin(e))^2\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x + a**2*c*d*x**2 + a**2*d**2*x**3/3 - 2*a*b*c**2*cos(
e + f*x)/f - 4*a*b*c*d*x*cos(e + f*x)/f + 4*a*b*c*d*sin(e + f*x)/f**2 - 2*a
*b*d**2*x**2*cos(e + f*x)/f + 4*a*b*d**2*x*sin(e + f*x)/f**2 + 4*a*b*d**2*c
os(e + f*x)/f**3 + b**2*c**2*x*sin(e + f*x)**2/2 + b**2*c**2*x*cos(e + f*x)
**2/2 - b**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) + b**2*c*d*x**2*sin(e + f
*x)**2/2 + b**2*c*d*x**2*cos(e + f*x)**2/2 - b**2*c*d*x*sin(e + f*x)*cos(e
+ f*x)/f - b**2*c*d*cos(e + f*x)**2/(2*f**2) + b**2*d**2*x**3*sin(e + f*x)*
*2/6 + b**2*d**2*x**3*cos(e + f*x)**2/6 - b**2*d**2*x**2*sin(e + f*x)*cos(e
 + f*x)/(2*f) + b**2*d**2*x*sin(e + f*x)**2/(4*f**2) - b**2*d**2*x*cos(e +
f*x)**2/(4*f**2) + b**2*d**2*sin(e + f*x)*cos(e + f*x)/(4*f**3), Ne(f, 0)),
 ((a + b*sin(e))**2*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Giac [A] time = 1.12622, size = 309, normalized size = 1.7

$$\frac{1}{3}a^2d^2x^3 + \frac{1}{6}b^2d^2x^3 + a^2cdx^2 + \frac{1}{2}b^2cdx^2 + a^2c^2x + \frac{1}{2}b^2c^2x - \frac{\left(b^2d^2fx + b^2cdf\right)\cos\left(2fx + 2e\right)}{4f^3} - \frac{2\left(abd^2f^2x^2 + 2abc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*d^2*x^3 + 1/6*b^2*d^2*x^3 + a^2*c*d*x^2 + 1/2*b^2*c*d*x^2 + a^2*c^2
*x + 1/2*b^2*c^2*x - 1/4*(b^2*d^2*f*x + b^2*c*d*f)*cos(2*f*x + 2*e)/f^3 - 2
*(a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 - 2*a*b*d^2)*cos(f*x + e)
/f^3 - 1/8*(2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 - b^2*d^2)*
sin(2*f*x + 2*e)/f^3 + 4*(a*b*d^2*f*x + a*b*c*d*f)*sin(f*x + e)/f^3
```

3.159 $\int (c+dx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=116

$$\frac{a^2(c+dx)^2}{2d}-\frac{2ab(c+dx)\cos(e+fx)}{f}+\frac{2abd\sin(e+fx)}{f^2}-\frac{b^2(c+dx)\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}b^2cx+\frac{b^2d\sin^2(e+}{4f^2}$$

```
[Out] (b^2*c*x)/2 + (b^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) - (2*a*b*(c + d*x)*Co
s[e + f*x])/f + (2*a*b*d*Sin[e + f*x])/f^2 - (b^2*(c + d*x)*Cos[e + f*x]*Si
n[e + f*x])/(2*f) + (b^2*d*Sin[e + f*x]^2)/(4*f^2)
```

Rubi [A] time = 0.0975296, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3317, 3296, 2637, 3310}

$$\frac{a^2(c+dx)^2}{2d}-\frac{2ab(c+dx)\cos(e+fx)}{f}+\frac{2abd\sin(e+fx)}{f^2}-\frac{b^2(c+dx)\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}b^2cx+\frac{b^2d\sin^2(e+}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (b^2*c*x)/2 + (b^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) - (2*a*b*(c + d*x)*Co
s[e + f*x])/f + (2*a*b*d*Sin[e + f*x])/f^2 - (b^2*(c + d*x)*Cos[e + f*x]*Si
n[e + f*x])/(2*f) + (b^2*d*Sin[e + f*x]^2)/(4*f^2)
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int (c+dx)(a+b\sin(e+fx))^2\,dx &= \int \left(a^2(c+dx)+2ab(c+dx)\sin(e+fx)+b^2(c+dx)\sin^2(e+fx)\right)\,dx\\ &= \frac{a^2(c+dx)^2}{2d}+(2ab)\int (c+dx)\sin(e+fx)\,dx+b^2\int (c+dx)\sin^2(e+fx)\,dx\\ &= \frac{a^2(c+dx)^2}{2d}-\frac{2ab(c+dx)\cos(e+fx)}{f}-\frac{b^2(c+dx)\cos(e+fx)\sin(e+fx)}{2f}+\frac{b^2d\sin \ldots}{4 \ldots}\\ &= \frac{1}{2}b^2cx+\frac{1}{4}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{2ab(c+dx)\cos(e+fx)}{f}+\frac{2abd\sin(e+fx)}{f^2}-\frac{b^2(c \ldots}{\ldots}\end{aligned}$$

Mathematica [A] time = 0.673374, size = 96, normalized size = 0.83

$$-\frac{2\left(2a^2+b^2\right)(e+fx)(d(e-fx)-2cf)+16abf(c+dx)\cos(e+fx)-16abd\sin(e+fx)+2b^2f(c+dx)\sin(2(e+fx))+\ldots}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -(2*(2*a^2 + b^2)*(e + f*x)*(-2*c*f + d*(e - f*x)) + 16*a*b*f*(c + d*x)*Cos
[e + f*x] + b^2*d*Cos[2*(e + f*x)] - 16*a*b*d*Sin[e + f*x] + 2*b^2*f*(c + d
*x)*Sin[2*(e + f*x)])/(8*f^2)
```

Maple [B] time = 0.013, size = 216, normalized size = 1.9

$$\frac{1}{f}\left(\frac{a^2d\left(fx+e\right)^2}{2f}+a^2c\left(fx+e\right)-\frac{a^2de\left(fx+e\right)}{f}+2\frac{abd\left(\sin\left(fx+e\right)-\left(fx+e\right)\cos\left(fx+e\right)\right)}{f}-2abc\cos\left(fx+e\right)+2\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(1/2*a^2/f*d*(f*x+e)^2+a^2*c*(f*x+e)-a^2/f*d*e*(f*x+e)+2/f*a*b*d*(sin(f
*x+e)-(f*x+e)*cos(f*x+e))-2*a*b*c*cos(f*x+e)+2/f*a*b*d*e*cos(f*x+e)+1/f*b^2
*d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*si
n(f*x+e)^2)+b^2*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/f*b^2*d*e*(-
1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A] time = 0.994603, size = 273, normalized size = 2.35

$$\frac{8\left(fx+e\right)a^2c+2\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)b^2c+\frac{4\left(fx+e\right)^2a^2d}{f}-\frac{8\left(fx+e\right)a^2de}{f}-\frac{2\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)b^2de}{f}-16\,abc\cos\left(f\right.}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(f*x + e)*a^2*c + 2*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c + 4*(f*x
+ e)^2*a^2*d/f - 8*(f*x + e)*a^2*d*e/f - 2*(2*f*x + 2*e - sin(2*f*x + 2*e))
*b^2*d*e/f - 16*a*b*c*cos(f*x + e) + 16*a*b*d*e*cos(f*x + e)/f - 16*((f*x +
 e)*cos(f*x + e) - sin(f*x + e))*a*b*d/f + (2*(f*x + e)^2 - 2*(f*x + e)*sin
(2*f*x + 2*e) - cos(2*f*x + 2*e))*b^2*d/f)/f
```

Fricas [A] time = 2.03659, size = 252, normalized size = 2.17

$$\frac{\left(2\,a^2+b^2\right)df^2x^2+2\left(2\,a^2+b^2\right)cf^2x-b^2d\cos\left(fx+e\right)^2-8\left(abdfx+abcf\right)\cos\left(fx+e\right)+2\left(4\,abd-\left(b^2dfx+b^2cf\right)\right.}{4\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*d*f^2*x^2 + 2*(2*a^2 + b^2)*c*f^2*x - b^2*d*cos(f*x + e)
^2 - 8*(a*b*d*f*x + a*b*c*f)*cos(f*x + e) + 2*(4*a*b*d - (b^2*d*f*x + b^2*c
*f)*cos(f*x + e))*sin(f*x + e))/f^2
```

Sympy [A] time = 0.84567, size = 219, normalized size = 1.89

$$\begin{cases} a^2cx + \frac{a^2dx^2}{2} - \frac{2abc\cos(e+fx)}{f} - \frac{2abdx\cos(e+fx)}{f} + \frac{2abd\sin(e+fx)}{f^2} + \frac{b^2cx\sin^2(e+fx)}{2} + \frac{b^2cx\cos^2(e+fx)}{2} - \frac{b^2c\sin(e+fx)\cos(e+fx)}{2f} + \frac{b^2dx \cdots}{} \\ (a+b\sin(e))^2\left(cx + \frac{dx^2}{2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c*x + a**2*d*x**2/2 - 2*a*b*c*cos(e + f*x)/f - 2*a*b*d*x*co
s(e + f*x)/f + 2*a*b*d*sin(e + f*x)/f**2 + b**2*c*x*sin(e + f*x)**2/2 + b**
2*c*x*cos(e + f*x)**2/2 - b**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + b**2*d*x
**2*sin(e + f*x)**2/4 + b**2*d*x**2*cos(e + f*x)**2/4 - b**2*d*x*sin(e + f*
x)*cos(e + f*x)/(2*f) - b**2*d*cos(e + f*x)**2/(4*f**2), Ne(f, 0)), ((a + b
*sin(e))**2*(c*x + d*x**2/2), True))
```

Giac [A] time = 1.10087, size = 161, normalized size = 1.39

$$\frac{1}{2}a^2dx^2 + \frac{1}{4}b^2dx^2 + a^2cx + \frac{1}{2}b^2cx - \frac{b^2d\cos(2fx+2e)}{8f^2} + \frac{2abd\sin(fx+e)}{f^2} - \frac{2(abdfx+abcf)\cos(fx+e)}{f^2} - \frac{(b^2df \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*d*x^2 + 1/4*b^2*d*x^2 + a^2*c*x + 1/2*b^2*c*x - 1/8*b^2*d*cos(2*f*x
 + 2*e)/f^2 + 2*a*b*d*sin(f*x + e)/f^2 - 2*(a*b*d*f*x + a*b*c*f)*cos(f*x +
e)/f^2 - 1/4*(b^2*d*f*x + b^2*c*f)*sin(2*f*x + 2*e)/f^2
```

3.160 $\int \frac{(a+b\sin(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=156

$$\frac{a^2\log(c+dx)}{d}+\frac{2ab\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}-\frac{b^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right.}{2d}$$

```
[Out] -(b^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (a^2*Log
[c + d*x])/d + (b^2*Log[c + d*x])/(2*d) + (2*a*b*CosIntegral[(c*f)/d + f*x]
*Sin[e - (c*f)/d])/d + (2*a*b*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/
d + (b^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rubi [A] time = 0.324364, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3317, 3303, 3299, 3302, 3312}

$$\frac{a^2\log(c+dx)}{d}+\frac{2ab\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}-\frac{b^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*x),x]
```

```
[Out] -(b^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (a^2*Log
[c + d*x])/d + (b^2*Log[c + d*x])/(2*d) + (2*a*b*CosIntegral[(c*f)/d + f*x]
*Sin[e - (c*f)/d])/d + (2*a*b*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/
d + (b^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\sin(e+fx))^2}{c+dx}\,dx &= \int \left(\frac{a^2}{c+dx}+\frac{2ab\sin(e+fx)}{c+dx}+\frac{b^2\sin^2(e+fx)}{c+dx}\right)dx \\
&= \frac{a^2\log(c+dx)}{d}+(2ab)\int \frac{\sin(e+fx)}{c+dx}\,dx+b^2\int \frac{\sin^2(e+fx)}{c+dx}\,dx \\
&= \frac{a^2\log(c+dx)}{d}+b^2\int \left(\frac{1}{2(c+dx)}-\frac{\cos(2e+2fx)}{2(c+dx)}\right)dx+\left(2ab\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+f\right.}{c+dx} \\
&= \frac{a^2\log(c+dx)}{d}+\frac{b^2\log(c+dx)}{2d}+\frac{2ab\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+\right.}{d} \\
&= \frac{a^2\log(c+dx)}{d}+\frac{b^2\log(c+dx)}{2d}+\frac{2ab\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+\right.}{d} \\
&= -\frac{b^2\cos\left(2e-\frac{2cf}{d}\right)\text{Ci}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{a^2\log(c+dx)}{d}+\frac{b^2\log(c+dx)}{2d}+\frac{2ab\text{Ci}\left(\frac{cf}{d}+fx\right)\sin}{d}
\end{aligned}$$

Mathematica [A] time = 0.291788, size = 134, normalized size = 0.86

$$\frac{2a^2\log(c+dx)+4ab\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+4ab\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)-b^2\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*x),x]
```

```
[Out] (-(b^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*f*(c + d*x))/d]) + 2*a^2*Log[c +
 d*x] + b^2*Log[c + d*x] + 4*a*b*CosIntegral[f*(c/d + x)]*Sin[e - (c*f)/d]
+ 4*a*b*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + b^2*Sin[2*e - (2*c*f)/d
]*SinIntegral[(2*f*(c + d*x))/d])/(2*d)
```

Maple [A] time = 0.019, size = 213, normalized size = 1.4

$$\frac{a^2\ln\left(\left(fx+e\right)d+cf-de\right)}{d}+2\,\frac{ab}{d}\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)-2\,\frac{ab}{d}\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)+\frac{b^2}{}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(d*x+c),x)
```

```
[Out] a^2*ln((f*x+e)*d+c*f-d*e)/d+2*a*b*Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-
2*a*b*Ci(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+1/2*b^2*ln((f*x+e)*d+c*f-d*e
)/d-1/2*b^2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d-1/2*b^2*Ci(2*f
*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d
```

Maxima [C] time = 1.28828, size = 451, normalized size = 2.89

$$\frac{\frac{4\,a^2f\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d}+\frac{4\left(f\left(-i\,E_1\left(\frac{i(fx+e)d-i\,de+i\,cf}{d}\right)+i\,E_1\left(-\frac{i(fx+e)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f\left(E_1\left(\frac{i(fx+e)d-i\,de+i\,cf}{d}\right)+E_1\left(-\frac{i(fx+e)d-i\,de+i\,cf}{d}\right)\right)\sin\left(-\frac{de-\cdots}{d}\right)\cdots}{d}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^2*f*log(c + (f*x + e)*d/f - d*e/f)/d + 4*(f*(-I*exp_integral_e(1,
(I*(f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(1, -(I*(f*x + e)*d -
I*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f*(exp_integral_e(1, (I*(f*x + e)*
d - I*d*e + I*c*f)/d) + exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I*c*f)/
d))*sin(-(d*e - c*f)/d))*a*b/d + (f*(exp_integral_e(1, (2*I*(f*x + e)*d - 2
*I*d*e + 2*I*c*f)/d) + exp_integral_e(1, -(2*I*(f*x + e)*d - 2*I*d*e + 2*I*
```

```
c*f)/d))*cos(-2*(d*e - c*f)/d) + f*(I*exp_integral_e(1, (2*I*(f*x + e)*d -
2*I*d*e + 2*I*c*f)/d) - I*exp_integral_e(1, -(2*I*(f*x + e)*d - 2*I*d*e + 2
*I*c*f)/d))*sin(-2*(d*e - c*f)/d) + 2*f*log((f*x + e)*d - d*e + c*f))*b^2/d
)/f
```

Fricas [A] time = 2.19746, size = 482, normalized size = 3.09

$$-\frac{2\,b^2\sin\left(-\frac{2\left(de-cf\right)}{d}\right)\mathrm{Si}\left(\frac{2\left(dfx+cf\right)}{d}\right)-8\,ab\cos\left(-\frac{de-cf}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)+\left(b^2\,\mathrm{Ci}\left(\frac{2\left(dfx+cf\right)}{d}\right)+b^2\,\mathrm{Ci}\left(-\frac{2\left(dfx+cf\right)}{d}\right)\right)\cos\left(-\frac{2\left(de-\right.}{d}\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*sin(-2*(d*e - c*f)/d)*sin_integral(2*(d*f*x + c*f)/d) - 8*a*b*c
os(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/d) + (b^2*cos_integral(2*(d*f
*x + c*f)/d) + b^2*cos_integral(-2*(d*f*x + c*f)/d))*cos(-2*(d*e - c*f)/d)
- 2*(2*a^2 + b^2)*log(d*x + c) + 4*(a*b*cos_integral((d*f*x + c*f)/d) + a*b
*cos_integral(-(d*f*x + c*f)/d))*sin(-(d*e - c*f)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(e+fx\right)\right)^2}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(d*x+c),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2/(c + d*x), x)
```

Giac [C] time = 1.60154, size = 9986, normalized size = 64.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(4*a*b*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)
^2*tan(1/2*e)^2*tan(e)^2 - 4*a*b*imag_part(cos_integral(-f*x - c*f/d))*tan(
c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a^2*log(abs(d*x + c))*t
an(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 2*b^2*log(abs(d*x + c)
)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - b^2*real_part(cos_i
ntegral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)
^2 - b^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f
/d)^2*tan(1/2*e)^2*tan(e)^2 + 8*a*b*sin_integral((d*f*x + c*f)/d)*tan(c*f/d
)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 2*b^2*imag_part(cos_integral(2
*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) + 2*b^2*
imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan
(1/2*e)^2*tan(e) - 4*b^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1
/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 8*a*b*real_part(cos_integral(f*x + c*f/d)
)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 - 8*a*b*real_part(cos_i
ntegral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 +
8*a*b*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(
1/2*e)^2*tan(e)^2 + 8*a*b*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^
2*tan(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 + 2*b^2*imag_part(cos_integral(2*f*x
 + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 2*b^2*imag
_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e
)^2*tan(e)^2 + 4*b^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(1/2*c*f
/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a*b*imag_part(cos_integral(f*x + c*f/d))*ta
n(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 4*a*b*imag_part(cos_integral(-f*
x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*a^2*log(abs(d*x
+ c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b^2*log(abs(d*x + c))*
tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + b^2*real_part(cos_integral(2*f
*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + b^2*real_part(c
os_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 +
 8*a*b*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*
e)^2 - 4*b^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*
f/d)^2*tan(1/2*e)^2*tan(e) - 4*b^2*real_part(cos_integral(-2*f*x - 2*c*f/d)
)*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 4*a*b*imag_part(cos_int
egral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + 4*a*b*imag_par
t(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + 4*a^
2*log(abs(d*x + c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + 2*b^2*log(abs(
d*x + c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - b^2*real_part(cos_integr
al(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - b^2*real_part
(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - 8
*a*b*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 +
 16*a*b*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*ta
n(1/2*e)*tan(e)^2 - 16*a*b*imag_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)
^2*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 + 32*a*b*sin_integral((d*f*x + c*f)/d
```

```
)*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 - 4*a*b*imag_part(cos_int
egral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a*b*imag_part(co
s_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a^2*log(ab
s(d*x + c))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 2*b^2*log(abs(d*x + c))*ta
n(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - b^2*real_part(cos_integral(2*f*x + 2*c*f
/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - b^2*real_part(cos_integral(-2*f*x
 - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 8*a*b*sin_integral((d*f*x
 + c*f)/d)*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a*b*imag_part(cos_integra
l(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 4*a*b*imag_part(co
s_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a^2*lo
g(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 2*b^2*log(abs(d*x
+ c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + b^2*real_part(cos_integral(2
*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + b^2*real_part(cos
_integral(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 8*a*b
*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 8*a
*b*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1
/2*e) - 8*a*b*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*
f/d)^2*tan(1/2*e) + 8*a*b*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2
*tan(1/2*c*f/d)*tan(1/2*e)^2 + 8*a*b*real_part(cos_integral(-f*x - c*f/d))*
tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 - 2*b^2*imag_part(cos_integral(2*f
*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b^2*imag_part(c
os_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 4
*b^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)
^2 - 2*b^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*
f/d)^2*tan(e) + 2*b^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^
2*tan(1/2*c*f/d)^2*tan(e) - 4*b^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d
)^2*tan(1/2*c*f/d)^2*tan(e) - 2*b^2*imag_part(cos_integral(2*f*x + 2*c*f/d)
)*tan(c*f/d)^2*tan(1/2*e)^2*tan(e) + 2*b^2*imag_part(cos_integral(-2*f*x -
2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e) - 4*b^2*sin_integral(2*(d*f*x +
c*f)/d)*tan(c*f/d)^2*tan(1/2*e)^2*tan(e) + 2*b^2*imag_part(cos_integral(2*f
*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 2*b^2*imag_part(cos_i
ntegral(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) + 4*b^2*sin
_integral(2*(d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 8*a*b*r
eal_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(e)^2 -
8*a*b*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan
(e)^2 + 2*b^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c
*f/d)^2*tan(e)^2 - 2*b^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/
d)*tan(1/2*c*f/d)^2*tan(e)^2 + 4*b^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*
f/d)*tan(1/2*c*f/d)^2*tan(e)^2 + 8*a*b*real_part(cos_integral(f*x + c*f/d))
*tan(c*f/d)^2*tan(1/2*e)*tan(e)^2 + 8*a*b*real_part(cos_integral(-f*x - c*f
/d))*tan(c*f/d)^2*tan(1/2*e)*tan(e)^2 - 8*a*b*real_part(cos_integral(f*x +
c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 - 8*a*b*real_part(cos_integral
(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 + 2*b^2*imag_part(cos_
integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*e)^2*tan(e)^2 - 2*b^2*imag_pa
rt(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*e)^2*tan(e)^2 + 4*b^2
```

```
*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(1/2*e)^2*tan(e)^2 + 8*a*b*r
eal_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 +
8*a*b*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2*tan
(e)^2 - 4*a*b*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f
/d)^2 + 4*a*b*imag_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*
f/d)^2 + 4*a^2*log(abs(d*x + c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 2*b^2*log(
abs(d*x + c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + b^2*real_part(cos_integral(2*
f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + b^2*real_part(cos_integral(
-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - 8*a*b*sin_integral((d*f*
x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 16*a*b*imag_part(cos_integral(f
*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e) - 16*a*b*imag_part(cos_
integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e) + 32*a*b*sin
_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e) - 4*a*b*i
mag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2 + 4*a*b*imag_
part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2 + 4*a^2*log(abs(
d*x + c))*tan(c*f/d)^2*tan(1/2*e)^2 + 2*b^2*log(abs(d*x + c))*tan(c*f/d)^2*
tan(1/2*e)^2 + b^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*ta
n(1/2*e)^2 + b^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan
(1/2*e)^2 - 8*a*b*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*e)^2 +
 4*a*b*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 -
 4*a*b*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2
+ 4*a^2*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*b^2*log(abs(d*x
 + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - b^2*real_part(cos_integral(2*f*x + 2
*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - b^2*real_part(cos_integral(-2*f*x
- 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 8*a*b*sin_integral((d*f*x + c*f
)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 4*b^2*real_part(cos_integral(2*f*x + 2
*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(e) - 4*b^2*real_part(cos_integral(
-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(e) - 4*b^2*real_part(cos
_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*e)^2*tan(e) - 4*b^2*real_par
t(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*e)^2*tan(e) + 4*a*b*im
ag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(e)^2 - 4*a*b*imag_part(
cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(e)^2 + 4*a^2*log(abs(d*x + c))
*tan(c*f/d)^2*tan(e)^2 + 2*b^2*log(abs(d*x + c))*tan(c*f/d)^2*tan(e)^2 - b^
2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(e)^2 - b^2*real
_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(e)^2 + 8*a*b*sin_int
egral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(e)^2 - 4*a*b*imag_part(cos_integral
(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(e)^2 + 4*a*b*imag_part(cos_integral(-f*
x - c*f/d))*tan(1/2*c*f/d)^2*tan(e)^2 + 4*a^2*log(abs(d*x + c))*tan(1/2*c*f
/d)^2*tan(e)^2 + 2*b^2*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(e)^2 + b^2*re
al_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(e)^2 + b^2*real
_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)^2*tan(e)^2 - 8*a*b*sin
_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(e)^2 + 16*a*b*imag_part(cos
_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 - 16*a*b*imag_pa
rt(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 + 32*a*b*
sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 - 4*a*b*im
```

```
ag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2*tan(e)^2 + 4*a*b*imag_part(
cos_integral(-f*x - c*f/d))*tan(1/2*e)^2*tan(e)^2 + 4*a^2*log(abs(d*x + c))
*tan(1/2*e)^2*tan(e)^2 + 2*b^2*log(abs(d*x + c))*tan(1/2*e)^2*tan(e)^2 + b^
2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(e)^2 + b^2*real
_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(e)^2 - 8*a*b*sin_int
egral((d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e)^2 - 8*a*b*real_part(cos_integral
(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d) - 8*a*b*real_part(cos_integral(-
f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d) - 2*b^2*imag_part(cos_integral(2*
f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2 + 2*b^2*imag_part(cos_integral(
-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2 - 4*b^2*sin_integral(2*(d*f*
x + c*f)/d)*tan(c*f/d)*tan(1/2*c*f/d)^2 + 8*a*b*real_part(cos_integral(f*x
+ c*f/d))*tan(c*f/d)^2*tan(1/2*e) + 8*a*b*real_part(cos_integral(-f*x - c*f
/d))*tan(c*f/d)^2*tan(1/2*e) - 8*a*b*real_part(cos_integral(f*x + c*f/d))*t
an(1/2*c*f/d)^2*tan(1/2*e) - 8*a*b*real_part(cos_integral(-f*x - c*f/d))*ta
n(1/2*c*f/d)^2*tan(1/2*e) - 2*b^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*
tan(c*f/d)*tan(1/2*e)^2 + 2*b^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*t
an(c*f/d)*tan(1/2*e)^2 - 4*b^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*t
an(1/2*e)^2 + 8*a*b*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan
(1/2*e)^2 + 8*a*b*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(
1/2*e)^2 - 2*b^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(
e) + 2*b^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(e) -
4*b^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)^2*tan(e) + 2*b^2*imag_part
(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(e) - 2*b^2*imag_part(c
os_integral(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)^2*tan(e) + 4*b^2*sin_integral
(2*(d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(e) + 2*b^2*imag_part(cos_integral(
2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(e) - 2*b^2*imag_part(cos_integral(-2*f*x
 - 2*c*f/d))*tan(1/2*e)^2*tan(e) + 4*b^2*sin_integral(2*(d*f*x + c*f)/d)*ta
n(1/2*e)^2*tan(e) + 2*b^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/
d)*tan(e)^2 - 2*b^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*ta
n(e)^2 + 4*b^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(e)^2 - 8*a*b*
real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(e)^2 - 8*a*b*real_p
art(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(e)^2 + 8*a*b*real_part(c
os_integral(f*x + c*f/d))*tan(1/2*e)*tan(e)^2 + 8*a*b*real_part(cos_integra
l(-f*x - c*f/d))*tan(1/2*e)*tan(e)^2 + 4*a*b*imag_part(cos_integral(f*x + c
*f/d))*tan(c*f/d)^2 - 4*a*b*imag_part(cos_integral(-f*x - c*f/d))*tan(c*f/d
)^2 + 4*a^2*log(abs(d*x + c))*tan(c*f/d)^2 + 2*b^2*log(abs(d*x + c))*tan(c*
f/d)^2 + b^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2 + b^2*re
al_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2 + 8*a*b*sin_integral((
d*f*x + c*f)/d)*tan(c*f/d)^2 - 4*a*b*imag_part(cos_integral(f*x + c*f/d))*t
an(1/2*c*f/d)^2 + 4*a*b*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d
)^2 + 4*a^2*log(abs(d*x + c))*tan(1/2*c*f/d)^2 + 2*b^2*log(abs(d*x + c))*ta
n(1/2*c*f/d)^2 - b^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d
)^2 - b^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)^2 - 8*a*
b*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2 + 16*a*b*imag_part(cos_int
egral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) - 16*a*b*imag_part(cos_integr
```

```
al(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) + 32*a*b*sin_integral((d*f*x +
c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e) - 4*a*b*imag_part(cos_integral(f*x + c*f/
d))*tan(1/2*e)^2 + 4*a*b*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2
 + 4*a^2*log(abs(d*x + c))*tan(1/2*e)^2 + 2*b^2*log(abs(d*x + c))*tan(1/2*e
)^2 - b^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2 - b^2*real_
part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2 - 8*a*b*sin_integral((d*f
*x + c*f)/d)*tan(1/2*e)^2 - 4*b^2*real_part(cos_integral(2*f*x + 2*c*f/d))*
tan(c*f/d)*tan(e) - 4*b^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f
/d)*tan(e) + 4*a*b*imag_part(cos_integral(f*x + c*f/d))*tan(e)^2 - 4*a*b*im
ag_part(cos_integral(-f*x - c*f/d))*tan(e)^2 + 4*a^2*log(abs(d*x + c))*tan(
e)^2 + 2*b^2*log(abs(d*x + c))*tan(e)^2 + b^2*real_part(cos_integral(2*f*x
+ 2*c*f/d))*tan(e)^2 + b^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(e)
^2 + 8*a*b*sin_integral((d*f*x + c*f)/d)*tan(e)^2 - 2*b^2*imag_part(cos_int
egral(2*f*x + 2*c*f/d))*tan(c*f/d) + 2*b^2*imag_part(cos_integral(-2*f*x -
2*c*f/d))*tan(c*f/d) - 4*b^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d) - 8
*a*b*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 8*a*b*real_part(
cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) + 8*a*b*real_part(cos_integral(f
*x + c*f/d))*tan(1/2*e) + 8*a*b*real_part(cos_integral(-f*x - c*f/d))*tan(1
/2*e) + 2*b^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(e) - 2*b^2*imag_
part(cos_integral(-2*f*x - 2*c*f/d))*tan(e) + 4*b^2*sin_integral(2*(d*f*x +
 c*f)/d)*tan(e) + 4*a*b*imag_part(cos_integral(f*x + c*f/d)) - 4*a*b*imag_p
art(cos_integral(-f*x - c*f/d)) + 4*a^2*log(abs(d*x + c)) + 2*b^2*log(abs(d
*x + c)) - b^2*real_part(cos_integral(2*f*x + 2*c*f/d)) - b^2*real_part(cos
_integral(-2*f*x - 2*c*f/d)) + 8*a*b*sin_integral((d*f*x + c*f)/d))/(d*tan(
c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + d*tan(c*f/d)^2*tan(1/2*c*
f/d)^2*tan(1/2*e)^2 + d*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + d*tan(c*f/
d)^2*tan(1/2*e)^2*tan(e)^2 + d*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + d*t
an(c*f/d)^2*tan(1/2*c*f/d)^2 + d*tan(c*f/d)^2*tan(1/2*e)^2 + d*tan(1/2*c*f/
d)^2*tan(1/2*e)^2 + d*tan(c*f/d)^2*tan(e)^2 + d*tan(1/2*c*f/d)^2*tan(e)^2 +
 d*tan(1/2*e)^2*tan(e)^2 + d*tan(c*f/d)^2 + d*tan(1/2*c*f/d)^2 + d*tan(1/2*
e)^2 + d*tan(e)^2 + d)
```

3.161 $\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=183

$$-\frac{a^2}{d(c+dx)}+\frac{2abf\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d^2}-\frac{2abf\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{2ab\sin(e+fx)}{d(c+dx)}+\frac{b^2f\text{CosInt}}{}$$

```
[Out] -(a^2/(d*(c + d*x))) + (2*a*b*f*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x]
)/d^2 + (b^2*f*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/d^2 - (
2*a*b*Sin[e + f*x])/(d*(c + d*x)) - (b^2*Sin[e + f*x]^2)/(d*(c + d*x)) - (2
*a*b*f*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d^2 + (b^2*f*Cos[2*e -
(2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/d^2
```

Rubi [A] time = 0.334011, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3317, 3297, 3303, 3299, 3302, 3313, 12}

$$-\frac{a^2}{d(c+dx)}+\frac{2abf\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d^2}-\frac{2abf\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{2ab\sin(e+fx)}{d(c+dx)}+\frac{b^2f\text{CosInt}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*x)^2,x]
```


```
[Out] -(a^2/(d*(c + d*x))) + (2*a*b*f*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x]
)/d^2 + (b^2*f*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/d^2 - (
2*a*b*Sin[e + f*x])/(d*(c + d*x)) - (b^2*Sin[e + f*x]^2)/(d*(c + d*x)) - (2
*a*b*f*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d^2 + (b^2*f*Cos[2*e -
(2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/d^2
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^2}\,dx = \int \left(\frac{a^2}{(c+dx)^2} + \frac{2ab\sin(e+fx)}{(c+dx)^2} + \frac{b^2\sin^2(e+fx)}{(c+dx)^2}\right) dx$$

$$= -\frac{a^2}{d(c+dx)} + (2ab)\int \frac{\sin(e+fx)}{(c+dx)^2}\,dx + b^2\int \frac{\sin^2(e+fx)}{(c+dx)^2}\,dx$$

$$= -\frac{a^2}{d(c+dx)} - \frac{2ab\sin(e+fx)}{d(c+dx)} - \frac{b^2\sin^2(e+fx)}{d(c+dx)} + \frac{(2abf)\int \frac{\cos(e+fx)}{c+dx}\,dx}{d} + \frac{\left(2b^2f\right)\int \frac{\sin(2e+2f}{2(c+dx)}}{d}$$

$$= -\frac{a^2}{d(c+dx)} - \frac{2ab\sin(e+fx)}{d(c+dx)} - \frac{b^2\sin^2(e+fx)}{d(c+dx)} + \frac{\left(b^2f\right)\int \frac{\sin(2e+2fx)}{c+dx}\,dx}{d} + \frac{\left(2abf\cos\left(e-\frac{cf}{d}\right.\right.}{}$$

$$= -\frac{a^2}{d(c+dx)} + \frac{2abf\cos\left(e-\frac{cf}{d}\right)\text{Ci}\left(\frac{cf}{d}+fx\right)}{d^2} - \frac{2ab\sin(e+fx)}{d(c+dx)} - \frac{b^2\sin^2(e+fx)}{d(c+dx)} - \frac{2abf\,\text{si}}{}$$

$$= -\frac{a^2}{d(c+dx)} + \frac{2abf\cos\left(e-\frac{cf}{d}\right)\text{Ci}\left(\frac{cf}{d}+fx\right)}{d^2} + \frac{b^2f\text{Ci}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2} - \frac{2ab\sin(}{d(c+}$$

Mathematica [A] time = 0.596356, size = 232, normalized size = 1.27

$$\frac{-2a^2d + 4abf(c+dx)\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\cos\left(e-\frac{cf}{d}\right) - 4abcf\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right) - 4abdfx\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (-2*a^2*d - b^2*d + b^2*d*Cos[2*(e + f*x)] + 4*a*b*f*(c + d*x)*Cos[e - (c*f
)/d]*CosIntegral[f*(c/d + x)] + 2*b^2*f*(c + d*x)*CosIntegral[(2*f*(c + d*x
))/d]*Sin[2*e - (2*c*f)/d] - 4*a*b*d*Sin[e + f*x] - 4*a*b*c*f*Sin[e - (c*f)
/d]*SinIntegral[f*(c/d + x)] - 4*a*b*d*f*x*Sin[e - (c*f)/d]*SinIntegral[f*(
c/d + x)] + 2*b^2*c*f*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] +
 2*b^2*d*f*x*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d])/(2*d^2*(c
 + d*x))
```

Maple [A] time = 0.018, size = 301, normalized size = 1.6

$$\frac{1}{f}\left(-\frac{a^2f^2}{\left(\left(fx+e\right)d+cf-de\right)d} + 2f^2ab\left(-\frac{\sin\left(fx+e\right)}{\left(\left(fx+e\right)d+cf-de\right)d} + \frac{1}{d}\left(\frac{1}{d}\text{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right) + \frac{1}{d}\text{Ci}\left(fx\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(d*x+c)^2,x)
```

```
[Out] 1/f*(-a^2*f^2/((f*x+e)*d+c*f-d*e)/d+2*f^2*a*b*(-sin(f*x+e)/((f*x+e)*d+c*f-d
*e)/d+(Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+Ci(f*x+e+(c*f-d*e)/d)*cos((
c*f-d*e)/d)/d)/d)-1/2*f^2*b^2/((f*x+e)*d+c*f-d*e)/d-1/4*f^2*b^2*(-2*cos(2*f
*x+2*e)/((f*x+e)*d+c*f-d*e)/d-2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d
*e)/d)/d-2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)/d))
```

Maxima [C] time = 1.50062, size = 498, normalized size = 2.72

$$-\frac{\frac{64\,a^2 f^2}{(fx+e)d^2-d^2e+cdf} - \frac{64\left(f^2\left(-i\,E_2\left(\frac{i(fx+e)d-i\,de+i\,cf}{d}\right)+i\,E_2\left(-\frac{i(fx+e)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f^2\left(E_2\left(\frac{i(fx+e)d-i\,de+i\,cf}{d}\right)+E_2\left(-\frac{i(fx+e)d-i\,de+i\,cf}{d}\right)\right)\sin\left(-\frac{de-c}{d}\right.}{(fx+e)d^2-d^2e+cdf}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/64*(64*a^2*f^2/((f*x + e)*d^2 - d^2*e + c*d*f) - 64*(f^2*(-I*exp_integra
l_e(2, (I*(f*x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(2, -(I*(f*x +
e)*d - I*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f^2*(exp_integral_e(2, (I*(
f*x + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(2, -(I*(f*x + e)*d - I*d*e
+ I*c*f)/d))*sin(-(d*e - c*f)/d))*a*b/((f*x + e)*d^2 - d^2*e + c*d*f) - (16
*f^2*(exp_integral_e(2, (2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d) + exp_inte
gral_e(2, -(2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d))*cos(-2*(d*e - c*f)/d)
+ f^2*(16*I*exp_integral_e(2, (2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d) - 16
*I*exp_integral_e(2, -(2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d))*sin(-2*(d*e
 - c*f)/d) - 32*f^2)*b^2/((f*x + e)*d^2 - d^2*e + c*d*f))/f
```

Fricas [A] time = 2.26381, size = 693, normalized size = 3.79

$$\frac{2\,b^2 d\cos\left(fx+e\right)^2 - 4\,abd\sin\left(fx+e\right) + 2\left(b^2 dfx + b^2 cf\right)\cos\left(-\frac{2\left(de-cf\right)}{d}\right)\operatorname{Si}\left(\frac{2\left(dfx+cf\right)}{d}\right) + 4\left(abdfx + abcf\right)\sin\left(-\frac{de-}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*d*cos(f*x + e)^2 - 4*a*b*d*sin(f*x + e) + 2*(b^2*d*f*x + b^2*c*f
)*cos(-2*(d*e - c*f)/d)*sin_integral(2*(d*f*x + c*f)/d) + 4*(a*b*d*f*x + a*
b*c*f)*sin(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/d) - 2*(a^2 + b^2)*d
+ 2*((a*b*d*f*x + a*b*c*f)*cos_integral((d*f*x + c*f)/d) + (a*b*d*f*x + a*b
*c*f)*cos_integral(-(d*f*x + c*f)/d))*cos(-(d*e - c*f)/d) - ((b^2*d*f*x + b
^2*c*f)*cos_integral(2*(d*f*x + c*f)/d) + (b^2*d*f*x + b^2*c*f)*cos_integra
l(-2*(d*f*x + c*f)/d))*sin(-2*(d*e - c*f)/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\sin\left(e + fx\right)\right)^2}{(c + dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2/(c + d*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.162 $\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^3}\,dx$

Optimal. Leaf size=245

$$-\frac{a^2}{2d(c+dx)^2}-\frac{abf^2\operatorname{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3}-\frac{abf^2\cos\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(xf+\frac{cf}{d}\right)}{d^3}-\frac{abf\cos(e+fx)}{d^2(c+dx)}-\frac{ab\sin(e}{d(c+}$$

```
[Out] -a^2/(2*d*(c + d*x)^2) - (a*b*f*Cos[e + f*x])/(d^2*(c + d*x)) + (b^2*f^2*Co
s[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/d^3 - (a*b*f^2*CosIntegr
al[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d^3 - (a*b*Sin[e + f*x])/(d*(c + d*x)^2
) - (b^2*f*Cos[e + f*x]*Sin[e + f*x])/(d^2*(c + d*x)) - (b^2*Sin[e + f*x]^2
)/(2*d*(c + d*x)^2) - (a*b*f^2*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])
/d^3 - (b^2*f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/d^3
```

Rubi [A] time = 0.424202, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3317, 3297, 3303, 3299, 3302, 3314, 31, 3312}

$$-\frac{a^2}{2d(c+dx)^2}-\frac{abf^2\operatorname{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3}-\frac{abf^2\cos\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(xf+\frac{cf}{d}\right)}{d^3}-\frac{abf\cos(e+fx)}{d^2(c+dx)}-\frac{ab\sin(e}{d(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] -a^2/(2*d*(c + d*x)^2) - (a*b*f*Cos[e + f*x])/(d^2*(c + d*x)) + (b^2*f^2*Co
s[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/d^3 - (a*b*f^2*CosIntegr
al[(c*f)/d + f*x]*Sin[e - (c*f)/d])/d^3 - (a*b*Sin[e + f*x])/(d*(c + d*x)^2
) - (b^2*f*Cos[e + f*x]*Sin[e + f*x])/(d^2*(c + d*x)) - (b^2*Sin[e + f*x]^2
)/(2*d*(c + d*x)^2) - (a*b*f^2*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])
/d^3 - (b^2*f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/d^3
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+dx)^3}\,dx = \int \left(\frac{a^2}{(c+dx)^3}+\frac{2ab\sin(e+fx)}{(c+dx)^3}+\frac{b^2\sin^2(e+fx)}{(c+dx)^3}\right)dx$$

$$= -\frac{a^2}{2d(c+dx)^2}+(2ab)\int \frac{\sin(e+fx)}{(c+dx)^3}\,dx+b^2\int \frac{\sin^2(e+fx)}{(c+dx)^3}\,dx$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\sin(e+fx)}{d(c+dx)^2}-\frac{b^2f\cos(e+fx)\sin(e+fx)}{d^2(c+dx)}-\frac{b^2\sin^2(e+fx)}{2d(c+dx)^2}+\frac{(abf)\int}{}$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cos(e+fx)}{d^2(c+dx)}+\frac{b^2f^2\log(c+dx)}{d^3}-\frac{ab\sin(e+fx)}{d(c+dx)^2}-\frac{b^2f\cos(e+fx)\sin}{d^2(c+dx)}$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cos(e+fx)}{d^2(c+dx)}-\frac{ab\sin(e+fx)}{d(c+dx)^2}-\frac{b^2f\cos(e+fx)\sin(e+fx)}{d^2(c+dx)}-\frac{b^2\sin^2(}{2d(c+}$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cos(e+fx)}{d^2(c+dx)}-\frac{abf^2\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^3}-\frac{ab\sin(e+fx)}{d(c+dx)^2}-\frac{b^2f\,c}{}$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cos(e+fx)}{d^2(c+dx)}+\frac{b^2f^2\cos\left(2e-\frac{2cf}{d}\right)\text{Ci}\left(\frac{2cf}{d}+2fx\right)}{d^3}-\frac{abf^2\text{Ci}\left(\frac{cf}{d}+fx\right.}{d^3}$$

Mathematica [A] time = 1.22926, size = 395, normalized size = 1.61

$$-\frac{2a^2d^2+4abc^2f^2\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)+4abf^2(c+dx)^2\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+4abd^2f^2x^2\cos\left(e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] -(2*a^2*d^2 + b^2*d^2 + 4*a*b*c*d*f*Cos[e + f*x] + 4*a*b*d^2*f*x*Cos[e + f*
x] - b^2*d^2*Cos[2*(e + f*x)] - 4*b^2*f^2*(c + d*x)^2*Cos[2*e - (2*c*f)/d]*
CosIntegral[(2*f*(c + d*x))/d] + 4*a*b*f^2*(c + d*x)^2*CosIntegral[f*(c/d +
 x)]*Sin[e - (c*f)/d] + 4*a*b*d^2*Sin[e + f*x] + 2*b^2*c*d*f*Sin[2*(e + f*x
)] + 2*b^2*d^2*f*x*Sin[2*(e + f*x)] + 4*a*b*c^2*f^2*Cos[e - (c*f)/d]*SinInt
egral[f*(c/d + x)] + 8*a*b*c*d*f^2*x*Cos[e - (c*f)/d]*SinIntegral[f*(c/d +
x)] + 4*a*b*d^2*f^2*x^2*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + 4*b^2*c
^2*f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 8*b^2*c*d*f^2*
x*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 4*b^2*d^2*f^2*x^2*S
```

```
in[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d])/(4*d^3*(c + d*x)^2)
```

Maple [A] time = 0.023, size = 374, normalized size = 1.5

$$\frac{1}{f}\left(-\frac{a^2f^3}{2\left(\left(fx+e\right)d+cf-de\right)^2d}+2f^3ab\left(-1/2\frac{\sin\left(fx+e\right)}{\left(\left(fx+e\right)d+cf-de\right)^2d}+1/2\frac{1}{d}\left(-\frac{\cos\left(fx+e\right)}{\left(\left(fx+e\right)d+cf-de\right)d}-\frac{1}{d}\left(\frac{1}{d}\mathrm{Si}\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(d*x+c)^3,x)
```

```
[Out] 1/f*(-1/2*a^2*f^3/((f*x+e)*d+c*f-d*e)^2/d+2*f^3*a*b*(-1/2*sin(f*x+e)/((f*x+
e)*d+c*f-d*e)^2/d+1/2*(-cos(f*x+e)/((f*x+e)*d+c*f-d*e)/d-(Si(f*x+e+(c*f-d*e
)/d)*cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)/d)/d)-1/4
*f^3*b^2/((f*x+e)*d+c*f-d*e)^2/d-1/4*f^3*b^2*(-cos(2*f*x+2*e)/((f*x+e)*d+c*
f-d*e)^2/d-(-2*sin(2*f*x+2*e)/((f*x+e)*d+c*f-d*e)/d+2*(2*Si(2*f*x+2*e+2*(c*
f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e
)/d)/d)/d)/d))
```

Maxima [C] time = 1.87744, size = 640, normalized size = 2.61

$$-\frac{\frac{32\,a^2f^3}{\left(fx+e\right)^2d^3+d^3e^2-2\,cd^2ef+c^2df^2-2\left(d^3e-cd^2f\right)\left(fx+e\right)}-\frac{64\left(f^3\left(-i\,E_3\left(\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)+i\,E_3\left(-\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f^3\left(E_3\left(\frac{i\left(fx+e\right)d-i\,de+i\,cf}{d}\right)+E\right.\right.}{\left(fx+e\right)^2d^3+d^3e^2-2\,cd^2ef+c^2df^2-2\left(d^3e-cd^2f\right)\left(fx+e\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(32*a^2*f^3/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*
(d^3*e - c*d^2*f)*(f*x + e)) - 64*(f^3*(-I*exp_integral_e(3, (I*(f*x + e)*d
 - I*d*e + I*c*f)/d) + I*exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)
/d))*cos(-(d*e - c*f)/d) + f^3*(exp_integral_e(3, (I*(f*x + e)*d - I*d*e +
I*c*f)/d) + exp_integral_e(3, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*sin(-(d*
e - c*f)/d))*a*b/((f*x + e)^2*d^3 + d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*(
d^3*e - c*d^2*f)*(f*x + e)) - (16*f^3*(exp_integral_e(3, (2*I*(f*x + e)*d -
 2*I*d*e + 2*I*c*f)/d) + exp_integral_e(3, -(2*I*(f*x + e)*d - 2*I*d*e + 2*
I*c*f)/d))*cos(-2*(d*e - c*f)/d) + f^3*(16*I*exp_integral_e(3, (2*I*(f*x +
```

```
e)*d - 2*I*d*e + 2*I*c*f)/d) - 16*I*exp_integral_e(3, -(2*I*(f*x + e)*d - 2
*I*d*e + 2*I*c*f)/d))*sin(-2*(d*e - c*f)/d) - 16*f^3)*b^2/((f*x + e)^2*d^3
+ d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*(d^3*e - c*d^2*f)*(f*x + e)))/f
```

Fricas [A] time = 2.50881, size = 1057, normalized size = 4.31

$$\frac{b^2d^2\cos\left(fx+e\right)^2-\left(a^2+b^2\right)d^2+2\left(b^2d^2f^2x^2+2\,b^2cdf^2x+b^2c^2f^2\right)\sin\left(-\frac{2\left(de-cf\right)}{d}\right)\operatorname{Si}\left(\frac{2\left(dfx+cf\right)}{d}\right)-2\left(abd^2f^2x^2+2\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*cos(f*x + e)^2 - (a^2 + b^2)*d^2 + 2*(b^2*d^2*f^2*x^2 + 2*b^2*
c*d*f^2*x + b^2*c^2*f^2)*sin(-2*(d*e - c*f)/d)*sin_integral(2*(d*f*x + c*f)
/d) - 2*(a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*cos(-(d*e - c*f)/
d)*sin_integral((d*f*x + c*f)/d) - 2*(a*b*d^2*f*x + a*b*c*d*f)*cos(f*x + e)
 + ((b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*cos_integral(2*(d*f*x
 + c*f)/d) + (b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*cos_integral
(-2*(d*f*x + c*f)/d))*cos(-2*(d*e - c*f)/d) - 2*(a*b*d^2 + (b^2*d^2*f*x + b
^2*c*d*f)*cos(f*x + e))*sin(f*x + e) + ((a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x
+ a*b*c^2*f^2)*cos_integral((d*f*x + c*f)/d) + (a*b*d^2*f^2*x^2 + 2*a*b*c*d
*f^2*x + a*b*c^2*f^2)*cos_integral(-(d*f*x + c*f)/d))*sin(-(d*e - c*f)/d))/
(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(e+fx\right)\right)^2}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2/(c + d*x)**3, x)
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.163 $\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=495

$$-\frac{6id^2(c+dx)\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}}+\frac{6id^2(c+dx)\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^3\sqrt{a^2-b^2}}-\frac{3d(c+dx)^2\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}+3d(c$$

```
[Out] ((-I)*(c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sq
rt[a^2 - b^2]*f) + (I*(c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a
^2 - b^2])])/(Sqrt[a^2 - b^2]*f) - (3*d*(c + d*x)^2*PolyLog[2, (I*b*E^(I*(e
 + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2) + (3*d*(c + d*x)^2*
PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f
^2) - ((6*I)*d^2*(c + d*x)*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 -
 b^2])])/(Sqrt[a^2 - b^2]*f^3) + ((6*I)*d^2*(c + d*x)*PolyLog[3, (I*b*E^(I*
(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^3) + (6*d^3*PolyLog[
4, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^4) - (6
*d^3*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b
^2]*f^4)
```

Rubi [A] time = 0.969198, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3323, 2264, 2190, 2531, 6609, 2282, 6589}

$$-\frac{6id^2(c+dx)\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}}+\frac{6id^2(c+dx)\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^3\sqrt{a^2-b^2}}-\frac{3d(c+dx)^2\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}+3d(c$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sq
rt[a^2 - b^2]*f) + (I*(c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a
^2 - b^2])])/(Sqrt[a^2 - b^2]*f) - (3*d*(c + d*x)^2*PolyLog[2, (I*b*E^(I*(e
 + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2) + (3*d*(c + d*x)^2*
PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f
^2) - ((6*I)*d^2*(c + d*x)*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 -
 b^2])])/(Sqrt[a^2 - b^2]*f^3) + ((6*I)*d^2*(c + d*x)*PolyLog[3, (I*b*E^(I*
(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^3) + (6*d^3*PolyLog[
4, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^4) - (6
```

```
*d^3*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b
^2]*f^4)
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx = 2\int \frac{e^{i(e+fx)}(c+dx)^3}{ib+2ae^{i(e+fx)}-ibe^{2i(e+fx)}}\,dx$$

$$= -\frac{(2ib)\int \frac{e^{i(e+fx)}(c+dx)^3}{2a-2\sqrt{a^2-b^2}-2ibe^{i(e+fx)}}\,dx}{\sqrt{a^2-b^2}} + \frac{(2ib)\int \frac{e^{i(e+fx)}(c+dx)^3}{2a+2\sqrt{a^2-b^2}-2ibe^{i(e+fx)}}\,dx}{\sqrt{a^2-b^2}}$$

$$= -\frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{(3id)\int(c+dx)^2\log\left(1-\frac{2ib \cdots}{2a-2 \cdots}\right)}{\sqrt{a^2-b^2}f}$$

$$= -\frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{3d(c+dx)^2\text{Li}_2\left(\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2} + \cdots$$

$$= -\frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{3d(c+dx)^2\text{Li}_2\left(\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2} + \cdots$$

$$= -\frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{3d(c+dx)^2\text{Li}_2\left(\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2} + \cdots$$

$$= -\frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{i(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{3d(c+dx)^2\text{Li}_2\left(\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2} + \cdots$$

Mathematica [A] time = 0.234814, size = 401, normalized size = 0.81

$$-\frac{i\left(\frac{3id\left(f^2(c+dx)^2\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)+2idf(c+dx)\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)-2d^2\text{PolyLog}\left(4,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)\right)}{f^3}+\frac{3d\left(2d\left(f(c+dx)\text{PolyLog}\left(3,-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}-a}\right)+id\text{Poly}\cdots\right.\right.}{\cdots}\right.}{f\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((-I)*((c + d*x)^3*Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2])] -
(c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])] + (3*d*((-
I)*f^2*(c + d*x)^2*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2
])] + 2*d*(f*(c + d*x)*PolyLog[3, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 -
 b^2])] + I*d*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])))/f^
3 + ((3*I)*d*(f^2*(c + d*x)^2*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^
2 - b^2])] + (2*I)*d*f*(c + d*x)*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt
[a^2 - b^2])] - 2*d^2*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2]
)]))/f^3))/(Sqrt[a^2 - b^2]*f)
```

Maple [F] time = 0.401, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{a+b\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*sin(f*x+e)),x)
```

```
[Out] int((d*x+c)^3/(a+b*sin(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.67475, size = 5222, normalized size = 10.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/4*(12*I*b*d^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(f*x + e)
- 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2
)/b^2))/b) - 12*I*b*d^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(f*
x + e) - 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^
2 - b^2)/b^2))/b) - 12*I*b*d^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(-2*I*
a*cos(f*x + e) - 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*s
qrt(-(a^2 - b^2)/b^2))/b) + 12*I*b*d^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/
2*(-2*I*a*cos(f*x + e) - 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) - I*b*sin(f*x
 + e))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(-3*I*b*d^3*f^2*x^2 - 6*I*b*c*d^2*f^2
*x - 3*I*b*c^2*d*f^2)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(f*x + e)
 + 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^
2)/b^2) + 2*b)/b + 1) + 2*(3*I*b*d^3*f^2*x^2 + 6*I*b*c*d^2*f^2*x + 3*I*b*c^
2*d*f^2)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(f*x + e) + 2*a*sin(f*
x + e) - 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b
)/b + 1) + 2*(3*I*b*d^3*f^2*x^2 + 6*I*b*c*d^2*f^2*x + 3*I*b*c^2*d*f^2)*sqrt
(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(
b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2
*(-3*I*b*d^3*f^2*x^2 - 6*I*b*c*d^2*f^2*x - 3*I*b*c^2*d*f^2)*sqrt(-(a^2 - b^
2)/b^2)*dilog(-1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) - 2*(b*cos(f*x +
 e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(b*d^3*e^3
 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*lo
g(2*b*cos(f*x + e) + 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*
a) + 2*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt(-(a
^2 - b^2)/b^2)*log(2*b*cos(f*x + e) - 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 -
 b^2)/b^2) - 2*I*a) - 2*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*
c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(f*x + e) + 2*I*b*sin(f*x + e)
+ 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*
b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(f*x + e) - 2
*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(b*d^3*f^3*x^3
+ 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c
^2*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(f*x + e) + 2*a*sin(f*
x + e) + 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b
)/b) + 2*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 -
 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*c
os(f*x + e) + 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt
(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c
^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt(-(a^2 - b^
2)/b^2)*log(1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(b*cos(f*x + e)
 + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b*d^3*f^3*x^3 +
3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2
```

```
*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x
 + e) - 2*(b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)
/b) + 12*(b*d^3*f*x + b*c*d^2*f)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I
*a*cos(f*x + e) - 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) + I*b*sin(f*x + e))*
sqrt(-(a^2 - b^2)/b^2))/b) - 12*(b*d^3*f*x + b*c*d^2*f)*sqrt(-(a^2 - b^2)/b
^2)*polylog(3, 1/2*(2*I*a*cos(f*x + e) - 2*a*sin(f*x + e) - 2*(b*cos(f*x +
e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(b*d^3*f*x + b*c*d^2
*f)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(-2*I*a*cos(f*x + e) - 2*a*sin(f*
x + e) + 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) -
 12*(b*d^3*f*x + b*c*d^2*f)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(-2*I*a*c
os(f*x + e) - 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt
(-(a^2 - b^2)/b^2))/b))/((a^2 - b^2)*f^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^3}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**3/(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*sin(f*x + e) + a), x)
```

3.164 $\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=367

$$-\frac{2d(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}+\frac{2d(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^2\sqrt{a^2-b^2}}-\frac{2id^2\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}}+\frac{2id^2\text{PolyLog}(}{f^3\sqrt{a^2}}$$

```
[Out] ((-I)*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sq
rt[a^2 - b^2]*f) + (I*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a
^2 - b^2])])/(Sqrt[a^2 - b^2]*f) - (2*d*(c + d*x)*PolyLog[2, (I*b*E^(I*(e +
 f*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2) + (2*d*(c + d*x)*Poly
Log[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2)
- ((2*I)*d^2*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt
[a^2 - b^2]*f^3) + ((2*I)*d^2*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^
2 - b^2])])/(Sqrt[a^2 - b^2]*f^3)
```

Rubi [A] time = 0.821562, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3323, 2264, 2190, 2531, 2282, 6589}

$$-\frac{2d(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}+\frac{2d(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^2\sqrt{a^2-b^2}}-\frac{2id^2\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}}+\frac{2id^2\text{PolyLog}(}{f^3\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sq
rt[a^2 - b^2]*f) + (I*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a
^2 - b^2])])/(Sqrt[a^2 - b^2]*f) - (2*d*(c + d*x)*PolyLog[2, (I*b*E^(I*(e +
 f*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2) + (2*d*(c + d*x)*Poly
Log[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2)
- ((2*I)*d^2*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt
[a^2 - b^2]*f^3) + ((2*I)*d^2*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^
2 - b^2])])/(Sqrt[a^2 - b^2]*f^3)
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
```

```
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx = 2\int \frac{e^{i(e+fx)}(c+dx)^2}{ib+2ae^{i(e+fx)}-ibe^{2i(e+fx)}}\,dx$$

$$= -\frac{(2ib)\int \frac{e^{i(e+fx)}(c+dx)^2}{2a-2\sqrt{a^2-b^2}-2ibe^{i(e+fx)}}\,dx}{\sqrt{a^2-b^2}} + \frac{(2ib)\int \frac{e^{i(e+fx)}(c+dx)^2}{2a+2\sqrt{a^2-b^2}-2ibe^{i(e+fx)}}\,dx}{\sqrt{a^2-b^2}}$$

$$= -\frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{(2id)\int(c+dx)\log\left(1-\frac{2ibe^{i}}{2a-2\cdots}\right.}{\sqrt{a^2-b^2}f}$$

$$= -\frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{2d(c+dx)\mathrm{Li}_2\left(\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2} + \cdots$$

$$= -\frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{2d(c+dx)\mathrm{Li}_2\left(\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2} + \cdots$$

$$= -\frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{i(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{2d(c+dx)\mathrm{Li}_2\left(\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2} + \cdots$$

Mathematica [A] time = 0.191191, size = 296, normalized size = 0.81

$$-\frac{i\left(\frac{2d\left(d\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)-if(c+dx)\mathrm{PolyLog}\left(2,-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}-a}\right)\right)}{f^2}+\frac{2id\left(f(c+dx)\mathrm{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)+id\,\mathrm{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)\right)}{f^2}+(c+dx)^2\log\left(1\cdots\right.\right.}{f\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((-I)*((c + d*x)^2*Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2])] -
(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])] + (2*d*((-
I)*f*(c + d*x)*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2])]
+ d*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])]))/f^2 + ((2*I)*
d*(f*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])] + I*
d*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]))/f^2))/(Sqrt[a^2
 - b^2]*f)
```

Maple [F] time = 0.283, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{a+b\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*sin(f*x+e)),x)
```

```
[Out] int((d*x+c)^2/(a+b*sin(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.14299, size = 3729, normalized size = 10.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(f*x + e) - 2
*a*sin(f*x + e) + 2*(b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b
^2))/b) - 2*b*d^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(f*x + e)
 - 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^
2)/b^2))/b) + 2*b*d^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(-2*I*a*cos(f*x
 + e) - 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2
 - b^2)/b^2))/b) - 2*b*d^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(-2*I*a*co
s(f*x + e) - 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(
-(a^2 - b^2)/b^2))/b) + (-2*I*b*d^2*f*x - 2*I*b*c*d*f)*sqrt(-(a^2 - b^2)/b^
2)*dilog(-1/2*(2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) -
```

```
I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (2*I*b*d^2*f*x + 2
*I*b*c*d*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(f*x + e) + 2*a*sin
(f*x + e) - 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) +
2*b)/b + 1) + (2*I*b*d^2*f*x + 2*I*b*c*d*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1
/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) + I*b*sin(f*
x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-2*I*b*d^2*f*x - 2*I*b*c*d*
f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e
) - 2*(b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b +
 1) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*
cos(f*x + e) + 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (
b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(f*x
 + e) - 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (b*d^2*e
^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(f*x + e)
+ 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b*d^2*e^2 - 2
*b*c*d*e*f + b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(f*x + e) - 2*I*
b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (b*d^2*f^2*x^2 + 2*b
*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a
*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sq
rt(-(a^2 - b^2)/b^2) + 2*b)/b) + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2
 + 2*b*c*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(f*x + e) + 2*a*si
n(f*x + e) - 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) +
 2*b)/b) - (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*sqrt(-
(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(b*cos
(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (b*d^2*f^2
*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*log(
1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) + I*b*sin(f
*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b))/((a^2 - b^2)*f^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^2}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**2/(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{b\sin(fx+e)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*sin(f*x + e) + a), x)
```

3.165 $\int \frac{c+dx}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=234

$$-\frac{d\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}+\frac{d\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^2\sqrt{a^2-b^2}}-\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}+\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f\sqrt{a^2-b^2}}$$

```
[Out] ((-I)*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt
[a^2 - b^2]*f) + (I*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 -
 b^2])])/(Sqrt[a^2 - b^2]*f) - (d*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a - Sqr
t[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2) + (d*PolyLog[2, (I*b*E^(I*(e + f*x)))
/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2)
```

Rubi [A] time = 0.452897, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3323, 2264, 2190, 2279, 2391}

$$-\frac{d\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}+\frac{d\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^2\sqrt{a^2-b^2}}-\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}+\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Sin[e + f*x]),x]
```


```
[Out] ((-I)*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt
[a^2 - b^2]*f) + (I*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 -
 b^2])])/(Sqrt[a^2 - b^2]*f) - (d*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a - Sqr
t[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2) + (d*PolyLog[2, (I*b*E^(I*(e + f*x)))
/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2)
```


Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx}{a+b\sin(e+fx)}\,dx &= 2\int \frac{e^{i(e+fx)}(c+dx)}{ib+2ae^{i(e+fx)}-ibe^{2i(e+fx)}}\,dx \\
&= -\frac{(2ib)\int \frac{e^{i(e+fx)}(c+dx)}{2a-2\sqrt{a^2-b^2}-2ibe^{i(e+fx)}}\,dx}{\sqrt{a^2-b^2}}+\frac{(2ib)\int \frac{e^{i(e+fx)}(c+dx)}{2a+2\sqrt{a^2-b^2}-2ibe^{i(e+fx)}}\,dx}{\sqrt{a^2-b^2}} \\
&= -\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f}+\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f}+\frac{(id)\int\log\left(1-\frac{2ibe^{i(e+fx)}}{2a-2\sqrt{a^2-b^2}}\right)dx}{\sqrt{a^2-b^2}f}- \\
&= -\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f}+\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f}+\frac{d\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{2ibx}{2a-2\sqrt{a^2-b^2}}\right)}{x}\,dx,x\right.}{\sqrt{a^2-b^2}f^2} \\
&= -\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f}+\frac{i(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f}-\frac{d\mathrm{Li}_2\left(\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2}+\frac{d\mathrm{Li}_2\left(\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b}}\right.}{\sqrt{a^2-b^2}f^2}
\end{aligned}
$$

Mathematica [A] time = 0.0408727, size = 182, normalized size = 0.78

$$\frac{-d\text{PolyLog}\left(2,-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}-a}\right)+d\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)-if(c+dx)\left(\log\left(1+\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}-a}\right)-\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)\right)}{f^2\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((-I)*f*(c + d*x)*(Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2])] -
Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]) - d*PolyLog[2, ((-I)*
b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2])] + d*PolyLog[2, (I*b*E^(I*(e + f*
x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*f^2)
```

Maple [B] time = 0.104, size = 492, normalized size = 2.1

$$\frac{2\,ic}{f}\arctan\left(\frac{2\,ib\mathrm{e}^{i(fx+e)}-2\,a}{2}\frac{1}{\sqrt{-a^2+b^2}}\right)\frac{1}{\sqrt{-a^2+b^2}}+\frac{dx}{f}\ln\left(\left(-ia-b\mathrm{e}^{i(fx+e)}+\sqrt{-a^2+b^2}\right)\left(-ia+\sqrt{-a^2+b^2}\right)^{-1}\right)\frac{1}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*sin(f*x+e)),x)
```

```
[Out] 2*I/f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(f*x+e))-2*a)/(-a^2+b^2)^(
1/2))+1/f*d/(-a^2+b^2)^(1/2)*ln((-I*a-b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(-
I*a+(-a^2+b^2)^(1/2)))*x+1/f^2*d/(-a^2+b^2)^(1/2)*ln((-I*a-b*exp(I*(f*x+e))
+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*e-1/f*d/(-a^2+b^2)^(1/2)*ln((I*
a+b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-1/f^2*d/(-a^
2+b^2)^(1/2)*ln((I*a+b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/
2)))*e-I/f^2*d/(-a^2+b^2)^(1/2)*dilog((-I*a-b*exp(I*(f*x+e))+(-a^2+b^2)^(1/
2))/(-I*a+(-a^2+b^2)^(1/2)))+I/f^2*d/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(f
*x+e))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2*I/f^2*d*e/(-a^2+b^2)^(1/
2)*arctan(1/2*(2*I*b*exp(I*(f*x+e))-2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.58934, size = 2461, normalized size = 10.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/4*(-2*I*b*d*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(f*x + e) + 2*a*
sin(f*x + e) + 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2)
 + 2*b)/b + 1) + 2*I*b*d*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(f*x +
 e) + 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 -
 b^2)/b^2) + 2*b)/b + 1) + 2*I*b*d*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*
a*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) + I*b*sin(f*x + e))*s
qrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*I*b*d*sqrt(-(a^2 - b^2)/b^2)*dilog(
-1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) + I*b*sin(
f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(b*d*e - b*c*f)*sqrt(-(a
^2 - b^2)/b^2)*log(2*b*cos(f*x + e) + 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 -
 b^2)/b^2) + 2*I*a) + 2*(b*d*e - b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(
f*x + e) - 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(b*
d*e - b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(f*x + e) + 2*I*b*sin(f*x +
 e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(b*d*e - b*c*f)*sqrt(-(a^2 -
b^2)/b^2)*log(-2*b*cos(f*x + e) - 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2
)/b^2) - 2*I*a) - 2*(b*d*f*x + b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a
*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sq
rt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b*d*f*x + b*d*e)*sqrt(-(a^2 - b^2)/b^2)
*log(1/2*(2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) - I*b*s
in(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(b*d*f*x + b*d*e)*sqrt(-(
a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(b*cos(
f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b*d*f*x
+ b*d*e)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x
+ e) - 2*(b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/
b))/((a^2 - b^2)*f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c + dx}{a + b\sin\left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((c + d*x)/(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{b\sin\left(fx + e\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*sin(f*x + e) + a), x)
```

3.166 $\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a+b\sin(e+fx))},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*Sin[e + f*x])), x]
```

Rubi [A] time = 0.0642031, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Sin[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + b*Sin[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx$$

Mathematica [A] time = 0.40193, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Sin[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + b*Sin[e + f*x])), x]
```

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a+b\sin\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*sin(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a+b*sin(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)*(b*sin(f*x + e) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{adx+ac+(bdx+bc)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c + (b*d*x + b*c)*sin(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sin\left(e+fx\right)\right)(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(1/((a + b*sin(e + f*x))*(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*sin(f*x + e) + a)), x)
```

3.167 $\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a+b\sin(e+fx))},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a + b*Sin[e + f*x])), x]
```

Rubi [A] time = 0.0610818, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Sin[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Sin[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx$$

Mathematica [A] time = 0.333187, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Sin[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*Sin[e + f*x])), x]
```

Maple [A] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx+c\right)^{2} \left(a+b\sin\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*sin(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*sin(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx+c\right)^{2}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^2*(b*sin(f*x + e) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{ad^{2}x^{2}+2\,acdx+ac^{2}+\left(bd^{2}x^{2}+2\,bcdx+bc^{2}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)
*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2(b\sin(fx+e)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*sin(f*x + e) + a)), x)
```

3.168 $\int \frac{(c+dx)^3}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=925

$$-\frac{6\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)d^3}{\left(a^2-b^2\right)f^4}-\frac{6\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)d^3}{\left(a^2-b^2\right)f^4}+\frac{6a\text{PolyLog}\left(4,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)d^3}{\left(a^2-b^2\right)^{3/2}f^4}-\frac{6a\text{PolyLog}\left(4,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)d^3}{\left(a^2-b^2\right)^{3/2}f^4}$$

```
[Out] (I*(c + d*x)^3)/((a^2 - b^2)*f) - (3*d*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f
*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) - (I*a*(c + d*x)^3*Log[1 -
(I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) - (3*d*
(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b
^2)*f^2) + (I*a*(c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b
^2])])/((a^2 - b^2)^(3/2)*f) + ((6*I)*d^2*(c + d*x)*PolyLog[2, (I*b*E^(I*(e
 + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^3) - (3*a*d*(c + d*x)^2*Po
lyLog[2, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f
^2) + ((6*I)*d^2*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 -
 b^2])])/((a^2 - b^2)*f^3) + (3*a*d*(c + d*x)^2*PolyLog[2, (I*b*E^(I*(e + f
*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^2) - (6*d^3*PolyLog[3, (
I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^4) - ((6*I)*a*d
^2*(c + d*x)*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2
 - b^2)^(3/2)*f^3) - (6*d^3*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2
- b^2])])/((a^2 - b^2)*f^4) + ((6*I)*a*d^2*(c + d*x)*PolyLog[3, (I*b*E^(I*(
e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^3) + (6*a*d^3*PolyL
og[4, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^4)
 - (6*a*d^3*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2
- b^2)^(3/2)*f^4) + (b*(c + d*x)^3*Cos[e + f*x])/((a^2 - b^2)*f*(a + b*Sin[
e + f*x]))
```

Rubi [A] time = 1.65487, antiderivative size = 925, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {3324, 3323, 2264, 2190, 2531, 6609, 2282, 6589, 4519}

$$-\frac{6\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)d^3}{\left(a^2-b^2\right)f^4}-\frac{6\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)d^3}{\left(a^2-b^2\right)f^4}+\frac{6a\text{PolyLog}\left(4,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)d^3}{\left(a^2-b^2\right)^{3/2}f^4}-\frac{6a\text{PolyLog}\left(4,\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)d^3}{\left(a^2-b^2\right)^{3/2}f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (I*(c + d*x)^3)/((a^2 - b^2)*f) - (3*d*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f
*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) - (I*a*(c + d*x)^3*Log[1 -
(I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) - (3*d*
(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b
^2)*f^2) + (I*a*(c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b
^2])])/((a^2 - b^2)^(3/2)*f) + ((6*I)*d^2*(c + d*x)*PolyLog[2, (I*b*E^(I*(e
 + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^3) - (3*a*d*(c + d*x)^2*Po
lyLog[2, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f
^2) + ((6*I)*d^2*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 -
 b^2])])/((a^2 - b^2)*f^3) + (3*a*d*(c + d*x)^2*PolyLog[2, (I*b*E^(I*(e + f
*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^2) - (6*d^3*PolyLog[3, (
I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^4) - ((6*I)*a*d
^2*(c + d*x)*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2
 - b^2)^(3/2)*f^3) - (6*d^3*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2
- b^2])])/((a^2 - b^2)*f^4) + ((6*I)*a*d^2*(c + d*x)*PolyLog[3, (I*b*E^(I*(
e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^3) + (6*a*d^3*PolyL
og[4, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^4)
 - (6*a*d^3*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2
- b^2)^(3/2)*f^4) + (b*(c + d*x)^3*Cos[e + f*x])/((a^2 - b^2)*f*(a + b*Sin[
e + f*x]))
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
```

```
& PosQ[a^2 - b^2]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+b\sin(e+fx))^2}\,dx = \frac{b(c+dx)^3\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{a\int \frac{(c+dx)^3}{a+b\sin(e+fx)}\,dx}{a^2-b^2} - \frac{(3bd)\int \frac{(c+dx)^2\cos(e+fx)}{a+b\sin(e+fx)}\,dx}{\left(a^2-b^2\right)f}$$

$$= \frac{i(c+dx)^3}{\left(a^2-b^2\right)f} + \frac{b(c+dx)^3\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{(2a)\int \frac{e^{i(e+fx)}(c+dx)^3}{ib+2ae^{i(e+fx)}-ibe^{2i(e+fx)}}\,dx}{a^2-b^2} - \frac{(3bd)\int \frac{e^{i(}}{a-\sqrt{}}}{\left(a^2\right.}$$

$$= \frac{i(c+dx)^3}{\left(a^2-b^2\right)f} - \frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} - \frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} + \frac{b(c+}{\left(a^2-b^2\right)}$$

$$= \frac{i(c+dx)^3}{\left(a^2-b^2\right)f} - \frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} - \frac{ia(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} - \frac{3d(c+dx)^2}{(a}$$

$$= \frac{i(c+dx)^3}{\left(a^2-b^2\right)f} - \frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} - \frac{ia(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} - \frac{3d(c+dx)^2}{(a}$$

$$= \frac{i(c+dx)^3}{\left(a^2-b^2\right)f} - \frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} - \frac{ia(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} - \frac{3d(c+dx)^2}{(a}$$

$$= \frac{i(c+dx)^3}{\left(a^2-b^2\right)f} - \frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} - \frac{ia(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} - \frac{3d(c+dx)^2}{(a}$$

$$= \frac{i(c+dx)^3}{\left(a^2-b^2\right)f} - \frac{3d(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} - \frac{ia(c+dx)^3\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} - \frac{3d(c+dx)^2}{(a}$$

Mathematica [A] time = 3.22821, size = 742, normalized size = 0.8

$$-\frac{\frac{ia\left(-3id\left(f^2(c+dx)^2\text{PolyLog}\left(2,-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}-a}\right)+2idf(c+dx)\text{PolyLog}\left(3,-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}-a}\right)-2d^2\text{PolyLog}\left(4,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)\right)+3id\left(f^2(c+dx)^2\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)+2idf(c+\right.\right.}{\sqrt{a^2-b^2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (I*f^3*(c + d*x)^3 - 3*d*f^2*(c + d*x)^2*Log[1 + (I*b*E^(I*(e + f*x)))/(-a
+ Sqrt[a^2 - b^2])] - 3*d*f^2*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a
+ Sqrt[a^2 - b^2])] + (6*I)*d^2*(f*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)
))/(a - Sqrt[a^2 - b^2])] + I*d*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[
a^2 - b^2])]) + (6*I)*d^2*(f*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a
+ Sqrt[a^2 - b^2])] + I*d*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 -
b^2])]) - (I*a*(f^3*(c + d*x)^3*Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sqrt[a^
2 - b^2])] - f^3*(c + d*x)^3*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 -
b^2])] - (3*I)*d*(f^2*(c + d*x)^2*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a +
 Sqrt[a^2 - b^2])] + (2*I)*d*f*(c + d*x)*PolyLog[3, ((-I)*b*E^(I*(e + f*x))
)/(-a + Sqrt[a^2 - b^2])] - 2*d^2*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a - Sqr
t[a^2 - b^2])]) + (3*I)*d*(f^2*(c + d*x)^2*PolyLog[2, (I*b*E^(I*(e + f*x)))
/(a + Sqrt[a^2 - b^2])] + (2*I)*d*f*(c + d*x)*PolyLog[3, (I*b*E^(I*(e + f*x
)))/(a + Sqrt[a^2 - b^2])] - 2*d^2*PolyLog[4, (I*b*E^(I*(e + f*x)))/(a + Sq
rt[a^2 - b^2])])))/Sqrt[a^2 - b^2] + (b*f^3*(c + d*x)^3*Cos[e + f*x])/(a +
b*Sin[e + f*x]))/((a^2 - b^2)*f^4)
```

Maple [F] time = 1.56, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{\left(a+b\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^3/(a+b*sin(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.28598, size = 11266, normalized size = 12.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(-6*I*a*b^2*d^3*sin(f*x + e) - 6*I*a^2*b*d^3)*sqrt(-(a^2 - b^2)/b^2)
*polylog(4, 1/2*(2*I*a*cos(f*x + e) - 2*a*sin(f*x + e) + 2*(b*cos(f*x + e)
+ I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(6*I*a*b^2*d^3*sin(f*x +
 e) + 6*I*a^2*b*d^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(f*x +
 e) - 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 -
 b^2)/b^2))/b) + 2*(6*I*a*b^2*d^3*sin(f*x + e) + 6*I*a^2*b*d^3)*sqrt(-(a^2
- b^2)/b^2)*polylog(4, 1/2*(-2*I*a*cos(f*x + e) - 2*a*sin(f*x + e) + 2*(b*c
os(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(-6*I*a*b^2*
d^3*sin(f*x + e) - 6*I*a^2*b*d^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(-2
*I*a*cos(f*x + e) - 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) - I*b*sin(f*x + e)
)*sqrt(-(a^2 - b^2)/b^2))/b) + 4*((a^2*b - b^3)*d^3*f^3*x^3 + 3*(a^2*b - b^
3)*c*d^2*f^3*x^2 + 3*(a^2*b - b^3)*c^2*d*f^3*x + (a^2*b - b^3)*c^3*f^3)*cos
(f*x + e) + (-12*I*(a^3 - a*b^2)*d^3*f*x - 12*I*(a^3 - a*b^2)*c*d^2*f + (-1
2*I*(a^2*b - b^3)*d^3*f*x - 12*I*(a^2*b - b^3)*c*d^2*f)*sin(f*x + e) + 2*(3
*I*a^2*b*d^3*f^2*x^2 + 6*I*a^2*b*c*d^2*f^2*x + 3*I*a^2*b*c^2*d*f^2 + (3*I*a
*b^2*d^3*f^2*x^2 + 6*I*a*b^2*c*d^2*f^2*x + 3*I*a*b^2*c^2*d*f^2)*sin(f*x + e
))*sqrt(-(a^2 - b^2)/b^2))*dilog(-1/2*(2*I*a*cos(f*x + e) + 2*a*sin(f*x + e
) + 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b +
 1) + (-12*I*(a^3 - a*b^2)*d^3*f*x - 12*I*(a^3 - a*b^2)*c*d^2*f + (-12*I*(a
^2*b - b^3)*d^3*f*x - 12*I*(a^2*b - b^3)*c*d^2*f)*sin(f*x + e) + 2*(-3*I*a^
2*b*d^3*f^2*x^2 - 6*I*a^2*b*c*d^2*f^2*x - 3*I*a^2*b*c^2*d*f^2 + (-3*I*a*b^2
*d^3*f^2*x^2 - 6*I*a*b^2*c*d^2*f^2*x - 3*I*a*b^2*c^2*d*f^2)*sin(f*x + e))*s
qrt(-(a^2 - b^2)/b^2))*dilog(-1/2*(2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) -
2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)
+ (12*I*(a^3 - a*b^2)*d^3*f*x + 12*I*(a^3 - a*b^2)*c*d^2*f + (12*I*(a^2*b -
 b^3)*d^3*f*x + 12*I*(a^2*b - b^3)*c*d^2*f)*sin(f*x + e) + 2*(-3*I*a^2*b*d^
3*f^2*x^2 - 6*I*a^2*b*c*d^2*f^2*x - 3*I*a^2*b*c^2*d*f^2 + (-3*I*a*b^2*d^3*f
^2*x^2 - 6*I*a*b^2*c*d^2*f^2*x - 3*I*a*b^2*c^2*d*f^2)*sin(f*x + e))*sqrt(-(
a^2 - b^2)/b^2))*dilog(-1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(b*
cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (12
*I*(a^3 - a*b^2)*d^3*f*x + 12*I*(a^3 - a*b^2)*c*d^2*f + (12*I*(a^2*b - b^3)
*d^3*f*x + 12*I*(a^2*b - b^3)*c*d^2*f)*sin(f*x + e) + 2*(3*I*a^2*b*d^3*f^2*
```

```
x^2 + 6*I*a^2*b*c*d^2*f^2*x + 3*I*a^2*b*c^2*d*f^2 + (3*I*a*b^2*d^3*f^2*x^2
+ 6*I*a*b^2*c*d^2*f^2*x + 3*I*a*b^2*c^2*d*f^2)*sin(f*x + e))*sqrt(-(a^2 - b
^2)/b^2))*dilog(-1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) - 2*(b*cos(f*x
 + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(3*(a^3
- a*b^2)*d^3*e^2 - 6*(a^3 - a*b^2)*c*d^2*e*f + 3*(a^3 - a*b^2)*c^2*d*f^2 +
3*((a^2*b - b^3)*d^3*e^2 - 2*(a^2*b - b^3)*c*d^2*e*f + (a^2*b - b^3)*c^2*d*
f^2)*sin(f*x + e) + (a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*
f^2 - a^2*b*c^3*f^3 + (a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*
e*f^2 - a*b^2*c^3*f^3)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(f*
x + e) + 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(3*(a
^3 - a*b^2)*d^3*e^2 - 6*(a^3 - a*b^2)*c*d^2*e*f + 3*(a^3 - a*b^2)*c^2*d*f^2
 + 3*((a^2*b - b^3)*d^3*e^2 - 2*(a^2*b - b^3)*c*d^2*e*f + (a^2*b - b^3)*c^2
*d*f^2)*sin(f*x + e) + (a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d
*e*f^2 - a^2*b*c^3*f^3 + (a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2
*d*e*f^2 - a*b^2*c^3*f^3)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos
(f*x + e) - 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(3
*(a^3 - a*b^2)*d^3*e^2 - 6*(a^3 - a*b^2)*c*d^2*e*f + 3*(a^3 - a*b^2)*c^2*d*
f^2 + 3*((a^2*b - b^3)*d^3*e^2 - 2*(a^2*b - b^3)*c*d^2*e*f + (a^2*b - b^3)*
c^2*d*f^2)*sin(f*x + e) - (a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^
2*d*e*f^2 - a^2*b*c^3*f^3 + (a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*
c^2*d*e*f^2 - a*b^2*c^3*f^3)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b
*cos(f*x + e) + 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) -
2*(3*(a^3 - a*b^2)*d^3*e^2 - 6*(a^3 - a*b^2)*c*d^2*e*f + 3*(a^3 - a*b^2)*c^
2*d*f^2 + 3*((a^2*b - b^3)*d^3*e^2 - 2*(a^2*b - b^3)*c*d^2*e*f + (a^2*b - b
^3)*c^2*d*f^2)*sin(f*x + e) - (a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*
b*c^2*d*e*f^2 - a^2*b*c^3*f^3 + (a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*
b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(
-2*b*cos(f*x + e) - 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a
) - 2*(3*(a^3 - a*b^2)*d^3*f^2*x^2 + 6*(a^3 - a*b^2)*c*d^2*f^2*x - 3*(a^3 -
 a*b^2)*d^3*e^2 + 6*(a^3 - a*b^2)*c*d^2*e*f + 3*((a^2*b - b^3)*d^3*f^2*x^2
+ 2*(a^2*b - b^3)*c*d^2*f^2*x - (a^2*b - b^3)*d^3*e^2 + 2*(a^2*b - b^3)*c*d
^2*e*f)*sin(f*x + e) - (a^2*b*d^3*f^3*x^3 + 3*a^2*b*c*d^2*f^3*x^2 + 3*a^2*b
*c^2*d*f^3*x + a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 +
(a*b^2*d^3*f^3*x^3 + 3*a*b^2*c*d^2*f^3*x^2 + 3*a*b^2*c^2*d*f^3*x + a*b^2*d^
3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2)*sin(f*x + e))*sqrt(-(a^2
 - b^2)/b^2))*log(1/2*(2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(b*cos(f*x
 + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(3*(a^3 - a*
b^2)*d^3*f^2*x^2 + 6*(a^3 - a*b^2)*c*d^2*f^2*x - 3*(a^3 - a*b^2)*d^3*e^2 +
6*(a^3 - a*b^2)*c*d^2*e*f + 3*((a^2*b - b^3)*d^3*f^2*x^2 + 2*(a^2*b - b^3)*
c*d^2*f^2*x - (a^2*b - b^3)*d^3*e^2 + 2*(a^2*b - b^3)*c*d^2*e*f)*sin(f*x +
e) + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*c*d^2*f^3*x^2 + 3*a^2*b*c^2*d*f^3*x + a^2
*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 + (a*b^2*d^3*f^3*x^3
 + 3*a*b^2*c*d^2*f^3*x^2 + 3*a*b^2*c^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*b^2*c*
d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(
1/2*(2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) - I*b*sin(f*
```

```
x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(3*(a^3 - a*b^2)*d^3*f^2*x^2 +
 6*(a^3 - a*b^2)*c*d^2*f^2*x - 3*(a^3 - a*b^2)*d^3*e^2 + 6*(a^3 - a*b^2)*c*
d^2*e*f + 3*((a^2*b - b^3)*d^3*f^2*x^2 + 2*(a^2*b - b^3)*c*d^2*f^2*x - (a^2
*b - b^3)*d^3*e^2 + 2*(a^2*b - b^3)*c*d^2*e*f)*sin(f*x + e) - (a^2*b*d^3*f^
3*x^3 + 3*a^2*b*c*d^2*f^3*x^2 + 3*a^2*b*c^2*d*f^3*x + a^2*b*d^3*e^3 - 3*a^2
*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*c*d^2*f
^3*x^2 + 3*a*b^2*c^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^
2*c^2*d*e*f^2)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(-2*I*a*cos(f*
x + e) + 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^
2 - b^2)/b^2) + 2*b)/b) - 2*(3*(a^3 - a*b^2)*d^3*f^2*x^2 + 6*(a^3 - a*b^2)*
c*d^2*f^2*x - 3*(a^3 - a*b^2)*d^3*e^2 + 6*(a^3 - a*b^2)*c*d^2*e*f + 3*((a^2
*b - b^3)*d^3*f^2*x^2 + 2*(a^2*b - b^3)*c*d^2*f^2*x - (a^2*b - b^3)*d^3*e^2
 + 2*(a^2*b - b^3)*c*d^2*e*f)*sin(f*x + e) + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*c
*d^2*f^3*x^2 + 3*a^2*b*c^2*d*f^3*x + a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f +
3*a^2*b*c^2*d*e*f^2 + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*c*d^2*f^3*x^2 + 3*a*b^2*
c^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2)*si
n(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(
f*x + e) - 2*(b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2
*b)/b) - 12*((a^2*b - b^3)*d^3*sin(f*x + e) + (a^3 - a*b^2)*d^3 + (a^2*b*d^
3*f*x + a^2*b*c*d^2*f + (a*b^2*d^3*f*x + a*b^2*c*d^2*f)*sin(f*x + e))*sqrt(
-(a^2 - b^2)/b^2))*polylog(3, 1/2*(2*I*a*cos(f*x + e) - 2*a*sin(f*x + e) +
2*(b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*((a^2
*b - b^3)*d^3*sin(f*x + e) + (a^3 - a*b^2)*d^3 - (a^2*b*d^3*f*x + a^2*b*c*d
^2*f + (a*b^2*d^3*f*x + a*b^2*c*d^2*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2)
)*polylog(3, 1/2*(2*I*a*cos(f*x + e) - 2*a*sin(f*x + e) - 2*(b*cos(f*x + e)
 + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*((a^2*b - b^3)*d^3*sin
(f*x + e) + (a^3 - a*b^2)*d^3 + (a^2*b*d^3*f*x + a^2*b*c*d^2*f + (a*b^2*d^3
*f*x + a*b^2*c*d^2*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*polylog(3, 1/2*
(-2*I*a*cos(f*x + e) - 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) - I*b*sin(f*x +
 e))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*((a^2*b - b^3)*d^3*sin(f*x + e) + (a^3
 - a*b^2)*d^3 - (a^2*b*d^3*f*x + a^2*b*c*d^2*f + (a*b^2*d^3*f*x + a*b^2*c*d
^2*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*polylog(3, 1/2*(-2*I*a*cos(f*x
+ e) - 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2
- b^2)/b^2))/b))/((a^4*b - 2*a^2*b^3 + b^5)*f^4*sin(f*x + e) + (a^5 - 2*a^3
*b^2 + a*b^4)*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^3}{\left(b \sin\left(fx + e\right) + a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*sin(f*x + e) + a)^2, x)
```

3.169 $\int \frac{(c+dx)^2}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=671

$$-\frac{2ad(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2\left(a^2-b^2\right)^{3/2}}+\frac{2ad(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^2\left(a^2-b^2\right)^{3/2}}+\frac{2id^2\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^3\left(a^2-b^2\right)}+\frac{2id^2\text{PolyLo}}{f^3\left(a\right.}$$

```
[Out] (I*(c + d*x)^2)/((a^2 - b^2)*f) - (2*d*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x
)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) - (I*a*(c + d*x)^2*Log[1 - (I
*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) - (2*d*(c
 + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*
f^2) + (I*a*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])
])/((a^2 - b^2)^(3/2)*f) + ((2*I)*d^2*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a -
 Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^3) - (2*a*d*(c + d*x)*PolyLog[2, (I*b*E^
(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^2) + ((2*I)*d^2
*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^3)
 + (2*a*d*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]
)/((a^2 - b^2)^(3/2)*f^2) - ((2*I)*a*d^2*PolyLog[3, (I*b*E^(I*(e + f*x)))/(
a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^3) + ((2*I)*a*d^2*PolyLog[3, (I
*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^3) + (b*(c
 + d*x)^2*Cos[e + f*x])/((a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

Rubi [A] time = 1.20514, antiderivative size = 671, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3324, 3323, 2264, 2190, 2531, 2282, 6589, 4519, 2279, 2391}

$$-\frac{2ad(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2\left(a^2-b^2\right)^{3/2}}+\frac{2ad(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^2\left(a^2-b^2\right)^{3/2}}+\frac{2id^2\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^3\left(a^2-b^2\right)}+\frac{2id^2\text{PolyLo}}{f^3\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (I*(c + d*x)^2)/((a^2 - b^2)*f) - (2*d*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x
)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) - (I*a*(c + d*x)^2*Log[1 - (I
*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) - (2*d*(c
 + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*
f^2) + (I*a*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])
])/((a^2 - b^2)^(3/2)*f) + ((2*I)*d^2*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a -
```

```
 Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^3) - (2*a*d*(c + d*x)*PolyLog[2, (I*b*E^
(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^2) + ((2*I)*d^2
*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^3)
 + (2*a*d*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]
)/((a^2 - b^2)^(3/2)*f^2) - ((2*I)*a*d^2*PolyLog[3, (I*b*E^(I*(e + f*x)))/(
a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^3) + ((2*I)*a*d^2*PolyLog[3, (I
*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f^3) + (b*(c
 + d*x)^2*Cos[e + f*x])/((a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+b\sin(e+fx))^2}\,dx = \frac{b(c+dx)^2\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{a\int \frac{(c+dx)^2}{a+b\sin(e+fx)}\,dx}{a^2-b^2} - \frac{(2bd)\int \frac{(c+dx)\cos(e+fx)}{a+b\sin(e+fx)}\,dx}{\left(a^2-b^2\right)f}$$

$$= \frac{i(c+dx)^2}{\left(a^2-b^2\right)f} + \frac{b(c+dx)^2\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{(2a)\int \frac{e^{i(e+fx)}(c+dx)^2}{ib+2ae^{i(e+fx)}-ibe^{2i(e+fx)}}\,dx}{a^2-b^2} - \frac{(2bd)\int \frac{e^{i(e-}}{a-\sqrt{a^2}}}{(a^2-}$$

$$= \frac{i(c+dx)^2}{\left(a^2-b^2\right)f} - \frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} - \frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} + \frac{b(c+dx)^2}{\left(a^2-b^2\right)f(a}$$

$$= \frac{i(c+dx)^2}{\left(a^2-b^2\right)f} - \frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} - \frac{ia(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} - \frac{2d(c+dx)\log}{(a^2-}$$

$$= \frac{i(c+dx)^2}{\left(a^2-b^2\right)f} - \frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} - \frac{ia(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} - \frac{2d(c+dx)\log}{(a^2-}$$

$$= \frac{i(c+dx)^2}{\left(a^2-b^2\right)f} - \frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} - \frac{ia(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} - \frac{2d(c+dx)\log}{(a^2-}$$

$$= \frac{i(c+dx)^2}{\left(a^2-b^2\right)f} - \frac{2d(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} - \frac{ia(c+dx)^2\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} - \frac{2d(c+dx)\log}{(a^2-}$$

Mathematica [A] time = 1.67465, size = 530, normalized size = 0.79

$$-\frac{\frac{ia\left(-2idf(c+dx)\text{PolyLog}\left(2,-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}-a}\right)+2idf(c+dx)\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)+2d^2\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)-2d^2\text{PolyLog}\left(3,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)+f^2(c+dx)^2\log\left(1+\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}-}\right.\right.}{\sqrt{a^2-b^2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (I*f^2*(c + d*x)^2 - 2*d*f*(c + d*x)*Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sq
rt[a^2 - b^2])] - 2*d*f*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a
^2 - b^2])] + (2*I)*d^2*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2
- b^2])] + (2*I)*d^2*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])
```

```
] - (I*a*(f^2*(c + d*x)^2*Log[1 + (I*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^
2])] - f^2*(c + d*x)^2*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]
 - (2*I)*d*f*(c + d*x)*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 -
 b^2])] + (2*I)*d*f*(c + d*x)*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^
2 - b^2])] + 2*d^2*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])]
- 2*d^2*PolyLog[3, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]))/Sqrt[a^2
- b^2] + (b*f^2*(c + d*x)^2*Cos[e + f*x])/(a + b*Sin[e + f*x]))/((a^2 - b^2
)*f^3)
```

Maple [F] time = 1.273, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{\left(a+b\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^2/(a+b*sin(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.36677, size = 7017, normalized size = 10.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a*b^2*d^2*sin(f*x + e) + a^2*b*d^2)*sqrt(-(a^2 - b^2)/b^2)*polylog
(3, 1/2*(2*I*a*cos(f*x + e) - 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) + I*b*si
n(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) - 4*(a*b^2*d^2*sin(f*x + e) + a^2*b*
d^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(f*x + e) - 2*a*sin(f*
x + e) - 2*(b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) +
 4*(a*b^2*d^2*sin(f*x + e) + a^2*b*d^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1
/2*(-2*I*a*cos(f*x + e) - 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) - I*b*sin(f*
x + e))*sqrt(-(a^2 - b^2)/b^2))/b) - 4*(a*b^2*d^2*sin(f*x + e) + a^2*b*d^2)
*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(-2*I*a*cos(f*x + e) - 2*a*sin(f*x +
 e) - 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))/b) - 4*
((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a^2*b - b^3)*c*d*f^2*x + (a^2*b - b^3)*c^2*
f^2)*cos(f*x + e) - (-4*I*(a^2*b - b^3)*d^2*sin(f*x + e) - 4*I*(a^3 - a*b^2
)*d^2 + 2*(2*I*a^2*b*d^2*f*x + 2*I*a^2*b*c*d*f + (2*I*a*b^2*d^2*f*x + 2*I*a
*b^2*c*d*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*dilog(-1/2*(2*I*a*cos(f*x
 + e) + 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2
 - b^2)/b^2) + 2*b)/b + 1) - (-4*I*(a^2*b - b^3)*d^2*sin(f*x + e) - 4*I*(a^
3 - a*b^2)*d^2 + 2*(-2*I*a^2*b*d^2*f*x - 2*I*a^2*b*c*d*f + (-2*I*a*b^2*d^2*
f*x - 2*I*a*b^2*c*d*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*dilog(-1/2*(2*
I*a*cos(f*x + e) + 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) - I*b*sin(f*x + e))
*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - (4*I*(a^2*b - b^3)*d^2*sin(f*x + e)
 + 4*I*(a^3 - a*b^2)*d^2 + 2*(-2*I*a^2*b*d^2*f*x - 2*I*a^2*b*c*d*f + (-2*I*
a*b^2*d^2*f*x - 2*I*a*b^2*c*d*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*dilo
g(-1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) + I*b*si
n(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - (4*I*(a^2*b - b^3)*d^2*s
in(f*x + e) + 4*I*(a^3 - a*b^2)*d^2 + 2*(2*I*a^2*b*d^2*f*x + 2*I*a^2*b*c*d*
f + (2*I*a*b^2*d^2*f*x + 2*I*a*b^2*c*d*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b
^2))*dilog(-1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) - 2*(b*cos(f*x + e)
 + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(2*(a^3 - a*b
^2)*d^2*e - 2*(a^3 - a*b^2)*c*d*f + 2*((a^2*b - b^3)*d^2*e - (a^2*b - b^3)*
c*d*f)*sin(f*x + e) + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2 + (a
*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*sin(f*x + e))*sqrt(-(a^2 -
b^2)/b^2))*log(2*b*cos(f*x + e) + 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2
)/b^2) + 2*I*a) - 2*(2*(a^3 - a*b^2)*d^2*e - 2*(a^3 - a*b^2)*c*d*f + 2*((a^
2*b - b^3)*d^2*e - (a^2*b - b^3)*c*d*f)*sin(f*x + e) + (a^2*b*d^2*e^2 - 2*a
^2*b*c*d*e*f + a^2*b*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2
*f^2)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(f*x + e) - 2*I*b*si
n(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(2*(a^3 - a*b^2)*d^2*e
 - 2*(a^3 - a*b^2)*c*d*f + 2*((a^2*b - b^3)*d^2*e - (a^2*b - b^3)*c*d*f)*si
n(f*x + e) - (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2 + (a*b^2*d^2*
e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2)
)*log(-2*b*cos(f*x + e) + 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) +
 2*I*a) - 2*(2*(a^3 - a*b^2)*d^2*e - 2*(a^3 - a*b^2)*c*d*f + 2*((a^2*b - b^
3)*d^2*e - (a^2*b - b^3)*c*d*f)*sin(f*x + e) - (a^2*b*d^2*e^2 - 2*a^2*b*c*d
*e*f + a^2*b*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*si
n(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(f*x + e) - 2*I*b*sin(f*x +
```

```
 e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(2*(a^3 - a*b^2)*d^2*f*x + 2*
(a^3 - a*b^2)*d^2*e + 2*((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*d^2*e)*sin(f
*x + e) - (a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*f^2*x - a^2*b*d^2*e^2 + 2*a^2*b*
c*d*e*f + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*
c*d*e*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(2*I*a*cos(f*x + e)
+ 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2
)/b^2) + 2*b)/b) + 2*(2*(a^3 - a*b^2)*d^2*f*x + 2*(a^3 - a*b^2)*d^2*e + 2*(
(a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*d^2*e)*sin(f*x + e) + (a^2*b*d^2*f^2*
x^2 + 2*a^2*b*c*d*f^2*x - a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f + (a*b^2*d^2*f^2*
x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*sin(f*x + e))*sq
rt(-(a^2 - b^2)/b^2))*log(1/2*(2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) - 2*(b
*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(2*(
a^3 - a*b^2)*d^2*f*x + 2*(a^3 - a*b^2)*d^2*e + 2*((a^2*b - b^3)*d^2*f*x + (
a^2*b - b^3)*d^2*e)*sin(f*x + e) - (a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*f^2*x -
 a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x -
 a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log
(1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) + I*b*sin(
f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(2*(a^3 - a*b^2)*d^2*f*x + 2
*(a^3 - a*b^2)*d^2*e + 2*((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*d^2*e)*sin(
f*x + e) + (a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*f^2*x - a^2*b*d^2*e^2 + 2*a^2*b
*c*d*e*f + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2
*c*d*e*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(-2*I*a*cos(f*x + e
) + 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b
^2)/b^2) + 2*b)/b))/((a^4*b - 2*a^2*b^3 + b^5)*f^3*sin(f*x + e) + (a^5 - 2*
a^3*b^2 + a*b^4)*f^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{\left(b\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*sin(f*x + e) + a)^2, x)
```

3.170 $\int \frac{c+dx}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=305

$$-\frac{ad\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2\left(a^2-b^2\right)^{3/2}}+\frac{ad\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^2\left(a^2-b^2\right)^{3/2}}-\frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}}+\frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f\left(a^2-b^2\right)^{3/2}}$$

```
[Out] ((-I)*a*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a
^2 - b^2)^(3/2)*f) + (I*a*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt
[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) - (d*Log[a + b*Sin[e + f*x]])/((a^2 -
b^2)*f^2) - (a*d*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(
(a^2 - b^2)^(3/2)*f^2) + (a*d*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^
2 - b^2])])/((a^2 - b^2)^(3/2)*f^2) + (b*(c + d*x)*Cos[e + f*x])/((a^2 - b^
2)*f*(a + b*Sin[e + f*x]))
```

Rubi [A] time = 0.550363, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3324, 3323, 2264, 2190, 2279, 2391, 2668, 31}

$$-\frac{ad\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f^2\left(a^2-b^2\right)^{3/2}}+\frac{ad\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f^2\left(a^2-b^2\right)^{3/2}}-\frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}}+\frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)}{f\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((-I)*a*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((a
^2 - b^2)^(3/2)*f) + (I*a*(c + d*x)*Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt
[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) - (d*Log[a + b*Sin[e + f*x]])/((a^2 -
b^2)*f^2) - (a*d*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a - Sqrt[a^2 - b^2])])/(
(a^2 - b^2)^(3/2)*f^2) + (a*d*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^
2 - b^2])])/((a^2 - b^2)^(3/2)*f^2) + (b*(c + d*x)*Cos[e + f*x])/((a^2 - b^
2)*f*(a + b*Sin[e + f*x]))
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
```

```
x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{c+dx}{(a+b\sin(e+fx))^2}\,dx = \frac{b(c+dx)\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{a\int \frac{c+dx}{a+b\sin(e+fx)}\,dx}{a^2-b^2} - \frac{(bd)\int \frac{\cos(e+fx)}{a+b\sin(e+fx)}\,dx}{\left(a^2-b^2\right)f}$$

$$= \frac{b(c+dx)\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{(2a)\int \frac{e^{i(e+fx)}(c+dx)}{ib+2ae^{i(e+fx)}-ibe^{2i(e+fx)}}\,dx}{a^2-b^2} - \frac{d\,\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\sin\right.}{\left(a^2-b^2\right)f^2}$$

$$= -\frac{d\log(a+b\sin(e+fx))}{\left(a^2-b^2\right)f^2} + \frac{b(c+dx)\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{(2iab)\int \frac{e^{i(e+fx)}(c+dx)}{2a-2\sqrt{a^2-b^2}-2ibe^{i(e+fx)}}\,d}{\left(a^2-b^2\right)^{3/2}}$$

$$= -\frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} + \frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} - \frac{d\log(a+b\sin(e+fx))}{\left(a^2-b^2\right)f^2} +$$

$$= -\frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} + \frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} - \frac{d\log(a+b\sin(e+fx))}{\left(a^2-b^2\right)f^2} +$$

$$= -\frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} + \frac{ia(c+dx)\log\left(1-\frac{ibe^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} - \frac{d\log(a+b\sin(e+fx))}{\left(a^2-b^2\right)f^2} -$$

Mathematica [A] time = 0.988389, size = 236, normalized size = 0.77

$$\frac{\frac{a\left(-d\,\text{PolyLog}\left(2,-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}-a}\right)+d\,\text{PolyLog}\left(2,\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)-if(c+dx)\left(\log\left(1+\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}-a}\right)-\log\left(1-\frac{ibe^{i(e+fx)}}{\sqrt{a^2-b^2}+a}\right)\right)\right)}{\sqrt{a^2-b^2}} + \frac{bf(c+dx)\cos(e+fx)}{a+b\sin(e+fx)} - d\log(a+b\sin(e+}{f^2\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-(d*Log[a + b*Sin[e + f*x]]) + (a*((-I)*f*(c + d*x)*(Log[1 + (I*b*E^(I*(e
+ f*x)))/(-a + Sqrt[a^2 - b^2])] - Log[1 - (I*b*E^(I*(e + f*x)))/(a + Sqrt[
```

```
a^2 - b^2])]) - d*PolyLog[2, ((-I)*b*E^(I*(e + f*x)))/(-a + Sqrt[a^2 - b^2]
)] + d*PolyLog[2, (I*b*E^(I*(e + f*x)))/(a + Sqrt[a^2 - b^2])]))/Sqrt[a^2 -
 b^2] + (b*f*(c + d*x)*Cos[e + f*x])/(a + b*Sin[e + f*x]))/((a^2 - b^2)*f^2
)
```

Maple [B] time = 0.789, size = 641, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*sin(f*x+e))^2,x)
```

```
[Out] 2*(d*x+c)*(I*b+a*exp(I*(f*x+e)))/f/(a^2-b^2)/(b*exp(2*I*(f*x+e))-b+2*I*a*ex
p(I*(f*x+e)))-2/f^2/(-a^2+b^2)*d*ln(exp(I*(f*x+e)))+1/f^2/(-a^2+b^2)*d*ln(I
*exp(2*I*(f*x+e))*b-I*b-2*a*exp(I*(f*x+e)))+I/f^2/(-a^2+b^2)^(3/2)*d*a*dilo
g((-I*a-b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))-I/f^2/(
-a^2+b^2)^(3/2)*d*a*dilog((I*a+b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(I*a+(-a^
2+b^2)^(1/2)))+1/f/(-a^2+b^2)^(3/2)*d*a*ln((I*a+b*exp(I*(f*x+e))+(-a^2+b^2)
^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/f^2/(-a^2+b^2)^(3/2)*d*a*ln((I*a+b*exp(
I*(f*x+e))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*e-1/f/(-a^2+b^2)^(3/2)
*d*a*ln((-I*a-b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(-I*a+(-a^2+b^2)^(1/2)))*x
-1/f^2/(-a^2+b^2)^(3/2)*d*a*ln((-I*a-b*exp(I*(f*x+e))+(-a^2+b^2)^(1/2))/(-I
*a+(-a^2+b^2)^(1/2)))*e-2*I/f/(-a^2+b^2)^(3/2)*a*c*arctan(1/2*(2*I*b*exp(I*
(f*x+e))-2*a)/(-a^2+b^2)^(1/2))+2*I/f^2/(-a^2+b^2)^(3/2)*a*d*e*arctan(1/2*(
2*I*b*exp(I*(f*x+e))-2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.46769, size = 3549, normalized size = 11.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((I*a*b^2*d*sin(f*x + e) + I*a^2*b*d)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2
*(2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) - I*b*sin(f*x +
 e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-I*a*b^2*d*sin(f*x + e) - I*a^
2*b*d)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(f*x + e) + 2*a*sin(f*x
+ e) - 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/
b + 1) + (-I*a*b^2*d*sin(f*x + e) - I*a^2*b*d)*sqrt(-(a^2 - b^2)/b^2)*dilog
(-1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) + I*b*sin
(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (I*a*b^2*d*sin(f*x + e) +
 I*a^2*b*d)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(f*x + e) + 2*a*si
n(f*x + e) - 2*(b*cos(f*x + e) + I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) +
 2*b)/b + 1) + (a^2*b*d*f*x + a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*sin(f*x
 + e))*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(f*x + e) + 2*a*sin(f*x + e
) + 2*(b*cos(f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)
- (a^2*b*d*f*x + a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*sin(f*x + e))*sqrt(-
(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) - 2*(b*cos(
f*x + e) - I*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (a^2*b*d*f*
x + a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b
^2)*log(1/2*(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) + 2*(b*cos(f*x + e) + I
*b*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - (a^2*b*d*f*x + a^2*b*d*
e + (a*b^2*d*f*x + a*b^2*d*e)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2)*log(1/2*
(-2*I*a*cos(f*x + e) + 2*a*sin(f*x + e) - 2*(b*cos(f*x + e) + I*b*sin(f*x +
 e))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*((a^2*b - b^3)*d*f*x + (a^2*b - b
^3)*c*f)*cos(f*x + e) - ((a^2*b - b^3)*d*sin(f*x + e) + (a^3 - a*b^2)*d + (
a^2*b*d*e - a^2*b*c*f + (a*b^2*d*e - a*b^2*c*f)*sin(f*x + e))*sqrt(-(a^2 -
b^2)/b^2))*log(2*b*cos(f*x + e) + 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2
)/b^2) + 2*I*a) - ((a^2*b - b^3)*d*sin(f*x + e) + (a^3 - a*b^2)*d + (a^2*b*
d*e - a^2*b*c*f + (a*b^2*d*e - a*b^2*c*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b
^2))*log(2*b*cos(f*x + e) - 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2)
 - 2*I*a) - ((a^2*b - b^3)*d*sin(f*x + e) + (a^3 - a*b^2)*d - (a^2*b*d*e -
a^2*b*c*f + (a*b^2*d*e - a*b^2*c*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*l
og(-2*b*cos(f*x + e) + 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*
I*a) - ((a^2*b - b^3)*d*sin(f*x + e) + (a^3 - a*b^2)*d - (a^2*b*d*e - a^2*b
*c*f + (a*b^2*d*e - a*b^2*c*f)*sin(f*x + e))*sqrt(-(a^2 - b^2)/b^2))*log(-2
*b*cos(f*x + e) - 2*I*b*sin(f*x + e) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a))
/((a^4*b - 2*a^2*b^3 + b^5)*f^2*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*f^
2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\left(b\sin\left(fx + e\right) + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*sin(f*x + e) + a)^2, x)
```

3.171 $\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a+b\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*Sin[e + f*x])^2), x]
```

Rubi [A] time = 0.0592222, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Sin[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + b*Sin[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx=\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx$$

Mathematica [A] time = 32.7934, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Sin[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + b*Sin[e + f*x])^2), x]
```

Maple [A] time = 3.118, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a+b\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)/(a+b*sin(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (2*a*b*cos(2*f*x + 2*e)*cos(f*x + e) + 2*a*b*cos(f*x + e) - ((a^2*b^2 - b^4
)*d*f*x + (a^2*b^2 - b^4)*c*f + ((a^2*b^2 - b^4)*d*f*x + (a^2*b^2 - b^4)*c*
f)*cos(2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d*f*x + (a^4 - a^2*b^2)*c*f)*cos
(f*x + e)^2 + 4*((a^3*b - a*b^3)*d*f*x + (a^3*b - a*b^3)*c*f)*cos(f*x + e)*
sin(2*f*x + 2*e) + ((a^2*b^2 - b^4)*d*f*x + (a^2*b^2 - b^4)*c*f)*sin(2*f*x
+ 2*e)^2 + 4*((a^4 - a^2*b^2)*d*f*x + (a^4 - a^2*b^2)*c*f)*sin(f*x + e)^2 -
 2*((a^2*b^2 - b^4)*d*f*x + (a^2*b^2 - b^4)*c*f + 2*((a^3*b - a*b^3)*d*f*x
+ (a^3*b - a*b^3)*c*f)*sin(f*x + e))*cos(2*f*x + 2*e) + 4*((a^3*b - a*b^3)*
d*f*x + (a^3*b - a*b^3)*c*f)*sin(f*x + e))*integrate(-2*(a*b*d*cos(f*x + e)
 + 2*(a^2*d*f*x + a^2*c*f)*cos(f*x + e)^2 + 2*(a^2*d*f*x + a^2*c*f)*sin(f*x
 + e)^2 + (a*b*d*cos(f*x + e) - (a*b*d*f*x + a*b*c*f)*sin(f*x + e))*cos(2*f
*x + 2*e) + (a*b*d*sin(f*x + e) + b^2*d + (a*b*d*f*x + a*b*c*f)*cos(f*x + e
))*sin(2*f*x + 2*e) + (a*b*d*f*x + a*b*c*f)*sin(f*x + e))/((a^2*b^2 - b^4)*
d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f + ((a^2*b^2 -
 b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f)*cos(2*
f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^2*f*x^2 + 2*(a^4 - a^2*b^2)*c*d*f*x + (
a^4 - a^2*b^2)*c^2*f)*cos(f*x + e)^2 + 4*((a^3*b - a*b^3)*d^2*f*x^2 + 2*(a^
3*b - a*b^3)*c*d*f*x + (a^3*b - a*b^3)*c^2*f)*cos(f*x + e)*sin(2*f*x + 2*e)
 + ((a^2*b^2 - b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)
*c^2*f)*sin(2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^2*f*x^2 + 2*(a^4 - a^2*b^
2)*c*d*f*x + (a^4 - a^2*b^2)*c^2*f)*sin(f*x + e)^2 - 2*((a^2*b^2 - b^4)*d^2
```

```
*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f + 2*((a^3*b - a*
b^3)*d^2*f*x^2 + 2*(a^3*b - a*b^3)*c*d*f*x + (a^3*b - a*b^3)*c^2*f)*sin(f*x
 + e))*cos(2*f*x + 2*e) + 4*((a^3*b - a*b^3)*d^2*f*x^2 + 2*(a^3*b - a*b^3)*
c*d*f*x + (a^3*b - a*b^3)*c^2*f)*sin(f*x + e)), x) + 2*(a*b*sin(f*x + e) +
b^2)*sin(2*f*x + 2*e))/((a^2*b^2 - b^4)*d*f*x + (a^2*b^2 - b^4)*c*f + ((a^2
*b^2 - b^4)*d*f*x + (a^2*b^2 - b^4)*c*f)*cos(2*f*x + 2*e)^2 + 4*((a^4 - a^2
*b^2)*d*f*x + (a^4 - a^2*b^2)*c*f)*cos(f*x + e)^2 + 4*((a^3*b - a*b^3)*d*f*
x + (a^3*b - a*b^3)*c*f)*cos(f*x + e)*sin(2*f*x + 2*e) + ((a^2*b^2 - b^4)*d
*f*x + (a^2*b^2 - b^4)*c*f)*sin(2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d*f*x +
 (a^4 - a^2*b^2)*c*f)*sin(f*x + e)^2 - 2*((a^2*b^2 - b^4)*d*f*x + (a^2*b^2
- b^4)*c*f + 2*((a^3*b - a*b^3)*d*f*x + (a^3*b - a*b^3)*c*f)*sin(f*x + e))*
cos(2*f*x + 2*e) + 4*((a^3*b - a*b^3)*d*f*x + (a^3*b - a*b^3)*c*f)*sin(f*x
+ e))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^2+b^2\right)dx-\left(b^2dx+b^2c\right)\cos\left(fx+e\right)^2+\left(a^2+b^2\right)c+2\left(abdx+abc\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2 + b^2)*d*x - (b^2*d*x + b^2*c)*cos(f*x + e)^2 + (a^2 + b^2
)*c + 2*(a*b*d*x + a*b*c)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(b\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*sin(f*x + e) + a)^2), x)
```

3.172 $\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a+b\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a + b*Sin[e + f*x])^2), x]
```

Rubi [A] time = 0.0568414, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Sin[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Sin[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx$$

Mathematica [A] time = 94.2385, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Sin[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*Sin[e + f*x])^2), x]
```

Maple [A] time = 5.75, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2 \left(a+b\sin\left(fx+e\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (2*a*b*cos(2*f*x + 2*e)*cos(f*x + e) + 2*a*b*cos(f*x + e) - ((a^2*b^2 - b^4
)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f + ((a^2*b^2
 - b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f)*cos(
2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^2*f*x^2 + 2*(a^4 - a^2*b^2)*c*d*f*x +
 (a^4 - a^2*b^2)*c^2*f)*cos(f*x + e)^2 + 4*((a^3*b - a*b^3)*d^2*f*x^2 + 2*(
a^3*b - a*b^3)*c*d*f*x + (a^3*b - a*b^3)*c^2*f)*cos(f*x + e)*sin(2*f*x + 2*
e) + ((a^2*b^2 - b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^
4)*c^2*f)*sin(2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^2*f*x^2 + 2*(a^4 - a^2*
b^2)*c*d*f*x + (a^4 - a^2*b^2)*c^2*f)*sin(f*x + e)^2 - 2*((a^2*b^2 - b^4)*d
^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f + 2*((a^3*b -
a*b^3)*d^2*f*x^2 + 2*(a^3*b - a*b^3)*c*d*f*x + (a^3*b - a*b^3)*c^2*f)*sin(f
*x + e))*cos(2*f*x + 2*e) + 4*((a^3*b - a*b^3)*d^2*f*x^2 + 2*(a^3*b - a*b^3
)*c*d*f*x + (a^3*b - a*b^3)*c^2*f)*sin(f*x + e))*integrate(-2*(2*a*b*d*cos(
f*x + e) + 2*(a^2*d*f*x + a^2*c*f)*cos(f*x + e)^2 + 2*(a^2*d*f*x + a^2*c*f)
*sin(f*x + e)^2 + (2*a*b*d*cos(f*x + e) - (a*b*d*f*x + a*b*c*f)*sin(f*x + e
))*cos(2*f*x + 2*e) + (2*a*b*d*sin(f*x + e) + 2*b^2*d + (a*b*d*f*x + a*b*c*
f)*cos(f*x + e))*sin(2*f*x + 2*e) + (a*b*d*f*x + a*b*c*f)*sin(f*x + e))/((a
^2*b^2 - b^4)*d^3*f*x^3 + 3*(a^2*b^2 - b^4)*c*d^2*f*x^2 + 3*(a^2*b^2 - b^4)
*c^2*d*f*x + (a^2*b^2 - b^4)*c^3*f + ((a^2*b^2 - b^4)*d^3*f*x^3 + 3*(a^2*b^
2 - b^4)*c*d^2*f*x^2 + 3*(a^2*b^2 - b^4)*c^2*d*f*x + (a^2*b^2 - b^4)*c^3*f)
*cos(2*f*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^3*f*x^3 + 3*(a^4 - a^2*b^2)*c*d^
```

```
2*f*x^2 + 3*(a^4 - a^2*b^2)*c^2*d*f*x + (a^4 - a^2*b^2)*c^3*f)*cos(f*x + e)
^2 + 4*((a^3*b - a*b^3)*d^3*f*x^3 + 3*(a^3*b - a*b^3)*c*d^2*f*x^2 + 3*(a^3*
b - a*b^3)*c^2*d*f*x + (a^3*b - a*b^3)*c^3*f)*cos(f*x + e)*sin(2*f*x + 2*e)
 + ((a^2*b^2 - b^4)*d^3*f*x^3 + 3*(a^2*b^2 - b^4)*c*d^2*f*x^2 + 3*(a^2*b^2
- b^4)*c^2*d*f*x + (a^2*b^2 - b^4)*c^3*f)*sin(2*f*x + 2*e)^2 + 4*((a^4 - a^
2*b^2)*d^3*f*x^3 + 3*(a^4 - a^2*b^2)*c*d^2*f*x^2 + 3*(a^4 - a^2*b^2)*c^2*d*
f*x + (a^4 - a^2*b^2)*c^3*f)*sin(f*x + e)^2 - 2*((a^2*b^2 - b^4)*d^3*f*x^3
+ 3*(a^2*b^2 - b^4)*c*d^2*f*x^2 + 3*(a^2*b^2 - b^4)*c^2*d*f*x + (a^2*b^2 -
b^4)*c^3*f + 2*((a^3*b - a*b^3)*d^3*f*x^3 + 3*(a^3*b - a*b^3)*c*d^2*f*x^2 +
 3*(a^3*b - a*b^3)*c^2*d*f*x + (a^3*b - a*b^3)*c^3*f)*sin(f*x + e))*cos(2*f
*x + 2*e) + 4*((a^3*b - a*b^3)*d^3*f*x^3 + 3*(a^3*b - a*b^3)*c*d^2*f*x^2 +
3*(a^3*b - a*b^3)*c^2*d*f*x + (a^3*b - a*b^3)*c^3*f)*sin(f*x + e)), x) + 2*
(a*b*sin(f*x + e) + b^2)*sin(2*f*x + 2*e))/((a^2*b^2 - b^4)*d^2*f*x^2 + 2*(
a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f + ((a^2*b^2 - b^4)*d^2*f*x^2
 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f)*cos(2*f*x + 2*e)^2 +
4*((a^4 - a^2*b^2)*d^2*f*x^2 + 2*(a^4 - a^2*b^2)*c*d*f*x + (a^4 - a^2*b^2)*
c^2*f)*cos(f*x + e)^2 + 4*((a^3*b - a*b^3)*d^2*f*x^2 + 2*(a^3*b - a*b^3)*c*
d*f*x + (a^3*b - a*b^3)*c^2*f)*cos(f*x + e)*sin(2*f*x + 2*e) + ((a^2*b^2 -
b^4)*d^2*f*x^2 + 2*(a^2*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f)*sin(2*f
*x + 2*e)^2 + 4*((a^4 - a^2*b^2)*d^2*f*x^2 + 2*(a^4 - a^2*b^2)*c*d*f*x + (a
^4 - a^2*b^2)*c^2*f)*sin(f*x + e)^2 - 2*((a^2*b^2 - b^4)*d^2*f*x^2 + 2*(a^2
*b^2 - b^4)*c*d*f*x + (a^2*b^2 - b^4)*c^2*f + 2*((a^3*b - a*b^3)*d^2*f*x^2
+ 2*(a^3*b - a*b^3)*c*d*f*x + (a^3*b - a*b^3)*c^2*f)*sin(f*x + e))*cos(2*f*
x + 2*e) + 4*((a^3*b - a*b^3)*d^2*f*x^2 + 2*(a^3*b - a*b^3)*c*d*f*x + (a^3*
b - a*b^3)*c^2*f)*sin(f*x + e))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^2+b^2\right)d^2x^2+2\left(a^2+b^2\right)cdx+\left(a^2+b^2\right)c^2-\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(fx+e\right)^2+2\left(abd^2x^2+2\,abcdx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2 + b^2)*d^2*x^2 + 2*(a^2 + b^2)*c*d*x + (a^2 + b^2)*c^2 - (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*
b*c*d*x + a*b*c^2)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(b\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*sin(f*x + e) + a)^2), x)
```

3.173 $\int (c+dx)^m (a+b\sin(e+fx))^n\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left((c+dx)^m(a+b\sin(e+fx))^n,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x]
```

Rubi [A] time = 0.0510695, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m(a+b\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + b*Sin[e + f*x])^n,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(a + b*Sin[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m(a+b\sin(e+fx))^n\,dx=\int (c+dx)^m(a+b\sin(e+fx))^n\,dx$$

Mathematica [A] time = 0.933121, size = 0, normalized size = 0.

$$\int (c+dx)^m(a+b\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + b*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x]
```

Maple [A] time = 0.375, size = 0, normalized size = 0.

$$\int (dx+c)^m \left(a+b\sin\left(fx+e\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sin(f*x+e))^n,x)
```

```
[Out] int((d*x+c)^m*(a+b*sin(f*x+e))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \left(b\sin\left(fx+e\right)+a\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*(b*sin(f*x + e) + a)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m \left(b\sin\left(fx+e\right)+a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*(b*sin(f*x + e) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \left(b\sin\left(fx+e\right)+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*(b*sin(f*x + e) + a)^n, x)
```

3.174 $\int (c+dx)^m(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=607

$$-\frac{3a^2be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{2f}-\frac{3a^2be^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if}{}\right.}{2f}$$

```
[Out] (a^3*(c + d*x)^(1 + m))/(d*(1 + m)) + (3*a*b^2*(c + d*x)^(1 + m))/(2*d*(1 +
 m)) - (3*a^2*b*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*f*(c + d
*x))/d])/(2*f*(((-I)*f*(c + d*x))/d)^m) - (3*b^3*E^(I*(e - (c*f)/d))*(c + d
*x)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d])/(8*f*(((-I)*f*(c + d*x))/d)^m) -
(3*a^2*b*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(2*E^(I*(e - (c*f)/d)
)*f*((I*f*(c + d*x))/d)^m) - (3*b^3*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x)
)/d])/(8*E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + ((3*I)*2^(-3 - m)*a
*b^2*E^((2*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/
d])/(f*(((-I)*f*(c + d*x))/d)^m) - ((3*I)*2^(-3 - m)*a*b^2*(c + d*x)^m*Gamm
a[1 + m, ((2*I)*f*(c + d*x))/d])/(E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x)
)/d)^m) + (3^(-1 - m)*b^3*E^((3*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m,
((-3*I)*f*(c + d*x))/d])/(8*f*(((-I)*f*(c + d*x))/d)^m) + (3^(-1 - m)*b^3*(
c + d*x)^m*Gamma[1 + m, ((3*I)*f*(c + d*x))/d])/(8*E^((3*I)*(e - (c*f)/d))*
f*((I*f*(c + d*x))/d)^m)
```

Rubi [A] time = 0.764204, antiderivative size = 607, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3317, 3308, 2181, 3312, 3307}

$$-\frac{3a^2be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{2f}-\frac{3a^2be^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if}{}\right.}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(c + d*x)^(1 + m))/(d*(1 + m)) + (3*a*b^2*(c + d*x)^(1 + m))/(2*d*(1 +
 m)) - (3*a^2*b*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*f*(c + d
*x))/d])/(2*f*(((-I)*f*(c + d*x))/d)^m) - (3*b^3*E^(I*(e - (c*f)/d))*(c + d
*x)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d])/(8*f*(((-I)*f*(c + d*x))/d)^m) -
(3*a^2*b*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(2*E^(I*(e - (c*f)/d)
)*f*((I*f*(c + d*x))/d)^m) - (3*b^3*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x)
)/d])/(8*E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + ((3*I)*2^(-3 - m)*a
```

```
*b^2*E^((2*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/
d])/(f*(((-I)*f*(c + d*x))/d)^m) - ((3*I)*2^(-3 - m)*a*b^2*(c + d*x)^m*Gamm
a[1 + m, ((2*I)*f*(c + d*x))/d])/(E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x)
)/d)^m) + (3^(-1 - m)*b^3*E^((3*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m,
((-3*I)*f*(c + d*x))/d])/(8*f*(((-I)*f*(c + d*x))/d)^m) + (3^(-1 - m)*b^3*(
c + d*x)^m*Gamma[1 + m, ((3*I)*f*(c + d*x))/d])/(8*E^((3*I)*(e - (c*f)/d))*
f*((I*f*(c + d*x))/d)^m)
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\int (c+dx)^m(a+b\sin(e+fx))^3\,dx = \int \left(a^3(c+dx)^m + 3a^2b(c+dx)^m\sin(e+fx) + 3ab^2(c+dx)^m\sin^2(e+fx) + b^3(c+\right.$$
$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \left(3a^2b\right)\int (c+dx)^m\sin(e+fx)\,dx + \left(3ab^2\right)\int (c+dx)^m\sin^2(e+f$$
$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{1}{2}\left(3ia^2b\right)\int e^{-i(e+fx)}(c+dx)^m\,dx - \frac{1}{2}\left(3ia^2b\right)\int e^{i(e+fx)}(c+dx)^m\,d$$
$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} - \frac{3a^2be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\right.}{2f}$$
$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} - \frac{3a^2be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\right.}{2f}$$
$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} - \frac{3a^2be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\right.}{2f}$$

Mathematica [A] time = 5.65666, size = 415, normalized size = 0.68

$$i(c+dx)^m\left(9ib\left(4a^2+b^2\right)e^{i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right) + 9ib\left(4a^2+b^2\right)e^{-i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((I/24)*(c + d*x)^m*(((-12*I)*a*(2*a^2 + 3*b^2)*f*(c + d*x))/(d*(1 + m)) +
((9*I)*b*(4*a^2 + b^2)*E^(I*(e - (c*f)/d))*Gamma[1 + m, ((-I)*f*(c + d*x))/
d])/(((-I)*f*(c + d*x))/d)^m + ((9*I)*b*(4*a^2 + b^2)*Gamma[1 + m, (I*f*(c
+ d*x))/d])/(E^(I*(e - (c*f)/d))*((I*f*(c + d*x))/d)^m) + (9*a*b^2*E^((2*I)
*(e - (c*f)/d))*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(2^m*(((-I)*f*(c + d*
x))/d)^m) - (9*a*b^2*Gamma[1 + m, ((2*I)*f*(c + d*x))/d])/(2^m*E^((2*I)*(e
- (c*f)/d))*((I*f*(c + d*x))/d)^m) - (I*b^3*E^((3*I)*(e - (c*f)/d))*Gamma[1
 + m, ((-3*I)*f*(c + d*x))/d])/(3^m*(((-I)*f*(c + d*x))/d)^m) - (I*b^3*Gamm
a[1 + m, ((3*I)*f*(c + d*x))/d])/(3^m*E^((3*I)*(e - (c*f)/d))*((I*f*(c + d*
x))/d)^m)))/f
```

Maple [F] time = 0.285, size = 0, normalized size = 0.

$$\int (dx+c)^m \left(a+b\sin\left(fx+e\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sin(f*x+e))^3,x)
```

```
[Out] int((d*x+c)^m*(a+b*sin(f*x+e))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.17111, size = 1045, normalized size = 1.72

$$\frac{\left(b^3dm+b^3d\right)e^{\left(-\frac{dm\log\left(\frac{3if}{d}\right)+3ide-3icf}{d}\right)}\Gamma\left(m+1,\frac{3idfx+3icf}{d}\right)+\left(-9iab^2dm-9iab^2d\right)e^{\left(-\frac{dm\log\left(\frac{2if}{d}\right)+2ide-2icf}{d}\right)}\Gamma\left(m+1,\frac{2idfx+2icf}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*((b^3*d*m + b^3*d)*e^(-(d*m*log(3*I*f/d) + 3*I*d*e - 3*I*c*f)/d)*gamma
(m + 1, (3*I*d*f*x + 3*I*c*f)/d) + (-9*I*a*b^2*d*m - 9*I*a*b^2*d)*e^(-(d*m*
log(2*I*f/d) + 2*I*d*e - 2*I*c*f)/d)*gamma(m + 1, (2*I*d*f*x + 2*I*c*f)/d)
- 9*((4*a^2*b + b^3)*d*m + (4*a^2*b + b^3)*d)*e^(-(d*m*log(I*f/d) + I*d*e -
 I*c*f)/d)*gamma(m + 1, (I*d*f*x + I*c*f)/d) - 9*((4*a^2*b + b^3)*d*m + (4*
a^2*b + b^3)*d)*e^(-(d*m*log(-I*f/d) - I*d*e + I*c*f)/d)*gamma(m + 1, (-I*d
*f*x - I*c*f)/d) + (9*I*a*b^2*d*m + 9*I*a*b^2*d)*e^(-(d*m*log(-2*I*f/d) - 2
```

```
*I*d*e + 2*I*c*f)/d)*gamma(m + 1, (-2*I*d*f*x - 2*I*c*f)/d) + (b^3*d*m + b^
3*d)*e^(-(d*m*log(-3*I*f/d) - 3*I*d*e + 3*I*c*f)/d)*gamma(m + 1, (-3*I*d*f*
x - 3*I*c*f)/d) + 12*((2*a^3 + 3*a*b^2)*d*f*x + (2*a^3 + 3*a*b^2)*c*f)*(d*x
 + c)^m)/(d*f*m + d*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^3 (dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*x + c)^m, x)
```

3.175 $\int (c+dx)^m(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=318

$$-\frac{abe^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{f}-\frac{abe^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if(c+}{d}\right.}{f}$$

```
[Out] (a^2*(c + d*x)^(1 + m))/(d*(1 + m)) + (b^2*(c + d*x)^(1 + m))/(2*d*(1 + m))
 - (a*b*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d])
/(f*(((-I)*f*(c + d*x))/d)^m) - (a*b*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x
))/d])/(E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + (I*2^(-3 - m)*b^2*E^
((2*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(f*
(((-I)*f*(c + d*x))/d)^m) - (I*2^(-3 - m)*b^2*(c + d*x)^m*Gamma[1 + m, ((2*
I)*f*(c + d*x))/d])/(E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)
```

Rubi [A] time = 0.392043, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3317, 3308, 2181, 3312, 3307}

$$-\frac{abe^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{f}-\frac{abe^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if(c+}{d}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(c + d*x)^(1 + m))/(d*(1 + m)) + (b^2*(c + d*x)^(1 + m))/(2*d*(1 + m))
 - (a*b*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*f*(c + d*x))/d])
/(f*(((-I)*f*(c + d*x))/d)^m) - (a*b*(c + d*x)^m*Gamma[1 + m, (I*f*(c + d*x
))/d])/(E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + (I*2^(-3 - m)*b^2*E^
((2*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(f*
(((-I)*f*(c + d*x))/d)^m) - (I*2^(-3 - m)*b^2*(c + d*x)^m*Gamma[1 + m, ((2*
I)*f*(c + d*x))/d])/(E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
```

```
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\int (c+dx)^m(a+b\sin(e+fx))^2\,dx = \int \left(a^2(c+dx)^m + 2ab(c+dx)^m\sin(e+fx) + b^2(c+dx)^m\sin^2(e+fx)\right)\,dx$$
$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + (2ab)\int (c+dx)^m\sin(e+fx)\,dx + b^2\int (c+dx)^m\sin^2(e+fx)\,dx$$
$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + (iab)\int e^{-i(e+fx)}(c+dx)^m\,dx - (iab)\int e^{i(e+fx)}(c+dx)^m\,dx + b^2\int$$
$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{b^2(c+dx)^{1+m}}{2d(1+m)} - \frac{abe^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c\cdot}{d}\right.}{f}$$
$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{b^2(c+dx)^{1+m}}{2d(1+m)} - \frac{abe^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c\cdot}{d}\right.}{f}$$
$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{b^2(c+dx)^{1+m}}{2d(1+m)} - \frac{abe^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c\cdot}{d}\right.}{f}$$

Mathematica [A] time = 3.93237, size = 268, normalized size = 0.84

$$-\frac{(c+dx)^m\left(8abe^{i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)+8abe^{-i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if(c+dx)}{d}\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((c + d*x)^m*((-4*(2*a^2 + b^2)*f*(c + d*x))/(d*(1 + m)) + (8*a*b*E^(I*(e
 - (c*f)/d))*Gamma[1 + m, ((-I)*f*(c + d*x))/d])/(((-I)*f*(c + d*x))/d)^m +
(8*a*b*Gamma[1 + m, (I*f*(c + d*x))/d])/(E^(I*(e - (c*f)/d))*((I*f*(c + d*x
))/d)^m) - (I*b^2*E^((2*I)*(e - (c*f)/d))*Gamma[1 + m, ((-2*I)*f*(c + d*x))
/d])/(2^m*(((-I)*f*(c + d*x))/d)^m) + (I*b^2*Gamma[1 + m, ((2*I)*f*(c + d*x
))/d])/(2^m*E^((2*I)*(e - (c*f)/d))*((I*f*(c + d*x))/d)^m)))/(8*f)
```

Maple [F] time = 0.198, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+b\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+b*sin(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97231, size = 662, normalized size = 2.08

$$\left(-i b^2 d m - i b^2 d\right) e^{\left(-\frac{d m \log\left(\frac{2 i f}{d}\right) + 2 i d e - 2 i c f}{d}\right)} \Gamma\left(m+1, \frac{2 i d f x + 2 i c f}{d}\right) - 8 \left(a b d m + a b d\right) e^{\left(-\frac{d m \log\left(\frac{i f}{d}\right) + i d e - i c f}{d}\right)} \Gamma\left(m+1, \frac{i d f x + i c f}{d}\right) - 8 (ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((-I*b^2*d*m - I*b^2*d)*e^(-(d*m*log(2*I*f/d) + 2*I*d*e - 2*I*c*f)/d)*g
amma(m + 1, (2*I*d*f*x + 2*I*c*f)/d) - 8*(a*b*d*m + a*b*d)*e^(-(d*m*log(I*f
/d) + I*d*e - I*c*f)/d)*gamma(m + 1, (I*d*f*x + I*c*f)/d) - 8*(a*b*d*m + a*
b*d)*e^(-(d*m*log(-I*f/d) - I*d*e + I*c*f)/d)*gamma(m + 1, (-I*d*f*x - I*c*
f)/d) + (I*b^2*d*m + I*b^2*d)*e^(-(d*m*log(-2*I*f/d) - 2*I*d*e + 2*I*c*f)/d
)*gamma(m + 1, (-2*I*d*f*x - 2*I*c*f)/d) + 4*((2*a^2 + b^2)*d*f*x + (2*a^2
+ b^2)*c*f)*(d*x + c)^m)/(d*f*m + d*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \sin\left(e + f x\right)\right)^2 (c + dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^2(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*x + c)^m, x)
```

3.176 $\int (c+dx)^m (a+b\sin(e+fx))\,dx$

Optimal. Leaf size=148

$$-\frac{be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{2f}-\frac{be^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if(c+dx)}{d}\right)}{2f}$$

```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) - (b*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gamm
a[1 + m, ((-I)*f*(c + d*x))/d])/(2*f*(((-I)*f*(c + d*x))/d)^m) - (b*(c + d*
x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(2*E^(I*(e - (c*f)/d))*f*((I*f*(c + d
*x))/d)^m)
```

Rubi [A] time = 0.148998, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3308, 2181}

$$-\frac{be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{2f}-\frac{be^{-i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if(c+dx)}{d}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Sin[e + f*x]),x]
```



```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) - (b*E^(I*(e - (c*f)/d))*(c + d*x)^m*Gamm
a[1 + m, ((-I)*f*(c + d*x))/d])/(2*f*(((-I)*f*(c + d*x))/d)^m) - (b*(c + d*
x)^m*Gamma[1 + m, (I*f*(c + d*x))/d])/(2*E^(I*(e - (c*f)/d))*f*((I*f*(c + d
*x))/d)^m)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m(a+b\sin(e+fx))\,dx &= \int \left(a(c+dx)^m+b(c+dx)^m\sin(e+fx)\right)dx\\ &= \frac{a(c+dx)^{1+m}}{d(1+m)}+b\int (c+dx)^m\sin(e+fx)\,dx\\ &= \frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{1}{2}(ib)\int e^{-i(e+fx)}(c+dx)^m\,dx-\frac{1}{2}(ib)\int e^{i(e+fx)}(c+dx)^m\,dx\\ &= \frac{a(c+dx)^{1+m}}{d(1+m)}-\frac{be^{i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{if(c+dx)}{d}\right)}{2f}-\frac{be^{-i\left(e-\frac{cf}{d}\right)}(c+\ldots}{}\end{aligned}$$

Mathematica [A] time = 0.187802, size = 138, normalized size = 0.93

$$\frac{1}{2}(c+dx)^m\left(-\frac{be^{i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{if(c+dx)}{d}\right)}{f}-\frac{be^{-i\left(e-\frac{cf}{d}\right)}\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{if(c+dx)}{d}\right)}{f}+\frac{2a\ldots}{d\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Sin[e + f*x]),x]
```

```
[Out] ((c + d*x)^m*((2*a*(c + d*x))/(d*(1 + m)) - (b*E^(I*(e - (c*f)/d))*Gamma[1
+ m, ((-I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (b*Gamma[1 + m,
(I*f*(c + d*x))/d])/(E^(I*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)))/2
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+b\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sin(f*x+e)),x)
```

```
[Out] int((d*x+c)^m*(a+b*sin(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82744, size = 319, normalized size = 2.16

$$-\frac{(bdm+bd)e^{\left(-\frac{dm\log\left(\frac{if}{d}\right)+ide-icf}{d}\right)}\Gamma\left(m+1,\frac{idfx+icf}{d}\right)+(bdm+bd)e^{\left(-\frac{dm\log\left(-\frac{if}{d}\right)-ide+icf}{d}\right)}\Gamma\left(m+1,\frac{-idfx-icf}{d}\right)-2\left(adfx+acf\right)}{2\left(dfm+df\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*((b*d*m + b*d)*e^(-(d*m*log(I*f/d) + I*d*e - I*c*f)/d)*gamma(m + 1, (I
*d*f*x + I*c*f)/d) + (b*d*m + b*d)*e^(-(d*m*log(-I*f/d) - I*d*e + I*c*f)/d)
*gamma(m + 1, (-I*d*f*x - I*c*f)/d) - 2*(a*d*f*x + a*c*f)*(d*x + c)^m)/(d*f
*m + d*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(e+fx\right)\right)(c+dx)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*x + c)^m, x)
```

3.177 $\int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(c+dx)^m}{a+b\sin(e+fx)},x\right)$$

```
[Out] Unintegrable[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
```

Rubi [A] time = 0.0573369, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + b*Sin[e + f*x]),x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + b*Sin[e + f*x]), x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx$$

Mathematica [A] time = 0.387383, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{a+b\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + b*Sin[e + f*x]),x]
```

```
[Out] Integrate[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
```

Maple [A] time = 0.078, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{a+b\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+b*sin(f*x+e)),x)
```

```
[Out] int((d*x+c)^m/(a+b*sin(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(b*sin(f*x + e) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^m}{b\sin\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m/(b*sin(f*x + e) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**m/(a + b*sin(e + f*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(b*sin(f*x + e) + a), x)
```

3.178 $\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(c+dx)^m}{(a+b\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable[(c + d*x)^m/(a + b*Sin[e + f*x])^2, x]
```

Rubi [A] time = 0.0551485, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + b*Sin[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx=\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx$$

Mathematica [A] time = 3.42937, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*x)^m/(a + b*Sin[e + f*x])^2, x]
```

Maple [A] time = 0.189, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(a+b\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+b*sin(f*x+e))^2,x)

[Out] int((d*x+c)^m/(a+b*sin(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(b\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sin(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((d*x + c)^m/(b*sin(f*x + e) + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(dx+c)^m}{b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sin(f*x+e))^2,x, algorithm="fricas")

[Out] integral(-(d*x + c)^m/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(b\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(b*sin(f*x + e) + a)^2, x)
```

3.179 $\int \frac{(e+fx)^3 \sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=164

$$\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{12f^3\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4}-\frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right.}{ad}$$

```
[Out] (I*(e + f*x)^3)/(a*d) + (e + f*x)^4/(4*a*f) + ((e + f*x)^3*Cot[c/2 + Pi/4 +
 (d*x)/2])/(a*d) - (6*f*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) + (
(12*I)*f^2*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - (12*f^3*PolyL
og[3, I*E^(I*(c + d*x))])/(a*d^4)
```

Rubi [A] time = 0.340199, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {4515, 32, 3318, 4184, 3717, 2190, 2531, 2282, 6589}

$$\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{12f^3\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4}-\frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```


```
[Out] (I*(e + f*x)^3)/(a*d) + (e + f*x)^4/(4*a*f) + ((e + f*x)^3*Cot[c/2 + Pi/4 +
 (d*x)/2])/(a*d) - (6*f*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) + (
(12*I)*f^2*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - (12*f^3*PolyL
og[3, I*E^(I*(c + d*x))])/(a*d^4)
```


Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\,dx}{a} - \int \frac{(e+fx)^3}{a+a\sin(c+dx)}\,dx$$

$$= \frac{(e+fx)^4}{4af} - \frac{\int (e+fx)^3\csc^2\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{dx}{2}\right)dx}{2a}$$

$$= \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(3f)\int (e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)dx}{ad}$$

$$= \frac{i(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(6f)\int \frac{e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}(e+fx)^2}{1-ie^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}dx}{ad}$$

$$= \frac{i(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2} + \frac{(12 \cdots}{\cdots}$$

$$= \frac{i(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2} + \frac{12i \cdots}{\cdots}$$

$$= \frac{i(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2} + \frac{12i \cdots}{\cdots}$$

$$= \frac{i(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6f(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad^2} + \frac{12i \cdots}{\cdots}$$

Mathematica [A] time = 1.83015, size = 261, normalized size = 1.59

$$\frac{\frac{24f(\cos(c)+i\sin(c))\left(\frac{2f(\cos(c)-i(\sin(c)+1))(d(e+fx)\text{PolyLog}(2,-\sin(c+dx)-i\cos(c+dx))-if\text{PolyLog}(3,-\sin(c+dx)-i\cos(c+dx)))}{d^3}-\frac{(\sin(c)+i\cos(c)+1)(e+fx)^2\log(\sin(c+dx)+i\cos(c+ \cdots}{d}\right.}{d(\cos(c)+i(\sin(c)+1))}\cdots}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) + (24*f*(Cos[c] + I*Sin[c])*
(((e + f*x)^3*(Cos[c] - I*Sin[c]))/(3*f) - ((e + f*x)^2*Log[1 + I*Cos[c + d
*x] + Sin[c + d*x]]*(1 + I*Cos[c] + Sin[c]))/d + (2*f*(d*(e + f*x)*PolyLog[
2, (-I)*Cos[c + d*x] - Sin[c + d*x]] - I*f*PolyLog[3, (-I)*Cos[c + d*x] - S
in[c + d*x]])*(Cos[c] - I*(1 + Sin[c])))/d^3))/(d*(Cos[c] + I*(1 + Sin[c]))
) - (8*(e + f*x)^3*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])))/(4*a)
```

Maple [B] time = 0.182, size = 526, normalized size = 3.2

$$\frac{f^3x^4}{4a}+\frac{ef^2x^3}{a}+\frac{3e^2fx^2}{2a}+\frac{e^3x}{a}+2\frac{f^3x^3+3ef^2x^2+3e^2fx+e^3}{da\left(e^{i(dx+c)}+i\right)}-12\frac{ef^2\ln\left(1-ie^{i(dx+c)}\right)x}{ad^2}-12\frac{ef^2\ln\left(1-ie^{i(dx+c)}\right)c}{d^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/a*f^3*x^4+1/a*e*f^2*x^3+3/2/a*e^2*f*x^2+1/a*e^3*x+2*(f^3*x^3+3*e*f^2*x^
2+3*e^2*f*x+e^3)/d/a/(exp(I*(d*x+c))+I)-12*f^2/d^2/a*e*ln(1-I*exp(I*(d*x+c)
))*x-12*f^2/d^3/a*e*ln(1-I*exp(I*(d*x+c)))*c+6*f/d^2/a*ln(exp(I*(d*x+c)))*e
^2-4*I*f^3/d^4/a*c^3-12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4+12*I*f^2/d^3/
a*e*polylog(2,I*exp(I*(d*x+c)))+6*I*f^2/d/a*e*x^2+12*I*f^3/d^3/a*polylog(2,
I*exp(I*(d*x+c)))*x+6*I*f^2/d^3/a*e*c^2+6*f^3/d^4/a*c^2*ln(exp(I*(d*x+c)))-
6*f/d^2/a*ln(exp(I*(d*x+c))+I)*e^2+2*I*f^3/d/a*x^3-6*f^3/d^2/a*ln(1-I*exp(I
*(d*x+c)))*x^2+6*f^3/d^4/a*ln(1-I*exp(I*(d*x+c)))*c^2+12*I*f^2/d^2/a*e*c*x-
6*f^3/d^4/a*c^2*ln(exp(I*(d*x+c))+I)-12*f^2/d^3/a*e*c*ln(exp(I*(d*x+c)))-6*
I*f^3/d^3/a*c^2*x+12*f^2/d^3/a*e*c*ln(exp(I*(d*x+c))+I)
```

Maxima [B] time = 2.08737, size = 1766, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(12*c^2*e*f^2*(1/(a*d^2 + a*d^2*sin(d*x + c)/(cos(d*x + c) + 1)) + arct
an(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d^2)) - 12*c*e^2*f*(1/(a*d + a*d*sin
(d*x + c)/(cos(d*x + c) + 1)) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a*
```

```
d)) - 6*((d*x + c)^2*cos(d*x + c)^2 + (d*x + c)^2*sin(d*x + c)^2 + 2*(d*x +
 c)^2*sin(d*x + c) + (d*x + c)^2 + 4*(d*x + c)*cos(d*x + c) - 2*(cos(d*x +
c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)*log(cos(d*x + c)^2 + sin(d*x +
c)^2 + 2*sin(d*x + c) + 1))*c*e*f^2/(a*d^2*cos(d*x + c)^2 + a*d^2*sin(d*x +
 c)^2 + 2*a*d^2*sin(d*x + c) + a*d^2) + 4*e^3*(arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a + 1/(a + a*sin(d*x + c)/(cos(d*x + c) + 1))) + 3*((d*x + c)^2
*cos(d*x + c)^2 + (d*x + c)^2*sin(d*x + c)^2 + 2*(d*x + c)^2*sin(d*x + c) +
 (d*x + c)^2 + 4*(d*x + c)*cos(d*x + c) - 2*(cos(d*x + c)^2 + sin(d*x + c)^
2 + 2*sin(d*x + c) + 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c
) + 1))*e^2*f/(a*d*cos(d*x + c)^2 + a*d*sin(d*x + c)^2 + 2*a*d*sin(d*x + c)
 + a*d) + 2*((d*x + c)^4*f^3 + 6*(d*x + c)^2*c^2*f^3 - 4*(d*x + c)*c^3*f^3
+ 8*I*c^3*f^3 + 4*(d*e*f^2 - c*f^3)*(d*x + c)^3 - (24*c^2*f^3*cos(d*x + c)
+ 24*I*c^2*f^3*sin(d*x + c) + 24*I*c^2*f^3)*arctan2(sin(d*x + c) + 1, cos(d
*x + c)) - (-24*I*(d*x + c)^2*f^3 + (-48*I*d*e*f^2 + 48*I*c*f^3)*(d*x + c)
- 24*((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*cos(d*x + c) + (-24*
I*(d*x + c)^2*f^3 + (-48*I*d*e*f^2 + 48*I*c*f^3)*(d*x + c))*sin(d*x + c))*a
rctan2(cos(d*x + c), sin(d*x + c) + 1) - (I*(d*x + c)^4*f^3 + (-4*I*c^3 - 2
4*c^2)*(d*x + c)*f^3 - 4*(-I*d*e*f^2 + (I*c + 2)*f^3)*(d*x + c)^3 - (24*d*e
*f^2 - (6*I*c^2 + 24*c)*f^3)*(d*x + c)^2)*cos(d*x + c) - (-48*I*d*e*f^2 - 4
8*I*(d*x + c)*f^3 + 48*I*c*f^3 - 48*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*cos(d
*x + c) + (-48*I*d*e*f^2 - 48*I*(d*x + c)*f^3 + 48*I*c*f^3)*sin(d*x + c))*d
ilog(I*e^(I*d*x + I*c)) - (12*(d*x + c)^2*f^3 + 12*c^2*f^3 + 24*(d*e*f^2 -
c*f^3)*(d*x + c) + (-12*I*(d*x + c)^2*f^3 - 12*I*c^2*f^3 + (-24*I*d*e*f^2 +
 24*I*c*f^3)*(d*x + c))*cos(d*x + c) + 12*((d*x + c)^2*f^3 + c^2*f^3 + 2*(d
*e*f^2 - c*f^3)*(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^
2 + 2*sin(d*x + c) + 1) + 48*(I*f^3*cos(d*x + c) - f^3*sin(d*x + c) - f^3)*
polylog(3, I*e^(I*d*x + I*c)) + ((d*x + c)^4*f^3 - 4*(c^3 - 6*I*c^2)*(d*x +
 c)*f^3 + (4*d*e*f^2 - (4*c - 8*I)*f^3)*(d*x + c)^3 - (-24*I*d*e*f^2 - 6*(c
^2 - 4*I*c)*f^3)*(d*x + c)^2)*sin(d*x + c))/(-4*I*a*d^3*cos(d*x + c) + 4*a*
d^3*sin(d*x + c) + 4*a*d^3))/d
```

Fricas [C] time = 2.24933, size = 2414, normalized size = 14.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(d^4*f^3*x^4 + 4*d^3*e^3 + 4*(d^4*e*f^2 + d^3*f^3)*x^3 + 6*(d^4*e^2*f +
 2*d^3*e*f^2)*x^2 + 4*(d^4*e^3 + 3*d^3*e^2*f)*x + (d^4*f^3*x^4 + 4*d^3*e^3
+ 4*(d^4*e*f^2 + d^3*f^3)*x^3 + 6*(d^4*e^2*f + 2*d^3*e*f^2)*x^2 + 4*(d^4*e^
```

```
3 + 3*d^3*e^2*f)*x)*cos(d*x + c) + (24*I*d*f^3*x + 24*I*d*e*f^2 + (24*I*d*f
^3*x + 24*I*d*e*f^2)*cos(d*x + c) + (24*I*d*f^3*x + 24*I*d*e*f^2)*sin(d*x +
 c))*dilog(I*cos(d*x + c) - sin(d*x + c)) + (-24*I*d*f^3*x - 24*I*d*e*f^2 +
 (-24*I*d*f^3*x - 24*I*d*e*f^2)*cos(d*x + c) + (-24*I*d*f^3*x - 24*I*d*e*f^
2)*sin(d*x + c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) - 12*(d^2*e^2*f - 2*
c*d*e*f^2 + c^2*f^3 + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c) + (d
^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*
x + c) + I) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3 + (d^
2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c) + (d^2*f^3*
x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*sin(d*x + c))*log(I*cos(d*x +
c) + sin(d*x + c) + 1) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^
2*f^3 + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c)
+ (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*sin(d*x + c))*log(-
I*cos(d*x + c) + sin(d*x + c) + 1) - 12*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3
+ (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c) + (d^2*e^2*f - 2*c*d*e*f
^2 + c^2*f^3)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) - 24*(f
^3*cos(d*x + c) + f^3*sin(d*x + c) + f^3)*polylog(3, I*cos(d*x + c) - sin(d
*x + c)) - 24*(f^3*cos(d*x + c) + f^3*sin(d*x + c) + f^3)*polylog(3, -I*cos
(d*x + c) - sin(d*x + c)) + (d^4*f^3*x^4 - 4*d^3*e^3 + 4*(d^4*e*f^2 - d^3*f
^3)*x^3 + 6*(d^4*e^2*f - 2*d^3*e*f^2)*x^2 + 4*(d^4*e^3 - 3*d^3*e^2*f)*x)*si
n(d*x + c))/(a*d^4*cos(d*x + c) + a*d^4*sin(d*x + c) + a*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^3\sin(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{f^3x^3\sin(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{3ef^2x^2\sin(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{3e^2fx\sin(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**3*sin(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*sin
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sin(c + d*x)/(sin
(c + d*x) + 1), x) + Integral(3*e**2*f*x*sin(c + d*x)/(sin(c + d*x) + 1), x
))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3\sin(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.180 $\int \frac{(e+fx)^2\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=129

$$\frac{4if^2\mathrm{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}+\frac{i(e+fx)^2}{ad}+\frac{(e+fx)^3}{3af}$$

```
[Out] (I*(e + f*x)^2)/(a*d) + (e + f*x)^3/(3*a*f) + ((e + f*x)^2*Cot[c/2 + Pi/4 +
 (d*x)/2])/(a*d) - (4*f*(e + f*x)*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) + ((4
*I)*f^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3)
```

Rubi [A] time = 0.257387, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4515, 32, 3318, 4184, 3717, 2190, 2279, 2391}

$$\frac{4if^2\mathrm{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}+\frac{i(e+fx)^2}{ad}+\frac{(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (I*(e + f*x)^2)/(a*d) + (e + f*x)^3/(3*a*f) + ((e + f*x)^2*Cot[c/2 + Pi/4 +
 (d*x)/2])/(a*d) - (4*f*(e + f*x)*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) + ((4
*I)*f^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3)
```

Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\,dx}{a} - \int \frac{(e+fx)^2}{a+a\sin(c+dx)}\,dx$$
$$= \frac{(e+fx)^3}{3af} - \frac{\int (e+fx)^2\csc^2\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{dx}{2}\right)dx}{2a}$$
$$= \frac{(e+fx)^3}{3af} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(2f)\int (e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)dx}{ad}$$
$$= \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(4f)\int \frac{e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}(e+fx)}{1-ie^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}dx}{ad}$$
$$= \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2} + \frac{(4\ldots}{\ldots}$$
$$= \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2} - \frac{(4i\ldots}{\ldots}$$
$$= \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2} + \frac{4i\ldots}{\ldots}$$

Mathematica [A] time = 1.25245, size = 213, normalized size = 1.65

$$\frac{\frac{12f(\cos(c)+i\sin(c))\left(\frac{f(\cos(c)-i(\sin(c)+1))\text{PolyLog}(2,-\sin(c+dx)-i\cos(c+dx))}{d^2}-\frac{(\sin(c)+i\cos(c)+1)(e+fx)\log(\sin(c+dx)+i\cos(c+dx)+1)}{d}+\frac{(\cos(c)-i\sin(c))(e+fx)^2}{2f}\right)}{d(\cos(c)+i(\sin(c)+1))} - \frac{\ldots}{d\left(\sin\left(\frac{c}{2}\ldots\right.\right.}\ldots}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2) + (12*f*(Cos[c] + I*Sin[c])*(((e + f*x)^2*(C
os[c] - I*Sin[c]))/(2*f) - ((e + f*x)*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]
]*(1 + I*Cos[c] + Sin[c]))/d + (f*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*
x]]*(Cos[c] - I*(1 + Sin[c])))/d^2))/(d*(Cos[c] + I*(1 + Sin[c]))) - (6*(e
+ f*x)^2*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])))/(3*a)
```

Maple [B] time = 0.108, size = 282, normalized size = 2.2

$$\frac{f^2x^3}{3a}+\frac{fex^2}{a}+\frac{e^2x}{a}+2\frac{f^2x^2+2fex+e^2}{da\left(e^{i(dx+c)}+i\right)}+4\frac{f\ln\left(e^{i(dx+c)}\right)e}{ad^2}-4\frac{f\ln\left(e^{i(dx+c)}+i\right)e}{ad^2}+\frac{2if^2x^2}{da}+\frac{4if^2cx}{ad^2}+\frac{2if^2c^2}{d^3a}-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/3/a*f^2*x^3+1/a*f*e*x^2+1/a*e^2*x+2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(I*(d*x
+c))+I)+4*f/d^2/a*ln(exp(I*(d*x+c)))*e-4*f/d^2/a*ln(exp(I*(d*x+c))+I)*e+2*I
*f^2/d/a*x^2+4*I*f^2/d^2/a*c*x+2*I*f^2/d^3/a*c^2-4*f^2/d^2/a*ln(1-I*exp(I*(
d*x+c)))*x-4*f^2/d^3/a*ln(1-I*exp(I*(d*x+c)))*c+4*I*f^2*polylog(2,I*exp(I*(
d*x+c)))/a/d^3-4*f^2/d^3/a*c*ln(exp(I*(d*x+c)))+4*f^2/d^3/a*c*ln(exp(I*(d*x
+c))+I)
```

Maxima [B] time = 1.8965, size = 545, normalized size = 4.22

$$\frac{d^3f^2x^3+3d^3efx^2+3d^3e^2x-6id^2e^2-\left(12\,def\cos(dx+c)+12i\,def\sin(dx+c)+12i\,def\right)\arctan(\sin(dx+c)+1,\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x - 6*I*d^2*e^2 - (12*d*e*f*cos(d*
x + c) + 12*I*d*e*f*sin(d*x + c) + 12*I*d*e*f)*arctan2(sin(d*x + c) + 1, co
s(d*x + c)) + (12*d*f^2*x*cos(d*x + c) + 12*I*d*f^2*x*sin(d*x + c) + 12*I*d
*f^2*x)*arctan2(cos(d*x + c), sin(d*x + c) + 1) - (I*d^3*f^2*x^3 + (3*I*d^3
*e*f - 6*d^2*f^2)*x^2 - 3*(-I*d^3*e^2 + 4*d^2*e*f)*x)*cos(d*x + c) + (12*f^
2*cos(d*x + c) + 12*I*f^2*sin(d*x + c) + 12*I*f^2)*dilog(I*e^(I*d*x + I*c))
 - (6*d*f^2*x + 6*d*e*f + (-6*I*d*f^2*x - 6*I*d*e*f)*cos(d*x + c) + 6*(d*f^
2*x + d*e*f)*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x
+ c) + 1) + (d^3*f^2*x^3 + 3*(d^3*e*f + 2*I*d^2*f^2)*x^2 + (3*d^3*e^2 + 12*
I*d^2*e*f)*x)*sin(d*x + c))/(-3*I*a*d^3*cos(d*x + c) + 3*a*d^3*sin(d*x + c)
 + 3*a*d^3)
```

Fricas [B] time = 1.98595, size = 1378, normalized size = 10.68

$$\frac{d^3 f^2 x^3 + 3\,d^2 e^2 + 3\,\left(d^3 e f + d^2 f^2\right) x^2 + 3\,\left(d^3 e^2 + 2\,d^2 e f\right) x + \left(d^3 f^2 x^3 + 3\,d^2 e^2 + 3\,\left(d^3 e f + d^2 f^2\right) x^2 + 3\,\left(d^3 e^2 + 2\,d^2 e f\right) x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(d^3*f^2*x^3 + 3*d^2*e^2 + 3*(d^3*e*f + d^2*f^2)*x^2 + 3*(d^3*e^2 + 2*d
^2*e*f)*x + (d^3*f^2*x^3 + 3*d^2*e^2 + 3*(d^3*e*f + d^2*f^2)*x^2 + 3*(d^3*e
^2 + 2*d^2*e*f)*x)*cos(d*x + c) + (6*I*f^2*cos(d*x + c) + 6*I*f^2*sin(d*x +
 c) + 6*I*f^2)*dilog(I*cos(d*x + c) - sin(d*x + c)) + (-6*I*f^2*cos(d*x + c
) - 6*I*f^2*sin(d*x + c) - 6*I*f^2)*dilog(-I*cos(d*x + c) - sin(d*x + c)) -
 6*(d*e*f - c*f^2 + (d*e*f - c*f^2)*cos(d*x + c) + (d*e*f - c*f^2)*sin(d*x
+ c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - 6*(d*f^2*x + c*f^2 + (d*f^2*
x + c*f^2)*cos(d*x + c) + (d*f^2*x + c*f^2)*sin(d*x + c))*log(I*cos(d*x + c
) + sin(d*x + c) + 1) - 6*(d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*cos(d*x + c)
 + (d*f^2*x + c*f^2)*sin(d*x + c))*log(-I*cos(d*x + c) + sin(d*x + c) + 1)
- 6*(d*e*f - c*f^2 + (d*e*f - c*f^2)*cos(d*x + c) + (d*e*f - c*f^2)*sin(d*x
 + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) + (d^3*f^2*x^3 - 3*d^2*e^2 +
 3*(d^3*e*f - d^2*f^2)*x^2 + 3*(d^3*e^2 - 2*d^2*e*f)*x)*sin(d*x + c))/(a*d^
3*cos(d*x + c) + a*d^3*sin(d*x + c) + a*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^2 \sin(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2 \sin(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx \sin(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*sin(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*sin
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sin(c + d*x)/(sin(c + d
*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sin(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.181 $\int \frac{(e+fx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=76

$$-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}+\frac{ex}{a}+\frac{fx^2}{2a}$$

```
[Out] (e*x)/a + (f*x^2)/(2*a) + ((e + f*x)*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - (2*
f*Log[Sin[c/2 + Pi/4 + (d*x)/2]])/(a*d^2)
```

Rubi [A] time = 0.095287, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4515, 3318, 4184, 3475}

$$-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}+\frac{ex}{a}+\frac{fx^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (e*x)/a + (f*x^2)/(2*a) + ((e + f*x)*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - (2*
f*Log[Sin[c/2 + Pi/4 + (d*x)/2]])/(a*d^2)
```

Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)\,dx}{a} - \int \frac{e+fx}{a+a\sin(c+dx)}\,dx\\
&= \frac{ex}{a} + \frac{fx^2}{2a} - \frac{\int (e+fx)\csc^2\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{dx}{2}\right)dx}{2a}\\
&= \frac{ex}{a} + \frac{fx^2}{2a} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{f\int \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)dx}{ad}\\
&= \frac{ex}{a} + \frac{fx^2}{2a} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}\end{aligned}$$

Mathematica [B] time = 0.496608, size = 199, normalized size = 2.62

$$\frac{\cos\left(\frac{dx}{2}\right)\left(d^2x(2e+fx)-4f\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+2d^2ex\sin\left(c+\frac{dx}{2}\right)+d^2fx^2\sin\left(c+\frac{dx}{2}\right)+2dfx\,\mathrm{co}}{2ad^2\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*d*f*x*Cos[c + (d*x)/2] + Cos[(d*x)/2]*(d^2*x*(2*e + f*x) - 4*f*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]) - 4*d*e*Sin[(d*x)/2] - 2*d*f*x*Sin[(d*x)/2
] + 2*d^2*e*x*Sin[c + (d*x)/2] + d^2*f*x^2*Sin[c + (d*x)/2] - 4*f*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + (d*x)/2])/(2*a*d^2*(Cos[c/2] + Sin[
c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [B] time = 0.076, size = 446, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 2/a*e/d*arctan(tan(1/2*d*x+1/2*c))+2/a*e/d/(tan(1/2*d*x+1/2*c)+1)+1/a*f/(1+
tan(1/2*d*x+1/2*c)^2)/(tan(1/2*d*x+1/2*c)+1)*x/d+1/a*f/(1+tan(1/2*d*x+1/2*c
)^2)/(tan(1/2*d*x+1/2*c)+1)*x/d*tan(1/2*d*x+1/2*c)^2+1/2/a*f/(1+tan(1/2*d*x
+1/2*c)^2)/(tan(1/2*d*x+1/2*c)+1)*x^2+1/2/a*f/(1+tan(1/2*d*x+1/2*c)^2)/(tan
(1/2*d*x+1/2*c)+1)*x^2*tan(1/2*d*x+1/2*c)+1/2/a*f/(1+tan(1/2*d*x+1/2*c)^2)/
(tan(1/2*d*x+1/2*c)+1)*x^2*tan(1/2*d*x+1/2*c)^2+1/2/a*f/(1+tan(1/2*d*x+1/2*
c)^2)/(tan(1/2*d*x+1/2*c)+1)*x^2*tan(1/2*d*x+1/2*c)^3-1/a*f/(1+tan(1/2*d*x+
1/2*c)^2)/(tan(1/2*d*x+1/2*c)+1)*x/d*tan(1/2*d*x+1/2*c)-1/a*f/(1+tan(1/2*d*
x+1/2*c)^2)/(tan(1/2*d*x+1/2*c)+1)*x/d*tan(1/2*d*x+1/2*c)^3+1/a*f/d^2*ln(1+
tan(1/2*d*x+1/2*c)^2)-2/a*f/d^2*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.47247, size = 369, normalized size = 4.86

$$-\frac{4\,cf\left(\frac{1}{ad+\frac{ad\sin(dx+c)}{\cos(dx+c)+1}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{ad}\right)-4\,e\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}}\right)-\frac{\left((dx+c)^2\cos(dx+c)^2+(dx+c)^2\sin(dx+c)^2+2\,(dx+c)^2}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(4*c*f*(1/(a*d + a*d*sin(d*x + c)/(cos(d*x + c) + 1)) + arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/(a*d)) - 4*e*(arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a + 1/(a + a*sin(d*x + c)/(cos(d*x + c) + 1))) - ((d*x + c)^2*cos(d*x
+ c)^2 + (d*x + c)^2*sin(d*x + c)^2 + 2*(d*x + c)^2*sin(d*x + c) + (d*x + c
)^2 + 4*(d*x + c)*cos(d*x + c) - 2*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin
(d*x + c) + 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1))*f
/(a*d*cos(d*x + c)^2 + a*d*sin(d*x + c)^2 + 2*a*d*sin(d*x + c) + a*d))/d
```

Fricas [B] time = 1.79171, size = 363, normalized size = 4.78

$$\frac{d^2fx^2+2\,de+2\left(d^2e+df\right)x+\left(d^2fx^2+2\,de+2\left(d^2e+df\right)x\right)\cos(dx+c)-2\left(f\cos(dx+c)+f\sin(dx+c)+f\right)\log}{2\left(ad^2\cos(dx+c)+ad^2\sin(dx+c)+ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(d^2*f*x^2 + 2*d*e + 2*(d^2*e + d*f)*x + (d^2*f*x^2 + 2*d*e + 2*(d^2*e
+ d*f)*x)*cos(d*x + c) - 2*(f*cos(d*x + c) + f*sin(d*x + c) + f)*log(sin(d*
x + c) + 1) + (d^2*f*x^2 - 2*d*e + 2*(d^2*e - d*f)*x)*sin(d*x + c))/(a*d^2*
cos(d*x + c) + a*d^2*sin(d*x + c) + a*d^2)
```

Sympy [A] time = 1.95445, size = 466, normalized size = 6.13

$$\begin{cases} \frac{2d^2ex\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2} + \frac{2d^2ex}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2} + \frac{d^2fx^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2} + \frac{d^2fx^2}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2} - \frac{4de\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2} - \frac{2dfx\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{2ad^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+} \\ \frac{\left(ex+\frac{fx^2}{2}\right)\sin(c)}{a\sin(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*d**2*e*x*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**
2) + 2*d**2*e*x/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + d**2*f*x**2*tan(c/
2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + d**2*f*x**2/(2*a*d**2*t
an(c/2 + d*x/2) + 2*a*d**2) - 4*d*e*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*
x/2) + 2*a*d**2) - 2*d*f*x*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2) + 2*
a*d**2) + 2*d*f*x/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 4*f*log(tan(c/2
+ d*x/2) + 1)*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 4*f
*log(tan(c/2 + d*x/2) + 1)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 2*f*log
(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a
*d**2) + 2*f*log(tan(c/2 + d*x/2)**2 + 1)/(2*a*d**2*tan(c/2 + d*x/2) + 2*a*
d**2), Ne(d, 0)), ((e*x + f*x**2/2)*sin(c)/(a*sin(c) + a), True))
```

Giac [B] time = 1.52989, size = 1042, normalized size = 13.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(d^2*f*x^2*tan(1/2*d*x)*tan(1/2*c) - d^2*f*x^2*tan(1/2*d*x) - d^2*f*x^2
*tan(1/2*c) + 2*d^2*x*e*tan(1/2*d*x)*tan(1/2*c) - d^2*f*x^2 - 2*d^2*x*e*tan
(1/2*d*x) - 2*d^2*x*e*tan(1/2*c) + 2*d*f*x*tan(1/2*d*x)*tan(1/2*c) - 2*d^2*
x*e + 2*d*f*x*tan(1/2*d*x) + 2*d*f*x*tan(1/2*c) + 2*d*e*tan(1/2*d*x)*tan(1/
2*c) - 2*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/
2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan
(1/2*d*x)*tan(1/2*c) - 2*d*f*x + 2*d*e*tan(1/2*d*x) + 2*f*log(2*(tan(1/2*c)
^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(
1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x) + 2*d*e*tan(1/2*c)
 + 2*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2
*c) - 2*d*e + 2*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4
 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) +
1)))/(a*d^2*tan(1/2*d*x)*tan(1/2*c) - a*d^2*tan(1/2*d*x) - a*d^2*tan(1/2*c)
 - a*d^2)
```

3.182 $\int \frac{\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{\cos(c+dx)}{d(a\sin(c+dx)+a)}+\frac{x}{a}$$

```
[Out] x/a + Cos[c + d*x]/(d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.0380084, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2735, 2648}

$$\frac{\cos(c+dx)}{d(a\sin(c+dx)+a)}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + Cos[c + d*x]/(d*(a + a*Sin[c + d*x]))
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{x}{a}-\int \frac{1}{a+a\sin(c+dx)}\,dx\\ &= \frac{x}{a}+\frac{\cos(c+dx)}{d(a+a\sin(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.108201, size = 72, normalized size = 2.57

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left((c+dx-2)\sin\left(\frac{1}{2}(c+dx)\right)+(c+dx)\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*((c + d*x)*Cos[(c + d*x)/2] + (-2 +
c + d*x)*Sin[(c + d*x)/2]))/(a*d*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.023, size = 41, normalized size = 1.5

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da}+2\,\frac{1}{da\left(\tan\left(1/2\,dx+c/2\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 2/a/d*arctan(tan(1/2*d*x+1/2*c))+2/a/d/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.42433, size = 68, normalized size = 2.43

$$\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*x + c)/(cos(d
*x + c) + 1)))/d
```

Fricas [A] time = 1.74005, size = 142, normalized size = 5.07

$$\frac{dx + (dx + 1)\cos(dx + c) + (dx - 1)\sin(dx + c) + 1}{ad\cos(dx + c) + ad\sin(dx + c) + ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x + (d*x + 1)*cos(d*x + c) + (d*x - 1)*sin(d*x + c) + 1)/(a*d*cos(d*x +
c) + a*d*sin(d*x + c) + a*d)
```

Sympy [A] time = 1.54983, size = 90, normalized size = 3.21

$$\begin{cases} \frac{dx\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{ad\tan\left(\frac{c}{2} + \frac{dx}{2}\right) + ad} + \frac{dx}{ad\tan\left(\frac{c}{2} + \frac{dx}{2}\right) + ad} - \frac{2\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{ad\tan\left(\frac{c}{2} + \frac{dx}{2}\right) + ad} & \text{for}\ d \neq 0 \\ \frac{x\sin(c)}{a\sin(c) + a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise(((d*x*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2) + a*d) + d*x/(a*d*tan
(c/2 + d*x/2) + a*d) - 2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2) + a*d), Ne(
d, 0)), (x*sin(c)/(a*sin(c) + a), True))
```

Giac [A] time = 1.12511, size = 43, normalized size = 1.54

$$\frac{\frac{dx+c}{a} + \frac{2}{a\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a + 2/(a*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.183 $\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sin(c+dx)}{(e+fx)(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Sin[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0480519, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Sin[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx=\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 8.57725, size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Sin[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 0.173, size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(fx+e)(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sin(dx+c)}{afx+ae+(afx+ae)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sin(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

3.184 $\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sin(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Sin[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0466772, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Sin[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 8.37545, size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Sin[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 0.238, size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(fx+e)^2 (a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(dx+c)}{af^2x^2+2\,aefx+ae^2+\left(af^2x^2+2\,aefx+ae^2\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e*f
*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

3.185 $\int \frac{(e+fx)^3 \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=247

$$-\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}+\frac{12f^3\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4}+\frac{6f^2(e+fx)\cos(c+dx)}{ad^3}+\frac{6f(e+fx)^2\log\left(1-ie^{i\ldots}\right.}{ad^2}$$

```
[Out] ((-I)*(e + f*x)^3)/(a*d) - (e + f*x)^4/(4*a*f) + (6*f^2*(e + f*x)*Cos[c + d
*x])/(a*d^3) - ((e + f*x)^3*Cos[c + d*x])/(a*d) - ((e + f*x)^3*Cot[c/2 + Pi
/4 + (d*x)/2])/(a*d) + (6*f*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*d^2)
 - ((12*I)*f^2*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) + (12*f^3*P
olyLog[3, I*E^(I*(c + d*x))])/(a*d^4) - (6*f^3*Sin[c + d*x])/(a*d^4) + (3*f
*(e + f*x)^2*Sin[c + d*x])/(a*d^2)
```

Rubi [A] time = 0.471918, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {4515, 3296, 2637, 32, 3318, 4184, 3717, 2190, 2531, 2282, 6589}

$$-\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}+\frac{12f^3\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4}+\frac{6f^2(e+fx)\cos(c+dx)}{ad^3}+\frac{6f(e+fx)^2\log\left(1-ie^{i\ldots}\right.}{ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```



```
[Out] ((-I)*(e + f*x)^3)/(a*d) - (e + f*x)^4/(4*a*f) + (6*f^2*(e + f*x)*Cos[c + d
*x])/(a*d^3) - ((e + f*x)^3*Cos[c + d*x])/(a*d) - ((e + f*x)^3*Cot[c/2 + Pi
/4 + (d*x)/2])/(a*d) + (6*f*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*d^2)
 - ((12*I)*f^2*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) + (12*f^3*P
olyLog[3, I*E^(I*(c + d*x))])/(a*d^4) - (6*f^3*Sin[c + d*x])/(a*d^4) + (3*f
*(e + f*x)^2*Sin[c + d*x])/(a*d^2)
```


Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3 \sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3 \sin(c+dx)\,dx}{a} - \int \frac{(e+fx)^3 \sin(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{\int (e+fx)^3\,dx}{a} + \frac{(3f)\int (e+fx)^2\cos(c+dx)\,dx}{ad} + \int \frac{(e+f}{a+a\sin(c}$$

$$= -\frac{(e+fx)^4}{4af} - \frac{(e+fx)^3\cos(c+dx)}{ad} + \frac{3f(e+fx)^2\sin(c+dx)}{ad^2} + \frac{\int (e+fx)^3\csc^2\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right.\right.}{2a}$$

$$= -\frac{(e+fx)^4}{4af} + \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} - \frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right.}{ad}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} + \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} - \frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{(e+fx)^3\,\mathrm{co}}{}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} + \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} - \frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{(e+fx)^3\,\mathrm{co}}{}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} + \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} - \frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{(e+fx)^3\,\mathrm{co}}{}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} + \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} - \frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{(e+fx)^3\,\mathrm{co}}{}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} + \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} - \frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{(e+fx)^3\,\mathrm{co}}{}$$

Mathematica [B] time = 3.00083, size = 1314, normalized size = 5.32

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-6 + 4*I)*d^3*e^3*Cos[(c + d*x)/2] + 6*d^2*e^2*f*Cos[(c + d*x)/2] + 12*d*
e*f^2*Cos[(c + d*x)/2] - 12*f^3*Cos[(c + d*x)/2] - 4*d^4*e^3*x*Cos[(c + d*x
)/2] - (18 - 12*I)*d^3*e^2*f*x*Cos[(c + d*x)/2] + 12*d^2*e*f^2*x*Cos[(c + d
*x)/2] + 12*d*f^3*x*Cos[(c + d*x)/2] - 6*d^4*e^2*f*x^2*Cos[(c + d*x)/2] - (
18 - 12*I)*d^3*e*f^2*x^2*Cos[(c + d*x)/2] + 6*d^2*f^3*x^2*Cos[(c + d*x)/2]
```

```
- 4*d^4*e*f^2*x^3*Cos[(c + d*x)/2] - (6 - 4*I)*d^3*f^3*x^3*Cos[(c + d*x)/2]
 - d^4*f^3*x^4*Cos[(c + d*x)/2] - 2*d^3*e^3*Cos[(3*(c + d*x))/2] - 6*d^2*e^
2*f*Cos[(3*(c + d*x))/2] + 12*d*e*f^2*Cos[(3*(c + d*x))/2] + 12*f^3*Cos[(3*
(c + d*x))/2] - 6*d^3*e^2*f*x*Cos[(3*(c + d*x))/2] - 12*d^2*e*f^2*x*Cos[(3*
(c + d*x))/2] + 12*d*f^3*x*Cos[(3*(c + d*x))/2] - 6*d^3*e*f^2*x^2*Cos[(3*(c
 + d*x))/2] - 6*d^2*f^3*x^2*Cos[(3*(c + d*x))/2] - 2*d^3*f^3*x^3*Cos[(3*(c
+ d*x))/2] + 24*d^2*e^2*f*Cos[(c + d*x)/2]*Log[1 + I*Cos[c + d*x] + Sin[c +
 d*x]] + 48*d^2*e*f^2*x*Cos[(c + d*x)/2]*Log[1 + I*Cos[c + d*x] + Sin[c + d
*x]] + 24*d^2*f^3*x^2*Cos[(c + d*x)/2]*Log[1 + I*Cos[c + d*x] + Sin[c + d*x
]] + (6 + 4*I)*d^3*e^3*Sin[(c + d*x)/2] + 6*d^2*e^2*f*Sin[(c + d*x)/2] - 12
*d*e*f^2*Sin[(c + d*x)/2] - 12*f^3*Sin[(c + d*x)/2] - 4*d^4*e^3*x*Sin[(c +
d*x)/2] + (18 + 12*I)*d^3*e^2*f*x*Sin[(c + d*x)/2] + 12*d^2*e*f^2*x*Sin[(c
+ d*x)/2] - 12*d*f^3*x*Sin[(c + d*x)/2] - 6*d^4*e^2*f*x^2*Sin[(c + d*x)/2]
+ (18 + 12*I)*d^3*e*f^2*x^2*Sin[(c + d*x)/2] + 6*d^2*f^3*x^2*Sin[(c + d*x)/
2] - 4*d^4*e*f^2*x^3*Sin[(c + d*x)/2] + (6 + 4*I)*d^3*f^3*x^3*Sin[(c + d*x)
/2] - d^4*f^3*x^4*Sin[(c + d*x)/2] + 24*d^2*e^2*f*Log[1 + I*Cos[c + d*x] +
Sin[c + d*x]]*Sin[(c + d*x)/2] + 48*d^2*e*f^2*x*Log[1 + I*Cos[c + d*x] + Si
n[c + d*x]]*Sin[(c + d*x)/2] + 24*d^2*f^3*x^2*Log[1 + I*Cos[c + d*x] + Sin[
c + d*x]]*Sin[(c + d*x)/2] + (48*I)*d*f^2*(e + f*x)*PolyLog[2, (-I)*Cos[c +
 d*x] - Sin[c + d*x]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 48*f^3*PolyLo
g[3, (-I)*Cos[c + d*x] - Sin[c + d*x]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
) - 2*d^3*e^3*Sin[(3*(c + d*x))/2] + 6*d^2*e^2*f*Sin[(3*(c + d*x))/2] + 12*
d*e*f^2*Sin[(3*(c + d*x))/2] - 12*f^3*Sin[(3*(c + d*x))/2] - 6*d^3*e^2*f*x*
Sin[(3*(c + d*x))/2] + 12*d^2*e*f^2*x*Sin[(3*(c + d*x))/2] + 12*d*f^3*x*Sin
[(3*(c + d*x))/2] - 6*d^3*e*f^2*x^2*Sin[(3*(c + d*x))/2] + 6*d^2*f^3*x^2*Si
n[(3*(c + d*x))/2] - 2*d^3*f^3*x^3*Sin[(3*(c + d*x))/2])/(4*a*d^4*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [B] time = 0.296, size = 748, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -2*I/d/a*f^3*x^3+4*I/d^4/a*f^3*c^3+12*f^2/d^2/a*e*ln(1-I*exp(I*(d*x+c)))*x-
1/4/a*f^3*x^4-1/a*e^3*x-6*f^3/d^4/a*c^2*ln(exp(I*(d*x+c)))+6*f/d^2/a*ln(exp
(I*(d*x+c))+I)*e^2+6*f^3/d^4/a*c^2*ln(exp(I*(d*x+c))+I)-6*f/d^2/a*ln(exp(I*
(d*x+c)))*e^2-1/a*e*f^2*x^3-3/2/a*e^2*f*x^2-2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*
x+e^3)/d/a/(exp(I*(d*x+c))+I)+12*f^2/d^3/a*e*ln(1-I*exp(I*(d*x+c)))*c+6*f^3
/d^2/a*ln(1-I*exp(I*(d*x+c)))*x^2-6*f^3/d^4/a*ln(1-I*exp(I*(d*x+c)))*c^2+12
```

```
*f^2/d^3/a*e*c*ln(exp(I*(d*x+c)))-12*f^2/d^3/a*e*c*ln(exp(I*(d*x+c))+I)-12*
I/d^3/a*f^3*polylog(2,I*exp(I*(d*x+c)))*x+6*I/d^3/a*f^3*c^2*x-6*I/d/a*e*f^2
*x^2-6*I/d^3/a*e*f^2*c^2-12*I/d^3/a*e*f^2*polylog(2,I*exp(I*(d*x+c)))-1/2*(
f^3*x^3*d^3-3*I*d^2*f^3*x^2+3*d^3*e*f^2*x^2-6*I*d^2*e*f^2*x+3*d^3*e^2*f*x-3
*I*d^2*e^2*f+d^3*e^3-6*d*f^3*x+6*I*f^3-6*f^2*e*d)/a/d^4*exp(-I*(d*x+c))-1/2
*(f^3*x^3*d^3+3*I*d^2*f^3*x^2+3*d^3*e*f^2*x^2+6*I*d^2*e*f^2*x+3*d^3*e^2*f*x
+3*I*d^2*e^2*f+d^3*e^3-6*d*f^3*x-6*I*f^3-6*f^2*e*d)/a/d^4*exp(I*(d*x+c))+12
*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4-12*I/d^2/a*e*f^2*c*x
```

Maxima [B] time = 3.0692, size = 6209, normalized size = 25.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(12*c^2*e*f^2*((sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 2)/(a*d^2 + a*d^2*sin(d*x + c)/(cos(d*x + c) + 1) + a*d^2
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*d^2*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d^2)) - 12*c*e^2*f*((si
n(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2)/(a
*d + a*d*sin(d*x + c)/(cos(d*x + c) + 1) + a*d*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + a*d*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + arctan(sin(d*x + c)/(c
os(d*x + c) + 1))/(a*d)) - 6*(((d*x + c)^2 - 1)*cos(d*x + c)^4 + ((d*x + c)
^2 - 1)*sin(d*x + c)^4 + ((d*x + c)*cos(d*x + c) + sin(d*x + c) + 1)*cos(2*
d*x + 2*c)^3 + 7*(d*x + c)*cos(d*x + c)^3 + (d*x + (d*x + c)*sin(d*x + c) +
 c - cos(d*x + c))*sin(2*d*x + 2*c)^3 + (2*(d*x + c)^2 - 3)*sin(d*x + c)^3
+ (((d*x + c)^2 - 1)*cos(d*x + c)^2 + ((d*x + c)^2 - 3)*sin(d*x + c)^2 + (d
*x + c)^2 + 6*(d*x + c)*cos(d*x + c) + 2*((d*x + c)^2 - (d*x + c)*cos(d*x +
 c) - 2)*sin(d*x + c) - 1)*cos(2*d*x + 2*c)^2 + ((d*x + c)^2 - 1)*cos(d*x +
 c)^2 + (((d*x + c)^2 - 3)*cos(d*x + c)^2 + ((d*x + c)^2 - 1)*sin(d*x + c)^
2 + (d*x + c)^2 + ((d*x + c)*cos(d*x + c) + sin(d*x + c) + 1)*cos(2*d*x + 2
*c) + 8*(d*x + c)*cos(d*x + c) + 2*((d*x + c)^2 + (d*x + c)*cos(d*x + c) -
1)*sin(d*x + c) - 1)*sin(2*d*x + 2*c)^2 + (2*((d*x + c)^2 - 1)*cos(d*x + c)
^2 + (d*x + c)^2 + 7*(d*x + c)*cos(d*x + c) - 3)*sin(d*x + c)^2 + ((d*x + c
)*cos(d*x + c)^3 - (2*(d*x + c)^2 - 3)*sin(d*x + c)^3 - (4*(d*x + c)^2 - (d
*x + c)*cos(d*x + c) - 6)*sin(d*x + c)^2 + 2*cos(d*x + c)^2 - ((2*(d*x + c)
^2 - 3)*cos(d*x + c)^2 + 2*(d*x + c)^2 + 12*(d*x + c)*cos(d*x + c) - 4)*sin
(d*x + c) + 1)*cos(2*d*x + 2*c) + (d*x + c)*cos(d*x + c) - 2*(cos(d*x + c)^
4 + sin(d*x + c)^4 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)
*cos(2*d*x + 2*c)^2 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1
```

)*sin(2*d*x + 2*c)^2 + 2*cos(d*x + c)^2*sin(d*x + c) + (2*cos(d*x + c)^2 + 1)*sin(d*x + c)^2 + 2*sin(d*x + c)^3 - 2*(sin(d*x + c)^3 + (cos(d*x + c)^2 + 1)*sin(d*x + c) + 2*sin(d*x + c)^2)*cos(2*d*x + 2*c) + cos(d*x + c)^2 + 2*(cos(d*x + c)^3 + cos(d*x + c)*sin(d*x + c)^2 + 2*cos(d*x + c)*sin(d*x + c) + cos(d*x + c))*sin(2*d*x + 2*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + ((2*(d*x + c)^2 - 3)*cos(d*x + c)^3 + (d*x + c)*sin(d*x + c)^3 + (d*x + (d*x + c)*sin(d*x + c) + c - cos(d*x + c))*cos(2*d*x + 2*c)^2 + 14*(d*x + c)*cos(d*x + c)^2 + (2*d*x + (2*(d*x + c)^2 - 3)*cos(d*x + c) + 2*c)*sin(d*x + c)^2 + d*x + 2*((d*x + c)*cos(d*x + c)^2 - (d*x + c)*sin(d*x + c)^2 - (d*x + c - 2*cos(d*x + c))*sin(d*x + c) + cos(d*x + c))*cos(2*d*x + 2*c) + 2*((d*x + c)^2 - 1)*cos(d*x + c) + ((d*x + c)*cos(d*x + c)^2 + 2*d*x + 4*((d*x + c)^2 - 1)*cos(d*x + c) + 2*c)*sin(d*x + c) + c)*sin(2*d*x + 2*c) + ((2*(d*x + c)^2 - 3)*cos(d*x + c)^2 + 2*(d*x + c)*cos(d*x + c) - 1)*sin(d*x + c))*c*e*f^2/(a*d^2*cos(d*x + c)^4 + a*d^2*sin(d*x + c)^4 + 2*a*d^2*cos(d*x + c)^2*sin(d*x + c) + 2*a*d^2*sin(d*x + c)^3 + a*d^2*cos(d*x + c)^2 + (a*d^2*cos(d*x + c)^2 + a*d^2*sin(d*x + c)^2 + 2*a*d^2*sin(d*x + c) + a*d^2)*cos(2*d*x + 2*c)^2 + (a*d^2*cos(d*x + c)^2 + a*d^2*sin(d*x + c)^2 + 2*a*d^2*sin(d*x + c) + a*d^2)*sin(2*d*x + 2*c)^2 + (2*a*d^2*cos(d*x + c)^2 + a*d^2)*sin(d*x + c)^2 - 2*(a*d^2*sin(d*x + c)^3 + 2*a*d^2*sin(d*x + c)^2 + (a*d^2*cos(d*x + c)^2 + a*d^2)*sin(d*x + c))*cos(2*d*x + 2*c) + 2*(a*d^2*cos(d*x + c)^3 + a*d^2*cos(d*x + c)*sin(d*x + c)^2 + 2*a*d^2*cos(d*x + c)*sin(d*x + c) + a*d^2*cos(d*x + c))*sin(2*d*x + 2*c)) + 4*e^3*((sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2)/(a + a*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a) + 3*(((d*x + c)^2 - 1)*cos(d*x + c)^4 + ((d*x + c)^2 - 1)*sin(d*x + c)^4 + ((d*x + c)*cos(d*x + c) + sin(d*x + c) + 1)*cos(2*d*x + 2*c)^3 + 7*(d*x + c)*cos(d*x + c)^3 + (d*x + (d*x + c)*sin(d*x + c) + c - cos(d*x + c))*sin(2*d*x + 2*c)^3 + (2*(d*x + c)^2 - 3)*sin(d*x + c)^3 + (((d*x + c)^2 - 1)*cos(d*x + c)^2 + ((d*x + c)^2 - 3)*sin(d*x + c)^2 + (d*x + c)^2 + 6*(d*x + c)*cos(d*x + c) + 2*((d*x + c)^2 - (d*x + c)*cos(d*x + c) - 2)*sin(d*x + c) - 1)*cos(2*d*x + 2*c)^2 + ((d*x + c)^2 - 1)*cos(d*x + c)^2 + (((d*x + c)^2 - 3)*cos(d*x + c)^2 + ((d*x + c)^2 - 1)*sin(d*x + c)^2 + (d*x + c)^2 + ((d*x + c)*cos(d*x + c) + sin(d*x + c) + 1)*cos(2*d*x + 2*c) + 8*(d*x + c)*cos(d*x + c) + 2*((d*x + c)^2 + (d*x + c)*cos(d*x + c) - 1)*sin(d*x + c) - 1)*sin(2*d*x + 2*c)^2 + (2*((d*x + c)^2 - 1)*cos(d*x + c)^2 + (d*x + c)^2 + 7*(d*x + c)*cos(d*x + c) - 3)*sin(d*x + c)^2 + ((d*x + c)*cos(d*x + c)^3 - (2*(d*x + c)^2 - 3)*sin(d*x + c)^3 - (4*(d*x + c)^2 - (d*x + c)*cos(d*x + c) - 6)*sin(d*x + c)^2 + 2*cos(d*x + c)^2 - ((2*(d*x + c)^2 - 3)*cos(d*x + c)^2 + 2*(d*x + c)^2 + 12*(d*x + c)*cos(d*x + c) - 4)*sin(d*x + c) + 1)*cos(2*d*x + 2*c) + (d*x + c)*cos(d*x + c) - 2*(cos(d*x + c)^4 + sin(d*x + c)^4 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)*cos(2*d*x + 2*c)^2 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)*sin(2*d*x + 2*c)^2 + 2*cos(d*x + c)^2*sin(d*x + c) + (2*cos(d*x + c)^2 + 1)*sin(d*x + c)^2 + 2*sin(d*x + c)^3 - 2*(sin(d*x + c)^3 + (cos(d*x + c)^2 + 1)*sin(d*x + c

```
) + 2*sin(d*x + c)^2)*cos(2*d*x + 2*c) + cos(d*x + c)^2 + 2*(cos(d*x + c)^3
 + cos(d*x + c)*sin(d*x + c)^2 + 2*cos(d*x + c)*sin(d*x + c) + cos(d*x + c)
)*sin(2*d*x + 2*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) +
1) + ((2*(d*x + c)^2 - 3)*cos(d*x + c)^3 + (d*x + c)*sin(d*x + c)^3 + (d*x
+ (d*x + c)*sin(d*x + c) + c - cos(d*x + c))*cos(2*d*x + 2*c)^2 + 14*(d*x +
 c)*cos(d*x + c)^2 + (2*d*x + (2*(d*x + c)^2 - 3)*cos(d*x + c) + 2*c)*sin(d
*x + c)^2 + d*x + 2*((d*x + c)*cos(d*x + c)^2 - (d*x + c)*sin(d*x + c)^2 -
(d*x + c - 2*cos(d*x + c))*sin(d*x + c) + cos(d*x + c))*cos(2*d*x + 2*c) +
2*((d*x + c)^2 - 1)*cos(d*x + c) + ((d*x + c)*cos(d*x + c)^2 + 2*d*x + 4*((
d*x + c)^2 - 1)*cos(d*x + c) + 2*c)*sin(d*x + c) + c)*sin(2*d*x + 2*c) + ((
2*(d*x + c)^2 - 3)*cos(d*x + c)^2 + 2*(d*x + c)*cos(d*x + c) - 1)*sin(d*x +
 c))*e^2*f/(a*d*cos(d*x + c)^4 + a*d*sin(d*x + c)^4 + 2*a*d*cos(d*x + c)^2*
sin(d*x + c) + 2*a*d*sin(d*x + c)^3 + a*d*cos(d*x + c)^2 + (a*d*cos(d*x + c
)^2 + a*d*sin(d*x + c)^2 + 2*a*d*sin(d*x + c) + a*d)*cos(2*d*x + 2*c)^2 + (
a*d*cos(d*x + c)^2 + a*d*sin(d*x + c)^2 + 2*a*d*sin(d*x + c) + a*d)*sin(2*d
*x + 2*c)^2 + (2*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c)^2 - 2*(a*d*sin(d*x
+ c)^3 + 2*a*d*sin(d*x + c)^2 + (a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c))*co
s(2*d*x + 2*c) + 2*(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)*sin(d*x + c)^2 +
2*a*d*cos(d*x + c)*sin(d*x + c) + a*d*cos(d*x + c))*sin(2*d*x + 2*c)) + 2*(
(d*x + c)^4*f^3 + (4*d*e*f^2 - (4*c + 2*I)*f^3)*(d*x + c)^3 + 12*I*d*e*f^2
- (-10*I*c^3 + 6*c^2 + 12*I*c - 12)*f^3 - (6*I*d*e*f^2 - 6*(c^2 + I*c - 1)*
f^3)*(d*x + c)^2 - (12*d*e*f^2 + (4*c^3 + 6*I*c^2 - 12*c - 12*I)*f^3)*(d*x
+ c) - (24*c^2*f^3*cos(d*x + c) + 24*I*c^2*f^3*sin(d*x + c) + 24*I*c^2*f^3)
*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - (-24*I*(d*x + c)^2*f^3 + (-48*I*
d*e*f^2 + 48*I*c*f^3)*(d*x + c) - 24*((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f^3)
*(d*x + c))*cos(d*x + c) + (-24*I*(d*x + c)^2*f^3 + (-48*I*d*e*f^2 + 48*I*c
*f^3)*(d*x + c))*sin(d*x + c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) - (2
*I*(d*x + c)^3*f^3 - 12*I*d*e*f^2 + (-2*I*c^3 - 6*c^2 + 12*I*c + 12)*f^3 -
6*(-I*d*e*f^2 + (I*c + 1)*f^3)*(d*x + c)^2 - (12*d*e*f^2 - (6*I*c^2 + 12*c
- 12*I)*f^3)*(d*x + c))*cos(2*d*x + 2*c) - (I*(d*x + c)^4*f^3 - 2*(-2*I*d*e
*f^2 + (2*I*c + 5)*f^3)*(d*x + c)^3 + 12*d*e*f^2 + (2*c^3 - 6*I*c^2 - 12*c
+ 12*I)*f^3 - (30*d*e*f^2 - (6*I*c^2 + 30*c - 6*I)*f^3)*(d*x + c)^2 + (-12*
I*d*e*f^2 + (-4*I*c^3 - 30*c^2 + 12*I*c + 12)*f^3)*(d*x + c))*cos(d*x + c)
- (-48*I*d*e*f^2 - 48*I*(d*x + c)*f^3 + 48*I*c*f^3 - 48*(d*e*f^2 + (d*x + c
)*f^3 - c*f^3)*cos(d*x + c) + (-48*I*d*e*f^2 - 48*I*(d*x + c)*f^3 + 48*I*c*
f^3)*sin(d*x + c))*dilog(I*e^(I*d*x + I*c)) - (12*(d*x + c)^2*f^3 + 12*c^2*
f^3 + 24*(d*e*f^2 - c*f^3)*(d*x + c) + (-12*I*(d*x + c)^2*f^3 - 12*I*c^2*f^
3 + (-24*I*d*e*f^2 + 24*I*c*f^3)*(d*x + c))*cos(d*x + c) + 12*((d*x + c)^2*
f^3 + c^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*sin(d*x + c))*log(cos(d*x +
c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 48*(I*f^3*cos(d*x + c) - f^3*
sin(d*x + c) - f^3)*polylog(3, I*e^(I*d*x + I*c)) + (2*(d*x + c)^3*f^3 - 12
*d*e*f^2 - (2*c^3 - 6*I*c^2 - 12*c + 12*I)*f^3 + (6*d*e*f^2 - (6*c - 6*I)*f
^3)*(d*x + c)^2 + 6*(2*I*d*e*f^2 + (c^2 - 2*I*c - 2)*f^3)*(d*x + c))*sin(2*
d*x + 2*c) + ((d*x + c)^4*f^3 + (4*d*e*f^2 - (4*c - 10*I)*f^3)*(d*x + c)^3
- 12*I*d*e*f^2 - (2*I*c^3 + 6*c^2 - 12*I*c - 12)*f^3 + 6*(5*I*d*e*f^2 + (c^
```

```
2 - 5*I*c - 1)*f^3)*(d*x + c)^2 - (12*d*e*f^2 + (4*c^3 - 30*I*c^2 - 12*c +
12*I)*f^3)*(d*x + c))*sin(d*x + c))/(-4*I*a*d^3*cos(d*x + c) + 4*a*d^3*sin(
d*x + c) + 4*a*d^3))/d
```

Fricas [C] time = 2.598, size = 2984, normalized size = 12.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(d^4*f^3*x^4 + 4*d^3*e^3 - 12*d^2*e^2*f + 4*(d^4*e*f^2 + d^3*f^3)*x^3
+ 24*f^3 + 6*(d^4*e^2*f + 2*d^3*e*f^2 - 2*d^2*f^3)*x^2 + 4*(d^3*f^3*x^3 + d
^3*e^3 + 3*d^2*e^2*f - 6*d*e*f^2 - 6*f^3 + 3*(d^3*e*f^2 + d^2*f^3)*x^2 + 3*
(d^3*e^2*f + 2*d^2*e*f^2 - 2*d*f^3)*x)*cos(d*x + c)^2 + 4*(d^4*e^3 + 3*d^3*
e^2*f - 6*d^2*e*f^2)*x + (d^4*f^3*x^4 + 8*d^3*e^3 - 24*d*e*f^2 + 4*(d^4*e*f
^2 + 2*d^3*f^3)*x^3 + 6*(d^4*e^2*f + 4*d^3*e*f^2)*x^2 + 4*(d^4*e^3 + 6*d^3*
e^2*f - 6*d*f^3)*x)*cos(d*x + c) - (-24*I*d*f^3*x - 24*I*d*e*f^2 + (-24*I*d
*f^3*x - 24*I*d*e*f^2)*cos(d*x + c) + (-24*I*d*f^3*x - 24*I*d*e*f^2)*sin(d*
x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) - (24*I*d*f^3*x + 24*I*d*e*f^2
 + (24*I*d*f^3*x + 24*I*d*e*f^2)*cos(d*x + c) + (24*I*d*f^3*x + 24*I*d*e*f^
2)*sin(d*x + c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) - 12*(d^2*e^2*f - 2*
c*d*e*f^2 + c^2*f^3 + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c) + (d
^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*
x + c) + I) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3 + (d^
2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c) + (d^2*f^3*
x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*sin(d*x + c))*log(I*cos(d*x +
c) + sin(d*x + c) + 1) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^
2*f^3 + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c)
+ (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*sin(d*x + c))*log(-
I*cos(d*x + c) + sin(d*x + c) + 1) - 12*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3
+ (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c) + (d^2*e^2*f - 2*c*d*e*f
^2 + c^2*f^3)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) - 24*(f
^3*cos(d*x + c) + f^3*sin(d*x + c) + f^3)*polylog(3, I*cos(d*x + c) - sin(d
*x + c)) - 24*(f^3*cos(d*x + c) + f^3*sin(d*x + c) + f^3)*polylog(3, -I*cos
(d*x + c) - sin(d*x + c)) + (d^4*f^3*x^4 - 4*d^3*e^3 - 12*d^2*e^2*f + 4*(d^
4*e*f^2 - d^3*f^3)*x^3 + 24*f^3 + 6*(d^4*e^2*f - 2*d^3*e*f^2 - 2*d^2*f^3)*x
^2 + 4*(d^4*e^3 - 3*d^3*e^2*f - 6*d^2*e*f^2)*x + 4*(d^3*f^3*x^3 + d^3*e^3 -
 3*d^2*e^2*f - 6*d*e*f^2 + 6*f^3 + 3*(d^3*e*f^2 - d^2*f^3)*x^2 + 3*(d^3*e^2
*f - 2*d^2*e*f^2 - 2*d*f^3)*x)*cos(d*x + c))*sin(d*x + c))/(a*d^4*cos(d*x +
 c) + a*d^4*sin(d*x + c) + a*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^3\sin^2(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{f^3x^3\sin^2(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{3ef^2x^2\sin^2(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{3e^2fx\sin^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**3*sin(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*
sin(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sin(c + d*x
)**2/(sin(c + d*x) + 1), x) + Integral(3*e**2*f*x*sin(c + d*x)**2/(sin(c +
d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3\sin(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

3.186 $\int \frac{(e+fx)^2 \sin^2(c+dx)}{a+a\sin(c+dx)}\, dx$

Optimal. Leaf size=188

$$-\frac{4if^2\text{PolyLog}\left(2, ie^{i(c+dx)}\right)}{ad^3}+\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{2f(e+fx)\sin(c+dx)}{ad^2}+\frac{2f^2\cos(c+dx)}{ad^3}-\frac{(e+fx)^2\cos}{ad}$$

```
[Out] ((-I)*(e + f*x)^2)/(a*d) - (e + f*x)^3/(3*a*f) + (2*f^2*Cos[c + d*x])/(a*d^
3) - ((e + f*x)^2*Cos[c + d*x])/(a*d) - ((e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)
/2])/(a*d) + (4*f*(e + f*x)*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) - ((4*I)*f^
2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) + (2*f*(e + f*x)*Sin[c + d*x])/(a*
d^2)
```

Rubi [A] time = 0.347819, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4515, 3296, 2638, 32, 3318, 4184, 3717, 2190, 2279, 2391}

$$-\frac{4if^2\text{PolyLog}\left(2, ie^{i(c+dx)}\right)}{ad^3}+\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{2f(e+fx)\sin(c+dx)}{ad^2}+\frac{2f^2\cos(c+dx)}{ad^3}-\frac{(e+fx)^2\cos}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```


```
[Out] ((-I)*(e + f*x)^2)/(a*d) - (e + f*x)^3/(3*a*f) + (2*f^2*Cos[c + d*x])/(a*d^
3) - ((e + f*x)^2*Cos[c + d*x])/(a*d) - ((e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)
/2])/(a*d) + (4*f*(e + f*x)*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) - ((4*I)*f^
2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) + (2*f*(e + f*x)*Sin[c + d*x])/(a*
d^2)
```


Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\sin(c+dx)\,dx}{a} - \int \frac{(e+fx)^2\sin(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{\int (e+fx)^2\,dx}{a} + \frac{(2f)\int (e+fx)\cos(c+dx)\,dx}{ad} + \int \frac{(e+f}{a+a\sin(}$$

$$= -\frac{(e+fx)^3}{3af} - \frac{(e+fx)^2\cos(c+dx)}{ad} + \frac{2f(e+fx)\sin(c+dx)}{ad^2} + \frac{\int (e+fx)^2\csc^2\left(\frac{1}{2}\left(c+\right.\right.}{2a}$$

$$= -\frac{(e+fx)^3}{3af} + \frac{2f^2\cos(c+dx)}{ad^3} - \frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{2}{}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} + \frac{2f^2\cos(c+dx)}{ad^3} - \frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{(e+fx)^2\cot\left(\frac{c}{2}+\right.}{ad}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} + \frac{2f^2\cos(c+dx)}{ad^3} - \frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{(e+fx)^2\cot\left(\frac{c}{2}+\right.}{ad}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} + \frac{2f^2\cos(c+dx)}{ad^3} - \frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{(e+fx)^2\cot\left(\frac{c}{2}+\right.}{ad}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} + \frac{2f^2\cos(c+dx)}{ad^3} - \frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{(e+fx)^2\cot\left(\frac{c}{2}+\right.}{ad}$$

Mathematica [A] time = 2.56786, size = 295, normalized size = 1.57

$$-\frac{\frac{12f(\cos(c)+i\sin(c))\left(\frac{f(\cos(c)-i(\sin(c)+1))\text{PolyLog}(2,-\sin(c+dx)-i\cos(c+dx))}{d^2}-\frac{(\sin(c)+i\cos(c)+1)(e+fx)\log(\sin(c+dx)+i\cos(c+dx)+1)}{d}+\frac{(\cos(c)-i\sin(c))(e+fx)^2}{2f}\right)}{d(\cos(c)+i(\sin(c)+1))}+\frac{3\cos}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x*(3*e^2 + 3*e*f*x + f^2*x^2) + (3*Cos[d*x]*((-2*f^2 + d^2*(e + f*x)^2)*C
os[c] - 2*d*f*(e + f*x)*Sin[c]))/d^3 + (12*f*(Cos[c] + I*Sin[c])*(((e + f*x
)^2*(Cos[c] - I*Sin[c]))/(2*f) - ((e + f*x)*Log[1 + I*Cos[c + d*x] + Sin[c
+ d*x]]*(1 + I*Cos[c] + Sin[c]))/d + (f*PolyLog[2, (-I)*Cos[c + d*x] - Sin[
c + d*x]]*(Cos[c] - I*(1 + Sin[c])))/d^2))/(d*(Cos[c] + I*(1 + Sin[c]))) -
(3*(2*d*f*(e + f*x)*Cos[c] + (-2*f^2 + d^2*(e + f*x)^2)*Sin[c])*Sin[d*x])/d
^3 - (6*(e + f*x)^2*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])))/(3*a)
```

Maple [B] time = 0.336, size = 408, normalized size = 2.2

$$-\frac{f^2x^3}{3a}-\frac{fex^2}{a}-\frac{e^2x}{a}-\frac{\left(f^2x^2d^2+2\,idf^2x+2\,d^2efx+2\,idef+d^2e^2-2\,f^2\right)\mathrm{e}^{i(dx+c)}}{2\,d^3a}-\frac{\left(f^2x^2d^2-2\,idf^2x+2\,d^2efx-2\,ide\right.}{2\,d^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3/a*f^2*x^3-1/a*f*e*x^2-1/a*e^2*x-1/2*(f^2*x^2*d^2+2*I*d*f^2*x+2*d^2*e*f
*x+2*I*d*e*f+d^2*e^2-2*f^2)/d^3/a*exp(I*(d*x+c))-1/2*(f^2*x^2*d^2-2*I*d*f^2
*x+2*d^2*e*f*x-2*I*d*e*f+d^2*e^2-2*f^2)/d^3/a*exp(-I*(d*x+c))-2*(f^2*x^2+2*
e*f*x+e^2)/d/a/(exp(I*(d*x+c))+I)+4*f/d^2/a*ln(exp(I*(d*x+c))+I)*e-4*f/d^2/
a*ln(exp(I*(d*x+c)))*e-4*I/d^2/a*f^2*c*x-2*I/d/a*f^2*x^2-4*I*f^2*polylog(2,
I*exp(I*(d*x+c)))/a/d^3+4*f^2/d^2/a*ln(1-I*exp(I*(d*x+c)))*x+4*f^2/d^3/a*ln
(1-I*exp(I*(d*x+c)))*c-2*I/d^3/a*f^2*c^2-4*f^2/d^3/a*c*ln(exp(I*(d*x+c))+I)
+4*f^2/d^3/a*c*ln(exp(I*(d*x+c)))
```

Maxima [B] time = 2.45847, size = 815, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*d^3*f^2*x^3 - 15*I*d^2*e^2 - 6*d*e*f + 3*(2*d^3*e*f - I*d^2*f^2)*x^2 +
6*I*f^2 + (6*d^3*e^2 - 6*I*d^2*e*f - 6*d*f^2)*x - (24*d*e*f*cos(d*x + c) +
24*I*d*e*f*sin(d*x + c) + 24*I*d*e*f)*arctan2(sin(d*x + c) + 1, cos(d*x + c
)) + (24*d*f^2*x*cos(d*x + c) + 24*I*d*f^2*x*sin(d*x + c) + 24*I*d*f^2*x)*a
rctan2(cos(d*x + c), sin(d*x + c) + 1) - (3*I*d^2*f^2*x^2 + 3*I*d^2*e^2 - 6
*d*e*f - 6*I*f^2 + (6*I*d^2*e*f - 6*d*f^2)*x)*cos(2*d*x + 2*c) - (2*I*d^3*f
^2*x^3 - 3*d^2*e^2 - 6*I*d*e*f + (6*I*d^3*e*f - 15*d^2*f^2)*x^2 + 6*f^2 + (
6*I*d^3*e^2 - 30*d^2*e*f - 6*I*d*f^2)*x)*cos(d*x + c) + (24*f^2*cos(d*x + c
) + 24*I*f^2*sin(d*x + c) + 24*I*f^2)*dilog(I*e^(I*d*x + I*c)) - (12*d*f^2*
x + 12*d*e*f + (-12*I*d*f^2*x - 12*I*d*e*f)*cos(d*x + c) + 12*(d*f^2*x + d*
e*f)*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1
) + (3*d^2*f^2*x^2 + 3*d^2*e^2 + 6*I*d*e*f - 6*f^2 + 6*(d^2*e*f + I*d*f^2)*
x)*sin(2*d*x + 2*c) + (2*d^3*f^2*x^3 + 3*I*d^2*e^2 - 6*d*e*f + 3*(2*d^3*e*f
 + 5*I*d^2*f^2)*x^2 - 6*I*f^2 + (6*d^3*e^2 + 30*I*d^2*e*f - 6*d*f^2)*x)*sin
(d*x + c))/(-6*I*a*d^3*cos(d*x + c) + 6*a*d^3*sin(d*x + c) + 6*a*d^3)
```

Fricas [B] time = 2.18301, size = 1674, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(d^3*f^2*x^3 + 3*d^2*e^2 - 6*d*e*f + 3*(d^3*e*f + d^2*f^2)*x^2 + 3*(d^
2*f^2*x^2 + d^2*e^2 + 2*d*e*f - 2*f^2 + 2*(d^2*e*f + d*f^2)*x)*cos(d*x + c)
^2 + 3*(d^3*e^2 + 2*d^2*e*f - 2*d*f^2)*x + (d^3*f^2*x^3 + 6*d^2*e^2 + 3*(d^
3*e*f + 2*d^2*f^2)*x^2 - 6*f^2 + 3*(d^3*e^2 + 4*d^2*e*f)*x)*cos(d*x + c) -
(-6*I*f^2*cos(d*x + c) - 6*I*f^2*sin(d*x + c) - 6*I*f^2)*dilog(I*cos(d*x +
c) - sin(d*x + c)) - (6*I*f^2*cos(d*x + c) + 6*I*f^2*sin(d*x + c) + 6*I*f^2
)*dilog(-I*cos(d*x + c) - sin(d*x + c)) - 6*(d*e*f - c*f^2 + (d*e*f - c*f^2
)*cos(d*x + c) + (d*e*f - c*f^2)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x
 + c) + I) - 6*(d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^2*x
 + c*f^2)*sin(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c) + 1) - 6*(d*f^2*x
 + c*f^2 + (d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^2*x + c*f^2)*sin(d*x + c))
*log(-I*cos(d*x + c) + sin(d*x + c) + 1) - 6*(d*e*f - c*f^2 + (d*e*f - c*f^
2)*cos(d*x + c) + (d*e*f - c*f^2)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d
*x + c) + I) + (d^3*f^2*x^3 - 3*d^2*e^2 - 6*d*e*f + 3*(d^3*e*f - d^2*f^2)*x
^2 + 3*(d^3*e^2 - 2*d^2*e*f - 2*d*f^2)*x + 3*(d^2*f^2*x^2 + d^2*e^2 - 2*d*e
*f - 2*f^2 + 2*(d^2*e*f - d*f^2)*x)*cos(d*x + c))*sin(d*x + c))/(a*d^3*cos(
d*x + c) + a*d^3*sin(d*x + c) + a*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^2\sin^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2\sin^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx\sin^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*sin(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*
sin(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sin(c + d*x)**2/(
sin(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2\sin(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

3.187 $\int \frac{(e+fx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=111

$$\frac{f\sin(c+dx)}{ad^2}+\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}-\frac{(e+fx)\cos(c+dx)}{ad}-\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{ex}{a}-\frac{fx^2}{2a}$$

```
[Out] -((e*x)/a) - (f*x^2)/(2*a) - ((e + f*x)*Cos[c + d*x])/(a*d) - ((e + f*x)*Co
t[c/2 + Pi/4 + (d*x)/2])/(a*d) + (2*f*Log[Sin[c/2 + Pi/4 + (d*x)/2]])/(a*d^
2) + (f*Sin[c + d*x])/(a*d^2)
```

Rubi [A] time = 0.159696, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4515, 3296, 2637, 3318, 4184, 3475}

$$\frac{f\sin(c+dx)}{ad^2}+\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}-\frac{(e+fx)\cos(c+dx)}{ad}-\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{ex}{a}-\frac{fx^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```


```
[Out] -((e*x)/a) - (f*x^2)/(2*a) - ((e + f*x)*Cos[c + d*x])/(a*d) - ((e + f*x)*Co
t[c/2 + Pi/4 + (d*x)/2])/(a*d) + (2*f*Log[Sin[c/2 + Pi/4 + (d*x)/2]])/(a*d^
2) + (f*Sin[c + d*x])/(a*d^2)
```


Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)\sin(c+dx)\,dx}{a}-\int \frac{(e+fx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx\\
&= -\frac{(e+fx)\cos(c+dx)}{ad}-\frac{\int (e+fx)\,dx}{a}+\frac{f\int \cos(c+dx)\,dx}{ad}+\int \frac{e+fx}{a+a\sin(c+dx)}\,dx\\
&= -\frac{ex}{a}-\frac{fx^2}{2a}-\frac{(e+fx)\cos(c+dx)}{ad}+\frac{f\sin(c+dx)}{ad^2}+\frac{\int (e+fx)\csc^2\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{dx}{2}\right)dx}{2a}\\
&= -\frac{ex}{a}-\frac{fx^2}{2a}-\frac{(e+fx)\cos(c+dx)}{ad}-\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}+\frac{f\sin(c+dx)}{ad^2}+\frac{f\int \cot\Big(
\end{aligned}
$$

$$
= -\frac{ex}{a}-\frac{fx^2}{2a}-\frac{(e+fx)\cos(c+dx)}{ad}-\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad}+\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right.}{ad^2}
$$

Mathematica [B] time = 0.787383, size = 236, normalized size = 2.13

$$
-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)\left(c^2(-f)+2d(e+fx)\cos(c+dx)+2cde-2f\sin(c+dx)-4f\log\right.\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Sin[(c + d*x)/2]*(-4*d*e + 2*c*d*e
 + 2*c*f - c^2*f + 2*d^2*e*x - 2*d*f*x + d^2*f*x^2 + 2*d*(e + f*x)*Cos[c +
d*x] - 4*f*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*f*Sin[c + d*x]) + C
os[(c + d*x)/2]*(2*c*d*e + 2*c*f - c^2*f + 2*d^2*e*x + 2*d*f*x + d^2*f*x^2
+ 2*d*(e + f*x)*Cos[c + d*x] - 4*f*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
 - 2*f*Sin[c + d*x])))/(2*a*d^2*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.116, size = 216, normalized size = 2.

$$-2\,\frac{e}{da\,(\tan(1/2\,dx+c/2)+1)}-\frac{fx}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{fx}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+2\,\frac{f\ln(\tan(1/2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/a*e/d/(tan(1/2*d*x+1/2*c)+1)-1/a/(tan(1/2*d*x+1/2*c)+1)/d*x*f+1/a/(tan(1
/2*d*x+1/2*c)+1)/d*x*f*tan(1/2*d*x+1/2*c)+2/a*f/d^2*ln(tan(1/2*d*x+1/2*c)+1
)-1/a*f/d^2*ln(1+tan(1/2*d*x+1/2*c)^2)-2/a*e/d/(1+tan(1/2*d*x+1/2*c)^2)-2/a
*e/d*arctan(tan(1/2*d*x+1/2*c))+f*sin(d*x+c)/a/d^2-1/a*f/d*cos(d*x+c)*x-1/2
*f*x^2/a+1/2/a*f/d^2*c^2
```

Maxima [B] time = 1.57833, size = 2379, normalized size = 21.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*c*f*((sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 2)/(a*d + a*d*sin(d*x + c)/(cos(d*x + c) + 1) + a*d*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + a*d*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + arctan(
sin(d*x + c)/(cos(d*x + c) + 1))/(a*d)) - 4*e*((sin(d*x + c)/(cos(d*x + c)
+ 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2)/(a + a*sin(d*x + c)/(cos(d*
```

```
x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a) - (((d*x + c
)^2 - 1)*cos(d*x + c)^4 + ((d*x + c)^2 - 1)*sin(d*x + c)^4 + ((d*x + c)*cos
(d*x + c) + sin(d*x + c) + 1)*cos(2*d*x + 2*c)^3 + 7*(d*x + c)*cos(d*x + c)
^3 + (d*x + (d*x + c)*sin(d*x + c) + c - cos(d*x + c))*sin(2*d*x + 2*c)^3 +
 (2*(d*x + c)^2 - 3)*sin(d*x + c)^3 + (((d*x + c)^2 - 1)*cos(d*x + c)^2 + (
(d*x + c)^2 - 3)*sin(d*x + c)^2 + (d*x + c)^2 + 6*(d*x + c)*cos(d*x + c) +
2*((d*x + c)^2 - (d*x + c)*cos(d*x + c) - 2)*sin(d*x + c) - 1)*cos(2*d*x +
2*c)^2 + ((d*x + c)^2 - 1)*cos(d*x + c)^2 + (((d*x + c)^2 - 3)*cos(d*x + c)
^2 + ((d*x + c)^2 - 1)*sin(d*x + c)^2 + (d*x + c)^2 + ((d*x + c)*cos(d*x +
c) + sin(d*x + c) + 1)*cos(2*d*x + 2*c) + 8*(d*x + c)*cos(d*x + c) + 2*((d*
x + c)^2 + (d*x + c)*cos(d*x + c) - 1)*sin(d*x + c) - 1)*sin(2*d*x + 2*c)^2
 + (2*((d*x + c)^2 - 1)*cos(d*x + c)^2 + (d*x + c)^2 + 7*(d*x + c)*cos(d*x
+ c) - 3)*sin(d*x + c)^2 + ((d*x + c)*cos(d*x + c)^3 - (2*(d*x + c)^2 - 3)*
sin(d*x + c)^3 - (4*(d*x + c)^2 - (d*x + c)*cos(d*x + c) - 6)*sin(d*x + c)^
2 + 2*cos(d*x + c)^2 - ((2*(d*x + c)^2 - 3)*cos(d*x + c)^2 + 2*(d*x + c)^2
+ 12*(d*x + c)*cos(d*x + c) - 4)*sin(d*x + c) + 1)*cos(2*d*x + 2*c) + (d*x
+ c)*cos(d*x + c) - 2*(cos(d*x + c)^4 + sin(d*x + c)^4 + (cos(d*x + c)^2 +
sin(d*x + c)^2 + 2*sin(d*x + c) + 1)*cos(2*d*x + 2*c)^2 + (cos(d*x + c)^2 +
 sin(d*x + c)^2 + 2*sin(d*x + c) + 1)*sin(2*d*x + 2*c)^2 + 2*cos(d*x + c)^2
*sin(d*x + c) + (2*cos(d*x + c)^2 + 1)*sin(d*x + c)^2 + 2*sin(d*x + c)^3 -
2*(sin(d*x + c)^3 + (cos(d*x + c)^2 + 1)*sin(d*x + c) + 2*sin(d*x + c)^2)*c
os(2*d*x + 2*c) + cos(d*x + c)^2 + 2*(cos(d*x + c)^3 + cos(d*x + c)*sin(d*x
 + c)^2 + 2*cos(d*x + c)*sin(d*x + c) + cos(d*x + c))*sin(2*d*x + 2*c))*log
(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + ((2*(d*x + c)^2 -
3)*cos(d*x + c)^3 + (d*x + c)*sin(d*x + c)^3 + (d*x + (d*x + c)*sin(d*x + c
) + c - cos(d*x + c))*cos(2*d*x + 2*c)^2 + 14*(d*x + c)*cos(d*x + c)^2 + (2
*d*x + (2*(d*x + c)^2 - 3)*cos(d*x + c) + 2*c)*sin(d*x + c)^2 + d*x + 2*((d
*x + c)*cos(d*x + c)^2 - (d*x + c)*sin(d*x + c)^2 - (d*x + c - 2*cos(d*x +
c))*sin(d*x + c) + cos(d*x + c))*cos(2*d*x + 2*c) + 2*((d*x + c)^2 - 1)*cos
(d*x + c) + ((d*x + c)*cos(d*x + c)^2 + 2*d*x + 4*((d*x + c)^2 - 1)*cos(d*x
 + c) + 2*c)*sin(d*x + c) + c)*sin(2*d*x + 2*c) + ((2*(d*x + c)^2 - 3)*cos(
d*x + c)^2 + 2*(d*x + c)*cos(d*x + c) - 1)*sin(d*x + c))*f/(a*d*cos(d*x + c
)^4 + a*d*sin(d*x + c)^4 + 2*a*d*cos(d*x + c)^2*sin(d*x + c) + 2*a*d*sin(d*
x + c)^3 + a*d*cos(d*x + c)^2 + (a*d*cos(d*x + c)^2 + a*d*sin(d*x + c)^2 +
2*a*d*sin(d*x + c) + a*d)*cos(2*d*x + 2*c)^2 + (a*d*cos(d*x + c)^2 + a*d*si
n(d*x + c)^2 + 2*a*d*sin(d*x + c) + a*d)*sin(2*d*x + 2*c)^2 + (2*a*d*cos(d*
x + c)^2 + a*d)*sin(d*x + c)^2 - 2*(a*d*sin(d*x + c)^3 + 2*a*d*sin(d*x + c)
^2 + (a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c))*cos(2*d*x + 2*c) + 2*(a*d*cos
(d*x + c)^3 + a*d*cos(d*x + c)*sin(d*x + c)^2 + 2*a*d*cos(d*x + c)*sin(d*x
+ c) + a*d*cos(d*x + c))*sin(2*d*x + 2*c)))/d
```

Fricas [B] time = 1.72685, size = 481, normalized size = 4.33

$$-\frac{d^2fx^2 + 2\left(dfx + de + f\right)\cos\left(dx + c\right)^2 + 2\,de + 2\left(d^2e + df\right)x + \left(d^2fx^2 + 4\,de + 2\left(d^2e + 2\,df\right)x\right)\cos\left(dx + c\right) - 2\left(f\,c\right.}{2\left(ad^2\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*f*x^2 + 2*(d*f*x + d*e + f)*cos(d*x + c)^2 + 2*d*e + 2*(d^2*e + d
*f)*x + (d^2*f*x^2 + 4*d*e + 2*(d^2*e + 2*d*f)*x)*cos(d*x + c) - 2*(f*cos(d
*x + c) + f*sin(d*x + c) + f)*log(sin(d*x + c) + 1) + (d^2*f*x^2 - 2*d*e +
2*(d^2*e - d*f)*x + 2*(d*f*x + d*e - f)*cos(d*x + c) - 2*f)*sin(d*x + c) -
2*f)/(a*d^2*cos(d*x + c) + a*d^2*sin(d*x + c) + a*d^2)
```

Sympy [A] time = 4.42202, size = 2086, normalized size = 18.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-2*d**2*e*x*tan(c/2 + d*x/2)**3/(2*a*d**2*tan(c/2 + d*x/2)**3 +
2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*d*
*2*e*x*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2
 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*d**2*e*x*tan(c/2 +
 d*x/2)/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*
d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*d**2*e*x/(2*a*d**2*tan(c/2 + d*x/2)**
3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) -
d**2*f*x**2*tan(c/2 + d*x/2)**3/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*ta
n(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - d**2*f*x**2*tan
(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)*
*2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - d**2*f*x**2*tan(c/2 + d*x/2)/(
2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(
c/2 + d*x/2) + 2*a*d**2) - d**2*f*x**2/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*
d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 6*d*e*ta
n(c/2 + d*x/2)**3/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)
**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 2*d*e*tan(c/2 + d*x/2)**2/(2*
a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/
2 + d*x/2) + 2*a*d**2) + 2*d*e*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2)*
```

```
*3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) -
 2*d*e/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d
**2*tan(c/2 + d*x/2) + 2*a*d**2) + 4*d*f*x*tan(c/2 + d*x/2)**3/(2*a*d**2*ta
n(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2
) + 2*a*d**2) - 4*d*f*x/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 +
d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 4*f*log(tan(c/2 + d*x/2
) + 1)*tan(c/2 + d*x/2)**3/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2
 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 4*f*log(tan(c/2 + d*
x/2) + 1)*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(
c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 4*f*log(tan(c/2 +
 d*x/2) + 1)*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(
c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 4*f*log(tan(c/2 +
 d*x/2) + 1)/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 +
 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*f*log(tan(c/2 + d*x/2)**2 + 1)*t
an(c/2 + d*x/2)**3/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2
)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*f*log(tan(c/2 + d*x/2)**2
+ 1)*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 +
 d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*f*log(tan(c/2 + d*x/
2)**2 + 1)*tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/
2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*f*log(tan(c/2 + d
*x/2)**2 + 1)/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2
+ 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*f*tan(c/2 + d*x/2)**3/(2*a*d**2
*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*
x/2) + 2*a*d**2) + 2*f*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**3 +
2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) + 2*f*
tan(c/2 + d*x/2)/(2*a*d**2*tan(c/2 + d*x/2)**3 + 2*a*d**2*tan(c/2 + d*x/2)*
*2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2) - 2*f/(2*a*d**2*tan(c/2 + d*x/2)
**3 + 2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2*tan(c/2 + d*x/2) + 2*a*d**2),
 Ne(d, 0)), ((e*x + f*x**2/2)*sin(c)**2/(a*sin(c) + a), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.188 $\int \frac{\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=45

$$-\frac{\cos(c+dx)}{ad}-\frac{\cos(c+dx)}{ad(\sin(c+dx)+1)}-\frac{x}{a}$$

```
[Out] -(x/a) - Cos[c + d*x]/(a*d) - Cos[c + d*x]/(a*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.0815769, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2746, 12, 2735, 2648}

$$-\frac{\cos(c+dx)}{ad}-\frac{\cos(c+dx)}{ad(\sin(c+dx)+1)}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```


```
[Out] -(x/a) - Cos[c + d*x]/(a*d) - Cos[c + d*x]/(a*d*(1 + Sin[c + d*x]))
```


Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\cos(c+dx)}{ad}-\frac{\int \frac{a\sin(c+dx)}{a+a\sin(c+dx)}\,dx}{a}\\ &= -\frac{\cos(c+dx)}{ad}-\int \frac{\sin(c+dx)}{a+a\sin(c+dx)}\,dx\\ &= -\frac{x}{a}-\frac{\cos(c+dx)}{ad}+\int \frac{1}{a+a\sin(c+dx)}\,dx\\ &= -\frac{x}{a}-\frac{\cos(c+dx)}{ad}-\frac{\cos(c+dx)}{d(a+a\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.139283, size = 85, normalized size = 1.89

$$-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+c+dx)+\sin\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+c+dx-2)\right.}{ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x)/2]*(c + d*x + Cos[c
 + d*x]) + (-2 + c + d*x + Cos[c + d*x])*Sin[(c + d*x)/2]))/(a*d*(1 + Sin[c
 + d*x])))
```

Maple [A] time = 0.026, size = 64, normalized size = 1.4

$$-2\,\frac{1}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)}-2\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da}-2\,\frac{1}{da\,(\tan(1/2\,dx+c/2)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/a/d/(1+tan(1/2*d*x+1/2*c)^2)-2/a/d*arctan(tan(1/2*d*x+1/2*c))-2/a/d/(tan
(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.45721, size = 174, normalized size = 3.87

$$-\frac{2\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+2}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*((sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 2)/(a + a*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.66073, size = 186, normalized size = 4.13

$$-\frac{dx+(dx+2)\cos(dx+c)+\cos(dx+c)^2+(dx+\cos(dx+c)-1)\sin(dx+c)+1}{ad\cos(dx+c)+ad\sin(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d*x + (d*x + 2)*cos(d*x + c) + cos(d*x + c)^2 + (d*x + cos(d*x + c) - 1)*
sin(d*x + c) + 1)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [A] time = 3.37656, size = 478, normalized size = 10.62

$$\begin{cases} -\frac{dx\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{dx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a} \\ \frac{x\sin^2(c)}{a\sin(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-d*x*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**3 + a*d*tan(c/2
+ d*x/2)**2 + a*d*tan(c/2 + d*x/2) + a*d) - d*x*tan(c/2 + d*x/2)**2/(a*d*ta
n(c/2 + d*x/2)**3 + a*d*tan(c/2 + d*x/2)**2 + a*d*tan(c/2 + d*x/2) + a*d) -
 d*x*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**3 + a*d*tan(c/2 + d*x/2)**2 +
a*d*tan(c/2 + d*x/2) + a*d) - d*x/(a*d*tan(c/2 + d*x/2)**3 + a*d*tan(c/2 +
d*x/2)**2 + a*d*tan(c/2 + d*x/2) + a*d) + 3*tan(c/2 + d*x/2)**3/(a*d*tan(c/
2 + d*x/2)**3 + a*d*tan(c/2 + d*x/2)**2 + a*d*tan(c/2 + d*x/2) + a*d) + tan
(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**3 + a*d*tan(c/2 + d*x/2)**2 + a*d*t
an(c/2 + d*x/2) + a*d) + tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**3 + a*d*ta
n(c/2 + d*x/2)**2 + a*d*tan(c/2 + d*x/2) + a*d) - 1/(a*d*tan(c/2 + d*x/2)**
3 + a*d*tan(c/2 + d*x/2)**2 + a*d*tan(c/2 + d*x/2) + a*d), Ne(d, 0)), (x*si
n(c)**2/(a*sin(c) + a), True))
```

Giac [A] time = 1.10098, size = 104, normalized size = 2.31

$$-\frac{\frac{dx+c}{a}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a + 2*(tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) + 2)/((tan
(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) + 1)*a)
)/d
```

3.189 $\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sin^2(c+dx)}{(e+fx)(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Sin[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.071648, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Sin[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 8.56318, size = 0, normalized size = 0.

$$\int \frac{\sin^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Sin[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 0.369, size = 0, normalized size = 0.

$$\int \frac{(\sin (dx + c))^2}{(fx + e)(a + a \sin (dx + c))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\cos (dx + c)^2 - 1}{afx + ae + (afx + ae)\sin (dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^2}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^2/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

3.190 $\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sin^2(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Sin[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0690869, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Sin[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 9.67415, size = 0, normalized size = 0.

$$\int \frac{\sin^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Sin[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 0.639, size = 0, normalized size = 0.

$$\int \frac{(\sin (dx+c))^{2}}{\left(fx+e\right)^{2} (a+a\sin (dx+c))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\cos (dx+c)^{2}-1}{af^{2}x^{2}+2\,aefx+ae^{2}+\left(af^{2}x^{2}+2\,aefx+ae^{2}\right)\sin (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2
+ 2*a*e*f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^2}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^2/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

3.191 $\int \frac{(e+fx)^3 \sin^3(c+dx)}{a+a\sin(c+dx)} dx$

Optimal. Leaf size=382

$$\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{12f^3\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4}-\frac{6f^2(e+fx)\cos(c+dx)}{ad^3}+\frac{3f^2(e+fx)\sin(c+dx)\,\text{c}}{4ad^3}$$

```
[Out] (-3*e*f^2*x)/(4*a*d^2) - (3*f^3*x^2)/(8*a*d^2) + (I*(e + f*x)^3)/(a*d) + (3
*(e + f*x)^4)/(8*a*f) - (6*f^2*(e + f*x)*Cos[c + d*x])/(a*d^3) + ((e + f*x)
^3*Cos[c + d*x])/(a*d) + ((e + f*x)^3*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - (6
*f*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) + ((12*I)*f^2*(e + f*x)*
PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - (12*f^3*PolyLog[3, I*E^(I*(c + d*x
))])/(a*d^4) + (6*f^3*Sin[c + d*x])/(a*d^4) - (3*f*(e + f*x)^2*Sin[c + d*x]
)/(a*d^2) + (3*f^2*(e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(4*a*d^3) - ((e + f
*x)^3*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - (3*f^3*Sin[c + d*x]^2)/(8*a*d^4)
 + (3*f*(e + f*x)^2*Sin[c + d*x]^2)/(4*a*d^2)
```

Rubi [A] time = 0.621256, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {4515, 3311, 32, 3310, 3296, 2637, 3318, 4184, 3717, 2190, 2531, 2282, 6589}

$$\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{12f^3\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^4}-\frac{6f^2(e+fx)\cos(c+dx)}{ad^3}+\frac{3f^2(e+fx)\sin(c+dx)\,\text{c}}{4ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-3*e*f^2*x)/(4*a*d^2) - (3*f^3*x^2)/(8*a*d^2) + (I*(e + f*x)^3)/(a*d) + (3
*(e + f*x)^4)/(8*a*f) - (6*f^2*(e + f*x)*Cos[c + d*x])/(a*d^3) + ((e + f*x)
^3*Cos[c + d*x])/(a*d) + ((e + f*x)^3*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - (6
*f*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) + ((12*I)*f^2*(e + f*x)*
PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - (12*f^3*PolyLog[3, I*E^(I*(c + d*x
))])/(a*d^4) + (6*f^3*Sin[c + d*x])/(a*d^4) - (3*f*(e + f*x)^2*Sin[c + d*x]
)/(a*d^2) + (3*f^2*(e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(4*a*d^3) - ((e + f
*x)^3*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - (3*f^3*Sin[c + d*x]^2)/(8*a*d^4)
 + (3*f*(e + f*x)^2*Sin[c + d*x]^2)/(4*a*d^2)
```

Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
```

```
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\sin^2(c+dx)\,dx}{a} - \int \frac{(e+fx)^3\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{(e+fx)^3\cos(c+dx)\sin(c+dx)}{2ad} + \frac{3f(e+fx)^2\sin^2(c+dx)}{4ad^2} + \frac{\int (e+fx)^3\,dx}{2a} - \frac{\int (e+f$$

$$= \frac{(e+fx)^4}{8af} + \frac{(e+fx)^3\cos(c+dx)}{ad} + \frac{3f^2(e+fx)\cos(c+dx)\sin(c+dx)}{4ad^3} - \frac{(e+fx)^3\cos(c$$

$$= -\frac{3ef^2x}{4ad^2} - \frac{3f^3x^2}{8ad^2} + \frac{3(e+fx)^4}{8af} + \frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{3f(e+fx)^2\sin(c+dx)}{ad^2} + \frac{3f^2(e$$

$$= -\frac{3ef^2x}{4ad^2} - \frac{3f^3x^2}{8ad^2} + \frac{3(e+fx)^4}{8af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} + \frac{(e+fx)^3\cos(c+dx)}{ad} + \frac{(e+$$

$$= -\frac{3ef^2x}{4ad^2} - \frac{3f^3x^2}{8ad^2} + \frac{i(e+fx)^3}{ad} + \frac{3(e+fx)^4}{8af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} + \frac{(e+fx)^3\cos(c-$$

$$= -\frac{3ef^2x}{4ad^2} - \frac{3f^3x^2}{8ad^2} + \frac{i(e+fx)^3}{ad} + \frac{3(e+fx)^4}{8af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} + \frac{(e+fx)^3\cos(c-$$

$$= -\frac{3ef^2x}{4ad^2} - \frac{3f^3x^2}{8ad^2} + \frac{i(e+fx)^3}{ad} + \frac{3(e+fx)^4}{8af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} + \frac{(e+fx)^3\cos(c-$$

$$= -\frac{3ef^2x}{4ad^2} - \frac{3f^3x^2}{8ad^2} + \frac{i(e+fx)^3}{ad} + \frac{3(e+fx)^4}{8af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} + \frac{(e+fx)^3\cos(c-$$

$$= -\frac{3ef^2x}{4ad^2} - \frac{3f^3x^2}{8ad^2} + \frac{i(e+fx)^3}{ad} + \frac{3(e+fx)^4}{8af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} + \frac{(e+fx)^3\cos(c-$$

Mathematica [A] time = 2.62818, size = 538, normalized size = 1.41

$$\frac{192f(\cos(c)+i\sin(c))\left(\frac{\frac{2f(\cos(c)-i(\sin(c)+1))(d(e+fx)\text{PolyLog}(2,-\sin(c+dx)-i\cos(c+dx))-if\text{PolyLog}(3,-\sin(c+dx)-i\cos(c+dx)))}{d^3}-\frac{(\sin(c)+i\cos(c)+1)(e+fx)^2\log(\sin(c+dx)+i\cos(c}{d}}{d(\cos(c)+i(\sin(c)+1))}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (48*e^3*x + 72*e^2*f*x^2 + 48*e*f^2*x^3 + 12*f^3*x^4 + (192*f*(Cos[c] + I*S
in[c])*(((e + f*x)^3*(Cos[c] - I*Sin[c]))/(3*f) - ((e + f*x)^2*Log[1 + I*Co
s[c + d*x] + Sin[c + d*x]]*(1 + I*Cos[c] + Sin[c]))/d + (2*f*(d*(e + f*x)*P
olyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]] - I*f*PolyLog[3, (-I)*Cos[c + d
*x] - Sin[c + d*x]])*(Cos[c] - I*(1 + Sin[c])))/d^3))/(d*(Cos[c] + I*(1 + S
in[c]))) - (64*(e + f*x)^3*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])) + (16*((6*I)*f^3 - 6*d*f^2*(e + f*x) - (3*I)*
d^2*f*(e + f*x)^2 + d^3*(e + f*x)^3)*(Cos[c + d*x] - I*Sin[c + d*x]))/d^4 +
 (16*((-6*I)*f^3 - 6*d*f^2*(e + f*x) + (3*I)*d^2*f*(e + f*x)^2 + d^3*(e + f
*x)^3)*(Cos[c + d*x] + I*Sin[c + d*x]))/d^4 + ((3*f^3 + (6*I)*d*f^2*(e + f*
x) - 6*d^2*f*(e + f*x)^2 - (4*I)*d^3*(e + f*x)^3)*(Cos[2*(c + d*x)] - I*Sin
[2*(c + d*x)]))/d^4 + ((3*f^3 - (6*I)*d*f^2*(e + f*x) - 6*d^2*f*(e + f*x)^2
 + (4*I)*d^3*(e + f*x)^3)*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]))/d^4)/(32
*a)
```

Maple [B] time = 0.18, size = 974, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -12*f^2/d^2/a*e*ln(1-I*exp(I*(d*x+c)))*x+3/8/a*f^3*x^4+3/2/a*e^3*x+6*f^3/d^
4/a*c^2*ln(exp(I*(d*x+c)))-6*f/d^2/a*ln(exp(I*(d*x+c))+I)*e^2-6*f^3/d^4/a*c
^2*ln(exp(I*(d*x+c))+I)-4*I*f^3/d^4/a*c^3+2*I*f^3/d/a*x^3+6*f/d^2/a*ln(exp(
I*(d*x+c)))*e^2+3/2/a*e*f^2*x^3+9/4/a*e^2*f*x^2+1/32*I*(4*f^3*x^3*d^3+6*I*d
^2*f^3*x^2+12*d^3*e*f^2*x^2+12*I*d^2*e*f^2*x+12*d^3*e^2*f*x+6*I*d^2*e^2*f+4
*d^3*e^3-6*d*f^3*x-3*I*f^3-6*f^2*e*d)/a/d^4*exp(2*I*(d*x+c))-1/32*I*(4*f^3*
x^3*d^3-6*I*d^2*f^3*x^2+12*d^3*e*f^2*x^2-12*I*d^2*e*f^2*x+12*d^3*e^2*f*x-6*
I*d^2*e^2*f+4*d^3*e^3-6*d*f^3*x+3*I*f^3-6*f^2*e*d)/a/d^4*exp(-2*I*(d*x+c))+
2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(I*(d*x+c))+I)+12*I*f^2/d^2/a
*e*c*x-12*f^2/d^3/a*e*ln(1-I*exp(I*(d*x+c)))*c-6*f^3/d^2/a*ln(1-I*exp(I*(d*
x+c)))*x^2+6*f^3/d^4/a*ln(1-I*exp(I*(d*x+c)))*c^2-12*f^2/d^3/a*e*c*ln(exp(I
*(d*x+c)))+12*f^2/d^3/a*e*c*ln(exp(I*(d*x+c))+I)-6*I*f^3/d^3/a*c^2*x+12*I*f
^3/d^3/a*polylog(2,I*exp(I*(d*x+c)))*x+12*I*f^2/d^3/a*e*polylog(2,I*exp(I*(
d*x+c)))+6*I*f^2/d/a*e*x^2+6*I*f^2/d^3/a*e*c^2+1/2*(f^3*x^3*d^3-3*I*d^2*f^3
*x^2+3*d^3*e*f^2*x^2-6*I*d^2*e*f^2*x+3*d^3*e^2*f*x-3*I*d^2*e^2*f+d^3*e^3-6*
d*f^3*x+6*I*f^3-6*f^2*e*d)/a/d^4*exp(-I*(d*x+c))+1/2*(f^3*x^3*d^3+3*I*d^2*f
^3*x^2+3*d^3*e*f^2*x^2+6*I*d^2*e*f^2*x+3*d^3*e^2*f*x+3*I*d^2*e^2*f+d^3*e^3-
```

```
6*d*f^3*x-6*I*f^3-6*f^2*e*d)/a/d^4*exp(I*(d*x+c))-12*f^3*polylog(3,I*exp(I*
(d*x+c)))/a/d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [C] time = 2.78325, size = 3534, normalized size = 9.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(6*d^4*f^3*x^4 + 16*d^3*e^3 - 42*d^2*e^2*f + 8*(3*d^4*e*f^2 + 2*d^3*f^
3)*x^3 + 2*(4*d^3*f^3*x^3 + 4*d^3*e^3 - 6*d^2*e^2*f - 6*d*e*f^2 + 3*f^3 + 6
*(2*d^3*e*f^2 - d^2*f^3)*x^2 + 6*(2*d^3*e^2*f - 2*d^2*e*f^2 - d*f^3)*x)*cos
(d*x + c)^3 + 93*f^3 + 6*(6*d^4*e^2*f + 8*d^3*e*f^2 - 7*d^2*f^3)*x^2 + 2*(8
*d^3*f^3*x^3 + 8*d^3*e^3 + 18*d^2*e^2*f - 48*d*e*f^2 - 45*f^3 + 6*(4*d^3*e*
f^2 + 3*d^2*f^3)*x^2 + 12*(2*d^3*e^2*f + 3*d^2*e*f^2 - 4*d*f^3)*x)*cos(d*x
+ c)^2 + 12*(2*d^4*e^3 + 4*d^3*e^2*f - 7*d^2*e*f^2)*x + 3*(2*d^4*f^3*x^4 +
8*d^3*e^3 + 2*d^2*e^2*f - 28*d*e*f^2 + 8*(d^4*e*f^2 + d^3*f^3)*x^3 - f^3 +
2*(6*d^4*e^2*f + 12*d^3*e*f^2 + d^2*f^3)*x^2 + 4*(2*d^4*e^3 + 6*d^3*e^2*f +
 d^2*e*f^2 - 7*d*f^3)*x)*cos(d*x + c) + (96*I*d*f^3*x + 96*I*d*e*f^2 + (96*
I*d*f^3*x + 96*I*d*e*f^2)*cos(d*x + c) + (96*I*d*f^3*x + 96*I*d*e*f^2)*sin(
d*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) + (-96*I*d*f^3*x - 96*I*d*e*
f^2 + (-96*I*d*f^3*x - 96*I*d*e*f^2)*cos(d*x + c) + (-96*I*d*f^3*x - 96*I*d
*e*f^2)*sin(d*x + c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) - 48*(d^2*e^2*f
 - 2*c*d*e*f^2 + c^2*f^3 + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c)
 + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*sin(d*x + c))*log(cos(d*x + c) + I*s
in(d*x + c) + I) - 48*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3
+ (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c) + (d^2
*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*sin(d*x + c))*log(I*cos(d
```

```
*x + c) + sin(d*x + c) + 1) - 48*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2
 - c^2*f^3 + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x
+ c) + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*sin(d*x + c))*
log(-I*cos(d*x + c) + sin(d*x + c) + 1) - 48*(d^2*e^2*f - 2*c*d*e*f^2 + c^2
*f^3 + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c) + (d^2*e^2*f - 2*c*
d*e*f^2 + c^2*f^3)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) -
96*(f^3*cos(d*x + c) + f^3*sin(d*x + c) + f^3)*polylog(3, I*cos(d*x + c) -
sin(d*x + c)) - 96*(f^3*cos(d*x + c) + f^3*sin(d*x + c) + f^3)*polylog(3, -
I*cos(d*x + c) - sin(d*x + c)) + (6*d^4*f^3*x^4 - 16*d^3*e^3 - 42*d^2*e^2*f
 + 8*(3*d^4*e*f^2 - 2*d^3*f^3)*x^3 + 93*f^3 + 6*(6*d^4*e^2*f - 8*d^3*e*f^2
- 7*d^2*f^3)*x^2 - 2*(4*d^3*f^3*x^3 + 4*d^3*e^3 + 6*d^2*e^2*f - 6*d*e*f^2 -
 3*f^3 + 6*(2*d^3*e*f^2 + d^2*f^3)*x^2 + 6*(2*d^3*e^2*f + 2*d^2*e*f^2 - d*f
^3)*x)*cos(d*x + c)^2 + 12*(2*d^4*e^3 - 4*d^3*e^2*f - 7*d^2*e*f^2)*x + 4*(2
*d^3*f^3*x^3 + 2*d^3*e^3 - 12*d^2*e^2*f - 21*d*e*f^2 + 24*f^3 + 6*(d^3*e*f^
2 - 2*d^2*f^3)*x^2 + 3*(2*d^3*e^2*f - 8*d^2*e*f^2 - 7*d*f^3)*x)*cos(d*x + c
))*sin(d*x + c))/(a*d^4*cos(d*x + c) + a*d^4*sin(d*x + c) + a*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^3\sin^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3\sin^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2\sin^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx\sin^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**3*sin(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*
sin(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sin(c + d*x
)**3/(sin(c + d*x) + 1), x) + Integral(3*e**2*f*x*sin(c + d*x)**3/(sin(c +
d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3\sin(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

3.192 $\int \frac{(e+fx)^2 \sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=278

$$\frac{4if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{f(e+fx)\sin^2(c+dx)}{2ad^2}-\frac{2f(e+fx)\sin(c+dx)}{ad^2}-\frac{2f^2\cos(}{ad}$$

```
[Out] -(f^2*x)/(4*a*d^2) + (I*(e + f*x)^2)/(a*d) + (e + f*x)^3/(2*a*f) - (2*f^2*C
os[c + d*x])/(a*d^3) + ((e + f*x)^2*Cos[c + d*x])/(a*d) + ((e + f*x)^2*Cot[
c/2 + Pi/4 + (d*x)/2])/(a*d) - (4*f*(e + f*x)*Log[1 - I*E^(I*(c + d*x))])/(
a*d^2) + ((4*I)*f^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - (2*f*(e + f*x)
*Sin[c + d*x])/(a*d^2) + (f^2*Cos[c + d*x]*Sin[c + d*x])/(4*a*d^3) - ((e +
f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) + (f*(e + f*x)*Sin[c + d*x]^2)/(2
*a*d^2)
```

Rubi [A] time = 0.492938, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {4515, 3311, 32, 2635, 8, 3296, 2638, 3318, 4184, 3717, 2190, 2279, 2391}

$$\frac{4if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{4f(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad^2}+\frac{f(e+fx)\sin^2(c+dx)}{2ad^2}-\frac{2f(e+fx)\sin(c+dx)}{ad^2}-\frac{2f^2\cos(}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] -(f^2*x)/(4*a*d^2) + (I*(e + f*x)^2)/(a*d) + (e + f*x)^3/(2*a*f) - (2*f^2*C
os[c + d*x])/(a*d^3) + ((e + f*x)^2*Cos[c + d*x])/(a*d) + ((e + f*x)^2*Cot[
c/2 + Pi/4 + (d*x)/2])/(a*d) - (4*f*(e + f*x)*Log[1 - I*E^(I*(c + d*x))])/(
a*d^2) + ((4*I)*f^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - (2*f*(e + f*x)
*Sin[c + d*x])/(a*d^2) + (f^2*Cos[c + d*x]*Sin[c + d*x])/(4*a*d^3) - ((e +
f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) + (f*(e + f*x)*Sin[c + d*x]^2)/(2
*a*d^2)
```


Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
```

```
IGtQ[n, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)^2\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\sin^2(c+dx)\,dx}{a} - \int \frac{(e+fx)^2\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{(e+fx)^2\cos(c+dx)\sin(c+dx)}{2ad} + \frac{f(e+fx)\sin^2(c+dx)}{2ad^2} + \frac{\int (e+fx)^2\,dx}{2a} - \frac{\int (e+fx)}{}$$

$$= \frac{(e+fx)^3}{6af} + \frac{(e+fx)^2\cos(c+dx)}{ad} + \frac{f^2\cos(c+dx)\sin(c+dx)}{4ad^3} - \frac{(e+fx)^2\cos(c+dx)\sin}{2ad}$$

$$= -\frac{f^2x}{4ad^2} + \frac{(e+fx)^3}{2af} + \frac{(e+fx)^2\cos(c+dx)}{ad} - \frac{2f(e+fx)\sin(c+dx)}{ad^2} + \frac{f^2\cos(c+dx)\sin}{4ad^3}$$

$$= -\frac{f^2x}{4ad^2} + \frac{(e+fx)^3}{2af} - \frac{2f^2\cos(c+dx)}{ad^3} + \frac{(e+fx)^2\cos(c+dx)}{ad} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d}{2}\right.}{ad}$$

$$= -\frac{f^2x}{4ad^2} + \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{2af} - \frac{2f^2\cos(c+dx)}{ad^3} + \frac{(e+fx)^2\cos(c+dx)}{ad} + \frac{(e+fx)^2\,\mathrm{co}}{}$$

$$= -\frac{f^2x}{4ad^2} + \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{2af} - \frac{2f^2\cos(c+dx)}{ad^3} + \frac{(e+fx)^2\cos(c+dx)}{ad} + \frac{(e+fx)^2\,\mathrm{co}}{}$$

$$= -\frac{f^2x}{4ad^2} + \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{2af} - \frac{2f^2\cos(c+dx)}{ad^3} + \frac{(e+fx)^2\cos(c+dx)}{ad} + \frac{(e+fx)^2\,\mathrm{co}}{}$$

$$= -\frac{f^2x}{4ad^2} + \frac{i(e+fx)^2}{ad} + \frac{(e+fx)^3}{2af} - \frac{2f^2\cos(c+dx)}{ad^3} + \frac{(e+fx)^2\cos(c+dx)}{ad} + \frac{(e+fx)^2\,\mathrm{co}}{}$$

Mathematica [B] time = 2.86717, size = 830, normalized size = 2.99

$$-\frac{-8f^2x^3\sin\left(\frac{1}{2}(c+dx)\right)d^3 - 24efx^2\sin\left(\frac{1}{2}(c+dx)\right)d^3 - 24e^2x\sin\left(\frac{1}{2}(c+dx)\right)d^3 - 6e^2\cos\left(\frac{3}{2}(c+dx)\right)d^2 - 6f^2x^2\cos\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(-6*d^2*e^2*Cos[(3*(c + d*x))/2] - 14*d*e*f*Cos[(3*(c + d*x))/2] + 15*f^2*
Cos[(3*(c + d*x))/2] - 12*d^2*e*f*x*Cos[(3*(c + d*x))/2] - 14*d*f^2*x*Cos[(
3*(c + d*x))/2] - 6*d^2*f^2*x^2*Cos[(3*(c + d*x))/2] - 2*d^2*e^2*Cos[(5*(c
+ d*x))/2] + 2*d*e*f*Cos[(5*(c + d*x))/2] + f^2*Cos[(5*(c + d*x))/2] - 4*d^
```

```
2*e*f*x*Cos[(5*(c + d*x))/2] + 2*d*f^2*x*Cos[(5*(c + d*x))/2] - 2*d^2*f^2*x
^2*Cos[(5*(c + d*x))/2] - 8*Cos[(c + d*x)/2]*(-2*f^2 - 2*d*f*(e + f*x) + (3
 - 2*I)*d^2*(e + f*x)^2 + d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2) - 8*d*f*(e + f*
x)*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]) + (24 + 16*I)*d^2*e^2*Sin[(c + d
*x)/2] + 16*d*e*f*Sin[(c + d*x)/2] - 16*f^2*Sin[(c + d*x)/2] - 24*d^3*e^2*x
*Sin[(c + d*x)/2] + (48 + 32*I)*d^2*e*f*x*Sin[(c + d*x)/2] + 16*d*f^2*x*Sin
[(c + d*x)/2] - 24*d^3*e*f*x^2*Sin[(c + d*x)/2] + (24 + 16*I)*d^2*f^2*x^2*S
in[(c + d*x)/2] - 8*d^3*f^2*x^3*Sin[(c + d*x)/2] + 64*d*e*f*Log[1 + I*Cos[c
 + d*x] + Sin[c + d*x]]*Sin[(c + d*x)/2] + 64*d*f^2*x*Log[1 + I*Cos[c + d*x
] + Sin[c + d*x]]*Sin[(c + d*x)/2] + (64*I)*f^2*PolyLog[2, (-I)*Cos[c + d*x
] - Sin[c + d*x]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 6*d^2*e^2*Sin[(3*
(c + d*x))/2] + 14*d*e*f*Sin[(3*(c + d*x))/2] + 15*f^2*Sin[(3*(c + d*x))/2]
 - 12*d^2*e*f*x*Sin[(3*(c + d*x))/2] + 14*d*f^2*x*Sin[(3*(c + d*x))/2] - 6*
d^2*f^2*x^2*Sin[(3*(c + d*x))/2] + 2*d^2*e^2*Sin[(5*(c + d*x))/2] + 2*d*e*f
*Sin[(5*(c + d*x))/2] - f^2*Sin[(5*(c + d*x))/2] + 4*d^2*e*f*x*Sin[(5*(c +
d*x))/2] + 2*d*f^2*x*Sin[(5*(c + d*x))/2] + 2*d^2*f^2*x^2*Sin[(5*(c + d*x))
/2])/(16*a*d^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [B] time = 0.447, size = 538, normalized size = 1.9

$$\frac{f^2x^3}{2a} + \frac{3fex^2}{2a} + \frac{3e^2x}{2a} + \frac{2if^2x^2}{da} + \frac{\left(f^2x^2d^2 + 2idf^2x + 2d^2efx + 2idef + d^2e^2 - 2f^2\right)e^{i(dx+c)}}{2d^3a} + \frac{\left(f^2x^2d^2 - 2idf^2x + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/a*f^2*x^3+3/2/a*f*e*x^2+3/2/a*e^2*x+2*I*f^2/d/a*x^2+1/2*(f^2*x^2*d^2+2*
I*d*f^2*x+2*d^2*e*f*x+2*I*d*e*f+d^2*e^2-2*f^2)/d^3/a*exp(I*(d*x+c))+1/2*(f^
2*x^2*d^2-2*I*d*f^2*x+2*d^2*e*f*x-2*I*d*e*f+d^2*e^2-2*f^2)/d^3/a*exp(-I*(d*
x+c))+4*I*f^2/d^2/a*c*x+2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(I*(d*x+c))+I)+4*f/
d^2/a*ln(exp(I*(d*x+c)))*e-4*f/d^2/a*ln(exp(I*(d*x+c))+I)*e-1/16*I*(2*f^2*x
^2*d^2-2*I*d*f^2*x+4*d^2*e*f*x-2*I*d*e*f+2*d^2*e^2-f^2)/d^3/a*exp(-2*I*(d*x
+c))+1/16*I*(2*f^2*x^2*d^2+2*I*d*f^2*x+4*d^2*e*f*x+2*I*d*e*f+2*d^2*e^2-f^2)
/d^3/a*exp(2*I*(d*x+c))+4*I*f^2*polylog(2,I*exp(I*(d*x+c)))/a/d^3-4*f^2/d^2
/a*ln(1-I*exp(I*(d*x+c)))*x-4*f^2/d^3/a*ln(1-I*exp(I*(d*x+c)))*c+2*I*f^2/d^
3/a*c^2-4*f^2/d^3/a*c*ln(exp(I*(d*x+c)))+4*f^2/d^3/a*c*ln(exp(I*(d*x+c))+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.3222, size = 1960, normalized size = 7.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d^3*f^2*x^3 + 4*d^2*e^2 + (2*d^2*f^2*x^2 + 2*d^2*e^2 - 2*d*e*f - f^2
 + 2*(2*d^2*e*f - d*f^2)*x)*cos(d*x + c)^3 - 7*d*e*f + 2*(3*d^3*e*f + 2*d^2
*f^2)*x^2 + 2*(2*d^2*f^2*x^2 + 2*d^2*e^2 + 3*d*e*f - 4*f^2 + (4*d^2*e*f + 3
*d*f^2)*x)*cos(d*x + c)^2 + (6*d^3*e^2 + 8*d^2*e*f - 7*d*f^2)*x + (2*d^3*f^
2*x^3 + 6*d^2*e^2 + d*e*f + 6*(d^3*e*f + d^2*f^2)*x^2 - 7*f^2 + (6*d^3*e^2
+ 12*d^2*e*f + d*f^2)*x)*cos(d*x + c) + (8*I*f^2*cos(d*x + c) + 8*I*f^2*sin
(d*x + c) + 8*I*f^2)*dilog(I*cos(d*x + c) - sin(d*x + c)) + (-8*I*f^2*cos(d
*x + c) - 8*I*f^2*sin(d*x + c) - 8*I*f^2)*dilog(-I*cos(d*x + c) - sin(d*x +
 c)) - 8*(d*e*f - c*f^2 + (d*e*f - c*f^2)*cos(d*x + c) + (d*e*f - c*f^2)*si
n(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - 8*(d*f^2*x + c*f^2 + (
d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^2*x + c*f^2)*sin(d*x + c))*log(I*cos(d
*x + c) + sin(d*x + c) + 1) - 8*(d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*cos(d*
x + c) + (d*f^2*x + c*f^2)*sin(d*x + c))*log(-I*cos(d*x + c) + sin(d*x + c)
 + 1) - 8*(d*e*f - c*f^2 + (d*e*f - c*f^2)*cos(d*x + c) + (d*e*f - c*f^2)*s
in(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) + (2*d^3*f^2*x^3 - 4*d
^2*e^2 - 7*d*e*f + 2*(3*d^3*e*f - 2*d^2*f^2)*x^2 - (2*d^2*f^2*x^2 + 2*d^2*e
^2 + 2*d*e*f - f^2 + 2*(2*d^2*e*f + d*f^2)*x)*cos(d*x + c)^2 + (6*d^3*e^2 -
 8*d^2*e*f - 7*d*f^2)*x + (2*d^2*f^2*x^2 + 2*d^2*e^2 - 8*d*e*f - 7*f^2 + 4*
(d^2*e*f - 2*d*f^2)*x)*cos(d*x + c))*sin(d*x + c))/(a*d^3*cos(d*x + c) + a*
d^3*sin(d*x + c) + a*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^2 \sin^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2 \sin^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx \sin^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*sin(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*
sin(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sin(c + d*x)**3/(
sin(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sin(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

3.193 $\int \frac{(e+fx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=158

$$\frac{f\sin^2(c+dx)}{4ad^2}-\frac{f\sin(c+dx)}{ad^2}-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}+\frac{(e+fx)\cos(c+dx)}{ad}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{(e+f}{}$$

```
[Out] (3*e*x)/(2*a) + (3*f*x^2)/(4*a) + ((e + f*x)*Cos[c + d*x])/(a*d) + ((e + f*
x)*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - (2*f*Log[Sin[c/2 + Pi/4 + (d*x)/2]])/
(a*d^2) - (f*Sin[c + d*x])/(a*d^2) - ((e + f*x)*Cos[c + d*x]*Sin[c + d*x])/
(2*a*d) + (f*Sin[c + d*x]^2)/(4*a*d^2)
```

Rubi [A] time = 0.219706, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4515, 3310, 3296, 2637, 3318, 4184, 3475}

$$\frac{f\sin^2(c+dx)}{4ad^2}-\frac{f\sin(c+dx)}{ad^2}-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}+\frac{(e+fx)\cos(c+dx)}{ad}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*e*x)/(2*a) + (3*f*x^2)/(4*a) + ((e + f*x)*Cos[c + d*x])/(a*d) + ((e + f*
x)*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - (2*f*Log[Sin[c/2 + Pi/4 + (d*x)/2]])/
(a*d^2) - (f*Sin[c + d*x])/(a*d^2) - ((e + f*x)*Cos[c + d*x]*Sin[c + d*x])/
(2*a*d) + (f*Sin[c + d*x]^2)/(4*a*d^2)
```

Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
```

```
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)\sin^2(c+dx)\,dx}{a} - \int \frac{(e+fx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{(e+fx)\cos(c+dx)\sin(c+dx)}{2ad} + \frac{f\sin^2(c+dx)}{4ad^2} + \frac{\int (e+fx)\,dx}{2a} - \frac{\int (e+fx)\sin(c+dx)}{a}$$

$$= \frac{ex}{2a} + \frac{fx^2}{4a} + \frac{(e+fx)\cos(c+dx)}{ad} - \frac{(e+fx)\cos(c+dx)\sin(c+dx)}{2ad} + \frac{f\sin^2(c+dx)}{4ad^2} + \int($$

$$= \frac{3ex}{2a} + \frac{3fx^2}{4a} + \frac{(e+fx)\cos(c+dx)}{ad} - \frac{f\sin(c+dx)}{ad^2} - \frac{(e+fx)\cos(c+dx)\sin(c+dx)}{2ad} + f$$

$$= \frac{3ex}{2a} + \frac{3fx^2}{4a} + \frac{(e+fx)\cos(c+dx)}{ad} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{f\sin(c+dx)}{ad^2} - \frac{(e+fx}{}$$

$$= \frac{3ex}{2a} + \frac{3fx^2}{4a} + \frac{(e+fx)\cos(c+dx)}{ad} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right.\right.}{ad^2}$$

Mathematica [A] time = 1.46918, size = 298, normalized size = 1.89

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)\left(2\left(-3c^2f-d(e+fx)\sin(2(c+dx))+6cde-4f\sin(c+dx)-8f\,\text{lo}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Sin[(c + d*x)/2]*(8*d*(e + f*x)*Cos
[c + d*x] - f*Cos[2*(c + d*x)] + 2*(-8*d*e + 6*c*d*e + 4*c*f - 3*c^2*f + 6*
d^2*e*x - 4*d*f*x + 3*d^2*f*x^2 - 8*f*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] - 4*f*Sin[c + d*x] - d*(e + f*x)*Sin[2*(c + d*x)])) + Cos[(c + d*x)/2]*
(8*d*(e + f*x)*Cos[c + d*x] - f*Cos[2*(c + d*x)] + 2*(6*c*d*e + 4*c*f - 3*c
^2*f + 6*d^2*e*x + 4*d*f*x + 3*d^2*f*x^2 - 8*f*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] - 4*f*Sin[c + d*x] - d*(e + f*x)*Sin[2*(c + d*x)]))))/(8*a*d^2
*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.149, size = 662, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 3/a*e/d*arctan(tan(1/2*d*x+1/2*c))+2/a*e/d/(tan(1/2*d*x+1/2*c)+1)+1/2/a*f/(
1+tan(1/2*d*x+1/2*c)^2)/(tan(1/2*d*x+1/2*c)+1)*x^2+1/a*f/(1+tan(1/2*d*x+1/2
*c)^2)/(tan(1/2*d*x+1/2*c)+1)*x/d+1/2/a*f/(1+tan(1/2*d*x+1/2*c)^2)/(tan(1/2
*d*x+1/2*c)+1)*x^2*tan(1/2*d*x+1/2*c)+1/2/a*f/(1+tan(1/2*d*x+1/2*c)^2)/(tan
(1/2*d*x+1/2*c)+1)*x^2*tan(1/2*d*x+1/2*c)^2+1/2/a*f/(1+tan(1/2*d*x+1/2*c)^2
)/(tan(1/2*d*x+1/2*c)+1)*x^2*tan(1/2*d*x+1/2*c)^3-1/a*f/(1+tan(1/2*d*x+1/2*
c)^2)/(tan(1/2*d*x+1/2*c)+1)*x/d*tan(1/2*d*x+1/2*c)+1/a*f/(1+tan(1/2*d*x+1/
2*c)^2)/(tan(1/2*d*x+1/2*c)+1)*x/d*tan(1/2*d*x+1/2*c)^2-1/a*f/(1+tan(1/2*d*
x+1/2*c)^2)/(tan(1/2*d*x+1/2*c)+1)*x/d*tan(1/2*d*x+1/2*c)^3-2/a*f/d^2*ln(ta
n(1/2*d*x+1/2*c)+1)+1/a*f/d^2*ln(1+tan(1/2*d*x+1/2*c)^2)+1/a*e/d/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+2/a*e/d/(1+tan(1/2*d*x+1/2*c)^2)^2*ta
n(1/2*d*x+1/2*c)^2-1/a*e/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+2/
a*e/d/(1+tan(1/2*d*x+1/2*c)^2)^2-1/2/a*f/d*sin(d*x+c)*cos(d*x+c)*x+1/4*f*x^
2/a-1/4/a*f/d^2*c^2+1/4*f*sin(d*x+c)^2/a/d^2-f*sin(d*x+c)/a/d^2+1/a*f/d*cos
(d*x+c)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.88373, size = 625, normalized size = 3.96

$$\frac{6\,d^2fx^2+2\left(2\,dfx+2\,de-f\right)\cos\left(dx+c\right)^3+2\left(4\,dfx+4\,de+3\,f\right)\cos\left(dx+c\right)^2+8\,de+4\left(3\,d^2e+2\,df\right)x+\left(6\,d^2fx^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(6*d^2*f*x^2 + 2*(2*d*f*x + 2*d*e - f)*cos(d*x + c)^3 + 2*(4*d*f*x + 4*
d*e + 3*f)*cos(d*x + c)^2 + 8*d*e + 4*(3*d^2*e + 2*d*f)*x + (6*d^2*f*x^2 +
```

```
12*d*e + 12*(d^2*e + d*f)*x + f)*cos(d*x + c) - 8*(f*cos(d*x + c) + f*sin(d
*x + c) + f)*log(sin(d*x + c) + 1) + (6*d^2*f*x^2 - 2*(2*d*f*x + 2*d*e + f)
*cos(d*x + c)^2 - 8*d*e + 4*(3*d^2*e - 2*d*f)*x + 4*(d*f*x + d*e - 2*f)*cos
(d*x + c) - 7*f)*sin(d*x + c) - 7*f)/(a*d^2*cos(d*x + c) + a*d^2*sin(d*x +
c) + a*d^2)
```

Sympy [A] time = 9.98242, size = 4869, normalized size = 30.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((18*d**2*e*x*tan(c/2 + d*x/2)**5/(12*a*d**2*tan(c/2 + d*x/2)**5 +
 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*
tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) + 18*d**2*e*x
*tan(c/2 + d*x/2)**4/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d
*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 +
12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) + 36*d**2*e*x*tan(c/2 + d*x/2)**3/(
12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*t
an(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*
x/2) + 12*a*d**2) + 36*d**2*e*x*tan(c/2 + d*x/2)**2/(12*a*d**2*tan(c/2 + d*
x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 2
4*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) + 18
*d**2*e*x*tan(c/2 + d*x/2)/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c
/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)
**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) + 18*d**2*e*x/(12*a*d**2*tan(
c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2
)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d*
*2) + 9*d**2*f*x**2*tan(c/2 + d*x/2)**5/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12
*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan
(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) + 9*d**2*f*x**2*
tan(c/2 + d*x/2)**4/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*
x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 1
2*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) + 18*d**2*f*x**2*tan(c/2 + d*x/2)**3
/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2
*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 +
d*x/2) + 12*a*d**2) + 18*d**2*f*x**2*tan(c/2 + d*x/2)**2/(12*a*d**2*tan(c/2
 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**
3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2)
 + 9*d**2*f*x**2*tan(c/2 + d*x/2)/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**
```

```
2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 +
 d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) + 9*d**2*f*x**2/(12*a*
d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/
2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2)
+ 12*a*d**2) - 36*d*e*tan(c/2 + d*x/2)**5/(12*a*d**2*tan(c/2 + d*x/2)**5 +
12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*t
an(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) - 36*d*e*tan(c
/2 + d*x/2)**3/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)*
*4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d
**2*tan(c/2 + d*x/2) + 12*a*d**2) - 12*d*e*tan(c/2 + d*x/2)**2/(12*a*d**2*t
an(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*
x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a
*d**2) - 24*d*e*tan(c/2 + d*x/2)/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2
*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 +
d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) + 12*d*e/(12*a*d**2*tan
(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/
2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d
**2) - 24*d*f*x*tan(c/2 + d*x/2)**5/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d
**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2
 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) + 12*d*f*x*tan(c/2 +
 d*x/2)**4/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 +
 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*
tan(c/2 + d*x/2) + 12*a*d**2) - 12*d*f*x*tan(c/2 + d*x/2)**3/(12*a*d**2*tan
(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/
2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d
**2) + 12*d*f*x*tan(c/2 + d*x/2)**2/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d
**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2
 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) - 12*d*f*x*tan(c/2 +
 d*x/2)/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24
*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan
(c/2 + d*x/2) + 12*a*d**2) + 24*d*f*x/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a
*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c
/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) - 24*f*log(tan(c/2
 + d*x/2) + 1)*tan(c/2 + d*x/2)**5/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d*
*2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2
+ d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) - 24*f*log(tan(c/2 +
d*x/2) + 1)*tan(c/2 + d*x/2)**4/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*
tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d
*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) - 48*f*log(tan(c/2 + d*x
/2) + 1)*tan(c/2 + d*x/2)**3/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan
(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/
2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) - 48*f*log(tan(c/2 + d*x/2)
 + 1)*tan(c/2 + d*x/2)**2/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/
2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)*
*2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) - 24*f*log(tan(c/2 + d*x/2) +
```

```
1)*tan(c/2 + d*x/2)/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*
x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 1
2*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) - 24*f*log(tan(c/2 + d*x/2) + 1)/(12
*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan
(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/
2) + 12*a*d**2) + 12*f*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**5/(12
*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan
(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/
2) + 12*a*d**2) + 12*f*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(12
*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan
(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/
2) + 12*a*d**2) + 24*f*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**3/(12
*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan
(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/
2) + 12*a*d**2) + 24*f*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**2/(12
*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan
(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/
2) + 12*a*d**2) + 12*f*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)/(12*a*
d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/
2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2)
+ 12*a*d**2) + 12*f*log(tan(c/2 + d*x/2)**2 + 1)/(12*a*d**2*tan(c/2 + d*x/2
)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a
*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) + 4*f*t
an(c/2 + d*x/2)**5/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x
/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12
*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) - 20*f*tan(c/2 + d*x/2)**4/(12*a*d**2
*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 +
d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12
*a*d**2) - 4*f*tan(c/2 + d*x/2)**3/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d*
*2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2
+ d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) - 4*f*tan(c/2 + d*x/2
)**2/(12*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*
d**2*tan(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/
2 + d*x/2) + 12*a*d**2) - 20*f*tan(c/2 + d*x/2)/(12*a*d**2*tan(c/2 + d*x/2)
**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**3 + 24*a*
d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x/2) + 12*a*d**2) + 4*f/(1
2*a*d**2*tan(c/2 + d*x/2)**5 + 12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*ta
n(c/2 + d*x/2)**3 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2*tan(c/2 + d*x
/2) + 12*a*d**2), Ne(d, 0)), ((e*x + f*x**2/2)*sin(c)**3/(a*sin(c) + a), Tr
ue))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.194 $\int \frac{\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2\cos(c+dx)}{ad}+\frac{\sin^2(c+dx)\cos(c+dx)}{d(a\sin(c+dx)+a)}-\frac{3\sin(c+dx)\cos(c+dx)}{2ad}+\frac{3x}{2a}$$

```
[Out] (3*x)/(2*a) + (2*Cos[c + d*x])/(a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*d
) + (Cos[c + d*x]*Sin[c + d*x]^2)/(d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.0619893, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2767, 2734}

$$\frac{2\cos(c+dx)}{ad}+\frac{\sin^2(c+dx)\cos(c+dx)}{d(a\sin(c+dx)+a)}-\frac{3\sin(c+dx)\cos(c+dx)}{2ad}+\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(2*a) + (2*Cos[c + d*x])/(a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*d
) + (Cos[c + d*x]*Sin[c + d*x]^2)/(d*(a + a*Sin[c + d*x]))
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\cos(c+dx)\sin^2(c+dx)}{d(a+a\sin(c+dx))} - \frac{\int \sin(c+dx)(2a-3a\sin(c+dx))\,dx}{a^2}$$
$$= \frac{3x}{2a} + \frac{2\cos(c+dx)}{ad} - \frac{3\cos(c+dx)\sin(c+dx)}{2ad} + \frac{\cos(c+dx)\sin^2(c+dx)}{d(a+a\sin(c+dx))}$$

Mathematica [A] time = 0.197634, size = 117, normalized size = 1.56

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)(-\sin(2(c+dx))+4\cos(c+dx)+6c+6dx-8)+\cos\left(\frac{1}{2}(c+dx)\right.\right.}{4ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Sin[(c + d*x)/2]*(-8 + 6*c + 6*d*x
+ 4*Cos[c + d*x] - Sin[2*(c + d*x)]) + Cos[(c + d*x)/2]*(6*c + 6*d*x + 4*Co
s[c + d*x] - Sin[2*(c + d*x)])))/(4*a*d*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.027, size = 163, normalized size = 2.2

$$\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}+2\,\frac{(\tan(1/2\,dx+c/2))^2}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}-\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+2/a/d/(1+tan(1/2*d*x+
1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d
*x+1/2*c)+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2+3/a/d*arctan(tan(1/2*d*x+1/2*c))
+2/a/d/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.46766, size = 286, normalized size = 3.81

$$\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+4}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + 4)/(a + a*sin(d*x + c)/(cos(d*x + c) + 1) + 2*a*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 3*arctan(s
in(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.79172, size = 247, normalized size = 3.29

$$\frac{\cos(dx+c)^3+3\,dx+3\,(dx+1)\cos(dx+c)+2\,\cos(dx+c)^2+\left(3\,dx-\cos(dx+c)^2+\cos(dx+c)-2\right)\sin(dx+c)+2}{2\,(ad\cos(dx+c)+ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(d*x + c)^3 + 3*d*x + 3*(d*x + 1)*cos(d*x + c) + 2*cos(d*x + c)^2 +
 (3*d*x - cos(d*x + c)^2 + cos(d*x + c) - 2)*sin(d*x + c) + 2)/(a*d*cos(d*x
 + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [A] time = 7.61435, size = 1127, normalized size = 15.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*d*x*tan(c/2 + d*x/2)**5/(2*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*tan
(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2 + d*x/2)**2 +
2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 3*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2
+ d*x/2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d
*tan(c/2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 6*d*x*tan(c/2 + d*
x/2)**3/(2*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(
c/2 + d*x/2)**3 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*
d) + 6*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*tan(c/2 +
 d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d*
tan(c/2 + d*x/2) + 2*a*d) + 3*d*x*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)*
*5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2
+ d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 3*d*x/(2*a*d*tan(c/2 + d*x/
2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c
/2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d) - 6*tan(c/2 + d*x/2)**5/(2
*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/
2)**3 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d) - 6*tan
(c/2 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4
*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/
2) + 2*a*d) - 2*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*x/2)**5 + 2*a*d*tan(
c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2 + d*x/2)**2 + 2
*a*d*tan(c/2 + d*x/2) + 2*a*d) - 4*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)
**5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2
 + d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d) + 2/(2*a*d*tan(c/2 + d*x/2)*
*5 + 2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**3 + 4*a*d*tan(c/2
+ d*x/2)**2 + 2*a*d*tan(c/2 + d*x/2) + 2*a*d), Ne(d, 0)), (x*sin(c)**3/(a*s
in(c) + a), True))
```

Giac [A] time = 1.10078, size = 123, normalized size = 1.64

$$\frac{\frac{3\,(dx+c)}{a} + \frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a} + \frac{4}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(3*(d*x + c)/a + 2*(tan(1/2*d*x + 1/2*c)^3 + 2*tan(1/2*d*x + 1/2*c)^2 -
 tan(1/2*d*x + 1/2*c) + 2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a) + 4/(a*(tan(1
/2*d*x + 1/2*c) + 1)))/d
```

3.195 $\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sin^3(c+dx)}{(e+fx)(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Sin[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0685952, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]

[Out] Defer[Int][Sin[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 6.20527, size = 0, normalized size = 0.

$$\int \frac{\sin^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]

[Out] Integrate[Sin[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 0.657, size = 0, normalized size = 0.

$$\int \frac{(\sin (dx+c))^{3}}{\left(fx+e\right)(a+a\sin (dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(\cos (dx+c)^{2}-1\right)\sin (dx+c)}{afx+ae+\left(afx+ae\right)\sin (dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*sin(d*x + c)/(a*f*x + a*e + (a*f*x + a*e)*si
n(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^3}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^3/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

3.196 $\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))} \, dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sin^3(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Sin[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0691858, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Sin[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))} \, dx = \int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))} \, dx$$

Mathematica [A] time = 5.84541, size = 0, normalized size = 0.

$$\int \frac{\sin^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Sin[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 0.973, size = 0, normalized size = 0.

$$\int \frac{(\sin (dx+c))^3}{(fx+e)^2 (a+a\sin (dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos (dx+c)^2-1\right)\sin (dx+c)}{af^2x^2+2\,aefx+ae^2+\left(af^2x^2+2\,aefx+ae^2\right)\sin (dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*sin(d*x + c)/(a*f^2*x^2 + 2*a*e*f*x + a*e^2
+ (a*f^2*x^2 + 2*a*e*f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^3}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^3/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

3.197 $\int \frac{(e+fx)^3 \csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=352

$$\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{6f^2(e+fx)\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}+\frac{6f^2(e+fx)\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3}+\frac{3if(e+f}{}$$

```
[Out] (I*(e + f*x)^3)/(a*d) - (2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((
e + f*x)^3*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - (6*f*(e + f*x)^2*Log[1 - I*E^
(I*(c + d*x))])/(a*d^2) + ((3*I)*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))]
)/(a*d^2) + ((12*I)*f^2*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) -
((3*I)*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) - (6*f^2*(e + f*x
)*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3) - (12*f^3*PolyLog[3, I*E^(I*(c + d*
x))])/(a*d^4) + (6*f^2*(e + f*x)*PolyLog[3, E^(I*(c + d*x))])/(a*d^3) - ((6
*I)*f^3*PolyLog[4, -E^(I*(c + d*x))])/(a*d^4) + ((6*I)*f^3*PolyLog[4, E^(I*
(c + d*x))])/(a*d^4)
```

Rubi [A] time = 0.46895, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {4535, 4183, 2531, 6609, 2282, 6589, 3318, 4184, 3717, 2190}

$$\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{6f^2(e+fx)\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}+\frac{6f^2(e+fx)\text{PolyLog}\left(3,e^{i(c+dx)}\right)}{ad^3}+\frac{3if(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```


```
[Out] (I*(e + f*x)^3)/(a*d) - (2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((
e + f*x)^3*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - (6*f*(e + f*x)^2*Log[1 - I*E^
(I*(c + d*x))])/(a*d^2) + ((3*I)*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))]
)/(a*d^2) + ((12*I)*f^2*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) -
((3*I)*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) - (6*f^2*(e + f*x
)*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3) - (12*f^3*PolyLog[3, I*E^(I*(c + d*
x))])/(a*d^4) + (6*f^2*(e + f*x)*PolyLog[3, E^(I*(c + d*x))])/(a*d^3) - ((6
*I)*f^3*PolyLog[4, -E^(I*(c + d*x))])/(a*d^4) + ((6*I)*f^3*PolyLog[4, E^(I*
(c + d*x))])/(a*d^4)
```


Rule 4535

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csc[c + d*x]^(n - 1))/(a + b*Si
n[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3 \csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3 \csc(c+dx)\,dx}{a} - \int \frac{(e+fx)^3}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{\int (e+fx)^3 \csc^2\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{dx}{2}\right)dx}{2a} - \frac{(3f)\int (e+fx)^2 \mathrm{lo}}{a}$$

$$= -\frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{3if(e+fx)^2 \mathrm{Li}_2\left(-e^{i(c+dx)}\right)}{ad^2}$$

$$= \frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{3if(e+fx)^2 \mathrm{Li}_2}{ad^2}$$

$$= \frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6f(e+fx)^2 \log}{ad^2}$$

$$= \frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6f(e+fx)^2 \log}{ad^2}$$

$$= \frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6f(e+fx)^2 \log}{ad^2}$$

$$= \frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3 \cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{6f(e+fx)^2 \log}{ad^2}$$

Mathematica [A] time = 2.56901, size = 443, normalized size = 1.26

$$\frac{3if\left(d^2(e+fx)^2 \mathrm{PolyLog}(2,-\cos(c+dx)-i\sin(c+dx))+2idf(e+fx)\mathrm{PolyLog}(3,-\cos(c+dx)-i\sin(c+dx))-2f^2\mathrm{PolyLog}(4,-\cos(c+dx)-i\sin(c+dx))\right)}{d^3} - \frac{3if\left(d^2(e+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)^3*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] + ((3*I)*f*(d^2*(e +
 f*x)^2*PolyLog[2, -Cos[c + d*x] - I*Sin[c + d*x]] + (2*I)*d*f*(e + f*x)*Po
lyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] - 2*f^2*PolyLog[4, -Cos[c + d*x] -
 I*Sin[c + d*x]]))/d^3 - ((3*I)*f*(d^2*(e + f*x)^2*PolyLog[2, Cos[c + d*x]
+ I*Sin[c + d*x]] + (2*I)*d*f*(e + f*x)*PolyLog[3, Cos[c + d*x] + I*Sin[c +
 d*x]] - 2*f^2*PolyLog[4, Cos[c + d*x] + I*Sin[c + d*x]]))/d^3 + (6*f*(Cos[
```

```
c] + I*Sin[c])*(((e + f*x)^3*(Cos[c] - I*Sin[c]))/(3*f) - ((e + f*x)^2*Log[
1 + I*Cos[c + d*x] + Sin[c + d*x]]*(1 + I*Cos[c] + Sin[c]))/d + (2*f*(d*(e
+ f*x)*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]] - I*f*PolyLog[3, (-I)*C
os[c + d*x] - Sin[c + d*x]])*(Cos[c] - I*(1 + Sin[c])))/d^3))/(Cos[c] + I*(
1 + Sin[c])) - (2*(e + f*x)^3*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])))/(a*d)
```

Maple [B] time = 0.285, size = 1151, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -12*f^2/d^2/a*e*ln(1-I*exp(I*(d*x+c)))*x-6*I*f^3*polylog(4,-exp(I*(d*x+c)))
/a/d^4+1/d/a*e^3*ln(exp(I*(d*x+c))-1)-1/d/a*e^3*ln(exp(I*(d*x+c))+1)+6*f^3/
d^4/a*c^2*ln(exp(I*(d*x+c)))-6*f/d^2/a*ln(exp(I*(d*x+c))+I)*e^2-6*f^3/d^4/a
*c^2*ln(exp(I*(d*x+c))+I)-4*I*f^3/d^4/a*c^3+2*I*f^3/d/a*x^3+6*f/d^2/a*ln(ex
p(I*(d*x+c)))*e^2+2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(I*(d*x+c))
+I)-3/d^2/a*e^2*f*c*ln(exp(I*(d*x+c))-1)+3/d^3/a*e*f^2*c^2*ln(exp(I*(d*x+c)
)-1)-1/d/a*f^3*ln(exp(I*(d*x+c))+1)*x^3+12*I*f^2/d^2/a*e*c*x+6*I*f^3*polylo
g(4,exp(I*(d*x+c)))/a/d^4-12*f^2/d^3/a*e*ln(1-I*exp(I*(d*x+c)))*c-6*f^3/d^2
/a*ln(1-I*exp(I*(d*x+c)))*x^2+6*f^3/d^4/a*ln(1-I*exp(I*(d*x+c)))*c^2-12*f^2
/d^3/a*e*c*ln(exp(I*(d*x+c)))+12*f^2/d^3/a*e*c*ln(exp(I*(d*x+c))+I)-6*I*f^3
/d^3/a*c^2*x+12*I*f^3/d^3/a*polylog(2,I*exp(I*(d*x+c)))*x+12*I*f^2/d^3/a*e*
polylog(2,I*exp(I*(d*x+c)))+6*I*f^2/d/a*e*x^2+6*I*f^2/d^3/a*e*c^2+6*I/d^2/a
*e*f^2*polylog(2,-exp(I*(d*x+c)))*x-6*I/d^2/a*e*f^2*polylog(2,exp(I*(d*x+c)
))*x+1/d/a*f^3*ln(1-exp(I*(d*x+c)))*x^3+1/d^4/a*f^3*ln(1-exp(I*(d*x+c)))*c^
3-3/d/a*e*f^2*ln(exp(I*(d*x+c))+1)*x^2+3/d/a*ln(1-exp(I*(d*x+c)))*e^2*f*x-3
/d/a*ln(exp(I*(d*x+c))+1)*e^2*f*x+3/d/a*e*f^2*ln(1-exp(I*(d*x+c)))*x^2-3/d^
3/a*e*f^2*ln(1-exp(I*(d*x+c)))*c^2+3/d^2/a*ln(1-exp(I*(d*x+c)))*c*e^2*f+3*I
/d^2/a*e^2*f*polylog(2,-exp(I*(d*x+c)))-3*I/d^2/a*e^2*f*polylog(2,exp(I*(d*
x+c)))+3*I/d^2/a*f^3*polylog(2,-exp(I*(d*x+c)))*x^2-3*I/d^2/a*f^3*polylog(2
,exp(I*(d*x+c)))*x^2-12*f^3*polylog(3,I*exp(I*(d*x+c)))/a/d^4+6/d^3/a*f^3*p
olylog(3,exp(I*(d*x+c)))*x-6/d^3/a*f^3*polylog(3,-exp(I*(d*x+c)))*x-1/d^4/a
*f^3*c^3*ln(exp(I*(d*x+c))-1)+6/d^3/a*e*f^2*polylog(3,exp(I*(d*x+c)))-6/d^3
/a*e*f^2*polylog(3,-exp(I*(d*x+c)))
```

Maxima [B] time = 3.40972, size = 3750, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(3*c*e^2*f*(2/(a*d + a*d*sin(d*x + c)/(cos(d*x + c) + 1)) + log(sin(d*x +
c)/(cos(d*x + c) + 1))/(a*d)) - e^3*(log(sin(d*x + c)/(cos(d*x + c) + 1))/a
 + 2/(a + a*sin(d*x + c)/(cos(d*x + c) + 1))) + (12*I*c^2*d*e*f^2 - 4*I*c^3
*f^3 + (12*I*d^2*e^2*f - 24*I*c*d*e*f^2 + 12*I*c^2*f^3 + 12*(d^2*e^2*f - 2*
c*d*e*f^2 + c^2*f^3)*cos(d*x + c) + (12*I*d^2*e^2*f - 24*I*c*d*e*f^2 + 12*I
*c^2*f^3)*sin(d*x + c))*arctan2(sin(d*x + c) + 1, cos(d*x + c)) + (-12*I*(d
*x + c)^2*f^3 + (-24*I*d*e*f^2 + 24*I*c*f^3)*(d*x + c) - 12*((d*x + c)^2*f^
3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*cos(d*x + c) + (-12*I*(d*x + c)^2*f^3 +
(-24*I*d*e*f^2 + 24*I*c*f^3)*(d*x + c))*sin(d*x + c))*arctan2(cos(d*x + c),
 sin(d*x + c) + 1) + (6*I*c^2*d*e*f^2 + 2*I*(d*x + c)^3*f^3 - 2*I*c^3*f^3 +
 (6*I*d*e*f^2 - 6*I*c*f^3)*(d*x + c)^2 + (6*I*d^2*e^2*f - 12*I*c*d*e*f^2 +
6*I*c^2*f^3)*(d*x + c) + 2*(3*c^2*d*e*f^2 + (d*x + c)^3*f^3 - c^3*f^3 + 3*(
d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*(d*x +
 c))*cos(d*x + c) + (6*I*c^2*d*e*f^2 + 2*I*(d*x + c)^3*f^3 - 2*I*c^3*f^3 +
(6*I*d*e*f^2 - 6*I*c*f^3)*(d*x + c)^2 + (6*I*d^2*e^2*f - 12*I*c*d*e*f^2 + 6
*I*c^2*f^3)*(d*x + c))*sin(d*x + c))*arctan2(sin(d*x + c), cos(d*x + c) + 1
) + (-6*I*c^2*d*e*f^2 + 2*I*c^3*f^3 - 2*(3*c^2*d*e*f^2 - c^3*f^3)*cos(d*x +
 c) + (-6*I*c^2*d*e*f^2 + 2*I*c^3*f^3)*sin(d*x + c))*arctan2(sin(d*x + c),
cos(d*x + c) - 1) + (2*I*(d*x + c)^3*f^3 + (6*I*d*e*f^2 - 6*I*c*f^3)*(d*x +
 c)^2 + (6*I*d^2*e^2*f - 12*I*c*d*e*f^2 + 6*I*c^2*f^3)*(d*x + c) + 2*((d*x
+ c)^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 +
 c^2*f^3)*(d*x + c))*cos(d*x + c) + (2*I*(d*x + c)^3*f^3 + (6*I*d*e*f^2 - 6
*I*c*f^3)*(d*x + c)^2 + (6*I*d^2*e^2*f - 12*I*c*d*e*f^2 + 6*I*c^2*f^3)*(d*x
 + c))*sin(d*x + c))*arctan2(sin(d*x + c), -cos(d*x + c) + 1) - 4*((d*x + c
)^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + c^
2*f^3)*(d*x + c))*cos(d*x + c) + (-24*I*d*e*f^2 - 24*I*(d*x + c)*f^3 + 24*I
*c*f^3 - 24*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*cos(d*x + c) + (-24*I*d*e*f^2
 - 24*I*(d*x + c)*f^3 + 24*I*c*f^3)*sin(d*x + c))*dilog(I*e^(I*d*x + I*c))
+ (-6*I*d^2*e^2*f + 12*I*c*d*e*f^2 - 6*I*(d*x + c)^2*f^3 - 6*I*c^2*f^3 + (-
12*I*d*e*f^2 + 12*I*c*f^3)*(d*x + c) - 6*(d^2*e^2*f - 2*c*d*e*f^2 + (d*x +
c)^2*f^3 + c^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*cos(d*x + c) + (-6*I*d^
2*e^2*f + 12*I*c*d*e*f^2 - 6*I*(d*x + c)^2*f^3 - 6*I*c^2*f^3 + (-12*I*d*e*f
^2 + 12*I*c*f^3)*(d*x + c))*sin(d*x + c))*dilog(-e^(I*d*x + I*c)) + (6*I*d^
2*e^2*f - 12*I*c*d*e*f^2 + 6*I*(d*x + c)^2*f^3 + 6*I*c^2*f^3 + (12*I*d*e*f^
2 - 12*I*c*f^3)*(d*x + c) + 6*(d^2*e^2*f - 2*c*d*e*f^2 + (d*x + c)^2*f^3 +
c^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*cos(d*x + c) + (6*I*d^2*e^2*f - 12
```

```
*I*c*d*e*f^2 + 6*I*(d*x + c)^2*f^3 + 6*I*c^2*f^3 + (12*I*d*e*f^2 - 12*I*c*f
^3)*(d*x + c))*sin(d*x + c))*dilog(e^(I*d*x + I*c)) + (3*c^2*d*e*f^2 + (d*x
 + c)^3*f^3 - c^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*
c*d*e*f^2 + c^2*f^3)*(d*x + c) + (-3*I*c^2*d*e*f^2 - I*(d*x + c)^3*f^3 + I*
c^3*f^3 + (-3*I*d*e*f^2 + 3*I*c*f^3)*(d*x + c)^2 + (-3*I*d^2*e^2*f + 6*I*c*
d*e*f^2 - 3*I*c^2*f^3)*(d*x + c))*cos(d*x + c) + (3*c^2*d*e*f^2 + (d*x + c)
^3*f^3 - c^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e
*f^2 + c^2*f^3)*(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^
2 + 2*cos(d*x + c) + 1) - (3*c^2*d*e*f^2 + (d*x + c)^3*f^3 - c^3*f^3 + 3*(d
*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*(d*x +
c) - (3*I*c^2*d*e*f^2 + I*(d*x + c)^3*f^3 - I*c^3*f^3 + (3*I*d*e*f^2 - 3*I*
c*f^3)*(d*x + c)^2 + (3*I*d^2*e^2*f - 6*I*c*d*e*f^2 + 3*I*c^2*f^3)*(d*x + c
))*cos(d*x + c) + (3*c^2*d*e*f^2 + (d*x + c)^3*f^3 - c^3*f^3 + 3*(d*e*f^2 -
 c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*(d*x + c))*sin(
d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1) + (6*d^
2*e^2*f - 12*c*d*e*f^2 + 6*(d*x + c)^2*f^3 + 6*c^2*f^3 + 12*(d*e*f^2 - c*f^
3)*(d*x + c) + (-6*I*d^2*e^2*f + 12*I*c*d*e*f^2 - 6*I*(d*x + c)^2*f^3 - 6*I
*c^2*f^3 + (-12*I*d*e*f^2 + 12*I*c*f^3)*(d*x + c))*cos(d*x + c) + 6*(d^2*e^
2*f - 2*c*d*e*f^2 + (d*x + c)^2*f^3 + c^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x +
c))*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)
 + (12*f^3*cos(d*x + c) + 12*I*f^3*sin(d*x + c) + 12*I*f^3)*polylog(4, -e^(
I*d*x + I*c)) - (12*f^3*cos(d*x + c) + 12*I*f^3*sin(d*x + c) + 12*I*f^3)*po
lylog(4, e^(I*d*x + I*c)) - 24*(I*f^3*cos(d*x + c) - f^3*sin(d*x + c) - f^3
)*polylog(3, I*e^(I*d*x + I*c)) + (12*d*e*f^2 + 12*(d*x + c)*f^3 - 12*c*f^3
 + (-12*I*d*e*f^2 - 12*I*(d*x + c)*f^3 + 12*I*c*f^3)*cos(d*x + c) + 12*(d*e
*f^2 + (d*x + c)*f^3 - c*f^3)*sin(d*x + c))*polylog(3, -e^(I*d*x + I*c)) -
(12*d*e*f^2 + 12*(d*x + c)*f^3 - 12*c*f^3 - (12*I*d*e*f^2 + 12*I*(d*x + c)*
f^3 - 12*I*c*f^3)*cos(d*x + c) + 12*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*sin(d
*x + c))*polylog(3, e^(I*d*x + I*c)) + (-4*I*(d*x + c)^3*f^3 + (-12*I*d*e*f
^2 + 12*I*c*f^3)*(d*x + c)^2 + (-12*I*d^2*e^2*f + 24*I*c*d*e*f^2 - 12*I*c^2
*f^3)*(d*x + c))*sin(d*x + c))/(-2*I*a*d^3*cos(d*x + c) + 2*a*d^3*sin(d*x +
 c) + 2*a*d^3))/d
```

Fricas [C] time = 3.22015, size = 6904, normalized size = 19.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^3*f^3*x^3 + 6*d^3*e*f^2*x^2 + 6*d^3*e^2*f*x + 2*d^3*e^3 + 2*(d^3*f
^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*cos(d*x + c) + (-3*I*d^
```

2*f^3*x^2 - 6*I*d^2*e*f^2*x - 3*I*d^2*e^2*f + (-3*I*d^2*f^3*x^2 - 6*I*d^2*e
*f^2*x - 3*I*d^2*e^2*f)*cos(d*x + c) + (-3*I*d^2*f^3*x^2 - 6*I*d^2*e*f^2*x
- 3*I*d^2*e^2*f)*sin(d*x + c))*dilog(cos(d*x + c) + I*sin(d*x + c)) + (3*I*
d^2*f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^2*f + (3*I*d^2*f^3*x^2 + 6*I*d^2*
e*f^2*x + 3*I*d^2*e^2*f)*cos(d*x + c) + (3*I*d^2*f^3*x^2 + 6*I*d^2*e*f^2*x
+ 3*I*d^2*e^2*f)*sin(d*x + c))*dilog(cos(d*x + c) - I*sin(d*x + c)) + (12*I
*d*f^3*x + 12*I*d*e*f^2 + (12*I*d*f^3*x + 12*I*d*e*f^2)*cos(d*x + c) + (12*
I*d*f^3*x + 12*I*d*e*f^2)*sin(d*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)
) + (-12*I*d*f^3*x - 12*I*d*e*f^2 + (-12*I*d*f^3*x - 12*I*d*e*f^2)*cos(d*x
+ c) + (-12*I*d*f^3*x - 12*I*d*e*f^2)*sin(d*x + c))*dilog(-I*cos(d*x + c) -
sin(d*x + c)) + (-3*I*d^2*f^3*x^2 - 6*I*d^2*e*f^2*x - 3*I*d^2*e^2*f + (-3*
I*d^2*f^3*x^2 - 6*I*d^2*e*f^2*x - 3*I*d^2*e^2*f)*cos(d*x + c) + (-3*I*d^2*f
^3*x^2 - 6*I*d^2*e*f^2*x - 3*I*d^2*e^2*f)*sin(d*x + c))*dilog(-cos(d*x + c)
+ I*sin(d*x + c)) + (3*I*d^2*f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^2*f + (
3*I*d^2*f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^2*f)*cos(d*x + c) + (3*I*d^2*
f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^2*f)*sin(d*x + c))*dilog(-cos(d*x + c
) - I*sin(d*x + c)) - (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*
e^3 + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*cos(d*x + c
) + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*sin(d*x + c))
*log(cos(d*x + c) + I*sin(d*x + c) + 1) - 6*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*
f^3 + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c) + (d^2*e^2*f - 2*c*d
*e*f^2 + c^2*f^3)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - (d
^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3 + (d^3*f^3*x^3 + 3*d
^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*cos(d*x + c) + (d^3*f^3*x^3 + 3*d^3
*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*sin(d*x + c))*log(cos(d*x + c) - I*si
n(d*x + c) + 1) - 6*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3 +
(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c) + (d^2*f
^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*sin(d*x + c))*log(I*cos(d*x
+ c) + sin(d*x + c) + 1) - 6*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 -
c^2*f^3 + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c
) + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*sin(d*x + c))*log
(-I*cos(d*x + c) + sin(d*x + c) + 1) + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e
*f^2 - c^3*f^3 + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*cos(d*
x + c) + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*sin(d*x + c))*
log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) + (d^3*e^3 - 3*c*d^2*e^2*
f + 3*c^2*d*e*f^2 - c^3*f^3 + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^
3*f^3)*cos(d*x + c) + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*s
in(d*x + c))*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) + (d^3*f^3*x
^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*
f^3 + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^
2*d*e*f^2 + c^3*f^3)*cos(d*x + c) + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*
e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*sin(d*x + c))*log(-cos(d
*x + c) + I*sin(d*x + c) + 1) - 6*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3 + (d^2
*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c) + (d^2*e^2*f - 2*c*d*e*f^2 + c
^2*f^3)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) + (d^3*f^3*x^

```
3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f
^3 + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2
*d*e*f^2 + c^3*f^3)*cos(d*x + c) + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e
^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*sin(d*x + c))*log(-cos(d*
x + c) - I*sin(d*x + c) + 1) + (6*I*f^3*cos(d*x + c) + 6*I*f^3*sin(d*x + c)
 + 6*I*f^3)*polylog(4, cos(d*x + c) + I*sin(d*x + c)) + (-6*I*f^3*cos(d*x +
 c) - 6*I*f^3*sin(d*x + c) - 6*I*f^3)*polylog(4, cos(d*x + c) - I*sin(d*x +
 c)) + (6*I*f^3*cos(d*x + c) + 6*I*f^3*sin(d*x + c) + 6*I*f^3)*polylog(4, -
cos(d*x + c) + I*sin(d*x + c)) + (-6*I*f^3*cos(d*x + c) - 6*I*f^3*sin(d*x +
 c) - 6*I*f^3)*polylog(4, -cos(d*x + c) - I*sin(d*x + c)) + 6*(d*f^3*x + d*
e*f^2 + (d*f^3*x + d*e*f^2)*cos(d*x + c) + (d*f^3*x + d*e*f^2)*sin(d*x + c)
)*polylog(3, cos(d*x + c) + I*sin(d*x + c)) + 6*(d*f^3*x + d*e*f^2 + (d*f^3
*x + d*e*f^2)*cos(d*x + c) + (d*f^3*x + d*e*f^2)*sin(d*x + c))*polylog(3, c
os(d*x + c) - I*sin(d*x + c)) - 12*(f^3*cos(d*x + c) + f^3*sin(d*x + c) + f
^3)*polylog(3, I*cos(d*x + c) - sin(d*x + c)) - 12*(f^3*cos(d*x + c) + f^3*
sin(d*x + c) + f^3)*polylog(3, -I*cos(d*x + c) - sin(d*x + c)) - 6*(d*f^3*x
 + d*e*f^2 + (d*f^3*x + d*e*f^2)*cos(d*x + c) + (d*f^3*x + d*e*f^2)*sin(d*x
 + c))*polylog(3, -cos(d*x + c) + I*sin(d*x + c)) - 6*(d*f^3*x + d*e*f^2 +
(d*f^3*x + d*e*f^2)*cos(d*x + c) + (d*f^3*x + d*e*f^2)*sin(d*x + c))*polylo
g(3, -cos(d*x + c) - I*sin(d*x + c)) - 2*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3
*d^3*e^2*f*x + d^3*e^3)*sin(d*x + c))/(a*d^4*cos(d*x + c) + a*d^4*sin(d*x +
 c) + a*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^3 \csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3 \csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2 \csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx \csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**3*csc(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*csc
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*csc(c + d*x)/(sin
(c + d*x) + 1), x) + Integral(3*e**2*f*x*csc(c + d*x)/(sin(c + d*x) + 1), x
))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3\csc(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.198 $\int \frac{(e+fx)^2 \csc(c+dx)}{a+a\sin(c+dx)} \, dx$

Optimal. Leaf size=249

$$\frac{2if(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{2if(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{4if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{2f^2\text{PolyLog}\left(3,-e^{i}\right.}{ad^3}$$

```
[Out] (I*(e + f*x)^2)/(a*d) - (2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((
e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - (4*f*(e + f*x)*Log[1 - I*E^(I
*(c + d*x))])/(a*d^2) + ((2*I)*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a
*d^2) + ((4*I)*f^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - ((2*I)*f*(e + f
*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) - (2*f^2*PolyLog[3, -E^(I*(c + d*x
))])/(a*d^3) + (2*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3)
```

Rubi [A] time = 0.329595, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {4535, 4183, 2531, 2282, 6589, 3318, 4184, 3717, 2190, 2279, 2391}

$$\frac{2if(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{2if(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{4if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{2f^2\text{PolyLog}\left(3,-e^{i}\right.}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```


```
[Out] (I*(e + f*x)^2)/(a*d) - (2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((
e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - (4*f*(e + f*x)*Log[1 - I*E^(I
*(c + d*x))])/(a*d^2) + ((2*I)*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a
*d^2) + ((4*I)*f^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - ((2*I)*f*(e + f
*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) - (2*f^2*PolyLog[3, -E^(I*(c + d*x
))])/(a*d^3) + (2*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3)
```


Rule 4535

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csc[c + d*x]^(n - 1))/(a + b*Si
n[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)^2\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\csc(c+dx)\,dx}{a} - \int \frac{(e+fx)^2}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{\int (e+fx)^2\csc^2\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{dx}{2}\right)dx}{2a} - \frac{(2f)\int (e+fx)\log}{ad}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{2if(e+fx)\mathrm{Li}_2\left(-e^{i(c+dx)}\right)}{ad^2}$$

$$= \frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{2if(e+fx)\mathrm{Li}_2}{ad^2}$$

$$= \frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{4f(e+fx)\log(}{ad^2}$$

$$= \frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{4f(e+fx)\log(}{ad^2}$$

$$= \frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{4f(e+fx)\log(}{ad^2}$$

Mathematica [A] time = 2.02965, size = 330, normalized size = 1.33

$$\frac{\frac{2if\left(d(e+fx)\mathrm{PolyLog}\left(2,-e^{i(c+dx)}\right)+if\,\mathrm{PolyLog}\left(3,-e^{i(c+dx)}\right)\right)}{d^2} + \frac{2f\left(f\,\mathrm{PolyLog}\left(3,e^{i(c+dx)}\right)-id(e+fx)\mathrm{PolyLog}\left(2,e^{i(c+dx)}\right)\right)}{d^2} + \frac{4f(\cos(c)+i\sin(c))\left(\frac{f(\cos(c)-i(\sin(c}{}\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((e + f*x)^2*Log[1 - E^(I*(c + d*x))] - (e + f*x)^2*Log[1 + E^(I*(c + d*x))
] + ((2*I)*f*(d*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))] + I*f*PolyLog[3, -E^
(I*(c + d*x))]))/d^2 + (2*f*((-I)*d*(e + f*x)*PolyLog[2, E^(I*(c + d*x))] +
 f*PolyLog[3, E^(I*(c + d*x))]))/d^2 + (4*f*(Cos[c] + I*Sin[c])*(((e + f*x)
^2*(Cos[c] - I*Sin[c]))/(2*f) - ((e + f*x)*Log[1 + I*Cos[c + d*x] + Sin[c +
 d*x]]*(1 + I*Cos[c] + Sin[c]))/d + (f*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c
 + d*x]]*(Cos[c] - I*(1 + Sin[c])))/d^2))/(Cos[c] + I*(1 + Sin[c])) - (2*(e
 + f*x)^2*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])))/(a*d)
```

Maple [B] time = 0.137, size = 643, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(I*(d*x+c))+I)+4*f/d^2/a*ln(exp(I*(d*x+c)))
*e-4*f/d^2/a*ln(exp(I*(d*x+c))+I)*e-4*f^2/d^2/a*ln(1-I*exp(I*(d*x+c)))*x-4*
f^2/d^3/a*ln(1-I*exp(I*(d*x+c)))*c-4*f^2/d^3/a*c*ln(exp(I*(d*x+c)))+4*f^2/d
^3/a*c*ln(exp(I*(d*x+c))+I)+2*I*f^2/d/a*x^2+2*I*f^2/d^3/a*c^2+1/a/d^3*f^2*c
^2*ln(exp(I*(d*x+c))-1)+1/a/d*f^2*ln(1-exp(I*(d*x+c)))*x^2-1/a/d^3*f^2*ln(1
-exp(I*(d*x+c)))*c^2-1/a/d*f^2*ln(exp(I*(d*x+c))+1)*x^2+1/a/d*e^2*ln(exp(I*
(d*x+c))-1)-1/a/d*e^2*ln(exp(I*(d*x+c))+1)+4*I*f^2*polylog(2,I*exp(I*(d*x+c
)))/a/d^3+4*I*f^2/d^2/a*c*x+2/a/d^2*ln(1-exp(I*(d*x+c)))*c*e*f+2/a/d*ln(1-e
xp(I*(d*x+c)))*e*f*x-2/a/d*ln(exp(I*(d*x+c))+1)*e*f*x-2/a/d^2*e*f*c*ln(exp(
I*(d*x+c))-1)-2*I/a/d^2*e*f*polylog(2,exp(I*(d*x+c)))+2*I/a/d^2*e*f*polylog
(2,-exp(I*(d*x+c)))-2*I/a/d^2*f^2*polylog(2,exp(I*(d*x+c)))*x+2*I/a/d^2*f^2
*polylog(2,-exp(I*(d*x+c)))*x-2*f^2*polylog(3,-exp(I*(d*x+c)))/a/d^3+2*f^2*
polylog(3,exp(I*(d*x+c)))/a/d^3
```

Maxima [B] time = 1.81162, size = 1904, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*c*e*f*(2/(a*d + a*d*sin(d*x + c)/(cos(d*x + c) + 1)) + log(sin(d*x + c)
/(cos(d*x + c) + 1))/(a*d)) - e^2*(log(sin(d*x + c)/(cos(d*x + c) + 1))/a +
 2/(a + a*sin(d*x + c)/(cos(d*x + c) + 1))) + (4*I*c^2*f^2 + (8*I*d*e*f - 8
*I*c*f^2 + 8*(d*e*f - c*f^2)*cos(d*x + c) + (8*I*d*e*f - 8*I*c*f^2)*sin(d*x
 + c))*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - (8*(d*x + c)*f^2*cos(d*x +
 c) + 8*I*(d*x + c)*f^2*sin(d*x + c) + 8*I*(d*x + c)*f^2)*arctan2(cos(d*x +
 c), sin(d*x + c) + 1) + (2*I*(d*x + c)^2*f^2 + 2*I*c^2*f^2 + (4*I*d*e*f -
4*I*c*f^2)*(d*x + c) + 2*((d*x + c)^2*f^2 + c^2*f^2 + 2*(d*e*f - c*f^2)*(d*
x + c))*cos(d*x + c) + (2*I*(d*x + c)^2*f^2 + 2*I*c^2*f^2 + (4*I*d*e*f - 4*
I*c*f^2)*(d*x + c))*sin(d*x + c))*arctan2(sin(d*x + c), cos(d*x + c) + 1) -
```

```
(2*c^2*f^2*cos(d*x + c) + 2*I*c^2*f^2*sin(d*x + c) + 2*I*c^2*f^2)*arctan2(
sin(d*x + c), cos(d*x + c) - 1) + (2*I*(d*x + c)^2*f^2 + (4*I*d*e*f - 4*I*c
*f^2)*(d*x + c) + 2*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*cos(d*x
 + c) + (2*I*(d*x + c)^2*f^2 + (4*I*d*e*f - 4*I*c*f^2)*(d*x + c))*sin(d*x +
 c))*arctan2(sin(d*x + c), -cos(d*x + c) + 1) - 4*((d*x + c)^2*f^2 + 2*(d*e
*f - c*f^2)*(d*x + c))*cos(d*x + c) - (8*f^2*cos(d*x + c) + 8*I*f^2*sin(d*x
 + c) + 8*I*f^2)*dilog(I*e^(I*d*x + I*c)) + (-4*I*d*e*f - 4*I*(d*x + c)*f^2
 + 4*I*c*f^2 - 4*(d*e*f + (d*x + c)*f^2 - c*f^2)*cos(d*x + c) + (-4*I*d*e*f
 - 4*I*(d*x + c)*f^2 + 4*I*c*f^2)*sin(d*x + c))*dilog(-e^(I*d*x + I*c)) + (
4*I*d*e*f + 4*I*(d*x + c)*f^2 - 4*I*c*f^2 + 4*(d*e*f + (d*x + c)*f^2 - c*f^
2)*cos(d*x + c) + (4*I*d*e*f + 4*I*(d*x + c)*f^2 - 4*I*c*f^2)*sin(d*x + c))
*dilog(e^(I*d*x + I*c)) + ((d*x + c)^2*f^2 + c^2*f^2 + 2*(d*e*f - c*f^2)*(d
*x + c) + (-I*(d*x + c)^2*f^2 - I*c^2*f^2 + (-2*I*d*e*f + 2*I*c*f^2)*(d*x +
 c))*cos(d*x + c) + ((d*x + c)^2*f^2 + c^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c
))*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)
- ((d*x + c)^2*f^2 + c^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c) - (I*(d*x + c)^2
*f^2 + I*c^2*f^2 + (2*I*d*e*f - 2*I*c*f^2)*(d*x + c))*cos(d*x + c) + ((d*x
+ c)^2*f^2 + c^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*sin(d*x + c))*log(cos(d
*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1) + (4*d*e*f + 4*(d*x + c)*f
^2 - 4*c*f^2 + (-4*I*d*e*f - 4*I*(d*x + c)*f^2 + 4*I*c*f^2)*cos(d*x + c) +
4*(d*e*f + (d*x + c)*f^2 - c*f^2)*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*
x + c)^2 + 2*sin(d*x + c) + 1) - 4*(I*f^2*cos(d*x + c) - f^2*sin(d*x + c) -
 f^2)*polylog(3, -e^(I*d*x + I*c)) - 4*(-I*f^2*cos(d*x + c) + f^2*sin(d*x +
 c) + f^2)*polylog(3, e^(I*d*x + I*c)) + (-4*I*(d*x + c)^2*f^2 + (-8*I*d*e*
f + 8*I*c*f^2)*(d*x + c))*sin(d*x + c))/(-2*I*a*d^2*cos(d*x + c) + 2*a*d^2*
sin(d*x + c) + 2*a*d^2))/d
```

Fricas [C] time = 2.61848, size = 4084, normalized size = 16.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*f^2*x^2 + 4*d^2*e*f*x + 2*d^2*e^2 + 2*(d^2*f^2*x^2 + 2*d^2*e*f*x
 + d^2*e^2)*cos(d*x + c) + (-2*I*d*f^2*x - 2*I*d*e*f + (-2*I*d*f^2*x - 2*I*
d*e*f)*cos(d*x + c) + (-2*I*d*f^2*x - 2*I*d*e*f)*sin(d*x + c))*dilog(cos(d*
x + c) + I*sin(d*x + c)) + (2*I*d*f^2*x + 2*I*d*e*f + (2*I*d*f^2*x + 2*I*d*
e*f)*cos(d*x + c) + (2*I*d*f^2*x + 2*I*d*e*f)*sin(d*x + c))*dilog(cos(d*x +
 c) - I*sin(d*x + c)) + (4*I*f^2*cos(d*x + c) + 4*I*f^2*sin(d*x + c) + 4*I*
f^2)*dilog(I*cos(d*x + c) - sin(d*x + c)) + (-4*I*f^2*cos(d*x + c) - 4*I*f^
```

```
2*sin(d*x + c) - 4*I*f^2)*dilog(-I*cos(d*x + c) - sin(d*x + c)) + (-2*I*d*f
^2*x - 2*I*d*e*f + (-2*I*d*f^2*x - 2*I*d*e*f)*cos(d*x + c) + (-2*I*d*f^2*x
- 2*I*d*e*f)*sin(d*x + c))*dilog(-cos(d*x + c) + I*sin(d*x + c)) + (2*I*d*f
^2*x + 2*I*d*e*f + (2*I*d*f^2*x + 2*I*d*e*f)*cos(d*x + c) + (2*I*d*f^2*x +
2*I*d*e*f)*sin(d*x + c))*dilog(-cos(d*x + c) - I*sin(d*x + c)) - (d^2*f^2*x
^2 + 2*d^2*e*f*x + d^2*e^2 + (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*cos(d*x
+ c) + (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*sin(d*x + c))*log(cos(d*x + c)
 + I*sin(d*x + c) + 1) - 4*(d*e*f - c*f^2 + (d*e*f - c*f^2)*cos(d*x + c) +
(d*e*f - c*f^2)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - (d^2
*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 + (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*co
s(d*x + c) + (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*sin(d*x + c))*log(cos(d*
x + c) - I*sin(d*x + c) + 1) - 4*(d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*cos(d
*x + c) + (d*f^2*x + c*f^2)*sin(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c)
 + 1) - 4*(d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^2*x + c*
f^2)*sin(d*x + c))*log(-I*cos(d*x + c) + sin(d*x + c) + 1) + (d^2*e^2 - 2*c
*d*e*f + c^2*f^2 + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*cos(d*x + c) + (d^2*e^2
- 2*c*d*e*f + c^2*f^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x
+ c) + 1/2) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2 + (d^2*e^2 - 2*c*d*e*f + c^2*f
^2)*cos(d*x + c) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*sin(d*x + c))*log(-1/2*c
os(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*
d*e*f - c^2*f^2 + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*cos(d*x
 + c) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*sin(d*x + c))*log
(-cos(d*x + c) + I*sin(d*x + c) + 1) - 4*(d*e*f - c*f^2 + (d*e*f - c*f^2)*c
os(d*x + c) + (d*e*f - c*f^2)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x +
 c) + I) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2 + (d^2*f^2*x^2
+ 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*cos(d*x + c) + (d^2*f^2*x^2 + 2*d^2*e*
f*x + 2*c*d*e*f - c^2*f^2)*sin(d*x + c))*log(-cos(d*x + c) - I*sin(d*x + c)
 + 1) + 2*(f^2*cos(d*x + c) + f^2*sin(d*x + c) + f^2)*polylog(3, cos(d*x +
c) + I*sin(d*x + c)) + 2*(f^2*cos(d*x + c) + f^2*sin(d*x + c) + f^2)*polylo
g(3, cos(d*x + c) - I*sin(d*x + c)) - 2*(f^2*cos(d*x + c) + f^2*sin(d*x + c
) + f^2)*polylog(3, -cos(d*x + c) + I*sin(d*x + c)) - 2*(f^2*cos(d*x + c) +
 f^2*sin(d*x + c) + f^2)*polylog(3, -cos(d*x + c) - I*sin(d*x + c)) - 2*(d^
2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*sin(d*x + c))/(a*d^3*cos(d*x + c) + a*d^
3*sin(d*x + c) + a*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^2 \csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2 \csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx \csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*csc(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*csc
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*csc(c + d*x)/(sin(c + d
*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \csc(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.199 $\int \frac{(e+fx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{ifPolyLog\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{ifPolyLog\left(2,e^{i(c+dx)}\right)}{ad^2}-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{2(e+fx)}{}$$

```
[Out] (-2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((e + f*x)*Cot[c/2 + Pi/4 +
 (d*x)/2])/(a*d) - (2*f*Log[Sin[c/2 + Pi/4 + (d*x)/2]])/(a*d^2) + (I*f*Poly
Log[2, -E^(I*(c + d*x))])/(a*d^2) - (I*f*PolyLog[2, E^(I*(c + d*x))])/(a*d^
2)
```

Rubi [A] time = 0.156537, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4535, 4183, 2279, 2391, 3318, 4184, 3475}

$$\frac{ifPolyLog\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{ifPolyLog\left(2,e^{i(c+dx)}\right)}{ad^2}-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}+\frac{(e+fx)\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)}{ad}-\frac{2(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((e + f*x)*Cot[c/2 + Pi/4 +
 (d*x)/2])/(a*d) - (2*f*Log[Sin[c/2 + Pi/4 + (d*x)/2]])/(a*d^2) + (I*f*Poly
Log[2, -E^(I*(c + d*x))])/(a*d^2) - (I*f*PolyLog[2, E^(I*(c + d*x))])/(a*d^
2)
```


Rule 4535

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csc[c + d*x]^(n - 1))/(a + b*Si
n[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)\csc(c+dx)\,dx}{a} - \int \frac{e+fx}{a+a\sin(c+dx)}\,dx$$
$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{\int (e+fx)\csc^2\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{dx}{2}\right)dx}{2a} - \frac{f\int \log\left(1-e^{i(c+dx)}\right)d}{ad}$$
$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{(if)\,\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}dx, x, e^{i(c+}\right.}{ad^2}$$
$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2} + \frac{if}{}$$

Mathematica [B] time = 1.06422, size = 300, normalized size = 2.24

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(f\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(i\left(\mathrm{PolyLog}\left(2,-e^{i(c+dx)}\right)-\mathrm{PolyLog}\left(2,e^{i(c+d}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-2*d*(e + f*x)*Sin[(c + d*x)/2] + f
*(c + d*x)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 2*f*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + d*e*Log[Tan[(c
 + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - c*f*Log[Tan[(c + d*x)/2
]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + f*((c + d*x)*(Log[1 - E^(I*(c +
d*x))] - Log[1 + E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c + d*x))] - Poly
Log[2, E^(I*(c + d*x))]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(a*d^2*(1
 + Sin[c + d*x]))
```

Maple [B] time = 0.144, size = 245, normalized size = 1.8

$$2\,\frac{fx+e}{da\left(e^{i(dx+c)}+i\right)} - 2\,\frac{f\ln\left(e^{i(dx+c)}+i\right)}{ad^2} + \frac{e\ln\left(e^{i(dx+c)}-1\right)}{da} - \frac{e\ln\left(e^{i(dx+c)}+1\right)}{da} - \frac{fc\ln\left(e^{i(dx+c)}-1\right)}{ad^2} - \frac{if\,\mathrm{polylog}\left(2,e^{i(dx+}\right.}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 2*(f*x+e)/d/a/(exp(I*(d*x+c))+I)-2/d^2/a*f*ln(exp(I*(d*x+c))+I)+1/d/a*e*ln(
exp(I*(d*x+c))-1)-1/d/a*e*ln(exp(I*(d*x+c))+1)-1/d^2/a*f*c*ln(exp(I*(d*x+c)
)-1)-I*f*polylog(2,exp(I*(d*x+c)))/a/d^2+I*f*polylog(2,-exp(I*(d*x+c)))/a/d
^2+2/d^2/a*f*ln(exp(I*(d*x+c)))+1/d/a*ln(1-exp(I*(d*x+c)))*f*x+1/d^2/a*ln(1
-exp(I*(d*x+c)))*c*f-1/d/a*ln(exp(I*(d*x+c))+1)*f*x
```

Maxima [B] time = 1.45336, size = 698, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (4*d*f*x*cos(d*x + c) + 4*I*d*f*x*sin(d*x + c) - 4*I*d*e - (4*f*cos(d*x + c
) + 4*I*f*sin(d*x + c) + 4*I*f)*arctan2(cos(c) + sin(d*x), cos(d*x) + sin(c
)) - (2*I*d*f*x + 2*I*d*e + 2*(d*f*x + d*e)*cos(d*x + c) + (2*I*d*f*x + 2*I
*d*e)*sin(d*x + c))*arctan2(sin(d*x + c), cos(d*x + c) + 1) + (2*d*e*cos(d*
x + c) + 2*I*d*e*sin(d*x + c) + 2*I*d*e)*arctan2(sin(d*x + c), cos(d*x + c)
 - 1) - (2*d*f*x*cos(d*x + c) + 2*I*d*f*x*sin(d*x + c) + 2*I*d*f*x)*arctan2
(sin(d*x + c), -cos(d*x + c) + 1) + (2*f*cos(d*x + c) + 2*I*f*sin(d*x + c)
+ 2*I*f)*dilog(-e^(I*d*x + I*c)) - (2*f*cos(d*x + c) + 2*I*f*sin(d*x + c) +
 2*I*f)*dilog(e^(I*d*x + I*c)) - (d*f*x + d*e + (-I*d*f*x - I*d*e)*cos(d*x
+ c) + (d*f*x + d*e)*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*
cos(d*x + c) + 1) + (d*f*x + d*e - (I*d*f*x + I*d*e)*cos(d*x + c) + (d*f*x
+ d*e)*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) +
 1) + 2*(I*f*cos(d*x + c) - f*sin(d*x + c) - f)*log(cos(d*x)^2 + cos(c)^2 +
 2*cos(c)*sin(d*x) + sin(d*x)^2 + 2*cos(d*x)*sin(c) + sin(c)^2))/(-2*I*a*d^
2*cos(d*x + c) + 2*a*d^2*sin(d*x + c) + 2*a*d^2)
```

Fricas [B] time = 2.10089, size = 1654, normalized size = 12.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*f*x + 2*d*e + 2*(d*f*x + d*e)*cos(d*x + c) + (-I*f*cos(d*x + c) -
I*f*sin(d*x + c) - I*f)*dilog(cos(d*x + c) + I*sin(d*x + c)) + (I*f*cos(d*x
```

```
 + c) + I*f*sin(d*x + c) + I*f)*dilog(cos(d*x + c) - I*sin(d*x + c)) + (-I*
f*cos(d*x + c) - I*f*sin(d*x + c) - I*f)*dilog(-cos(d*x + c) + I*sin(d*x +
c)) + (I*f*cos(d*x + c) + I*f*sin(d*x + c) + I*f)*dilog(-cos(d*x + c) - I*s
in(d*x + c)) - (d*f*x + d*e + (d*f*x + d*e)*cos(d*x + c) + (d*f*x + d*e)*si
n(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + 1) - (d*f*x + d*e + (d*f*x
+ d*e)*cos(d*x + c) + (d*f*x + d*e)*sin(d*x + c))*log(cos(d*x + c) - I*sin(
d*x + c) + 1) + (d*e - c*f + (d*e - c*f)*cos(d*x + c) + (d*e - c*f)*sin(d*x
 + c))*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) + (d*e - c*f + (d*
e - c*f)*cos(d*x + c) + (d*e - c*f)*sin(d*x + c))*log(-1/2*cos(d*x + c) - 1
/2*I*sin(d*x + c) + 1/2) + (d*f*x + c*f + (d*f*x + c*f)*cos(d*x + c) + (d*f
*x + c*f)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + 1) + (d*f*x +
c*f + (d*f*x + c*f)*cos(d*x + c) + (d*f*x + c*f)*sin(d*x + c))*log(-cos(d*x
 + c) - I*sin(d*x + c) + 1) - 2*(f*cos(d*x + c) + f*sin(d*x + c) + f)*log(s
in(d*x + c) + 1) - 2*(d*f*x + d*e)*sin(d*x + c))/(a*d^2*cos(d*x + c) + a*d^
2*sin(d*x + c) + a*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e\csc(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{fx\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e*csc(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f*x*csc(c + d*x)
/(sin(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\csc(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.200 $\int \frac{\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{\cos(c+dx)}{d(a\sin(c+dx)+a)}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + Cos[c + d*x]/(d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.0537851, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2747, 3770, 2648}

$$\frac{\cos(c+dx)}{d(a\sin(c+dx)+a)}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + Cos[c + d*x]/(d*(a + a*Sin[c + d*x]))
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \csc(c+dx)\,dx}{a} - \int \frac{1}{a+a\sin(c+dx)}\,dx$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\cos(c+dx)}{d(a+a\sin(c+dx))}$$

Mathematica [A] time = 0.0660189, size = 48, normalized size = 1.26

$$-\frac{\sec(c+dx)\left(\sin(c+dx)+\sqrt{\cos^2(c+dx)}\tanh^{-1}\left(\sqrt{\cos^2(c+dx)}\right)-1\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((Sec[c + d*x]*(-1 + ArcTanh[Sqrt[Cos[c + d*x]^2]]*Sqrt[Cos[c + d*x]^2] +
Sin[c + d*x]))/(a*d))
```

Maple [A] time = 0.036, size = 40, normalized size = 1.1

$$2\,\frac{1}{da\,(\tan(1/2\,dx+c/2)+1)} + \frac{1}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 2/a/d/(tan(1/2*d*x+1/2*c)+1)+1/a/d*ln(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.00464, size = 69, normalized size = 1.82

$$\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{2}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c)/(cos(d*x + c) + 1))/a + 2/(a + a*sin(d*x + c)/(cos(d*x +
c) + 1)))/d
```

Fricas [B] time = 1.60375, size = 293, normalized size = 7.71

$$-\frac{(\cos(dx+c)+\sin(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(\cos(dx+c)+\sin(dx+c)+1)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2\,(ad\cos(dx+c)+ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(d*x + c) + sin(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(
d*x + c) + sin(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) - 2*cos(d*x + c)
+ 2*sin(d*x + c) - 2)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.18004, size = 51, normalized size = 1.34

$$\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{2}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] (log(abs(tan(1/2*d*x + 1/2*c)))/a + 2/(a*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.201 $\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\csc(c+dx)}{(e+fx)(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Csc[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.05991, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Csc[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 10.959, size = 0, normalized size = 0.

$$\int \frac{\csc(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Csc[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 3.595, size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)}{(fx+e)(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\csc(dx+c)}{afx+ae+(afx+ae)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

3.202 $\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\csc(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Csc[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0552685, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Csc[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 12.5782, size = 0, normalized size = 0.

$$\int \frac{\csc(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Csc[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 7.668, size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)}{(fx+e)^2 (a+a\sin(dx+c))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(dx+c)}{af^2x^2 + 2\,aefx + ae^2 + (af^2x^2 + 2\,aefx + ae^2)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e*f
*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) + 2*
e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.203 $\int \frac{(e+fx)^3\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=463

$$-\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^3}+\frac{6f^2(e+fx)\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}-\frac{6f^2(e}{}$$

```
[Out] ((-2*I)*(e + f*x)^3)/(a*d) + (2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a*d)
 - ((e + f*x)^3*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - ((e + f*x)^3*Cot[c + d*x
])/(a*d) + (6*f*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) + (3*f*(e +
 f*x)^2*Log[1 - E^((2*I)*(c + d*x))])/(a*d^2) - ((3*I)*f*(e + f*x)^2*PolyLo
g[2, -E^(I*(c + d*x))])/(a*d^2) - ((12*I)*f^2*(e + f*x)*PolyLog[2, I*E^(I*(
c + d*x))])/(a*d^3) + ((3*I)*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a*
d^2) - ((3*I)*f^2*(e + f*x)*PolyLog[2, E^((2*I)*(c + d*x))])/(a*d^3) + (6*f
^2*(e + f*x)*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3) + (12*f^3*PolyLog[3, I*E
^(I*(c + d*x))])/(a*d^4) - (6*f^2*(e + f*x)*PolyLog[3, E^(I*(c + d*x))])/(a
*d^3) + (3*f^3*PolyLog[3, E^((2*I)*(c + d*x))])/(2*a*d^4) + ((6*I)*f^3*Poly
Log[4, -E^(I*(c + d*x))])/(a*d^4) - ((6*I)*f^3*PolyLog[4, E^(I*(c + d*x))])
/(a*d^4)
```

Rubi [A] time = 0.775845, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4535, 4184, 3717, 2190, 2531, 2282, 6589, 4183, 6609, 3318}

$$-\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^3}+\frac{6f^2(e+fx)\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}-\frac{6f^2(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-2*I)*(e + f*x)^3)/(a*d) + (2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a*d)
 - ((e + f*x)^3*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - ((e + f*x)^3*Cot[c + d*x
])/(a*d) + (6*f*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) + (3*f*(e +
 f*x)^2*Log[1 - E^((2*I)*(c + d*x))])/(a*d^2) - ((3*I)*f*(e + f*x)^2*PolyLo
g[2, -E^(I*(c + d*x))])/(a*d^2) - ((12*I)*f^2*(e + f*x)*PolyLog[2, I*E^(I*(
c + d*x))])/(a*d^3) + ((3*I)*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a*
d^2) - ((3*I)*f^2*(e + f*x)*PolyLog[2, E^((2*I)*(c + d*x))])/(a*d^3) + (6*f
^2*(e + f*x)*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3) + (12*f^3*PolyLog[3, I*E
^(I*(c + d*x))])/(a*d^4) - (6*f^2*(e + f*x)*PolyLog[3, E^(I*(c + d*x))])/(a
```

```
*d^3) + (3*f^3*PolyLog[3, E^((2*I)*(c + d*x))])/(2*a*d^4) + ((6*I)*f^3*Poly
Log[4, -E^(I*(c + d*x))])/(a*d^4) - ((6*I)*f^3*PolyLog[4, E^(I*(c + d*x))])
/(a*d^4)
```

Rule 4535

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csc[c + d*x]^(n - 1))/(a + b*Si
n[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(e+fx)^3\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\csc^2(c+dx)\,dx}{a} - \int \frac{(e+fx)^3\csc(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{(e+fx)^3\cot(c+dx)}{ad} - \frac{\int (e+fx)^3\csc(c+dx)\,dx}{a} + \frac{(3f)\int (e+fx)^2\cot(c+dx)\,dx}{ad} + \int$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^3\cot(c+dx)}{ad} + \frac{\int (e+fx)^3\csc^2\left(\frac{1}{2}\left(\right.\right.}{2a}$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)^3\cot(c}{ad}$$

$$= -\frac{2i(e+fx)^3}{ad} + \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)^3\cot(}{ad}$$

$$= -\frac{2i(e+fx)^3}{ad} + \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)^3\cot(}{ad}$$

$$= -\frac{2i(e+fx)^3}{ad} + \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)^3\cot(}{ad}$$

$$= -\frac{2i(e+fx)^3}{ad} + \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)^3\cot(}{ad}$$

$$= -\frac{2i(e+fx)^3}{ad} + \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^3\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)^3\cot(}{ad}$$

<u>Mathematica</u> [B] time = 10.7448, size = 1013, normalized size = 2.19

$$\frac{-d^3x^3\log\left(1-e^{-i(c+dx)}\right)f^3 + d^3x^3\log\left(1+e^{-i(c+dx)}\right)f^3 + 3\left(id^2\text{PolyLog}\left(2,-e^{-i(c+dx)}\right)x^2 + 2d\text{PolyLog}\left(3,-e^{-i(c+dx)}\right)x -\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (((-2*I)*d^3*(e + f*x)^3)/(-1 + E^((2*I)*c)) - 3*d^2*e*(d*e - 2*f)*f*x*Log[
1 - E^((-I)*(c + d*x))] - 3*d^2*(d*e - f)*f^2*x^2*Log[1 - E^((-I)*(c + d*x)
)] - d^3*f^3*x^3*Log[1 - E^((-I)*(c + d*x))] + 3*d^2*e*f*(d*e + 2*f)*x*Log[
1 + E^((-I)*(c + d*x))] + 3*d^2*f^2*(d*e + f)*x^2*Log[1 + E^((-I)*(c + d*x)
```

```
)] + d^3*f^3*x^3*Log[1 + E^((-I)*(c + d*x))] + I*d^2*e^2*(d*e - 3*f)*(d*x +
 I*Log[1 - E^(I*(c + d*x))]) + d^2*e^2*(d*e + 3*f)*((-I)*d*x + Log[1 + E^(I
*(c + d*x))]) + (3*I)*d*e*f*(d*e + 2*f)*PolyLog[2, -E^((-I)*(c + d*x))] - (
3*I)*d*e*(d*e - 2*f)*f*PolyLog[2, E^((-I)*(c + d*x))] + 6*f^2*(d*e + f)*(I*
d*x*PolyLog[2, -E^((-I)*(c + d*x))] + PolyLog[3, -E^((-I)*(c + d*x))]) - (6
*I)*(d*e - f)*f^2*(d*x*PolyLog[2, E^((-I)*(c + d*x))] - I*PolyLog[3, E^((-I
)*(c + d*x))]) + 3*f^3*(I*d^2*x^2*PolyLog[2, -E^((-I)*(c + d*x))] + 2*d*x*P
olyLog[3, -E^((-I)*(c + d*x))] - (2*I)*PolyLog[4, -E^((-I)*(c + d*x))]) - (
3*I)*f^3*(d^2*x^2*PolyLog[2, E^((-I)*(c + d*x))] - (2*I)*d*x*PolyLog[3, E^(
(-I)*(c + d*x))] - 2*PolyLog[4, E^((-I)*(c + d*x))])))/(a*d^4) - (6*f*(Cos[c
] + I*Sin[c])*(((e + f*x)^3*(Cos[c] - I*Sin[c]))/(3*f) - ((e + f*x)^2*Log[1
 + I*Cos[c + d*x] + Sin[c + d*x]]*(1 + I*Cos[c] + Sin[c]))/d + (2*f*(d*(e +
 f*x)*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]] - I*f*PolyLog[3, (-I)*Co
s[c + d*x] - Sin[c + d*x]])*(Cos[c] - I*(1 + Sin[c])))/d^3))/(a*d*(Cos[c] +
 I*(1 + Sin[c]))) + (Csc[c/2]*Csc[c/2 + (d*x)/2]*(e^3*Sin[(d*x)/2] + 3*e^2*
f*x*Sin[(d*x)/2] + 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/2]))/(2*a*d
) + (Sec[c/2]*Sec[c/2 + (d*x)/2]*(e^3*Sin[(d*x)/2] + 3*e^2*f*x*Sin[(d*x)/2]
 + 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/2]))/(2*a*d) + (2*(e^3*Sin[
(d*x)/2] + 3*e^2*f*x*Sin[(d*x)/2] + 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[
(d*x)/2]))/(a*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)
/2]))
```

Maple [B] time = 0.307, size = 1705, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -2*(-2*f^3*x^3+I*exp(I*(d*x+c))*f^3*x^3-6*e*f^2*x^2+3*I*exp(I*(d*x+c))*e*f^
2*x^2-6*e^2*f*x+3*I*exp(I*(d*x+c))*e^2*f*x-2*e^3+I*exp(I*(d*x+c))*e^3+f^3*x
^3*exp(2*I*(d*x+c))+3*e*f^2*x^2*exp(2*I*(d*x+c))+3*e^2*f*x*exp(2*I*(d*x+c))
+e^3*exp(2*I*(d*x+c)))/(exp(2*I*(d*x+c))-1)/(exp(I*(d*x+c))+I)/d/a+3/d^2/a*
f^3*ln(exp(I*(d*x+c))+1)*x^2+3/d^2/a*f^3*ln(1-exp(I*(d*x+c)))*x^2-3/d^4/a*f
^3*ln(1-exp(I*(d*x+c)))*c^2+3/d^4/a*f^3*c^2*ln(exp(I*(d*x+c))-1)+3/d^2/a*e^
2*f*ln(exp(I*(d*x+c))-1)+3/d^2/a*e^2*f*ln(exp(I*(d*x+c))+1)-4*I/d/a*f^3*x^3
+8*I/d^4/a*f^3*c^3+12*f^2/d^2/a*e*ln(1-I*exp(I*(d*x+c)))*x-6*I*f^3*polylog(
4,exp(I*(d*x+c)))/a/d^4-1/d/a*e^3*ln(exp(I*(d*x+c))-1)+1/d/a*e^3*ln(exp(I*(
d*x+c))+1)-6/d^3/a*e*f^2*c*ln(exp(I*(d*x+c))-1)-12*f^3/d^4/a*c^2*ln(exp(I*(
d*x+c)))+6*f/d^2/a*ln(exp(I*(d*x+c))+I)*e^2+6*f^3/d^4/a*c^2*ln(exp(I*(d*x+c
))+I)-12*f/d^2/a*ln(exp(I*(d*x+c)))*e^2+3/d^2/a*e^2*f*c*ln(exp(I*(d*x+c))-1
```

```
)-3/d^3/a*e*f^2*c^2*ln(exp(I*(d*x+c))-1)+1/d/a*f^3*ln(exp(I*(d*x+c))+1)*x^3
+6*I*f^3*polylog(4,-exp(I*(d*x+c)))/a/d^4+12*f^2/d^3/a*e*ln(1-I*exp(I*(d*x+
c)))*c+6*f^3/d^2/a*ln(1-I*exp(I*(d*x+c)))*x^2-6*f^3/d^4/a*ln(1-I*exp(I*(d*x
+c)))*c^2+24*f^2/d^3/a*e*c*ln(exp(I*(d*x+c)))-12*f^2/d^3/a*e*c*ln(exp(I*(d*
x+c))+I)-12*I/d^3/a*f^3*polylog(2,I*exp(I*(d*x+c)))*x-12*I/d^3/a*e*f^2*poly
log(2,I*exp(I*(d*x+c)))+6*f^3*polylog(3,-exp(I*(d*x+c)))/a/d^4+6*f^3*polylo
g(3,exp(I*(d*x+c)))/a/d^4-24*I/d^2/a*e*f^2*c*x+6*I/d^2/a*e*f^2*polylog(2,ex
p(I*(d*x+c)))*x-6*I/d^2/a*e*f^2*polylog(2,-exp(I*(d*x+c)))*x-1/d/a*f^3*ln(1
-exp(I*(d*x+c)))*x^3-1/d^4/a*f^3*ln(1-exp(I*(d*x+c)))*c^3+3/d/a*e*f^2*ln(ex
p(I*(d*x+c))+1)*x^2-3/d/a*ln(1-exp(I*(d*x+c)))*e^2*f*x+3/d/a*ln(exp(I*(d*x+
c))+1)*e^2*f*x-3/d/a*e*f^2*ln(1-exp(I*(d*x+c)))*x^2+3/d^3/a*e*f^2*ln(1-exp(
I*(d*x+c)))*c^2-3/d^2/a*ln(1-exp(I*(d*x+c)))*c*e^2*f+12*f^3*polylog(3,I*exp
(I*(d*x+c)))/a/d^4-6/d^3/a*f^3*polylog(3,exp(I*(d*x+c)))*x+6/d^3/a*f^3*poly
log(3,-exp(I*(d*x+c)))*x+1/d^4/a*f^3*c^3*ln(exp(I*(d*x+c))-1)-6/d^3/a*e*f^2
*polylog(3,exp(I*(d*x+c)))+6/d^3/a*e*f^2*polylog(3,-exp(I*(d*x+c)))+6/d^2/a
*e*f^2*ln(1-exp(I*(d*x+c)))*x+6/d^3/a*e*f^2*ln(1-exp(I*(d*x+c)))*c+6/d^2/a*
e*f^2*ln(exp(I*(d*x+c))+1)*x+3*I/d^2/a*e^2*f*polylog(2,exp(I*(d*x+c)))-3*I/
d^2/a*e^2*f*polylog(2,-exp(I*(d*x+c)))-12*I/d/a*e*f^2*x^2-3*I/d^2/a*f^3*pol
ylog(2,-exp(I*(d*x+c)))*x^2-6*I/d^3/a*e*f^2*polylog(2,exp(I*(d*x+c)))-6*I/d
^3/a*e*f^2*polylog(2,-exp(I*(d*x+c)))-12*I/d^3/a*e*f^2*c^2+12*I/d^3/a*f^3*c
^2*x-6*I/d^3/a*f^3*polylog(2,-exp(I*(d*x+c)))*x-6*I/d^3/a*f^3*polylog(2,exp
(I*(d*x+c)))*x+3*I/d^2/a*f^3*polylog(2,exp(I*(d*x+c)))*x^2
```

Maxima [B] time = 15.8953, size = 10242, normalized size = 22.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(3*c*e^2*f*((5*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/(a*d*sin(d*x + c)/(
cos(d*x + c) + 1) + a*d*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 2*log(sin(d*
x + c)/(cos(d*x + c) + 1))/(a*d) - sin(d*x + c)/(a*d*(cos(d*x + c) + 1))) -
 e^3*((5*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/(a*sin(d*x + c)/(cos(d*x + c)
 + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 2*log(sin(d*x + c)/(cos(d*
x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))) + 2*(-24*I*c^2*d*e*f^
2 + 8*I*c^3*f^3 + (-12*I*d^2*e^2*f + 24*I*c*d*e*f^2 - 12*I*c^2*f^3 + 12*(d^
2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(3*d*x + 3*c) + (12*I*d^2*e^2*f - 24*I*
c*d*e*f^2 + 12*I*c^2*f^3)*cos(2*d*x + 2*c) - 12*(d^2*e^2*f - 2*c*d*e*f^2 +
c^2*f^3)*cos(d*x + c) + (12*I*d^2*e^2*f - 24*I*c*d*e*f^2 + 12*I*c^2*f^3)*si
n(3*d*x + 3*c) - 12*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*sin(2*d*x + 2*c) +
```

```
(-12*I*d^2*e^2*f + 24*I*c*d*e*f^2 - 12*I*c^2*f^3)*sin(d*x + c))*arctan2(sin
(d*x + c) + 1, cos(d*x + c)) + (12*I*(d*x + c)^2*f^3 + (24*I*d*e*f^2 - 24*I
*c*f^3)*(d*x + c) - 12*((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*co
s(3*d*x + 3*c) + (-12*I*(d*x + c)^2*f^3 + (-24*I*d*e*f^2 + 24*I*c*f^3)*(d*x
 + c))*cos(2*d*x + 2*c) + 12*((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x +
c))*cos(d*x + c) + (-12*I*(d*x + c)^2*f^3 + (-24*I*d*e*f^2 + 24*I*c*f^3)*(d
*x + c))*sin(3*d*x + 3*c) + 12*((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x
+ c))*sin(2*d*x + 2*c) + (12*I*(d*x + c)^2*f^3 + (24*I*d*e*f^2 - 24*I*c*f^3
)*(d*x + c))*sin(d*x + c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) + (-2*I*
(d*x + c)^3*f^3 - 6*I*d^2*e^2*f + (-6*I*c^2 + 12*I*c)*d*e*f^2 + (2*I*c^3 -
6*I*c^2)*f^3 + (-6*I*d*e*f^2 + (6*I*c - 6*I)*f^3)*(d*x + c)^2 + (-6*I*d^2*e
^2*f + (12*I*c - 12*I)*d*e*f^2 + (-6*I*c^2 + 12*I*c)*f^3)*(d*x + c) + 2*((d
*x + c)^3*f^3 + 3*d^2*e^2*f + 3*(c^2 - 2*c)*d*e*f^2 - (c^3 - 3*c^2)*f^3 + 3
*(d*e*f^2 - (c - 1)*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*(c - 1)*d*e*f^2 + (
c^2 - 2*c)*f^3)*(d*x + c))*cos(3*d*x + 3*c) + (2*I*(d*x + c)^3*f^3 + 6*I*d^
2*e^2*f + (6*I*c^2 - 12*I*c)*d*e*f^2 + (-2*I*c^3 + 6*I*c^2)*f^3 + (6*I*d*e*
f^2 + (-6*I*c + 6*I)*f^3)*(d*x + c)^2 + (6*I*d^2*e^2*f + (-12*I*c + 12*I)*d
*e*f^2 + (6*I*c^2 - 12*I*c)*f^3)*(d*x + c))*cos(2*d*x + 2*c) - 2*((d*x + c)
^3*f^3 + 3*d^2*e^2*f + 3*(c^2 - 2*c)*d*e*f^2 - (c^3 - 3*c^2)*f^3 + 3*(d*e*f
^2 - (c - 1)*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*(c - 1)*d*e*f^2 + (c^2 - 2
*c)*f^3)*(d*x + c))*cos(d*x + c) + (2*I*(d*x + c)^3*f^3 + 6*I*d^2*e^2*f + (
6*I*c^2 - 12*I*c)*d*e*f^2 + (-2*I*c^3 + 6*I*c^2)*f^3 + (6*I*d*e*f^2 + (-6*I
*c + 6*I)*f^3)*(d*x + c)^2 + (6*I*d^2*e^2*f + (-12*I*c + 12*I)*d*e*f^2 + (6
*I*c^2 - 12*I*c)*f^3)*(d*x + c))*sin(3*d*x + 3*c) - 2*((d*x + c)^3*f^3 + 3*
d^2*e^2*f + 3*(c^2 - 2*c)*d*e*f^2 - (c^3 - 3*c^2)*f^3 + 3*(d*e*f^2 - (c - 1
)*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*(c - 1)*d*e*f^2 + (c^2 - 2*c)*f^3)*(d
*x + c))*sin(2*d*x + 2*c) + (-2*I*(d*x + c)^3*f^3 - 6*I*d^2*e^2*f + (-6*I*c
^2 + 12*I*c)*d*e*f^2 + (2*I*c^3 - 6*I*c^2)*f^3 + (-6*I*d*e*f^2 + (6*I*c - 6
*I)*f^3)*(d*x + c)^2 + (-6*I*d^2*e^2*f + (12*I*c - 12*I)*d*e*f^2 + (-6*I*c^
2 + 12*I*c)*f^3)*(d*x + c))*sin(d*x + c))*arctan2(sin(d*x + c), cos(d*x + c
) + 1) + (-6*I*d^2*e^2*f + (6*I*c^2 + 12*I*c)*d*e*f^2 + (-2*I*c^3 - 6*I*c^2
)*f^3 + 2*(3*d^2*e^2*f - 3*(c^2 + 2*c)*d*e*f^2 + (c^3 + 3*c^2)*f^3)*cos(3*d
*x + 3*c) + (6*I*d^2*e^2*f + (-6*I*c^2 - 12*I*c)*d*e*f^2 + (2*I*c^3 + 6*I*c
^2)*f^3)*cos(2*d*x + 2*c) - 2*(3*d^2*e^2*f - 3*(c^2 + 2*c)*d*e*f^2 + (c^3 +
 3*c^2)*f^3)*cos(d*x + c) + (6*I*d^2*e^2*f + (-6*I*c^2 - 12*I*c)*d*e*f^2 +
(2*I*c^3 + 6*I*c^2)*f^3)*sin(3*d*x + 3*c) - 2*(3*d^2*e^2*f - 3*(c^2 + 2*c)*
d*e*f^2 + (c^3 + 3*c^2)*f^3)*sin(2*d*x + 2*c) + (-6*I*d^2*e^2*f + (6*I*c^2
+ 12*I*c)*d*e*f^2 + (-2*I*c^3 - 6*I*c^2)*f^3)*sin(d*x + c))*arctan2(sin(d*x
 + c), cos(d*x + c) - 1) + (-2*I*(d*x + c)^3*f^3 + (-6*I*d*e*f^2 + (6*I*c +
 6*I)*f^3)*(d*x + c)^2 + (-6*I*d^2*e^2*f + (12*I*c + 12*I)*d*e*f^2 + (-6*I*
c^2 - 12*I*c)*f^3)*(d*x + c) + 2*((d*x + c)^3*f^3 + 3*(d*e*f^2 - (c + 1)*f^
3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*(c + 1)*d*e*f^2 + (c^2 + 2*c)*f^3)*(d*x +
 c))*cos(3*d*x + 3*c) + (2*I*(d*x + c)^3*f^3 + (6*I*d*e*f^2 + (-6*I*c - 6*I
)*f^3)*(d*x + c)^2 + (6*I*d^2*e^2*f + (-12*I*c - 12*I)*d*e*f^2 + (6*I*c^2 +
 12*I*c)*f^3)*(d*x + c))*cos(2*d*x + 2*c) - 2*((d*x + c)^3*f^3 + 3*(d*e*f^2
```

```
 - (c + 1)*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*(c + 1)*d*e*f^2 + (c^2 + 2*c
)*f^3)*(d*x + c))*cos(d*x + c) + (2*I*(d*x + c)^3*f^3 + (6*I*d*e*f^2 + (-6*
I*c - 6*I)*f^3)*(d*x + c)^2 + (6*I*d^2*e^2*f + (-12*I*c - 12*I)*d*e*f^2 + (
6*I*c^2 + 12*I*c)*f^3)*(d*x + c))*sin(3*d*x + 3*c) - 2*((d*x + c)^3*f^3 + 3
*(d*e*f^2 - (c + 1)*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*(c + 1)*d*e*f^2 + (
c^2 + 2*c)*f^3)*(d*x + c))*sin(2*d*x + 2*c) + (-2*I*(d*x + c)^3*f^3 + (-6*I
*d*e*f^2 + (6*I*c + 6*I)*f^3)*(d*x + c)^2 + (-6*I*d^2*e^2*f + (12*I*c + 12*
I)*d*e*f^2 + (-6*I*c^2 - 12*I*c)*f^3)*(d*x + c))*sin(d*x + c))*arctan2(sin(
d*x + c), -cos(d*x + c) + 1) - 8*((d*x + c)^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*
x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*(d*x + c))*cos(3*d*x + 3*c
) + (12*I*c^2*d*e*f^2 - 4*I*(d*x + c)^3*f^3 - 4*I*c^3*f^3 + (-12*I*d*e*f^2
+ 12*I*c*f^3)*(d*x + c)^2 + (-12*I*d^2*e^2*f + 24*I*c*d*e*f^2 - 12*I*c^2*f^
3)*(d*x + c))*cos(2*d*x + 2*c) - 4*(3*c^2*d*e*f^2 - (d*x + c)^3*f^3 - c^3*f
^3 - 3*(d*e*f^2 - c*f^3)*(d*x + c)^2 - 3*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3
)*(d*x + c))*cos(d*x + c) + (24*I*d*e*f^2 + 24*I*(d*x + c)*f^3 - 24*I*c*f^3
 - 24*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*cos(3*d*x + 3*c) + (-24*I*d*e*f^2 -
 24*I*(d*x + c)*f^3 + 24*I*c*f^3)*cos(2*d*x + 2*c) + 24*(d*e*f^2 + (d*x + c
)*f^3 - c*f^3)*cos(d*x + c) + (-24*I*d*e*f^2 - 24*I*(d*x + c)*f^3 + 24*I*c*
f^3)*sin(3*d*x + 3*c) + 24*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*sin(2*d*x + 2*
c) + (24*I*d*e*f^2 + 24*I*(d*x + c)*f^3 - 24*I*c*f^3)*sin(d*x + c))*dilog(I
*e^(I*d*x + I*c)) + (6*I*d^2*e^2*f + (-12*I*c + 12*I)*d*e*f^2 + 6*I*(d*x +
c)^2*f^3 + (6*I*c^2 - 12*I*c)*f^3 + (12*I*d*e*f^2 + (-12*I*c + 12*I)*f^3)*(
d*x + c) - 6*(d^2*e^2*f - 2*(c - 1)*d*e*f^2 + (d*x + c)^2*f^3 + (c^2 - 2*c)
*f^3 + 2*(d*e*f^2 - (c - 1)*f^3)*(d*x + c))*cos(3*d*x + 3*c) + (-6*I*d^2*e^
2*f + (12*I*c - 12*I)*d*e*f^2 - 6*I*(d*x + c)^2*f^3 + (-6*I*c^2 + 12*I*c)*f
^3 + (-12*I*d*e*f^2 + (12*I*c - 12*I)*f^3)*(d*x + c))*cos(2*d*x + 2*c) + 6*
(d^2*e^2*f - 2*(c - 1)*d*e*f^2 + (d*x + c)^2*f^3 + (c^2 - 2*c)*f^3 + 2*(d*e
*f^2 - (c - 1)*f^3)*(d*x + c))*cos(d*x + c) + (-6*I*d^2*e^2*f + (12*I*c - 1
2*I)*d*e*f^2 - 6*I*(d*x + c)^2*f^3 + (-6*I*c^2 + 12*I*c)*f^3 + (-12*I*d*e*f
^2 + (12*I*c - 12*I)*f^3)*(d*x + c))*sin(3*d*x + 3*c) + 6*(d^2*e^2*f - 2*(c
 - 1)*d*e*f^2 + (d*x + c)^2*f^3 + (c^2 - 2*c)*f^3 + 2*(d*e*f^2 - (c - 1)*f^
3)*(d*x + c))*sin(2*d*x + 2*c) + (6*I*d^2*e^2*f + (-12*I*c + 12*I)*d*e*f^2
+ 6*I*(d*x + c)^2*f^3 + (6*I*c^2 - 12*I*c)*f^3 + (12*I*d*e*f^2 + (-12*I*c +
 12*I)*f^3)*(d*x + c))*sin(d*x + c))*dilog(-e^(I*d*x + I*c)) + (-6*I*d^2*e^
2*f + (12*I*c + 12*I)*d*e*f^2 - 6*I*(d*x + c)^2*f^3 + (-6*I*c^2 - 12*I*c)*f
^3 + (-12*I*d*e*f^2 + (12*I*c + 12*I)*f^3)*(d*x + c) + 6*(d^2*e^2*f - 2*(c
+ 1)*d*e*f^2 + (d*x + c)^2*f^3 + (c^2 + 2*c)*f^3 + 2*(d*e*f^2 - (c + 1)*f^3
)*(d*x + c))*cos(3*d*x + 3*c) + (6*I*d^2*e^2*f + (-12*I*c - 12*I)*d*e*f^2 +
 6*I*(d*x + c)^2*f^3 + (6*I*c^2 + 12*I*c)*f^3 + (12*I*d*e*f^2 + (-12*I*c -
12*I)*f^3)*(d*x + c))*cos(2*d*x + 2*c) - 6*(d^2*e^2*f - 2*(c + 1)*d*e*f^2 +
 (d*x + c)^2*f^3 + (c^2 + 2*c)*f^3 + 2*(d*e*f^2 - (c + 1)*f^3)*(d*x + c))*c
os(d*x + c) + (6*I*d^2*e^2*f + (-12*I*c - 12*I)*d*e*f^2 + 6*I*(d*x + c)^2*f
^3 + (6*I*c^2 + 12*I*c)*f^3 + (12*I*d*e*f^2 + (-12*I*c - 12*I)*f^3)*(d*x +
c))*sin(3*d*x + 3*c) - 6*(d^2*e^2*f - 2*(c + 1)*d*e*f^2 + (d*x + c)^2*f^3 +
 (c^2 + 2*c)*f^3 + 2*(d*e*f^2 - (c + 1)*f^3)*(d*x + c))*sin(2*d*x + 2*c) +
```

(-6*I*d^2*e^2*f + (12*I*c + 12*I)*d*e*f^2 - 6*I*(d*x + c)^2*f^3 + (-6*I*c^2 - 12*I*c)*f^3 + (-12*I*d*e*f^2 + (12*I*c + 12*I)*f^3)*(d*x + c))*sin(d*x + c))*dilog(e^(I*d*x + I*c)) - ((d*x + c)^3*f^3 + 3*d^2*e^2*f + 3*(c^2 - 2*c)*d*e*f^2 - (c^3 - 3*c^2)*f^3 + 3*(d*e*f^2 - (c - 1)*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*(c - 1)*d*e*f^2 + (c^2 - 2*c)*f^3)*(d*x + c) - (-I*(d*x + c)^3*f^3 - 3*I*d^2*e^2*f + (-3*I*c^2 + 6*I*c)*d*e*f^2 + (I*c^3 - 3*I*c^2)*f^3 + (-3*I*d*e*f^2 + (3*I*c - 3*I)*f^3)*(d*x + c)^2 + (-3*I*d^2*e^2*f + (6*I*c - 6*I)*d*e*f^2 + (-3*I*c^2 + 6*I*c)*f^3)*(d*x + c))*cos(3*d*x + 3*c) - ((d*x + c)^3*f^3 + 3*d^2*e^2*f + 3*(c^2 - 2*c)*d*e*f^2 - (c^3 - 3*c^2)*f^3 + 3*(d*e*f^2 - (c - 1)*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*(c - 1)*d*e*f^2 + (c^2 - 2*c)*f^3)*(d*x + c))*cos(2*d*x + 2*c) - (I*(d*x + c)^3*f^3 + 3*I*d^2*e^2*f + (3*I*c^2 - 6*I*c)*d*e*f^2 + (-I*c^3 + 3*I*c^2)*f^3 + (3*I*d*e*f^2 + (-3*I*c + 3*I)*f^3)*(d*x + c)^2 + (3*I*d^2*e^2*f + (-6*I*c + 6*I)*d*e*f^2 + (3*I*c^2 - 6*I*c)*f^3)*(d*x + c))*cos(d*x + c) - ((d*x + c)^3*f^3 + 3*d^2*e^2*f + 3*(c^2 - 2*c)*d*e*f^2 - (c^3 - 3*c^2)*f^3 + 3*(d*e*f^2 - (c - 1)*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*(c - 1)*d*e*f^2 + (c^2 - 2*c)*f^3)*(d*x + c))*sin(3*d*x + 3*c) - (I*(d*x + c)^3*f^3 + 3*I*d^2*e^2*f + (3*I*c^2 - 6*I*c)*d*e*f^2 + (-I*c^3 + 3*I*c^2)*f^3 + (3*I*d*e*f^2 + (-3*I*c + 3*I)*f^3)*(d*x + c)^2 + (3*I*d^2*e^2*f + (-6*I*c + 6*I)*d*e*f^2 + (3*I*c^2 - 6*I*c)*f^3)*(d*x + c))*sin(2*d*x + 2*c) + ((d*x + c)^3*f^3 + 3*d^2*e^2*f + 3*(c^2 - 2*c)*d*e*f^2 - (c^3 - 3*c^2)*f^3 + 3*(d*e*f^2 - (c - 1)*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*(c - 1)*d*e*f^2 + (c^2 - 2*c)*f^3)*(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1) + ((d*x + c)^3*f^3 - 3*d^2*e^2*f + 3*(c^2 + 2*c)*d*e*f^2 - (c^3 + 3*c^2)*f^3 + 3*(d*e*f^2 - (c + 1)*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*(c + 1)*d*e*f^2 + (c^2 + 2*c)*f^3)*(d*x + c) + (I*(d*x + c)^3*f^3 - 3*I*d^2*e^2*f + (3*I*c^2 + 6*I*c)*d*e*f^2 + (-I*c^3 - 3*I*c^2)*f^3 + (3*I*d*e*f^2 + (-3*I*c - 3*I)*f^3)*(d*x + c)^2 + (3*I*d^2*e^2*f + (-6*I*c - 6*I)*d*e*f^2 + (3*I*c^2 + 6*I*c)*f^3)*(d*x + c))*cos(3*d*x + 3*c) - ((d*x + c)^3*f^3 - 3*d^2*e^2*f + 3*(c^2 + 2*c)*d*e*f^2 - (c^3 + 3*c^2)*f^3 + 3*(d*e*f^2 - (c + 1)*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*(c + 1)*d*e*f^2 + (c^2 + 2*c)*f^3)*(d*x + c))*cos(2*d*x + 2*c) + (-I*(d*x + c)^3*f^3 + 3*I*d^2*e^2*f + (-3*I*c^2 - 6*I*c)*d*e*f^2 + (I*c^3 + 3*I*c^2)*f^3 + (-3*I*d*e*f^2 + (3*I*c + 3*I)*f^3)*(d*x + c)^2 + (-3*I*d^2*e^2*f + (6*I*c + 6*I)*d*e*f^2 + (-3*I*c^2 - 6*I*c)*f^3)*(d*x + c))*cos(d*x + c) - ((d*x + c)^3*f^3 - 3*d^2*e^2*f + 3*(c^2 + 2*c)*d*e*f^2 - (c^3 + 3*c^2)*f^3 + 3*(d*e*f^2 - (c + 1)*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*(c + 1)*d*e*f^2 + (c^2 + 2*c)*f^3)*(d*x + c))*sin(3*d*x + 3*c) + (-I*(d*x + c)^3*f^3 + 3*I*d^2*e^2*f + (-3*I*c^2 - 6*I*c)*d*e*f^2 + (I*c^3 + 3*I*c^2)*f^3 + (-3*I*d*e*f^2 + (3*I*c + 3*I)*f^3)*(d*x + c)^2 + (-3*I*d^2*e^2*f + (6*I*c + 6*I)*d*e*f^2 + (-3*I*c^2 - 6*I*c)*f^3)*(d*x + c))*sin(2*d*x + 2*c) + ((d*x + c)^3*f^3 - 3*d^2*e^2*f + 3*(c^2 + 2*c)*d*e*f^2 - (c^3 + 3*c^2)*f^3 + 3*(d*e*f^2 - (c + 1)*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*(c + 1)*d*e*f^2 + (c^2 + 2*c)*f^3)*(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1) - (6*d^2*e^2*f - 12*c*d*e*f^2 + 6*(d*x + c)^2*f^3 + 6*c^2*f^3 + 12*(d*e*f^2 - c*f^3)*(d*x + c) - (-6*I*d^2*e^2*f + 12*I*c*d*e*f^2

```
 - 6*I*(d*x + c)^2*f^3 - 6*I*c^2*f^3 + (-12*I*d*e*f^2 + 12*I*c*f^3)*(d*x +
c))*cos(3*d*x + 3*c) - 6*(d^2*e^2*f - 2*c*d*e*f^2 + (d*x + c)^2*f^3 + c^2*f
^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*cos(2*d*x + 2*c) - (6*I*d^2*e^2*f - 12*
I*c*d*e*f^2 + 6*I*(d*x + c)^2*f^3 + 6*I*c^2*f^3 + (12*I*d*e*f^2 - 12*I*c*f^
3)*(d*x + c))*cos(d*x + c) - 6*(d^2*e^2*f - 2*c*d*e*f^2 + (d*x + c)^2*f^3 +
 c^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*sin(3*d*x + 3*c) - (6*I*d^2*e^2*f
 - 12*I*c*d*e*f^2 + 6*I*(d*x + c)^2*f^3 + 6*I*c^2*f^3 + (12*I*d*e*f^2 - 12*
I*c*f^3)*(d*x + c))*sin(2*d*x + 2*c) + 6*(d^2*e^2*f - 2*c*d*e*f^2 + (d*x +
c)^2*f^3 + c^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*sin(d*x + c))*log(cos(d
*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + (12*f^3*cos(3*d*x + 3*c)
 + 12*I*f^3*cos(2*d*x + 2*c) - 12*f^3*cos(d*x + c) + 12*I*f^3*sin(3*d*x + 3
*c) - 12*f^3*sin(2*d*x + 2*c) - 12*I*f^3*sin(d*x + c) - 12*I*f^3)*polylog(4
, -e^(I*d*x + I*c)) - (12*f^3*cos(3*d*x + 3*c) + 12*I*f^3*cos(2*d*x + 2*c)
- 12*f^3*cos(d*x + c) + 12*I*f^3*sin(3*d*x + 3*c) - 12*f^3*sin(2*d*x + 2*c)
 - 12*I*f^3*sin(d*x + c) - 12*I*f^3)*polylog(4, e^(I*d*x + I*c)) + (-24*I*f
^3*cos(3*d*x + 3*c) + 24*f^3*cos(2*d*x + 2*c) + 24*I*f^3*cos(d*x + c) + 24*
f^3*sin(3*d*x + 3*c) + 24*I*f^3*sin(2*d*x + 2*c) - 24*f^3*sin(d*x + c) - 24
*f^3)*polylog(3, I*e^(I*d*x + I*c)) - (12*d*e*f^2 + 12*(d*x + c)*f^3 - 12*(
c - 1)*f^3 - (-12*I*d*e*f^2 - 12*I*(d*x + c)*f^3 + (12*I*c - 12*I)*f^3)*cos
(3*d*x + 3*c) - 12*(d*e*f^2 + (d*x + c)*f^3 - (c - 1)*f^3)*cos(2*d*x + 2*c)
 - (12*I*d*e*f^2 + 12*I*(d*x + c)*f^3 + (-12*I*c + 12*I)*f^3)*cos(d*x + c)
- 12*(d*e*f^2 + (d*x + c)*f^3 - (c - 1)*f^3)*sin(3*d*x + 3*c) - (12*I*d*e*f
^2 + 12*I*(d*x + c)*f^3 + (-12*I*c + 12*I)*f^3)*sin(2*d*x + 2*c) + 12*(d*e*
f^2 + (d*x + c)*f^3 - (c - 1)*f^3)*sin(d*x + c))*polylog(3, -e^(I*d*x + I*c
)) + (12*d*e*f^2 + 12*(d*x + c)*f^3 - 12*(c + 1)*f^3 + (12*I*d*e*f^2 + 12*I
*(d*x + c)*f^3 + (-12*I*c - 12*I)*f^3)*cos(3*d*x + 3*c) - 12*(d*e*f^2 + (d*
x + c)*f^3 - (c + 1)*f^3)*cos(2*d*x + 2*c) + (-12*I*d*e*f^2 - 12*I*(d*x + c
)*f^3 + (12*I*c + 12*I)*f^3)*cos(d*x + c) - 12*(d*e*f^2 + (d*x + c)*f^3 - (
c + 1)*f^3)*sin(3*d*x + 3*c) + (-12*I*d*e*f^2 - 12*I*(d*x + c)*f^3 + (12*I*
c + 12*I)*f^3)*sin(2*d*x + 2*c) + 12*(d*e*f^2 + (d*x + c)*f^3 - (c + 1)*f^3
)*sin(d*x + c))*polylog(3, e^(I*d*x + I*c)) + (-8*I*(d*x + c)^3*f^3 + (-24*
I*d*e*f^2 + 24*I*c*f^3)*(d*x + c)^2 + (-24*I*d^2*e^2*f + 48*I*c*d*e*f^2 - 2
4*I*c^2*f^3)*(d*x + c))*sin(3*d*x + 3*c) - 4*(3*c^2*d*e*f^2 - (d*x + c)^3*f
^3 - c^3*f^3 - 3*(d*e*f^2 - c*f^3)*(d*x + c)^2 - 3*(d^2*e^2*f - 2*c*d*e*f^2
 + c^2*f^3)*(d*x + c))*sin(2*d*x + 2*c) + (-12*I*c^2*d*e*f^2 + 4*I*(d*x + c
)^3*f^3 + 4*I*c^3*f^3 + (12*I*d*e*f^2 - 12*I*c*f^3)*(d*x + c)^2 + (12*I*d^2
*e^2*f - 24*I*c*d*e*f^2 + 12*I*c^2*f^3)*(d*x + c))*sin(d*x + c))/(-2*I*a*d^
3*cos(3*d*x + 3*c) + 2*a*d^3*cos(2*d*x + 2*c) + 2*I*a*d^3*cos(d*x + c) + 2*
a*d^3*sin(3*d*x + 3*c) + 2*I*a*d^3*sin(2*d*x + 2*c) - 2*a*d^3*sin(d*x + c)
- 2*a*d^3))/d
```

Fricas [C] time = 3.96971, size = 10928, normalized size = 23.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^3*f^3*x^3 + 6*d^3*e*f^2*x^2 + 6*d^3*e^2*f*x + 2*d^3*e^3 - 4*(d^3*
f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*cos(d*x + c)^2 - 2*(d^
3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*cos(d*x + c) + (3*I*
d^2*f^3*x^2 + 3*I*d^2*e^2*f - 6*I*d*e*f^2 + (-3*I*d^2*f^3*x^2 - 3*I*d^2*e^2
*f + 6*I*d*e*f^2 - 6*I*(d^2*e*f^2 - d*f^3)*x)*cos(d*x + c)^2 + 6*I*(d^2*e*f
^2 - d*f^3)*x + (3*I*d^2*f^3*x^2 + 3*I*d^2*e^2*f - 6*I*d*e*f^2 + 6*I*(d^2*e
*f^2 - d*f^3)*x + (3*I*d^2*f^3*x^2 + 3*I*d^2*e^2*f - 6*I*d*e*f^2 + 6*I*(d^2
*e*f^2 - d*f^3)*x)*cos(d*x + c))*sin(d*x + c))*dilog(cos(d*x + c) + I*sin(d
*x + c)) + (-3*I*d^2*f^3*x^2 - 3*I*d^2*e^2*f + 6*I*d*e*f^2 + (3*I*d^2*f^3*x
^2 + 3*I*d^2*e^2*f - 6*I*d*e*f^2 + 6*I*(d^2*e*f^2 - d*f^3)*x)*cos(d*x + c)^
2 - 6*I*(d^2*e*f^2 - d*f^3)*x + (-3*I*d^2*f^3*x^2 - 3*I*d^2*e^2*f + 6*I*d*e
*f^2 - 6*I*(d^2*e*f^2 - d*f^3)*x + (-3*I*d^2*f^3*x^2 - 3*I*d^2*e^2*f + 6*I*
d*e*f^2 - 6*I*(d^2*e*f^2 - d*f^3)*x)*cos(d*x + c))*sin(d*x + c))*dilog(cos(
d*x + c) - I*sin(d*x + c)) + (-12*I*d*f^3*x - 12*I*d*e*f^2 + (12*I*d*f^3*x
+ 12*I*d*e*f^2)*cos(d*x + c)^2 + (-12*I*d*f^3*x - 12*I*d*e*f^2 + (-12*I*d*f
^3*x - 12*I*d*e*f^2)*cos(d*x + c))*sin(d*x + c))*dilog(I*cos(d*x + c) - sin
(d*x + c)) + (12*I*d*f^3*x + 12*I*d*e*f^2 + (-12*I*d*f^3*x - 12*I*d*e*f^2)*
cos(d*x + c)^2 + (12*I*d*f^3*x + 12*I*d*e*f^2 + (12*I*d*f^3*x + 12*I*d*e*f^
2)*cos(d*x + c))*sin(d*x + c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) + (3*I
*d^2*f^3*x^2 + 3*I*d^2*e^2*f + 6*I*d*e*f^2 + (-3*I*d^2*f^3*x^2 - 3*I*d^2*e^
2*f - 6*I*d*e*f^2 - 6*I*(d^2*e*f^2 + d*f^3)*x)*cos(d*x + c)^2 + 6*I*(d^2*e*
f^2 + d*f^3)*x + (3*I*d^2*f^3*x^2 + 3*I*d^2*e^2*f + 6*I*d*e*f^2 + 6*I*(d^2*
e*f^2 + d*f^3)*x + (3*I*d^2*f^3*x^2 + 3*I*d^2*e^2*f + 6*I*d*e*f^2 + 6*I*(d^
2*e*f^2 + d*f^3)*x)*cos(d*x + c))*sin(d*x + c))*dilog(-cos(d*x + c) + I*sin
(d*x + c)) + (-3*I*d^2*f^3*x^2 - 3*I*d^2*e^2*f - 6*I*d*e*f^2 + (3*I*d^2*f^3
*x^2 + 3*I*d^2*e^2*f + 6*I*d*e*f^2 + 6*I*(d^2*e*f^2 + d*f^3)*x)*cos(d*x + c
)^2 - 6*I*(d^2*e*f^2 + d*f^3)*x + (-3*I*d^2*f^3*x^2 - 3*I*d^2*e^2*f - 6*I*d
*e*f^2 - 6*I*(d^2*e*f^2 + d*f^3)*x + (-3*I*d^2*f^3*x^2 - 3*I*d^2*e^2*f - 6*
I*d*e*f^2 - 6*I*(d^2*e*f^2 + d*f^3)*x)*cos(d*x + c))*sin(d*x + c))*dilog(-c
os(d*x + c) - I*sin(d*x + c)) + (d^3*f^3*x^3 + d^3*e^3 + 3*d^2*e^2*f + 3*(d
^3*e*f^2 + d^2*f^3)*x^2 - (d^3*f^3*x^3 + d^3*e^3 + 3*d^2*e^2*f + 3*(d^3*e*f
^2 + d^2*f^3)*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2)*x)*cos(d*x + c)^2 + 3*(d^3*
e^2*f + 2*d^2*e*f^2)*x + (d^3*f^3*x^3 + d^3*e^3 + 3*d^2*e^2*f + 3*(d^3*e*f^
2 + d^2*f^3)*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2)*x + (d^3*f^3*x^3 + d^3*e^3 +
 3*d^2*e^2*f + 3*(d^3*e*f^2 + d^2*f^3)*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2)*x)
*cos(d*x + c))*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + 1) + 6*(d^
```

```
2*e^2*f - 2*c*d*e*f^2 + c^2*f^3 - (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d
*x + c)^2 + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3 + (d^2*e^2*f - 2*c*d*e*f^2 +
 c^2*f^3)*cos(d*x + c))*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I
) + (d^3*f^3*x^3 + d^3*e^3 + 3*d^2*e^2*f + 3*(d^3*e*f^2 + d^2*f^3)*x^2 - (d
^3*f^3*x^3 + d^3*e^3 + 3*d^2*e^2*f + 3*(d^3*e*f^2 + d^2*f^3)*x^2 + 3*(d^3*e
^2*f + 2*d^2*e*f^2)*x)*cos(d*x + c)^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2)*x + (d^
3*f^3*x^3 + d^3*e^3 + 3*d^2*e^2*f + 3*(d^3*e*f^2 + d^2*f^3)*x^2 + 3*(d^3*e^
2*f + 2*d^2*e*f^2)*x + (d^3*f^3*x^3 + d^3*e^3 + 3*d^2*e^2*f + 3*(d^3*e*f^2
+ d^2*f^3)*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2)*x)*cos(d*x + c))*sin(d*x + c))
*log(cos(d*x + c) - I*sin(d*x + c) + 1) + 6*(d^2*f^3*x^2 + 2*d^2*e*f^2*x +
2*c*d*e*f^2 - c^2*f^3 - (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^
3)*cos(d*x + c)^2 + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3 +
(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c))*sin(d*x
 + c))*log(I*cos(d*x + c) + sin(d*x + c) + 1) + 6*(d^2*f^3*x^2 + 2*d^2*e*f^
2*x + 2*c*d*e*f^2 - c^2*f^3 - (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 -
c^2*f^3)*cos(d*x + c)^2 + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*
f^3 + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c))*s
in(d*x + c))*log(-I*cos(d*x + c) + sin(d*x + c) + 1) - (d^3*e^3 - 3*(c + 1)
*d^2*e^2*f + 3*(c^2 + 2*c)*d*e*f^2 - (c^3 + 3*c^2)*f^3 - (d^3*e^3 - 3*(c +
1)*d^2*e^2*f + 3*(c^2 + 2*c)*d*e*f^2 - (c^3 + 3*c^2)*f^3)*cos(d*x + c)^2 +
(d^3*e^3 - 3*(c + 1)*d^2*e^2*f + 3*(c^2 + 2*c)*d*e*f^2 - (c^3 + 3*c^2)*f^3
+ (d^3*e^3 - 3*(c + 1)*d^2*e^2*f + 3*(c^2 + 2*c)*d*e*f^2 - (c^3 + 3*c^2)*f^
3)*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) +
 1/2) - (d^3*e^3 - 3*(c + 1)*d^2*e^2*f + 3*(c^2 + 2*c)*d*e*f^2 - (c^3 + 3*c
^2)*f^3 - (d^3*e^3 - 3*(c + 1)*d^2*e^2*f + 3*(c^2 + 2*c)*d*e*f^2 - (c^3 + 3
*c^2)*f^3)*cos(d*x + c)^2 + (d^3*e^3 - 3*(c + 1)*d^2*e^2*f + 3*(c^2 + 2*c)*
d*e*f^2 - (c^3 + 3*c^2)*f^3 + (d^3*e^3 - 3*(c + 1)*d^2*e^2*f + 3*(c^2 + 2*c
)*d*e*f^2 - (c^3 + 3*c^2)*f^3)*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d*x
 + c) - 1/2*I*sin(d*x + c) + 1/2) - (d^3*f^3*x^3 + 3*c*d^2*e^2*f - 3*(c^2 +
 2*c)*d*e*f^2 + (c^3 + 3*c^2)*f^3 + 3*(d^3*e*f^2 - d^2*f^3)*x^2 - (d^3*f^3*
x^3 + 3*c*d^2*e^2*f - 3*(c^2 + 2*c)*d*e*f^2 + (c^3 + 3*c^2)*f^3 + 3*(d^3*e*
f^2 - d^2*f^3)*x^2 + 3*(d^3*e^2*f - 2*d^2*e*f^2)*x)*cos(d*x + c)^2 + 3*(d^3
*e^2*f - 2*d^2*e*f^2)*x + (d^3*f^3*x^3 + 3*c*d^2*e^2*f - 3*(c^2 + 2*c)*d*e*
f^2 + (c^3 + 3*c^2)*f^3 + 3*(d^3*e*f^2 - d^2*f^3)*x^2 + 3*(d^3*e^2*f - 2*d^
2*e*f^2)*x + (d^3*f^3*x^3 + 3*c*d^2*e^2*f - 3*(c^2 + 2*c)*d*e*f^2 + (c^3 +
3*c^2)*f^3 + 3*(d^3*e*f^2 - d^2*f^3)*x^2 + 3*(d^3*e^2*f - 2*d^2*e*f^2)*x)*c
os(d*x + c))*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + 1) + 6*(d^2
*e^2*f - 2*c*d*e*f^2 + c^2*f^3 - (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*
x + c)^2 + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3 + (d^2*e^2*f - 2*c*d*e*f^2 +
c^2*f^3)*cos(d*x + c))*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I
) - (d^3*f^3*x^3 + 3*c*d^2*e^2*f - 3*(c^2 + 2*c)*d*e*f^2 + (c^3 + 3*c^2)*f^
3 + 3*(d^3*e*f^2 - d^2*f^3)*x^2 - (d^3*f^3*x^3 + 3*c*d^2*e^2*f - 3*(c^2 + 2
*c)*d*e*f^2 + (c^3 + 3*c^2)*f^3 + 3*(d^3*e*f^2 - d^2*f^3)*x^2 + 3*(d^3*e^2*
f - 2*d^2*e*f^2)*x)*cos(d*x + c)^2 + 3*(d^3*e^2*f - 2*d^2*e*f^2)*x + (d^3*f
^3*x^3 + 3*c*d^2*e^2*f - 3*(c^2 + 2*c)*d*e*f^2 + (c^3 + 3*c^2)*f^3 + 3*(d^3
```

```
*e*f^2 - d^2*f^3)*x^2 + 3*(d^3*e^2*f - 2*d^2*e*f^2)*x + (d^3*f^3*x^3 + 3*c*
d^2*e^2*f - 3*(c^2 + 2*c)*d*e*f^2 + (c^3 + 3*c^2)*f^3 + 3*(d^3*e*f^2 - d^2*
f^3)*x^2 + 3*(d^3*e^2*f - 2*d^2*e*f^2)*x)*cos(d*x + c))*sin(d*x + c))*log(-
cos(d*x + c) - I*sin(d*x + c) + 1) + (6*I*f^3*cos(d*x + c)^2 - 6*I*f^3 + (-
6*I*f^3*cos(d*x + c) - 6*I*f^3)*sin(d*x + c))*polylog(4, cos(d*x + c) + I*s
in(d*x + c)) + (-6*I*f^3*cos(d*x + c)^2 + 6*I*f^3 + (6*I*f^3*cos(d*x + c) +
 6*I*f^3)*sin(d*x + c))*polylog(4, cos(d*x + c) - I*sin(d*x + c)) + (6*I*f^
3*cos(d*x + c)^2 - 6*I*f^3 + (-6*I*f^3*cos(d*x + c) - 6*I*f^3)*sin(d*x + c)
)*polylog(4, -cos(d*x + c) + I*sin(d*x + c)) + (-6*I*f^3*cos(d*x + c)^2 + 6
*I*f^3 + (6*I*f^3*cos(d*x + c) + 6*I*f^3)*sin(d*x + c))*polylog(4, -cos(d*x
 + c) - I*sin(d*x + c)) - 6*(d*f^3*x + d*e*f^2 - f^3 - (d*f^3*x + d*e*f^2 -
 f^3)*cos(d*x + c)^2 + (d*f^3*x + d*e*f^2 - f^3 + (d*f^3*x + d*e*f^2 - f^3)
*cos(d*x + c))*sin(d*x + c))*polylog(3, cos(d*x + c) + I*sin(d*x + c)) - 6*
(d*f^3*x + d*e*f^2 - f^3 - (d*f^3*x + d*e*f^2 - f^3)*cos(d*x + c)^2 + (d*f^
3*x + d*e*f^2 - f^3 + (d*f^3*x + d*e*f^2 - f^3)*cos(d*x + c))*sin(d*x + c))
*polylog(3, cos(d*x + c) - I*sin(d*x + c)) - 12*(f^3*cos(d*x + c)^2 - f^3 -
 (f^3*cos(d*x + c) + f^3)*sin(d*x + c))*polylog(3, I*cos(d*x + c) - sin(d*x
 + c)) - 12*(f^3*cos(d*x + c)^2 - f^3 - (f^3*cos(d*x + c) + f^3)*sin(d*x +
c))*polylog(3, -I*cos(d*x + c) - sin(d*x + c)) + 6*(d*f^3*x + d*e*f^2 + f^3
 - (d*f^3*x + d*e*f^2 + f^3)*cos(d*x + c)^2 + (d*f^3*x + d*e*f^2 + f^3 + (d
*f^3*x + d*e*f^2 + f^3)*cos(d*x + c))*sin(d*x + c))*polylog(3, -cos(d*x + c
) + I*sin(d*x + c)) + 6*(d*f^3*x + d*e*f^2 + f^3 - (d*f^3*x + d*e*f^2 + f^3
)*cos(d*x + c)^2 + (d*f^3*x + d*e*f^2 + f^3 + (d*f^3*x + d*e*f^2 + f^3)*cos
(d*x + c))*sin(d*x + c))*polylog(3, -cos(d*x + c) - I*sin(d*x + c)) - 2*(d^
3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3 + 2*(d^3*f^3*x^3 + 3*
d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*cos(d*x + c))*sin(d*x + c))/(a*d^4
*cos(d*x + c)^2 - a*d^4 - (a*d^4*cos(d*x + c) + a*d^4)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.204 $\int \frac{(e+fx)^2\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=327

$$-\frac{2if(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}+\frac{2if(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}-\frac{4if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{if^2\text{PolyLog}\left(2,e\right.}{ad^3}$$

```
[Out] ((-2*I)*(e + f*x)^2)/(a*d) + (2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d)
 - ((e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - ((e + f*x)^2*Cot[c + d*x
])/(a*d) + (4*f*(e + f*x)*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) + (2*f*(e + f
*x)*Log[1 - E^((2*I)*(c + d*x))])/(a*d^2) - ((2*I)*f*(e + f*x)*PolyLog[2, -
E^(I*(c + d*x))])/(a*d^2) - ((4*I)*f^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^
3) + ((2*I)*f*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) - (I*f^2*PolyL
og[2, E^((2*I)*(c + d*x))])/(a*d^3) + (2*f^2*PolyLog[3, -E^(I*(c + d*x))])/
(a*d^3) - (2*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3)
```

Rubi [A] time = 0.508825, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {4535, 4184, 3717, 2190, 2279, 2391, 4183, 2531, 2282, 6589, 3318}

$$-\frac{2if(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}+\frac{2if(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}-\frac{4if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{if^2\text{PolyLog}\left(2,e\right.}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```


```
[Out] ((-2*I)*(e + f*x)^2)/(a*d) + (2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d)
 - ((e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)/2])/(a*d) - ((e + f*x)^2*Cot[c + d*x
])/(a*d) + (4*f*(e + f*x)*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) + (2*f*(e + f
*x)*Log[1 - E^((2*I)*(c + d*x))])/(a*d^2) - ((2*I)*f*(e + f*x)*PolyLog[2, -
E^(I*(c + d*x))])/(a*d^2) - ((4*I)*f^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^
3) + ((2*I)*f*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) - (I*f^2*PolyL
og[2, E^((2*I)*(c + d*x))])/(a*d^3) + (2*f^2*PolyLog[3, -E^(I*(c + d*x))])/
(a*d^3) - (2*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3)
```


Rule 4535

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
```

```
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csc[c + d*x]^(n - 1))/(a + b*Si
n[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\csc^2(c+dx)\,dx}{a} - \int \frac{(e+fx)^2\csc(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= -\frac{(e+fx)^2\cot(c+dx)}{ad} - \frac{\int (e+fx)^2\csc(c+dx)\,dx}{a} + \frac{(2f)\int (e+fx)\cot(c+dx)\,dx}{ad} + \int$$
$$= -\frac{i(e+fx)^2}{ad} + \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^2\cot(c+dx)}{ad} + \frac{\int (e+fx)^2\csc^2\left(\frac{1}{2}\left(\right.\right.}{2a}$$
$$= -\frac{i(e+fx)^2}{ad} + \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)^2\cot(c}{ad}$$
$$= -\frac{2i(e+fx)^2}{ad} + \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)^2\cot(}{ad}$$
$$= -\frac{2i(e+fx)^2}{ad} + \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)^2\cot(}{ad}$$
$$= -\frac{2i(e+fx)^2}{ad} + \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)^2\cot(}{ad}$$
$$= -\frac{2i(e+fx)^2}{ad} + \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)^2\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)^2\cot(}{ad}$$

Mathematica [B] time = 8.33988, size = 693, normalized size = 2.12

$$\frac{2if(de+f)\text{PolyLog}\left(2,-e^{-i(c+dx)}\right) - 2if(de-f)\text{PolyLog}\left(2,e^{-i(c+dx)}\right) + 2f^2\left(idx\text{PolyLog}\left(2,-e^{-i(c+dx)}\right) + \text{PolyLog}\left(3,\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (((-2*I)*d^2*(e + f*x)^2)/(-1 + E^((2*I)*c)) - 2*d*(d*e - f)*f*x*Log[1 - E^
((-I)*(c + d*x))] - d^2*f^2*x^2*Log[1 - E^((-I)*(c + d*x))] + 2*d*f*(d*e +
f)*x*Log[1 + E^((-I)*(c + d*x))] + d^2*f^2*x^2*Log[1 + E^((-I)*(c + d*x))]
+ I*d*e*(d*e - 2*f)*(d*x + I*Log[1 - E^(I*(c + d*x))]) + d*e*(d*e + 2*f)*((
-I)*d*x + Log[1 + E^(I*(c + d*x))]) + (2*I)*f*(d*e + f)*PolyLog[2, -E^((-I)
*(c + d*x))] - (2*I)*(d*e - f)*f*PolyLog[2, E^((-I)*(c + d*x))] + 2*f^2*(I*
d*x*PolyLog[2, -E^((-I)*(c + d*x))] + PolyLog[3, -E^((-I)*(c + d*x))]) - (2
```

```
*I)*f^2*(d*x*PolyLog[2, E^((-I)*(c + d*x))] - I*PolyLog[3, E^((-I)*(c + d*x
))]))/(a*d^3) - (4*f*(Cos[c] + I*Sin[c])*(((e + f*x)^2*(Cos[c] - I*Sin[c]))
/(2*f) - ((e + f*x)*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(1 + I*Cos[c] +
Sin[c]))/d + (f*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*(1
 + Sin[c])))/d^2))/(a*d*(Cos[c] + I*(1 + Sin[c]))) + (Csc[c/2]*Csc[c/2 + (d
*x)/2]*(e^2*Sin[(d*x)/2] + 2*e*f*x*Sin[(d*x)/2] + f^2*x^2*Sin[(d*x)/2]))/(2
*a*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]*(e^2*Sin[(d*x)/2] + 2*e*f*x*Sin[(d*x)/
2] + f^2*x^2*Sin[(d*x)/2]))/(2*a*d) + (2*(e^2*Sin[(d*x)/2] + 2*e*f*x*Sin[(d
*x)/2] + f^2*x^2*Sin[(d*x)/2]))/(a*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] time = 0.187, size = 942, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -2*(-2*f^2*x^2+I*exp(I*(d*x+c))*f^2*x^2-4*f*e*x+2*I*exp(I*(d*x+c))*e*f*x-2*
e^2+I*exp(I*(d*x+c))*e^2+f^2*x^2*exp(2*I*(d*x+c))+2*e*f*x*exp(2*I*(d*x+c))+
e^2*exp(2*I*(d*x+c)))/(exp(2*I*(d*x+c))-1)/(exp(I*(d*x+c))+I)/d/a-4*I*f^2*p
olylog(2,I*exp(I*(d*x+c)))/a/d^3-8*f/d^2/a*ln(exp(I*(d*x+c)))*e+4*f/d^2/a*l
n(exp(I*(d*x+c))+I)*e+4*f^2/d^2/a*ln(1-I*exp(I*(d*x+c)))*x+4*f^2/d^3/a*ln(1
-I*exp(I*(d*x+c)))*c+8*f^2/d^3/a*c*ln(exp(I*(d*x+c)))-4*f^2/d^3/a*c*ln(exp(
I*(d*x+c))+I)-1/a/d^3*f^2*c^2*ln(exp(I*(d*x+c))-1)-1/a/d*f^2*ln(1-exp(I*(d*
x+c)))*x^2+1/a/d^3*f^2*ln(1-exp(I*(d*x+c)))*c^2+1/a/d*f^2*ln(exp(I*(d*x+c))
+1)*x^2-1/a/d*e^2*ln(exp(I*(d*x+c))-1)+1/a/d*e^2*ln(exp(I*(d*x+c))+1)-2/a/d
^2*ln(1-exp(I*(d*x+c)))*c*e*f-2/a/d*ln(1-exp(I*(d*x+c)))*e*f*x+2/a/d*ln(exp
(I*(d*x+c))+1)*e*f*x+2/a/d^2*e*f*c*ln(exp(I*(d*x+c))-1)+2*f^2*polylog(3,-ex
p(I*(d*x+c)))/a/d^3-2*f^2*polylog(3,exp(I*(d*x+c)))/a/d^3-2*I*f^2*polylog(2
,exp(I*(d*x+c)))/a/d^3+2*I/a/d^2*f^2*polylog(2,exp(I*(d*x+c)))*x-2*I/a/d^2*
f^2*polylog(2,-exp(I*(d*x+c)))*x-8*I/a/d^2*f^2*c*x+2*I/a/d^2*e*f*polylog(2,
exp(I*(d*x+c)))-2*I/a/d^2*e*f*polylog(2,-exp(I*(d*x+c)))+2/a/d^2*f^2*ln(exp
(I*(d*x+c))+1)*x-2/a/d^3*f^2*c*ln(exp(I*(d*x+c))-1)+2/a/d^2*e*f*ln(exp(I*(d
*x+c))-1)+2/a/d^2*e*f*ln(exp(I*(d*x+c))+1)-4*I/a/d^3*f^2*c^2-2*I/a/d^3*f^2*
polylog(2,-exp(I*(d*x+c)))-4*I/a/d*f^2*x^2+2/a/d^2*f^2*ln(1-exp(I*(d*x+c)))
*x+2/a/d^3*f^2*ln(1-exp(I*(d*x+c)))*c
```

Maxima [B] time = 4.0475, size = 5003, normalized size = 15.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c*e*f*((5*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/(a*d*sin(d*x + c)/(co
s(d*x + c) + 1) + a*d*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 2*log(sin(d*x
+ c)/(cos(d*x + c) + 1))/(a*d) - sin(d*x + c)/(a*d*(cos(d*x + c) + 1))) - e
^2*((5*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/(a*sin(d*x + c)/(cos(d*x + c) +
 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 2*log(sin(d*x + c)/(cos(d*x
+ c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))) + 2*(-8*I*c^2*f^2 + (-8
*I*d*e*f + 8*I*c*f^2 + 8*(d*e*f - c*f^2)*cos(3*d*x + 3*c) + (8*I*d*e*f - 8*
I*c*f^2)*cos(2*d*x + 2*c) - 8*(d*e*f - c*f^2)*cos(d*x + c) + (8*I*d*e*f - 8
*I*c*f^2)*sin(3*d*x + 3*c) - 8*(d*e*f - c*f^2)*sin(2*d*x + 2*c) + (-8*I*d*e
*f + 8*I*c*f^2)*sin(d*x + c))*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - (8*
(d*x + c)*f^2*cos(3*d*x + 3*c) + 8*I*(d*x + c)*f^2*cos(2*d*x + 2*c) - 8*(d*
x + c)*f^2*cos(d*x + c) + 8*I*(d*x + c)*f^2*sin(3*d*x + 3*c) - 8*(d*x + c)*
f^2*sin(2*d*x + 2*c) - 8*I*(d*x + c)*f^2*sin(d*x + c) - 8*I*(d*x + c)*f^2)*
arctan2(cos(d*x + c), sin(d*x + c) + 1) + (-2*I*(d*x + c)^2*f^2 - 4*I*d*e*f
 + (-2*I*c^2 + 4*I*c)*f^2 + (-4*I*d*e*f + (4*I*c - 4*I)*f^2)*(d*x + c) + 2*
((d*x + c)^2*f^2 + 2*d*e*f + (c^2 - 2*c)*f^2 + 2*(d*e*f - (c - 1)*f^2)*(d*x
 + c))*cos(3*d*x + 3*c) + (2*I*(d*x + c)^2*f^2 + 4*I*d*e*f + (2*I*c^2 - 4*I
*c)*f^2 + (4*I*d*e*f + (-4*I*c + 4*I)*f^2)*(d*x + c))*cos(2*d*x + 2*c) - 2*
((d*x + c)^2*f^2 + 2*d*e*f + (c^2 - 2*c)*f^2 + 2*(d*e*f - (c - 1)*f^2)*(d*x
 + c))*cos(d*x + c) + (2*I*(d*x + c)^2*f^2 + 4*I*d*e*f + (2*I*c^2 - 4*I*c)*
f^2 + (4*I*d*e*f + (-4*I*c + 4*I)*f^2)*(d*x + c))*sin(3*d*x + 3*c) - 2*((d*
x + c)^2*f^2 + 2*d*e*f + (c^2 - 2*c)*f^2 + 2*(d*e*f - (c - 1)*f^2)*(d*x + c
))*sin(2*d*x + 2*c) + (-2*I*(d*x + c)^2*f^2 - 4*I*d*e*f + (-2*I*c^2 + 4*I*c
)*f^2 + (-4*I*d*e*f + (4*I*c - 4*I)*f^2)*(d*x + c))*sin(d*x + c))*arctan2(s
in(d*x + c), cos(d*x + c) + 1) + (-4*I*d*e*f + (2*I*c^2 + 4*I*c)*f^2 + 2*(2
*d*e*f - (c^2 + 2*c)*f^2)*cos(3*d*x + 3*c) + (4*I*d*e*f + (-2*I*c^2 - 4*I*c
)*f^2)*cos(2*d*x + 2*c) - 2*(2*d*e*f - (c^2 + 2*c)*f^2)*cos(d*x + c) + (4*I
*d*e*f + (-2*I*c^2 - 4*I*c)*f^2)*sin(3*d*x + 3*c) - 2*(2*d*e*f - (c^2 + 2*c
)*f^2)*sin(2*d*x + 2*c) + (-4*I*d*e*f + (2*I*c^2 + 4*I*c)*f^2)*sin(d*x + c)
)*arctan2(sin(d*x + c), cos(d*x + c) - 1) + (-2*I*(d*x + c)^2*f^2 + (-4*I*d
*e*f + (4*I*c + 4*I)*f^2)*(d*x + c) + 2*((d*x + c)^2*f^2 + 2*(d*e*f - (c +
1)*f^2)*(d*x + c))*cos(3*d*x + 3*c) + (2*I*(d*x + c)^2*f^2 + (4*I*d*e*f + (
-4*I*c - 4*I)*f^2)*(d*x + c))*cos(2*d*x + 2*c) - 2*((d*x + c)^2*f^2 + 2*(d*
e*f - (c + 1)*f^2)*(d*x + c))*cos(d*x + c) + (2*I*(d*x + c)^2*f^2 + (4*I*d*
e*f + (-4*I*c - 4*I)*f^2)*(d*x + c))*sin(3*d*x + 3*c) - 2*((d*x + c)^2*f^2
+ 2*(d*e*f - (c + 1)*f^2)*(d*x + c))*sin(2*d*x + 2*c) + (-2*I*(d*x + c)^2*f
```

```
^2 + (-4*I*d*e*f + (4*I*c + 4*I)*f^2)*(d*x + c))*sin(d*x + c))*arctan2(sin(
d*x + c), -cos(d*x + c) + 1) - 8*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x
+ c))*cos(3*d*x + 3*c) + (-4*I*(d*x + c)^2*f^2 + 4*I*c^2*f^2 + (-8*I*d*e*f
+ 8*I*c*f^2)*(d*x + c))*cos(2*d*x + 2*c) + 4*((d*x + c)^2*f^2 - c^2*f^2 + 2
*(d*e*f - c*f^2)*(d*x + c))*cos(d*x + c) - (8*f^2*cos(3*d*x + 3*c) + 8*I*f^
2*cos(2*d*x + 2*c) - 8*f^2*cos(d*x + c) + 8*I*f^2*sin(3*d*x + 3*c) - 8*f^2*
sin(2*d*x + 2*c) - 8*I*f^2*sin(d*x + c) - 8*I*f^2)*dilog(I*e^(I*d*x + I*c))
 + (4*I*d*e*f + 4*I*(d*x + c)*f^2 + (-4*I*c + 4*I)*f^2 - 4*(d*e*f + (d*x +
c)*f^2 - (c - 1)*f^2)*cos(3*d*x + 3*c) + (-4*I*d*e*f - 4*I*(d*x + c)*f^2 +
(4*I*c - 4*I)*f^2)*cos(2*d*x + 2*c) + 4*(d*e*f + (d*x + c)*f^2 - (c - 1)*f^
2)*cos(d*x + c) + (-4*I*d*e*f - 4*I*(d*x + c)*f^2 + (4*I*c - 4*I)*f^2)*sin(
3*d*x + 3*c) + 4*(d*e*f + (d*x + c)*f^2 - (c - 1)*f^2)*sin(2*d*x + 2*c) + (
4*I*d*e*f + 4*I*(d*x + c)*f^2 + (-4*I*c + 4*I)*f^2)*sin(d*x + c))*dilog(-e^
(I*d*x + I*c)) + (-4*I*d*e*f - 4*I*(d*x + c)*f^2 + (4*I*c + 4*I)*f^2 + 4*(d
*e*f + (d*x + c)*f^2 - (c + 1)*f^2)*cos(3*d*x + 3*c) + (4*I*d*e*f + 4*I*(d*
x + c)*f^2 + (-4*I*c - 4*I)*f^2)*cos(2*d*x + 2*c) - 4*(d*e*f + (d*x + c)*f^
2 - (c + 1)*f^2)*cos(d*x + c) + (4*I*d*e*f + 4*I*(d*x + c)*f^2 + (-4*I*c -
4*I)*f^2)*sin(3*d*x + 3*c) - 4*(d*e*f + (d*x + c)*f^2 - (c + 1)*f^2)*sin(2*
d*x + 2*c) + (-4*I*d*e*f - 4*I*(d*x + c)*f^2 + (4*I*c + 4*I)*f^2)*sin(d*x +
 c))*dilog(e^(I*d*x + I*c)) - ((d*x + c)^2*f^2 + 2*d*e*f + (c^2 - 2*c)*f^2
+ 2*(d*e*f - (c - 1)*f^2)*(d*x + c) - (-I*(d*x + c)^2*f^2 - 2*I*d*e*f + (-I
*c^2 + 2*I*c)*f^2 + (-2*I*d*e*f + (2*I*c - 2*I)*f^2)*(d*x + c))*cos(3*d*x +
 3*c) - ((d*x + c)^2*f^2 + 2*d*e*f + (c^2 - 2*c)*f^2 + 2*(d*e*f - (c - 1)*f
^2)*(d*x + c))*cos(2*d*x + 2*c) - (I*(d*x + c)^2*f^2 + 2*I*d*e*f + (I*c^2 -
 2*I*c)*f^2 + (2*I*d*e*f + (-2*I*c + 2*I)*f^2)*(d*x + c))*cos(d*x + c) - ((
d*x + c)^2*f^2 + 2*d*e*f + (c^2 - 2*c)*f^2 + 2*(d*e*f - (c - 1)*f^2)*(d*x +
 c))*sin(3*d*x + 3*c) - (I*(d*x + c)^2*f^2 + 2*I*d*e*f + (I*c^2 - 2*I*c)*f^
2 + (2*I*d*e*f + (-2*I*c + 2*I)*f^2)*(d*x + c))*sin(2*d*x + 2*c) + ((d*x +
c)^2*f^2 + 2*d*e*f + (c^2 - 2*c)*f^2 + 2*(d*e*f - (c - 1)*f^2)*(d*x + c))*s
in(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1) + ((
d*x + c)^2*f^2 - 2*d*e*f + (c^2 + 2*c)*f^2 + 2*(d*e*f - (c + 1)*f^2)*(d*x +
 c) + (I*(d*x + c)^2*f^2 - 2*I*d*e*f + (I*c^2 + 2*I*c)*f^2 + (2*I*d*e*f + (
-2*I*c - 2*I)*f^2)*(d*x + c))*cos(3*d*x + 3*c) - ((d*x + c)^2*f^2 - 2*d*e*f
 + (c^2 + 2*c)*f^2 + 2*(d*e*f - (c + 1)*f^2)*(d*x + c))*cos(2*d*x + 2*c) +
(-I*(d*x + c)^2*f^2 + 2*I*d*e*f + (-I*c^2 - 2*I*c)*f^2 + (-2*I*d*e*f + (2*I
*c + 2*I)*f^2)*(d*x + c))*cos(d*x + c) - ((d*x + c)^2*f^2 - 2*d*e*f + (c^2
+ 2*c)*f^2 + 2*(d*e*f - (c + 1)*f^2)*(d*x + c))*sin(3*d*x + 3*c) + (-I*(d*x
 + c)^2*f^2 + 2*I*d*e*f + (-I*c^2 - 2*I*c)*f^2 + (-2*I*d*e*f + (2*I*c + 2*I
)*f^2)*(d*x + c))*sin(2*d*x + 2*c) + ((d*x + c)^2*f^2 - 2*d*e*f + (c^2 + 2*
c)*f^2 + 2*(d*e*f - (c + 1)*f^2)*(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^
2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1) - (4*d*e*f + 4*(d*x + c)*f^2 - 4*c
*f^2 - (-4*I*d*e*f - 4*I*(d*x + c)*f^2 + 4*I*c*f^2)*cos(3*d*x + 3*c) - 4*(d
*e*f + (d*x + c)*f^2 - c*f^2)*cos(2*d*x + 2*c) - (4*I*d*e*f + 4*I*(d*x + c)
*f^2 - 4*I*c*f^2)*cos(d*x + c) - 4*(d*e*f + (d*x + c)*f^2 - c*f^2)*sin(3*d*
x + 3*c) - (4*I*d*e*f + 4*I*(d*x + c)*f^2 - 4*I*c*f^2)*sin(2*d*x + 2*c) + 4
```

```
*(d*e*f + (d*x + c)*f^2 - c*f^2)*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x
 + c)^2 + 2*sin(d*x + c) + 1) + (-4*I*f^2*cos(3*d*x + 3*c) + 4*f^2*cos(2*d*
x + 2*c) + 4*I*f^2*cos(d*x + c) + 4*f^2*sin(3*d*x + 3*c) + 4*I*f^2*sin(2*d*
x + 2*c) - 4*f^2*sin(d*x + c) - 4*f^2)*polylog(3, -e^(I*d*x + I*c)) + (4*I*
f^2*cos(3*d*x + 3*c) - 4*f^2*cos(2*d*x + 2*c) - 4*I*f^2*cos(d*x + c) - 4*f^
2*sin(3*d*x + 3*c) - 4*I*f^2*sin(2*d*x + 2*c) + 4*f^2*sin(d*x + c) + 4*f^2)
*polylog(3, e^(I*d*x + I*c)) + (-8*I*(d*x + c)^2*f^2 + (-16*I*d*e*f + 16*I*
c*f^2)*(d*x + c))*sin(3*d*x + 3*c) + 4*((d*x + c)^2*f^2 - c^2*f^2 + 2*(d*e*
f - c*f^2)*(d*x + c))*sin(2*d*x + 2*c) + (4*I*(d*x + c)^2*f^2 - 4*I*c^2*f^2
 + (8*I*d*e*f - 8*I*c*f^2)*(d*x + c))*sin(d*x + c))/(-2*I*a*d^2*cos(3*d*x +
 3*c) + 2*a*d^2*cos(2*d*x + 2*c) + 2*I*a*d^2*cos(d*x + c) + 2*a*d^2*sin(3*d
*x + 3*c) + 2*I*a*d^2*sin(2*d*x + 2*c) - 2*a*d^2*sin(d*x + c) - 2*a*d^2))/d
```

Fricas [C] time = 2.81162, size = 6126, normalized size = 18.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*f^2*x^2 + 4*d^2*e*f*x + 2*d^2*e^2 - 4*(d^2*f^2*x^2 + 2*d^2*e*f*
x + d^2*e^2)*cos(d*x + c)^2 - 2*(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*cos(d
*x + c) + (2*I*d*f^2*x + 2*I*d*e*f + (-2*I*d*f^2*x - 2*I*d*e*f + 2*I*f^2)*c
os(d*x + c)^2 - 2*I*f^2 + (2*I*d*f^2*x + 2*I*d*e*f - 2*I*f^2 + (2*I*d*f^2*x
 + 2*I*d*e*f - 2*I*f^2)*cos(d*x + c))*sin(d*x + c))*dilog(cos(d*x + c) + I*
sin(d*x + c)) + (-2*I*d*f^2*x - 2*I*d*e*f + (2*I*d*f^2*x + 2*I*d*e*f - 2*I*
f^2)*cos(d*x + c)^2 + 2*I*f^2 + (-2*I*d*f^2*x - 2*I*d*e*f + 2*I*f^2 + (-2*I
*d*f^2*x - 2*I*d*e*f + 2*I*f^2)*cos(d*x + c))*sin(d*x + c))*dilog(cos(d*x +
 c) - I*sin(d*x + c)) + (4*I*f^2*cos(d*x + c)^2 - 4*I*f^2 + (-4*I*f^2*cos(d
*x + c) - 4*I*f^2)*sin(d*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) + (-4
*I*f^2*cos(d*x + c)^2 + 4*I*f^2 + (4*I*f^2*cos(d*x + c) + 4*I*f^2)*sin(d*x
+ c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) + (2*I*d*f^2*x + 2*I*d*e*f + (-
2*I*d*f^2*x - 2*I*d*e*f - 2*I*f^2)*cos(d*x + c)^2 + 2*I*f^2 + (2*I*d*f^2*x
+ 2*I*d*e*f + 2*I*f^2 + (2*I*d*f^2*x + 2*I*d*e*f + 2*I*f^2)*cos(d*x + c))*s
in(d*x + c))*dilog(-cos(d*x + c) + I*sin(d*x + c)) + (-2*I*d*f^2*x - 2*I*d*
e*f + (2*I*d*f^2*x + 2*I*d*e*f + 2*I*f^2)*cos(d*x + c)^2 - 2*I*f^2 + (-2*I*
d*f^2*x - 2*I*d*e*f - 2*I*f^2 + (-2*I*d*f^2*x - 2*I*d*e*f - 2*I*f^2)*cos(d*
x + c))*sin(d*x + c))*dilog(-cos(d*x + c) - I*sin(d*x + c)) + (d^2*f^2*x^2
+ d^2*e^2 + 2*d*e*f - (d^2*f^2*x^2 + d^2*e^2 + 2*d*e*f + 2*(d^2*e*f + d*f^2
)*x)*cos(d*x + c)^2 + 2*(d^2*e*f + d*f^2)*x + (d^2*f^2*x^2 + d^2*e^2 + 2*d*
e*f + 2*(d^2*e*f + d*f^2)*x + (d^2*f^2*x^2 + d^2*e^2 + 2*d*e*f + 2*(d^2*e*f
```

```
 + d*f^2)*x)*cos(d*x + c))*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c)
+ 1) + 4*(d*e*f - c*f^2 - (d*e*f - c*f^2)*cos(d*x + c)^2 + (d*e*f - c*f^2 +
 (d*e*f - c*f^2)*cos(d*x + c))*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x +
 c) + I) + (d^2*f^2*x^2 + d^2*e^2 + 2*d*e*f - (d^2*f^2*x^2 + d^2*e^2 + 2*d*
e*f + 2*(d^2*e*f + d*f^2)*x)*cos(d*x + c)^2 + 2*(d^2*e*f + d*f^2)*x + (d^2*
f^2*x^2 + d^2*e^2 + 2*d*e*f + 2*(d^2*e*f + d*f^2)*x + (d^2*f^2*x^2 + d^2*e^
2 + 2*d*e*f + 2*(d^2*e*f + d*f^2)*x)*cos(d*x + c))*sin(d*x + c))*log(cos(d*
x + c) - I*sin(d*x + c) + 1) + 4*(d*f^2*x + c*f^2 - (d*f^2*x + c*f^2)*cos(d
*x + c)^2 + (d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*cos(d*x + c))*sin(d*x + c)
)*log(I*cos(d*x + c) + sin(d*x + c) + 1) + 4*(d*f^2*x + c*f^2 - (d*f^2*x +
c*f^2)*cos(d*x + c)^2 + (d*f^2*x + c*f^2 + (d*f^2*x + c*f^2)*cos(d*x + c))*
sin(d*x + c))*log(-I*cos(d*x + c) + sin(d*x + c) + 1) - (d^2*e^2 - 2*(c + 1
)*d*e*f + (c^2 + 2*c)*f^2 - (d^2*e^2 - 2*(c + 1)*d*e*f + (c^2 + 2*c)*f^2)*c
os(d*x + c)^2 + (d^2*e^2 - 2*(c + 1)*d*e*f + (c^2 + 2*c)*f^2 + (d^2*e^2 - 2
*(c + 1)*d*e*f + (c^2 + 2*c)*f^2)*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(
d*x + c) + 1/2*I*sin(d*x + c) + 1/2) - (d^2*e^2 - 2*(c + 1)*d*e*f + (c^2 +
2*c)*f^2 - (d^2*e^2 - 2*(c + 1)*d*e*f + (c^2 + 2*c)*f^2)*cos(d*x + c)^2 + (
d^2*e^2 - 2*(c + 1)*d*e*f + (c^2 + 2*c)*f^2 + (d^2*e^2 - 2*(c + 1)*d*e*f +
(c^2 + 2*c)*f^2)*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d*x + c) - 1/2*I*
sin(d*x + c) + 1/2) - (d^2*f^2*x^2 + 2*c*d*e*f - (c^2 + 2*c)*f^2 - (d^2*f^2
*x^2 + 2*c*d*e*f - (c^2 + 2*c)*f^2 + 2*(d^2*e*f - d*f^2)*x)*cos(d*x + c)^2
+ 2*(d^2*e*f - d*f^2)*x + (d^2*f^2*x^2 + 2*c*d*e*f - (c^2 + 2*c)*f^2 + 2*(d
^2*e*f - d*f^2)*x + (d^2*f^2*x^2 + 2*c*d*e*f - (c^2 + 2*c)*f^2 + 2*(d^2*e*f
 - d*f^2)*x)*cos(d*x + c))*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c)
 + 1) + 4*(d*e*f - c*f^2 - (d*e*f - c*f^2)*cos(d*x + c)^2 + (d*e*f - c*f^2
+ (d*e*f - c*f^2)*cos(d*x + c))*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x
 + c) + I) - (d^2*f^2*x^2 + 2*c*d*e*f - (c^2 + 2*c)*f^2 - (d^2*f^2*x^2 + 2*
c*d*e*f - (c^2 + 2*c)*f^2 + 2*(d^2*e*f - d*f^2)*x)*cos(d*x + c)^2 + 2*(d^2*
e*f - d*f^2)*x + (d^2*f^2*x^2 + 2*c*d*e*f - (c^2 + 2*c)*f^2 + 2*(d^2*e*f -
d*f^2)*x + (d^2*f^2*x^2 + 2*c*d*e*f - (c^2 + 2*c)*f^2 + 2*(d^2*e*f - d*f^2)
*x)*cos(d*x + c))*sin(d*x + c))*log(-cos(d*x + c) - I*sin(d*x + c) + 1) + 2
*(f^2*cos(d*x + c)^2 - f^2 - (f^2*cos(d*x + c) + f^2)*sin(d*x + c))*polylog
(3, cos(d*x + c) + I*sin(d*x + c)) + 2*(f^2*cos(d*x + c)^2 - f^2 - (f^2*cos
(d*x + c) + f^2)*sin(d*x + c))*polylog(3, cos(d*x + c) - I*sin(d*x + c)) -
2*(f^2*cos(d*x + c)^2 - f^2 - (f^2*cos(d*x + c) + f^2)*sin(d*x + c))*polylo
g(3, -cos(d*x + c) + I*sin(d*x + c)) - 2*(f^2*cos(d*x + c)^2 - f^2 - (f^2*c
os(d*x + c) + f^2)*sin(d*x + c))*polylog(3, -cos(d*x + c) - I*sin(d*x + c))
 - 2*(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 + 2*(d^2*f^2*x^2 + 2*d^2*e*f*x +
d^2*e^2)*cos(d*x + c))*sin(d*x + c))/(a*d^3*cos(d*x + c)^2 - a*d^3 - (a*d^3
*cos(d*x + c) + a*d^3)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^2\csc^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2\csc^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx\csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*csc(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*
csc(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*csc(c + d*x)**2/(
sin(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2\csc(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

3.205 $\int \frac{(e+fx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=169

$$-\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}+\frac{if\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}+\frac{f\log(\sin(c+dx))}{ad^2}-\frac{(e+fx)\cot\left(\right.}{ad}$$

```
[Out] (2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) - ((e + f*x)*Cot[c/2 + Pi/4 +
(d*x)/2])/(a*d) - ((e + f*x)*Cot[c + d*x])/(a*d) + (2*f*Log[Sin[c/2 + Pi/4
+ (d*x)/2]])/(a*d^2) + (f*Log[Sin[c + d*x]])/(a*d^2) - (I*f*PolyLog[2, -E^(
I*(c + d*x))])/(a*d^2) + (I*f*PolyLog[2, E^(I*(c + d*x))])/(a*d^2)
```

Rubi [A] time = 0.189351, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4535, 4184, 3475, 4183, 2279, 2391, 3318}

$$-\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}+\frac{if\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}+\frac{f\log(\sin(c+dx))}{ad^2}-\frac{(e+fx)\cot\left(\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) - ((e + f*x)*Cot[c/2 + Pi/4 +
(d*x)/2])/(a*d) - ((e + f*x)*Cot[c + d*x])/(a*d) + (2*f*Log[Sin[c/2 + Pi/4
+ (d*x)/2]])/(a*d^2) + (f*Log[Sin[c + d*x]])/(a*d^2) - (I*f*PolyLog[2, -E^(
I*(c + d*x))])/(a*d^2) + (I*f*PolyLog[2, E^(I*(c + d*x))])/(a*d^2)
```

Rule 4535

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csc[c + d*x]^(n - 1))/(a + b*Si
n[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
```

```
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)\csc^2(c+dx)\,dx}{a} - \int \frac{(e+fx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= -\frac{(e+fx)\cot(c+dx)}{ad} - \frac{\int (e+fx)\csc(c+dx)\,dx}{a} + \frac{f\int \cot(c+dx)\,dx}{ad} + \int \frac{e+fx}{a+a\sin(c-}$$
$$= \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)\cot(c+dx)}{ad} + \frac{f\log(\sin(c+dx))}{ad^2} + \frac{\int (e+fx)\csc^2}{}$$
$$= \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)\cot(c+dx)}{ad} + \frac{f\log(\mathrm{si}}{}$$
$$= \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(e+fx)\cot(c+dx)}{ad} + \frac{2f\log\left(\right.}{}$$

Mathematica [B] time = 1.70638, size = 396, normalized size = 2.34

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-2f\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(i\left(\mathrm{PolyLog}\left(2,-e^{i(c+dx)}\right)-\mathrm{PolyLog}\left(2,e\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-(d*(e + f*x)*Cos[(c + d*x)/2]*(1 +
 Cot[(c + d*x)/2])) + 4*d*(e + f*x)*Sin[(c + d*x)/2] - 2*f*(c + d*x)*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]) + 4*f*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 2*f*Log[Sin[c + d*x]]*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]) - 2*d*e*Log[Tan[(c + d*x)/2]]*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]) + 2*c*f*Log[Tan[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]) - 2*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c
 + d*x))]) + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])
)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + d*(e + f*x)*Sin[(c + d*x)/2]*(1 +
 Tan[(c + d*x)/2])))/(2*a*d^2*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.185, size = 351, normalized size = 2.1

$$-2\,\frac{-2\,fx+i\mathrm{e}^{i(dx+c)}fx-2\,e+i\mathrm{e}^{i(dx+c)}e+fx\mathrm{e}^{2\,i(dx+c)}+e\mathrm{e}^{2\,i(dx+c)}}{\left(\mathrm{e}^{2\,i(dx+c)}-1\right)\left(\mathrm{e}^{i(dx+c)}+i\right)da}+\frac{fc\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{ad^2}+\frac{if\mathrm{polylog}\left(2,\mathrm{e}^{i(dx+c)}\right)}{ad^2}-\frac{if\mathrm{poly}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -2*(-2*f*x+I*exp(I*(d*x+c))*f*x-2*e+I*exp(I*(d*x+c))*e+f*x*exp(2*I*(d*x+c))
+e*exp(2*I*(d*x+c)))/(exp(2*I*(d*x+c))-1)/(exp(I*(d*x+c))+I)/d/a+1/d^2/a*f*
c*ln(exp(I*(d*x+c))-1)+I*f*polylog(2,exp(I*(d*x+c)))/a/d^2-I*f*polylog(2,-e
xp(I*(d*x+c)))/a/d^2-1/d/a*e*ln(exp(I*(d*x+c))-1)+1/d/a*e*ln(exp(I*(d*x+c))
+1)-4/d^2/a*f*ln(exp(I*(d*x+c)))+1/d^2/a*f*ln(exp(I*(d*x+c))-1)+1/d^2/a*f*l
n(exp(I*(d*x+c))+1)+2/d^2/a*f*ln(exp(I*(d*x+c))+I)-1/d/a*ln(1-exp(I*(d*x+c)
))*f*x-1/d^2/a*ln(1-exp(I*(d*x+c)))*c*f+1/d/a*ln(exp(I*(d*x+c))+1)*f*x
```

Maxima [B] time = 2.0077, size = 1727, normalized size = 10.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(8*d*f*x*cos(3*d*x + 3*c) + 8*I*d*f*x*sin(3*d*x + 3*c) + 8*I*d*e - (4*f*co
s(3*d*x + 3*c) + 4*I*f*cos(2*d*x + 2*c) - 4*f*cos(d*x + c) + 4*I*f*sin(3*d*
x + 3*c) - 4*f*sin(2*d*x + 2*c) - 4*I*f*sin(d*x + c) - 4*I*f)*arctan2(cos(c
) + sin(d*x), cos(d*x) + sin(c)) - (-2*I*d*f*x - 2*I*d*e + 2*(d*f*x + d*e +
 f)*cos(3*d*x + 3*c) + (2*I*d*f*x + 2*I*d*e + 2*I*f)*cos(2*d*x + 2*c) - 2*(
d*f*x + d*e + f)*cos(d*x + c) + (2*I*d*f*x + 2*I*d*e + 2*I*f)*sin(3*d*x + 3
*c) - 2*(d*f*x + d*e + f)*sin(2*d*x + 2*c) + (-2*I*d*f*x - 2*I*d*e - 2*I*f)
*sin(d*x + c) - 2*I*f)*arctan2(sin(d*x + c), cos(d*x + c) + 1) - (2*I*d*e -
 2*(d*e - f)*cos(3*d*x + 3*c) + (-2*I*d*e + 2*I*f)*cos(2*d*x + 2*c) + 2*(d*
e - f)*cos(d*x + c) + (-2*I*d*e + 2*I*f)*sin(3*d*x + 3*c) + 2*(d*e - f)*sin
(2*d*x + 2*c) + (2*I*d*e - 2*I*f)*sin(d*x + c) - 2*I*f)*arctan2(sin(d*x + c
), cos(d*x + c) - 1) - (2*d*f*x*cos(3*d*x + 3*c) + 2*I*d*f*x*cos(2*d*x + 2*
c) - 2*d*f*x*cos(d*x + c) + 2*I*d*f*x*sin(3*d*x + 3*c) - 2*d*f*x*sin(2*d*x
+ 2*c) - 2*I*d*f*x*sin(d*x + c) - 2*I*d*f*x)*arctan2(sin(d*x + c), -cos(d*x
 + c) + 1) - (-4*I*d*f*x + 4*I*d*e)*cos(2*d*x + 2*c) - 4*(d*f*x - d*e)*cos(
d*x + c) + (2*f*cos(3*d*x + 3*c) + 2*I*f*cos(2*d*x + 2*c) - 2*f*cos(d*x + c
) + 2*I*f*sin(3*d*x + 3*c) - 2*f*sin(2*d*x + 2*c) - 2*I*f*sin(d*x + c) - 2*
I*f)*dilog(-e^(I*d*x + I*c)) - (2*f*cos(3*d*x + 3*c) + 2*I*f*cos(2*d*x + 2*
c) - 2*f*cos(d*x + c) + 2*I*f*sin(3*d*x + 3*c) - 2*f*sin(2*d*x + 2*c) - 2*I
*f*sin(d*x + c) - 2*I*f)*dilog(e^(I*d*x + I*c)) + (d*f*x + d*e - (-I*d*f*x
- I*d*e - I*f)*cos(3*d*x + 3*c) - (d*f*x + d*e + f)*cos(2*d*x + 2*c) - (I*d
*f*x + I*d*e + I*f)*cos(d*x + c) - (d*f*x + d*e + f)*sin(3*d*x + 3*c) - (I*
```

```
d*f*x + I*d*e + I*f)*sin(2*d*x + 2*c) + (d*f*x + d*e + f)*sin(d*x + c) + f)
*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1) - (d*f*x + d*e +
 (I*d*f*x + I*d*e - I*f)*cos(3*d*x + 3*c) - (d*f*x + d*e - f)*cos(2*d*x + 2
*c) + (-I*d*f*x - I*d*e + I*f)*cos(d*x + c) - (d*f*x + d*e - f)*sin(3*d*x +
 3*c) + (-I*d*f*x - I*d*e + I*f)*sin(2*d*x + 2*c) + (d*f*x + d*e - f)*sin(d
*x + c) - f)*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1) - (-
2*I*f*cos(3*d*x + 3*c) + 2*f*cos(2*d*x + 2*c) + 2*I*f*cos(d*x + c) + 2*f*si
n(3*d*x + 3*c) + 2*I*f*sin(2*d*x + 2*c) - 2*f*sin(d*x + c) - 2*f)*log(cos(d
*x)^2 + cos(c)^2 + 2*cos(c)*sin(d*x) + sin(d*x)^2 + 2*cos(d*x)*sin(c) + sin
(c)^2) - 4*(d*f*x - d*e)*sin(2*d*x + 2*c) - (4*I*d*f*x - 4*I*d*e)*sin(d*x +
 c))/(-2*I*a*d^2*cos(3*d*x + 3*c) + 2*a*d^2*cos(2*d*x + 2*c) + 2*I*a*d^2*co
s(d*x + c) + 2*a*d^2*sin(3*d*x + 3*c) + 2*I*a*d^2*sin(2*d*x + 2*c) - 2*a*d^
2*sin(d*x + c) - 2*a*d^2)
```

Fricas [B] time = 2.34288, size = 2284, normalized size = 13.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*f*x - 4*(d*f*x + d*e)*cos(d*x + c)^2 + 2*d*e - 2*(d*f*x + d*e)*co
s(d*x + c) + (-I*f*cos(d*x + c)^2 + (I*f*cos(d*x + c) + I*f)*sin(d*x + c) +
 I*f)*dilog(cos(d*x + c) + I*sin(d*x + c)) + (I*f*cos(d*x + c)^2 + (-I*f*co
s(d*x + c) - I*f)*sin(d*x + c) - I*f)*dilog(cos(d*x + c) - I*sin(d*x + c))
+ (-I*f*cos(d*x + c)^2 + (I*f*cos(d*x + c) + I*f)*sin(d*x + c) + I*f)*dilog
(-cos(d*x + c) + I*sin(d*x + c)) + (I*f*cos(d*x + c)^2 + (-I*f*cos(d*x + c)
 - I*f)*sin(d*x + c) - I*f)*dilog(-cos(d*x + c) - I*sin(d*x + c)) + (d*f*x
- (d*f*x + d*e + f)*cos(d*x + c)^2 + d*e + (d*f*x + d*e + (d*f*x + d*e + f)
*cos(d*x + c) + f)*sin(d*x + c) + f)*log(cos(d*x + c) + I*sin(d*x + c) + 1)
 + (d*f*x - (d*f*x + d*e + f)*cos(d*x + c)^2 + d*e + (d*f*x + d*e + (d*f*x
+ d*e + f)*cos(d*x + c) + f)*sin(d*x + c) + f)*log(cos(d*x + c) - I*sin(d*x
 + c) + 1) + ((d*e - (c + 1)*f)*cos(d*x + c)^2 - d*e + (c + 1)*f - (d*e - (
c + 1)*f + (d*e - (c + 1)*f)*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d*x +
 c) + 1/2*I*sin(d*x + c) + 1/2) + ((d*e - (c + 1)*f)*cos(d*x + c)^2 - d*e +
 (c + 1)*f - (d*e - (c + 1)*f + (d*e - (c + 1)*f)*cos(d*x + c))*sin(d*x + c
))*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) - (d*f*x - (d*f*x + c*
f)*cos(d*x + c)^2 + c*f + (d*f*x + c*f + (d*f*x + c*f)*cos(d*x + c))*sin(d*
x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + 1) - (d*f*x - (d*f*x + c*f)*co
s(d*x + c)^2 + c*f + (d*f*x + c*f + (d*f*x + c*f)*cos(d*x + c))*sin(d*x + c
))*log(-cos(d*x + c) - I*sin(d*x + c) + 1) - 2*(f*cos(d*x + c)^2 - (f*cos(d
```

```
*x + c) + f)*sin(d*x + c) - f)*log(sin(d*x + c) + 1) - 2*(d*f*x + d*e + 2*(
d*f*x + d*e)*cos(d*x + c))*sin(d*x + c))/(a*d^2*cos(d*x + c)^2 - a*d^2 - (a
*d^2*cos(d*x + c) + a*d^2)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e\csc^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{fx\csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e*csc(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f*x*csc(c + d
*x)**2/(sin(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\csc(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

3.206 $\int \frac{\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=51

$$-\frac{2\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\cot(c+dx)}{d(a\sin(c+dx)+a)}$$

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - (2*Cot[c + d*x])/(a*d) + Cot[c + d*x]/(d*(a +
 a*Sin[c + d*x]))
```

Rubi [A] time = 0.0766124, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2768, 2748, 3767, 8, 3770}

$$-\frac{2\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\cot(c+dx)}{d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - (2*Cot[c + d*x])/(a*d) + Cot[c + d*x]/(d*(a +
 a*Sin[c + d*x]))
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\cot(c+dx)}{d(a+a\sin(c+dx))} - \frac{\int \csc^2(c+dx)(-2a+a\sin(c+dx))\,dx}{a^2}\\
&= \frac{\cot(c+dx)}{d(a+a\sin(c+dx))} - \frac{\int \csc(c+dx)\,dx}{a} + \frac{2\int \csc^2(c+dx)\,dx}{a}\\
&= \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\cot(c+dx)}{d(a+a\sin(c+dx))} - \frac{2\,\text{Subst}\left(\int 1\,dx, x, \cot(c+dx)\right)}{ad}\\
&= \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{2\cot(c+dx)}{ad} + \frac{\cot(c+dx)}{d(a+a\sin(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.185737, size = 57, normalized size = 1.12

$$\frac{\sec(c+dx)\left(2\sin(c+dx)-\csc(c+dx)+\sqrt{\cos^2(c+dx)}\tanh^{-1}\left(\sqrt{\cos^2(c+dx)}\right)-1\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*(-1 + ArcTanh[Sqrt[Cos[c + d*x]^2]]*Sqrt[Cos[c + d*x]^2] - Cs
c[c + d*x] + 2*Sin[c + d*x]))/(a*d)
```

Maple [A] time = 0.042, size = 77, normalized size = 1.5

$$\frac{1}{2\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{1}{da\,(\tan\left(1/2\,dx+c/2\right)+1)}-\frac{1}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{1}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/a/d*tan(1/2*d*x+1/2*c)-2/a/d/(tan(1/2*d*x+1/2*c)+1)-1/2/a/d/tan(1/2*d*x
+1/2*c)-1/a/d*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 0.990806, size = 151, normalized size = 2.96

$$-\frac{\frac{\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}+1}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{2\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((5*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/(a*sin(d*x + c)/(cos(d*x + c)
 + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 2*log(sin(d*x + c)/(cos(d*
x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [B] time = 1.8429, size = 433, normalized size = 8.49

$$\frac{4\,\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)^2-\left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)-1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(\cos\left(dx+c\right)^2-(\cos(d}{2\left(ad\cos\left(dx+c\right)^2-ad-(ad\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*cos(d*x + c)^2 + (cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) -
 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(
```

```
d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(2*cos(d*x + c) + 1)*sin(d*x
 + c) + 2*cos(d*x + c) - 2)/(a*d*cos(d*x + c)^2 - a*d - (a*d*cos(d*x + c) +
 a*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.17431, size = 119, normalized size = 2.33

$$-\frac{\frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a - tan(1/2*d*x + 1/2*c)/a - (tan(1/
2*d*x + 1/2*c)^2 - 4*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c)^2 + t
an(1/2*d*x + 1/2*c))*a))/d
```

3.207 $\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))} dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\csc^2(c+dx)}{(e+fx)(a\sin(c+dx)+a)}, x\right)$$

```
[Out] Unintegrable[Csc[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0692805, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))} dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Csc[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))} dx = \int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))} dx$$

Mathematica [A] time = 23.3752, size = 0, normalized size = 0.

$$\int \frac{\csc^2(c+dx)}{(e+fx)(a+a\sin(c+dx))} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Csc[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 3.372, size = 0, normalized size = 0.

$$\int \frac{(\csc (dx + c))^2}{\left(fx + e\right)(a + a \sin (dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc (dx + c)^2}{afx + ae + \left(afx + ae\right)\sin (dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)^2/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^2 (c+dx)}{e \sin (c+dx)+e+fx \sin (c+dx)+fx} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/
a
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.208 $\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\csc^2(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Csc[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0681111, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Csc[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 43.6554, size = 0, normalized size = 0.

$$\int \frac{\csc^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Csc[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 7.221, size = 0, normalized size = 0.

$$\int \frac{(\csc(dx+c))^2}{(fx+e)^2 (a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(dx+c)^2}{af^2x^2+2\,aefx+ae^2+\left(af^2x^2+2\,aefx+ae^2\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)^2/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e
*f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^2(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) +
 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.209 $\int \frac{(e+fx)^3 \csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=600

$$\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}+\frac{3if^2(e+fx)\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^3}-\frac{9f^2(e+fx)\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}+\frac{9f^2(e}{}$$

```
[Out] ((2*I)*(e + f*x)^3)/(a*d) - (6*f^2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d
^3) - (3*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((e + f*x)^3*Cot[c/2
 + Pi/4 + (d*x)/2])/(a*d) + ((e + f*x)^3*Cot[c + d*x])/(a*d) - (3*f*(e + f*
x)^2*Csc[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Cot[c + d*x]*Csc[c + d*x])/(2*a
*d) - (6*f*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) - (3*f*(e + f*x)
^2*Log[1 - E^((2*I)*(c + d*x))])/(a*d^2) + ((3*I)*f^3*PolyLog[2, -E^(I*(c +
 d*x))])/(a*d^4) + (((9*I)/2)*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))])/(
a*d^2) + ((12*I)*f^2*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - ((3
*I)*f^3*PolyLog[2, E^(I*(c + d*x))])/(a*d^4) - (((9*I)/2)*f*(e + f*x)^2*Pol
yLog[2, E^(I*(c + d*x))])/(a*d^2) + ((3*I)*f^2*(e + f*x)*PolyLog[2, E^((2*I
)*(c + d*x))])/(a*d^3) - (9*f^2*(e + f*x)*PolyLog[3, -E^(I*(c + d*x))])/(a*
d^3) - (12*f^3*PolyLog[3, I*E^(I*(c + d*x))])/(a*d^4) + (9*f^2*(e + f*x)*Po
lyLog[3, E^(I*(c + d*x))])/(a*d^3) - (3*f^3*PolyLog[3, E^((2*I)*(c + d*x))]
)/(2*a*d^4) - ((9*I)*f^3*PolyLog[4, -E^(I*(c + d*x))])/(a*d^4) + ((9*I)*f^3
*PolyLog[4, E^(I*(c + d*x))])/(a*d^4)
```

Rubi [A] time = 1.10824, antiderivative size = 600, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {4535, 4186, 4183, 2279, 2391, 2531, 6609, 2282, 6589, 4184, 3717, 2190, 3318}

$$\frac{12if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}+\frac{3if^2(e+fx)\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{ad^3}-\frac{9f^2(e+fx)\text{PolyLog}\left(3,-e^{i(c+dx)}\right)}{ad^3}+\frac{9f^2(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] ((2*I)*(e + f*x)^3)/(a*d) - (6*f^2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d
^3) - (3*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((e + f*x)^3*Cot[c/2
 + Pi/4 + (d*x)/2])/(a*d) + ((e + f*x)^3*Cot[c + d*x])/(a*d) - (3*f*(e + f*
x)^2*Csc[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Cot[c + d*x]*Csc[c + d*x])/(2*a
*d) - (6*f*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*d^2) - (3*f*(e + f*x)
```

```
^2*Log[1 - E^((2*I)*(c + d*x))])/(a*d^2) + ((3*I)*f^3*PolyLog[2, -E^(I*(c +
 d*x))])/(a*d^4) + (((9*I)/2)*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))])/(
a*d^2) + ((12*I)*f^2*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - ((3
*I)*f^3*PolyLog[2, E^(I*(c + d*x))])/(a*d^4) - (((9*I)/2)*f*(e + f*x)^2*Pol
yLog[2, E^(I*(c + d*x))])/(a*d^2) + ((3*I)*f^2*(e + f*x)*PolyLog[2, E^((2*I
)*(c + d*x))])/(a*d^3) - (9*f^2*(e + f*x)*PolyLog[3, -E^(I*(c + d*x))])/(a*
d^3) - (12*f^3*PolyLog[3, I*E^(I*(c + d*x))])/(a*d^4) + (9*f^2*(e + f*x)*Po
lyLog[3, E^(I*(c + d*x))])/(a*d^3) - (3*f^3*PolyLog[3, E^((2*I)*(c + d*x))]
)/(2*a*d^4) - ((9*I)*f^3*PolyLog[4, -E^(I*(c + d*x))])/(a*d^4) + ((9*I)*f^3
*PolyLog[4, E^(I*(c + d*x))])/(a*d^4)
```

Rule 4535

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csc[c + d*x]^(n - 1))/(a + b*Si
n[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
```

```
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(e+fx)^3\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\csc^3(c+dx)\,dx}{a} - \int \frac{(e+fx)^3\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{3f(e+fx)^2\csc(c+dx)}{2ad^2} - \frac{(e+fx)^3\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\int (e+fx)^3\csc(c+dx)\,d}{2a}$$

$$= -\frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\cot(c+dx)}{ad} - 3$$

$$= \frac{i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\cot}{ad}$$

$$= \frac{i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\cot}{a}$$

$$= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\,\mathrm{co}}{}$$

$$= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\,\mathrm{co}}{}$$

$$= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\,\mathrm{co}}{}$$

$$= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\,\mathrm{co}}{}$$

$$= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\,\mathrm{co}}{}$$

Mathematica [B] time = 31.369, size = 1485, normalized size = 2.48

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*e^3*Log[Tan[(c + d*x)/2]])/(2*a*d) + (3*e*f^2*Log[Tan[(c + d*x)/2]])/(a*
d^3) + (9*e^2*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c + d*
x))]) - c*Log[Tan[(c + d*x)/2]] + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog
[2, E^(I*(c + d*x))])))/(2*a*d^2) + (3*f^3*((c + d*x)*(Log[1 - E^(I*(c + d*
```

```
x))] - Log[1 + E^(I*(c + d*x))]) - c*Log[Tan[(c + d*x)/2]] + I*(PolyLog[2,
-E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a*d^4) + (E^(I*c)*f^3*C
sc[c]*((2*d^3*x^3)/E^((2*I)*c) + (3*I)*d^2*(1 - E^((-2*I)*c))*x^2*Log[1 - E
^((-I)*(c + d*x))] + (3*I)*d^2*(1 - E^((-2*I)*c))*x^2*Log[1 + E^((-I)*(c +
d*x))] - (6*(-1 + E^((2*I)*c))*(d*x*PolyLog[2, -E^((-I)*(c + d*x))] - I*Pol
yLog[3, -E^((-I)*(c + d*x))]))/E^((2*I)*c) - (6*(-1 + E^((2*I)*c))*(d*x*Pol
yLog[2, E^((-I)*(c + d*x))] - I*PolyLog[3, E^((-I)*(c + d*x))]))/E^((2*I)*c
)))/(2*a*d^4) - (9*e*f^2*(d^2*x^2*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] -
I*d*x*PolyLog[2, -Cos[c + d*x] - I*Sin[c + d*x]] + I*d*x*PolyLog[2, Cos[c +
 d*x] + I*Sin[c + d*x]] + PolyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] - Poly
Log[3, Cos[c + d*x] + I*Sin[c + d*x]]))/(a*d^3) + (3*f^3*(-2*d^3*x^3*ArcTan
h[Cos[c + d*x] + I*Sin[c + d*x]] + (3*I)*d^2*x^2*PolyLog[2, -Cos[c + d*x] -
 I*Sin[c + d*x]] - (3*I)*d^2*x^2*PolyLog[2, Cos[c + d*x] + I*Sin[c + d*x]]
- 6*d*x*PolyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] + 6*d*x*PolyLog[3, Cos[c
 + d*x] + I*Sin[c + d*x]] - (6*I)*PolyLog[4, -Cos[c + d*x] - I*Sin[c + d*x]
] + (6*I)*PolyLog[4, Cos[c + d*x] + I*Sin[c + d*x]]))/(2*a*d^4) - (3*e^2*f*
Csc[c]*(-(d*x*Cos[c]) + Log[Cos[d*x]*Sin[c] + Cos[c]*Sin[d*x]]*Sin[c]))/(a*
d^2*(Cos[c]^2 + Sin[c]^2)) + (6*f*(Cos[c] + I*Sin[c])*(((e + f*x)^3*(Cos[c]
 - I*Sin[c]))/(3*f) - ((e + f*x)^2*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(
1 + I*Cos[c] + Sin[c]))/d + (2*f*(d*(e + f*x)*PolyLog[2, (-I)*Cos[c + d*x]
- Sin[c + d*x]] - I*f*PolyLog[3, (-I)*Cos[c + d*x] - Sin[c + d*x]])*(Cos[c]
 - I*(1 + Sin[c])))/d^3))/(a*d*(Cos[c] + I*(1 + Sin[c]))) + (Csc[c]*Csc[c +
 d*x]^2*(e^3*Sin[d*x] + 3*e^2*f*x*Sin[d*x] + 3*e*f^2*x^2*Sin[d*x] + f^3*x^3
*Sin[d*x]))/(2*a*d) + (Csc[c]*Csc[c + d*x]*(-(d*e^3*Cos[c]) - 3*d*e^2*f*x*C
os[c] - 3*d*e*f^2*x^2*Cos[c] - d*f^3*x^3*Cos[c] - 3*e^2*f*Sin[c] - 6*e*f^2*
x*Sin[c] - 3*f^3*x^2*Sin[c] - 2*d*e^3*Sin[d*x] - 6*d*e^2*f*x*Sin[d*x] - 6*d
*e*f^2*x^2*Sin[d*x] - 2*d*f^3*x^3*Sin[d*x]))/(2*a*d^2) - (2*(e^3*Sin[(d*x)/
2] + 3*e^2*f*x*Sin[(d*x)/2] + 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/
2]))/(a*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
+ (3*e*f^2*Csc[c]*Sec[c]*(d^2*E^(I*ArcTan[Tan[c]])*x^2 + ((I*d*x*(-Pi + 2*A
rcTan[Tan[c]]) - Pi*Log[1 + E^((-2*I)*d*x)] - 2*(d*x + ArcTan[Tan[c]])*Log[
1 - E^((2*I)*(d*x + ArcTan[Tan[c]]))] + Pi*Log[Cos[d*x]] + 2*ArcTan[Tan[c]]
*Log[Sin[d*x + ArcTan[Tan[c]]]] + I*PolyLog[2, E^((2*I)*(d*x + ArcTan[Tan[c
]]))])*Tan[c])/Sqrt[1 + Tan[c]^2]))/(a*d^3*Sqrt[Sec[c]^2*(Cos[c]^2 + Sin[c]
^2)])
```

Maple [B] time = 0.286, size = 2257, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -3/d^4/a*f^3*c*ln(exp(I*(d*x+c))-1)+3/d^3/a*e*f^2*ln(exp(I*(d*x+c))-1)-3/d^
3/a*e*f^2*ln(exp(I*(d*x+c))+1)-3/d^2/a*f^3*ln(exp(I*(d*x+c))+1)*x^2-3/d^2/a
*f^3*ln(1-exp(I*(d*x+c)))*x^2+3/d^4/a*f^3*ln(1-exp(I*(d*x+c)))*c^2-3/d^4/a*
f^3*c^2*ln(exp(I*(d*x+c))-1)-3/d^2/a*e^2*f*ln(exp(I*(d*x+c))-1)-3/d^2/a*e^2
*f*ln(exp(I*(d*x+c))+1)-12*f^2/d^2/a*e*ln(1-I*exp(I*(d*x+c)))*x+24*I/d^2/a*
c*e*f^2*x+3/2/d/a*e^3*ln(exp(I*(d*x+c))-1)-3/2/d/a*e^3*ln(exp(I*(d*x+c))+1)
-9/2*I/d^2/a*f^3*polylog(2,exp(I*(d*x+c)))*x^2+12*I/d^3/a*c^2*e*f^2-12*I/d^
3/a*f^3*c^2*x+9/2*I/d^2/a*f^3*polylog(2,-exp(I*(d*x+c)))*x^2+6*I/d^3/a*f^3*
polylog(2,-exp(I*(d*x+c)))*x+6*I/d^3/a*f^3*polylog(2,exp(I*(d*x+c)))*x+12*I
/d/a*e*f^2*x^2-9/2*I/d^2/a*e^2*f*polylog(2,exp(I*(d*x+c)))+9/2*I/d^2/a*e^2*
f*polylog(2,-exp(I*(d*x+c)))+6*I/d^3/a*e*f^2*polylog(2,exp(I*(d*x+c)))+6*I/
d^3/a*e*f^2*polylog(2,-exp(I*(d*x+c)))+6/d^3/a*e*f^2*c*ln(exp(I*(d*x+c))-1)
+3/d^3/a*f^3*ln(1-exp(I*(d*x+c)))*x+3/d^4/a*f^3*ln(1-exp(I*(d*x+c)))*c-3/d^
3/a*f^3*ln(exp(I*(d*x+c))+1)*x-8*I/d^4/a*f^3*c^3+4*I/d/a*f^3*x^3+12*f^3/d^4
/a*c^2*ln(exp(I*(d*x+c)))-6*f/d^2/a*ln(exp(I*(d*x+c))+I)*e^2-6*f^3/d^4/a*c^
2*ln(exp(I*(d*x+c))+I)+12*f/d^2/a*ln(exp(I*(d*x+c)))*e^2-9/2/d^2/a*e^2*f*c*
ln(exp(I*(d*x+c))-1)+9/2/d^3/a*e*f^2*c^2*ln(exp(I*(d*x+c))-1)-3/2/d/a*f^3*l
n(exp(I*(d*x+c))+1)*x^3-12*f^2/d^3/a*e*ln(1-I*exp(I*(d*x+c)))*c-6*f^3/d^2/a
*ln(1-I*exp(I*(d*x+c)))*x^2+6*f^3/d^4/a*ln(1-I*exp(I*(d*x+c)))*c^2-24*f^2/d
^3/a*e*c*ln(exp(I*(d*x+c)))+12*f^2/d^3/a*e*c*ln(exp(I*(d*x+c))+I)+12*I*f^3/
d^3/a*polylog(2,I*exp(I*(d*x+c)))*x+12*I*f^2/d^3/a*e*polylog(2,I*exp(I*(d*x
+c)))+3*I*f^3*polylog(2,-exp(I*(d*x+c)))/a/d^4+9*I*f^3*polylog(4,exp(I*(d*x
+c)))/a/d^4-6*f^3*polylog(3,-exp(I*(d*x+c)))/a/d^4-6*f^3*polylog(3,exp(I*(d
*x+c)))/a/d^4+3/2/d/a*f^3*ln(1-exp(I*(d*x+c)))*x^3+3/2/d^4/a*f^3*ln(1-exp(I
*(d*x+c)))*c^3-9/2/d/a*e*f^2*ln(exp(I*(d*x+c))+1)*x^2+9/2/d/a*ln(1-exp(I*(d
*x+c)))*e^2*f*x-9/2/d/a*ln(exp(I*(d*x+c))+1)*e^2*f*x+9/2/d/a*e*f^2*ln(1-exp
(I*(d*x+c)))*x^2-9/2/d^3/a*e*f^2*ln(1-exp(I*(d*x+c)))*c^2+9/2/d^2/a*ln(1-ex
p(I*(d*x+c)))*c*e^2*f-3*I*f^3*polylog(2,exp(I*(d*x+c)))/a/d^4+9*I/d^2/a*pol
ylog(2,-exp(I*(d*x+c)))*e*f^2*x-9*I/d^2/a*polylog(2,exp(I*(d*x+c)))*e*f^2*x
-9*I*f^3*polylog(4,-exp(I*(d*x+c)))/a/d^4-12*f^3*polylog(3,I*exp(I*(d*x+c))
)/a/d^4+9/d^3/a*f^3*polylog(3,exp(I*(d*x+c)))*x-9/d^3/a*f^3*polylog(3,-exp(
I*(d*x+c)))*x-3/2/d^4/a*f^3*c^3*ln(exp(I*(d*x+c))-1)+9/d^3/a*e*f^2*polylog(
3,exp(I*(d*x+c)))-9/d^3/a*e*f^2*polylog(3,-exp(I*(d*x+c)))-6/d^2/a*e*f^2*ln
(1-exp(I*(d*x+c)))*x-6/d^3/a*e*f^2*ln(1-exp(I*(d*x+c)))*c-6/d^2/a*e*f^2*ln(
exp(I*(d*x+c))+1)*x+(3*d*f^3*x^3*exp(4*I*(d*x+c))+6*e*f^2*x*exp(3*I*(d*x+c)
)+3*I*d*e^3*exp(3*I*(d*x+c))+9*I*d*e*f^2*x^2*exp(3*I*(d*x+c))-3*I*f^3*x^2*e
xp(4*I*(d*x+c))-3*I*e^2*f*exp(4*I*(d*x+c))-5*d*f^3*x^3*exp(2*I*(d*x+c))+3*I
*e^2*f*exp(2*I*(d*x+c))+3*I*f^3*x^2*exp(2*I*(d*x+c))+4*d*e^3-5*d*e^3*exp(2*
I*(d*x+c))+3*f^3*x^2*exp(3*I*(d*x+c))+3*d*e^3*exp(4*I*(d*x+c))+3*e^2*f*exp(
3*I*(d*x+c))-3*I*d*e*f^2*x^2*exp(I*(d*x+c))-3*I*d*e^2*f*x*exp(I*(d*x+c))+4*
d*f^3*x^3-3*f^3*x^2*exp(I*(d*x+c))-3*exp(I*(d*x+c))*e^2*f-I*d*e^3*exp(I*(d*
x+c))-6*e*f^2*x*exp(I*(d*x+c))+12*d*e*f^2*x^2+12*d*e^2*f*x-I*d*f^3*x^3*exp(
I*(d*x+c))+9*d*e^2*f*x*exp(4*I*(d*x+c))-6*I*e*f^2*x*exp(4*I*(d*x+c))+3*I*d*
```

```
f^3*x^3*exp(3*I*(d*x+c))+6*I*e*f^2*x*exp(2*I*(d*x+c))-15*d*e*f^2*x^2*exp(2*
I*(d*x+c))-15*d*e^2*f*x*exp(2*I*(d*x+c))+9*d*e*f^2*x^2*exp(4*I*(d*x+c))+9*I
*d*e^2*f*x*exp(3*I*(d*x+c)))/(exp(2*I*(d*x+c))-1)^2/d^2/(exp(I*(d*x+c))+I)/
a
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 5.57135, size = 17747, normalized size = 29.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*d^3*f^3*x^3 + 4*d^3*e^3 - 6*d^2*e^2*f - 8*(d^3*f^3*x^3 + 3*d^3*e*f^
2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*cos(d*x + c)^3 + 6*(2*d^3*e*f^2 - d^2*f^3)
*x^2 - 6*(d^3*f^3*x^3 + d^3*e^3 - d^2*e^2*f + (3*d^3*e*f^2 - d^2*f^3)*x^2 +
 (3*d^3*e^2*f - 2*d^2*e*f^2)*x)*cos(d*x + c)^2 + 12*(d^3*e^2*f - d^2*e*f^2)
*x + 6*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*cos(d*x +
c) - (9*I*d^2*f^3*x^2 + 9*I*d^2*e^2*f - 12*I*d*e*f^2 + (-9*I*d^2*f^3*x^2 -
9*I*d^2*e^2*f + 12*I*d*e*f^2 - 6*I*f^3 - 6*I*(3*d^2*e*f^2 - 2*d*f^3)*x)*cos
(d*x + c)^3 + 6*I*f^3 + (-9*I*d^2*f^3*x^2 - 9*I*d^2*e^2*f + 12*I*d*e*f^2 -
6*I*f^3 - 6*I*(3*d^2*e*f^2 - 2*d*f^3)*x)*cos(d*x + c)^2 + 6*I*(3*d^2*e*f^2
- 2*d*f^3)*x + (9*I*d^2*f^3*x^2 + 9*I*d^2*e^2*f - 12*I*d*e*f^2 + 6*I*f^3 +
6*I*(3*d^2*e*f^2 - 2*d*f^3)*x)*cos(d*x + c) + (9*I*d^2*f^3*x^2 + 9*I*d^2*e^
2*f - 12*I*d*e*f^2 + 6*I*f^3 + (-9*I*d^2*f^3*x^2 - 9*I*d^2*e^2*f + 12*I*d*e
*f^2 - 6*I*f^3 - 6*I*(3*d^2*e*f^2 - 2*d*f^3)*x)*cos(d*x + c)^2 + 6*I*(3*d^2
*e*f^2 - 2*d*f^3)*x)*sin(d*x + c))*dilog(cos(d*x + c) + I*sin(d*x + c)) - (
-9*I*d^2*f^3*x^2 - 9*I*d^2*e^2*f + 12*I*d*e*f^2 + (9*I*d^2*f^3*x^2 + 9*I*d^
2*e^2*f - 12*I*d*e*f^2 + 6*I*f^3 + 6*I*(3*d^2*e*f^2 - 2*d*f^3)*x)*cos(d*x +
 c)^3 - 6*I*f^3 + (9*I*d^2*f^3*x^2 + 9*I*d^2*e^2*f - 12*I*d*e*f^2 + 6*I*f^3
```

```
 + 6*I*(3*d^2*e*f^2 - 2*d*f^3)*x)*cos(d*x + c)^2 - 6*I*(3*d^2*e*f^2 - 2*d*f
^3)*x + (-9*I*d^2*f^3*x^2 - 9*I*d^2*e^2*f + 12*I*d*e*f^2 - 6*I*f^3 - 6*I*(3
*d^2*e*f^2 - 2*d*f^3)*x)*cos(d*x + c) + (-9*I*d^2*f^3*x^2 - 9*I*d^2*e^2*f +
 12*I*d*e*f^2 - 6*I*f^3 + (9*I*d^2*f^3*x^2 + 9*I*d^2*e^2*f - 12*I*d*e*f^2 +
 6*I*f^3 + 6*I*(3*d^2*e*f^2 - 2*d*f^3)*x)*cos(d*x + c)^2 - 6*I*(3*d^2*e*f^2
 - 2*d*f^3)*x)*sin(d*x + c))*dilog(cos(d*x + c) - I*sin(d*x + c)) - (-24*I*
d*f^3*x - 24*I*d*e*f^2 + (24*I*d*f^3*x + 24*I*d*e*f^2)*cos(d*x + c)^3 + (24
*I*d*f^3*x + 24*I*d*e*f^2)*cos(d*x + c)^2 + (-24*I*d*f^3*x - 24*I*d*e*f^2)*
cos(d*x + c) + (-24*I*d*f^3*x - 24*I*d*e*f^2 + (24*I*d*f^3*x + 24*I*d*e*f^2
)*cos(d*x + c)^2)*sin(d*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) - (24*
I*d*f^3*x + 24*I*d*e*f^2 + (-24*I*d*f^3*x - 24*I*d*e*f^2)*cos(d*x + c)^3 +
(-24*I*d*f^3*x - 24*I*d*e*f^2)*cos(d*x + c)^2 + (24*I*d*f^3*x + 24*I*d*e*f^
2)*cos(d*x + c) + (24*I*d*f^3*x + 24*I*d*e*f^2 + (-24*I*d*f^3*x - 24*I*d*e*
f^2)*cos(d*x + c)^2)*sin(d*x + c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) -
(9*I*d^2*f^3*x^2 + 9*I*d^2*e^2*f + 12*I*d*e*f^2 + (-9*I*d^2*f^3*x^2 - 9*I*d
^2*e^2*f - 12*I*d*e*f^2 - 6*I*f^3 - 6*I*(3*d^2*e*f^2 + 2*d*f^3)*x)*cos(d*x
+ c)^3 + 6*I*f^3 + (-9*I*d^2*f^3*x^2 - 9*I*d^2*e^2*f - 12*I*d*e*f^2 - 6*I*f
^3 - 6*I*(3*d^2*e*f^2 + 2*d*f^3)*x)*cos(d*x + c)^2 + 6*I*(3*d^2*e*f^2 + 2*d
*f^3)*x + (9*I*d^2*f^3*x^2 + 9*I*d^2*e^2*f + 12*I*d*e*f^2 + 6*I*f^3 + 6*I*(
3*d^2*e*f^2 + 2*d*f^3)*x)*cos(d*x + c) + (9*I*d^2*f^3*x^2 + 9*I*d^2*e^2*f +
 12*I*d*e*f^2 + 6*I*f^3 + (-9*I*d^2*f^3*x^2 - 9*I*d^2*e^2*f - 12*I*d*e*f^2
- 6*I*f^3 - 6*I*(3*d^2*e*f^2 + 2*d*f^3)*x)*cos(d*x + c)^2 + 6*I*(3*d^2*e*f^
2 + 2*d*f^3)*x)*sin(d*x + c))*dilog(-cos(d*x + c) + I*sin(d*x + c)) - (-9*I
*d^2*f^3*x^2 - 9*I*d^2*e^2*f - 12*I*d*e*f^2 + (9*I*d^2*f^3*x^2 + 9*I*d^2*e^
2*f + 12*I*d*e*f^2 + 6*I*f^3 + 6*I*(3*d^2*e*f^2 + 2*d*f^3)*x)*cos(d*x + c)^
3 - 6*I*f^3 + (9*I*d^2*f^3*x^2 + 9*I*d^2*e^2*f + 12*I*d*e*f^2 + 6*I*f^3 + 6
*I*(3*d^2*e*f^2 + 2*d*f^3)*x)*cos(d*x + c)^2 - 6*I*(3*d^2*e*f^2 + 2*d*f^3)*
x + (-9*I*d^2*f^3*x^2 - 9*I*d^2*e^2*f - 12*I*d*e*f^2 - 6*I*f^3 - 6*I*(3*d^2
*e*f^2 + 2*d*f^3)*x)*cos(d*x + c) + (-9*I*d^2*f^3*x^2 - 9*I*d^2*e^2*f - 12*
I*d*e*f^2 - 6*I*f^3 + (9*I*d^2*f^3*x^2 + 9*I*d^2*e^2*f + 12*I*d*e*f^2 + 6*I
*f^3 + 6*I*(3*d^2*e*f^2 + 2*d*f^3)*x)*cos(d*x + c)^2 - 6*I*(3*d^2*e*f^2 + 2
*d*f^3)*x)*sin(d*x + c))*dilog(-cos(d*x + c) - I*sin(d*x + c)) - 3*(d^3*f^3
*x^3 + d^3*e^3 + 2*d^2*e^2*f + 2*d*e*f^2 - (d^3*f^3*x^3 + d^3*e^3 + 2*d^2*e
^2*f + 2*d*e*f^2 + (3*d^3*e*f^2 + 2*d^2*f^3)*x^2 + (3*d^3*e^2*f + 4*d^2*e*f
^2 + 2*d*f^3)*x)*cos(d*x + c)^3 + (3*d^3*e*f^2 + 2*d^2*f^3)*x^2 - (d^3*f^3*
x^3 + d^3*e^3 + 2*d^2*e^2*f + 2*d*e*f^2 + (3*d^3*e*f^2 + 2*d^2*f^3)*x^2 + (
3*d^3*e^2*f + 4*d^2*e*f^2 + 2*d*f^3)*x)*cos(d*x + c)^2 + (3*d^3*e^2*f + 4*d
^2*e*f^2 + 2*d*f^3)*x + (d^3*f^3*x^3 + d^3*e^3 + 2*d^2*e^2*f + 2*d*e*f^2 +
(3*d^3*e*f^2 + 2*d^2*f^3)*x^2 + (3*d^3*e^2*f + 4*d^2*e*f^2 + 2*d*f^3)*x)*co
s(d*x + c) + (d^3*f^3*x^3 + d^3*e^3 + 2*d^2*e^2*f + 2*d*e*f^2 + (3*d^3*e*f^
2 + 2*d^2*f^3)*x^2 - (d^3*f^3*x^3 + d^3*e^3 + 2*d^2*e^2*f + 2*d*e*f^2 + (3*
d^3*e*f^2 + 2*d^2*f^3)*x^2 + (3*d^3*e^2*f + 4*d^2*e*f^2 + 2*d*f^3)*x)*cos(d
*x + c)^2 + (3*d^3*e^2*f + 4*d^2*e*f^2 + 2*d*f^3)*x)*sin(d*x + c))*log(cos(
d*x + c) + I*sin(d*x + c) + 1) - 12*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3 - (d
^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c)^3 - (d^2*e^2*f - 2*c*d*e*f^2
```

```
 + c^2*f^3)*cos(d*x + c)^2 + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x +
c) + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3 - (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^
3)*cos(d*x + c)^2)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - 3
*(d^3*f^3*x^3 + d^3*e^3 + 2*d^2*e^2*f + 2*d*e*f^2 - (d^3*f^3*x^3 + d^3*e^3
+ 2*d^2*e^2*f + 2*d*e*f^2 + (3*d^3*e*f^2 + 2*d^2*f^3)*x^2 + (3*d^3*e^2*f +
4*d^2*e*f^2 + 2*d*f^3)*x)*cos(d*x + c)^3 + (3*d^3*e*f^2 + 2*d^2*f^3)*x^2 -
(d^3*f^3*x^3 + d^3*e^3 + 2*d^2*e^2*f + 2*d*e*f^2 + (3*d^3*e*f^2 + 2*d^2*f^3
)*x^2 + (3*d^3*e^2*f + 4*d^2*e*f^2 + 2*d*f^3)*x)*cos(d*x + c)^2 + (3*d^3*e^
2*f + 4*d^2*e*f^2 + 2*d*f^3)*x + (d^3*f^3*x^3 + d^3*e^3 + 2*d^2*e^2*f + 2*d
*e*f^2 + (3*d^3*e*f^2 + 2*d^2*f^3)*x^2 + (3*d^3*e^2*f + 4*d^2*e*f^2 + 2*d*f
^3)*x)*cos(d*x + c) + (d^3*f^3*x^3 + d^3*e^3 + 2*d^2*e^2*f + 2*d*e*f^2 + (3
*d^3*e*f^2 + 2*d^2*f^3)*x^2 - (d^3*f^3*x^3 + d^3*e^3 + 2*d^2*e^2*f + 2*d*e*
f^2 + (3*d^3*e*f^2 + 2*d^2*f^3)*x^2 + (3*d^3*e^2*f + 4*d^2*e*f^2 + 2*d*f^3)
*x)*cos(d*x + c)^2 + (3*d^3*e^2*f + 4*d^2*e*f^2 + 2*d*f^3)*x)*sin(d*x + c))
*log(cos(d*x + c) - I*sin(d*x + c) + 1) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x +
 2*c*d*e*f^2 - c^2*f^3 - (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f
^3)*cos(d*x + c)^3 - (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*
cos(d*x + c)^2 + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(
d*x + c) + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3 - (d^2*f^3*
x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c)^2)*sin(d*x + c))*
log(I*cos(d*x + c) + sin(d*x + c) + 1) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x +
2*c*d*e*f^2 - c^2*f^3 - (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^
3)*cos(d*x + c)^3 - (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*c
os(d*x + c)^2 + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d
*x + c) + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3 - (d^2*f^3*x
^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c)^2)*sin(d*x + c))*l
og(-I*cos(d*x + c) + sin(d*x + c) + 1) + 3*(d^3*e^3 - (3*c + 2)*d^2*e^2*f +
 (3*c^2 + 4*c + 2)*d*e*f^2 - (c^3 + 2*c^2 + 2*c)*f^3 - (d^3*e^3 - (3*c + 2)
*d^2*e^2*f + (3*c^2 + 4*c + 2)*d*e*f^2 - (c^3 + 2*c^2 + 2*c)*f^3)*cos(d*x +
 c)^3 - (d^3*e^3 - (3*c + 2)*d^2*e^2*f + (3*c^2 + 4*c + 2)*d*e*f^2 - (c^3 +
 2*c^2 + 2*c)*f^3)*cos(d*x + c)^2 + (d^3*e^3 - (3*c + 2)*d^2*e^2*f + (3*c^2
 + 4*c + 2)*d*e*f^2 - (c^3 + 2*c^2 + 2*c)*f^3)*cos(d*x + c) + (d^3*e^3 - (3
*c + 2)*d^2*e^2*f + (3*c^2 + 4*c + 2)*d*e*f^2 - (c^3 + 2*c^2 + 2*c)*f^3 - (
d^3*e^3 - (3*c + 2)*d^2*e^2*f + (3*c^2 + 4*c + 2)*d*e*f^2 - (c^3 + 2*c^2 +
2*c)*f^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2*I*sin(d
*x + c) + 1/2) + 3*(d^3*e^3 - (3*c + 2)*d^2*e^2*f + (3*c^2 + 4*c + 2)*d*e*f
^2 - (c^3 + 2*c^2 + 2*c)*f^3 - (d^3*e^3 - (3*c + 2)*d^2*e^2*f + (3*c^2 + 4*
c + 2)*d*e*f^2 - (c^3 + 2*c^2 + 2*c)*f^3)*cos(d*x + c)^3 - (d^3*e^3 - (3*c
+ 2)*d^2*e^2*f + (3*c^2 + 4*c + 2)*d*e*f^2 - (c^3 + 2*c^2 + 2*c)*f^3)*cos(d
*x + c)^2 + (d^3*e^3 - (3*c + 2)*d^2*e^2*f + (3*c^2 + 4*c + 2)*d*e*f^2 - (c
^3 + 2*c^2 + 2*c)*f^3)*cos(d*x + c) + (d^3*e^3 - (3*c + 2)*d^2*e^2*f + (3*c
^2 + 4*c + 2)*d*e*f^2 - (c^3 + 2*c^2 + 2*c)*f^3 - (d^3*e^3 - (3*c + 2)*d^2*
e^2*f + (3*c^2 + 4*c + 2)*d*e*f^2 - (c^3 + 2*c^2 + 2*c)*f^3)*cos(d*x + c)^2
)*sin(d*x + c))*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) + 3*(d^3*
f^3*x^3 + 3*c*d^2*e^2*f - (3*c^2 + 4*c)*d*e*f^2 + (c^3 + 2*c^2 + 2*c)*f^3 -
```

(d^3*f^3*x^3 + 3*c*d^2*e^2*f - (3*c^2 + 4*c)*d*e*f^2 + (c^3 + 2*c^2 + 2*c)
*f^3 + (3*d^3*e*f^2 - 2*d^2*f^3)*x^2 + (3*d^3*e^2*f - 4*d^2*e*f^2 + 2*d*f^3
)*x)*cos(d*x + c)^3 + (3*d^3*e*f^2 - 2*d^2*f^3)*x^2 - (d^3*f^3*x^3 + 3*c*d^
2*e^2*f - (3*c^2 + 4*c)*d*e*f^2 + (c^3 + 2*c^2 + 2*c)*f^3 + (3*d^3*e*f^2 -
2*d^2*f^3)*x^2 + (3*d^3*e^2*f - 4*d^2*e*f^2 + 2*d*f^3)*x)*cos(d*x + c)^2 +
(3*d^3*e^2*f - 4*d^2*e*f^2 + 2*d*f^3)*x + (d^3*f^3*x^3 + 3*c*d^2*e^2*f - (3
*c^2 + 4*c)*d*e*f^2 + (c^3 + 2*c^2 + 2*c)*f^3 + (3*d^3*e*f^2 - 2*d^2*f^3)*x
^2 + (3*d^3*e^2*f - 4*d^2*e*f^2 + 2*d*f^3)*x)*cos(d*x + c) + (d^3*f^3*x^3 +
3*c*d^2*e^2*f - (3*c^2 + 4*c)*d*e*f^2 + (c^3 + 2*c^2 + 2*c)*f^3 + (3*d^3*e
*f^2 - 2*d^2*f^3)*x^2 - (d^3*f^3*x^3 + 3*c*d^2*e^2*f - (3*c^2 + 4*c)*d*e*f^
2 + (c^3 + 2*c^2 + 2*c)*f^3 + (3*d^3*e*f^2 - 2*d^2*f^3)*x^2 + (3*d^3*e^2*f
- 4*d^2*e*f^2 + 2*d*f^3)*x)*cos(d*x + c)^2 + (3*d^3*e^2*f - 4*d^2*e*f^2 + 2
*d*f^3)*x)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + 1) - 12*(d^2*
e^2*f - 2*c*d*e*f^2 + c^2*f^3 - (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x
+ c)^3 - (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c)^2 + (d^2*e^2*f -
2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c) + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3 -
(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-cos
(d*x + c) + I*sin(d*x + c) + I) + 3*(d^3*f^3*x^3 + 3*c*d^2*e^2*f - (3*c^2 +
4*c)*d*e*f^2 + (c^3 + 2*c^2 + 2*c)*f^3 - (d^3*f^3*x^3 + 3*c*d^2*e^2*f - (3
*c^2 + 4*c)*d*e*f^2 + (c^3 + 2*c^2 + 2*c)*f^3 + (3*d^3*e*f^2 - 2*d^2*f^3)*x
^2 + (3*d^3*e^2*f - 4*d^2*e*f^2 + 2*d*f^3)*x)*cos(d*x + c)^3 + (3*d^3*e*f^2
- 2*d^2*f^3)*x^2 - (d^3*f^3*x^3 + 3*c*d^2*e^2*f - (3*c^2 + 4*c)*d*e*f^2 +
(c^3 + 2*c^2 + 2*c)*f^3 + (3*d^3*e*f^2 - 2*d^2*f^3)*x^2 + (3*d^3*e^2*f - 4*
d^2*e*f^2 + 2*d*f^3)*x)*cos(d*x + c)^2 + (3*d^3*e^2*f - 4*d^2*e*f^2 + 2*d*f
^3)*x + (d^3*f^3*x^3 + 3*c*d^2*e^2*f - (3*c^2 + 4*c)*d*e*f^2 + (c^3 + 2*c^2
+ 2*c)*f^3 + (3*d^3*e*f^2 - 2*d^2*f^3)*x^2 + (3*d^3*e^2*f - 4*d^2*e*f^2 +
2*d*f^3)*x)*cos(d*x + c) + (d^3*f^3*x^3 + 3*c*d^2*e^2*f - (3*c^2 + 4*c)*d*e
*f^2 + (c^3 + 2*c^2 + 2*c)*f^3 + (3*d^3*e*f^2 - 2*d^2*f^3)*x^2 - (d^3*f^3*x
^3 + 3*c*d^2*e^2*f - (3*c^2 + 4*c)*d*e*f^2 + (c^3 + 2*c^2 + 2*c)*f^3 + (3*d
^3*e*f^2 - 2*d^2*f^3)*x^2 + (3*d^3*e^2*f - 4*d^2*e*f^2 + 2*d*f^3)*x)*cos(d*
x + c)^2 + (3*d^3*e^2*f - 4*d^2*e*f^2 + 2*d*f^3)*x)*sin(d*x + c))*log(-cos(
d*x + c) - I*sin(d*x + c) + 1) - (18*I*f^3*cos(d*x + c)^3 + 18*I*f^3*cos(d*
x + c)^2 - 18*I*f^3*cos(d*x + c) - 18*I*f^3 + (18*I*f^3*cos(d*x + c)^2 - 18
*I*f^3)*sin(d*x + c))*polylog(4, cos(d*x + c) + I*sin(d*x + c)) - (-18*I*f^
3*cos(d*x + c)^3 - 18*I*f^3*cos(d*x + c)^2 + 18*I*f^3*cos(d*x + c) + 18*I*f
^3 + (-18*I*f^3*cos(d*x + c)^2 + 18*I*f^3)*sin(d*x + c))*polylog(4, cos(d*x
+ c) - I*sin(d*x + c)) - (18*I*f^3*cos(d*x + c)^3 + 18*I*f^3*cos(d*x + c)^
2 - 18*I*f^3*cos(d*x + c) - 18*I*f^3 + (18*I*f^3*cos(d*x + c)^2 - 18*I*f^3)
*sin(d*x + c))*polylog(4, -cos(d*x + c) + I*sin(d*x + c)) - (-18*I*f^3*cos(
d*x + c)^3 - 18*I*f^3*cos(d*x + c)^2 + 18*I*f^3*cos(d*x + c) + 18*I*f^3 + (
-18*I*f^3*cos(d*x + c)^2 + 18*I*f^3)*sin(d*x + c))*polylog(4, -cos(d*x + c)
- I*sin(d*x + c)) + 6*(3*d*f^3*x + 3*d*e*f^2 - (3*d*f^3*x + 3*d*e*f^2 - 2*
f^3)*cos(d*x + c)^3 - 2*f^3 - (3*d*f^3*x + 3*d*e*f^2 - 2*f^3)*cos(d*x + c)^
2 + (3*d*f^3*x + 3*d*e*f^2 - 2*f^3)*cos(d*x + c) + (3*d*f^3*x + 3*d*e*f^2 -
2*f^3 - (3*d*f^3*x + 3*d*e*f^2 - 2*f^3)*cos(d*x + c)^2)*sin(d*x + c))*poly

```
log(3, cos(d*x + c) + I*sin(d*x + c)) + 6*(3*d*f^3*x + 3*d*e*f^2 - (3*d*f^3
*x + 3*d*e*f^2 - 2*f^3)*cos(d*x + c)^3 - 2*f^3 - (3*d*f^3*x + 3*d*e*f^2 - 2
*f^3)*cos(d*x + c)^2 + (3*d*f^3*x + 3*d*e*f^2 - 2*f^3)*cos(d*x + c) + (3*d*
f^3*x + 3*d*e*f^2 - 2*f^3 - (3*d*f^3*x + 3*d*e*f^2 - 2*f^3)*cos(d*x + c)^2)
*sin(d*x + c))*polylog(3, cos(d*x + c) - I*sin(d*x + c)) + 24*(f^3*cos(d*x
+ c)^3 + f^3*cos(d*x + c)^2 - f^3*cos(d*x + c) - f^3 + (f^3*cos(d*x + c)^2
- f^3)*sin(d*x + c))*polylog(3, I*cos(d*x + c) - sin(d*x + c)) + 24*(f^3*co
s(d*x + c)^3 + f^3*cos(d*x + c)^2 - f^3*cos(d*x + c) - f^3 + (f^3*cos(d*x +
 c)^2 - f^3)*sin(d*x + c))*polylog(3, -I*cos(d*x + c) - sin(d*x + c)) - 6*(
3*d*f^3*x + 3*d*e*f^2 - (3*d*f^3*x + 3*d*e*f^2 + 2*f^3)*cos(d*x + c)^3 + 2*
f^3 - (3*d*f^3*x + 3*d*e*f^2 + 2*f^3)*cos(d*x + c)^2 + (3*d*f^3*x + 3*d*e*f
^2 + 2*f^3)*cos(d*x + c) + (3*d*f^3*x + 3*d*e*f^2 + 2*f^3 - (3*d*f^3*x + 3*
d*e*f^2 + 2*f^3)*cos(d*x + c)^2)*sin(d*x + c))*polylog(3, -cos(d*x + c) + I
*sin(d*x + c)) - 6*(3*d*f^3*x + 3*d*e*f^2 - (3*d*f^3*x + 3*d*e*f^2 + 2*f^3)
*cos(d*x + c)^3 + 2*f^3 - (3*d*f^3*x + 3*d*e*f^2 + 2*f^3)*cos(d*x + c)^2 +
(3*d*f^3*x + 3*d*e*f^2 + 2*f^3)*cos(d*x + c) + (3*d*f^3*x + 3*d*e*f^2 + 2*f
^3 - (3*d*f^3*x + 3*d*e*f^2 + 2*f^3)*cos(d*x + c)^2)*sin(d*x + c))*polylog(
3, -cos(d*x + c) - I*sin(d*x + c)) - 2*(2*d^3*f^3*x^3 + 2*d^3*e^3 + 3*d^2*e
^2*f + 3*(2*d^3*e*f^2 + d^2*f^3)*x^2 - 4*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3
*d^3*e^2*f*x + d^3*e^3)*cos(d*x + c)^2 + 6*(d^3*e^2*f + d^2*e*f^2)*x - (d^3
*f^3*x^3 + d^3*e^3 - 3*d^2*e^2*f + 3*(d^3*e*f^2 - d^2*f^3)*x^2 + 3*(d^3*e^2
*f - 2*d^2*e*f^2)*x)*cos(d*x + c))*sin(d*x + c))/(a*d^4*cos(d*x + c)^3 + a*
d^4*cos(d*x + c)^2 - a*d^4*cos(d*x + c) - a*d^4 + (a*d^4*cos(d*x + c)^2 - a
*d^4)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.210 $\int \frac{(e+fx)^2\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=392

$$\frac{3if(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{3if(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{4if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}+\frac{if^2\text{PolyLog}\left(2,e^{2i(c}\right.}{ad^3}$$

```
[Out] ((2*I)*(e + f*x)^2)/(a*d) - (3*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d)
- (f^2*ArcTanh[Cos[c + d*x]])/(a*d^3) + ((e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)
/2])/(a*d) + ((e + f*x)^2*Cot[c + d*x])/(a*d) - (f*(e + f*x)*Csc[c + d*x])/
(a*d^2) - ((e + f*x)^2*Cot[c + d*x]*Csc[c + d*x])/(2*a*d) - (4*f*(e + f*x)*
Log[1 - I*E^(I*(c + d*x))])/(a*d^2) - (2*f*(e + f*x)*Log[1 - E^((2*I)*(c +
d*x))])/(a*d^2) + ((3*I)*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2)
+ ((4*I)*f^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - ((3*I)*f*(e + f*x)*Po
lyLog[2, E^(I*(c + d*x))])/(a*d^2) + (I*f^2*PolyLog[2, E^((2*I)*(c + d*x))]
)/(a*d^3) - (3*f^2*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3) + (3*f^2*PolyLog[3
, E^(I*(c + d*x))])/(a*d^3)
```

Rubi [A] time = 0.722168, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {4535, 4186, 3770, 4183, 2531, 2282, 6589, 4184, 3717, 2190, 2279, 2391, 3318}

$$\frac{3if(e+fx)\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{3if(e+fx)\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{4if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}+\frac{if^2\text{PolyLog}\left(2,e^{2i(c}\right.}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] ((2*I)*(e + f*x)^2)/(a*d) - (3*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d)
- (f^2*ArcTanh[Cos[c + d*x]])/(a*d^3) + ((e + f*x)^2*Cot[c/2 + Pi/4 + (d*x)
/2])/(a*d) + ((e + f*x)^2*Cot[c + d*x])/(a*d) - (f*(e + f*x)*Csc[c + d*x])/
(a*d^2) - ((e + f*x)^2*Cot[c + d*x]*Csc[c + d*x])/(2*a*d) - (4*f*(e + f*x)*
Log[1 - I*E^(I*(c + d*x))])/(a*d^2) - (2*f*(e + f*x)*Log[1 - E^((2*I)*(c +
d*x))])/(a*d^2) + ((3*I)*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2)
+ ((4*I)*f^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^3) - ((3*I)*f*(e + f*x)*Po
lyLog[2, E^(I*(c + d*x))])/(a*d^2) + (I*f^2*PolyLog[2, E^((2*I)*(c + d*x))]
)/(a*d^3) - (3*f^2*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3) + (3*f^2*PolyLog[3
, E^(I*(c + d*x))])/(a*d^3)
```

Rule 4535

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csc[c + d*x]^(n - 1))/(a + b*Si
n[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
```

```
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(e+fx)^2\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\csc^3(c+dx)\,dx}{a} - \int \frac{(e+fx)^2\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{f(e+fx)\csc(c+dx)}{ad^2} - \frac{(e+fx)^2\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\int (e+fx)^2\csc(c+dx)\,dx}{2a}$$

$$= -\frac{(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3} + \frac{(e+fx)^2\cot(c+dx)}{ad} - \frac{f(e+}{}$$

$$= \frac{i(e+fx)^2}{ad} - \frac{3(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3} + \frac{(e+fx)^2\cot(c+}{ad}$$

$$= \frac{i(e+fx)^2}{ad} - \frac{3(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3} + \frac{(e+fx)^2\cot\left(\frac{c}{2}+\right.}{ad}$$

$$= \frac{2i(e+fx)^2}{ad} - \frac{3(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3} + \frac{(e+fx)^2\cot\left(\frac{c}{2}\right.}{ad}$$

$$= \frac{2i(e+fx)^2}{ad} - \frac{3(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3} + \frac{(e+fx)^2\cot\left(\frac{c}{2}\right.}{ad}$$

$$= \frac{2i(e+fx)^2}{ad} - \frac{3(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3} + \frac{(e+fx)^2\cot\left(\frac{c}{2}\right.}{ad}$$

$$= \frac{2i(e+fx)^2}{ad} - \frac{3(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f^2\tanh^{-1}(\cos(c+dx))}{ad^3} + \frac{(e+fx)^2\cot\left(\frac{c}{2}\right.}{ad}$$

Mathematica [B] time = 17.4976, size = 951, normalized size = 2.43

$$\frac{\frac{32f(\cos(c)+i\sin(c))\left(\frac{(\cos(c)-i\sin(c))(e+fx)^2}{2f}-\frac{\log(i\cos(c+dx)+\sin(c+dx)+1)(i\cos(c)+\sin(c)+1)(e+fx)}{d}+\frac{f\,\mathrm{PolyLog}(2,-i\cos(c+dx)-\sin(c+dx))(\cos(c)-i(\sin(c)+1))}{d^2}\right)d^2}{\cos(c)+i(\sin(c)+1)} - \frac{(e+fx)}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (8*((2*I)*d^2*e*f*x + I*d^2*f^2*x^2 - 3*d^2*e^2*ArcTanh[Cos[c + d*x] + I*Si
n[c + d*x]] - 2*f^2*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] - 6*d^2*e*f*x*Ar
cTanh[Cos[c + d*x] + I*Sin[c + d*x]] - 3*d^2*f^2*x^2*ArcTanh[Cos[c + d*x] +
 I*Sin[c + d*x]] + 2*d^2*e*f*x*Cot[c] + d^2*f^2*x^2*Cot[c] - 2*d*e*f*Log[1
- Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]] - 2*d*f^2*x*Log[1 - Cos[2*(c + d*x
)] - I*Sin[2*(c + d*x)]] + (3*I)*d*f*(e + f*x)*PolyLog[2, -Cos[c + d*x] - I
*Sin[c + d*x]] - (3*I)*d*f*(e + f*x)*PolyLog[2, Cos[c + d*x] + I*Sin[c + d*
x]] + I*f^2*PolyLog[2, Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]] - 3*f^2*PolyL
og[3, -Cos[c + d*x] - I*Sin[c + d*x]] + 3*f^2*PolyLog[3, Cos[c + d*x] + I*S
in[c + d*x]]) + (32*d^2*f*(Cos[c] + I*Sin[c])*(((e + f*x)^2*(Cos[c] - I*Sin
[c]))/(2*f) - ((e + f*x)*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(1 + I*Cos[
c] + Sin[c]))/d + (f*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] -
 I*(1 + Sin[c])))/d^2))/(Cos[c] + I*(1 + Sin[c])) - (d*(e + f*x)*Csc[c]*Csc
[c + d*x]^2*(2*f*Cos[(d*x)/2] + 2*f*Cos[(3*d*x)/2] + 5*d*e*Cos[c - (d*x)/2]
 + 5*d*f*x*Cos[c - (d*x)/2] - d*e*Cos[c + (d*x)/2] - d*f*x*Cos[c + (d*x)/2]
 - 2*f*Cos[2*c + (d*x)/2] + d*e*Cos[c + (3*d*x)/2] + d*f*x*Cos[c + (3*d*x)/
2] - 2*f*Cos[2*c + (3*d*x)/2] - 3*d*e*Cos[3*c + (3*d*x)/2] - 3*d*f*x*Cos[3*
c + (3*d*x)/2] - 4*d*e*Cos[c + (5*d*x)/2] - 4*d*f*x*Cos[c + (5*d*x)/2] + 2*
d*e*Cos[3*c + (5*d*x)/2] + 2*d*f*x*Cos[3*c + (5*d*x)/2] + d*e*Sin[(d*x)/2]
+ d*f*x*Sin[(d*x)/2] + d*e*Sin[(3*d*x)/2] + d*f*x*Sin[(3*d*x)/2] + 2*f*Sin[
c - (d*x)/2] + 2*f*Sin[c + (d*x)/2] + 3*d*e*Sin[2*c + (d*x)/2] + 3*d*f*x*Si
n[2*c + (d*x)/2] + 2*f*Sin[c + (3*d*x)/2] + d*e*Sin[2*c + (3*d*x)/2] + d*f*
x*Sin[2*c + (3*d*x)/2] - 2*f*Sin[3*c + (3*d*x)/2] - 2*d*e*Sin[2*c + (5*d*x)
/2] - 2*d*f*x*Sin[2*c + (5*d*x)/2]))/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])))/(8*a*d^3)
```

Maple [B] time = 0.231, size = 1215, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/a/d^3*f^2*ln(exp(I*(d*x+c))-1)-1/a/d^3*f^2*ln(exp(I*(d*x+c))+1)+8*f/d^2/a
*ln(exp(I*(d*x+c)))*e-4*f/d^2/a*ln(exp(I*(d*x+c))+I)*e-4*f^2/d^2/a*ln(1-I*e
xp(I*(d*x+c)))*x-4*f^2/d^3/a*ln(1-I*exp(I*(d*x+c)))*c-8*f^2/d^3/a*c*ln(exp(
I*(d*x+c)))+4*f^2/d^3/a*c*ln(exp(I*(d*x+c))+I)+3/2/a/d^3*f^2*c^2*ln(exp(I*(
d*x+c))-1)+3/2/a/d*f^2*ln(1-exp(I*(d*x+c)))*x^2-3/2/a/d^3*f^2*ln(1-exp(I*(d
*x+c)))*c^2-3/2/a/d*f^2*ln(exp(I*(d*x+c))+1)*x^2+3/2/a/d*e^2*ln(exp(I*(d*x+
c))-1)-3/2/a/d*e^2*ln(exp(I*(d*x+c))+1)+(3*d*f^2*x^2*exp(4*I*(d*x+c))+6*d*e
*f*x*exp(4*I*(d*x+c))+3*d*e^2*exp(4*I*(d*x+c))-5*d*f^2*x^2*exp(2*I*(d*x+c))
```

```
-I*d*f^2*x^2*exp(I*(d*x+c))-10*d*e*f*x*exp(2*I*(d*x+c))+2*f^2*x*exp(3*I*(d*
x+c))+2*I*f^2*x*exp(2*I*(d*x+c))+2*I*e*f*exp(2*I*(d*x+c))-5*d*e^2*exp(2*I*(
d*x+c))+4*d*f^2*x^2+2*e*f*exp(3*I*(d*x+c))-I*d*e^2*exp(I*(d*x+c))+3*I*d*f^2
*x^2*exp(3*I*(d*x+c))+6*I*d*e*f*x*exp(3*I*(d*x+c))+8*d*e*f*x-2*f^2*x*exp(I*
(d*x+c))-2*I*d*e*f*x*exp(I*(d*x+c))+3*I*d*e^2*exp(3*I*(d*x+c))+4*d*e^2-2*ex
p(I*(d*x+c))*e*f-2*I*f^2*x*exp(4*I*(d*x+c))-2*I*e*f*exp(4*I*(d*x+c)))/(exp(
2*I*(d*x+c))-1)^2/d^2/(exp(I*(d*x+c))+I)/a+8*I/a/d^2*c*f^2*x-3*I/a/d^2*poly
log(2,exp(I*(d*x+c)))*f^2*x-3*I/a/d^2*e*f*polylog(2,exp(I*(d*x+c)))+3*I/a/d
^2*e*f*polylog(2,-exp(I*(d*x+c)))+3*I/a/d^2*polylog(2,-exp(I*(d*x+c)))*f^2*
x+4*I*f^2*polylog(2,I*exp(I*(d*x+c)))/a/d^3+3/a/d^2*ln(1-exp(I*(d*x+c)))*c*
e*f+3/a/d*ln(1-exp(I*(d*x+c)))*e*f*x-3/a/d*ln(exp(I*(d*x+c))+1)*e*f*x-3/a/d
^2*e*f*c*ln(exp(I*(d*x+c))-1)-3*f^2*polylog(3,-exp(I*(d*x+c)))/a/d^3+3*f^2*
polylog(3,exp(I*(d*x+c)))/a/d^3+2*I*f^2*polylog(2,-exp(I*(d*x+c)))/a/d^3+2*
I/a/d^3*f^2*polylog(2,exp(I*(d*x+c)))+4*I/a/d^3*c^2*f^2+4*I/a/d*f^2*x^2-2/a
/d^2*f^2*ln(exp(I*(d*x+c))+1)*x+2/a/d^3*f^2*c*ln(exp(I*(d*x+c))-1)-2/a/d^2*
e*f*ln(exp(I*(d*x+c))-1)-2/a/d^2*e*f*ln(exp(I*(d*x+c))+1)-2/a/d^2*f^2*ln(1-
exp(I*(d*x+c)))*x-2/a/d^3*f^2*ln(1-exp(I*(d*x+c)))*c
```

Maxima [B] time = 13.3787, size = 8266, normalized size = 21.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*(2*c*e*f*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 - 1)/(a*d*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*d*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3) - (4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)/(a*d) + 12*log(sin(d*x + c)/(cos(d*x + c) +
1))/(a*d)) + e^2*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(
d*x + c) + 1)^2)/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 - 1)/(a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3) - 12*log(sin(d*x + c)/(cos(d*x + c) + 1))/a
) + 8*(16*I*c^2*f^2 + (16*I*d*e*f - 16*I*c*f^2 + 16*(d*e*f - c*f^2)*cos(5*d
*x + 5*c) + (16*I*d*e*f - 16*I*c*f^2)*cos(4*d*x + 4*c) - 32*(d*e*f - c*f^2)
*cos(3*d*x + 3*c) + (-32*I*d*e*f + 32*I*c*f^2)*cos(2*d*x + 2*c) + 16*(d*e*f
 - c*f^2)*cos(d*x + c) + (16*I*d*e*f - 16*I*c*f^2)*sin(5*d*x + 5*c) - 16*(d
*e*f - c*f^2)*sin(4*d*x + 4*c) + (-32*I*d*e*f + 32*I*c*f^2)*sin(3*d*x + 3*c
) + 32*(d*e*f - c*f^2)*sin(2*d*x + 2*c) + (16*I*d*e*f - 16*I*c*f^2)*sin(d*x
 + c))*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - (16*(d*x + c)*f^2*cos(5*d*
x + 5*c) + 16*I*(d*x + c)*f^2*cos(4*d*x + 4*c) - 32*(d*x + c)*f^2*cos(3*d*x
```

+ 3*c) - 32*I*(d*x + c)*f^2*cos(2*d*x + 2*c) + 16*(d*x + c)*f^2*cos(d*x +
c) + 16*I*(d*x + c)*f^2*sin(5*d*x + 5*c) - 16*(d*x + c)*f^2*sin(4*d*x + 4*c
) - 32*I*(d*x + c)*f^2*sin(3*d*x + 3*c) + 32*(d*x + c)*f^2*sin(2*d*x + 2*c)
+ 16*I*(d*x + c)*f^2*sin(d*x + c) + 16*I*(d*x + c)*f^2)*arctan2(cos(d*x +
c), sin(d*x + c) + 1) + (6*I*(d*x + c)^2*f^2 + 8*I*d*e*f + (6*I*c^2 - 8*I*c
+ 4*I)*f^2 + (12*I*d*e*f + (-12*I*c + 8*I)*f^2)*(d*x + c) + 2*(3*(d*x + c)
^2*f^2 + 4*d*e*f + (3*c^2 - 4*c + 2)*f^2 + 2*(3*d*e*f - (3*c - 2)*f^2)*(d*x
+ c))*cos(5*d*x + 5*c) + (6*I*(d*x + c)^2*f^2 + 8*I*d*e*f + (6*I*c^2 - 8*I
*c + 4*I)*f^2 + (12*I*d*e*f + (-12*I*c + 8*I)*f^2)*(d*x + c))*cos(4*d*x + 4
c) - 4(3*(d*x + c)^2*f^2 + 4*d*e*f + (3*c^2 - 4*c + 2)*f^2 + 2*(3*d*e*f -
(3*c - 2)*f^2)*(d*x + c))*cos(3*d*x + 3*c) + (-12*I*(d*x + c)^2*f^2 - 16*I
*d*e*f + (-12*I*c^2 + 16*I*c - 8*I)*f^2 + (-24*I*d*e*f + (24*I*c - 16*I)*f^
2)*(d*x + c))*cos(2*d*x + 2*c) + 2*(3*(d*x + c)^2*f^2 + 4*d*e*f + (3*c^2 -
4*c + 2)*f^2 + 2*(3*d*e*f - (3*c - 2)*f^2)*(d*x + c))*cos(d*x + c) + (6*I*(
d*x + c)^2*f^2 + 8*I*d*e*f + (6*I*c^2 - 8*I*c + 4*I)*f^2 + (12*I*d*e*f + (-
12*I*c + 8*I)*f^2)*(d*x + c))*sin(5*d*x + 5*c) - 2*(3*(d*x + c)^2*f^2 + 4*d
*e*f + (3*c^2 - 4*c + 2)*f^2 + 2*(3*d*e*f - (3*c - 2)*f^2)*(d*x + c))*sin(4
*d*x + 4*c) + (-12*I*(d*x + c)^2*f^2 - 16*I*d*e*f + (-12*I*c^2 + 16*I*c - 8
*I)*f^2 + (-24*I*d*e*f + (24*I*c - 16*I)*f^2)*(d*x + c))*sin(3*d*x + 3*c) +
4*(3*(d*x + c)^2*f^2 + 4*d*e*f + (3*c^2 - 4*c + 2)*f^2 + 2*(3*d*e*f - (3*c
- 2)*f^2)*(d*x + c))*sin(2*d*x + 2*c) + (6*I*(d*x + c)^2*f^2 + 8*I*d*e*f +
(6*I*c^2 - 8*I*c + 4*I)*f^2 + (12*I*d*e*f + (-12*I*c + 8*I)*f^2)*(d*x + c)
)*sin(d*x + c))*arctan2(sin(d*x + c), cos(d*x + c) + 1) + (8*I*d*e*f + (-6*
I*c^2 - 8*I*c - 4*I)*f^2 + 2*(4*d*e*f - (3*c^2 + 4*c + 2)*f^2)*cos(5*d*x +
5*c) + (8*I*d*e*f + (-6*I*c^2 - 8*I*c - 4*I)*f^2)*cos(4*d*x + 4*c) - 4*(4*d
*e*f - (3*c^2 + 4*c + 2)*f^2)*cos(3*d*x + 3*c) + (-16*I*d*e*f + (12*I*c^2 +
16*I*c + 8*I)*f^2)*cos(2*d*x + 2*c) + 2*(4*d*e*f - (3*c^2 + 4*c + 2)*f^2)*
cos(d*x + c) + (8*I*d*e*f + (-6*I*c^2 - 8*I*c - 4*I)*f^2)*sin(5*d*x + 5*c)
- 2*(4*d*e*f - (3*c^2 + 4*c + 2)*f^2)*sin(4*d*x + 4*c) + (-16*I*d*e*f + (12
*I*c^2 + 16*I*c + 8*I)*f^2)*sin(3*d*x + 3*c) + 4*(4*d*e*f - (3*c^2 + 4*c +
2)*f^2)*sin(2*d*x + 2*c) + (8*I*d*e*f + (-6*I*c^2 - 8*I*c - 4*I)*f^2)*sin(d
*x + c))*arctan2(sin(d*x + c), cos(d*x + c) - 1) + (6*I*(d*x + c)^2*f^2 + (
12*I*d*e*f + (-12*I*c - 8*I)*f^2)*(d*x + c) + 2*(3*(d*x + c)^2*f^2 + 2*(3*d
*e*f - (3*c + 2)*f^2)*(d*x + c))*cos(5*d*x + 5*c) + (6*I*(d*x + c)^2*f^2 +
(12*I*d*e*f + (-12*I*c - 8*I)*f^2)*(d*x + c))*cos(4*d*x + 4*c) - 4*(3*(d*x
+ c)^2*f^2 + 2*(3*d*e*f - (3*c + 2)*f^2)*(d*x + c))*cos(3*d*x + 3*c) + (-12
I(d*x + c)^2*f^2 + (-24*I*d*e*f + (24*I*c + 16*I)*f^2)*(d*x + c))*cos(2*d
*x + 2*c) + 2*(3*(d*x + c)^2*f^2 + 2*(3*d*e*f - (3*c + 2)*f^2)*(d*x + c))*c
os(d*x + c) + (6*I*(d*x + c)^2*f^2 + (12*I*d*e*f + (-12*I*c - 8*I)*f^2)*(d*
x + c))*sin(5*d*x + 5*c) - 2*(3*(d*x + c)^2*f^2 + 2*(3*d*e*f - (3*c + 2)*f^
2)*(d*x + c))*sin(4*d*x + 4*c) + (-12*I*(d*x + c)^2*f^2 + (-24*I*d*e*f + (2
4*I*c + 16*I)*f^2)*(d*x + c))*sin(3*d*x + 3*c) + 4*(3*(d*x + c)^2*f^2 + 2*(
3*d*e*f - (3*c + 2)*f^2)*(d*x + c))*sin(2*d*x + 2*c) + (6*I*(d*x + c)^2*f^2
+ (12*I*d*e*f + (-12*I*c - 8*I)*f^2)*(d*x + c))*sin(d*x + c))*arctan2(sin(
d*x + c), -cos(d*x + c) + 1) - 16*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x

```
 + c))*cos(5*d*x + 5*c) + (-4*I*(d*x + c)^2*f^2 + 8*d*e*f + (12*I*c^2 - 8*c
)*f^2 - 8*(I*d*e*f + (-I*c - 1)*f^2)*(d*x + c))*cos(4*d*x + 4*c) + (20*(d*x
 + c)^2*f^2 + 8*I*d*e*f - 4*(3*c^2 + 2*I*c)*f^2 + (40*d*e*f - (40*c - 8*I)*
f^2)*(d*x + c))*cos(3*d*x + 3*c) + (12*I*(d*x + c)^2*f^2 - 8*d*e*f + (-20*I
*c^2 + 8*c)*f^2 - 8*(-3*I*d*e*f + (3*I*c + 1)*f^2)*(d*x + c))*cos(2*d*x + 2
*c) - (12*(d*x + c)^2*f^2 + 8*I*d*e*f - 4*(c^2 + 2*I*c)*f^2 + (24*d*e*f - (
24*c - 8*I)*f^2)*(d*x + c))*cos(d*x + c) - (16*f^2*cos(5*d*x + 5*c) + 16*I*
f^2*cos(4*d*x + 4*c) - 32*f^2*cos(3*d*x + 3*c) - 32*I*f^2*cos(2*d*x + 2*c)
+ 16*f^2*cos(d*x + c) + 16*I*f^2*sin(5*d*x + 5*c) - 16*f^2*sin(4*d*x + 4*c)
 - 32*I*f^2*sin(3*d*x + 3*c) + 32*f^2*sin(2*d*x + 2*c) + 16*I*f^2*sin(d*x +
 c) + 16*I*f^2)*dilog(I*e^(I*d*x + I*c)) + (-12*I*d*e*f - 12*I*(d*x + c)*f^
2 + (12*I*c - 8*I)*f^2 - 4*(3*d*e*f + 3*(d*x + c)*f^2 - (3*c - 2)*f^2)*cos(
5*d*x + 5*c) + (-12*I*d*e*f - 12*I*(d*x + c)*f^2 + (12*I*c - 8*I)*f^2)*cos(
4*d*x + 4*c) + 8*(3*d*e*f + 3*(d*x + c)*f^2 - (3*c - 2)*f^2)*cos(3*d*x + 3*
c) + (24*I*d*e*f + 24*I*(d*x + c)*f^2 + (-24*I*c + 16*I)*f^2)*cos(2*d*x + 2
*c) - 4*(3*d*e*f + 3*(d*x + c)*f^2 - (3*c - 2)*f^2)*cos(d*x + c) + (-12*I*d
*e*f - 12*I*(d*x + c)*f^2 + (12*I*c - 8*I)*f^2)*sin(5*d*x + 5*c) + 4*(3*d*e
*f + 3*(d*x + c)*f^2 - (3*c - 2)*f^2)*sin(4*d*x + 4*c) + (24*I*d*e*f + 24*I
*(d*x + c)*f^2 + (-24*I*c + 16*I)*f^2)*sin(3*d*x + 3*c) - 8*(3*d*e*f + 3*(d
*x + c)*f^2 - (3*c - 2)*f^2)*sin(2*d*x + 2*c) + (-12*I*d*e*f - 12*I*(d*x +
c)*f^2 + (12*I*c - 8*I)*f^2)*sin(d*x + c))*dilog(-e^(I*d*x + I*c)) + (12*I*
d*e*f + 12*I*(d*x + c)*f^2 + (-12*I*c - 8*I)*f^2 + 4*(3*d*e*f + 3*(d*x + c)
*f^2 - (3*c + 2)*f^2)*cos(5*d*x + 5*c) + (12*I*d*e*f + 12*I*(d*x + c)*f^2 +
 (-12*I*c - 8*I)*f^2)*cos(4*d*x + 4*c) - 8*(3*d*e*f + 3*(d*x + c)*f^2 - (3*
c + 2)*f^2)*cos(3*d*x + 3*c) + (-24*I*d*e*f - 24*I*(d*x + c)*f^2 + (24*I*c
+ 16*I)*f^2)*cos(2*d*x + 2*c) + 4*(3*d*e*f + 3*(d*x + c)*f^2 - (3*c + 2)*f^
2)*cos(d*x + c) + (12*I*d*e*f + 12*I*(d*x + c)*f^2 + (-12*I*c - 8*I)*f^2)*s
in(5*d*x + 5*c) - 4*(3*d*e*f + 3*(d*x + c)*f^2 - (3*c + 2)*f^2)*sin(4*d*x +
 4*c) + (-24*I*d*e*f - 24*I*(d*x + c)*f^2 + (24*I*c + 16*I)*f^2)*sin(3*d*x
+ 3*c) + 8*(3*d*e*f + 3*(d*x + c)*f^2 - (3*c + 2)*f^2)*sin(2*d*x + 2*c) + (
12*I*d*e*f + 12*I*(d*x + c)*f^2 + (-12*I*c - 8*I)*f^2)*sin(d*x + c))*dilog(
e^(I*d*x + I*c)) + (3*(d*x + c)^2*f^2 + 4*d*e*f + (3*c^2 - 4*c + 2)*f^2 + 2
*(3*d*e*f - (3*c - 2)*f^2)*(d*x + c) + (-3*I*(d*x + c)^2*f^2 - 4*I*d*e*f +
(-3*I*c^2 + 4*I*c - 2*I)*f^2 + (-6*I*d*e*f + (6*I*c - 4*I)*f^2)*(d*x + c))*
cos(5*d*x + 5*c) + (3*(d*x + c)^2*f^2 + 4*d*e*f + (3*c^2 - 4*c + 2)*f^2 + 2
*(3*d*e*f - (3*c - 2)*f^2)*(d*x + c))*cos(4*d*x + 4*c) + (6*I*(d*x + c)^2*f
^2 + 8*I*d*e*f + (6*I*c^2 - 8*I*c + 4*I)*f^2 + (12*I*d*e*f + (-12*I*c + 8*I
)*f^2)*(d*x + c))*cos(3*d*x + 3*c) - 2*(3*(d*x + c)^2*f^2 + 4*d*e*f + (3*c^
2 - 4*c + 2)*f^2 + 2*(3*d*e*f - (3*c - 2)*f^2)*(d*x + c))*cos(2*d*x + 2*c)
+ (-3*I*(d*x + c)^2*f^2 - 4*I*d*e*f + (-3*I*c^2 + 4*I*c - 2*I)*f^2 + (-6*I*
d*e*f + (6*I*c - 4*I)*f^2)*(d*x + c))*cos(d*x + c) + (3*(d*x + c)^2*f^2 + 4
*d*e*f + (3*c^2 - 4*c + 2)*f^2 + 2*(3*d*e*f - (3*c - 2)*f^2)*(d*x + c))*sin
(5*d*x + 5*c) + (3*I*(d*x + c)^2*f^2 + 4*I*d*e*f + (3*I*c^2 - 4*I*c + 2*I)*
f^2 + (6*I*d*e*f + (-6*I*c + 4*I)*f^2)*(d*x + c))*sin(4*d*x + 4*c) - 2*(3*(
d*x + c)^2*f^2 + 4*d*e*f + (3*c^2 - 4*c + 2)*f^2 + 2*(3*d*e*f - (3*c - 2)*f
```

```
^2)*(d*x + c))*sin(3*d*x + 3*c) + (-6*I*(d*x + c)^2*f^2 - 8*I*d*e*f + (-6*I
*c^2 + 8*I*c - 4*I)*f^2 + (-12*I*d*e*f + (12*I*c - 8*I)*f^2)*(d*x + c))*sin
(2*d*x + 2*c) + (3*(d*x + c)^2*f^2 + 4*d*e*f + (3*c^2 - 4*c + 2)*f^2 + 2*(3
*d*e*f - (3*c - 2)*f^2)*(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d
*x + c)^2 + 2*cos(d*x + c) + 1) - (3*(d*x + c)^2*f^2 - 4*d*e*f + (3*c^2 + 4
*c + 2)*f^2 + 2*(3*d*e*f - (3*c + 2)*f^2)*(d*x + c) - (3*I*(d*x + c)^2*f^2
- 4*I*d*e*f + (3*I*c^2 + 4*I*c + 2*I)*f^2 + (6*I*d*e*f + (-6*I*c - 4*I)*f^2
)*(d*x + c))*cos(5*d*x + 5*c) + (3*(d*x + c)^2*f^2 - 4*d*e*f + (3*c^2 + 4*c
 + 2)*f^2 + 2*(3*d*e*f - (3*c + 2)*f^2)*(d*x + c))*cos(4*d*x + 4*c) - (-6*I
*(d*x + c)^2*f^2 + 8*I*d*e*f + (-6*I*c^2 - 8*I*c - 4*I)*f^2 + (-12*I*d*e*f
+ (12*I*c + 8*I)*f^2)*(d*x + c))*cos(3*d*x + 3*c) - 2*(3*(d*x + c)^2*f^2 -
4*d*e*f + (3*c^2 + 4*c + 2)*f^2 + 2*(3*d*e*f - (3*c + 2)*f^2)*(d*x + c))*co
s(2*d*x + 2*c) - (3*I*(d*x + c)^2*f^2 - 4*I*d*e*f + (3*I*c^2 + 4*I*c + 2*I)
*f^2 + (6*I*d*e*f + (-6*I*c - 4*I)*f^2)*(d*x + c))*cos(d*x + c) + (3*(d*x +
 c)^2*f^2 - 4*d*e*f + (3*c^2 + 4*c + 2)*f^2 + 2*(3*d*e*f - (3*c + 2)*f^2)*(
d*x + c))*sin(5*d*x + 5*c) - (-3*I*(d*x + c)^2*f^2 + 4*I*d*e*f + (-3*I*c^2
- 4*I*c - 2*I)*f^2 + (-6*I*d*e*f + (6*I*c + 4*I)*f^2)*(d*x + c))*sin(4*d*x
+ 4*c) - 2*(3*(d*x + c)^2*f^2 - 4*d*e*f + (3*c^2 + 4*c + 2)*f^2 + 2*(3*d*e*
f - (3*c + 2)*f^2)*(d*x + c))*sin(3*d*x + 3*c) - (6*I*(d*x + c)^2*f^2 - 8*I
*d*e*f + (6*I*c^2 + 8*I*c + 4*I)*f^2 + (12*I*d*e*f + (-12*I*c - 8*I)*f^2)*(
d*x + c))*sin(2*d*x + 2*c) + (3*(d*x + c)^2*f^2 - 4*d*e*f + (3*c^2 + 4*c +
2)*f^2 + 2*(3*d*e*f - (3*c + 2)*f^2)*(d*x + c))*sin(d*x + c))*log(cos(d*x +
 c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1) + (8*d*e*f + 8*(d*x + c)*f^2 -
 8*c*f^2 + (-8*I*d*e*f - 8*I*(d*x + c)*f^2 + 8*I*c*f^2)*cos(5*d*x + 5*c) +
8*(d*e*f + (d*x + c)*f^2 - c*f^2)*cos(4*d*x + 4*c) + (16*I*d*e*f + 16*I*(d*
x + c)*f^2 - 16*I*c*f^2)*cos(3*d*x + 3*c) - 16*(d*e*f + (d*x + c)*f^2 - c*f
^2)*cos(2*d*x + 2*c) + (-8*I*d*e*f - 8*I*(d*x + c)*f^2 + 8*I*c*f^2)*cos(d*x
 + c) + 8*(d*e*f + (d*x + c)*f^2 - c*f^2)*sin(5*d*x + 5*c) + (8*I*d*e*f + 8
*I*(d*x + c)*f^2 - 8*I*c*f^2)*sin(4*d*x + 4*c) - 16*(d*e*f + (d*x + c)*f^2
- c*f^2)*sin(3*d*x + 3*c) + (-16*I*d*e*f - 16*I*(d*x + c)*f^2 + 16*I*c*f^2)
*sin(2*d*x + 2*c) + 8*(d*e*f + (d*x + c)*f^2 - c*f^2)*sin(d*x + c))*log(cos
(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + (-12*I*f^2*cos(5*d*x +
 5*c) + 12*f^2*cos(4*d*x + 4*c) + 24*I*f^2*cos(3*d*x + 3*c) - 24*f^2*cos(2*
d*x + 2*c) - 12*I*f^2*cos(d*x + c) + 12*f^2*sin(5*d*x + 5*c) + 12*I*f^2*sin
(4*d*x + 4*c) - 24*f^2*sin(3*d*x + 3*c) - 24*I*f^2*sin(2*d*x + 2*c) + 12*f^
2*sin(d*x + c) + 12*f^2)*polylog(3, -e^(I*d*x + I*c)) + (12*I*f^2*cos(5*d*x
 + 5*c) - 12*f^2*cos(4*d*x + 4*c) - 24*I*f^2*cos(3*d*x + 3*c) + 24*f^2*cos(
2*d*x + 2*c) + 12*I*f^2*cos(d*x + c) - 12*f^2*sin(5*d*x + 5*c) - 12*I*f^2*s
in(4*d*x + 4*c) + 24*f^2*sin(3*d*x + 3*c) + 24*I*f^2*sin(2*d*x + 2*c) - 12*
f^2*sin(d*x + c) - 12*f^2)*polylog(3, e^(I*d*x + I*c)) + (-16*I*(d*x + c)^2
*f^2 + (-32*I*d*e*f + 32*I*c*f^2)*(d*x + c))*sin(5*d*x + 5*c) + (4*(d*x + c
)^2*f^2 + 8*I*d*e*f - 4*(3*c^2 + 2*I*c)*f^2 + (8*d*e*f - (8*c - 8*I)*f^2)*(
d*x + c))*sin(4*d*x + 4*c) + (20*I*(d*x + c)^2*f^2 - 8*d*e*f + (-12*I*c^2 +
 8*c)*f^2 - 8*(-5*I*d*e*f + (5*I*c + 1)*f^2)*(d*x + c))*sin(3*d*x + 3*c) -
(12*(d*x + c)^2*f^2 + 8*I*d*e*f - 4*(5*c^2 + 2*I*c)*f^2 + (24*d*e*f - (24*c
```

```
 - 8*I)*f^2)*(d*x + c))*sin(2*d*x + 2*c) + (-12*I*(d*x + c)^2*f^2 + 8*d*e*f
 + (4*I*c^2 - 8*c)*f^2 - 8*(3*I*d*e*f + (-3*I*c - 1)*f^2)*(d*x + c))*sin(d*
x + c))/(-4*I*a*d^2*cos(5*d*x + 5*c) + 4*a*d^2*cos(4*d*x + 4*c) + 8*I*a*d^2
*cos(3*d*x + 3*c) - 8*a*d^2*cos(2*d*x + 2*c) - 4*I*a*d^2*cos(d*x + c) + 4*a
*d^2*sin(5*d*x + 5*c) + 4*I*a*d^2*sin(4*d*x + 4*c) - 8*a*d^2*sin(3*d*x + 3*
c) - 8*I*a*d^2*sin(2*d*x + 2*c) + 4*a*d^2*sin(d*x + c) + 4*a*d^2))/d
```

Fricas [C] time = 3.54559, size = 9528, normalized size = 24.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*d^2*f^2*x^2 + 4*d^2*e^2 - 8*(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*c
os(d*x + c)^3 - 4*d*e*f - 2*(3*d^2*f^2*x^2 + 3*d^2*e^2 - 2*d*e*f + 2*(3*d^2
*e*f - d*f^2)*x)*cos(d*x + c)^2 + 4*(2*d^2*e*f - d*f^2)*x + 6*(d^2*f^2*x^2
+ 2*d^2*e*f*x + d^2*e^2)*cos(d*x + c) - (6*I*d*f^2*x + (-6*I*d*f^2*x - 6*I*
d*e*f + 4*I*f^2)*cos(d*x + c)^3 + 6*I*d*e*f + (-6*I*d*f^2*x - 6*I*d*e*f + 4
*I*f^2)*cos(d*x + c)^2 - 4*I*f^2 + (6*I*d*f^2*x + 6*I*d*e*f - 4*I*f^2)*cos(
d*x + c) + (6*I*d*f^2*x + 6*I*d*e*f + (-6*I*d*f^2*x - 6*I*d*e*f + 4*I*f^2)*
cos(d*x + c)^2 - 4*I*f^2)*sin(d*x + c))*dilog(cos(d*x + c) + I*sin(d*x + c)
) - (-6*I*d*f^2*x + (6*I*d*f^2*x + 6*I*d*e*f - 4*I*f^2)*cos(d*x + c)^3 - 6*
I*d*e*f + (6*I*d*f^2*x + 6*I*d*e*f - 4*I*f^2)*cos(d*x + c)^2 + 4*I*f^2 + (-
6*I*d*f^2*x - 6*I*d*e*f + 4*I*f^2)*cos(d*x + c) + (-6*I*d*f^2*x - 6*I*d*e*f
 + (6*I*d*f^2*x + 6*I*d*e*f - 4*I*f^2)*cos(d*x + c)^2 + 4*I*f^2)*sin(d*x +
c))*dilog(cos(d*x + c) - I*sin(d*x + c)) - (8*I*f^2*cos(d*x + c)^3 + 8*I*f^
2*cos(d*x + c)^2 - 8*I*f^2*cos(d*x + c) - 8*I*f^2 + (8*I*f^2*cos(d*x + c)^2
 - 8*I*f^2)*sin(d*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) - (-8*I*f^2*
cos(d*x + c)^3 - 8*I*f^2*cos(d*x + c)^2 + 8*I*f^2*cos(d*x + c) + 8*I*f^2 +
(-8*I*f^2*cos(d*x + c)^2 + 8*I*f^2)*sin(d*x + c))*dilog(-I*cos(d*x + c) - s
in(d*x + c)) - (6*I*d*f^2*x + (-6*I*d*f^2*x - 6*I*d*e*f - 4*I*f^2)*cos(d*x
+ c)^3 + 6*I*d*e*f + (-6*I*d*f^2*x - 6*I*d*e*f - 4*I*f^2)*cos(d*x + c)^2 +
4*I*f^2 + (6*I*d*f^2*x + 6*I*d*e*f + 4*I*f^2)*cos(d*x + c) + (6*I*d*f^2*x +
 6*I*d*e*f + (-6*I*d*f^2*x - 6*I*d*e*f - 4*I*f^2)*cos(d*x + c)^2 + 4*I*f^2)
*sin(d*x + c))*dilog(-cos(d*x + c) + I*sin(d*x + c)) - (-6*I*d*f^2*x + (6*I
*d*f^2*x + 6*I*d*e*f + 4*I*f^2)*cos(d*x + c)^3 - 6*I*d*e*f + (6*I*d*f^2*x +
 6*I*d*e*f + 4*I*f^2)*cos(d*x + c)^2 - 4*I*f^2 + (-6*I*d*f^2*x - 6*I*d*e*f
- 4*I*f^2)*cos(d*x + c) + (-6*I*d*f^2*x - 6*I*d*e*f + (6*I*d*f^2*x + 6*I*d*
e*f + 4*I*f^2)*cos(d*x + c)^2 - 4*I*f^2)*sin(d*x + c))*dilog(-cos(d*x + c)
- I*sin(d*x + c)) - (3*d^2*f^2*x^2 + 3*d^2*e^2 - (3*d^2*f^2*x^2 + 3*d^2*e^2
```

+ 4*d*e*f + 2*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x)*cos(d*x + c)^3 + 4*d*e*f - (3*d^2*f^2*x^2 + 3*d^2*e^2 + 4*d*e*f + 2*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x)*cos(d*x + c)^2 + 2*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x + (3*d^2*f^2*x^2 + 3*d^2*e^2 + 4*d*e*f + 2*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x)*cos(d*x + c) + (3*d^2*f^2*x^2 + 3*d^2*e^2 + 4*d*e*f - (3*d^2*f^2*x^2 + 3*d^2*e^2 + 4*d*e*f + 2*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x)*cos(d*x + c)^2 + 2*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + 1) + 8*((d*e*f - c*f^2)*cos(d*x + c)^3 - d*e*f + c*f^2 + (d*e*f - c*f^2)*cos(d*x + c)^2 - (d*e*f - c*f^2)*cos(d*x + c) - (d*e*f - c*f^2 - (d*e*f - c*f^2)*cos(d*x + c)^2)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) - (3*d^2*f^2*x^2 + 3*d^2*e^2 - (3*d^2*f^2*x^2 + 3*d^2*e^2 + 4*d*e*f + 2*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x)*cos(d*x + c)^3 + 4*d*e*f - (3*d^2*f^2*x^2 + 3*d^2*e^2 + 4*d*e*f + 2*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x)*cos(d*x + c)^2 + 2*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x + (3*d^2*f^2*x^2 + 3*d^2*e^2 + 4*d*e*f + 2*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x)*cos(d*x + c) + (3*d^2*f^2*x^2 + 3*d^2*e^2 + 4*d*e*f - (3*d^2*f^2*x^2 + 3*d^2*e^2 + 4*d*e*f + 2*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x)*cos(d*x + c)^2 + 2*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x)*sin(d*x + c))*log(cos(d*x + c) - I*sin(d*x + c) + 1) - 8*(d*f^2*x - (d*f^2*x + c*f^2)*cos(d*x + c)^3 + c*f^2 - (d*f^2*x + c*f^2)*cos(d*x + c)^2 + (d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^2*x + c*f^2 - (d*f^2*x + c*f^2)*cos(d*x + c)^2)*sin(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c) + 1) - 8*(d*f^2*x - (d*f^2*x + c*f^2)*cos(d*x + c)^3 + c*f^2 - (d*f^2*x + c*f^2)*cos(d*x + c)^2 + (d*f^2*x + c*f^2)*cos(d*x + c) + (d*f^2*x + c*f^2 - (d*f^2*x + c*f^2)*cos(d*x + c)^2)*sin(d*x + c))*log(-I*cos(d*x + c) + sin(d*x + c) + 1) + (3*d^2*e^2 - 2*(3*c + 2)*d*e*f - (3*d^2*e^2 - 2*(3*c + 2)*d*e*f + (3*c^2 + 4*c + 2)*f^2)*cos(d*x + c)^3 + (3*c^2 + 4*c + 2)*f^2 - (3*d^2*e^2 - 2*(3*c + 2)*d*e*f + (3*c^2 + 4*c + 2)*f^2)*cos(d*x + c)^2 + (3*d^2*e^2 - 2*(3*c + 2)*d*e*f + (3*c^2 + 4*c + 2)*f^2)*cos(d*x + c) + (3*d^2*e^2 - 2*(3*c + 2)*d*e*f + (3*c^2 + 4*c + 2)*f^2 - (3*d^2*e^2 - 2*(3*c + 2)*d*e*f + (3*c^2 + 4*c + 2)*f^2)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) + (3*d^2*e^2 - 2*(3*c + 2)*d*e*f - (3*d^2*e^2 - 2*(3*c + 2)*d*e*f + (3*c^2 + 4*c + 2)*f^2)*cos(d*x + c)^3 + (3*c^2 + 4*c + 2)*f^2 - (3*d^2*e^2 - 2*(3*c + 2)*d*e*f + (3*c^2 + 4*c + 2)*f^2)*cos(d*x + c)^2 + (3*d^2*e^2 - 2*(3*c + 2)*d*e*f + (3*c^2 + 4*c + 2)*f^2)*cos(d*x + c) + (3*d^2*e^2 - 2*(3*c + 2)*d*e*f + (3*c^2 + 4*c + 2)*f^2 - (3*d^2*e^2 - 2*(3*c + 2)*d*e*f + (3*c^2 + 4*c + 2)*f^2)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) + (3*d^2*f^2*x^2 + 6*c*d*e*f - (3*d^2*f^2*x^2 + 6*c*d*e*f - (3*c^2 + 4*c)*f^2 + 2*(3*d^2*e*f - 2*d*f^2)*x)*cos(d*x + c)^3 - (3*c^2 + 4*c)*f^2 - (3*d^2*f^2*x^2 + 6*c*d*e*f - (3*c^2 + 4*c)*f^2 + 2*(3*d^2*e*f - 2*d*f^2)*x)*cos(d*x + c)^2 + 2*(3*d^2*e*f - 2*d*f^2)*x + (3*d^2*f^2*x^2 + 6*c*d*e*f - (3*c^2 + 4*c)*f^2 + 2*(3*d^2*e*f - 2*d*f^2)*x)*cos(d*x + c) + (3*d^2*f^2*x^2 + 6*c*d*e*f - (3*c^2 + 4*c)*f^2 - (3*d^2*f^2*x^2 + 6*c*d*e*f - (3*c^2 + 4*c)*f^2 + 2*(3*d^2*e*f - 2*d*f^2)*x)*cos(d*x + c)^2 + 2*(3*d^2*e*f - 2*d*f^2)*x)*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + 1) + 8*((d*e*f - c*f^2)*cos(d*x + c)^3 - d*e*f + c*f^2 + (d*e*f - c*f^2)*cos(d*x + c)^2 - (d*e*f - c*

```
f^2)*cos(d*x + c) - (d*e*f - c*f^2 - (d*e*f - c*f^2)*cos(d*x + c)^2)*sin(d*
x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) + (3*d^2*f^2*x^2 + 6*c*d*e*
f - (3*d^2*f^2*x^2 + 6*c*d*e*f - (3*c^2 + 4*c)*f^2 + 2*(3*d^2*e*f - 2*d*f^2
)*x)*cos(d*x + c)^3 - (3*c^2 + 4*c)*f^2 - (3*d^2*f^2*x^2 + 6*c*d*e*f - (3*c
^2 + 4*c)*f^2 + 2*(3*d^2*e*f - 2*d*f^2)*x)*cos(d*x + c)^2 + 2*(3*d^2*e*f -
2*d*f^2)*x + (3*d^2*f^2*x^2 + 6*c*d*e*f - (3*c^2 + 4*c)*f^2 + 2*(3*d^2*e*f
- 2*d*f^2)*x)*cos(d*x + c) + (3*d^2*f^2*x^2 + 6*c*d*e*f - (3*c^2 + 4*c)*f^2
 - (3*d^2*f^2*x^2 + 6*c*d*e*f - (3*c^2 + 4*c)*f^2 + 2*(3*d^2*e*f - 2*d*f^2)
*x)*cos(d*x + c)^2 + 2*(3*d^2*e*f - 2*d*f^2)*x)*sin(d*x + c))*log(-cos(d*x
+ c) - I*sin(d*x + c) + 1) - 6*(f^2*cos(d*x + c)^3 + f^2*cos(d*x + c)^2 - f
^2*cos(d*x + c) - f^2 + (f^2*cos(d*x + c)^2 - f^2)*sin(d*x + c))*polylog(3,
 cos(d*x + c) + I*sin(d*x + c)) - 6*(f^2*cos(d*x + c)^3 + f^2*cos(d*x + c)^
2 - f^2*cos(d*x + c) - f^2 + (f^2*cos(d*x + c)^2 - f^2)*sin(d*x + c))*polyl
og(3, cos(d*x + c) - I*sin(d*x + c)) + 6*(f^2*cos(d*x + c)^3 + f^2*cos(d*x
+ c)^2 - f^2*cos(d*x + c) - f^2 + (f^2*cos(d*x + c)^2 - f^2)*sin(d*x + c))*
polylog(3, -cos(d*x + c) + I*sin(d*x + c)) + 6*(f^2*cos(d*x + c)^3 + f^2*co
s(d*x + c)^2 - f^2*cos(d*x + c) - f^2 + (f^2*cos(d*x + c)^2 - f^2)*sin(d*x
+ c))*polylog(3, -cos(d*x + c) - I*sin(d*x + c)) - 2*(2*d^2*f^2*x^2 + 2*d^2
*e^2 + 2*d*e*f - 4*(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*cos(d*x + c)^2 + 2
*(2*d^2*e*f + d*f^2)*x - (d^2*f^2*x^2 + d^2*e^2 - 2*d*e*f + 2*(d^2*e*f - d*
f^2)*x)*cos(d*x + c))*sin(d*x + c))/(a*d^3*cos(d*x + c)^3 + a*d^3*cos(d*x +
 c)^2 - a*d^3*cos(d*x + c) - a*d^3 + (a*d^3*cos(d*x + c)^2 - a*d^3)*sin(d*x
 + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^2 \csc^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2 \csc^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx \csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*csc(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*
csc(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*csc(c + d*x)**3/(
sin(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \csc(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*csc(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

3.211 $\int \frac{(e+fx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=216

$$\frac{3ifPolyLog\left(2,-e^{i(c+dx)}\right)}{2ad^2}-\frac{3ifPolyLog\left(2,e^{i(c+dx)}\right)}{2ad^2}-\frac{f\csc(c+dx)}{2ad^2}-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}-\frac{f\log(\sin(c+dx))}{ad^2}$$

```
[Out] (-3*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((e + f*x)*Cot[c/2 + Pi/4 +
 (d*x)/2])/(a*d) + ((e + f*x)*Cot[c + d*x])/(a*d) - (f*Csc[c + d*x])/(2*a*d
^2) - ((e + f*x)*Cot[c + d*x]*Csc[c + d*x])/(2*a*d) - (2*f*Log[Sin[c/2 + Pi
/4 + (d*x)/2]])/(a*d^2) - (f*Log[Sin[c + d*x]])/(a*d^2) + (((3*I)/2)*f*Poly
Log[2, -E^(I*(c + d*x))])/(a*d^2) - (((3*I)/2)*f*PolyLog[2, E^(I*(c + d*x))
])/(a*d^2)
```

Rubi [A] time = 0.283136, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4535, 4185, 4183, 2279, 2391, 4184, 3475, 3318}

$$\frac{3ifPolyLog\left(2,-e^{i(c+dx)}\right)}{2ad^2}-\frac{3ifPolyLog\left(2,e^{i(c+dx)}\right)}{2ad^2}-\frac{f\csc(c+dx)}{2ad^2}-\frac{2f\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\right)}{ad^2}-\frac{f\log(\sin(c+dx))}{ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-3*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((e + f*x)*Cot[c/2 + Pi/4 +
 (d*x)/2])/(a*d) + ((e + f*x)*Cot[c + d*x])/(a*d) - (f*Csc[c + d*x])/(2*a*d
^2) - ((e + f*x)*Cot[c + d*x]*Csc[c + d*x])/(2*a*d) - (2*f*Log[Sin[c/2 + Pi
/4 + (d*x)/2]])/(a*d^2) - (f*Log[Sin[c + d*x]])/(a*d^2) + (((3*I)/2)*f*Poly
Log[2, -E^(I*(c + d*x))])/(a*d^2) - (((3*I)/2)*f*PolyLog[2, E^(I*(c + d*x))
])/(a*d^2)
```


Rule 4535

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csc[c + d*x]^(n - 1))/(a + b*Si
n[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(e+fx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)\csc^3(c+dx)\,dx}{a} - \int \frac{(e+fx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

$$= -\frac{f\csc(c+dx)}{2ad^2} - \frac{(e+fx)\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\int (e+fx)\csc(c+dx)\,dx}{2a} - \frac{\int (e+fx$$

$$= -\frac{(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)\cot(c+dx)}{ad} - \frac{f\csc(c+dx)}{2ad^2} - \frac{(e+fx)\cot(c+dx)}{2ad}$$

$$= -\frac{3(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)\cot(c+dx)}{ad} - \frac{f\csc(c+dx)}{2ad^2} - \frac{(e+fx)\cot(c+dx}{2ad}$$

$$= -\frac{3(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{(e+fx)\cot(c+dx)}{ad} - \frac{f\csc(}{2a}$$

$$= -\frac{3(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{(e+fx)\cot(c+dx)}{ad} - \frac{f\csc(}{2a}$$

Mathematica [B] time = 3.57546, size = 484, normalized size = 2.24

$$\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(12f\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(i\left(\text{PolyLog}\left(2,-e^{i(c+dx)}\right)-\text{PolyLog}\left(2,e\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-(d*(e + f*x)*(1 + Cot[(c + d*x)/2]
)*Csc[(c + d*x)/2]) - 16*d*(e + f*x)*Sin[(c + d*x)/2] + 8*f*(c + d*x)*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]) + 2*(-f + 2*d*(e + f*x))*Cot[(c + d*x)/2]*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 16*f*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 8*f*Log[Sin[c + d*x]]*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]) + 12*d*e*Log[Tan[(c + d*x)/2]]*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]) - 12*c*f*Log[Tan[(c + d*x)/2]]*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]) + 12*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] - Log[1
 + E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c
 + d*x))]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 2*(f + 2*d*(e + f*x))*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*Tan[(c + d*x)/2] + d*(e + f*x)*Sec[(c
+ d*x)/2]*(1 + Tan[(c + d*x)/2])))/(8*a*d^2*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.227, size = 468, normalized size = 2.2

$$\frac{3\,dfx\mathrm{e}^{4\,i(dx+c)}+3\,de\mathrm{e}^{4\,i(dx+c)}-5\,dfx\mathrm{e}^{2\,i(dx+c)}+3\,idfx\mathrm{e}^{3\,i(dx+c)}-5\,de\mathrm{e}^{2\,i(dx+c)}+f\mathrm{e}^{3\,i(dx+c)}+3\,ide\mathrm{e}^{3\,i(dx+c)}-if\mathrm{e}^{4\,i(dx+c)}+4\,}{\left(\mathrm{e}^{2\,i(dx+c)}-1\right)^2 d^2\left(\mathrm{e}^{i(dx+c)}+i\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] (3*d*f*x*exp(4*I*(d*x+c))+3*d*e*exp(4*I*(d*x+c))-5*d*f*x*exp(2*I*(d*x+c))+3
*I*d*f*x*exp(3*I*(d*x+c))-5*d*e*exp(2*I*(d*x+c))+f*exp(3*I*(d*x+c))+3*I*d*e
*exp(3*I*(d*x+c))-I*f*exp(4*I*(d*x+c))+4*d*f*x-I*d*f*x*exp(I*(d*x+c))+4*d*e
-exp(I*(d*x+c))*f-I*d*e*exp(I*(d*x+c))+I*f*exp(2*I*(d*x+c)))/(exp(2*I*(d*x+
c))-1)^2/d^2/(exp(I*(d*x+c))+I)/a-3/2/d^2/a*f*c*ln(exp(I*(d*x+c))-1)-3/2*I*
f*polylog(2,exp(I*(d*x+c)))/a/d^2+3/2*I*f*polylog(2,-exp(I*(d*x+c)))/a/d^2+
3/2/d/a*e*ln(exp(I*(d*x+c))-1)-3/2/d/a*e*ln(exp(I*(d*x+c))+1)+4/d^2/a*f*ln(
exp(I*(d*x+c)))-1/d^2/a*f*ln(exp(I*(d*x+c))-1)-1/d^2/a*f*ln(exp(I*(d*x+c))+
1)-2/d^2/a*f*ln(exp(I*(d*x+c))+I)+3/2/d/a*ln(1-exp(I*(d*x+c)))*f*x+3/2/d^2/
a*ln(1-exp(I*(d*x+c)))*c*f-3/2/d/a*ln(exp(I*(d*x+c))+1)*f*x
```

Maxima [B] time = 4.08092, size = 2817, normalized size = 13.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (16*d*f*x*cos(5*d*x + 5*c) + 16*I*d*f*x*sin(5*d*x + 5*c) - 16*I*d*e - (8*f*
cos(5*d*x + 5*c) + 8*I*f*cos(4*d*x + 4*c) - 16*f*cos(3*d*x + 3*c) - 16*I*f*
cos(2*d*x + 2*c) + 8*f*cos(d*x + c) + 8*I*f*sin(5*d*x + 5*c) - 8*f*sin(4*d*
x + 4*c) - 16*I*f*sin(3*d*x + 3*c) + 16*f*sin(2*d*x + 2*c) + 8*I*f*sin(d*x
+ c) + 8*I*f)*arctan2(cos(c) + sin(d*x), cos(d*x) + sin(c)) - (6*I*d*f*x +
6*I*d*e + 2*(3*d*f*x + 3*d*e + 2*f)*cos(5*d*x + 5*c) + (6*I*d*f*x + 6*I*d*e
 + 4*I*f)*cos(4*d*x + 4*c) - 4*(3*d*f*x + 3*d*e + 2*f)*cos(3*d*x + 3*c) + (
-12*I*d*f*x - 12*I*d*e - 8*I*f)*cos(2*d*x + 2*c) + 2*(3*d*f*x + 3*d*e + 2*f
)*cos(d*x + c) + (6*I*d*f*x + 6*I*d*e + 4*I*f)*sin(5*d*x + 5*c) - 2*(3*d*f*
x + 3*d*e + 2*f)*sin(4*d*x + 4*c) + (-12*I*d*f*x - 12*I*d*e - 8*I*f)*sin(3*
d*x + 3*c) + 4*(3*d*f*x + 3*d*e + 2*f)*sin(2*d*x + 2*c) + (6*I*d*f*x + 6*I*
d*e + 4*I*f)*sin(d*x + c) + 4*I*f)*arctan2(sin(d*x + c), cos(d*x + c) + 1)
- (-6*I*d*e - 2*(3*d*e - 2*f)*cos(5*d*x + 5*c) + (-6*I*d*e + 4*I*f)*cos(4*d
```

```
*x + 4*c) + 4*(3*d*e - 2*f)*cos(3*d*x + 3*c) + (12*I*d*e - 8*I*f)*cos(2*d*x
 + 2*c) - 2*(3*d*e - 2*f)*cos(d*x + c) + (-6*I*d*e + 4*I*f)*sin(5*d*x + 5*c
) + 2*(3*d*e - 2*f)*sin(4*d*x + 4*c) + (12*I*d*e - 8*I*f)*sin(3*d*x + 3*c)
- 4*(3*d*e - 2*f)*sin(2*d*x + 2*c) + (-6*I*d*e + 4*I*f)*sin(d*x + c) + 4*I*
f)*arctan2(sin(d*x + c), cos(d*x + c) - 1) - (6*d*f*x*cos(5*d*x + 5*c) + 6*
I*d*f*x*cos(4*d*x + 4*c) - 12*d*f*x*cos(3*d*x + 3*c) - 12*I*d*f*x*cos(2*d*x
 + 2*c) + 6*d*f*x*cos(d*x + c) + 6*I*d*f*x*sin(5*d*x + 5*c) - 6*d*f*x*sin(4
*d*x + 4*c) - 12*I*d*f*x*sin(3*d*x + 3*c) + 12*d*f*x*sin(2*d*x + 2*c) + 6*I
*d*f*x*sin(d*x + c) + 6*I*d*f*x)*arctan2(sin(d*x + c), -cos(d*x + c) + 1) -
 (-4*I*d*f*x + 12*I*d*e + 4*f)*cos(4*d*x + 4*c) - (20*d*f*x - 12*d*e + 4*I*
f)*cos(3*d*x + 3*c) - (12*I*d*f*x - 20*I*d*e - 4*f)*cos(2*d*x + 2*c) + (12*
d*f*x - 4*d*e + 4*I*f)*cos(d*x + c) + (6*f*cos(5*d*x + 5*c) + 6*I*f*cos(4*d
*x + 4*c) - 12*f*cos(3*d*x + 3*c) - 12*I*f*cos(2*d*x + 2*c) + 6*f*cos(d*x +
 c) + 6*I*f*sin(5*d*x + 5*c) - 6*f*sin(4*d*x + 4*c) - 12*I*f*sin(3*d*x + 3*
c) + 12*f*sin(2*d*x + 2*c) + 6*I*f*sin(d*x + c) + 6*I*f)*dilog(-e^(I*d*x +
I*c)) - (6*f*cos(5*d*x + 5*c) + 6*I*f*cos(4*d*x + 4*c) - 12*f*cos(3*d*x + 3
*c) - 12*I*f*cos(2*d*x + 2*c) + 6*f*cos(d*x + c) + 6*I*f*sin(5*d*x + 5*c) -
 6*f*sin(4*d*x + 4*c) - 12*I*f*sin(3*d*x + 3*c) + 12*f*sin(2*d*x + 2*c) + 6
*I*f*sin(d*x + c) + 6*I*f)*dilog(e^(I*d*x + I*c)) - (3*d*f*x + 3*d*e + (-3*
I*d*f*x - 3*I*d*e - 2*I*f)*cos(5*d*x + 5*c) + (3*d*f*x + 3*d*e + 2*f)*cos(4
*d*x + 4*c) + (6*I*d*f*x + 6*I*d*e + 4*I*f)*cos(3*d*x + 3*c) - 2*(3*d*f*x +
 3*d*e + 2*f)*cos(2*d*x + 2*c) + (-3*I*d*f*x - 3*I*d*e - 2*I*f)*cos(d*x + c
) + (3*d*f*x + 3*d*e + 2*f)*sin(5*d*x + 5*c) + (3*I*d*f*x + 3*I*d*e + 2*I*f
)*sin(4*d*x + 4*c) - 2*(3*d*f*x + 3*d*e + 2*f)*sin(3*d*x + 3*c) + (-6*I*d*f
*x - 6*I*d*e - 4*I*f)*sin(2*d*x + 2*c) + (3*d*f*x + 3*d*e + 2*f)*sin(d*x +
c) + 2*f)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1) + (3*d*
f*x + 3*d*e - (3*I*d*f*x + 3*I*d*e - 2*I*f)*cos(5*d*x + 5*c) + (3*d*f*x + 3
*d*e - 2*f)*cos(4*d*x + 4*c) - (-6*I*d*f*x - 6*I*d*e + 4*I*f)*cos(3*d*x + 3
*c) - 2*(3*d*f*x + 3*d*e - 2*f)*cos(2*d*x + 2*c) - (3*I*d*f*x + 3*I*d*e - 2
*I*f)*cos(d*x + c) + (3*d*f*x + 3*d*e - 2*f)*sin(5*d*x + 5*c) - (-3*I*d*f*x
 - 3*I*d*e + 2*I*f)*sin(4*d*x + 4*c) - 2*(3*d*f*x + 3*d*e - 2*f)*sin(3*d*x
+ 3*c) - (6*I*d*f*x + 6*I*d*e - 4*I*f)*sin(2*d*x + 2*c) + (3*d*f*x + 3*d*e
- 2*f)*sin(d*x + c) - 2*f)*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x
+ c) + 1) - (-4*I*f*cos(5*d*x + 5*c) + 4*f*cos(4*d*x + 4*c) + 8*I*f*cos(3*d
*x + 3*c) - 8*f*cos(2*d*x + 2*c) - 4*I*f*cos(d*x + c) + 4*f*sin(5*d*x + 5*c
) + 4*I*f*sin(4*d*x + 4*c) - 8*f*sin(3*d*x + 3*c) - 8*I*f*sin(2*d*x + 2*c)
+ 4*f*sin(d*x + c) + 4*f)*log(cos(d*x)^2 + cos(c)^2 + 2*cos(c)*sin(d*x) + s
in(d*x)^2 + 2*cos(d*x)*sin(c) + sin(c)^2) - (4*d*f*x - 12*d*e + 4*I*f)*sin(
4*d*x + 4*c) - (20*I*d*f*x - 12*I*d*e - 4*f)*sin(3*d*x + 3*c) + (12*d*f*x -
 20*d*e + 4*I*f)*sin(2*d*x + 2*c) - (-12*I*d*f*x + 4*I*d*e + 4*f)*sin(d*x +
 c))/(-4*I*a*d^2*cos(5*d*x + 5*c) + 4*a*d^2*cos(4*d*x + 4*c) + 8*I*a*d^2*co
s(3*d*x + 3*c) - 8*a*d^2*cos(2*d*x + 2*c) - 4*I*a*d^2*cos(d*x + c) + 4*a*d^
2*sin(5*d*x + 5*c) + 4*I*a*d^2*sin(4*d*x + 4*c) - 8*a*d^2*sin(3*d*x + 3*c)
- 8*I*a*d^2*sin(2*d*x + 2*c) + 4*a*d^2*sin(d*x + c) + 4*a*d^2)
```

Fricas [B] time = 2.37466, size = 3522, normalized size = 16.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(8*(d*f*x + d*e)*cos(d*x + c)^3 - 4*d*f*x + 2*(3*d*f*x + 3*d*e - f)*cos
(d*x + c)^2 - 4*d*e - 6*(d*f*x + d*e)*cos(d*x + c) + (-3*I*f*cos(d*x + c)^3
 - 3*I*f*cos(d*x + c)^2 + 3*I*f*cos(d*x + c) + (-3*I*f*cos(d*x + c)^2 + 3*I
*f)*sin(d*x + c) + 3*I*f)*dilog(cos(d*x + c) + I*sin(d*x + c)) + (3*I*f*cos
(d*x + c)^3 + 3*I*f*cos(d*x + c)^2 - 3*I*f*cos(d*x + c) + (3*I*f*cos(d*x +
c)^2 - 3*I*f)*sin(d*x + c) - 3*I*f)*dilog(cos(d*x + c) - I*sin(d*x + c)) +
(-3*I*f*cos(d*x + c)^3 - 3*I*f*cos(d*x + c)^2 + 3*I*f*cos(d*x + c) + (-3*I*
f*cos(d*x + c)^2 + 3*I*f)*sin(d*x + c) + 3*I*f)*dilog(-cos(d*x + c) + I*sin
(d*x + c)) + (3*I*f*cos(d*x + c)^3 + 3*I*f*cos(d*x + c)^2 - 3*I*f*cos(d*x +
 c) + (3*I*f*cos(d*x + c)^2 - 3*I*f)*sin(d*x + c) - 3*I*f)*dilog(-cos(d*x +
 c) - I*sin(d*x + c)) - ((3*d*f*x + 3*d*e + 2*f)*cos(d*x + c)^3 - 3*d*f*x +
 (3*d*f*x + 3*d*e + 2*f)*cos(d*x + c)^2 - 3*d*e - (3*d*f*x + 3*d*e + 2*f)*c
os(d*x + c) - (3*d*f*x - (3*d*f*x + 3*d*e + 2*f)*cos(d*x + c)^2 + 3*d*e + 2
*f)*sin(d*x + c) - 2*f)*log(cos(d*x + c) + I*sin(d*x + c) + 1) - ((3*d*f*x
+ 3*d*e + 2*f)*cos(d*x + c)^3 - 3*d*f*x + (3*d*f*x + 3*d*e + 2*f)*cos(d*x +
 c)^2 - 3*d*e - (3*d*f*x + 3*d*e + 2*f)*cos(d*x + c) - (3*d*f*x - (3*d*f*x
+ 3*d*e + 2*f)*cos(d*x + c)^2 + 3*d*e + 2*f)*sin(d*x + c) - 2*f)*log(cos(d*
x + c) - I*sin(d*x + c) + 1) + ((3*d*e - (3*c + 2)*f)*cos(d*x + c)^3 + (3*d
*e - (3*c + 2)*f)*cos(d*x + c)^2 - 3*d*e + (3*c + 2)*f - (3*d*e - (3*c + 2)
*f)*cos(d*x + c) + ((3*d*e - (3*c + 2)*f)*cos(d*x + c)^2 - 3*d*e + (3*c + 2
)*f)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) + ((3*
d*e - (3*c + 2)*f)*cos(d*x + c)^3 + (3*d*e - (3*c + 2)*f)*cos(d*x + c)^2 -
3*d*e + (3*c + 2)*f - (3*d*e - (3*c + 2)*f)*cos(d*x + c) + ((3*d*e - (3*c +
 2)*f)*cos(d*x + c)^2 - 3*d*e + (3*c + 2)*f)*sin(d*x + c))*log(-1/2*cos(d*x
 + c) - 1/2*I*sin(d*x + c) + 1/2) + 3*((d*f*x + c*f)*cos(d*x + c)^3 - d*f*x
 + (d*f*x + c*f)*cos(d*x + c)^2 - c*f - (d*f*x + c*f)*cos(d*x + c) - (d*f*x
 - (d*f*x + c*f)*cos(d*x + c)^2 + c*f)*sin(d*x + c))*log(-cos(d*x + c) + I*
sin(d*x + c) + 1) + 3*((d*f*x + c*f)*cos(d*x + c)^3 - d*f*x + (d*f*x + c*f)
*cos(d*x + c)^2 - c*f - (d*f*x + c*f)*cos(d*x + c) - (d*f*x - (d*f*x + c*f)
*cos(d*x + c)^2 + c*f)*sin(d*x + c))*log(-cos(d*x + c) - I*sin(d*x + c) + 1
) - 4*(f*cos(d*x + c)^3 + f*cos(d*x + c)^2 - f*cos(d*x + c) + (f*cos(d*x +
c)^2 - f)*sin(d*x + c) - f)*log(sin(d*x + c) + 1) + 2*(2*d*f*x - 4*(d*f*x +
 d*e)*cos(d*x + c)^2 + 2*d*e - (d*f*x + d*e - f)*cos(d*x + c) + f)*sin(d*x
+ c) + 2*f)/(a*d^2*cos(d*x + c)^3 + a*d^2*cos(d*x + c)^2 - a*d^2*cos(d*x +
```

```
c) - a*d^2 + (a*d^2*cos(d*x + c)^2 - a*d^2)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e\csc^3(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{fx\csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e*csc(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f*x*csc(c + d
*x)**3/(sin(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\csc(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*csc(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

3.212 $\int \frac{\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{2\cot(c+dx)}{ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{3\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{d(a\sin(c+dx)+a)}$$

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(2*a*d) + (2*Cot[c + d*x])/(a*d) - (3*Cot[c + d*
x]*Csc[c + d*x])/(2*a*d) + (Cot[c + d*x]*Csc[c + d*x])/(d*(a + a*Sin[c + d*
x]))
```

Rubi [A] time = 0.0895943, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2768, 2748, 3768, 3770, 3767, 8}

$$\frac{2\cot(c+dx)}{ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{3\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(2*a*d) + (2*Cot[c + d*x])/(a*d) - (3*Cot[c + d*
x]*Csc[c + d*x])/(2*a*d) + (Cot[c + d*x]*Csc[c + d*x])/(d*(a + a*Sin[c + d*
x]))
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\cot(c+dx)\csc(c+dx)}{d(a+a\sin(c+dx))} - \frac{\int \csc^3(c+dx)(-3a+2a\sin(c+dx))\,dx}{a^2}\\ &= \frac{\cot(c+dx)\csc(c+dx)}{d(a+a\sin(c+dx))} - \frac{2\int \csc^2(c+dx)\,dx}{a} + \frac{3\int \csc^3(c+dx)\,dx}{a}\\ &= -\frac{3\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\cot(c+dx)\csc(c+dx)}{d(a+a\sin(c+dx))} + \frac{3\int \csc(c+dx)\,dx}{2a} + \frac{2\,\mathrm{Subst}(\int 1\,dx,\ldots}{a\ldots}\\ &= -\frac{3\tanh^{-1}(\cos(c+dx))}{2ad} + \frac{2\cot(c+dx)}{ad} - \frac{3\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\cot(c+dx)\csc(c+d\ldots}{d(a+a\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.511085, size = 85, normalized size = 1.04

$$-\frac{4\tan(c+dx)-4\csc(2(c+dx))-3\sec(c+dx)+\csc^2(c+dx)\sec(c+dx)+3\sqrt{\cos^2(c+dx)}\sec(c+dx)\tanh^{-1}\left(\sqrt{\mathrm{co}\ldots}\right.}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(-4*Csc[2*(c + d*x)] - 3*Sec[c + d*x] + 3*ArcTanh[Sqrt[Cos[c + d*x]^2]]*Sq
rt[Cos[c + d*x]^2]*Sec[c + d*x] + Csc[c + d*x]^2*Sec[c + d*x] + 4*Tan[c + d
*x])/(2*a*d)
```

Maple [A] time = 0.05, size = 115, normalized size = 1.4

$$\frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}-\frac{1}{2\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{1}{da\,(\tan(1/2\,dx+c/2)+1)}-\frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2}+\frac{1}{2\,da}\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/8/a/d*tan(1/2*d*x+1/2*c)^2-1/2/a/d*tan(1/2*d*x+1/2*c)+2/a/d/(tan(1/2*d*x+
1/2*c)+1)-1/8/a/d/tan(1/2*d*x+1/2*c)^2+1/2/a/d/tan(1/2*d*x+1/2*c)+3/2/a/d*l
n(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 0.985118, size = 212, normalized size = 2.59

$$-\frac{\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}}{a}-\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-1}{\frac{a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 1)/(a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3) - 12*log(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [B] time = 1.81147, size = 636, normalized size = 7.76

$$\frac{8\cos(dx+c)^{3}+6\cos(dx+c)^{2}-3\left(\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\text{lo}}{4\left(ad\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(8*cos(d*x + c)^3 + 6*cos(d*x + c)^2 - 3*(cos(d*x + c)^3 + cos(d*x + c)
^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)*log(1/2*cos(d*x
+ c) + 1/2) + 3*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin
(d*x + c) - cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) - 2*(4*cos(d*x +
 c)^2 + cos(d*x + c) - 2)*sin(d*x + c) - 6*cos(d*x + c) - 4)/(a*d*cos(d*x +
 c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d + (a*d*cos(d*x + c)^2 -
 a*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.13823, size = 151, normalized size = 1.84

$$\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}+\frac{16}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a*tan(1/2*d*x + 1/2*c)^2 - 4*a*
tan(1/2*d*x + 1/2*c))/a^2 + 16/(a*(tan(1/2*d*x + 1/2*c) + 1)) - (18*tan(1/2
*d*x + 1/2*c)^2 - 4*tan(1/2*d*x + 1/2*c) + 1)/(a*tan(1/2*d*x + 1/2*c)^2))/d
```

3.213 $\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\csc^3(c+dx)}{(e+fx)(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Csc[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0669059, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Csc[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 73.5326, size = 0, normalized size = 0.

$$\int \frac{\csc^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Csc[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 7.305, size = 0, normalized size = 0.

$$\int \frac{(\csc (dx + c))^3}{(fx + e)(a + a\sin (dx + c))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc (dx + c)^3}{afx + ae + (afx + ae)\sin (dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)^3/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^3 (c+dx)}{e\sin (c+dx)+e+fx\sin (c+dx)+fx}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/
a
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.214 $\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\csc^3(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Csc[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0664905, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Csc[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 150.7, size = 0, normalized size = 0.

$$\int \frac{\csc^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Csc[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 9.902, size = 0, normalized size = 0.

$$\int \frac{(\csc (dx+c))^3}{(fx+e)^2 (a+a\sin (dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc (dx+c)^3}{af^2x^2+2\,aefx+ae^2+\left(af^2x^2+2\,aefx+ae^2\right)\sin (dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)^3/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e
*f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^3(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) +
 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.215 $\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)} dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sin^2(c+dx)(e+fx)^m}{a\sin(c+dx)+a},x\right)$$

[Out] Unintegrable[((e + f*x)^m*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]), x]

Rubi [A] time = 0.0623214, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Int[((e + f*x)^m*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]

[Out] Defer[Int][((e + f*x)^m*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]), x]

Rubi steps

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)} dx = \int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)} dx$$

Mathematica [A] time = 7.32844, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+a\sin(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[((e + f*x)^m*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]

[Out] Integrate[((e + f*x)^m*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]), x]

Maple [A] time = 0.325, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \left(\sin \left(dx+c\right)\right)^{2}}{a+a\sin \left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \sin \left(dx+c\right)^{2}}{a\sin \left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*sin(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(\cos \left(dx+c\right)^{2}-1\right)\left(fx+e\right)^{m}}{a\sin \left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(f*x + e)^m/(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \sin(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*sin(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

3.216 $\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sin(c+dx)(e+fx)^m}{a\sin(c+dx)+a},x\right)$$

```
[Out] Unintegrable[((e + f*x)^m*Sin[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0391888, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((e + f*x)^m*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Defer[Int][((e + f*x)^m*Sin[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx$$

Mathematica [A] time = 1.73902, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Integrate[((e + f*x)^m*Sin[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Maple [A] time = 0.134, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m}\sin\left(dx+c\right)}{a+a\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m}\sin\left(dx+c\right)}{a\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*sin(d*x + c)/(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^{m}\sin\left(dx+c\right)}{a\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*sin(d*x + c)/(a*sin(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(e+fx)^m \sin(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m*sin(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \sin(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*sin(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.217 $\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(e+fx)^m}{a\sin(c+dx)+a},x\right)$$

```
[Out] Unintegrable[(e + f*x)^m/(a + a*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0508557, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)^m/(a + a*Sin[c + d*x]),x]
```

```
[Out] Defer[Int][(e + f*x)^m/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$$

Mathematica [A] time = 0.762998, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^m/(a + a*Sin[c + d*x]),x]
```

```
[Out] Integrate[(e + f*x)^m/(a + a*Sin[c + d*x]), x]
```

Maple [A] time = 0.105, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m}}{a+a\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m}}{a\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^{m}}{a\sin\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(e+fx)^m}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

3.218 $\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\csc(c+dx)(e+fx)^m}{a\sin(c+dx)+a},x\right)$$

```
[Out] Unintegrable[((e + f*x)^m*Csc[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0413263, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((e + f*x)^m*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Defer[Int][((e + f*x)^m*Csc[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)}\,dx$$

Mathematica [A] time = 5.39632, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Integrate[((e + f*x)^m*Csc[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Maple [A] time = 0.083, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m}\csc\left(dx+c\right)}{a+a\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m}\csc\left(dx+c\right)}{a\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^{m}\csc\left(dx+c\right)}{a\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(e+fx)^m \csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m*csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \csc(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*csc(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.219 $\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\csc^2(c+dx)(e+fx)^m}{a\sin(c+dx)+a},x\right)$$

```
[Out] Unintegrable[((e + f*x)^m*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0646949, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((e + f*x)^m*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]

[Out] Defer[Int][((e + f*x)^m*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

Mathematica [A] time = 9.89653, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]

[Out] Integrate[((e + f*x)^m*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]), x]
```

Maple [A] time = 0.106, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \left(\csc\left(dx+c\right)\right)^{2}}{a+a\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \csc\left(dx+c\right)^{2}}{a\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^{m} \csc\left(dx+c\right)^{2}}{a\sin\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \csc(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*csc(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

3.220 $\int \frac{(e+fx)^3 \sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=544

$$\frac{6iaf^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\sqrt{a^2-b^2}}-\frac{6iaf^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3\sqrt{a^2-b^2}}+\frac{3af(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}-\ldots$$

```
[Out] (e + f*x)^4/(4*b*f) + (I*a*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) - (I*a*(e + f*x)^3*Log[1 - (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) + (3*a*f*(e + f
*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2
- b^2]*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) + ((6*I)*a*f^2*(e + f*x)*PolyLog[3,
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3) - ((6
*I)*a*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]
)/(b*Sqrt[a^2 - b^2]*d^3) - (6*a*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a -
Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^4) + (6*a*f^3*PolyLog[4, (I*b*E^(I*
(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^4)
```

Rubi [A] time = 0.967603, antiderivative size = 544, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {4515, 32, 3323, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{6iaf^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\sqrt{a^2-b^2}}-\frac{6iaf^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3\sqrt{a^2-b^2}}+\frac{3af(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (e + f*x)^4/(4*b*f) + (I*a*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) - (I*a*(e + f*x)^3*Log[1 - (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) + (3*a*f*(e + f
*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2
- b^2]*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) + ((6*I)*a*f^2*(e + f*x)*PolyLog[3,
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3) - ((6
*I)*a*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]
)/(b*Sqrt[a^2 - b^2]*d^3) - (6*a*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a -
Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^4) + (6*a*f^3*PolyLog[4, (I*b*E^(I*
```

```
(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^4)
```

Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\,dx}{b} - \frac{a\int \frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx}{b}$$
$$= \frac{(e+fx)^4}{4bf} - \frac{(2a)\int \frac{e^{i(c+dx)}(e+fx)^3}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b}$$
$$= \frac{(e+fx)^4}{4bf} + \frac{(2ia)\int \frac{e^{i(c+dx)}(e+fx)^3}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2ia)\int \frac{e^{i(c+dx)}(e+fx)^3}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}}$$
$$= \frac{(e+fx)^4}{4bf} + \frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{(3iaf)\int(e+ \cdots}{\cdots}$$
$$= \frac{(e+fx)^4}{4bf} + \frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} + \frac{3af(e+fx)^2 \cdots}{b\sqrt{a^2 \cdots}}$$
$$= \frac{(e+fx)^4}{4bf} + \frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} + \frac{3af(e+fx)^2 \cdots}{b\sqrt{a^2 \cdots}}$$
$$= \frac{(e+fx)^4}{4bf} + \frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} + \frac{3af(e+fx)^2 \cdots}{b\sqrt{a^2 \cdots}}$$
$$= \frac{(e+fx)^4}{4bf} + \frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} + \frac{3af(e+fx)^2 \cdots}{b\sqrt{a^2 \cdots}}$$

Mathematica [A] time = 3.29527, size = 956, normalized size = 1.76

$$\frac{x\left(4e^3+6fxe^2+4f^2x^2e+f^3x^3\right)}{4b} - \frac{a\left(2\sqrt{b^2-a^2}e^3\tan^{-1}\left(\frac{ia+be^{i(c+dx)}}{\sqrt{a^2-b^2}}\right)d^3+\sqrt{a^2-b^2}f^3x^3\log\left(1-\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}-ia}\right)d^3+3\sqrt{a^2-b \cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(4*b) - (a*(2*Sqrt[-a^2 + b
^2]*d^3*e^3*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 3*Sqrt[a^2
- b^2]*d^3*e^2*f*x*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])]
 + 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sq
rt[-a^2 + b^2])] + Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 - (b*E^(I*(c + d*x)))/
```

```
((-I)*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 + (b*E^(
I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*L
og[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - Sqrt[a^2 - b^2]*d^3*
f^3*x^3*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - (3*I)*Sqrt[
a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[
-a^2 + b^2])] + (3*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(
I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] + 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLo
g[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 6*Sqrt[a^2 - b^2]*d
*f^3*x*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 6*Sqrt
[a^2 - b^2]*d*e*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2
]))] - 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sq
rt[-a^2 + b^2]))] + (6*I)*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*E^(I*(c + d*x))
)/((-I)*a + Sqrt[-a^2 + b^2])] - (6*I)*Sqrt[a^2 - b^2]*f^3*PolyLog[4, -((b*
E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))]))/(b*Sqrt[-(a^2 - b^2)^2]*d^4)
```

Maple [F] time = 0.843, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3}\sin\left(dx+c\right)}{a+b\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.31947, size = 5493, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 - b^2)*d^4*f^3*x^4 + 4*(a^2 - b^2)*d^4*e*f^2*x^3 + 6*(a^2 - b^2)*
d^4*e^2*f*x^2 + 4*(a^2 - b^2)*d^4*e^3*x + 12*I*a*b*f^3*sqrt(-(a^2 - b^2)/b^
2)*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c
) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*I*a*b*f^3*sqrt(-(a^2
- b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*co
s(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*I*a*b*f^3*sq
rt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*co
s(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*I*a*b*f^3*sq
rt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*co
s(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 2*(3*I*a*b*d^2*
f^3*x^2 + 6*I*a*b*d^2*e*f^2*x + 3*I*a*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*d
ilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(-3*I*a*b*d^2*f^3*x^
2 - 6*I*a*b*d^2*e*f^2*x - 3*I*a*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-
1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(-3*I*a*b*d^2*f^3*x^2 - 6*
I*a*b*d^2*e*f^2*x - 3*I*a*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-
2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(3*I*a*b*d^2*f^3*x^2 + 6*I*a*b*
d^2*e*f^2*x + 3*I*a*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*
cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(a*b*d^3*e^3 - 3*a*b*c*d^2*e^2*f + 3*
a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c)
+ 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(a*b*d^3*e^3
 - 3*a*b*c*d^2*e^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*sqrt(-(a^2 - b^2)/b
^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
- 2*I*a) + 2*(a*b*d^3*e^3 - 3*a*b*c*d^2*e^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3
*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2
*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(a*b*d^3*e^3 - 3*a*b*c*d^2*e^2*f + 3
*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c
) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(a*b*d^3*f
^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*
b*c^2*d*e*f^2 + a*b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x
+ c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) + 2*b)/b) + 2*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^
3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)*sqrt(-(a^2
```

```
 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x
+ c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(a*b*d^3*f^3*
x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c
^2*d*e*f^2 + a*b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x +
c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2) + 2*b)/b) + 2*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*
e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)*sqrt(-(a^2 -
 b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x +
 c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 12*(a*b*d*f^3*x
+ a*b*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) -
2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2))/b) - 12*(a*b*d*f^3*x + a*b*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3,
 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*(a*b*d*f^3*x + a*b*d*e*f^2)*sqrt(-
(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*
x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(a*b*d*f^3*x + a
*b*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*
x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b))/((
a^2*b - b^3)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \sin(dx+c)}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

3.221 $\int \frac{(e+fx)^2 \sin(c+dx)}{a+b\sin(c+dx)} dx$

Optimal. Leaf size=408

$$\frac{2af(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}-\frac{2af(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{2iaf^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\sqrt{a^2-b^2}}-\frac{2iaf^2\text{PolyLo}}{bd^3\sqrt{a}}$$

```
[Out] (e + f*x)^3/(3*b*f) + (I*a*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) - (I*a*(e + f*x)^2*Log[1 - (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) + (2*a*f*(e + f
*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 -
b^2]*d^2) - (2*a*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) + ((2*I)*a*f^2*PolyLog[3, (I*b*E^(I*(c +
 d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3) - ((2*I)*a*f^2*Poly
Log[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3
)
```

Rubi [A] time = 0.858607, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4515, 32, 3323, 2264, 2190, 2531, 2282, 6589}

$$\frac{2af(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}-\frac{2af(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{2iaf^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\sqrt{a^2-b^2}}-\frac{2iaf^2\text{PolyLo}}{bd^3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```


```
[Out] (e + f*x)^3/(3*b*f) + (I*a*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) - (I*a*(e + f*x)^2*Log[1 - (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) + (2*a*f*(e + f
*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 -
b^2]*d^2) - (2*a*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) + ((2*I)*a*f^2*PolyLog[3, (I*b*E^(I*(c +
 d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3) - ((2*I)*a*f^2*Poly
Log[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3
)
```


Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\,dx}{b} - \frac{a\int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{(2a)\int \frac{e^{i(c+dx)}(e+fx)^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b}$$

$$= \frac{(e+fx)^3}{3bf} + \frac{(2ia)\int \frac{e^{i(c+dx)}(e+fx)^2}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2ia)\int \frac{e^{i(c+dx)}(e+fx)^2}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}}$$

$$= \frac{(e+fx)^3}{3bf} + \frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{(2iaf)\int(e+}{}$$

$$= \frac{(e+fx)^3}{3bf} + \frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} + \frac{2af(e+fx)\mathrm{I}}{b\sqrt{a^2}}$$

$$= \frac{(e+fx)^3}{3bf} + \frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} + \frac{2af(e+fx)\mathrm{I}}{b\sqrt{a^2}}$$

$$= \frac{(e+fx)^3}{3bf} + \frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} + \frac{2af(e+fx)\mathrm{I}}{b\sqrt{a^2}}$$

Mathematica [A] time = 2.05572, size = 445, normalized size = 1.09

$$\frac{x\left(3e^2+3efx+f^2x^2\right)}{3b} - \frac{ia\left(-i\left(2f^2\sqrt{a^2-b^2}\mathrm{PolyLog}\left(3,\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}-ia}\right)-2f^2\sqrt{a^2-b^2}\mathrm{PolyLog}\left(3,-\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}+ia}\right)+d^2\left(2e^2\sqrt{b}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) - (I*a*(-2*Sqrt[a^2 - b^2]*d*f*(e + f
*x)*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 2*Sqrt[a^
2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 +
b^2]))] - I*(d^2*(2*Sqrt[-a^2 + b^2]*e^2*ArcTan[(I*a + b*E^(I*(c + d*x)))/S
qrt[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(Log[1 - (b*E^(I*(c + d*x
)))/((-I)*a + Sqrt[-a^2 + b^2])] - Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[
-a^2 + b^2])])) + 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*E^(I*(c + d*x)))/((-I
)*a + Sqrt[-a^2 + b^2])] - 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, -((b*E^(I*(c +
d*x)))/(I*a + Sqrt[-a^2 + b^2]))])))/(b*Sqrt[-(a^2 - b^2)^2]*d^3)
```

Maple [F] time = 0.664, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sin(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.05961, size = 3943, normalized size = 9.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(a^2 - b^2)*d^3*f^2*x^3 + 6*(a^2 - b^2)*d^3*e*f*x^2 + 6*(a^2 - b^2)*
d^3*e^2*x + 6*a*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x
+ c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2))/b) - 6*a*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*co
s(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2))/b) - 6*a*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*c
os(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2))/b) + 6*a*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(
d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2))/b) - (6*I*a*b*d*f^2*x + 6*I*a*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*
dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - (-6*I*a*b*d*f^2*x - 6
*I*a*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*s
in(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
+ 2*b)/b + 1) - (-6*I*a*b*d*f^2*x - 6*I*a*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*d
ilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - (6*I*a*b*d*f^2*x + 6*
I*a*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*s
in(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
+ 2*b)/b + 1) - 3*(a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*sqrt(-(a^2 -
b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)
/b^2) + 2*I*a) - 3*(a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*sqrt(-(a^2 -
 b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) - 2*I*a) + 3*(a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*sqrt(-(a^2
- b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b
^2)/b^2) + 2*I*a) + 3*(a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*sqrt(-(a^
2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 -
 b^2)/b^2) - 2*I*a) - 3*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f
- a*b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin
(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) +
2*b)/b) + 3*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^
2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) -
2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 3*
(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*sqrt(-(a^
2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*
x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 3*(a*b*d^2*f^
2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^
2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*
b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b))/((a^2*b - b^3)*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sin(dx+c)}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

3.222 $\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=267

$$\frac{af\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}-\frac{af\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd\sqrt{a^2-b^2}}-\frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd\sqrt{a^2-b^2}}+\frac{e}{}$$

```
[Out] (e*x)/b + (f*x^2)/(2*b) + (I*a*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a -
 Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) - (I*a*(e + f*x)*Log[1 - (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) + (a*f*PolyLog[
2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) -
(a*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 -
 b^2]*d^2)
```

Rubi [A] time = 0.584555, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4515, 3323, 2264, 2190, 2279, 2391}

$$\frac{af\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}-\frac{af\,\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd\sqrt{a^2-b^2}}-\frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd\sqrt{a^2-b^2}}+\frac{e}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```


```
[Out] (e*x)/b + (f*x^2)/(2*b) + (I*a*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a -
 Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) - (I*a*(e + f*x)*Log[1 - (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) + (a*f*PolyLog[
2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) -
(a*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 -
 b^2]*d^2)
```


Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\,dx}{b} - \frac{a\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{b}$$
$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{(2a)\int \frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b}$$
$$= \frac{ex}{b} + \frac{fx^2}{2b} + \frac{(2ia)\int \frac{e^{i(c+dx)}(e+fx)}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2ia)\int \frac{e^{i(c+dx)}(e+fx)}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}}$$
$$= \frac{ex}{b} + \frac{fx^2}{2b} + \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{(iaf)\int \log\left(1-\right.}{b\sqrt{a^2}}$$
$$= \frac{ex}{b} + \frac{fx^2}{2b} + \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{(af)\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{ex}{b} + \frac{fx^2}{2b} + \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} + \frac{af\,\text{Li}_2\left(\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right.}{b\sqrt{a^2-b^2}d^2}$$

Mathematica [A] time = 1.58663, size = 299, normalized size = 1.12

$$\frac{x(2e+fx)}{2b} - \frac{ia\left(-f\sqrt{a^2-b^2}\text{PolyLog}\left(2,\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}-ia}\right)+f\sqrt{a^2-b^2}\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}+ia}\right)-id\left(2e\sqrt{b^2-a^2}\tan^{-1}\left(\frac{ia+be^{i(c}}{\sqrt{a^2-}}\right.\right.\right.}{bd^2\sqrt{-\left(a^2-b^2\right)^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (x*(2*e + f*x))/(2*b) - (I*a*((-I)*d*(2*Sqrt[-a^2 + b^2]*e*ArcTan[(I*a + b*
E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*x*(Log[1 - (b*E^(I*(c
 + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - Log[1 + (b*E^(I*(c + d*x)))/(I*a +
 Sqrt[-a^2 + b^2])])) - Sqrt[a^2 - b^2]*f*PolyLog[2, (b*E^(I*(c + d*x)))/((
-I)*a + Sqrt[-a^2 + b^2])] + Sqrt[a^2 - b^2]*f*PolyLog[2, -((b*E^(I*(c + d*
x)))/(I*a + Sqrt[-a^2 + b^2]))]))/(b*Sqrt[-(a^2 - b^2)^2]*d^2)
```

Maple [B] time = 0.141, size = 548, normalized size = 2.1

$$\frac{fx^2}{2b} + \frac{ex}{b} - \frac{2iae}{bd}\arctan\left(\frac{2ibe^{i(dx+c)}-2a}{2}\frac{1}{\sqrt{-a^2+b^2}}\right)\frac{1}{\sqrt{-a^2+b^2}} - \frac{afx}{bd}\ln\left(\left(ia+be^{i(dx+c)}-\sqrt{-a^2+b^2}\right)\left(ia-\sqrt{-a^2+b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2*f*x^2/b+e*x/b-2*I/b*a/d*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x
+c))-2*a)/(-a^2+b^2)^(1/2))-1/b*a/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x
+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-1/b*a/d^2*f/(-a^2+b^2)^(1/
2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+1/b
*a/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a
^2+b^2)^(1/2)))*x+1/b*a/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a
^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+I/b*a/d^2*f/(-a^2+b^2)^(1/2)*dilog
((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-I/b*a/d^2*
f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2
+b^2)^(1/2)))+2*I/b*a/d^2*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x
+c))-2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.13899, size = 2583, normalized size = 9.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^2 - b^2)*d^2*f*x^2 + 4*(a^2 - b^2)*d^2*e*x - 2*I*a*b*f*sqrt(-(a^2
 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d
*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*a*b*
f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c)
- 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1
```

```
) + 2*I*a*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*
sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
 + 2*b)/b + 1) - 2*I*a*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*
x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2) + 2*b)/b + 1) - 2*(a*b*d*e - a*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*
log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*
I*a) - 2*(a*b*d*e - a*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) -
2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(a*b*d*e - a*b
*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2
*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(a*b*d*e - a*b*c*f)*sqrt(-(a^2 - b^2
)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b
^2) - 2*I*a) - 2*(a*b*d*f*x + a*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*
a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(a*b*d*f*x + a*b*c*f)*sqrt(-(a^2 - b^2)
/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) -
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(a*b*d*f*x + a*b*c*f
)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) +
2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*
(a*b*d*f*x + a*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) +
 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2) + 2*b)/b))/((a^2*b - b^3)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\sin(dx+c)}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

3.223 $\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=57

$$\frac{x}{b}-\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\sqrt{a^2-b^2}}$$

```
[Out] x/b - (2*a*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*Sqrt[a^2 -
b^2]*d)
```

Rubi [A] time = 0.0676604, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2735, 2660, 618, 204}

$$\frac{x}{b}-\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] x/b - (2*a*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*Sqrt[a^2 -
b^2]*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+b\sin(c+dx)}\,dx}{b}\\
&= \frac{x}{b}-\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\\
&= \frac{x}{b}+\frac{(4a)\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\\
&= \frac{x}{b}-\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d}
\end{aligned}
$$

Mathematica [A] time = 0.109685, size = 59, normalized size = 1.04

$$
\frac{-\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}+\frac{c}{d}+x}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] (c/d + x - (2*a*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2
 - b^2]*d))/b
```

Maple [A] time = 0., size = 70, normalized size = 1.2

$$2\,{\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{bd}}-2\,{\frac{a}{bd\sqrt{a^2-b^2}}\arctan\left(1/2\,{\frac{2\,a\tan\left(1/2\,dx+c/2\right)+2\,b}{\sqrt{a^2-b^2}}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 2/d/b*arctan(tan(1/2*d*x+1/2*c))-2/d/b*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89226, size = 510, normalized size = 8.95

$$\left[\frac{2\left(a^2-b^2\right)dx-\sqrt{-a^2+b^2}a\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)}{2\left(a^2b-b^3\right)d},\ \frac{\left(a^2-b^2\right)dx}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*a*log(-((2*a^2 - b^2)*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*
cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
a^2 - b^2)))/((a^2*b - b^3)*d), ((a^2 - b^2)*d*x + sqrt(a^2 - b^2)*a*arctan
(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))))/((a^2*b - b^3)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10345, size = 104, normalized size = 1.82

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\sqrt{a^2-b^2}b}-\frac{dx+c}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*a/(sqrt(a^2 - b^2)*b) - (d*x + c)/b)/d
```

3.224 $\int \frac{(e+fx)^3 \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=643

$$-\frac{6ia^2f^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3\sqrt{a^2-b^2}}+\frac{6ia^2f^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^3\sqrt{a^2-b^2}}-\frac{3a^2f(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2\sqrt{a^2-b^2}}+$$

```
[Out] -(a*(e + f*x)^4)/(4*b^2*f) + (6*f^2*(e + f*x)*Cos[c + d*x])/(b*d^3) - ((e +
 f*x)^3*Cos[c + d*x])/(b*d) - (I*a^2*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d) + (I*a^2*(e + f*x)^3*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d)
 - (3*a^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(b^2*Sqrt[a^2 - b^2]*d^2) + (3*a^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I
*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^2) - ((6*I)*a^2
*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^
2*Sqrt[a^2 - b^2]*d^3) + ((6*I)*a^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^3) + (6*a^2*f^3*Poly
Log[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d
^4) - (6*a^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(
b^2*Sqrt[a^2 - b^2]*d^4) - (6*f^3*Sin[c + d*x])/(b*d^4) + (3*f*(e + f*x)^2*
Sin[c + d*x])/(b*d^2)
```

Rubi [A] time = 1.17592, antiderivative size = 643, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {4515, 3296, 2637, 32, 3323, 2264, 2190, 2531, 6609, 2282, 6589}

$$-\frac{6ia^2f^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3\sqrt{a^2-b^2}}+\frac{6ia^2f^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^3\sqrt{a^2-b^2}}-\frac{3a^2f(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2\sqrt{a^2-b^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] -(a*(e + f*x)^4)/(4*b^2*f) + (6*f^2*(e + f*x)*Cos[c + d*x])/(b*d^3) - ((e +
 f*x)^3*Cos[c + d*x])/(b*d) - (I*a^2*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d) + (I*a^2*(e + f*x)^3*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d)
 - (3*a^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(b^2*Sqrt[a^2 - b^2]*d^2) + (3*a^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I
```

```
*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^2) - ((6*I)*a^2
*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^
2*Sqrt[a^2 - b^2]*d^3) + ((6*I)*a^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^3) + (6*a^2*f^3*Poly
Log[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d
^4) - (6*a^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(
b^2*Sqrt[a^2 - b^2]*d^4) - (6*f^3*Sin[c + d*x])/(b*d^4) + (3*f*(e + f*x)^2*
Sin[c + d*x])/(b*d^2)
```

Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
```

```
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\sin(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{a\int (e+fx)^3\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx}{b^2} + \frac{(3f)\int (e+fx)^2\cos(c$$

$$= -\frac{a(e+fx)^4}{4b^2f} - \frac{(e+fx)^3\cos(c+dx)}{bd} + \frac{3f(e+fx)^2\sin(c+dx)}{bd^2} + \frac{\left(2a^2\right)\int \frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i}}}{b^2}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} - \frac{(e+fx)^3\cos(c+dx)}{bd} + \frac{3f(e+fx)^2\sin(c+d}{bd^2}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} - \frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{ia^2(e+fx)^3\log\left(1-\right.}{b^2\sqrt{a^2-b^2}d}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} - \frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{ia^2(e+fx)^3\log\left(1-\right.}{b^2\sqrt{a^2-b^2}d}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} - \frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{ia^2(e+fx)^3\log\left(1-\right.}{b^2\sqrt{a^2-b^2}d}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} - \frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{ia^2(e+fx)^3\log\left(1-\right.}{b^2\sqrt{a^2-b^2}d}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} - \frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{ia^2(e+fx)^3\log\left(1-\right.}{b^2\sqrt{a^2-b^2}d}$$

Mathematica [A] time = 6.67317, size = 1020, normalized size = 1.59

$$-ax\left(4e^3+6fxe^2+4f^2x^2e+f^3x^3\right)d^4 - 4b(e+fx)\left(d^2(e+fx)^2-6f^2\right)\cos(c+dx)d + \frac{4a^2\left(2\sqrt{b^2-a^2}e^3\tan^{-1}\left(\frac{ia+be^{i(c+dx)}}{\sqrt{a^2-b^2}}\right)d^3+\sqrt{a}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(a*d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)) - 4*b*d*(e + f*x)*(
-6*f^2 + d^2*(e + f*x)^2)*Cos[c + d*x] + (4*a^2*(2*Sqrt[-a^2 + b^2]*d^3*e^3
```

```
*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 3*Sqrt[a^2 - b^2]*d^3*
e^2*f*x*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 3*Sqrt[a
^2 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b
^2])] + Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + S
qrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 + (b*E^(I*(c + d*x)
))/(I*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 + (b*E
^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log
[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - (3*I)*Sqrt[a^2 - b^2]*
d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2]
)] + (3*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(I*(c + d*x)
))/(I*a + Sqrt[-a^2 + b^2]))] + 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, (b*E^(
I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 6*Sqrt[a^2 - b^2]*d*f^3*x*Poly
Log[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 6*Sqrt[a^2 - b^2]
*d*e*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] - 6*Sq
rt[a^2 - b^2]*d*f^3*x*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b
^2]))] + (6*I)*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*E^(I*(c + d*x)))/((-I)*a +
 Sqrt[-a^2 + b^2])] - (6*I)*Sqrt[a^2 - b^2]*f^3*PolyLog[4, -((b*E^(I*(c + d
*x)))/(I*a + Sqrt[-a^2 + b^2]))]))/Sqrt[-(a^2 - b^2)^2] + 12*b*f*(-2*f^2 +
d^2*(e + f*x)^2)*Sin[c + d*x])/(4*b^2*d^4)
```

Maple [F] time = 0.894, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3} \left(\sin \left(dx+c\right)\right)^{2}}{a+b\sin \left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.40277, size = 6095, normalized size = 9.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*((a^3 - a*b^2)*d^4*f^3*x^4 + 4*(a^3 - a*b^2)*d^4*e*f^2*x^3 + 6*(a^3 -
a*b^2)*d^4*e^2*f*x^2 + 4*(a^3 - a*b^2)*d^4*e^3*x + 12*I*a^2*b*f^3*sqrt(-(a^
2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*
cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*I*a^2*b*f^
3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x +
 c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12
*I*a^2*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d
*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) -
12*I*a^2*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin
(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
+ 2*(-3*I*a^2*b*d^2*f^3*x^2 - 6*I*a^2*b*d^2*e*f^2*x - 3*I*a^2*b*d^2*e^2*f)*
sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) +
2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)
+ 2*(3*I*a^2*b*d^2*f^3*x^2 + 6*I*a^2*b*d^2*e*f^2*x + 3*I*a^2*b*d^2*e^2*f)*s
qrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2
*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) +
 2*(3*I*a^2*b*d^2*f^3*x^2 + 6*I*a^2*b*d^2*e*f^2*x + 3*I*a^2*b*d^2*e^2*f)*sq
rt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2
*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) +
 2*(-3*I*a^2*b*d^2*f^3*x^2 - 6*I*a^2*b*d^2*e*f^2*x - 3*I*a^2*b*d^2*e^2*f)*s
qrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) -
2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)
- 2*(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*
f^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b
*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f +
 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*
x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(a^2*
b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*sqrt
(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(
a^2 - b^2)/b^2) + 2*I*a) + 2*(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b
*c^2*d*e*f^2 - a^2*b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c)
- 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(a^2*b*d^3*f
```

```
^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f
 - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*
a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*
x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2
*b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*
x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b
)/b) - 2*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x +
 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*sqrt(-(a^2 - b^
2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c)
 + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(a^2*b*d^3*f^3*x^
3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a
^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*co
s(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2) + 2*b)/b) + 12*(a^2*b*d*f^3*x + a^2*b*d*e*f^2)*sqrt(-(a^2
 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*c
os(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*(a^2*b*d*f^
3*x + a^2*b*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x +
 c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2))/b) - 12*(a^2*b*d*f^3*x + a^2*b*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*
polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(a^2*b*d*f^3*x + a^2*b*d*e*f^2)*s
qrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*c
os(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 4*((a^2*b - b^
3)*d^3*f^3*x^3 + 3*(a^2*b - b^3)*d^3*e*f^2*x^2 + (a^2*b - b^3)*d^3*e^3 - 6*
(a^2*b - b^3)*d*e*f^2 + 3*((a^2*b - b^3)*d^3*e^2*f - 2*(a^2*b - b^3)*d*f^3)
*x)*cos(d*x + c) - 12*((a^2*b - b^3)*d^2*f^3*x^2 + 2*(a^2*b - b^3)*d^2*e*f^
2*x + (a^2*b - b^3)*d^2*e^2*f - 2*(a^2*b - b^3)*f^3)*sin(d*x + c))/((a^2*b^
2 - b^4)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \sin(dx+c)^2}{b\sin(dx+c)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

3.225 $\int \frac{(e+fx)^2 \sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=479

$$-\frac{2a^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2\sqrt{a^2-b^2}}+\frac{2a^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2\sqrt{a^2-b^2}}-\frac{2ia^2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3\sqrt{a^2-b^2}}+\frac{2ia^2f^2\text{Po}}{b}$$

```
[Out] -(a*(e + f*x)^3)/(3*b^2*f) + (2*f^2*Cos[c + d*x])/(b*d^3) - ((e + f*x)^2*Co
s[c + d*x])/(b*d) - (I*a^2*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d) + (I*a^2*(e + f*x)^2*Log[1 - (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d) - (2*a^2*
f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*S
qrt[a^2 - b^2]*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^2) - ((2*I)*a^2*f^2*PolyLog[3
, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^3) +
 ((2*I)*a^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b
^2*Sqrt[a^2 - b^2]*d^3) + (2*f*(e + f*x)*Sin[c + d*x])/(b*d^2)
```

Rubi [A] time = 1.03808, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4515, 3296, 2638, 32, 3323, 2264, 2190, 2531, 2282, 6589}

$$-\frac{2a^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2\sqrt{a^2-b^2}}+\frac{2a^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2\sqrt{a^2-b^2}}-\frac{2ia^2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3\sqrt{a^2-b^2}}+\frac{2ia^2f^2\text{Po}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] -(a*(e + f*x)^3)/(3*b^2*f) + (2*f^2*Cos[c + d*x])/(b*d^3) - ((e + f*x)^2*Co
s[c + d*x])/(b*d) - (I*a^2*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d) + (I*a^2*(e + f*x)^2*Log[1 - (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d) - (2*a^2*
f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*S
qrt[a^2 - b^2]*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^2) - ((2*I)*a^2*f^2*PolyLog[3
, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^3) +
 ((2*I)*a^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b
^2*Sqrt[a^2 - b^2]*d^3) + (2*f*(e + f*x)*Sin[c + d*x])/(b*d^2)
```

Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\sin(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^2\cos(c+dx)}{bd} - \frac{a\int (e+fx)^2\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{b^2} + \frac{(2f)\int (e+fx)\cos(c \cdots}{bd}$$

$$= -\frac{a(e+fx)^3}{3b^2f} - \frac{(e+fx)^2\cos(c+dx)}{bd} + \frac{2f(e+fx)\sin(c+dx)}{bd^2} + \frac{(2a^2)\int \frac{e^{i(c+dx)}(e+fx)^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c \cdots}}}{b^2}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cos(c+dx)}{bd^3} - \frac{(e+fx)^2\cos(c+dx)}{bd} + \frac{2f(e+fx)\sin(c+dx)}{bd^2} - \frac{(2ia \cdots}{}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cos(c+dx)}{bd^3} - \frac{(e+fx)^2\cos(c+dx)}{bd} - \frac{ia^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right) \cdots}{b^2\sqrt{a^2-b^2}\,d}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cos(c+dx)}{bd^3} - \frac{(e+fx)^2\cos(c+dx)}{bd} - \frac{ia^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right) \cdots}{b^2\sqrt{a^2-b^2}\,d}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cos(c+dx)}{bd^3} - \frac{(e+fx)^2\cos(c+dx)}{bd} - \frac{ia^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right) \cdots}{b^2\sqrt{a^2-b^2}\,d}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cos(c+dx)}{bd^3} - \frac{(e+fx)^2\cos(c+dx)}{bd} - \frac{ia^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right) \cdots}{b^2\sqrt{a^2-b^2}\,d}$$

Mathematica [A] time = 3.1866, size = 531, normalized size = 1.11

$$\frac{3ia^2\left(-i\left(2f^2\sqrt{a^2-b^2}\text{PolyLog}\left(3,\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}-ia}\right)-2f^2\sqrt{a^2-b^2}\text{PolyLog}\left(3,-\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}+ia}\right)+d^2\left(2e^2\sqrt{b^2-a^2}\tan^{-1}\left(\frac{ia+be^{i(c+dx)}}{\sqrt{a^2-b^2}}\right)+fx\sqrt{a^2-b^2}(2e+fx)\left(\log\left(1-\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}-ia}\right)-\cdots\right.\right.\right.\right.}{d^3\sqrt{-\left(a^2-b^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(a*x*(3*e^2 + 3*e*f*x + f^2*x^2)) + ((3*I)*a^2*(-2*Sqrt[a^2 - b^2]*d*f*(e
 + f*x)*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 2*Sqr
t[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^
2 + b^2]))] - I*(d^2*(2*Sqrt[-a^2 + b^2]*e^2*ArcTan[(I*a + b*E^(I*(c + d*x)
))/Sqrt[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(Log[1 - (b*E^(I*(c +
```

```
 d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - Log[1 + (b*E^(I*(c + d*x)))/(I*a + S
qrt[-a^2 + b^2])])) + 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*E^(I*(c + d*x)))/
((-I)*a + Sqrt[-a^2 + b^2])] - 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, -((b*E^(I*(
c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))])))/(Sqrt[-(a^2 - b^2)^2]*d^3) - (3*b*
Cos[d*x]*((-2*f^2 + d^2*(e + f*x)^2)*Cos[c] - 2*d*f*(e + f*x)*Sin[c]))/d^3
+ (3*b*(2*d*f*(e + f*x)*Cos[c] + (-2*f^2 + d^2*(e + f*x)^2)*Sin[c])*Sin[d*x
])/d^3)/(3*b^2)
```

Maple [F] time = 0.976, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2}\left(\sin\left(dx+c\right)\right)^{2}}{a+b\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.45094, size = 4311, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(a^3 - a*b^2)*d^3*f^2*x^3 + 6*(a^3 - a*b^2)*d^3*e*f*x^2 + 6*(a^3 -
a*b^2)*d^3*e^2*x + 6*a^2*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a
*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2))/b) - 6*a^2*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/
2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*a^2*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polyl
og(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*a^2*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylo
g(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2))/b) + (-6*I*a^2*b*d*f^2*x - 6*I*a^2*b*d*e*f)*sqr
t(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(
b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (
6*I*a^2*b*d*f^2*x + 6*I*a^2*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I
*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (6*I*a^2*b*d*f^2*x + 6*I*a^2*b*d*e*f
)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c)
 + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b +
1) + (-6*I*a^2*b*d*f^2*x - 6*I*a^2*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1
/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 3*(a^2*b*d^2*e^2 - 2*a^2*b*c
*d*e*f + a^2*b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b
*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 3*(a^2*b*d^2*e^2 - 2*
a^2*b*c*d*e*f + a^2*b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c)
- 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 3*(a^2*b*d^2*e
^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d
*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 3*(a^2
*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2
*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)
- 3*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^
2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) +
2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 3*
(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*s
qrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b
*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 3*(a^2
*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*sqrt(
-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*co
s(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 3*(a^2*b*
d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*sqrt(-(a
^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d
*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 6*((a^2*b -
b^3)*d^2*f^2*x^2 + 2*(a^2*b - b^3)*d^2*e*f*x + (a^2*b - b^3)*d^2*e^2 - 2*(a
^2*b - b^3)*f^2)*cos(d*x + c) - 12*((a^2*b - b^3)*d*f^2*x + (a^2*b - b^3)*d
*e*f)*sin(d*x + c))/((a^2*b^2 - b^4)*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sin(dx+c)^2}{b \sin(dx+c)+a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

3.226 $\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=311

$$-\frac{a^2 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2\sqrt{a^2-b^2}}+\frac{a^2 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2\sqrt{a^2-b^2}}-\frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d\sqrt{a^2-b^2}}+\frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+d}}{\sqrt{a^2-b^2}}\right.}{b^2d\sqrt{a^2-b^2}}$$

```
[Out] -((a*e*x)/b^2) - (a*f*x^2)/(2*b^2) - ((e + f*x)*Cos[c + d*x])/(b*d) - (I*a^
2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*Sqrt
[a^2 - b^2]*d) + (I*a^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a
^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d) - (a^2*f*PolyLog[2, (I*b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^2) + (a^2*f*PolyLog[2, (
I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^2) + (f
*Sin[c + d*x])/(b*d^2)
```

Rubi [A] time = 0.550827, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4515, 3296, 2637, 3323, 2264, 2190, 2279, 2391}

$$-\frac{a^2 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2\sqrt{a^2-b^2}}+\frac{a^2 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2\sqrt{a^2-b^2}}-\frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d\sqrt{a^2-b^2}}+\frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+d}}{\sqrt{a^2-b^2}}\right.}{b^2d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] -((a*e*x)/b^2) - (a*f*x^2)/(2*b^2) - ((e + f*x)*Cos[c + d*x])/(b*d) - (I*a^
2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*Sqrt
[a^2 - b^2]*d) + (I*a^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a
^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d) - (a^2*f*PolyLog[2, (I*b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^2) + (a^2*f*PolyLog[2, (
I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*Sqrt[a^2 - b^2]*d^2) + (f
*Sin[c + d*x])/(b*d^2)
```


Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
```

```
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\sin(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)\cos(c+dx)}{bd} - \frac{a\int (e+fx)\,dx}{b^2} + \frac{a^2\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{b^2} + \frac{f\int \cos(c+dx)\,dx}{bd}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} - \frac{(e+fx)\cos(c+dx)}{bd} + \frac{f\sin(c+dx)}{bd^2} + \frac{\left(2a^2\right)\int \frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b^2}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} - \frac{(e+fx)\cos(c+dx)}{bd} + \frac{f\sin(c+dx)}{bd^2} - \frac{\left(2ia^2\right)\int \frac{e^{i(c+dx)}(e+fx)}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{b\sqrt{a^2-b^2}} +$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} - \frac{(e+fx)\cos(c+dx)}{bd} - \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}d} + \frac{ia^2(e+fx)\log\left(1\right.}{b^2\sqrt{a^2-\right.}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} - \frac{(e+fx)\cos(c+dx)}{bd} - \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}d} + \frac{ia^2(e+fx)\log\left(1\right.}{b^2\sqrt{a^2-\right.}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} - \frac{(e+fx)\cos(c+dx)}{bd} - \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}d} + \frac{ia^2(e+fx)\log\left(1\right.}{b^2\sqrt{a^2-\right.}$$

Mathematica [B] time = 6.73369, size = 709, normalized size = 2.28

$$2a^2d(e+fx)\left(-\frac{if\left(\text{PolyLog}\left(2,\frac{a\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a+i\left(\sqrt{b^2-a^2}+b\right)}\right)+\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{\sqrt{b^2-a^2}+a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{b^2-a^2}-ia+b}\right)\right)}{\sqrt{b^2-a^2}} + \frac{if\left(\text{PolyLog}\left(2,\frac{a\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a-i\left(\sqrt{b^2-a^2}+b\right)}\right)+\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\right.}{\sqrt{b^2-a^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x)*(c*f - d*(2*e + f*x)) - 2*b*d*(e + f*x)*Cos[c + d*x] + (2*a^2*
d*(e + f*x)*((2*(d*e - c*f)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
```

```
])/Sqrt[a^2 - b^2] - (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 +
 b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (
a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b
^2] + (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(
c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(c +
d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1
- I*Tan[(c + d*x)/2]]*Log[(-b + Sqrt[-a^2 + b^2] - a*Tan[(c + d*x)/2])/(I*a
 - b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b
+ Sqrt[-a^2 + b^2])]))/Sqrt[-a^2 + b^2] - (I*f*(Log[1 + I*Tan[(c + d*x)/2]]
*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2
])] + PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))
]))/Sqrt[-a^2 + b^2]))/(d*e - c*f + I*f*Log[1 - I*Tan[(c + d*x)/2]] - I*f*L
og[1 + I*Tan[(c + d*x)/2]]) + 2*b*f*Sin[c + d*x])/(2*b^2*d^2)
```

Maple [B] time = 0.296, size = 625, normalized size = 2.

$$-\frac{afx^2}{2b^2}-\frac{aex}{b^2}-\frac{\left(dfx+if+de\right)e^{i(dx+c)}}{2bd^2}-\frac{\left(dfx-if+de\right)e^{-i(dx+c)}}{2bd^2}+\frac{2ia^2e}{b^2d}\arctan\left(\frac{2ibe^{i(dx+c)}-2a}{2}\frac{1}{\sqrt{-a^2+b^2}}\right)\frac{1}{\sqrt{-a^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2*a*f*x^2/b^2-a*e*x/b^2-1/2*(d*f*x+I*f+d*e)/b/d^2*exp(I*(d*x+c))-1/2*(d*
f*x-I*f+d*e)/b/d^2*exp(-I*(d*x+c))+2*I*a^2/b^2/d*e/(-a^2+b^2)^(1/2)*arctan(
1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+a^2/b^2/d*f/(-a^2+b^2)^(1/
2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+a^2
/b^2/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a
-(-a^2+b^2)^(1/2)))*c-a^2/b^2/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))
+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-a^2/b^2/d^2*f/(-a^2+b^2)^(1/2)
*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-I*a^2
/b^2/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(
I*a-(-a^2+b^2)^(1/2)))+I*a^2/b^2/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*
(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2*I*a^2/b^2/d^2*f*c/(-a^
2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.25883, size = 2770, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^3 - a*b^2)*d^2*f*x^2 - 2*I*a^2*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog(
-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*a^2*b*f*sqrt(-(a^2 - b^
2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x +
c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*a^2*b*f*s
qrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) +
2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)
- 2*I*a^2*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*
sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
 + 2*b)/b + 1) + 4*(a^3 - a*b^2)*d^2*e*x - 4*(a^2*b - b^3)*f*sin(d*x + c) -
 2*(a^2*b*d*e - a^2*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*
I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(a^2*b*d*e - a^2
*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) +
2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(a^2*b*d*e - a^2*b*c*f)*sqrt(-(a^2
- b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b
^2)/b^2) + 2*I*a) + 2*(a^2*b*d*e - a^2*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2
*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)
- 2*(a^2*b*d*f*x + a^2*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x
 + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) + 2*b)/b) + 2*(a^2*b*d*f*x + a^2*b*c*f)*sqrt(-(a^2 - b^2)/b^2)
*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(a^2*b*d*f*x + a^2*b*c*f)
*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2
*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(
a^2*b*d*f*x + a^2*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c
) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2) + 2*b)/b) + 4*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*d*e)*cos(d*x +
```

```
c))/((a^2*b^2 - b^4)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\sin(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] integrate((f*x + e)*sin(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

3.227 $\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\sqrt{a^2-b^2}}-\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}$$

```
[Out] -((a*x)/b^2) + (2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^
2*Sqrt[a^2 - b^2]*d) - Cos[c + d*x]/(b*d)
```

Rubi [A] time = 0.106451, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2746, 12, 2735, 2660, 618, 204}

$$\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\sqrt{a^2-b^2}}-\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) + (2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^
2*Sqrt[a^2 - b^2]*d) - Cos[c + d*x]/(b*d)
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx &= -\frac{\cos(c+dx)}{bd}-\frac{\int \frac{a\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}\\
&= -\frac{\cos(c+dx)}{bd}-\frac{a\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}\\
&= -\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}+\frac{a^2\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^2}\\
&= -\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}+\frac{\left(2a^2\right)\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}\\
&= -\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}-\frac{\left(4a^2\right)\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}\\
&= -\frac{ax}{b^2}+\frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d}-\frac{\cos(c+dx)}{bd}
\end{aligned}
$$

Mathematica [A] time = 0.179816, size = 71, normalized size = 0.95

$$-\frac{-\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+a(c+dx)+b\cos(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*(c + d*x) - (2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] + b*Cos[c + d*x])/(b^2*d))
```

Maple [A] time = 0.027, size = 96, normalized size = 1.3

$$-2\frac{1}{bd\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)}-2\frac{a\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{b^2d}+2\frac{a^2}{b^2d\sqrt{a^2-b^2}}\arctan\left(1/2\frac{2\,a\tan\left(1/2\,dx+c/2\right)+}{\sqrt{a^2-b^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/d/b/(1+tan(1/2*d*x+1/2*c)^2)-2/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))+2/d/b^
2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*
a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87542, size = 609, normalized size = 8.12

$$\left[-\frac{\sqrt{-a^2+b^2}a^2\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2(a^3-ab^2)dx+2(a^2b-}{2(a^2b^2-b^4)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a^2*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d
*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt
(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(a
^3 - a*b^2)*d*x + 2*(a^2*b - b^3)*cos(d*x + c))/((a^2*b^2 - b^4)*d), -(sqrt
(a^2 - b^2)*a^2*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))
) + (a^3 - a*b^2)*d*x + (a^2*b - b^3)*cos(d*x + c))/((a^2*b^2 - b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12139, size = 134, normalized size = 1.79

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2}{\sqrt{a^2-b^2}b^2}-\frac{(dx+c)a}{b^2}-\frac{2}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c
) + b)/sqrt(a^2 - b^2)))*a^2/(sqrt(a^2 - b^2)*b^2) - (d*x + c)*a/b^2 - 2/((
tan(1/2*d*x + 1/2*c)^2 + 1)*b))/d
```

3.228 $\int \frac{(e+fx)^3 \sin^3(c+dx)}{a+b\sin(c+dx)}\, dx$

Optimal. Leaf size=802

$$\frac{(e+fx)^4}{8bf}+\frac{a^2(e+fx)^4}{4b^3f}+\frac{a\cos(c+dx)(e+fx)^3}{b^2d}+\frac{ia^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)(e+fx)^3}{b^3\sqrt{a^2-b^2}\,d}-\frac{ia^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)(e+fx)^3}{b^3\sqrt{a^2-b^2}\,d}-$$

```
[Out] (-3*e*f^2*x)/(4*b*d^2) - (3*f^3*x^2)/(8*b*d^2) + (a^2*(e + f*x)^4)/(4*b^3*f
) + (e + f*x)^4/(8*b*f) - (6*a*f^2*(e + f*x)*Cos[c + d*x])/(b^2*d^3) + (a*(
e + f*x)^3*Cos[c + d*x])/(b^2*d) + (I*a^3*(e + f*x)^3*Log[1 - (I*b*E^(I*(c
+ d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d) - (I*a^3*(e + f*x)
^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^
2]*d) + (3*a^3*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^2) - (3*a^3*f*(e + f*x)^2*PolyLog[2, (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^2) + ((6*I
)*a^3*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])]
)/(b^3*Sqrt[a^2 - b^2]*d^3) - ((6*I)*a^3*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I
*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^3) - (6*a^3*f^3
*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b
^2]*d^4) + (6*a^3*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]
)])/(b^3*Sqrt[a^2 - b^2]*d^4) + (6*a*f^3*Sin[c + d*x])/(b^2*d^4) - (3*a*f*(
e + f*x)^2*Sin[c + d*x])/(b^2*d^2) + (3*f^2*(e + f*x)*Cos[c + d*x]*Sin[c +
d*x])/(4*b*d^3) - ((e + f*x)^3*Cos[c + d*x]*Sin[c + d*x])/(2*b*d) - (3*f^3*
Sin[c + d*x]^2)/(8*b*d^4) + (3*f*(e + f*x)^2*Sin[c + d*x]^2)/(4*b*d^2)
```

Rubi [A] time = 1.3412, antiderivative size = 802, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {4515, 3311, 32, 3310, 3296, 2637, 3323, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{(e+fx)^4}{8bf}+\frac{a^2(e+fx)^4}{4b^3f}+\frac{a\cos(c+dx)(e+fx)^3}{b^2d}+\frac{ia^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)(e+fx)^3}{b^3\sqrt{a^2-b^2}\,d}-\frac{ia^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)(e+fx)^3}{b^3\sqrt{a^2-b^2}\,d}-$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*e*f^2*x)/(4*b*d^2) - (3*f^3*x^2)/(8*b*d^2) + (a^2*(e + f*x)^4)/(4*b^3*f
) + (e + f*x)^4/(8*b*f) - (6*a*f^2*(e + f*x)*Cos[c + d*x])/(b^2*d^3) + (a*(
e + f*x)^3*Cos[c + d*x])/(b^2*d) + (I*a^3*(e + f*x)^3*Log[1 - (I*b*E^(I*(c
```

```
+ d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d) - (I*a^3*(e + f*x)
^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^
2]*d) + (3*a^3*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^2) - (3*a^3*f*(e + f*x)^2*PolyLog[2, (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^2) + ((6*I
)*a^3*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])]
)/(b^3*Sqrt[a^2 - b^2]*d^3) - ((6*I)*a^3*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I
*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^3) - (6*a^3*f^3
*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b
^2]*d^4) + (6*a^3*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]
)])/(b^3*Sqrt[a^2 - b^2]*d^4) + (6*a*f^3*Sin[c + d*x])/(b^2*d^4) - (3*a*f*(
e + f*x)^2*Sin[c + d*x])/(b^2*d^2) + (3*f^2*(e + f*x)*Cos[c + d*x]*Sin[c +
d*x])/(4*b*d^3) - ((e + f*x)^3*Cos[c + d*x]*Sin[c + d*x])/(2*b*d) - (3*f^3*
Sin[c + d*x]^2)/(8*b*d^4) + (3*f*(e + f*x)^2*Sin[c + d*x]^2)/(4*b*d^2)
```

Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
```

```
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\sin^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^3\cos(c+dx)\sin(c+dx)}{2bd} + \frac{3f(e+fx)^2\sin^2(c+dx)}{4bd^2} - \frac{a\int (e+fx)^3\sin(c+dx)}{b^2}$$

$$= \frac{(e+fx)^4}{8bf} + \frac{a(e+fx)^3\cos(c+dx)}{b^2d} + \frac{3f^2(e+fx)\cos(c+dx)\sin(c+dx)}{4bd^3} - \frac{(e+fx)^3\cos}{}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} + \frac{a(e+fx)^3\cos(c+dx)}{b^2d} - \frac{3af(e+fx)^2\sin(}{b^2d^2}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^3\cos(}{b^2d}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^3\cos(}{b^2d}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^3\cos(}{b^2d}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^3\cos(}{b^2d}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^3\cos(}{b^2d}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^3\cos(}{b^2d}$$

Mathematica [B] time = 5.50605, size = 1923, normalized size = 2.4

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (16*a^2*Sqrt[-(a^2 - b^2)^2]*d^4*e^3*x + 8*b^2*Sqrt[-(-a^2 + b^2)^2]*d^4*e^
3*x + 24*a^2*Sqrt[-(a^2 - b^2)^2]*d^4*e^2*f*x^2 + 12*b^2*Sqrt[-(-a^2 + b^2)
```

```
^2]*d^4*e^2*f*x^2 + 16*a^2*Sqrt[-(a^2 - b^2)^2]*d^4*e*f^2*x^3 + 8*b^2*Sqrt[
-(-a^2 + b^2)^2]*d^4*e*f^2*x^3 + 4*a^2*Sqrt[-(a^2 - b^2)^2]*d^4*f^3*x^4 + 2
*b^2*Sqrt[-(-a^2 + b^2)^2]*d^4*f^3*x^4 - 32*a^3*Sqrt[-a^2 + b^2]*d^3*e^3*Ar
cTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 16*a*b*Sqrt[-(a^2 - b^2)^
2]*d^3*e^3*Cos[c + d*x] - 96*a*b*Sqrt[-(a^2 - b^2)^2]*d*e*f^2*Cos[c + d*x]
+ 48*a*b*Sqrt[-(a^2 - b^2)^2]*d^3*e^2*f*x*Cos[c + d*x] - 96*a*b*Sqrt[-(a^2
- b^2)^2]*d*f^3*x*Cos[c + d*x] + 48*a*b*Sqrt[-(a^2 - b^2)^2]*d^3*e*f^2*x^2*
Cos[c + d*x] + 16*a*b*Sqrt[-(a^2 - b^2)^2]*d^3*f^3*x^3*Cos[c + d*x] - 6*b^2
*Sqrt[-(a^2 - b^2)^2]*d^2*e^2*f*Cos[2*(c + d*x)] + 3*b^2*Sqrt[-(a^2 - b^2)^
2]*f^3*Cos[2*(c + d*x)] - 12*b^2*Sqrt[-(a^2 - b^2)^2]*d^2*e*f^2*x*Cos[2*(c
+ d*x)] - 6*b^2*Sqrt[-(a^2 - b^2)^2]*d^2*f^3*x^2*Cos[2*(c + d*x)] - 48*a^3*
Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2
 + b^2])] - 48*a^3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*E^(I*(c + d*x))
)/((-I)*a + Sqrt[-a^2 + b^2])] - 16*a^3*Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 -
 (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 48*a^3*Sqrt[a^2 - b^2]*
d^3*e^2*f*x*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] + 48*a^3*
Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2
+ b^2])] + 16*a^3*Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 + (b*E^(I*(c + d*x)))/(
I*a + Sqrt[-a^2 + b^2])] + (48*I)*a^3*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*Pol
yLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - (48*I)*a^3*Sqrt[
a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-
a^2 + b^2]))] - 96*a^3*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, (b*E^(I*(c + d*x)
))/((-I)*a + Sqrt[-a^2 + b^2])] - 96*a^3*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3,
 (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 96*a^3*Sqrt[a^2 - b^2]*
d*e*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] + 96*a^
3*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2
 + b^2]))] - (96*I)*a^3*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*E^(I*(c + d*x)))/
((-I)*a + Sqrt[-a^2 + b^2])] + (96*I)*a^3*Sqrt[a^2 - b^2]*f^3*PolyLog[4, -(
(b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] - 48*a*b*Sqrt[-(a^2 - b^2)^2
]*d^2*e^2*f*Sin[c + d*x] + 96*a*b*Sqrt[-(a^2 - b^2)^2]*f^3*Sin[c + d*x] - 9
6*a*b*Sqrt[-(a^2 - b^2)^2]*d^2*e*f^2*x*Sin[c + d*x] - 48*a*b*Sqrt[-(a^2 - b
^2)^2]*d^2*f^3*x^2*Sin[c + d*x] - 4*b^2*Sqrt[-(a^2 - b^2)^2]*d^3*e^3*Sin[2*
(c + d*x)] + 6*b^2*Sqrt[-(a^2 - b^2)^2]*d*e*f^2*Sin[2*(c + d*x)] - 12*b^2*S
qrt[-(a^2 - b^2)^2]*d^3*e^2*f*x*Sin[2*(c + d*x)] + 6*b^2*Sqrt[-(a^2 - b^2)^
2]*d*f^3*x*Sin[2*(c + d*x)] - 12*b^2*Sqrt[-(a^2 - b^2)^2]*d^3*e*f^2*x^2*Sin
[2*(c + d*x)] - 4*b^2*Sqrt[-(a^2 - b^2)^2]*d^3*f^3*x^3*Sin[2*(c + d*x)])/(1
6*b^3*Sqrt[-(a^2 - b^2)^2]*d^4)
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3}\left(\sin\left(dx+c\right)\right)^{3}}{a+b\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 5.39924, size = 6724, normalized size = 8.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*((2*a^4 - a^2*b^2 - b^4)*d^4*f^3*x^4 + 4*(2*a^4 - a^2*b^2 - b^4)*d^4*e*
f^2*x^3 + 24*I*a^3*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(d
*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2))/b) - 24*I*a^3*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(
2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2))/b) + 24*I*a^3*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polyl
og(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2))/b) - 24*I*a^3*b*f^3*sqrt(-(a^2 - b^2)/b^2)*pol
ylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 3*(2*(2*a^4 - a^2*b^2 - b^4)*d^4*e^2*f
+ (a^2*b^2 - b^4)*d^2*f^3)*x^2 - 3*(2*(a^2*b^2 - b^4)*d^2*f^3*x^2 + 4*(a^2*
b^2 - b^4)*d^2*e*f^2*x + 2*(a^2*b^2 - b^4)*d^2*e^2*f - (a^2*b^2 - b^4)*f^3)
*cos(d*x + c)^2 - 2*(6*I*a^3*b*d^2*f^3*x^2 + 12*I*a^3*b*d^2*e*f^2*x + 6*I*a
^3*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a
*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
```

) + 2*b)/b + 1) - 2*(-6*I*a^3*b*d^2*f^3*x^2 - 12*I*a^3*b*d^2*e*f^2*x - 6*I*
a^3*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*
a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2) + 2*b)/b + 1) - 2*(-6*I*a^3*b*d^2*f^3*x^2 - 12*I*a^3*b*d^2*e*f^2*x - 6*I
*a^3*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) +
2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2) + 2*b)/b + 1) - 2*(6*I*a^3*b*d^2*f^3*x^2 + 12*I*a^3*b*d^2*e*f^2*x + 6*
I*a^3*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) +
2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2) + 2*b)/b + 1) - 4*(a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*
d*e*f^2 - a^3*b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*
b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 4*(a^3*b*d^3*e^3 - 3
*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*sqrt(-(a^2 - b^2)
/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2
) - 2*I*a) + 4*(a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 -
a^3*b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*
x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 4*(a^3*b*d^3*e^3 - 3*a^3*b*c
*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*lo
g(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I
a) - 4(a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x +
3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*sqrt(-(a^2 - b^2
)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) -
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 4*(a^3*b*d^3*f^3*x^3
+ 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3
*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d
*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2) + 2*b)/b) - 4*(a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3
*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*
f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c)
+ 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) +
4*(a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3
*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2
)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 24*(a^3*b*d*f^3*x + a^3*b
*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*s
in(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))
/b) - 24*(a^3*b*d*f^3*x + a^3*b*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3,
1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 24*(a^3*b*d*f^3*x + a^3*b*d*e*f^2)*sqr
t(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos
(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 24*(a^3*b*d*f^3*
x + a^3*b*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a
*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))
/b) + 2*(2*(2*a^4 - a^2*b^2 - b^4)*d^4*e^3 + 3*(a^2*b^2 - b^4)*d^2*e*f^2)*x
+ 8*((a^3*b - a*b^3)*d^3*f^3*x^3 + 3*(a^3*b - a*b^3)*d^3*e*f^2*x^2 + (a^3*

```
b - a*b^3)*d^3*e^3 - 6*(a^3*b - a*b^3)*d*e*f^2 + 3*((a^3*b - a*b^3)*d^3*e^2
*f - 2*(a^3*b - a*b^3)*d*f^3)*x)*cos(d*x + c) - 2*(12*(a^3*b - a*b^3)*d^2*f
^3*x^2 + 24*(a^3*b - a*b^3)*d^2*e*f^2*x + 12*(a^3*b - a*b^3)*d^2*e^2*f - 24
*(a^3*b - a*b^3)*f^3 + (2*(a^2*b^2 - b^4)*d^3*f^3*x^3 + 6*(a^2*b^2 - b^4)*d
^3*e*f^2*x^2 + 2*(a^2*b^2 - b^4)*d^3*e^3 - 3*(a^2*b^2 - b^4)*d*e*f^2 + 3*(2
*(a^2*b^2 - b^4)*d^3*e^2*f - (a^2*b^2 - b^4)*d*f^3)*x)*cos(d*x + c))*sin(d*
x + c))/((a^2*b^3 - b^5)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \sin(dx+c)^3}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

3.229 $\int \frac{(e+fx)^2 \sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=592

$$\frac{2a^3 f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2\sqrt{a^2-b^2}}-\frac{2a^3 f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2\sqrt{a^2-b^2}}+\frac{2ia^3 f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3\sqrt{a^2-b^2}}-\frac{2ia^3 f^2\text{Po}}{b}$$

```
[Out] -(f^2*x)/(4*b*d^2) + (a^2*(e + f*x)^3)/(3*b^3*f) + (e + f*x)^3/(6*b*f) - (2
*a*f^2*Cos[c + d*x])/(b^2*d^3) + (a*(e + f*x)^2*Cos[c + d*x])/(b^2*d) + (I*
a^3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*
Sqrt[a^2 - b^2]*d) - (I*a^3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a +
Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d) + (2*a^3*f*(e + f*x)*PolyLog[2,
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^2) - (
2*a^3*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/
(b^3*Sqrt[a^2 - b^2]*d^2) + ((2*I)*a^3*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))
/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^3) - ((2*I)*a^3*f^2*PolyLog
[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^3)
 - (2*a*f*(e + f*x)*Sin[c + d*x])/(b^2*d^2) + (f^2*Cos[c + d*x]*Sin[c + d*x
])/(4*b*d^3) - ((e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(2*b*d) + (f*(e + f*
x)*Sin[c + d*x]^2)/(2*b*d^2)
```

Rubi [A] time = 1.18028, antiderivative size = 592, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {4515, 3311, 32, 2635, 8, 3296, 2638, 3323, 2264, 2190, 2531, 2282, 6589}

$$\frac{2a^3 f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2\sqrt{a^2-b^2}}-\frac{2a^3 f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2\sqrt{a^2-b^2}}+\frac{2ia^3 f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3\sqrt{a^2-b^2}}-\frac{2ia^3 f^2\text{Po}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```


```
[Out] -(f^2*x)/(4*b*d^2) + (a^2*(e + f*x)^3)/(3*b^3*f) + (e + f*x)^3/(6*b*f) - (2
*a*f^2*Cos[c + d*x])/(b^2*d^3) + (a*(e + f*x)^2*Cos[c + d*x])/(b^2*d) + (I*
a^3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*
Sqrt[a^2 - b^2]*d) - (I*a^3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a +
Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d) + (2*a^3*f*(e + f*x)*PolyLog[2,
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^2) - (
2*a^3*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/
```

```
(b^3*Sqrt[a^2 - b^2]*d^2) + ((2*I)*a^3*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))
/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^3) - ((2*I)*a^3*f^2*PolyLog
[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d^3)
 - (2*a*f*(e + f*x)*Sin[c + d*x])/(b^2*d^2) + (f^2*Cos[c + d*x]*Sin[c + d*x
])/(4*b*d^3) - ((e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(2*b*d) + (f*(e + f*
x)*Sin[c + d*x]^2)/(2*b*d^2)
```

Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\sin^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^2\cos(c+dx)\sin(c+dx)}{2bd} + \frac{f(e+fx)\sin^2(c+dx)}{2bd^2} - \frac{a\int (e+fx)^2\sin(c+dx)\,dx}{b^2}$$

$$= \frac{(e+fx)^3}{6bf} + \frac{a(e+fx)^2\cos(c+dx)}{b^2d} + \frac{f^2\cos(c+dx)\sin(c+dx)}{4bd^3} - \frac{(e+fx)^2\cos(c+dx)\,\mathrm{si}}{2bd}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} + \frac{a(e+fx)^2\cos(c+dx)}{b^2d} - \frac{2af(e+fx)\sin(c+dx)}{b^2d^2} + \frac{f}{}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^2\cos(c+dx)}{b^2d} - \frac{2af(e+}{}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^2\cos(c+dx)}{b^2d} + \frac{ia^3(e+}{}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^2\cos(c+dx)}{b^2d} + \frac{ia^3(e+}{}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^2\cos(c+dx)}{b^2d} + \frac{ia^3(e+}{}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cos(c+dx)}{b^2d^3} + \frac{a(e+fx)^2\cos(c+dx)}{b^2d} + \frac{ia^3(e+}{}$$

Mathematica [A] time = 4.43864, size = 1166, normalized size = 1.97

$$\frac{-48\sqrt{b^2-a^2}d^2e^2\tan^{-1}\left(\frac{ia+be^{i(c+dx)}}{\sqrt{a^2-b^2}}\right)a^3-24\sqrt{a^2-b^2}d^2f^2x^2\log\left(1-\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}-ia}\right)a^3-48\sqrt{a^2-b^2}d^2efx\log\left(1-\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}-ia}\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (24*a^2*Sqrt[-(a^2 - b^2)^2]*d^3*e^2*x + 12*b^2*Sqrt[-(-a^2 + b^2)^2]*d^3*e
^2*x + 24*a^2*Sqrt[-(a^2 - b^2)^2]*d^3*e*f*x^2 + 12*b^2*Sqrt[-(-a^2 + b^2)^
2]*d^3*e*f*x^2 + 8*a^2*Sqrt[-(a^2 - b^2)^2]*d^3*f^2*x^3 + 4*b^2*Sqrt[-(-a^2
 + b^2)^2]*d^3*f^2*x^3 - 48*a^3*Sqrt[-a^2 + b^2]*d^2*e^2*ArcTan[(I*a + b*E^
(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 24*a*b*Sqrt[-(a^2 - b^2)^2]*d^2*e^2*Cos[c
 + d*x] - 48*a*b*Sqrt[-(a^2 - b^2)^2]*f^2*Cos[c + d*x] + 48*a*b*Sqrt[-(a^2
- b^2)^2]*d^2*e*f*x*Cos[c + d*x] + 24*a*b*Sqrt[-(a^2 - b^2)^2]*d^2*f^2*x^2*
Cos[c + d*x] - 6*b^2*Sqrt[-(a^2 - b^2)^2]*d*e*f*Cos[2*(c + d*x)] - 6*b^2*Sq
rt[-(a^2 - b^2)^2]*d*f^2*x*Cos[2*(c + d*x)] - 48*a^3*Sqrt[a^2 - b^2]*d^2*e*
f*x*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 24*a^3*Sqrt[
a^2 - b^2]*d^2*f^2*x^2*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^
2])] + 48*a^3*Sqrt[a^2 - b^2]*d^2*e*f*x*Log[1 + (b*E^(I*(c + d*x)))/(I*a +
Sqrt[-a^2 + b^2])] + 24*a^3*Sqrt[a^2 - b^2]*d^2*f^2*x^2*Log[1 + (b*E^(I*(c
+ d*x)))/(I*a + Sqrt[-a^2 + b^2])] + (48*I)*a^3*Sqrt[a^2 - b^2]*d*f*(e + f*
x)*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - (48*I)*a^3
*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt
[-a^2 + b^2]))] - 48*a^3*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*E^(I*(c + d*x)))
/((-I)*a + Sqrt[-a^2 + b^2])] + 48*a^3*Sqrt[a^2 - b^2]*f^2*PolyLog[3, -((b*
E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] - 48*a*b*Sqrt[-(a^2 - b^2)^2]*d
*e*f*Sin[c + d*x] - 48*a*b*Sqrt[-(a^2 - b^2)^2]*d*f^2*x*Sin[c + d*x] - 6*b^
2*Sqrt[-(a^2 - b^2)^2]*d^2*e^2*Sin[2*(c + d*x)] + 3*b^2*Sqrt[-(a^2 - b^2)^2
]*f^2*Sin[2*(c + d*x)] - 12*b^2*Sqrt[-(a^2 - b^2)^2]*d^2*e*f*x*Sin[2*(c + d
*x)] - 6*b^2*Sqrt[-(a^2 - b^2)^2]*d^2*f^2*x^2*Sin[2*(c + d*x)])/(24*b^3*Sqr
t[-(a^2 - b^2)^2]*d^3)
```

Maple [F] time = 0.388, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2(\sin(dx+c))^3}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.03307, size = 4691, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*(2*a^4 - a^2*b^2 - b^4)*d^3*f^2*x^3 + 6*(2*a^4 - a^2*b^2 - b^4)*d^3
*e*f*x^2 + 12*a^3*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*
x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2))/b) - 12*a^3*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I
*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2))/b) - 12*a^3*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3,
 -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2))/b) + 12*a^3*b*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3,
 -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2))/b) - 6*((a^2*b^2 - b^4)*d*f^2*x + (a^2*b^2 - b^4)*d
*e*f)*cos(d*x + c)^2 - 2*(6*I*a^3*b*d*f^2*x + 6*I*a^3*b*d*e*f)*sqrt(-(a^2 -
 b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x
 + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(-6*I*a^
3*b*d*f^2*x - 6*I*a^3*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos
(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(-6*I*a^3*b*d*f^2*x - 6*I*a^3*b*d*e*f)*s
qrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) +
```

```
2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)
- 2*(6*I*a^3*b*d*f^2*x + 6*I*a^3*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2
*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 6*(a^3*b*d^2*e^2 - 2*a^3*b*c*d
*e*f + a^3*b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*s
in(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 6*(a^3*b*d^2*e^2 - 2*a^
3*b*c*d*e*f + a^3*b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) -
2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 6*(a^3*b*d^2*e^2
 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x
 + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 6*(a^3*b
*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b
*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) -
6*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)
*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*
(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 6*(a
^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*sqr
t(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*c
os(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 6*(a^3*b
*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*sqrt(-(
a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(
d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 6*(a^3*b*d^
2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*sqrt(-(a^2
 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x
 + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 3*(2*(2*a^4 -
a^2*b^2 - b^4)*d^3*e^2 + (a^2*b^2 - b^4)*d*f^2)*x + 12*((a^3*b - a*b^3)*d^2
*f^2*x^2 + 2*(a^3*b - a*b^3)*d^2*e*f*x + (a^3*b - a*b^3)*d^2*e^2 - 2*(a^3*b
 - a*b^3)*f^2)*cos(d*x + c) - 3*(8*(a^3*b - a*b^3)*d*f^2*x + 8*(a^3*b - a*b
^3)*d*e*f + (2*(a^2*b^2 - b^4)*d^2*f^2*x^2 + 4*(a^2*b^2 - b^4)*d^2*e*f*x +
2*(a^2*b^2 - b^4)*d^2*e^2 - (a^2*b^2 - b^4)*f^2)*cos(d*x + c))*sin(d*x + c)
)/((a^2*b^3 - b^5)*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sin(dx+c)^3}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

3.230 $\int \frac{(e+fx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=382

$$\frac{a^3 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2\sqrt{a^2-b^2}}-\frac{a^3 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2\sqrt{a^2-b^2}}+\frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d\sqrt{a^2-b^2}}-\frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d\sqrt{a^2-b^2}}$$

```
[Out] (a^2*e*x)/b^3 + (e*x)/(2*b) + (a^2*f*x^2)/(2*b^3) + (f*x^2)/(4*b) + (a*(e +
 f*x)*Cos[c + d*x])/(b^2*d) + (I*a^3*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x))
)/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d) - (I*a^3*(e + f*x)*Log[1
- (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d) + (
a^3*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^
2 - b^2]*d^2) - (a^3*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2
])])/(b^3*Sqrt[a^2 - b^2]*d^2) - (a*f*Sin[c + d*x])/(b^2*d^2) - ((e + f*x)*
Cos[c + d*x]*Sin[c + d*x])/(2*b*d) + (f*Sin[c + d*x]^2)/(4*b*d^2)
```

Rubi [A] time = 0.665739, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {4515, 3310, 3296, 2637, 3323, 2264, 2190, 2279, 2391}

$$\frac{a^3 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2\sqrt{a^2-b^2}}-\frac{a^3 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2\sqrt{a^2-b^2}}+\frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d\sqrt{a^2-b^2}}-\frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```


```
[Out] (a^2*e*x)/b^3 + (e*x)/(2*b) + (a^2*f*x^2)/(2*b^3) + (f*x^2)/(4*b) + (a*(e +
 f*x)*Cos[c + d*x])/(b^2*d) + (I*a^3*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x))
)/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d) - (I*a^3*(e + f*x)*Log[1
- (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^2 - b^2]*d) + (
a^3*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*Sqrt[a^
2 - b^2]*d^2) - (a^3*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2
])])/(b^3*Sqrt[a^2 - b^2]*d^2) - (a*f*Sin[c + d*x])/(b^2*d^2) - ((e + f*x)*
Cos[c + d*x]*Sin[c + d*x])/(2*b*d) + (f*Sin[c + d*x]^2)/(4*b*d^2)
```


Rule 4515

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sin[c +
```

```
d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sin[c + d*x]^(n - 1))/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
IGtQ[n, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\sin^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)\cos(c+dx)\sin(c+dx)}{2bd} + \frac{f\sin^2(c+dx)}{4bd^2} - \frac{a\int (e+fx)\sin(c+dx)\,dx}{b^2} + \frac{a^2\int \ldots}{\ldots}$$

$$= \frac{ex}{2b} + \frac{fx^2}{4b} + \frac{a(e+fx)\cos(c+dx)}{b^2d} - \frac{(e+fx)\cos(c+dx)\sin(c+dx)}{2bd} + \frac{f\sin^2(c+dx)}{4bd^2} + \ldots$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} + \frac{a(e+fx)\cos(c+dx)}{b^2d} - \frac{af\sin(c+dx)}{b^2d^2} - \frac{(e+fx)\cos(c+d\ldots}{2bd} \ldots$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} + \frac{a(e+fx)\cos(c+dx)}{b^2d} - \frac{af\sin(c+dx)}{b^2d^2} - \frac{(e+fx)\cos(c+d\ldots}{2bd} \ldots$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} + \frac{a(e+fx)\cos(c+dx)}{b^2d} + \frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d} - \frac{ia^3\ldots}{\ldots}$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} + \frac{a(e+fx)\cos(c+dx)}{b^2d} + \frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d} - \frac{ia^3\ldots}{\ldots}$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} + \frac{a(e+fx)\cos(c+dx)}{b^2d} + \frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d} - \frac{ia^3\ldots}{\ldots}$$

Mathematica [A] time = 8.03769, size = 752, normalized size = 1.97

$$\frac{8a^3d(e+fx)\left(-\frac{if\left(\text{PolyLog}\left(2,\frac{a\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a+i\left(\sqrt{b^2-a^2}+b\right)}\right)+\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{\sqrt{b^2-a^2}+a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{b^2-a^2}-ia+b}\right)\right)}{\sqrt{b^2-a^2}}+\frac{if\left(\text{PolyLog}\left(2,\frac{a\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a-i\left(\sqrt{b^2-a^2}+b\right)}\right)+\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\right.}{\sqrt{b^2-a^2}}\right.}{}$$

$$-\frac{}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(2*(2*a^2 + b^2)*(c + d*x)*(c*f - d*(2*e + f*x)) - 8*a*b*d*(e + f*x)*Cos[c
 + d*x] + b^2*f*Cos[2*(c + d*x)] + (8*a^3*d*(e + f*x)*((2*(d*e - c*f)*ArcTa
n[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*(Log[1
- I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)
*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a +
I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 + I*Tan[(c + d*
x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2
 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 +
 b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(-b + S
qrt[-a^2 + b^2] - a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2])] + PolyL
og[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]))/Sqrt[-a^2
+ b^2] - (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Ta
n[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] + PolyLog[2, (a + I*a*Tan[(c
+ d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2]))/(d*e - c*f
 + I*f*Log[1 - I*Tan[(c + d*x)/2]] - I*f*Log[1 + I*Tan[(c + d*x)/2]]) + 8*a
*b*f*Sin[c + d*x] + 2*b^2*d*(e + f*x)*Sin[2*(c + d*x)])/(8*b^3*d^2)
```

Maple [B] time = 0.43, size = 710, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2*a^2*f*x^2/b^3+1/4*f*x^2/b+a^2*e*x/b^3+1/2*e*x/b-1/16*I*(2*d*f*x-I*f+2*d
*e)/b/d^2*exp(-2*I*(d*x+c))+1/2*a*(d*f*x+I*f+d*e)/b^2/d^2*exp(I*(d*x+c))+1/
2*a*(d*f*x-I*f+d*e)/b^2/d^2*exp(-I*(d*x+c))-I*a^3/b^3/d^2*f/(-a^2+b^2)^(1/2
```

```
)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+2*I
*a^3/b^3/d^2*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a
^2+b^2)^(1/2))-a^3/b^3/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+
b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-a^3/b^3/d^2*f/(-a^2+b^2)^(1/2)*ln((I*
a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+a^3/b^3/d*f/
(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)
^(1/2)))*x+a^3/b^3/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^
2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-2*I*a^3/b^3/d*e/(-a^2+b^2)^(1/2)*arctan
(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+1/16*I*(2*d*f*x+I*f+2*d*e
)/b/d^2*exp(2*I*(d*x+c))+I*a^3/b^3/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(
I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.59615, size = 2938, normalized size = 7.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*I*a^3*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2
*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) + 2*b)/b + 1) - 2*I*a^3*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*co
s(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*I*a^3*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog(
-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*a^3*b*f*sqrt(-(a^2 - b
^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x
+ c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - (2*a^4 - a^
2*b^2 - b^4)*d^2*f*x^2 - 2*(2*a^4 - a^2*b^2 - b^4)*d^2*e*x + (a^2*b^2 - b^4
```

```
)*f*cos(d*x + c)^2 + 2*(a^3*b*d*e - a^3*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2
*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)
+ 2*(a^3*b*d*e - a^3*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2
*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(a^3*b*d*e - a^
3*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c)
+ 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(a^3*b*d*e - a^3*b*c*f)*sqrt(-(a^
2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 -
 b^2)/b^2) - 2*I*a) + 2*(a^3*b*d*f*x + a^3*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*lo
g(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(a^3*b*d*f*x + a^3*b*c*f)*sq
rt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*
cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(a^3*
b*d*f*x + a^3*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) +
2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2) + 2*b)/b) - 2*(a^3*b*d*f*x + a^3*b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2
*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 4*((a^3*b - a*b^3)*d*f*x + (a^3*b
- a*b^3)*d*e)*cos(d*x + c) + 2*(2*(a^3*b - a*b^3)*f + ((a^2*b^2 - b^4)*d*f*
x + (a^2*b^2 - b^4)*d*e)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\sin(dx+c)^3}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

3.231 $\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=107

$$-\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d\sqrt{a^2-b^2}}+\frac{x\left(2a^2+b^2\right)}{2b^3}+\frac{a\cos(c+dx)}{b^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) - (2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(b^3*Sqrt[a^2 - b^2]*d) + (a*Cos[c + d*x])/(b^2*d) - (Cos[c + d*x
]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.18554, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2793, 3023, 2735, 2660, 618, 204}

$$-\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d\sqrt{a^2-b^2}}+\frac{x\left(2a^2+b^2\right)}{2b^3}+\frac{a\cos(c+dx)}{b^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) - (2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(b^3*Sqrt[a^2 - b^2]*d) + (a*Cos[c + d*x])/(b^2*d) - (Cos[c + d*x
]*Sin[c + d*x])/(2*b*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
```

```
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{a+b\sin(c+dx)-2a\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx}{2b}$$
$$= \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{ab+(2a^2+b^2)\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b^2}$$
$$= \frac{(2a^2+b^2)x}{2b^3} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{a^3\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^3}$$
$$= \frac{(2a^2+b^2)x}{2b^3} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\right.}{b^3d}$$
$$= \frac{(2a^2+b^2)x}{2b^3} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{(4a^3)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2\right.}{b^3d}$$
$$= \frac{(2a^2+b^2)x}{2b^3} - \frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd}$$

Mathematica [A] time = 0.24686, size = 97, normalized size = 0.91

$$\frac{2(2a^2+b^2)(c+dx) - \frac{8a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 4ab\cos(c+dx) - b^2\sin(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*(2*a^2 + b^2)*(c + d*x) - (8*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/Sqrt[a^2 - b^2] + 4*a*b*Cos[c + d*x] - b^2*Sin[2*(c + d*x)])/(4*
b^3*d)
```

Maple [B] time = 0.028, size = 216, normalized size = 2.

$$\frac{1}{bd}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2} + 2\,\frac{(\tan(1/2\,dx+c/2))^2\,a}{b^2d\left(1+(\tan(1/2\,dx+c/2))^2\right)^2} - \frac{1}{bd}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+2/d/b^2/(1+tan(1/2*d*
x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a-1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1
/2*d*x+1/2*c)+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*a+2/d/b^3*arctan(tan(1/2*d
*x+1/2*c))*a^2+1/d/b*arctan(tan(1/2*d*x+1/2*c))-2/d*a^3/b^3/(a^2-b^2)^(1/2)
*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27219, size = 768, normalized size = 7.18

$$\left[-\frac{\sqrt{-a^2+b^2}a^3\log\left(-\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-\left(2a^4-a^2b^2-b^4\right)dx+\left(\right.}{2\left(a^2b^3-b^5\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a^3*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqr
t(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - (2*
a^4 - a^2*b^2 - b^4)*d*x + (a^2*b^2 - b^4)*cos(d*x + c)*sin(d*x + c) - 2*(a
^3*b - a*b^3)*cos(d*x + c))/((a^2*b^3 - b^5)*d), 1/2*(2*sqrt(a^2 - b^2)*a^3
*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (2*a^4 - a^
2*b^2 - b^4)*d*x - (a^2*b^2 - b^4)*cos(d*x + c)*sin(d*x + c) + 2*(a^3*b - a
*b^3)*cos(d*x + c))/((a^2*b^3 - b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12006, size = 204, normalized size = 1.91

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^3}{\sqrt{a^2-b^2}b^3}-\frac{(2a^2+b^2)(dx+c)}{b^3}-\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2a\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*a^3/(sqrt(a^2 - b^2)*b^3) - (2*a^2 + b^2)*(d*
x + c)/b^3 - 2*(b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 - b*t
an(1/2*d*x + 1/2*c) + 2*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

3.232 $\int \frac{(e+fx)^3 \csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=732

$$\frac{6ibf^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^3\sqrt{a^2-b^2}}-\frac{6ibf^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ad^3\sqrt{a^2-b^2}}+\frac{3bf(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^2\sqrt{a^2-b^2}}-\frac{3bf}{}$$

```
[Out] (-2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a*d) + (I*b*(e + f*x)^3*Log[1 -
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d) - (I*b*
(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a
^2 - b^2]*d) + ((3*I)*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2) -
 ((3*I)*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) + (3*b*f*(e + f*
x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 -
 b^2]*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[
a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d^2) - (6*f^2*(e + f*x)*PolyLog[3, -E^(I*(
c + d*x))])/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[3, E^(I*(c + d*x))])/(a*d^3)
 + ((6*I)*b*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
b^2])])/(a*Sqrt[a^2 - b^2]*d^3) - ((6*I)*b*f^2*(e + f*x)*PolyLog[3, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d^3) - ((6*I)*f^3
*PolyLog[4, -E^(I*(c + d*x))])/(a*d^4) + ((6*I)*f^3*PolyLog[4, E^(I*(c + d*
x))])/(a*d^4) - (6*b*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b
^2])])/(a*Sqrt[a^2 - b^2]*d^4) + (6*b*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/
(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d^4)
```

Rubi [A] time = 1.11721, antiderivative size = 732, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {4535, 4183, 2531, 6609, 2282, 6589, 3323, 2264, 2190}

$$\frac{6ibf^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^3\sqrt{a^2-b^2}}-\frac{6ibf^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ad^3\sqrt{a^2-b^2}}+\frac{3bf(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^2\sqrt{a^2-b^2}}-\frac{3bf}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```


```
[Out] (-2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a*d) + (I*b*(e + f*x)^3*Log[1 -
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d) - (I*b*
(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a
^2 - b^2]*d) + ((3*I)*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2) -
 ((3*I)*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) + (3*b*f*(e + f*
```

```
x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 -
 b^2]*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[
a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d^2) - (6*f^2*(e + f*x)*PolyLog[3, -E^(I*(
c + d*x))])/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[3, E^(I*(c + d*x))])/(a*d^3)
 + ((6*I)*b*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
b^2])])/(a*Sqrt[a^2 - b^2]*d^3) - ((6*I)*b*f^2*(e + f*x)*PolyLog[3, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d^3) - ((6*I)*f^3
*PolyLog[4, -E^(I*(c + d*x))])/(a*d^4) + ((6*I)*f^3*PolyLog[4, E^(I*(c + d*
x))])/(a*d^4) - (6*b*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b
^2])])/(a*Sqrt[a^2 - b^2]*d^4) + (6*b*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/
(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d^4)
```

Rule 4535

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csc[c + d*x]^(n - 1))/(a + b*Si
n[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3 \csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3 \csc(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(2b)\int \frac{e^{i(c+dx)}(e+fx)^3}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{a} - \frac{(3f)\int (e+fx)^2 \log\left(1-e^{\cdots}\right.}{ad}$$

$$= -\frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{3if(e+fx)^2 \mathrm{Li}_2\left(-e^{i(c+dx)}\right)}{ad^2} - \frac{3if(e+fx)^2 \mathrm{Li}_2\left(e^{i(c+dx)}\right)}{ad^2} + \cdots$$

$$= -\frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^3 \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d} - \frac{ib(e+fx)^3 \log\left(1-\frac{ibe^{i(c\cdots}}{a+\sqrt{a\cdots}}\right.}{a\sqrt{a^2-b^2}d}$$

$$= -\frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^3 \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d} - \frac{ib(e+fx)^3 \log\left(1-\frac{ibe^{i(c\cdots}}{a+\sqrt{a\cdots}}\right.}{a\sqrt{a^2-b^2}d}$$

$$= -\frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^3 \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d} - \frac{ib(e+fx)^3 \log\left(1-\frac{ibe^{i(c\cdots}}{a+\sqrt{a\cdots}}\right.}{a\sqrt{a^2-b^2}d}$$

$$= -\frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^3 \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d} - \frac{ib(e+fx)^3 \log\left(1-\frac{ibe^{i(c\cdots}}{a+\sqrt{a\cdots}}\right.}{a\sqrt{a^2-b^2}d}$$

$$= -\frac{2(e+fx)^3 \tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^3 \log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d} - \frac{ib(e+fx)^3 \log\left(1-\frac{ibe^{i(c\cdots}}{a+\sqrt{a\cdots}}\right.}{a\sqrt{a^2-b^2}d}$$

Mathematica [A] time = 2.60291, size = 894, normalized size = 1.22

$$\frac{-2d^3\tanh^{-1}(\cos(c+dx)+i\sin(c+dx))(e+fx)^3 + \frac{b\left(3d^2 f\,\mathrm{PolyLog}\left(2,-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}-a}\right)(e+fx)^2 + i\left(2ie^3\tan^{-1}\left(\frac{ia+be^{i(c+dx)}}{\sqrt{a^2-b^2}}\right)d^3 + f^3x^3\log\left(\frac{ie^{i(c+dx)}}{\sqrt{a^2-b^2\cdots}}\right.\right.\right.}{\cdots}}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*d^3*(e + f*x)^3*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] + (b*(3*d^2*f*(e
 + f*x)^2*PolyLog[2, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] + I*(
(2*I)*d^3*e^3*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 3*d^3*e^2
*f*x*Log[1 + (I*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] + 3*d^3*e*f^2*x^
2*Log[1 + (I*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] + d^3*f^3*x^3*Log[1
 + (I*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] - 3*d^3*e^2*f*x*Log[1 - (I
```

```
*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] - 3*d^3*e*f^2*x^2*Log[1 - (I*b*E
^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] - d^3*f^3*x^3*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])] + (3*I)*d^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] + 6*d*f^2*(e + f*x)*PolyLog[3, ((-I)*
b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] - 6*d*e*f^2*PolyLog[3, (I*b*E^(I
*(c + d*x)))/(a + Sqrt[a^2 - b^2])] - 6*d*f^3*x*PolyLog[3, (I*b*E^(I*(c + d
*x)))/(a + Sqrt[a^2 - b^2])] + (6*I)*f^3*PolyLog[4, ((-I)*b*E^(I*(c + d*x))
)/(-a + Sqrt[a^2 - b^2])] - (6*I)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a +
 Sqrt[a^2 - b^2])])))/Sqrt[a^2 - b^2] + (3*I)*f*(d^2*(e + f*x)^2*PolyLog[2,
 -Cos[c + d*x] - I*Sin[c + d*x]] + (2*I)*d*f*(e + f*x)*PolyLog[3, -Cos[c +
d*x] - I*Sin[c + d*x]] - 2*f^2*PolyLog[4, -Cos[c + d*x] - I*Sin[c + d*x]])
- (3*I)*f*(d^2*(e + f*x)^2*PolyLog[2, Cos[c + d*x] + I*Sin[c + d*x]] + (2*I
)*d*f*(e + f*x)*PolyLog[3, Cos[c + d*x] + I*Sin[c + d*x]] - 2*f^2*PolyLog[4
, Cos[c + d*x] + I*Sin[c + d*x]]))/(a*d^4)
```

Maple [F] time = 0.864, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3}\csc\left(dx+c\right)}{a+b\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.77588, size = 8263, normalized size = 11.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(-12*I*b^2*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(d*x +
c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2))/b) + 12*I*b^2*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*c
os(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2))/b) - 12*I*b^2*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I
*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2))/b) + 12*I*b^2*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I
*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2))/b) - 12*I*(a^2 - b^2)*f^3*polylog(4, cos(d*x + c) + I*s
in(d*x + c)) + 12*I*(a^2 - b^2)*f^3*polylog(4, cos(d*x + c) - I*sin(d*x + c
)) - 12*I*(a^2 - b^2)*f^3*polylog(4, -cos(d*x + c) + I*sin(d*x + c)) + 12*I
*(a^2 - b^2)*f^3*polylog(4, -cos(d*x + c) - I*sin(d*x + c)) + 2*(3*I*b^2*d^
2*f^3*x^2 + 6*I*b^2*d^2*e*f^2*x + 3*I*b^2*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)
*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*
b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(-3*I*b^2*d^2*f^3*
x^2 - 6*I*b^2*d^2*e*f^2*x - 3*I*b^2*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog
(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(-3*I*b^2*d^2*f^3*x^2 -
6*I*b^2*d^2*e*f^2*x - 3*I*b^2*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*
(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(3*I*b^2*d^2*f^3*x^2 + 6*I*b^
2*d^2*e*f^2*x + 3*I*b^2*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*
a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f +
3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c
) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(b^2*d^3*e
^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*sqrt(-(a^2 - b^2)
/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2
) - 2*I*a) - 2*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c
^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) +
 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f +
 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x +
 c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(b^2*d^3
*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*
b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*
x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^
```

2 - b^2)/b^2) + 2*b)/b) - 2*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 12*(b^2*d*f^3*x + b^2*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(b^2*d*f^3*x + b^2*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(b^2*d*f^3*x + b^2*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*(b^2*d*f^3*x + b^2*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + (6*I*(a^2 - b^2)*d^2*f^3*x^2 + 12*I*(a^2 - b^2)*d^2*e*f^2*x + 6*I*(a^2 - b^2)*d^2*e^2*f)*dilog(cos(d*x + c) + I*sin(d*x + c)) + (-6*I*(a^2 - b^2)*d^2*f^3*x^2 - 12*I*(a^2 - b^2)*d^2*e*f^2*x - 6*I*(a^2 - b^2)*d^2*e^2*f)*dilog(cos(d*x + c) - I*sin(d*x + c)) + (6*I*(a^2 - b^2)*d^2*f^3*x^2 + 12*I*(a^2 - b^2)*d^2*e*f^2*x + 6*I*(a^2 - b^2)*d^2*e^2*f)*dilog(-cos(d*x + c) + I*sin(d*x + c)) + (-6*I*(a^2 - b^2)*d^2*f^3*x^2 - 12*I*(a^2 - b^2)*d^2*e*f^2*x - 6*I*(a^2 - b^2)*d^2*e^2*f)*dilog(-cos(d*x + c) - I*sin(d*x + c)) + 2*((a^2 - b^2)*d^3*f^3*x^3 + 3*(a^2 - b^2)*d^3*e*f^2*x^2 + 3*(a^2 - b^2)*d^3*e^2*f*x + (a^2 - b^2)*d^3*e^3)*log(cos(d*x + c) + I*sin(d*x + c) + 1) + 2*((a^2 - b^2)*d^3*f^3*x^3 + 3*(a^2 - b^2)*d^3*e*f^2*x^2 + 3*(a^2 - b^2)*d^3*e^2*f*x + (a^2 - b^2)*d^3*e^3)*log(cos(d*x + c) - I*sin(d*x + c) + 1) - 2*((a^2 - b^2)*d^3*e^3 - 3*(a^2 - b^2)*c*d^2*e^2*f + 3*(a^2 - b^2)*c^2*d*e*f^2 - (a^2 - b^2)*c^3*f^3)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) - 2*((a^2 - b^2)*d^3*e^3 - 3*(a^2 - b^2)*c*d^2*e^2*f + 3*(a^2 - b^2)*c^2*d*e*f^2 - (a^2 - b^2)*c^3*f^3)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) - 2*((a^2 - b^2)*d^3*f^3*x^3 + 3*(a^2 - b^2)*d^3*e*f^2*x^2 + 3*(a^2 - b^2)*d^3*e^2*f*x + 3*(a^2 - b^2)*c*d^2*e^2*f - 3*(a^2 - b^2)*c^2*d*e*f^2 + (a^2 - b^2)*c^3*f^3)*log(-cos(d*x + c) + I*sin(d*x + c) + 1) - 2*((a^2 - b^2)*d^3*f^3*x^3 + 3*(a^2 - b^2)*d^3*e*f^2*x^2 + 3*(a^2 - b^2)*d^3*e^2*f*x + 3*(a^2 - b^2)*c*d^2*e^2*f - 3*(a^2 - b^2)*c^2*d*e*f^2 + (a^2 - b^2)*c^3*f^3)*log(-cos(d*x + c) - I*sin(d*x + c) + 1) - 12*((a^2 - b^2)*d*f^3*x + (a^2 - b^2)*d*e*f^2)*polylog(3, cos(d*x + c) + I*sin(d*x + c)) - 12*((a^2 - b^2)*d*f^3*x + (a^2 - b^2)*d*e*f^2)*polylog(3, cos(d*x + c) - I*sin(d*x + c)) + 12*((a^2 - b^2)*d*f^3*x + (a^2 - b^2)*d*e*f^2)*polylog(3, -cos(d*x + c) + I*sin(d*x + c)) + 12*((a^2 - b^2)*d*f^3*x + (a^2 - b^2)*d*e*f^2)*polylog(3, -cos(d*x

```
+ c) - I*sin(d*x + c)))/((a^3 - a*b^2)*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csc(d*x+c)/(a+b*sin(d*x+c)),x)

[Out] Integral((e + f*x)**3*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] Timed out
```

3.233 $\int \frac{(e+fx)^2 \csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=528

$$\frac{2bf(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^2\sqrt{a^2-b^2}}-\frac{2bf(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ad^2\sqrt{a^2-b^2}}+\frac{2ibf^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^3\sqrt{a^2-b^2}}-\frac{2ibf^2\text{PolyLog}}{ad^3\sqrt{a}}$$

```
[Out] (-2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d) + (I*b*(e + f*x)^2*Log[1 -
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d) - (I*b*
(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a
^2 - b^2]*d) + ((2*I)*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2) - (
(2*I)*f*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) + (2*b*f*(e + f*x)*P
olyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*
d^2) - (2*b*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^
2])])/(a*Sqrt[a^2 - b^2]*d^2) - (2*f^2*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3
) + (2*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3) + ((2*I)*b*f^2*PolyLog[3, (
I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d^3) - ((2*
I)*b*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a
^2 - b^2]*d^3)
```

Rubi [A] time = 0.946378, antiderivative size = 528, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4535, 4183, 2531, 2282, 6589, 3323, 2264, 2190}

$$\frac{2bf(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^2\sqrt{a^2-b^2}}-\frac{2bf(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ad^2\sqrt{a^2-b^2}}+\frac{2ibf^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^3\sqrt{a^2-b^2}}-\frac{2ibf^2\text{PolyLog}}{ad^3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```


```
[Out] (-2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d) + (I*b*(e + f*x)^2*Log[1 -
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d) - (I*b*
(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a
^2 - b^2]*d) + ((2*I)*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2) - (
(2*I)*f*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) + (2*b*f*(e + f*x)*P
olyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*
d^2) - (2*b*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^
2])])/(a*Sqrt[a^2 - b^2]*d^2) - (2*f^2*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3
) + (2*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3) + ((2*I)*b*f^2*PolyLog[3, (
```

```
I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d^3) - ((2*
I)*b*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a
^2 - b^2]*d^3)
```

Rule 4535

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csc[c + d*x]^(n - 1))/(a + b*Si
n[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\csc(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(2b)\int \frac{e^{i(c+dx)}(e+fx)^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{a} - \frac{(2f)\int (e+fx)\log\left(1-e^{i(\cdot}\right.}{ad}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{2if(e+fx)\mathrm{Li}_2\left(-e^{i(c+dx)}\right)}{ad^2} - \frac{2if(e+fx)\mathrm{Li}_2\left(e^{i(c+dx)}\right)}{ad^2} + \cdots$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d} - \frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c}}{a+\sqrt{a}}\right.}{a\sqrt{a^2-b^2}d}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d} - \frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c}}{a+\sqrt{a}}\right.}{a\sqrt{a^2-b^2}d}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d} - \frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c}}{a+\sqrt{a}}\right.}{a\sqrt{a^2-b^2}d}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d} - \frac{ib(e+fx)^2\log\left(1-\frac{ibe^{i(c}}{a+\sqrt{a}}\right.}{a\sqrt{a^2-b^2}d}$$

Mathematica [A] time = 1.68982, size = 573, normalized size = 1.09

$$\frac{\frac{b\left(i\left(2idf(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)+2f^2\text{PolyLog}\left(3,-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}-a}\right)-2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)+2id^2e^2\tan^{-1}\left(\frac{ia+be^{i(c+dx)}}{\sqrt{a^2-b^2}}\right)+2d^2efx\log\left(1+\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}-a}\right)-2d^2ef\cdots}{\sqrt{a^2-b^2}}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (d^2*(e + f*x)^2*Log[1 - E^(I*(c + d*x))] - d^2*(e + f*x)^2*Log[1 + E^(I*(c
 + d*x))] + (2*I)*d*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))] - (2*I)*d*f*(e
 + f*x)*PolyLog[2, E^(I*(c + d*x))] - 2*f^2*PolyLog[3, -E^(I*(c + d*x))] +
2*f^2*PolyLog[3, E^(I*(c + d*x))] + (b*(2*d*f*(e + f*x)*PolyLog[2, ((-I)*b*
E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] + I*((2*I)*d^2*e^2*ArcTan[(I*a + b
*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 2*d^2*e*f*x*Log[1 + (I*b*E^(I*(c + d*x
)))/(-a + Sqrt[a^2 - b^2])] + d^2*f^2*x^2*Log[1 + (I*b*E^(I*(c + d*x)))/(-a
 + Sqrt[a^2 - b^2])] - 2*d^2*e*f*x*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[
a^2 - b^2])] - d^2*f^2*x^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^
```

```
2])] + (2*I)*d*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
 b^2])] + 2*f^2*PolyLog[3, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])]
 - 2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])))/Sqrt[a^
2 - b^2])/(a*d^3)
```

Maple [F] time = 0.642, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2}\csc\left(dx+c\right)}{a+b\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.98515, size = 5696, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) -
2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2))/b) - 4*b^2*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x +
```

```
 c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2))/b) - 4*b^2*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x
+ c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^
2)/b^2))/b) + 4*b^2*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c
) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2))/b) + 4*(a^2 - b^2)*f^2*polylog(3, cos(d*x + c) + I*sin(d*x + c)) + 4*
(a^2 - b^2)*f^2*polylog(3, cos(d*x + c) - I*sin(d*x + c)) - 4*(a^2 - b^2)*f
^2*polylog(3, -cos(d*x + c) + I*sin(d*x + c)) - 4*(a^2 - b^2)*f^2*polylog(3
, -cos(d*x + c) - I*sin(d*x + c)) - 2*(2*I*b^2*d*f^2*x + 2*I*b^2*d*e*f)*sqr
t(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(
b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2
*(-2*I*b^2*d*f^2*x - 2*I*b^2*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*
a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(-2*I*b^2*d*f^2*x - 2*I*b^2*d*e*f)*
sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) +
 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)
 - 2*(2*I*b^2*d*f^2*x + 2*I*b^2*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-
2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f +
b^2*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x +
c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f +
 b^2*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x +
 c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f
+ b^2*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x
 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*
f + b^2*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d
*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(b^2*d^2*f^2*x^2 + 2*b^2*
d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*
I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x +
2*b^2*c*d*e*f - b^2*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x
+ c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) + 2*b)/b) - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*
f - b^2*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*
sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
 + 2*b)/b) + 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2
*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c
) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)
- (4*I*(a^2 - b^2)*d*f^2*x + 4*I*(a^2 - b^2)*d*e*f)*dilog(cos(d*x + c) + I*
sin(d*x + c)) - (-4*I*(a^2 - b^2)*d*f^2*x - 4*I*(a^2 - b^2)*d*e*f)*dilog(co
s(d*x + c) - I*sin(d*x + c)) - (4*I*(a^2 - b^2)*d*f^2*x + 4*I*(a^2 - b^2)*d
*e*f)*dilog(-cos(d*x + c) + I*sin(d*x + c)) - (-4*I*(a^2 - b^2)*d*f^2*x - 4
*I*(a^2 - b^2)*d*e*f)*dilog(-cos(d*x + c) - I*sin(d*x + c)) - 2*((a^2 - b^2
)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x + (a^2 - b^2)*d^2*e^2)*log(cos(d*x
+ c) + I*sin(d*x + c) + 1) - 2*((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2
```

```
*e*f*x + (a^2 - b^2)*d^2*e^2)*log(cos(d*x + c) - I*sin(d*x + c) + 1) + 2*((
a^2 - b^2)*d^2*e^2 - 2*(a^2 - b^2)*c*d*e*f + (a^2 - b^2)*c^2*f^2)*log(-1/2*
cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) + 2*((a^2 - b^2)*d^2*e^2 - 2*(a^2
- b^2)*c*d*e*f + (a^2 - b^2)*c^2*f^2)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x
 + c) + 1/2) + 2*((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x + 2*(a^
2 - b^2)*c*d*e*f - (a^2 - b^2)*c^2*f^2)*log(-cos(d*x + c) + I*sin(d*x + c)
+ 1) + 2*((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x + 2*(a^2 - b^2)
*c*d*e*f - (a^2 - b^2)*c^2*f^2)*log(-cos(d*x + c) - I*sin(d*x + c) + 1))/((
a^3 - a*b^2)*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e+fx\right)^2\csc\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.234 $\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=325

$$\frac{bf\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^2\sqrt{a^2-b^2}}-\frac{bf\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ad^2\sqrt{a^2-b^2}}+\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{if\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{ib(e+fx}{a}$$

```
[Out] (-2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) + (I*b*(e + f*x)*Log[1 - (I*b
*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d) - (I*b*(e +
 f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b
^2]*d) + (I*f*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2) - (I*f*PolyLog[2, E^(I*
(c + d*x))])/(a*d^2) + (b*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
- b^2])])/(a*Sqrt[a^2 - b^2]*d^2) - (b*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d^2)
```

Rubi [A] time = 0.615057, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4535, 4183, 2279, 2391, 3323, 2264, 2190}

$$\frac{bf\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ad^2\sqrt{a^2-b^2}}-\frac{bf\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ad^2\sqrt{a^2-b^2}}+\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{if\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}+\frac{ib(e+fx}{a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```


```
[Out] (-2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) + (I*b*(e + f*x)*Log[1 - (I*b
*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d) - (I*b*(e +
 f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b
^2]*d) + (I*f*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2) - (I*f*PolyLog[2, E^(I*
(c + d*x))])/(a*d^2) + (b*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
- b^2])])/(a*Sqrt[a^2 - b^2]*d^2) - (b*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d^2)
```


Rule 4535

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csc[c + d*x]^(n - 1))/(a + b*Si
n[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
```

```
, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\csc(c+dx)\,dx}{a} - \frac{b\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{(2b)\int \frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{a} - \frac{f\int \log\left(1-e^{i(c+dx)}\right)dx}{ad} + \cdots$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{\left(2ib^2\right)\int \frac{e^{i(c+dx)}(e+fx)}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{a\sqrt{a^2-b^2}} - \frac{\left(2ib^2\right)\int \frac{e^{i(c+dx)}(e+fx)}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+\cdots}}}{a\sqrt{a^2-b^2}}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right.}{a\sqrt{a^2-b^2}\,d}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right.}{a\sqrt{a^2-b^2}\,d}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{ib(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{ib(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right.}{a\sqrt{a^2-b^2}\,d}$$

Mathematica [B] time = 6.39248, size = 764, normalized size = 2.35

$$-\frac{bd(e+fx)\left(-\frac{if\left(\text{PolyLog}\left(2,\frac{a\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a+i\left(\sqrt{b^2-a^2}+b\right)}\right)+\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{\sqrt{b^2-a^2}+a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{b^2-a^2}-ia+b}\right)\right)}{\sqrt{b^2-a^2}}+\frac{if\left(\text{PolyLog}\left(2,\frac{a\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a-i\left(\sqrt{b^2-a^2}+b\right)}\right)+\log\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\cdots\right.}{\sqrt{b^2-a^2}}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (d*e*Log[Tan[(c + d*x)/2]] - c*f*Log[Tan[(c + d*x)/2]] + f*((c + d*x)*(Log[
1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c +
 d*x))] - PolyLog[2, E^(I*(c + d*x))])) - (b*d*(e + f*x)*((2*(d*e - c*f)*Ar
cTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*(Log
[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((
-I)*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a
 + I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 + I*Tan[(c +
```

```
 d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-
a^2 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^
2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(-b
+ Sqrt[-a^2 + b^2] - a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2])] + Po
lyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]))/Sqrt[-a
^2 + b^2] - (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a
*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] + PolyLog[2, (a + I*a*Tan[
(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2]))/(d*e -
c*f + I*f*Log[1 - I*Tan[(c + d*x)/2]] - I*f*Log[1 + I*Tan[(c + d*x)/2]]))/(
a*d^2)
```

Maple [B] time = 0.169, size = 660, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -2*I/d*e/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b
^2)^(1/2))-1/d/a*e*ln(exp(I*(d*x+c))+1)+1/d/a*e*ln(exp(I*(d*x+c))-1)-1/d^2/
a*f*c*ln(exp(I*(d*x+c))-1)-1/d*f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+
c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-1/d^2*f*b/a/(-a^2+b^2)^(1/2
)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+1/d*
f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^
2+b^2)^(1/2)))*x+1/d^2*f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^
2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+I/d^2*f*b/a/(-a^2+b^2)^(1/2)*dilog(
(I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-I/d^2*f*b/a
/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+
b^2)^(1/2)))+I/d^2*f/a*dilog(exp(I*(d*x+c)))-1/d/a*ln(exp(I*(d*x+c))+1)*f*x
+I/d^2*f/a*dilog(exp(I*(d*x+c))+1)+2*I/d^2*f*c/a*b/(-a^2+b^2)^(1/2)*arctan(
1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.9389, size = 3510, normalized size = 10.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*I*b^2*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a
*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
) + 2*b)/b + 1) - 2*I*b^2*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*
x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2) + 2*b)/b + 1) - 2*I*b^2*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(
-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*b^2*f*sqrt(-(a^2 - b^2)/b^2)
*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*(a^2 - b^2)*f*d
ilog(cos(d*x + c) + I*sin(d*x + c)) - 2*I*(a^2 - b^2)*f*dilog(cos(d*x + c)
- I*sin(d*x + c)) + 2*I*(a^2 - b^2)*f*dilog(-cos(d*x + c) + I*sin(d*x + c))
 - 2*I*(a^2 - b^2)*f*dilog(-cos(d*x + c) - I*sin(d*x + c)) + 2*(b^2*d*e - b
^2*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) +
2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(b^2*d*e - b^2*c*f)*sqrt(-(a^2 - b^
2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b
^2) - 2*I*a) - 2*(b^2*d*e - b^2*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*
x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(b^2*
d*e - b^2*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x
 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(b^2*d*f*x + b^2*c*f)*sqrt(
-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos
(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(b^2*d*f
*x + b^2*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(
d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2
*b)/b) + 2*(b^2*d*f*x + b^2*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos
(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) + 2*b)/b) - 2*(b^2*d*f*x + b^2*c*f)*sqrt(-(a^2 - b^2)/b^2)
*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*((a^2 - b^2)*d*f*x + (a^
2 - b^2)*d*e)*log(cos(d*x + c) + I*sin(d*x + c) + 1) + 2*((a^2 - b^2)*d*f*x
 + (a^2 - b^2)*d*e)*log(cos(d*x + c) - I*sin(d*x + c) + 1) - 2*((a^2 - b^2)
*d*e - (a^2 - b^2)*c*f)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) -
 2*((a^2 - b^2)*d*e - (a^2 - b^2)*c*f)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*
```

```
x + c) + 1/2) - 2*((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(-cos(d*x + c) +
 I*sin(d*x + c) + 1) - 2*((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(-cos(d*x
 + c) - I*sin(d*x + c) + 1))/((a^3 - a*b^2)*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.235 $\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=67

$$-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] (-2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*Sqrt[a^2 - b^2]*
d) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0829515, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2747, 3770, 2660, 618, 204}

$$-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*Sqrt[a^2 - b^2]*
d) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\int \csc(c+dx)\,dx}{a} - \frac{b\int \frac{1}{a+b\sin(c+dx)}\,dx}{a}\\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{(2b)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{(4b)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\\
&= -\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0700474, size = 77, normalized size = 1.15

$$\frac{-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] -
Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]])/(a*d)
```

Maple [A] time = 0.001, size = 69, normalized size = 1.

$$-2\,{\frac {b}{da\sqrt {{a}^{2}-{b}^{2}}}\arctan \left(1/2\,{\frac {2\,a\tan \left(1/2\,dx+c/2 \right) +2\,b}{\sqrt {{a}^{2}-{b}^{2}}}} \right) }+{\frac {1}{da}\ln \left(\tan \left({\frac {dx}{2}}+{\frac {c}{2}} \right) \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/d*b/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^
(1/2))+1/a/d*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.75549, size = 698, normalized size = 10.42

$$\left[-\frac{\sqrt{-a^2+b^2}b\log\left(-\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+\left(a^2-b^2\right)\log\left(\frac{1}{2}\cos(dx\right.}{2\left(a^3-ab^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*b*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*
x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(
```

```
-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + (a^2
- b^2)*log(1/2*cos(d*x + c) + 1/2) - (a^2 - b^2)*log(-1/2*cos(d*x + c) + 1/
2))/((a^3 - a*b^2)*d), 1/2*(2*sqrt(a^2 - b^2)*b*arctan(-(a*sin(d*x + c) + b
)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^2 - b^2)*log(1/2*cos(d*x + c) + 1/2)
 + (a^2 - b^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^3 - a*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.49542, size = 112, normalized size = 1.67

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b}{\sqrt{a^2-b^2}\,a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*b/(sqrt(a^2 - b^2)*a) - log(abs(tan(1/2*d*x + 1/2
*c)))/a)/d
```

3.236 $\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=882

$$\frac{3\text{PolyLog}\left(3,e^{2i(c+dx)}\right)f^3}{2ad^4}+\frac{6ib\text{PolyLog}\left(4,-e^{i(c+dx)}\right)f^3}{a^2d^4}-\frac{6ib\text{PolyLog}\left(4,e^{i(c+dx)}\right)f^3}{a^2d^4}+\frac{6b^2\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f^3}{a^2\sqrt{a^2-b^2}d^4}$$

```
[Out] ((-I)*(e + f*x)^3)/(a*d) + (2*b*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a^2*
d) - ((e + f*x)^3*Cot[c + d*x])/(a*d) - (I*b^2*(e + f*x)^3*Log[1 - (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d) + (I*b^2*(e +
 f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2
 - b^2]*d) + (3*f*(e + f*x)^2*Log[1 - E^((2*I)*(c + d*x))])/(a*d^2) - ((3*I
)*b*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))])/(a^2*d^2) + ((3*I)*b*f*(e +
 f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a^2*d^2) - (3*b^2*f*(e + f*x)^2*PolyL
og[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^
2) + (3*b^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
b^2])])/(a^2*Sqrt[a^2 - b^2]*d^2) - ((3*I)*f^2*(e + f*x)*PolyLog[2, E^((2*I
)*(c + d*x))])/(a*d^3) + (6*b*f^2*(e + f*x)*PolyLog[3, -E^(I*(c + d*x))])/(
a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, E^(I*(c + d*x))])/(a^2*d^3) - ((6*
I)*b^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])
])/(a^2*Sqrt[a^2 - b^2]*d^3) + ((6*I)*b^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^3) + (3*f^3*Po
lyLog[3, E^((2*I)*(c + d*x))])/(2*a*d^4) + ((6*I)*b*f^3*PolyLog[4, -E^(I*(c
 + d*x))])/(a^2*d^4) - ((6*I)*b*f^3*PolyLog[4, E^(I*(c + d*x))])/(a^2*d^4)
+ (6*b^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*
Sqrt[a^2 - b^2]*d^4) - (6*b^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqr
t[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^4)
```

Rubi [A] time = 1.55123, antiderivative size = 882, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {4535, 4184, 3717, 2190, 2531, 2282, 6589, 4183, 6609, 3323, 2264}

$$\frac{3\text{PolyLog}\left(3,e^{2i(c+dx)}\right)f^3}{2ad^4}+\frac{6ib\text{PolyLog}\left(4,-e^{i(c+dx)}\right)f^3}{a^2d^4}-\frac{6ib\text{PolyLog}\left(4,e^{i(c+dx)}\right)f^3}{a^2d^4}+\frac{6b^2\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f^3}{a^2\sqrt{a^2-b^2}d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-I)*(e + f*x)^3)/(a*d) + (2*b*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a^2*
d) - ((e + f*x)^3*Cot[c + d*x])/(a*d) - (I*b^2*(e + f*x)^3*Log[1 - (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d) + (I*b^2*(e +
 f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2
 - b^2]*d) + (3*f*(e + f*x)^2*Log[1 - E^((2*I)*(c + d*x))])/(a*d^2) - ((3*I
)*b*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))])/(a^2*d^2) + ((3*I)*b*f*(e +
 f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a^2*d^2) - (3*b^2*f*(e + f*x)^2*PolyL
og[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^
2) + (3*b^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
b^2])])/(a^2*Sqrt[a^2 - b^2]*d^2) - ((3*I)*f^2*(e + f*x)*PolyLog[2, E^((2*I
)*(c + d*x))])/(a*d^3) + (6*b*f^2*(e + f*x)*PolyLog[3, -E^(I*(c + d*x))])/(
a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, E^(I*(c + d*x))])/(a^2*d^3) - ((6*
I)*b^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])
])/(a^2*Sqrt[a^2 - b^2]*d^3) + ((6*I)*b^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^3) + (3*f^3*Po
lyLog[3, E^((2*I)*(c + d*x))])/(2*a*d^4) + ((6*I)*b*f^3*PolyLog[4, -E^(I*(c
 + d*x))])/(a^2*d^4) - ((6*I)*b*f^3*PolyLog[4, E^(I*(c + d*x))])/(a^2*d^4)
+ (6*b^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*
Sqrt[a^2 - b^2]*d^4) - (6*b^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqr
t[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^4)
```

Rule 4535

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csc[c + d*x]^(n - 1))/(a + b*Si
n[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
```

```
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\csc^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^3\cot(c+dx)}{ad} - \frac{b\int (e+fx)^3\csc(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx}{a^2} + \frac{(3f)\int(e+\ldots}{}$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^3\cot(c+dx)}{ad} + \frac{\left(2b^2\right)\int \frac{e^{i(c+dx)}(e+\ldots}{ib+2ae^{i(c+dx)}-\ldots}}{a^2}$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^3\cot(c+dx)}{ad} + \frac{3f(e+fx)^2\log\left(1-\ldots\right.}{ad^2}$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^3\cot(c+dx)}{ad} - \frac{ib^2(e+fx)^3\log\left(1-\ldots\right.}{a^2\sqrt{a^2-b^2}\ldots}$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^3\cot(c+dx)}{ad} - \frac{ib^2(e+fx)^3\log\left(1-\ldots\right.}{a^2\sqrt{a^2-b^2}\ldots}$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^3\cot(c+dx)}{ad} - \frac{ib^2(e+fx)^3\log\left(1-\ldots\right.}{a^2\sqrt{a^2-b^2}\ldots}$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^3\cot(c+dx)}{ad} - \frac{ib^2(e+fx)^3\log\left(1-\ldots\right.}{a^2\sqrt{a^2-b^2}\ldots}$$

$$= -\frac{i(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^3\cot(c+dx)}{ad} - \frac{ib^2(e+fx)^3\log\left(1-\ldots\right.}{a^2\sqrt{a^2-b^2}\ldots}$$

Mathematica [A] time = 43.0345, size = 1680, normalized size = 1.9

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((-2*I)*a*d^3*(e + f*x)^3)/(-1 + E^((2*I)*c)) - 3*d^2*e*f*(b*d*e - 2*a*f)*
x*Log[1 - E^((-I)*(c + d*x))] - 3*d^2*f^2*(b*d*e - a*f)*x^2*Log[1 - E^((-I)
*(c + d*x))] - b*d^3*f^3*x^3*Log[1 - E^((-I)*(c + d*x))] + 3*d^2*e*f*(b*d*e
 + 2*a*f)*x*Log[1 + E^((-I)*(c + d*x))] + 3*d^2*f^2*(b*d*e + a*f)*x^2*Log[1
 + E^((-I)*(c + d*x))] + b*d^3*f^3*x^3*Log[1 + E^((-I)*(c + d*x))] + I*d^2*
e^2*(b*d*e - 3*a*f)*(d*x + I*Log[1 - E^(I*(c + d*x))]) + d^2*e^2*(b*d*e + 3
*a*f)*((-I)*d*x + Log[1 + E^(I*(c + d*x))]) + (3*I)*d*e*f*(b*d*e + 2*a*f)*P
olyLog[2, -E^((-I)*(c + d*x))] - (3*I)*d*e*f*(b*d*e - 2*a*f)*PolyLog[2, E^(
(-I)*(c + d*x))] + 6*f^2*(b*d*e + a*f)*(I*d*x*PolyLog[2, -E^((-I)*(c + d*x)
)] + PolyLog[3, -E^((-I)*(c + d*x))]) + 6*f^2*(-(b*d*e) + a*f)*(I*d*x*PolyL
og[2, E^((-I)*(c + d*x))] + PolyLog[3, E^((-I)*(c + d*x))]) + 3*b*f^3*(I*d^
2*x^2*PolyLog[2, -E^((-I)*(c + d*x))] + 2*d*x*PolyLog[3, -E^((-I)*(c + d*x)
)] - (2*I)*PolyLog[4, -E^((-I)*(c + d*x))]) - (3*I)*b*f^3*(d^2*x^2*PolyLog[
2, E^((-I)*(c + d*x))] - (2*I)*d*x*PolyLog[3, E^((-I)*(c + d*x))] - 2*PolyL
og[4, E^((-I)*(c + d*x))]))/(a^2*d^4) + (b^2*(2*Sqrt[-a^2 + b^2]*d^3*e^3*Ar
cTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 3*Sqrt[a^2 - b^2]*d^3*e^2
*f*x*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 3*Sqrt[a^2
- b^2]*d^3*e*f^2*x^2*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2]
)] + Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt
[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 + (b*E^(I*(c + d*x)))/
(I*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(I
*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1
+ (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - (3*I)*Sqrt[a^2 - b^2]*d^2
*f*(e + f*x)^2*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])]
+ (3*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(I*(c + d*x)))/
(I*a + Sqrt[-a^2 + b^2]))] + 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, (b*E^(I*(
c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog
[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 6*Sqrt[a^2 - b^2]*d*
e*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] - 6*Sqrt[
a^2 - b^2]*d*f^3*x*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]
))] + (6*I)*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*E^(I*(c + d*x)))/((-I)*a + Sq
rt[-a^2 + b^2])] - (6*I)*Sqrt[a^2 - b^2]*f^3*PolyLog[4, -((b*E^(I*(c + d*x)
))/(I*a + Sqrt[-a^2 + b^2]))]))/(a^2*Sqrt[-(a^2 - b^2)^2]*d^4) + (Csc[c/2]*
Csc[c/2 + (d*x)/2]*(e^3*Sin[(d*x)/2] + 3*e^2*f*x*Sin[(d*x)/2] + 3*e*f^2*x^2
*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/2]))/(2*a*d) + (Sec[c/2]*Sec[c/2 + (d*x)/
2]*(e^3*Sin[(d*x)/2] + 3*e^2*f*x*Sin[(d*x)/2] + 3*e*f^2*x^2*Sin[(d*x)/2] +
```

```
f^3*x^3*Sin[(d*x)/2]))/(2*a*d)
```

Maple [F] time = 2.592, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\csc(dx+c))^2}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.45504, size = 10330, normalized size = 11.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(12*I*b^3*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(d*x + c
) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2))/b)*sin(d*x + c) - 12*I*b^3*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4,
1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 12*I*b^3*f^3*sqrt(-(a^2 -
 b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c)
```

```
 - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 12*I*b^3*f^3
*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b
*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) +
 12*I*(a^2*b - b^3)*f^3*polylog(4, cos(d*x + c) + I*sin(d*x + c))*sin(d*x +
 c) - 12*I*(a^2*b - b^3)*f^3*polylog(4, cos(d*x + c) - I*sin(d*x + c))*sin(
d*x + c) + 12*I*(a^2*b - b^3)*f^3*polylog(4, -cos(d*x + c) + I*sin(d*x + c)
)*sin(d*x + c) - 12*I*(a^2*b - b^3)*f^3*polylog(4, -cos(d*x + c) - I*sin(d*
x + c))*sin(d*x + c) + 2*(-3*I*b^3*d^2*f^3*x^2 - 6*I*b^3*d^2*e*f^2*x - 3*I*
b^3*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*
sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
 + 2*b)/b + 1)*sin(d*x + c) + 2*(3*I*b^3*d^2*f^3*x^2 + 6*I*b^3*d^2*e*f^2*x
+ 3*I*b^3*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c)
+ 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) + 2*b)/b + 1)*sin(d*x + c) + 2*(3*I*b^3*d^2*f^3*x^2 + 6*I*b^3*d^2*e*
f^2*x + 3*I*b^3*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*
x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) + 2*(-3*I*b^3*d^2*f^3*x^2 - 6*I*b^
3*d^2*e*f^2*x - 3*I*b^3*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*
a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) - 2*(b^3*d^3*e^3 - 3*b^3*c
*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*
b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*s
in(d*x + c) - 2*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*
c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) +
 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) + 2*(b^3*d^3*e^3 - 3*b^3*
c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(-
2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)
*sin(d*x + c) + 2*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^
3*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c
) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) - 2*(b^3*d^3*f^3*x^3 +
 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*
e*f^2 + b^3*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2
*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) + 2*b)/b)*sin(d*x + c) + 2*(b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b
^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*sqrt(
-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos
(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c)
 - 2*(b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d
^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*
(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) + 2*(b^3*d^3*f^3*x^3 + 3
*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*
f^2 + b^3*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*
a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2) + 2*b)/b)*sin(d*x + c) + 12*(b^3*d*f^3*x + b^3*d*e*f^2)*sqrt(-(a^2 - b^2
```

)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 12*(b^3*d*f^3*x + b^3*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 12*(b^3*d*f^3*x + b^3*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 12*(b^3*d*f^3*x + b^3*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + (-6*I*(a^2*b - b^3)*d^2*f^3*x^2 - 6*I*(a^2*b - b^3)*d^2*e^2*f + 12*I*(a^3 - a*b^2)*d*e*f^2 - 12*I*((a^2*b - b^3)*d^2*e*f^2 - (a^3 - a*b^2)*d*f^3)*x)*dilog(cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + (6*I*(a^2*b - b^3)*d^2*f^3*x^2 + 6*I*(a^2*b - b^3)*d^2*e^2*f - 12*I*(a^3 - a*b^2)*d*e*f^2 + 12*I*((a^2*b - b^3)*d^2*e*f^2 - (a^3 - a*b^2)*d*f^3)*x)*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + (-6*I*(a^2*b - b^3)*d^2*f^3*x^2 - 6*I*(a^2*b - b^3)*d^2*e^2*f - 12*I*(a^3 - a*b^2)*d*e*f^2 - 12*I*((a^2*b - b^3)*d^2*e*f^2 + (a^3 - a*b^2)*d*f^3)*x)*dilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + (6*I*(a^2*b - b^3)*d^2*f^3*x^2 + 6*I*(a^2*b - b^3)*d^2*e^2*f + 12*I*(a^3 - a*b^2)*d*e*f^2 + 12*I*((a^2*b - b^3)*d^2*e*f^2 + (a^3 - a*b^2)*d*f^3)*x)*dilog(-cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - 2*((a^2*b - b^3)*d^3*f^3*x^3 + (a^2*b - b^3)*d^3*e^3 + 3*(a^3 - a*b^2)*d^2*e^2*f + 3*((a^2*b - b^3)*d^3*e*f^2 + (a^3 - a*b^2)*d^2*f^3)*x^2 + 3*((a^2*b - b^3)*d^3*e^2*f + 2*(a^3 - a*b^2)*d^2*e*f^2)*x)*log(cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) - 2*((a^2*b - b^3)*d^3*f^3*x^3 + (a^2*b - b^3)*d^3*e^3 + 3*(a^3 - a*b^2)*d^2*e^2*f + 3*((a^2*b - b^3)*d^3*e*f^2 + (a^3 - a*b^2)*d^2*f^3)*x^2 + 3*((a^2*b - b^3)*d^3*e^2*f + 2*(a^3 - a*b^2)*d^2*e*f^2)*x)*log(cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) + 2*((a^2*b - b^3)*d^3*e^3 - 3*(a^3 - a*b^2 + (a^2*b - b^3)*c)*d^2*e^2*f + 3*((a^2*b - b^3)*c^2 + 2*(a^3 - a*b^2)*c)*d*e*f^2 - ((a^2*b - b^3)*c^3 + 3*(a^3 - a*b^2)*c^2)*f^3)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + 2*((a^2*b - b^3)*d^3*e^3 - 3*(a^3 - a*b^2 + (a^2*b - b^3)*c)*d^2*e^2*f + 3*((a^2*b - b^3)*c^2 + 2*(a^3 - a*b^2)*c)*d*e*f^2 - ((a^2*b - b^3)*c^3 + 3*(a^3 - a*b^2)*c^2)*f^3)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + 2*((a^2*b - b^3)*d^3*f^3*x^3 + 3*(a^2*b - b^3)*c*d^2*e^2*f - 3*((a^2*b - b^3)*c^2 + 2*(a^3 - a*b^2)*c)*d*e*f^2 + ((a^2*b - b^3)*c^3 + 3*(a^3 - a*b^2)*c^2)*f^3 + 3*((a^2*b - b^3)*d^3*e*f^2 - (a^3 - a*b^2)*d^2*f^3)*x^2 + 3*((a^2*b - b^3)*d^3*e^2*f - 2*(a^3 - a*b^2)*d^2*e*f^2)*x)*log(-cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) + 2*((a^2*b - b^3)*d^3*f^3*x^3 + 3*(a^2*b - b^3)*c*d^2*e^2*f - 3*((a^2*b - b^3)*c^2 + 2*(a^3 - a*b^2)*c)*d*e*f^2 + ((a^2*b - b^3)*c^3 + 3*(a^3 - a*b^2)*c^2)*f^3 + 3*((a^2*b - b^3)*d^3*e*f^2 - (a^3 - a*b^2)*d^2*f^3)*x^2 + 3*((a^2*b - b^3)*d^3*e^2*f - 2*(a^3 - a*b^2)*d^2*e*f^2)*x)*log(-cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) + 12*((a^2*b - b^3)*d*f^3*x + (a^2*b - b^3)*d*e*f^2 - (a^3 - a*b^2)*f^3)*polylog(3, cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 12*((a^2*b - b^3)*d*f^3*x + (a^2*b - b^3)*d*e*f^2 - (a^3 - a*b^2)*f^3)*polylog(3, cos(d*x + c) - I*s

```
in(d*x + c))*sin(d*x + c) - 12*((a^2*b - b^3)*d*f^3*x + (a^2*b - b^3)*d*e*f
^2 + (a^3 - a*b^2)*f^3)*polylog(3, -cos(d*x + c) + I*sin(d*x + c))*sin(d*x
+ c) - 12*((a^2*b - b^3)*d*f^3*x + (a^2*b - b^3)*d*e*f^2 + (a^3 - a*b^2)*f^
3)*polylog(3, -cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 4*((a^3 - a*b^
2)*d^3*f^3*x^3 + 3*(a^3 - a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 - a*b^2)*d^3*e^2*f*
x + (a^3 - a*b^2)*d^3*e^3)*cos(d*x + c))/((a^4 - a^2*b^2)*d^4*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^3 \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.237 $\int \frac{(e+fx)^2\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=639

$$-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2d^2\sqrt{a^2-b^2}}+\frac{2b^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2d^2\sqrt{a^2-b^2}}-\frac{2ib^2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2d^3\sqrt{a^2-b^2}}+\frac{2ib^2f^2\text{Po}}{a}$$

```
[Out] ((-I)*(e + f*x)^2)/(a*d) + (2*b*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a^2*
d) - ((e + f*x)^2*Cot[c + d*x])/(a*d) - (I*b^2*(e + f*x)^2*Log[1 - (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d) + (I*b^2*(e +
 f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2
 - b^2]*d) + (2*f*(e + f*x)*Log[1 - E^((2*I)*(c + d*x))])/(a*d^2) - ((2*I)*
b*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a^2*d^2) + ((2*I)*b*f*(e + f*x
)*PolyLog[2, E^(I*(c + d*x))])/(a^2*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, (I
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^2) + (2*
b^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a
^2*Sqrt[a^2 - b^2]*d^2) - (I*f^2*PolyLog[2, E^((2*I)*(c + d*x))])/(a*d^3) +
 (2*b*f^2*PolyLog[3, -E^(I*(c + d*x))])/(a^2*d^3) - (2*b*f^2*PolyLog[3, E^(
I*(c + d*x))])/(a^2*d^3) - ((2*I)*b^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/
(a - Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^3) + ((2*I)*b^2*f^2*PolyLog[
3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^3)
```

Rubi [A] time = 1.20559, antiderivative size = 639, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4535, 4184, 3717, 2190, 2279, 2391, 4183, 2531, 2282, 6589, 3323, 2264}

$$-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2d^2\sqrt{a^2-b^2}}+\frac{2b^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2d^2\sqrt{a^2-b^2}}-\frac{2ib^2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2d^3\sqrt{a^2-b^2}}+\frac{2ib^2f^2\text{Po}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-I)*(e + f*x)^2)/(a*d) + (2*b*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a^2*
d) - ((e + f*x)^2*Cot[c + d*x])/(a*d) - (I*b^2*(e + f*x)^2*Log[1 - (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d) + (I*b^2*(e +
 f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2
 - b^2]*d) + (2*f*(e + f*x)*Log[1 - E^((2*I)*(c + d*x))])/(a*d^2) - ((2*I)*
b*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a^2*d^2) + ((2*I)*b*f*(e + f*x
)*PolyLog[2, E^(I*(c + d*x))])/(a^2*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, (I
```

```
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^2) + (2*
b^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a
^2*Sqrt[a^2 - b^2]*d^2) - (I*f^2*PolyLog[2, E^((2*I)*(c + d*x))])/(a*d^3) +
 (2*b*f^2*PolyLog[3, -E^(I*(c + d*x))])/(a^2*d^3) - (2*b*f^2*PolyLog[3, E^(
I*(c + d*x))])/(a^2*d^3) - ((2*I)*b^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/
(a - Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^3) + ((2*I)*b^2*f^2*PolyLog[
3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^3)
```

Rule 4535

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csc[c + d*x]^(n - 1))/(a + b*Si
n[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
```

```
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\csc^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^2\cot(c+dx)}{ad} - \frac{b\int (e+fx)^2\csc(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{a^2} + \frac{(2f)\int (e+$$

$$= -\frac{i(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^2\cot(c+dx)}{ad} + \frac{\left(2b^2\right)\int \frac{e^{i(c+dx)}(}{ib+2ae^{i(c+dx)}}}{a^2}$$

$$= -\frac{i(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^2\cot(c+dx)}{ad} + \frac{2f(e+fx)\log\left(1\right.}{ad^2}$$

$$= -\frac{i(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^2\cot(c+dx)}{ad} - \frac{ib^2(e+fx)^2\log\left(1\right.}{a^2\sqrt{a^2-}}$$

$$= -\frac{i(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^2\cot(c+dx)}{ad} - \frac{ib^2(e+fx)^2\log\left(1\right.}{a^2\sqrt{a^2-}}$$

$$= -\frac{i(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^2\cot(c+dx)}{ad} - \frac{ib^2(e+fx)^2\log\left(1\right.}{a^2\sqrt{a^2-}}$$

$$= -\frac{i(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)^2\cot(c+dx)}{ad} - \frac{ib^2(e+fx)^2\log\left(1\right.}{a^2\sqrt{a^2-}}$$

Mathematica [A] time = 11.9213, size = 911, normalized size = 1.43

$$\frac{i\left(-2\sqrt{a^2-b^2}df(e+fx)\text{PolyLog}\left(2,\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}-ia}\right)+2\sqrt{a^2-b^2}df(e+fx)\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{ia+\sqrt{b^2-a^2}}\right)-i\left(\left(2\sqrt{b^2-a^2}\tan^{-1}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((-2*I)*a*d^2*(e + f*x)^2)/(-1 + E^((2*I)*c)) - 2*d*f*(b*d*e - a*f)*x*Log[
1 - E^((-I)*(c + d*x))] - b*d^2*f^2*x^2*Log[1 - E^((-I)*(c + d*x))] + 2*d*f
```

```
*(b*d*e + a*f)*x*Log[1 + E^((-I)*(c + d*x))] + b*d^2*f^2*x^2*Log[1 + E^((-I
)*(c + d*x))] + I*d*e*(b*d*e - 2*a*f)*(d*x + I*Log[1 - E^(I*(c + d*x))]) +
d*e*(b*d*e + 2*a*f)*((-I)*d*x + Log[1 + E^(I*(c + d*x))]) + (2*I)*f*(b*d*e
+ a*f)*PolyLog[2, -E^((-I)*(c + d*x))] + (2*I)*f*(-(b*d*e) + a*f)*PolyLog[2
, E^((-I)*(c + d*x))] + 2*b*f^2*(I*d*x*PolyLog[2, -E^((-I)*(c + d*x))] + Po
lyLog[3, -E^((-I)*(c + d*x))]) - (2*I)*b*f^2*(d*x*PolyLog[2, E^((-I)*(c + d
*x))] - I*PolyLog[3, E^((-I)*(c + d*x))]))/(a^2*d^3) + (I*b^2*(-2*Sqrt[a^2
- b^2]*d*f*(e + f*x)*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b
^2])] + 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*E^(I*(c + d*x)))/(I
*a + Sqrt[-a^2 + b^2]))] - I*(d^2*(2*Sqrt[-a^2 + b^2]*e^2*ArcTan[(I*a + b*E
^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(Log[1 -
 (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - Log[1 + (b*E^(I*(c + d*
x)))/(I*a + Sqrt[-a^2 + b^2])])) + 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*E^(I
*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3
, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))])))/(a^2*Sqrt[-(a^2 - b^2
)^2]*d^3) + (Csc[c/2]*Csc[c/2 + (d*x)/2]*(e^2*Sin[(d*x)/2] + 2*e*f*x*Sin[(d
*x)/2] + f^2*x^2*Sin[(d*x)/2]))/(2*a*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]*(e^2
*Sin[(d*x)/2] + 2*e*f*x*Sin[(d*x)/2] + f^2*x^2*Sin[(d*x)/2]))/(2*a*d)
```

Maple [F] time = 2.214, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2}\left(\csc\left(dx+c\right)\right)^{2}}{a+b\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.9288, size = 6947, normalized size = 10.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) -
 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2))/b)*sin(d*x + c) - 4*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2
*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 4*b^3*f^2*sqrt(-(a^2 - b^2)/b^2
)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 4*b^3*f^2*sqrt(-(a^2
- b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c
) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 4*(a^2*b -
b^3)*f^2*polylog(3, cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 4*(a^2*b
- b^3)*f^2*polylog(3, cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - 4*(a^2*
b - b^3)*f^2*polylog(3, -cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - 4*(a
^2*b - b^3)*f^2*polylog(3, -cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 2
*(-2*I*b^3*d*f^2*x - 2*I*b^3*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*
a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) + 2*(2*I*b^3*d*f^2*x + 2*I
*b^3*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin
(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) +
2*b)/b + 1)*sin(d*x + c) + 2*(2*I*b^3*d*f^2*x + 2*I*b^3*d*e*f)*sqrt(-(a^2 -
 b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*
x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c
) + 2*(-2*I*b^3*d*f^2*x - 2*I*b^3*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*
(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) - 2*(b^3*d^2*e^2 - 2
*b^3*c*d*e*f + b^3*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2
*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) - 2*(b
^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*co
s(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d
*x + c) + 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sqrt(-(a^2 - b^2)/b
^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
 + 2*I*a)*sin(d*x + c) + 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sqrt
(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(
a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) - 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*
x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(
d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(
```

```
a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f
*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos
(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) - 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*
f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*c
os(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*
e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a
*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) + (-4*I*(a^2*b - b^3)*d*f^2*x -
 4*I*(a^2*b - b^3)*d*e*f + 4*I*(a^3 - a*b^2)*f^2)*dilog(cos(d*x + c) + I*si
n(d*x + c))*sin(d*x + c) + (4*I*(a^2*b - b^3)*d*f^2*x + 4*I*(a^2*b - b^3)*d
*e*f - 4*I*(a^3 - a*b^2)*f^2)*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x
+ c) + (-4*I*(a^2*b - b^3)*d*f^2*x - 4*I*(a^2*b - b^3)*d*e*f - 4*I*(a^3 - a
*b^2)*f^2)*dilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + (4*I*(a^2*b
 - b^3)*d*f^2*x + 4*I*(a^2*b - b^3)*d*e*f + 4*I*(a^3 - a*b^2)*f^2)*dilog(-c
os(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - 2*((a^2*b - b^3)*d^2*f^2*x^2 +
 (a^2*b - b^3)*d^2*e^2 + 2*(a^3 - a*b^2)*d*e*f + 2*((a^2*b - b^3)*d^2*e*f +
 (a^3 - a*b^2)*d*f^2)*x)*log(cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c
) - 2*((a^2*b - b^3)*d^2*f^2*x^2 + (a^2*b - b^3)*d^2*e^2 + 2*(a^3 - a*b^2)*
d*e*f + 2*((a^2*b - b^3)*d^2*e*f + (a^3 - a*b^2)*d*f^2)*x)*log(cos(d*x + c)
 - I*sin(d*x + c) + 1)*sin(d*x + c) + 2*((a^2*b - b^3)*d^2*e^2 - 2*(a^3 - a
*b^2 + (a^2*b - b^3)*c)*d*e*f + ((a^2*b - b^3)*c^2 + 2*(a^3 - a*b^2)*c)*f^2
)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + 2*((a^2*
b - b^3)*d^2*e^2 - 2*(a^3 - a*b^2 + (a^2*b - b^3)*c)*d*e*f + ((a^2*b - b^3)
*c^2 + 2*(a^3 - a*b^2)*c)*f^2)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) +
 1/2)*sin(d*x + c) + 2*((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a^2*b - b^3)*c*d*e*f
 - ((a^2*b - b^3)*c^2 + 2*(a^3 - a*b^2)*c)*f^2 + 2*((a^2*b - b^3)*d^2*e*f -
 (a^3 - a*b^2)*d*f^2)*x)*log(-cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x +
c) + 2*((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a^2*b - b^3)*c*d*e*f - ((a^2*b - b^3
)*c^2 + 2*(a^3 - a*b^2)*c)*f^2 + 2*((a^2*b - b^3)*d^2*e*f - (a^3 - a*b^2)*d
*f^2)*x)*log(-cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) + 4*((a^3 - a
*b^2)*d^2*f^2*x^2 + 2*(a^3 - a*b^2)*d^2*e*f*x + (a^3 - a*b^2)*d^2*e^2)*cos(
d*x + c))/((a^4 - a^2*b^2)*d^3*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e+fx\right)^{2}\csc^{2}\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.238 $\int \frac{(e+fx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=370

$$-\frac{b^2 f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2d^2\sqrt{a^2-b^2}}+\frac{b^2 f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2d^2\sqrt{a^2-b^2}}-\frac{ibf\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{a^2d^2}+\frac{ibf\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{a^2d^2}-\frac{ib^2(e}{}$$

```
[Out] (2*b*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a^2*d) - ((e + f*x)*Cot[c + d*x])
/(a*d) - (I*b^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2
])])/(a^2*Sqrt[a^2 - b^2]*d) + (I*b^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d) + (f*Log[Sin[c + d*x]])/
(a*d^2) - (I*b*f*PolyLog[2, -E^(I*(c + d*x))])/(a^2*d^2) + (I*b*f*PolyLog[2
, E^(I*(c + d*x))])/(a^2*d^2) - (b^2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^2) + (b^2*f*PolyLog[2, (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^2)
```

Rubi [A] time = 0.616411, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {4535, 4184, 3475, 4183, 2279, 2391, 3323, 2264, 2190}

$$-\frac{b^2 f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2d^2\sqrt{a^2-b^2}}+\frac{b^2 f\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2d^2\sqrt{a^2-b^2}}-\frac{ibf\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{a^2d^2}+\frac{ibf\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{a^2d^2}-\frac{ib^2(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] (2*b*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a^2*d) - ((e + f*x)*Cot[c + d*x])
/(a*d) - (I*b^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2
])])/(a^2*Sqrt[a^2 - b^2]*d) + (I*b^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d) + (f*Log[Sin[c + d*x]])/
(a*d^2) - (I*b*f*PolyLog[2, -E^(I*(c + d*x))])/(a^2*d^2) + (I*b*f*PolyLog[2
, E^(I*(c + d*x))])/(a^2*d^2) - (b^2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^2) + (b^2*f*PolyLog[2, (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*Sqrt[a^2 - b^2]*d^2)
```


Rule 4535

```
Int[(Csc[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csc[c +
```

```
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csc[c + d*x]^(n - 1))/(a + b*Si
n[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n
, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
```

```
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\csc^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)\cot(c+dx)}{ad} - \frac{b\int (e+fx)\csc(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{a^2} + \frac{f\int \cot(c+dx)}{ad}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)\cot(c+dx)}{ad} + \frac{f\log(\sin(c+dx))}{ad^2} + \frac{\left(2b^2\right)\int \frac{e^{i(c+}}{ib+2ae^{i(c+}}}{a^2}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)\cot(c+dx)}{ad} + \frac{f\log(\sin(c+dx))}{ad^2} - \frac{\left(2ib^3\right)\int \frac{e^{i(c+}}{2a-2\sqrt{a^2}}}{a^2\sqrt{a^2}}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)\cot(c+dx)}{ad} - \frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d} + \frac{ib^2(e}{}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)\cot(c+dx)}{ad} - \frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d} + \frac{ib^2(e}{}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} - \frac{(e+fx)\cot(c+dx)}{ad} - \frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d} + \frac{ib^2(e}{}$$

Mathematica [B] time = 11.2584, size = 933, normalized size = 2.52

$$\frac{(de+dfx)\left(\frac{2(de-cf)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{if\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt{b^2-a^2}}{-ia+b+\sqrt{b^2-a^2}}\right)+\text{PolyLog}\left(2,\frac{a\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a+i\left(b+\sqrt{b^2-a^2}\right)}\right)\right)}{\sqrt{b^2-a^2}}+\frac{if\left(\log\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-(d*e*Cos[(c + d*x)/2]) + c*f*Cos[(c + d*x)/2] - f*(c + d*x)*Cos[(c + d*x
)/2])*Csc[(c + d*x)/2])/(2*a*d^2) + (f*Log[Sin[c + d*x]])/(a*d^2) - (b*e*Lo
g[Tan[(c + d*x)/2]])/(a^2*d) + (b*c*f*Log[Tan[(c + d*x)/2]])/(a^2*d^2) - (b
*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c + d*x))]) + I*(Po
lyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a^2*d^2) + (b^
2*(d*e + d*f*x)*((2*(d*e - c*f)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/Sqrt[a^2 - b^2] - (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a
^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] + PolyLog[
2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2
 + b^2] + (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*T
an[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(
c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Lo
g[1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])
/(I*a - b + Sqrt[-a^2 + b^2]))] + PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*
a - b + Sqrt[-a^2 + b^2])]))/Sqrt[-a^2 + b^2] - (I*f*(Log[1 + I*Tan[(c + d*
x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2
 + b^2])] + PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 +
b^2]))]))/Sqrt[-a^2 + b^2]))/(a^2*d^2*(d*e - c*f + I*f*Log[1 - I*Tan[(c + d
*x)/2]] - I*f*Log[1 + I*Tan[(c + d*x)/2]])) + (Sec[(c + d*x)/2]*(d*e*Sin[(c
 + d*x)/2] - c*f*Sin[(c + d*x)/2] + f*(c + d*x)*Sin[(c + d*x)/2]))/(2*a*d^2
)
```

Maple [B] time = 0.194, size = 766, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 2*I/d/a^2*b^2*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^
2+b^2)^(1/2))-I/d^2/a^2*b*f*dilog(exp(I*(d*x+c))+1)-2*I*(f*x+e)/d/a/(exp(2*
I*(d*x+c))-1)-2/d^2/a*f*ln(exp(I*(d*x+c)))+I/d^2/a^2*b^2*f/(-a^2+b^2)^(1/2)
*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-I/d^
2/a^2*b*f*dilog(exp(I*(d*x+c)))+1/d/a^2*b^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*ex
p(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+1/d^2/a^2*b^2*f/(-
a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(
1/2)))*c+1/d^2/a*f*ln(exp(I*(d*x+c))-1)+1/d^2/a*f*ln(exp(I*(d*x+c))+1)-1/d/
a^2*b*e*ln(exp(I*(d*x+c))-1)+1/d/a^2*b*e*ln(exp(I*(d*x+c))+1)+1/d^2/a^2*b*f
*c*ln(exp(I*(d*x+c))-1)+1/d/a^2*b*f*ln(exp(I*(d*x+c))+1)*x-2*I/d^2/a^2*b^2*
f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2)
)-1/d/a^2*b^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))
/(I*a+(-a^2+b^2)^(1/2)))*x-1/d^2/a^2*b^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I
*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-I/d^2/a^2*b^2*f/(-a^2
+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(
1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.48821, size = 4132, normalized size = 11.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(-2*I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*
a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2) + 2*b)/b + 1)*sin(d*x + c) + 2*I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2
*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) + 2*I*b^3*f*sqrt(-(a
```

```
^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*co
s(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x
 + c) - 2*I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) +
2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2) + 2*b)/b + 1)*sin(d*x + c) - 2*I*(a^2*b - b^3)*f*dilog(cos(d*x + c) +
I*sin(d*x + c))*sin(d*x + c) + 2*I*(a^2*b - b^3)*f*dilog(cos(d*x + c) - I*s
in(d*x + c))*sin(d*x + c) - 2*I*(a^2*b - b^3)*f*dilog(-cos(d*x + c) + I*sin
(d*x + c))*sin(d*x + c) + 2*I*(a^2*b - b^3)*f*dilog(-cos(d*x + c) - I*sin(d
*x + c))*sin(d*x + c) - 2*(b^3*d*e - b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*
b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*s
in(d*x + c) - 2*(b^3*d*e - b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x
+ c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c
) + 2*(b^3*d*e - b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*
I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) + 2*(b^
3*d*e - b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d
*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) - 2*(b^3*d*f*x +
 b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x
+ c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/
b)*sin(d*x + c) + 2*(b^3*d*f*x + b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2
*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) - 2*(b^3*d*f*x + b^3*c*f)*s
qrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(
b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x
 + c) + 2*(b^3*d*f*x + b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(
d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) - 2*((a^2*b - b^3)*d*f*x + (a^2*b -
b^3)*d*e + (a^3 - a*b^2)*f)*log(cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x
+ c) - 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*d*e + (a^3 - a*b^2)*f)*log(co
s(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) + 2*((a^2*b - b^3)*d*e - (a^3
 - a*b^2 + (a^2*b - b^3)*c)*f)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) +
 1/2)*sin(d*x + c) + 2*((a^2*b - b^3)*d*e - (a^3 - a*b^2 + (a^2*b - b^3)*c)
*f)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + 2*((a^
2*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)*log(-cos(d*x + c) + I*sin(d*x + c) +
1)*sin(d*x + c) + 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)*log(-cos(d*x
+ c) - I*sin(d*x + c) + 1)*sin(d*x + c) + 4*((a^3 - a*b^2)*d*f*x + (a^3 - a
*b^2)*d*e)*cos(d*x + c))/((a^4 - a^2*b^2)*d^2*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)\csc^2(c + dx)}{a + b\sin(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.239 $\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\sqrt{a^2-b^2}}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] (2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*Sqrt[a^2 - b^
2]*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d)
```

Rubi [A] time = 0.128176, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2802, 12, 2747, 3770, 2660, 618, 204}

$$\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\sqrt{a^2-b^2}}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```


```
[Out] (2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*Sqrt[a^2 - b^
2]*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d)
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)}{ad} - \frac{\int \frac{b\csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$
$$= -\frac{\cot(c+dx)}{ad} - \frac{b\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$
$$= -\frac{\cot(c+dx)}{ad} - \frac{b\int \csc(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2}$$
$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2 d} - \frac{\cot(c+dx)}{ad} + \frac{(2b^2)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2 d}$$
$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2 d} - \frac{\cot(c+dx)}{ad} - \frac{(4b^2)\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c+\right.\right.}{a^2 d}$$
$$= \frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2 d} - \frac{\cot(c+dx)}{ad}$$

Mathematica [A] time = 0.44458, size = 111, normalized size = 1.34

$$\frac{\frac{4b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + a\tan\left(\frac{1}{2}(c+dx)\right) - a\cot\left(\frac{1}{2}(c+dx)\right) - 2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((4*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] -
 a*Cot[(c + d*x)/2] + 2*b*Log[Cos[(c + d*x)/2]] - 2*b*Log[Sin[(c + d*x)/2]]
 + a*Tan[(c + d*x)/2])/(2*a^2*d)
```

Maple [A] time = 0.001, size = 109, normalized size = 1.3

$$\frac{1}{2\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + 2\,\frac{b^2}{da^2\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(1/2\,dx+c/2)+2\,b}{\sqrt{a^2-b^2}}\right) - \frac{1}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1} - \frac{b}{da^2}\ln\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/a/d*tan(1/2*d*x+1/2*c)+2/d*b^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/2/a/d/tan(1/2*d*x+1/2*c)-1/d/a^2*b*l
n(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.70463, size = 944, normalized size = 11.37

$$\left[-\frac{\sqrt{-a^2+b^2}b^2\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)\sin(dx+c)-\left(a^2b-b^3\right)\log}{2\left(a^4-a^2b^2\right)d\,\mathrm{si}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*b^2*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d
*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt
(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*
x + c) - (a^2*b - b^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (a^2*b -
b^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 2*(a^3 - a*b^2)*cos(d*x +
c))/((a^4 - a^2*b^2)*d*sin(d*x + c)), -1/2*(2*sqrt(a^2 - b^2)*b^2*arctan(-(
a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - (a^2*b -
 b^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (a^2*b - b^3)*log(-1/2*cos
(d*x + c) + 1/2)*sin(d*x + c) + 2*(a^3 - a*b^2)*cos(d*x + c))/((a^4 - a^2*b
^2)*d*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.26924, size = 176, normalized size = 2.12

$$\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^2}{\sqrt{a^2-b^2}a^2}-\frac{2b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*b^2/(sqrt(a^2 - b^2)*a^2) - 2*b*log(abs(tan(1/
2*d*x + 1/2*c)))/a^2 + tan(1/2*d*x + 1/2*c)/a + (2*b*tan(1/2*d*x + 1/2*c) -
 a)/(a^2*tan(1/2*d*x + 1/2*c)))/d
```

3.240 $\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)} \, dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sin^2(c+dx)(e+fx)^m}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable[((e + f*x)^m*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0683369, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[((e + f*x)^m*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] Defer[Int][((e + f*x)^m*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)} \, dx = \int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)} \, dx$$

Mathematica [A] time = 7.66317, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m \sin^2(c+dx)}{a+b\sin(c+dx)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] Integrate[((e + f*x)^m*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Maple [A] time = 0.296, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \left(\sin \left(dx+c\right)\right)^{2}}{a+b\sin \left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \sin \left(dx+c\right)^{2}}{b\sin \left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*sin(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(\cos \left(dx+c\right)^{2}-1\right)\left(fx+e\right)^{m}}{b\sin \left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(f*x + e)^m/(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \sin(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*sin(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

3.241 $\int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sin(c+dx)(e+fx)^m}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable[((e + f*x)^m*Sin[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0415453, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((e + f*x)^m*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] Defer[Int][((e + f*x)^m*Sin[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)}\,dx$$

Mathematica [A] time = 0.798472, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m \sin(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] Integrate[((e + f*x)^m*Sin[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Maple [A] time = 0.154, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \sin \left(dx+c\right)}{a+b \sin \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \sin \left(dx+c\right)}{b \sin \left(dx+c\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^{m} \sin \left(dx+c\right)}{b \sin \left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e+fx\right)^m \sin\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m*sin(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^m \sin\left(dx+c\right)}{b\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*sin(d*x + c)/(b*sin(d*x + c) + a), x)
```

3.242 $\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(e+fx)^m}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable[(e + f*x)^m/(a + b*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0536151, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)^m/(a + b*Sin[c + d*x]),x]

[Out] Defer[Int][(e + f*x)^m/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

Mathematica [A] time = 0.306019, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^m/(a + b*Sin[c + d*x]),x]

[Out] Integrate[(e + f*x)^m/(a + b*Sin[c + d*x]), x]
```

Maple [A] time = 0.067, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m}}{a+b\sin\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m}}{b\sin\left(dx+c\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(fx+e\right)^{m}}{b\sin\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e+fx\right)^m}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^m}{b\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

3.243 $\int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\csc(c+dx)(e+fx)^m}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable[((e + f*x)^m*Csc[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0416573, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((e + f*x)^m*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] Defer[Int][((e + f*x)^m*Csc[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Mathematica [A] time = 36.3038, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m \csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] Integrate[((e + f*x)^m*Csc[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Maple [A] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m}\csc\left(dx+c\right)}{a+b\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m}\csc\left(dx+c\right)}{b\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*csc(d*x + c)/(b*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^{m}\csc\left(dx+c\right)}{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*csc(d*x + c)/(b*sin(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e+fx\right)^m \csc\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^m \csc\left(dx+c\right)}{b\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*csc(d*x + c)/(b*sin(d*x + c) + a), x)
```

3.244 $\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\csc^2(c+dx)(e+fx)^m}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable[((e + f*x)^m*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0678792, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((e + f*x)^m*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]

[Out] Defer[Int][((e + f*x)^m*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Mathematica [A] time = 6.38634, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m \csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]

[Out] Integrate[((e + f*x)^m*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Maple [A] time = 0.108, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \left(\csc \left(dx+c\right)\right)^{2}}{a+b\sin \left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \csc \left(dx+c\right)^{2}}{b\sin \left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*csc(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx+e\right)^{m} \csc \left(dx+c\right)^{2}}{b\sin \left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*csc(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \csc(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*csc(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

3.245 $\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=574

$$\frac{a^2 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\left(a^2-b^2\right)^{3/2}}-\frac{a^2 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\left(a^2-b^2\right)^{3/2}}-\frac{f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{af\log}{}$$

```
[Out] (I*a^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(
a^2 - b^2)^(3/2)*d) - (I*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[
a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) - (I*a^2*(e + f*x)*Log[1 - (I*b*E^(I*(c
 + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (I*(e + f*x)*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) +
 (a*f*Log[a + b*Sin[c + d*x]])/(b*(a^2 - b^2)*d^2) + (a^2*f*PolyLog[2, (I*b
*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) - (f*Po
lyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d
^2) - (a^2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a
^2 - b^2)^(3/2)*d^2) + (f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
b^2])])/(b*Sqrt[a^2 - b^2]*d^2) - (a*(e + f*x)*Cos[c + d*x])/((a^2 - b^2)*d
*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.61675, antiderivative size = 574, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6742, 3324, 3323, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{a^2 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\left(a^2-b^2\right)^{3/2}}-\frac{a^2 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\left(a^2-b^2\right)^{3/2}}-\frac{f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{af\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (I*a^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(
a^2 - b^2)^(3/2)*d) - (I*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[
a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) - (I*a^2*(e + f*x)*Log[1 - (I*b*E^(I*(c
 + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (I*(e + f*x)*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d) +
 (a*f*Log[a + b*Sin[c + d*x]])/(b*(a^2 - b^2)*d^2) + (a^2*f*PolyLog[2, (I*b
*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) - (f*Po
lyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d
```

```
^2) - (a^2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a
^2 - b^2)^(3/2)*d^2) + (f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
b^2])])/(b*Sqrt[a^2 - b^2]*d^2) - (a*(e + f*x)*Cos[c + d*x])/((a^2 - b^2)*d
*(a + b*Sin[c + d*x]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(-\frac{a(e+fx)}{b(a+b\sin(c+dx))^2} + \frac{e+fx}{b(a+b\sin(c+dx))}\right)dx$$

$$= \frac{\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{b} - \frac{a\int \frac{e+fx}{(a+b\sin(c+dx))^2}\,dx}{b}$$

$$= -\frac{a(e+fx)\cos(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{2\int \frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b} - \frac{a^2\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)} + \frac{(af)\int}{(a}$$

$$= -\frac{a(e+fx)\cos(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\left(2a^2\right)\int \frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b\left(a^2-b^2\right)} - \frac{(2i)\int \frac{e^{i(c+dx)}(e+fx)}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}}{\sqrt{a^2-b^2}}$$

$$= -\frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} + \frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} + \frac{af\log(a+b\sin(c+dx))}{b\left(a^2-b^2\right)d^2} - $$

$$= \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} - \frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d}$$

$$= \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} - \frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d}$$

$$= \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} - \frac{i(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} - \frac{ia^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d}$$

Mathematica [B] time = 15.4048, size = 2141, normalized size = 3.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-(a*d*e*Cos[c + d*x]) + a*c*f*Cos[c + d*x] - a*f*(c + d*x)*Cos[c + d*x])/(
(a - b)*(a + b)*d^2*(a + b*Sin[c + d*x])) + (((2*a*f*ArcTan[(b + a*Tan[(c +
 d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (2*(-(b*d*e) + a*f + b*c*f)*A
rcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (a*f*Log
[Sec[(c + d*x)/2]^2])/b - (a*f*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])]
)/b - (I*b*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan
```

```
[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 - I*Tan[
(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]]))/Sqrt[-a^2 + b^2] + (I*b*f*
(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]
)/(I*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(
a - I*(b + Sqrt[-a^2 + b^2]))]]))/Sqrt[-a^2 + b^2] + (I*b*f*(Log[1 - I*Tan[(
c + d*x)/2]]*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + S
qrt[-a^2 + b^2]))] + PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[
-a^2 + b^2])]]))/Sqrt[-a^2 + b^2] - (I*b*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[
(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] +
 PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]]))/S
qrt[-a^2 + b^2])*(-((b*e)/((a^2 - b^2)*(a + b*Sin[c + d*x]))) + (b*c*f)/((a
^2 - b^2)*d*(a + b*Sin[c + d*x])) - (b*f*(c + d*x))/((a^2 - b^2)*d*(a + b*S
in[c + d*x])) + (a*f*Cos[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))))/(
d*((a*f*Tan[(c + d*x)/2])/b - (a*f*Cos[(c + d*x)/2]^2*(b*Cos[c + d*x]*Sec[(
c + d*x)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c + d*x)/2]))/
(b*(a + b*Sin[c + d*x])) + (a^2*f*Sec[(c + d*x)/2]^2)/((a^2 - b^2)*(1 + (b
+ a*Tan[(c + d*x)/2])^2/(a^2 - b^2))) - (a*(-(b*d*e) + a*f + b*c*f)*Sec[(c
+ d*x)/2]^2)/((a^2 - b^2)*(1 + (b + a*Tan[(c + d*x)/2])^2/(a^2 - b^2))) + (
I*b*f*(((-I/2)*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b +
 Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) - (Log[1
- (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]
^2)/(2*(I + Tan[(c + d*x)/2])) + (a*Log[1 - I*Tan[(c + d*x)/2]]*Sec[(c + d*
x)/2]^2)/(2*(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]))))/Sqrt[-a^2 + b^2]
 - (I*b*f*(((I/2)*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b
- Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - ((I/2)*
a*Log[1 - (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]*Sec[(
c + d*x)/2]^2)/(a + I*a*Tan[(c + d*x)/2]) + (a*Log[1 + I*Tan[(c + d*x)/2]]*
Sec[(c + d*x)/2]^2)/(2*(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]))))/Sqrt[
-a^2 + b^2] - (I*b*f*(((I/2)*Log[1 - (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b
 + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) - ((I/2
)*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 +
 b^2])]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) + (a*Log[1 - I*Tan[(c
+ d*x)/2]]*Sec[(c + d*x)/2]^2)/(2*(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2
]))))/Sqrt[-a^2 + b^2] + (I*b*f*(((-I/2)*Log[1 - (a*(1 + I*Tan[(c + d*x)/2]
))/(a - I*(b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)
/2]) + ((I/2)*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sq
rt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) + (a*Log[1 +
I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(2*(b + Sqrt[-a^2 + b^2] + a*Tan[(c
 + d*x)/2]))))/Sqrt[-a^2 + b^2]))
```

Maple [A] time = 0.917, size = 750, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 2*I*a*(f*x+e)*(b-I*a*exp(I*(d*x+c)))/b/(-a^2+b^2)/d/(b*exp(2*I*(d*x+c))-b+2
*I*a*exp(I*(d*x+c)))-2/(a^2-b^2)/d^2/b*a*f*ln(exp(I*(d*x+c)))+1/(a^2-b^2)/d
^2/b*a*f*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-2*I/(a^2-b^2)/d*b*
e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))-
1/(a^2-b^2)/d*b*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2
))/(I*a-(-a^2+b^2)^(1/2)))*x-1/(a^2-b^2)/d^2*b*f/(-a^2+b^2)^(1/2)*ln((I*a+b
*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+1/(a^2-b^2)/d*b
*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b
^2)^(1/2)))*x+1/(a^2-b^2)/d^2*b*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))
+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-I/(a^2-b^2)/d^2*b*f/(-a^2+b^2)
^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2))
)+I/(a^2-b^2)/d^2*b*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^
2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))+2*I/(a^2-b^2)/d^2*b*c*f/(-a^2+b^2)^(1/2)*
arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.06444, size = 3528, normalized size = 6.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((-I*b^4*f*sin(d*x + c) - I*a*b^3*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*
(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x +
```

```
c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (I*b^4*f*sin(d*x + c) + I*a*b^3*
f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c)
 - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b +
1) + (I*b^4*f*sin(d*x + c) + I*a*b^3*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(
-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-I*b^4*f*sin(d*x + c) - I*a*b^3
*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x +
c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b
+ 1) - (a*b^3*d*f*x + a*b^3*c*f + (b^4*d*f*x + b^4*c*f)*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos
(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (a*b^3*d*f
*x + a*b^3*c*f + (b^4*d*f*x + b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - (a*b^3*d*f*x + a*b^3*c*f +
(b^4*d*f*x + b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*
cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) + 2*b)/b) + (a*b^3*d*f*x + a*b^3*c*f + (b^4*d*f*x + b^4
*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2
*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) + 2*b)/b) - 2*((a^3*b - a*b^3)*d*f*x + (a^3*b - a*b^3)*d*e)*cos(d*x + c
) + ((a^3*b - a*b^3)*f*sin(d*x + c) + (a^4 - a^2*b^2)*f - (a*b^3*d*e - a*b^
3*c*f + (b^4*d*e - b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*c
os(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + ((
a^3*b - a*b^3)*f*sin(d*x + c) + (a^4 - a^2*b^2)*f - (a*b^3*d*e - a*b^3*c*f
+ (b^4*d*e - b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x
 + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + ((a^3*b
- a*b^3)*f*sin(d*x + c) + (a^4 - a^2*b^2)*f + (a*b^3*d*e - a*b^3*c*f + (b^4
*d*e - b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c)
 + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + ((a^3*b - a*b
^3)*f*sin(d*x + c) + (a^4 - a^2*b^2)*f + (a*b^3*d*e - a*b^3*c*f + (b^4*d*e
- b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) - 2*
I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a))/((a^4*b^2 - 2*a^2*b
^4 + b^6)*d^2*sin(d*x + c) + (a^5*b - 2*a^3*b^3 + a*b^5)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\sin(dx+c)}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

3.246 $\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=1106

$$\frac{i(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^2}{b\left(a^2-b^2\right)^{3/2}d}-\frac{i(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^2}{b\left(a^2-b^2\right)^{3/2}d}+\frac{2f(e+fx)\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^2}{b\left(a^2-b^2\right)^{3/2}d^2}-\frac{2f(e+fx)\mathrm{P}}{b}$$

```
[Out] ((-I)*a*(e + f*x)^2)/(b*(a^2 - b^2)*d) + (2*a*f*(e + f*x)*Log[1 - (I*b*E^(I
*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) + (I*a^2*(e + f*x)
^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/
2)*d) - (I*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])]
)/(b*Sqrt[a^2 - b^2]*d) + (2*a*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a
 + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) - (I*a^2*(e + f*x)^2*Log[1 - (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (I*(e +
 f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 -
 b^2]*d) - ((2*I)*a*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(b*(a^2 - b^2)*d^3) + (2*a^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*
x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) - (2*f*(e + f*x)*Pol
yLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^
2) - ((2*I)*a*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/
(b*(a^2 - b^2)*d^3) - (2*a^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) + (2*f*(e + f*x)*PolyLog[2
, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) + (
(2*I)*a^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(
a^2 - b^2)^(3/2)*d^3) - ((2*I)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3) - ((2*I)*a^2*f^2*PolyLog[3, (I*b*E
^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + ((2*I)*
f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 -
b^2]*d^3) - (a*(e + f*x)^2*Cos[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x]
))
```

Rubi [A] time = 2.55663, antiderivative size = 1106, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {6742, 3324, 3323, 2264, 2190, 2531, 2282, 6589, 4519, 2279, 2391}

$$\frac{i(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^2}{b\left(a^2-b^2\right)^{3/2}d}-\frac{i(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^2}{b\left(a^2-b^2\right)^{3/2}d}+\frac{2f(e+fx)\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^2}{b\left(a^2-b^2\right)^{3/2}d^2}-\frac{2f(e+fx)\mathrm{P}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-I)*a*(e + f*x)^2)/(b*(a^2 - b^2)*d) + (2*a*f*(e + f*x)*Log[1 - (I*b*E^(I
*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) + (I*a^2*(e + f*x)
^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/
2)*d) - (I*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])]
)/(b*Sqrt[a^2 - b^2]*d) + (2*a*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a
 + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) - (I*a^2*(e + f*x)^2*Log[1 - (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (I*(e +
 f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 -
 b^2]*d) - ((2*I)*a*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(b*(a^2 - b^2)*d^3) + (2*a^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*
x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) - (2*f*(e + f*x)*Pol
yLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^
2) - ((2*I)*a*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/
(b*(a^2 - b^2)*d^3) - (2*a^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) + (2*f*(e + f*x)*PolyLog[2
, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) + (
(2*I)*a^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(
a^2 - b^2)^(3/2)*d^3) - ((2*I)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3) - ((2*I)*a^2*f^2*PolyLog[3, (I*b*E
^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + ((2*I)*
f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 -
b^2]*d^3) - (a*(e + f*x)^2*Cos[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x]
))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
```

```
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
```

```
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int\left(-\frac{a(e+fx)^2}{b(a+b\sin(c+dx))^2}+\frac{(e+fx)^2}{b(a+b\sin(c+dx))}\right)dx$$

$$= \frac{\int\frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{b} - \frac{a\int\frac{(e+fx)^2}{(a+b\sin(c+dx))^2}\,dx}{b}$$

$$= -\frac{a(e+fx)^2\cos(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{2\int\frac{e^{i(c+dx)}(e+fx)^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b} - \frac{a^2\int\frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)} + \frac{(2af}{}$$

$$= -\frac{ia(e+fx)^2}{b\left(a^2-b^2\right)d} - \frac{a(e+fx)^2\cos(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\left(2a^2\right)\int\frac{e^{i(c+dx)}(e+fx)^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b\left(a^2-b^2\right)} - \frac{(2i)\int}{}$$

$$= -\frac{ia(e+fx)^2}{b\left(a^2-b^2\right)d} + \frac{2af(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^2} - \frac{i(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d} + \frac{2af(e}{}$$

$$= -\frac{ia(e+fx)^2}{b\left(a^2-b^2\right)d} + \frac{2af(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^2} + \frac{ia^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} - \frac{i(e+}{}$$

$$= -\frac{ia(e+fx)^2}{b\left(a^2-b^2\right)d} + \frac{2af(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^2} + \frac{ia^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} - \frac{i(e+}{}$$

$$= -\frac{ia(e+fx)^2}{b\left(a^2-b^2\right)d} + \frac{2af(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^2} + \frac{ia^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} - \frac{i(e+}{}$$

$$= -\frac{ia(e+fx)^2}{b\left(a^2-b^2\right)d} + \frac{2af(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^2} + \frac{ia^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} - \frac{i(e+}{}$$

Mathematica [B] time = 24.8248, size = 3757, normalized size = 3.4

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b*e*f*((Pi*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
b^2] + (2*(-c + Pi/2 - d*x)*ArcTanh[((a + b)*Cot[(-c + Pi/2 - d*x)/2])/Sqrt
```

[-a^2 + b^2]] - 2*(-c + ArcCos[-(a/b)])*ArcTanh[((-a + b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]] + (ArcCos[-(a/b)] - (2*I)*(ArcTanh[((a + b)*Cot[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]] - ArcTanh[((-a + b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]]))*Log[Sqrt[-a^2 + b^2]/(Sqrt[2]*Sqrt[b]*E^((I/2)*(-c + Pi/2 - d*x))*Sqrt[a + b*Sin[c + d*x]])] + (ArcCos[-(a/b)] + (2*I)*(ArcTanh[((a + b)*Cot[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]] - ArcTanh[((-a + b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]]))*Log[(Sqrt[-a^2 + b^2]*E^((I/2)*(-c + Pi/2 - d*x)))/(Sqrt[2]*Sqrt[b]*Sqrt[a + b*Sin[c + d*x]])] - (ArcCos[-(a/b)] + (2*I)*ArcTanh[((-a + b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]])*Log[1 - ((a - I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))] + (-ArcCos[-(a/b)] + (2*I)*ArcTanh[((-a + b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]])*Log[1 - ((a + I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))] + I*(PolyLog[2, ((a - I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))] - PolyLog[2, ((a + I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))]))/Sqrt[-a^2 + b^2]))/((-a^2 + b^2)*d^2) + (2*a^2*f^2*Cot[c]*((Pi*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (2*(-c + Pi/2 - d*x)*ArcTanh[((a + b)*Cot[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]] - 2*(-c + ArcCos[-(a/b)])*ArcTanh[((-a + b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]] + (ArcCos[-(a/b)] - (2*I)*(ArcTanh[((a + b)*Cot[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]] - ArcTanh[((-a + b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]]))*Log[Sqrt[-a^2 + b^2]/(Sqrt[2]*Sqrt[b]*E^((I/2)*(-c + Pi/2 - d*x))*Sqrt[a + b*Sin[c + d*x]])] + (ArcCos[-(a/b)] + (2*I)*(ArcTanh[((a + b)*Cot[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]] - ArcTanh[((-a + b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]]))*Log[(Sqrt[-a^2 + b^2]*E^((I/2)*(-c + Pi/2 - d*x)))/(Sqrt[2]*Sqrt[b]*Sqrt[a + b*Sin[c + d*x]])] - (ArcCos[-(a/b)] + (2*I)*ArcTanh[((-a + b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]])*Log[1 - ((a - I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))] + (-ArcCos[-(a/b)] + (2*I)*ArcTanh[((-a + b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[-a^2 + b^2]])*Log[1 - ((a + I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))] + I*(PolyLog[2, ((a - I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))] - PolyLog[2, ((a + I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(-c + Pi/2 - d*x)/2]))]))/Sqrt[-a^2 + b^2]))/(b*(-a^2 + b^2)*d^3) + (b*E^(I*c)*f^2*(d^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - d^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (2*I)*d*x*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (2*I)*d*x*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 2*PolyLog[3, (I

```
*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 2*P
olyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)
*c)]))]))/((-a^2 + b^2)*d^3*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]) + ((2*I)*b*e^2*
ArcTan[(I*b*Cos[c] - I*(-a + b*Sin[c])*Tan[(d*x)/2])/Sqrt[-a^2 + b^2*Cos[c]
^2 + b^2*Sin[c]^2]])/((-a^2 + b^2)*d*Sqrt[-a^2 + b^2*Cos[c]^2 + b^2*Sin[c]^
2]) + ((4*I)*a^2*e*f*ArcTan[(I*b*Cos[c] - I*(-a + b*Sin[c])*Tan[(d*x)/2])/S
qrt[-a^2 + b^2*Cos[c]^2 + b^2*Sin[c]^2]]*Cot[c])/(b*(-a^2 + b^2)*d^2*Sqrt[-
a^2 + b^2*Cos[c]^2 + b^2*Sin[c]^2]) + (2*a*f^2*Csc[c]*(-(x^2*Cos[c])/(2*b)
+ (x*(d*x*Cos[c] - (2*a*ArcTan[(Sec[(d*x)/2]*(Cos[c] - I*Sin[c])*(b*Cos[c +
 (d*x)/2] + a*Sin[(d*x)/2]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2])]
*Cos[c]*(Cos[c] - I*Sin[c]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2])
- Log[a + b*Sin[c + d*x]]*Sin[c]))/(b*d) + (-((a*Cos[c]*((-I)*d*x*(Log[1 +
(I*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] - Log[1 - (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])]) - PolyLog[2, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt
[a^2 - b^2])] + PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]))/(
Sqrt[a^2 - b^2]*d)) + (2*a*x*ArcTan[(Sec[(d*x)/2]*(Cos[c] - I*Sin[c])*(b*Co
s[c + (d*x)/2] + a*Sin[(d*x)/2]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])
^2])]*Cos[c]*(Cos[c] - I*Sin[c]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])
^2]) + ((c + d*x)*Log[a + b*Sin[c + d*x]]*Sin[c])/d - (b*(((c + d*x)*Log[a
+ b*Sin[c + d*x]])/b - ((-I/2)*(-c + Pi/2 - d*x)^2 + (4*I)*ArcSin[Sqrt[(a +
 b)/b]/Sqrt[2]]*ArcTan[((a - b)*Tan[(-c + Pi/2 - d*x)/2])/Sqrt[a^2 - b^2]]
+ (-c + Pi/2 - d*x + 2*ArcSin[Sqrt[(a + b)/b]/Sqrt[2]])*Log[1 + ((a - Sqrt[
a^2 - b^2])*E^(I*(-c + Pi/2 - d*x)))/b] + (-c + Pi/2 - d*x - 2*ArcSin[Sqrt[
(a + b)/b]/Sqrt[2]])*Log[1 + ((a + Sqrt[a^2 - b^2])*E^(I*(-c + Pi/2 - d*x))
)/b] - (-c + Pi/2 - d*x)*Log[a + b*Sin[c + d*x]] - I*(PolyLog[2, -(((a - Sq
rt[a^2 - b^2])*E^(I*(-c + Pi/2 - d*x)))/b)] + PolyLog[2, -(((a + Sqrt[a^2 -
 b^2])*E^(I*(-c + Pi/2 - d*x)))/b)]))/b)*Sin[c])/d)/(b*d)))/((-a^2 + b^2)*d
) - (2*a*e*f*Csc[c]*(-(b*d*x*Cos[c]) + b*Log[a + b*Cos[d*x]*Sin[c] + b*Cos[
c]*Sin[d*x]]*Sin[c] + ((2*I)*a*b*ArcTan[(I*b*Cos[c] - I*(-a + b*Sin[c])*Tan
[(d*x)/2])/Sqrt[-a^2 + b^2*Cos[c]^2 + b^2*Sin[c]^2]]*Cos[c])/Sqrt[-a^2 + b^
2*Cos[c]^2 + b^2*Sin[c]^2]))/((-a^2 + b^2)*d^2*(b^2*Cos[c]^2 + b^2*Sin[c]^2
)) + (Csc[c/2]*Sec[c/2]*(a^2*e^2*Cos[c] + 2*a^2*e*f*x*Cos[c] + a^2*f^2*x^2*
Cos[c] + a*b*e^2*Sin[d*x] + 2*a*b*e*f*x*Sin[d*x] + a*b*f^2*x^2*Sin[d*x]))/(
2*(a - b)*b*(a + b)*d*(a + b*Sin[c + d*x]))
```

Maple [F] time = 1.519, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sin(dx+c)}{(a+b\sin(dx+c))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.88813, size = 7002, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^4*f^2*sin(d*x + c) + a*b^3*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3,
 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 4*(b^4*f^2*sin(d*x + c) + a*b^3*f^2)*
sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c
) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 4*(b
^4*f^2*sin(d*x + c) + a*b^3*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*co
s(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2))/b) + 4*(b^4*f^2*sin(d*x + c) + a*b^3*f^2)*sqrt(-(a^2 - b^2)/
b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 4*((a^3*b - a*b^3)*d^2*f^2*x^2
+ 2*(a^3*b - a*b^3)*d^2*e*f*x + (a^3*b - a*b^3)*d^2*e^2)*cos(d*x + c) + (4*
I*(a^3*b - a*b^3)*f^2*sin(d*x + c) + 4*I*(a^4 - a^2*b^2)*f^2 + 2*(-2*I*a*b^
3*d*f^2*x - 2*I*a*b^3*d*e*f + (-2*I*b^4*d*f^2*x - 2*I*b^4*d*e*f)*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2))*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x +
c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b
+ 1) + (4*I*(a^3*b - a*b^3)*f^2*sin(d*x + c) + 4*I*(a^4 - a^2*b^2)*f^2 + 2*
(2*I*a*b^3*d*f^2*x + 2*I*a*b^3*d*e*f + (2*I*b^4*d*f^2*x + 2*I*b^4*d*e*f)*si
```

n(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*si
n(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) +
 2*b)/b + 1) + (-4*I*(a^3*b - a*b^3)*f^2*sin(d*x + c) - 4*I*(a^4 - a^2*b^2)
f^2 + 2(2*I*a*b^3*d*f^2*x + 2*I*a*b^3*d*e*f + (2*I*b^4*d*f^2*x + 2*I*b^4*
d*e*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog(-1/2*(-2*I*a*cos(d*x + c
) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2) + 2*b)/b + 1) + (-4*I*(a^3*b - a*b^3)*f^2*sin(d*x + c) - 4*I*(a^4
- a^2*b^2)*f^2 + 2*(-2*I*a*b^3*d*f^2*x - 2*I*a*b^3*d*e*f + (-2*I*b^4*d*f^2*
x - 2*I*b^4*d*e*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog(-1/2*(-2*I*a
*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(2*(a^4 - a^2*b^2)*d*e*f - 2*(a^4 -
a^2*b^2)*c*f^2 + 2*((a^3*b - a*b^3)*d*e*f - (a^3*b - a*b^3)*c*f^2)*sin(d*x
+ c) - (a*b^3*d^2*e^2 - 2*a*b^3*c*d*e*f + a*b^3*c^2*f^2 + (b^4*d^2*e^2 - 2*
b^4*c*d*e*f + b^4*c^2*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*co
s(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(
2*(a^4 - a^2*b^2)*d*e*f - 2*(a^4 - a^2*b^2)*c*f^2 + 2*((a^3*b - a*b^3)*d*e*
f - (a^3*b - a*b^3)*c*f^2)*sin(d*x + c) - (a*b^3*d^2*e^2 - 2*a*b^3*c*d*e*f
+ a*b^3*c^2*f^2 + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sq
rt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(2*(a^4 - a^2*b^2)*d*e*f - 2*(a^4 - a^2*b
^2)*c*f^2 + 2*((a^3*b - a*b^3)*d*e*f - (a^3*b - a*b^3)*c*f^2)*sin(d*x + c)
+ (a*b^3*d^2*e^2 - 2*a*b^3*c*d*e*f + a*b^3*c^2*f^2 + (b^4*d^2*e^2 - 2*b^4*c
*d*e*f + b^4*c^2*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*
x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(2*(a
^4 - a^2*b^2)*d*e*f - 2*(a^4 - a^2*b^2)*c*f^2 + 2*((a^3*b - a*b^3)*d*e*f -
(a^3*b - a*b^3)*c*f^2)*sin(d*x + c) + (a*b^3*d^2*e^2 - 2*a*b^3*c*d*e*f + a*
b^3*c^2*f^2 + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(
-(a^2 - b^2)/b^2) - 2*I*a) + 2*(2*(a^4 - a^2*b^2)*d*f^2*x + 2*(a^4 - a^2*b^
2)*c*f^2 + 2*((a^3*b - a*b^3)*d*f^2*x + (a^3*b - a*b^3)*c*f^2)*sin(d*x + c)
 - (a*b^3*d^2*f^2*x^2 + 2*a*b^3*d^2*e*f*x + 2*a*b^3*c*d*e*f - a*b^3*c^2*f^2
 + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x +
 c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b
) + 2*(2*(a^4 - a^2*b^2)*d*f^2*x + 2*(a^4 - a^2*b^2)*c*f^2 + 2*((a^3*b - a*
b^3)*d*f^2*x + (a^3*b - a*b^3)*c*f^2)*sin(d*x + c) + (a*b^3*d^2*f^2*x^2 + 2
*a*b^3*d^2*e*f*x + 2*a*b^3*c*d*e*f - a*b^3*c^2*f^2 + (b^4*d^2*f^2*x^2 + 2*b
^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2))*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) -
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(2*(a^4 - a^2*b^2)*
d*f^2*x + 2*(a^4 - a^2*b^2)*c*f^2 + 2*((a^3*b - a*b^3)*d*f^2*x + (a^3*b - a
*b^3)*c*f^2)*sin(d*x + c) - (a*b^3*d^2*f^2*x^2 + 2*a*b^3*d^2*e*f*x + 2*a*b^
3*c*d*e*f - a*b^3*c^2*f^2 + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*
e*f - b^4*c^2*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(-2*I*a*co
s(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(

```
-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(2*(a^4 - a^2*b^2)*d*f^2*x + 2*(a^4 - a^2*b
^2)*c*f^2 + 2*((a^3*b - a*b^3)*d*f^2*x + (a^3*b - a*b^3)*c*f^2)*sin(d*x + c
) + (a*b^3*d^2*f^2*x^2 + 2*a*b^3*d^2*e*f*x + 2*a*b^3*c*d*e*f - a*b^3*c^2*f^
2 + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x
 + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)
/b))/((a^4*b^2 - 2*a^2*b^4 + b^6)*d^3*sin(d*x + c) + (a^5*b - 2*a^3*b^3 + a
*b^5)*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sin(dx+c)}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

3.247 $\int \frac{(e+fx)^3 \sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=1512

result too large to display

```
[Out] ((-I)*a*(e + f*x)^3)/(b*(a^2 - b^2)*d) + (3*a*f*(e + f*x)^2*Log[1 - (I*b*E^
(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) + (I*a^2*(e + f*
x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(
3/2)*d) - (I*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]
)])/(b*Sqrt[a^2 - b^2]*d) + (3*a*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x))
)/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) - (I*a^2*(e + f*x)^3*Log[1 -
(I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (I*
(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a
^2 - b^2]*d) - ((6*I)*a*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a -
 Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) + (3*a^2*f*(e + f*x)^2*PolyLog[2, (
I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) - (3
*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*
Sqrt[a^2 - b^2]*d^2) - ((6*I)*a*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x
)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) - (3*a^2*f*(e + f*x)^2*Poly
Log[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d
^2) + (3*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2
])])/(b*Sqrt[a^2 - b^2]*d^2) + (6*a*f^3*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a
 - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^4) + ((6*I)*a^2*f^2*(e + f*x)*PolyLo
g[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3
) - ((6*I)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b
^2])])/(b*Sqrt[a^2 - b^2]*d^3) + (6*a*f^3*PolyLog[3, (I*b*E^(I*(c + d*x)))/
(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^4) - ((6*I)*a^2*f^2*(e + f*x)*Poly
Log[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d
^3) + ((6*I)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
 b^2])])/(b*Sqrt[a^2 - b^2]*d^3) - (6*a^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^4) + (6*f^3*PolyLog[4, (
I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^4) + (6*a
^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b
^2)^(3/2)*d^4) - (6*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^
2])])/(b*Sqrt[a^2 - b^2]*d^4) - (a*(e + f*x)^3*Cos[c + d*x])/((a^2 - b^2)*d
*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 3.06644, antiderivative size = 1512, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$

= 0.385, Rules used = {6742, 3324, 3323, 2264, 2190, 2531, 6609, 2282, 6589, 4519}

$$\frac{6a\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f^3}{b\left(a^2-b^2\right)d^4}+\frac{6a\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)f^3}{b\left(a^2-b^2\right)d^4}+\frac{6\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f^3}{b\sqrt{a^2-b^2}d^4}-\frac{6a^2\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f^3}{b\left(a^2-b^2\right)^{3/2}d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-I)*a*(e + f*x)^3)/(b*(a^2 - b^2)*d) + (3*a*f*(e + f*x)^2*Log[1 - (I*b*E^
(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) + (I*a^2*(e + f*
x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(
3/2)*d) - (I*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]
)])/(b*Sqrt[a^2 - b^2]*d) + (3*a*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x))
)/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) - (I*a^2*(e + f*x)^3*Log[1 -
(I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (I*
(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a
^2 - b^2]*d) - ((6*I)*a*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a -
 Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) + (3*a^2*f*(e + f*x)^2*PolyLog[2, (
I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) - (3
*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*
Sqrt[a^2 - b^2]*d^2) - ((6*I)*a*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x
)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) - (3*a^2*f*(e + f*x)^2*Poly
Log[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d
^2) + (3*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2
])])/(b*Sqrt[a^2 - b^2]*d^2) + (6*a*f^3*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a
 - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^4) + ((6*I)*a^2*f^2*(e + f*x)*PolyLo
g[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3
) - ((6*I)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b
^2])])/(b*Sqrt[a^2 - b^2]*d^3) + (6*a*f^3*PolyLog[3, (I*b*E^(I*(c + d*x)))/
(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^4) - ((6*I)*a^2*f^2*(e + f*x)*Poly
Log[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d
^3) + ((6*I)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
 b^2])])/(b*Sqrt[a^2 - b^2]*d^3) - (6*a^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^4) + (6*f^3*PolyLog[4, (
I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^4) + (6*a
^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b
^2)^(3/2)*d^4) - (6*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^
2])])/(b*Sqrt[a^2 - b^2]*d^4) - (a*(e + f*x)^3*Cos[c + d*x])/((a^2 - b^2)*d
*(a + b*Sin[c + d*x]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
```

]

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rubi steps

$$\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int\left(-\frac{a(e+fx)^3}{b(a+b\sin(c+dx))^2}+\frac{(e+fx)^3}{b(a+b\sin(c+dx))}\right)dx$$

$$= \frac{\int \frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx}{b} - \frac{a\int \frac{(e+fx)^3}{(a+b\sin(c+dx))^2}\,dx}{b}$$

$$= -\frac{a(e+fx)^3\cos(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{2\int \frac{e^{i(c+dx)}(e+fx)^3}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b} - \frac{a^2\int \frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx}{b(a^2-b^2)} + \frac{(3af}{}$$

$$= -\frac{ia(e+fx)^3}{b(a^2-b^2)\,d} - \frac{a(e+fx)^3\cos(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{(2a^2)\int \frac{e^{i(c+dx)}(e+fx)^3}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b(a^2-b^2)} - \frac{(2i)\int}{}$$

$$= -\frac{ia(e+fx)^3}{b(a^2-b^2)\,d} + \frac{3af(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)\,d^2} - \frac{i(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d} + \frac{3af(e}{}$$

$$= -\frac{ia(e+fx)^3}{b(a^2-b^2)\,d} + \frac{3af(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)\,d^2} + \frac{ia^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}\,d} - \frac{i(e}{}$$

$$= -\frac{ia(e+fx)^3}{b(a^2-b^2)\,d} + \frac{3af(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)\,d^2} + \frac{ia^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}\,d} - \frac{i(e}{}$$

$$= -\frac{ia(e+fx)^3}{b(a^2-b^2)\,d} + \frac{3af(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)\,d^2} + \frac{ia^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}\,d} - \frac{i(e}{}$$

$$= -\frac{ia(e+fx)^3}{b(a^2-b^2)\,d} + \frac{3af(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)\,d^2} + \frac{ia^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}\,d} - \frac{i(e}{}$$

$$= -\frac{ia(e+fx)^3}{b(a^2-b^2)\,d} + \frac{3af(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)\,d^2} + \frac{ia^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}\,d} - \frac{i(e}{}$$

Mathematica [B] time = 21.9973, size = 5444, normalized size = 3.6

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 1.612, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \sin(dx+c)}{(a+b\sin(dx+c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.11266, size = 11271, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(6*I*b^4*f^3*sin(d*x + c) + 6*I*a*b^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*po
lylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(-6*I*b^4*f^3*sin(d*x + c)
```

```
- 6*I*a*b^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(d*x + c)
- 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2))/b) + 2*(6*I*b^4*f^3*sin(d*x + c) + 6*I*a*b^3*f^3)*sqrt(-(a^2 - b^2)
/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*
b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(-6*I*b^4*f^3*sin(d*x + c) -
 6*I*a*b^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*si
n(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
 - 4*((a^3*b - a*b^3)*d^3*f^3*x^3 + 3*(a^3*b - a*b^3)*d^3*e*f^2*x^2 + 3*(a^
3*b - a*b^3)*d^3*e^2*f*x + (a^3*b - a*b^3)*d^3*e^3)*cos(d*x + c) + (12*I*(a
^4 - a^2*b^2)*d*f^3*x + 12*I*(a^4 - a^2*b^2)*d*e*f^2 + (12*I*(a^3*b - a*b^3
)*d*f^3*x + 12*I*(a^3*b - a*b^3)*d*e*f^2)*sin(d*x + c) + 2*(-3*I*a*b^3*d^2*
f^3*x^2 - 6*I*a*b^3*d^2*e*f^2*x - 3*I*a*b^3*d^2*e^2*f + (-3*I*b^4*d^2*f^3*x
^2 - 6*I*b^4*d^2*e*f^2*x - 3*I*b^4*d^2*e^2*f)*sin(d*x + c))*sqrt(-(a^2 - b^
2)/b^2))*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x +
 c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (12*I*(a^4 -
 a^2*b^2)*d*f^3*x + 12*I*(a^4 - a^2*b^2)*d*e*f^2 + (12*I*(a^3*b - a*b^3)*d*
f^3*x + 12*I*(a^3*b - a*b^3)*d*e*f^2)*sin(d*x + c) + 2*(3*I*a*b^3*d^2*f^3*x
^2 + 6*I*a*b^3*d^2*e*f^2*x + 3*I*a*b^3*d^2*e^2*f + (3*I*b^4*d^2*f^3*x^2 + 6
*I*b^4*d^2*e*f^2*x + 3*I*b^4*d^2*e^2*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
))*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) -
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-12*I*(a^4 - a^2*
b^2)*d*f^3*x - 12*I*(a^4 - a^2*b^2)*d*e*f^2 + (-12*I*(a^3*b - a*b^3)*d*f^3*
x - 12*I*(a^3*b - a*b^3)*d*e*f^2)*sin(d*x + c) + 2*(3*I*a*b^3*d^2*f^3*x^2 +
 6*I*a*b^3*d^2*e*f^2*x + 3*I*a*b^3*d^2*e^2*f + (3*I*b^4*d^2*f^3*x^2 + 6*I*b
^4*d^2*e*f^2*x + 3*I*b^4*d^2*e^2*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*d
ilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-12*I*(a^4 - a^2*b^2
)*d*f^3*x - 12*I*(a^4 - a^2*b^2)*d*e*f^2 + (-12*I*(a^3*b - a*b^3)*d*f^3*x -
 12*I*(a^3*b - a*b^3)*d*e*f^2)*sin(d*x + c) + 2*(-3*I*a*b^3*d^2*f^3*x^2 - 6
*I*a*b^3*d^2*e*f^2*x - 3*I*a*b^3*d^2*e^2*f + (-3*I*b^4*d^2*f^3*x^2 - 6*I*b^
4*d^2*e*f^2*x - 3*I*b^4*d^2*e^2*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*di
log(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(3*(a^4 - a^2*b^2)*d
^2*e^2*f - 6*(a^4 - a^2*b^2)*c*d*e*f^2 + 3*(a^4 - a^2*b^2)*c^2*f^3 + 3*((a^
3*b - a*b^3)*d^2*e^2*f - 2*(a^3*b - a*b^3)*c*d*e*f^2 + (a^3*b - a*b^3)*c^2*
f^3)*sin(d*x + c) - (a*b^3*d^3*e^3 - 3*a*b^3*c*d^2*e^2*f + 3*a*b^3*c^2*d*e*
f^2 - a*b^3*c^3*f^3 + (b^4*d^3*e^3 - 3*b^4*c*d^2*e^2*f + 3*b^4*c^2*d*e*f^2
- b^4*c^3*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) +
 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(3*(a^4 - a^2
*b^2)*d^2*e^2*f - 6*(a^4 - a^2*b^2)*c*d*e*f^2 + 3*(a^4 - a^2*b^2)*c^2*f^3 +
 3*((a^3*b - a*b^3)*d^2*e^2*f - 2*(a^3*b - a*b^3)*c*d*e*f^2 + (a^3*b - a*b^
3)*c^2*f^3)*sin(d*x + c) - (a*b^3*d^3*e^3 - 3*a*b^3*c*d^2*e^2*f + 3*a*b^3*c
^2*d*e*f^2 - a*b^3*c^3*f^3 + (b^4*d^3*e^3 - 3*b^4*c*d^2*e^2*f + 3*b^4*c^2*d
*e*f^2 - b^4*c^3*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x
 + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(3*(a^
```

```
4 - a^2*b^2)*d^2*e^2*f - 6*(a^4 - a^2*b^2)*c*d*e*f^2 + 3*(a^4 - a^2*b^2)*c^
2*f^3 + 3*((a^3*b - a*b^3)*d^2*e^2*f - 2*(a^3*b - a*b^3)*c*d*e*f^2 + (a^3*b
 - a*b^3)*c^2*f^3)*sin(d*x + c) + (a*b^3*d^3*e^3 - 3*a*b^3*c*d^2*e^2*f + 3*
a*b^3*c^2*d*e*f^2 - a*b^3*c^3*f^3 + (b^4*d^3*e^3 - 3*b^4*c*d^2*e^2*f + 3*b^
4*c^2*d*e*f^2 - b^4*c^3*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b
*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) +
2*(3*(a^4 - a^2*b^2)*d^2*e^2*f - 6*(a^4 - a^2*b^2)*c*d*e*f^2 + 3*(a^4 - a^2
*b^2)*c^2*f^3 + 3*((a^3*b - a*b^3)*d^2*e^2*f - 2*(a^3*b - a*b^3)*c*d*e*f^2
+ (a^3*b - a*b^3)*c^2*f^3)*sin(d*x + c) + (a*b^3*d^3*e^3 - 3*a*b^3*c*d^2*e^
2*f + 3*a*b^3*c^2*d*e*f^2 - a*b^3*c^3*f^3 + (b^4*d^3*e^3 - 3*b^4*c*d^2*e^2*
f + 3*b^4*c^2*d*e*f^2 - b^4*c^3*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*
log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2
*I*a) + 2*(3*(a^4 - a^2*b^2)*d^2*f^3*x^2 + 6*(a^4 - a^2*b^2)*d^2*e*f^2*x +
6*(a^4 - a^2*b^2)*c*d*e*f^2 - 3*(a^4 - a^2*b^2)*c^2*f^3 + 3*((a^3*b - a*b^3
)*d^2*f^3*x^2 + 2*(a^3*b - a*b^3)*d^2*e*f^2*x + 2*(a^3*b - a*b^3)*c*d*e*f^2
 - (a^3*b - a*b^3)*c^2*f^3)*sin(d*x + c) - (a*b^3*d^3*f^3*x^3 + 3*a*b^3*d^3
*e*f^2*x^2 + 3*a*b^3*d^3*e^2*f*x + 3*a*b^3*c*d^2*e^2*f - 3*a*b^3*c^2*d*e*f^
2 + a*b^3*c^3*f^3 + (b^4*d^3*f^3*x^3 + 3*b^4*d^3*e*f^2*x^2 + 3*b^4*d^3*e^2*
f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^4*c^3*f^3)*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2))*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b
*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(3*(
a^4 - a^2*b^2)*d^2*f^3*x^2 + 6*(a^4 - a^2*b^2)*d^2*e*f^2*x + 6*(a^4 - a^2*b
^2)*c*d*e*f^2 - 3*(a^4 - a^2*b^2)*c^2*f^3 + 3*((a^3*b - a*b^3)*d^2*f^3*x^2
+ 2*(a^3*b - a*b^3)*d^2*e*f^2*x + 2*(a^3*b - a*b^3)*c*d*e*f^2 - (a^3*b - a*
b^3)*c^2*f^3)*sin(d*x + c) + (a*b^3*d^3*f^3*x^3 + 3*a*b^3*d^3*e*f^2*x^2 + 3
*a*b^3*d^3*e^2*f*x + 3*a*b^3*c*d^2*e^2*f - 3*a*b^3*c^2*d*e*f^2 + a*b^3*c^3*
f^3 + (b^4*d^3*f^3*x^3 + 3*b^4*d^3*e*f^2*x^2 + 3*b^4*d^3*e^2*f*x + 3*b^4*c*
d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^4*c^3*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2))*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c)
- I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(3*(a^4 - a^2*b^2)
*d^2*f^3*x^2 + 6*(a^4 - a^2*b^2)*d^2*e*f^2*x + 6*(a^4 - a^2*b^2)*c*d*e*f^2
- 3*(a^4 - a^2*b^2)*c^2*f^3 + 3*((a^3*b - a*b^3)*d^2*f^3*x^2 + 2*(a^3*b - a
*b^3)*d^2*e*f^2*x + 2*(a^3*b - a*b^3)*c*d*e*f^2 - (a^3*b - a*b^3)*c^2*f^3)*
sin(d*x + c) - (a*b^3*d^3*f^3*x^3 + 3*a*b^3*d^3*e*f^2*x^2 + 3*a*b^3*d^3*e^2
*f*x + 3*a*b^3*c*d^2*e^2*f - 3*a*b^3*c^2*d*e*f^2 + a*b^3*c^3*f^3 + (b^4*d^3
*f^3*x^3 + 3*b^4*d^3*e*f^2*x^2 + 3*b^4*d^3*e^2*f*x + 3*b^4*c*d^2*e^2*f - 3*
b^4*c^2*d*e*f^2 + b^4*c^3*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(1/
2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(3*(a^4 - a^2*b^2)*d^2*f^3*x^2
+ 6*(a^4 - a^2*b^2)*d^2*e*f^2*x + 6*(a^4 - a^2*b^2)*c*d*e*f^2 - 3*(a^4 - a^
2*b^2)*c^2*f^3 + 3*((a^3*b - a*b^3)*d^2*f^3*x^2 + 2*(a^3*b - a*b^3)*d^2*e*f
^2*x + 2*(a^3*b - a*b^3)*c*d*e*f^2 - (a^3*b - a*b^3)*c^2*f^3)*sin(d*x + c)
+ (a*b^3*d^3*f^3*x^3 + 3*a*b^3*d^3*e*f^2*x^2 + 3*a*b^3*d^3*e^2*f*x + 3*a*b^
3*c*d^2*e^2*f - 3*a*b^3*c^2*d*e*f^2 + a*b^3*c^3*f^3 + (b^4*d^3*f^3*x^3 + 3*
b^4*d^3*e*f^2*x^2 + 3*b^4*d^3*e^2*f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*c^2*d*e*f
```

```
^2 + b^4*c^3*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(-2*I*a*cos
(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) + 2*b)/b) + 12*((a^3*b - a*b^3)*f^3*sin(d*x + c) + (a^4 -
a^2*b^2)*f^3 + (a*b^3*d*f^3*x + a*b^3*d*e*f^2 + (b^4*d*f^3*x + b^4*d*e*f^2)
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*polylog(3, 1/2*(2*I*a*cos(d*x + c) -
 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2))/b) + 12*((a^3*b - a*b^3)*f^3*sin(d*x + c) + (a^4 - a^2*b^2)*f^3 - (a
*b^3*d*f^3*x + a*b^3*d*e*f^2 + (b^4*d*f^3*x + b^4*d*e*f^2)*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2))*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c)
 - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*((
a^3*b - a*b^3)*f^3*sin(d*x + c) + (a^4 - a^2*b^2)*f^3 - (a*b^3*d*f^3*x + a*
b^3*d*e*f^2 + (b^4*d*f^3*x + b^4*d*e*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2))*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*((a^3*b - a*b^3)*f^3*sin(d*x
 + c) + (a^4 - a^2*b^2)*f^3 + (a*b^3*d*f^3*x + a*b^3*d*e*f^2 + (b^4*d*f^3*x
 + b^4*d*e*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*polylog(3, -(I*a*cos(
d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2))/b))/((a^4*b^2 - 2*a^2*b^4 + b^6)*d^4*sin(d*x + c) + (a^5*b - 2
*a^3*b^3 + a*b^5)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \sin(dx+c)}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

3.248 $\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=751

$$\frac{3a^3f\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2bd^2\left(a^2-b^2\right)^{5/2}}-\frac{3a^3f\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{2bd^2\left(a^2-b^2\right)^{5/2}}-\frac{3af\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2bd^2\left(a^2-b^2\right)^{3/2}}+\frac{3af\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{2bd^2\left(a^2-b^2\right)^{3/2}}-$$

```
[Out] (((3*I)/2)*a^3*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]
)])/(b*(a^2 - b^2)^(5/2)*d) - (((3*I)/2)*a*(e + f*x)*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) - (((3*I)/2)*a^3*(e
 + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2
)^(5/2)*d) + (((3*I)/2)*a*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (3*a^2*f*Log[a + b*Sin[c + d*x]])/
(2*b*(a^2 - b^2)^2*d^2) - (f*Log[a + b*Sin[c + d*x]])/(b*(a^2 - b^2)*d^2) +
 (3*a^3*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(2*b*(a^
2 - b^2)^(5/2)*d^2) - (3*a*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
 - b^2])])/(2*b*(a^2 - b^2)^(3/2)*d^2) - (3*a^3*f*PolyLog[2, (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^(5/2)*d^2) + (3*a*f*PolyLo
g[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^(3/2)*d
^2) - (a*(e + f*x)*Cos[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) -
 (a*f)/(2*b*(a^2 - b^2)*d^2*(a + b*Sin[c + d*x])) - (3*a^2*(e + f*x)*Cos[c
+ d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) + ((e + f*x)*Cos[c + d*x])
/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 2.95513, antiderivative size = 751, normalized size of antiderivative = 1., number of steps used = 48, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6742, 3325, 3324, 3323, 2264, 2190, 2279, 2391, 2668, 31, 32}

$$\frac{3a^3f\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2bd^2\left(a^2-b^2\right)^{5/2}}-\frac{3a^3f\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{2bd^2\left(a^2-b^2\right)^{5/2}}-\frac{3af\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2bd^2\left(a^2-b^2\right)^{3/2}}+\frac{3af\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{2bd^2\left(a^2-b^2\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (((3*I)/2)*a^3*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]
)])/(b*(a^2 - b^2)^(5/2)*d) - (((3*I)/2)*a*(e + f*x)*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) - (((3*I)/2)*a^3*(e
 + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2
```

```
)^(5/2)*d) + (((3*I)/2)*a*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (3*a^2*f*Log[a + b*Sin[c + d*x]])/
(2*b*(a^2 - b^2)^2*d^2) - (f*Log[a + b*Sin[c + d*x]])/(b*(a^2 - b^2)*d^2) +
 (3*a^3*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(2*b*(a^
2 - b^2)^(5/2)*d^2) - (3*a*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
 - b^2])])/(2*b*(a^2 - b^2)^(3/2)*d^2) - (3*a^3*f*PolyLog[2, (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^(5/2)*d^2) + (3*a*f*PolyLo
g[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^(3/2)*d
^2) - (a*(e + f*x)*Cos[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) -
 (a*f)/(2*b*(a^2 - b^2)*d^2*(a + b*Sin[c + d*x])) - (3*a^2*(e + f*x)*Cos[c
+ d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) + ((e + f*x)*Cos[c + d*x])
/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3325

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> -Simp[(b*(c + d*x)^m*Cos[e + f*x]*(a + b*Sin[e + f*x])^(n + 1
))/(f*(n + 1)*(a^2 - b^2)), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m*(a +
b*Sin[e + f*x])^(n + 1), x], x] - Dist[(b*(n + 2))/((n + 1)*(a^2 - b^2)), I
nt[(c + d*x)^m*Sin[e + f*x]*(a + b*Sin[e + f*x])^(n + 1), x], x] + Dist[(b*
d*m)/(f*(n + 1)*(a^2 - b^2)), Int[(c + d*x)^(m - 1)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && ILtQ[n, -2] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(-\frac{a(e+fx)}{b(a+b\sin(c+dx))^3} + \frac{e+fx}{b(a+b\sin(c+dx))^2}\right) dx$$

$$= \frac{\int \frac{e+fx}{(a+b\sin(c+dx))^2}\,dx}{b} - \frac{a\int \frac{e+fx}{(a+b\sin(c+dx))^3}\,dx}{b}$$

$$= -\frac{a(e+fx)\cos(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{(e+fx)\cos(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{a\int \frac{(e+fx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{2\left(a^2-b^2\right)} + \cdots$$

$$= -\frac{a(e+fx)\cos(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{a^2(e+fx)\cos(c+dx)}{\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{(e+fx)\cos(c+\cdots}{\left(a^2-b^2\right)d(a+b\sin(c\cdots}$$

$$= -\frac{f\log(a+b\sin(c+dx))}{b\left(a^2-b^2\right)d^2} - \frac{a(e+fx)\cos(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{af}{2b\left(a^2-b^2\right)d^2(a+b\sin(c+\cdots}$$

$$= -\frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} + \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} + \frac{a^2f\log(a+b\sin(c+dx))}{b\left(a^2-b^2\right)^2d^2}$$

$$= \frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}d} - \frac{ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} - \frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\cdots\right.}{b\left(a^2-b^2\right)^{5/2}d}$$

$$= \frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}d} - \frac{3ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{3/2}d} - \frac{ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+d\cdots}}{a+\sqrt{a^2-\cdots}}\right.}{b\left(a^2-b^2\right)^{5/2}d}$$

$$= \frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{5/2}d} - \frac{3ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{3/2}d} - \frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(\cdots}}{a+\sqrt{\cdots}}\right.}{2b\left(a^2-b^2\right)^{5/2}d}$$

$$= \frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{5/2}d} - \frac{3ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{3/2}d} - \frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(\cdots}}{a+\sqrt{\cdots}}\right.}{2b\left(a^2-b^2\right)^{5/2}d}$$

$$= \frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{5/2}d} - \frac{3ia(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{3/2}d} - \frac{3ia^3(e+fx)\log\left(1-\frac{ibe^{i(\cdots}}{a+\sqrt{\cdots}}\right.}{2b\left(a^2-b^2\right)^{5/2}d}$$

Mathematica [B] time = 15.4607, size = 2408, normalized size = 3.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-(a*d*e*Cos[c + d*x]) + a*c*f*Cos[c + d*x] - a*f*(c + d*x)*Cos[c + d*x])/(
2*(a - b)*(a + b)*d^2*(a + b*Sin[c + d*x])^2) + (-(a^3*f) + a*b^2*f - a^2*b
*d*e*Cos[c + d*x] - 2*b^3*d*e*Cos[c + d*x] + a^2*b*c*f*Cos[c + d*x] + 2*b^3
*c*f*Cos[c + d*x] - a^2*b*f*(c + d*x)*Cos[c + d*x] - 2*b^3*f*(c + d*x)*Cos[
c + d*x])/(2*(a - b)^2*b*(a + b)^2*d^2*(a + b*Sin[c + d*x])) + (((-2*(a^2 +
 2*b^2)*f*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
 + (2*(a^2*f + 2*b^2*f + a*b*(-3*d*e + 3*c*f))*ArcTan[(b + a*Tan[(c + d*x)/
2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - ((a^2 + 2*b^2)*f*Log[Sec[(c + d*x)/
2]^2])/b + ((a^2 + 2*b^2)*f*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])])/b
 + ((3*I)*a*b*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*
Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 - I*T
an[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] - ((3*
I)*a*b*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c
+ d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(c + d*
x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] - ((3*I)*a*b*f*(
Log[1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2
])/(I*a - b + Sqrt[-a^2 + b^2]))] + PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(
I*a - b + Sqrt[-a^2 + b^2])]))/Sqrt[-a^2 + b^2] + ((3*I)*a*b*f*(Log[1 + I*T
an[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b -
 Sqrt[-a^2 + b^2])] + PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sq
rt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2])*((-3*a*b*e)/(2*(a^2 - b^2)^2*(a + b*S
in[c + d*x])) + (3*a*b*c*f)/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (3*a
*b*f*(c + d*x))/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) + (a^2*f*Cos[c + d
*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) + (b^2*f*Cos[c + d*x])/((a^2
- b^2)^2*d*(a + b*Sin[c + d*x]))))/(d*(-(((a^2 + 2*b^2)*f*Tan[(c + d*x)/2])
/b) + ((a^2 + 2*b^2)*f*Cos[(c + d*x)/2]^2*(b*Cos[c + d*x]*Sec[(c + d*x)/2]^
2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c + d*x)/2]))/(b*(a + b*Si
n[c + d*x])) - (a*(a^2 + 2*b^2)*f*Sec[(c + d*x)/2]^2)/((a^2 - b^2)*(1 + (b
+ a*Tan[(c + d*x)/2])^2/(a^2 - b^2))) + (a*(a^2*f + 2*b^2*f + a*b*(-3*d*e +
 3*c*f))*Sec[(c + d*x)/2]^2)/((a^2 - b^2)*(1 + (b + a*Tan[(c + d*x)/2])^2/(
a^2 - b^2))) - ((3*I)*a*b*f*(((-I/2)*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c
 + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(
c + d*x)/2]) - (Log[1 - (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b
^2])]*Sec[(c + d*x)/2]^2)/(2*(I + Tan[(c + d*x)/2])) + (a*Log[1 - I*Tan[(c
+ d*x)/2]]*Sec[(c + d*x)/2]^2)/(2*(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2
]))))/Sqrt[-a^2 + b^2] + ((3*I)*a*b*f*(((I/2)*Log[(b - Sqrt[-a^2 + b^2] + a
```

```
*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 + I
*Tan[(c + d*x)/2]) - ((I/2)*a*Log[1 - (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b
 + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(a + I*a*Tan[(c + d*x)/2]) + (a*
Log[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(2*(b - Sqrt[-a^2 + b^2] +
a*Tan[(c + d*x)/2]))))/Sqrt[-a^2 + b^2] + ((3*I)*a*b*f*(((I/2)*Log[1 - (a*(
1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2
)/(1 - I*Tan[(c + d*x)/2]) - ((I/2)*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c +
d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c
 + d*x)/2]) + (a*Log[1 - I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(2*(b + Sq
rt[-a^2 + b^2] + a*Tan[(c + d*x)/2]))))/Sqrt[-a^2 + b^2] - ((3*I)*a*b*f*(((
-I/2)*Log[1 - (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]*
Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) + ((I/2)*Log[(b + Sqrt[-a^2 +
b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2
)/(1 + I*Tan[(c + d*x)/2]) + (a*Log[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2
]^2)/(2*(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]))))/Sqrt[-a^2 + b^2]))
```

Maple [A] time = 1.699, size = 1084, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] I*(4*I*b*a^3*d*f*x*exp(I*(d*x+c))+5*I*b^3*a*d*f*x*exp(I*(d*x+c))+4*I*b*a^3*
d*e*exp(I*(d*x+c))-3*I*b^3*a*d*e*exp(3*I*(d*x+c))+2*a^4*d*f*x*exp(2*I*(d*x+
c))+5*b^2*d*f*x*exp(2*I*(d*x+c))*a^2+2*b^4*d*f*x*exp(2*I*(d*x+c))-2*I*b^2*f
*exp(2*I*(d*x+c))*a^2+5*I*b^3*a*d*e*exp(I*(d*x+c))-3*I*b^3*a*d*f*x*exp(3*I*
(d*x+c))+2*I*a^4*f*exp(2*I*(d*x+c))+2*a^4*d*e*exp(2*I*(d*x+c))+b*a^3*f*exp(
3*I*(d*x+c))+5*b^2*d*e*exp(2*I*(d*x+c))*a^2-b^3*a*f*exp(3*I*(d*x+c))+2*b^4*
d*e*exp(2*I*(d*x+c))-a^2*b^2*d*f*x-2*b^4*d*f*x-b*a^3*f*exp(I*(d*x+c))-a^2*b
^2*d*e+b^3*a*f*exp(I*(d*x+c))-2*b^4*d*e)/(I*b+2*a*exp(I*(d*x+c))-I*b*exp(2*
I*(d*x+c)))^2/(a^2-b^2)^2/d^2/b-1/(-a^2+b^2)^2/d^2/b*a^2*f*ln(exp(I*(d*x+c)
))+1/2/(-a^2+b^2)^2/d^2/b*a^2*f*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-
I*b)-2/(-a^2+b^2)^2/d^2*b*f*ln(exp(I*(d*x+c)))+1/(-a^2+b^2)^2/d^2*b*f*ln(I*
b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+3/2*I/(-a^2+b^2)^(5/2)/d^2*b*a*f
*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-3*I/
(-a^2+b^2)^(5/2)/d*b*a*e*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(
1/2))+3*I/(-a^2+b^2)^(5/2)/d^2*b*a*f*c*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a
)/(-a^2+b^2)^(1/2))-3/2*I/(-a^2+b^2)^(5/2)/d^2*b*a*f*dilog((I*a+b*exp(I*(d*
x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+3/2/(-a^2+b^2)^(5/2)/d*b*a*
f*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+3/2/
```

```
(-a^2+b^2)^(5/2)/d^2*b*a*f*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+
(-a^2+b^2)^(1/2)))*c-3/2/(-a^2+b^2)^(5/2)/d*b*a*f*ln((I*a+b*exp(I*(d*x+c))-
(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-3/2/(-a^2+b^2)^(5/2)/d^2*b*a*f*
ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.517, size = 5449, normalized size = 7.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((-3*I*a*b^5*f*cos(d*x + c)^2 + 6*I*a^2*b^4*f*sin(d*x + c) + 3*I*(a^3*b
^3 + a*b^5)*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*
sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
 + 2*b)/b + 1) + (3*I*a*b^5*f*cos(d*x + c)^2 - 6*I*a^2*b^4*f*sin(d*x + c) -
 3*I*(a^3*b^3 + a*b^5)*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x
+ c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) + 2*b)/b + 1) + (3*I*a*b^5*f*cos(d*x + c)^2 - 6*I*a^2*b^4*f*sin
(d*x + c) - 3*I*(a^3*b^3 + a*b^5)*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*
I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-3*I*a*b^5*f*cos(d*x + c)^2 + 6*I*
a^2*b^4*f*sin(d*x + c) + 3*I*(a^3*b^3 + a*b^5)*f)*sqrt(-(a^2 - b^2)/b^2)*di
log(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 3*((a^3*b^3 + a*b^5)*d
*f*x + (a^3*b^3 + a*b^5)*c*f - (a*b^5*d*f*x + a*b^5*c*f)*cos(d*x + c)^2 + 2
*(a^2*b^4*d*f*x + a^2*b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(1/2
*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 3*((a^3*b^3 + a*b^5)*d*f*x + (a^3*b
```

```
^3 + a*b^5)*c*f - (a*b^5*d*f*x + a*b^5*c*f)*cos(d*x + c)^2 + 2*(a^2*b^4*d*f
*x + a^2*b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d
*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2) + 2*b)/b) + 3*((a^3*b^3 + a*b^5)*d*f*x + (a^3*b^3 + a*b^5)*c
*f - (a*b^5*d*f*x + a*b^5*c*f)*cos(d*x + c)^2 + 2*(a^2*b^4*d*f*x + a^2*b^4*
c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*
a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2) + 2*b)/b) - 3*((a^3*b^3 + a*b^5)*d*f*x + (a^3*b^3 + a*b^5)*c*f - (a*b^5*
d*f*x + a*b^5*c*f)*cos(d*x + c)^2 + 2*(a^2*b^4*d*f*x + a^2*b^4*c*f)*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x +
c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)
 + 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*f + 2*((2*a^5*b - a^3*b^3 - a*b^5)*d*f*x +
 (2*a^5*b - a^3*b^3 - a*b^5)*d*e)*cos(d*x + c) + ((a^4*b^2 + a^2*b^4 - 2*b^
6)*f*cos(d*x + c)^2 - 2*(a^5*b + a^3*b^3 - 2*a*b^5)*f*sin(d*x + c) - (a^6 +
 2*a^4*b^2 - a^2*b^4 - 2*b^6)*f + 3*((a^3*b^3 + a*b^5)*d*e - (a^3*b^3 + a*b
^5)*c*f - (a*b^5*d*e - a*b^5*c*f)*cos(d*x + c)^2 + 2*(a^2*b^4*d*e - a^2*b^4
*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) + 2*I*b*si
n(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + ((a^4*b^2 + a^2*b^4 - 2*
b^6)*f*cos(d*x + c)^2 - 2*(a^5*b + a^3*b^3 - 2*a*b^5)*f*sin(d*x + c) - (a^6
 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*f + 3*((a^3*b^3 + a*b^5)*d*e - (a^3*b^3 + a
*b^5)*c*f - (a*b^5*d*e - a*b^5*c*f)*cos(d*x + c)^2 + 2*(a^2*b^4*d*e - a^2*b
^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) - 2*I*b*
sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + ((a^4*b^2 + a^2*b^4 -
2*b^6)*f*cos(d*x + c)^2 - 2*(a^5*b + a^3*b^3 - 2*a*b^5)*f*sin(d*x + c) - (a
^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*f - 3*((a^3*b^3 + a*b^5)*d*e - (a^3*b^3 +
 a*b^5)*c*f - (a*b^5*d*e - a*b^5*c*f)*cos(d*x + c)^2 + 2*(a^2*b^4*d*e - a^2
*b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) + 2*I
*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + ((a^4*b^2 + a^2*b^4
 - 2*b^6)*f*cos(d*x + c)^2 - 2*(a^5*b + a^3*b^3 - 2*a*b^5)*f*sin(d*x + c) -
 (a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*f - 3*((a^3*b^3 + a*b^5)*d*e - (a^3*b^
3 + a*b^5)*c*f - (a*b^5*d*e - a*b^5*c*f)*cos(d*x + c)^2 + 2*(a^2*b^4*d*e -
a^2*b^4*c*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) -
2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*((a^5*b - 2*a^
3*b^3 + a*b^5)*f + ((a^4*b^2 + a^2*b^4 - 2*b^6)*d*f*x + (a^4*b^2 + a^2*b^4
- 2*b^6)*d*e)*cos(d*x + c))*sin(d*x + c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7
 - b^9)*d^2*cos(d*x + c)^2 - 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d^
2*sin(d*x + c) - (a^8*b - 2*a^6*b^3 + 2*a^2*b^7 - b^9)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\sin(dx+c)}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

3.249 $\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=1584

result too large to display

```
[Out] (((-3*I)/2)*a^2*(e + f*x)^2)/(b*(a^2 - b^2)^2*d) + (I*(e + f*x)^2)/(b*(a^2
- b^2)*d) + (2*a*f^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(
a^2 - b^2)^(3/2)*d^3) + (3*a^2*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a
 - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^2) - (2*f*(e + f*x)*Log[1 - (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) + (((3*I)/2)*a^
3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2
 - b^2)^(5/2)*d) - (((3*I)/2)*a*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(
a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (3*a^2*f*(e + f*x)*Log[1 -
 (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^2) - (2*f
*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 -
b^2)*d^2) - (((3*I)/2)*a^3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + S
qrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d) + (((3*I)/2)*a*(e + f*x)^2*Log[1
- (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) - (
(3*I)*a^2*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(
a^2 - b^2)^2*d^3) + ((2*I)*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a
^2 - b^2])])/(b*(a^2 - b^2)*d^3) + (3*a^3*f*(e + f*x)*PolyLog[2, (I*b*E^(I*
(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d^2) - (3*a*f*(e +
 f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^
2)^(3/2)*d^2) - ((3*I)*a^2*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a
^2 - b^2])])/(b*(a^2 - b^2)^2*d^3) + ((2*I)*f^2*PolyLog[2, (I*b*E^(I*(c + d
*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) - (3*a^3*f*(e + f*x)*Poly
Log[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d
^2) + (3*a*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2
])])/(b*(a^2 - b^2)^(3/2)*d^2) + ((3*I)*a^3*f^2*PolyLog[3, (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d^3) - ((3*I)*a*f^2*Poly
Log[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d
^3) - ((3*I)*a^3*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])
])/(b*(a^2 - b^2)^(5/2)*d^3) + ((3*I)*a*f^2*PolyLog[3, (I*b*E^(I*(c + d*x))
)/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) - (a*(e + f*x)^2*Cos[c
+ d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) - (a*f*(e + f*x))/(b*(a^2
- b^2)*d^2*(a + b*Sin[c + d*x])) - (3*a^2*(e + f*x)^2*Cos[c + d*x])/(2*(a^2
 - b^2)^2*d*(a + b*Sin[c + d*x])) + ((e + f*x)^2*Cos[c + d*x])/((a^2 - b^2)
*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 5.94317, antiderivative size = 1584, normalized size of antiderivative = 1., number of steps used = 73, number of rules used = 16, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ =

0.615, Rules used = {6742, 3325, 3324, 3323, 2264, 2190, 2531, 2282, 6589, 4519, 2279, 2391, 4422, 2660, 618, 204}

$$\frac{3i(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^3}{2b\left(a^2-b^2\right)^{5/2}d}-\frac{3i(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)a^3}{2b\left(a^2-b^2\right)^{5/2}d}+\frac{3f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)a^3}{b\left(a^2-b^2\right)^{5/2}d^2}-\frac{3f(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (((-3*I)/2)*a^2*(e + f*x)^2)/(b*(a^2 - b^2)^2*d) + (I*(e + f*x)^2)/(b*(a^2
- b^2)*d) + (2*a*f^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(
a^2 - b^2)^(3/2)*d^3) + (3*a^2*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a
 - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^2) - (2*f*(e + f*x)*Log[1 - (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) + (((3*I)/2)*a^
3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2
 - b^2)^(5/2)*d) - (((3*I)/2)*a*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(
a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + (3*a^2*f*(e + f*x)*Log[1 -
 (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^2) - (2*f
*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 -
b^2)*d^2) - (((3*I)/2)*a^3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + S
qrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d) + (((3*I)/2)*a*(e + f*x)^2*Log[1
- (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) - (
(3*I)*a^2*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(
a^2 - b^2)^2*d^3) + ((2*I)*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a
^2 - b^2])])/(b*(a^2 - b^2)*d^3) + (3*a^3*f*(e + f*x)*PolyLog[2, (I*b*E^(I*
(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d^2) - (3*a*f*(e +
 f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^
2)^(3/2)*d^2) - ((3*I)*a^2*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a
^2 - b^2])])/(b*(a^2 - b^2)^2*d^3) + ((2*I)*f^2*PolyLog[2, (I*b*E^(I*(c + d
*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) - (3*a^3*f*(e + f*x)*Poly
Log[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d
^2) + (3*a*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2
])])/(b*(a^2 - b^2)^(3/2)*d^2) + ((3*I)*a^3*f^2*PolyLog[3, (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d^3) - ((3*I)*a*f^2*Poly
Log[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d
^3) - ((3*I)*a^3*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])
])/(b*(a^2 - b^2)^(5/2)*d^3) + ((3*I)*a*f^2*PolyLog[3, (I*b*E^(I*(c + d*x))
)/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) - (a*(e + f*x)^2*Cos[c
+ d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) - (a*f*(e + f*x))/(b*(a^2
- b^2)*d^2*(a + b*Sin[c + d*x])) - (3*a^2*(e + f*x)^2*Cos[c + d*x])/(2*(a^2
 - b^2)^2*d*(a + b*Sin[c + d*x])) + ((e + f*x)^2*Cos[c + d*x])/((a^2 - b^2)
*d*(a + b*Sin[c + d*x]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3325

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> -Simp[(b*(c + d*x)^m*Cos[e + f*x]*(a + b*Sin[e + f*x])^(n + 1
))/(f*(n + 1)*(a^2 - b^2)), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m*(a +
b*Sin[e + f*x])^(n + 1), x], x] - Dist[(b*(n + 2))/((n + 1)*(a^2 - b^2)), I
nt[(c + d*x)^m*Sin[e + f*x]*(a + b*Sin[e + f*x])^(n + 1), x], x] + Dist[(b*
d*m)/(f*(n + 1)*(a^2 - b^2)), Int[(c + d*x)^(m - 1)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && ILtQ[n, -2] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int\left(-\frac{a(e+fx)^2}{b(a+b\sin(c+dx))^3}+\frac{(e+fx)^2}{b(a+b\sin(c+dx))^2}\right)dx$$

$$= \frac{\int \frac{(e+fx)^2}{(a+b\sin(c+dx))^2}\,dx}{b} - \frac{a\int \frac{(e+fx)^2}{(a+b\sin(c+dx))^3}\,dx}{b}$$

$$= -\frac{a(e+fx)^2\cos(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2}+\frac{(e+fx)^2\cos(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))}+\frac{a\int \frac{(e+fx)^2\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{i(e+fx)^2}{b\left(a^2-b^2\right)d}-\frac{a(e+fx)^2\cos(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2}-\frac{af(e+fx)}{b\left(a^2-b^2\right)d^2(a+b\sin(c+dx))}-$$

$$= -\frac{ia^2(e+fx)^2}{b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^2}{b\left(a^2-b^2\right)d}-\frac{2f(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^2}-\frac{2f(e+fx)\log\left(1-\right.}{b\left(a^2-b^2\right)d^2}$$

$$= -\frac{ia^2(e+fx)^2}{b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^2}{b\left(a^2-b^2\right)d}+\frac{2a^2f(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^2d^2}-\frac{2f(e+fx)\log\left(1-\right.}{b\left(a^2-b^2\right)d^2}$$

$$= -\frac{3ia^2(e+fx)^2}{2b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^2}{b\left(a^2-b^2\right)d}+\frac{2af^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^3}+\frac{2a^2f(e+fx)\log\left(1-\right.}{b\left(a^2-b^2\right)^2d^2}$$

$$= -\frac{3ia^2(e+fx)^2}{2b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^2}{b\left(a^2-b^2\right)d}+\frac{2af^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^3}+\frac{3a^2f(e+fx)\log\left(1-\right.}{b\left(a^2-b^2\right)^2d^2}$$

$$= -\frac{3ia^2(e+fx)^2}{2b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^2}{b\left(a^2-b^2\right)d}+\frac{2af^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^3}+\frac{3a^2f(e+fx)\log\left(1-\right.}{b\left(a^2-b^2\right)^2d^2}$$

$$= -\frac{3ia^2(e+fx)^2}{2b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^2}{b\left(a^2-b^2\right)d}+\frac{2af^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^3}+\frac{3a^2f(e+fx)\log\left(1-\right.}{b\left(a^2-b^2\right)^2d^2}$$

$$= -\frac{3ia^2(e+fx)^2}{2b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^2}{b\left(a^2-b^2\right)d}+\frac{2af^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^3}+\frac{3a^2f(e+fx)\log\left(1-\right.}{b\left(a^2-b^2\right)^2d^2}$$

$$= -\frac{3ia^2(e+fx)^2}{2b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^2}{b\left(a^2-b^2\right)d}+\frac{2af^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^3}+\frac{3a^2f(e+fx)\log\left(1-\right.}{b\left(a^2-b^2\right)^2d^2}$$

Mathematica [B] time = 25.016, size = 13567, normalized size = 8.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 3.147, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \sin \left(dx+c\right)}{\left(a+b \sin \left(dx+c\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.4687, size = 12407, normalized size = 7.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/8*(8*(a^6 - 2*a^4*b^2 + a^2*b^4)*d*f^2*x + 8*(a^6 - 2*a^4*b^2 + a^2*b^4)*
d*e*f + 12*(a*b^5*f^2*cos(d*x + c)^2 - 2*a^2*b^4*f^2*sin(d*x + c) - (a^3*b^
3 + a*b^5)*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) -
 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2))/b) - 12*(a*b^5*f^2*cos(d*x + c)^2 - 2*a^2*b^4*f^2*sin(d*x + c) - (a^
3*b^3 + a*b^5)*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x +
c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2))/b) - 12*(a*b^5*f^2*cos(d*x + c)^2 - 2*a^2*b^4*f^2*sin(d*x + c) -
 (a^3*b^3 + a*b^5)*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c
) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2))/b) + 12*(a*b^5*f^2*cos(d*x + c)^2 - 2*a^2*b^4*f^2*sin(d*x + c) - (a^3
*b^3 + a*b^5)*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a
*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))
/b) + 4*((2*a^5*b - a^3*b^3 - a*b^5)*d^2*f^2*x^2 + 2*(2*a^5*b - a^3*b^3 - a
*b^5)*d^2*e*f*x + (2*a^5*b - a^3*b^3 - a*b^5)*d^2*e^2)*cos(d*x + c) + (4*I*
(a^4*b^2 + a^2*b^4 - 2*b^6)*f^2*cos(d*x + c)^2 - 8*I*(a^5*b + a^3*b^3 - 2*a
*b^5)*f^2*sin(d*x + c) - 4*I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*f^2 + 2*(6
*I*(a^3*b^3 + a*b^5)*d*f^2*x + 6*I*(a^3*b^3 + a*b^5)*d*e*f + (-6*I*a*b^5*d*
f^2*x - 6*I*a*b^5*d*e*f)*cos(d*x + c)^2 + (12*I*a^2*b^4*d*f^2*x + 12*I*a^2*
b^4*d*e*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog(-1/2*(2*I*a*cos(d*x
+ c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) + 2*b)/b + 1) + (4*I*(a^4*b^2 + a^2*b^4 - 2*b^6)*f^2*cos(d*x +
c)^2 - 8*I*(a^5*b + a^3*b^3 - 2*a*b^5)*f^2*sin(d*x + c) - 4*I*(a^6 + 2*a^4*
b^2 - a^2*b^4 - 2*b^6)*f^2 + 2*(-6*I*(a^3*b^3 + a*b^5)*d*f^2*x - 6*I*(a^3*b
^3 + a*b^5)*d*e*f + (6*I*a*b^5*d*f^2*x + 6*I*a*b^5*d*e*f)*cos(d*x + c)^2 +
(-12*I*a^2*b^4*d*f^2*x - 12*I*a^2*b^4*d*e*f)*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2))*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x +
c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-4*I*(a^4*b^
2 + a^2*b^4 - 2*b^6)*f^2*cos(d*x + c)^2 + 8*I*(a^5*b + a^3*b^3 - 2*a*b^5)*f
^2*sin(d*x + c) + 4*I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*f^2 + 2*(-6*I*(a^
3*b^3 + a*b^5)*d*f^2*x - 6*I*(a^3*b^3 + a*b^5)*d*e*f + (6*I*a*b^5*d*f^2*x +
 6*I*a*b^5*d*e*f)*cos(d*x + c)^2 + (-12*I*a^2*b^4*d*f^2*x - 12*I*a^2*b^4*d*
e*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog(-1/2*(-2*I*a*cos(d*x + c)
+ 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) + 2*b)/b + 1) + (-4*I*(a^4*b^2 + a^2*b^4 - 2*b^6)*f^2*cos(d*x + c)^2
 + 8*I*(a^5*b + a^3*b^3 - 2*a*b^5)*f^2*sin(d*x + c) + 4*I*(a^6 + 2*a^4*b^2
- a^2*b^4 - 2*b^6)*f^2 + 2*(6*I*(a^3*b^3 + a*b^5)*d*f^2*x + 6*I*(a^3*b^3 +
a*b^5)*d*e*f + (-6*I*a*b^5*d*f^2*x - 6*I*a*b^5*d*e*f)*cos(d*x + c)^2 + (12*
I*a^2*b^4*d*f^2*x + 12*I*a^2*b^4*d*e*f)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
))*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) +
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(2*(a^6 + 2*a^4
*b^2 - a^2*b^4 - 2*b^6)*d*e*f - 2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f^2
```

- 2*((a^4*b^2 + a^2*b^4 - 2*b^6)*d*e*f - (a^4*b^2 + a^2*b^4 - 2*b^6)*c*f^2
)*cos(d*x + c)^2 + 4*((a^5*b + a^3*b^3 - 2*a*b^5)*d*e*f - (a^5*b + a^3*b^3
- 2*a*b^5)*c*f^2)*sin(d*x + c) - (3*(a^3*b^3 + a*b^5)*d^2*e^2 - 6*(a^3*b^3
+ a*b^5)*c*d*e*f - (2*a^5*b - 2*a*b^5 - 3*(a^3*b^3 + a*b^5)*c^2)*f^2 - (3*a
*b^5*d^2*e^2 - 6*a*b^5*c*d*e*f + (3*a*b^5*c^2 - 2*a^3*b^3 + 2*a*b^5)*f^2)*c
os(d*x + c)^2 + 2*(3*a^2*b^4*d^2*e^2 - 6*a^2*b^4*c*d*e*f + (3*a^2*b^4*c^2 -
2*a^4*b^2 + 2*a^2*b^4)*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*
cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2
(2(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e*f - 2*(a^6 + 2*a^4*b^2 - a^2*b^
4 - 2*b^6)*c*f^2 - 2*((a^4*b^2 + a^2*b^4 - 2*b^6)*d*e*f - (a^4*b^2 + a^2*b^
4 - 2*b^6)*c*f^2)*cos(d*x + c)^2 + 4*((a^5*b + a^3*b^3 - 2*a*b^5)*d*e*f - (
a^5*b + a^3*b^3 - 2*a*b^5)*c*f^2)*sin(d*x + c) - (3*(a^3*b^3 + a*b^5)*d^2*e
^2 - 6*(a^3*b^3 + a*b^5)*c*d*e*f - (2*a^5*b - 2*a*b^5 - 3*(a^3*b^3 + a*b^5)
*c^2)*f^2 - (3*a*b^5*d^2*e^2 - 6*a*b^5*c*d*e*f + (3*a*b^5*c^2 - 2*a^3*b^3 +
2*a*b^5)*f^2)*cos(d*x + c)^2 + 2*(3*a^2*b^4*d^2*e^2 - 6*a^2*b^4*c*d*e*f +
(3*a^2*b^4*c^2 - 2*a^4*b^2 + 2*a^2*b^4)*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2))*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b
^2) - 2*I*a) - 2*(2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e*f - 2*(a^6 + 2*
a^4*b^2 - a^2*b^4 - 2*b^6)*c*f^2 - 2*((a^4*b^2 + a^2*b^4 - 2*b^6)*d*e*f - (
a^4*b^2 + a^2*b^4 - 2*b^6)*c*f^2)*cos(d*x + c)^2 + 4*((a^5*b + a^3*b^3 - 2*
a*b^5)*d*e*f - (a^5*b + a^3*b^3 - 2*a*b^5)*c*f^2)*sin(d*x + c) + (3*(a^3*b^
3 + a*b^5)*d^2*e^2 - 6*(a^3*b^3 + a*b^5)*c*d*e*f - (2*a^5*b - 2*a*b^5 - 3*(
a^3*b^3 + a*b^5)*c^2)*f^2 - (3*a*b^5*d^2*e^2 - 6*a*b^5*c*d*e*f + (3*a*b^5*c
^2 - 2*a^3*b^3 + 2*a*b^5)*f^2)*cos(d*x + c)^2 + 2*(3*a^2*b^4*d^2*e^2 - 6*a^
2*b^4*c*d*e*f + (3*a^2*b^4*c^2 - 2*a^4*b^2 + 2*a^2*b^4)*f^2)*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sq
rt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*
e*f - 2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f^2 - 2*((a^4*b^2 + a^2*b^4 -
2*b^6)*d*e*f - (a^4*b^2 + a^2*b^4 - 2*b^6)*c*f^2)*cos(d*x + c)^2 + 4*((a^5
*b + a^3*b^3 - 2*a*b^5)*d*e*f - (a^5*b + a^3*b^3 - 2*a*b^5)*c*f^2)*sin(d*x
+ c) + (3*(a^3*b^3 + a*b^5)*d^2*e^2 - 6*(a^3*b^3 + a*b^5)*c*d*e*f - (2*a^5*
b - 2*a*b^5 - 3*(a^3*b^3 + a*b^5)*c^2)*f^2 - (3*a*b^5*d^2*e^2 - 6*a*b^5*c*d
*e*f + (3*a*b^5*c^2 - 2*a^3*b^3 + 2*a*b^5)*f^2)*cos(d*x + c)^2 + 2*(3*a^2*b
^4*d^2*e^2 - 6*a^2*b^4*c*d*e*f + (3*a^2*b^4*c^2 - 2*a^4*b^2 + 2*a^2*b^4)*f^
2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) - 2*I*b*sin(
d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(2*(a^6 + 2*a^4*b^2 - a^
2*b^4 - 2*b^6)*d*f^2*x + 2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f^2 - 2*((
a^4*b^2 + a^2*b^4 - 2*b^6)*d*f^2*x + (a^4*b^2 + a^2*b^4 - 2*b^6)*c*f^2)*cos
(d*x + c)^2 + 4*((a^5*b + a^3*b^3 - 2*a*b^5)*d*f^2*x + (a^5*b + a^3*b^3 - 2
*a*b^5)*c*f^2)*sin(d*x + c) - 3*((a^3*b^3 + a*b^5)*d^2*f^2*x^2 + 2*(a^3*b^3
+ a*b^5)*d^2*e*f*x + 2*(a^3*b^3 + a*b^5)*c*d*e*f - (a^3*b^3 + a*b^5)*c^2*f
^2 - (a*b^5*d^2*f^2*x^2 + 2*a*b^5*d^2*e*f*x + 2*a*b^5*c*d*e*f - a*b^5*c^2*f
^2)*cos(d*x + c)^2 + 2*(a^2*b^4*d^2*f^2*x^2 + 2*a^2*b^4*d^2*e*f*x + 2*a^2*b
^4*c*d*e*f - a^2*b^4*c^2*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(1/2
*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x +

```
c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2
*b^6)*d*f^2*x + 2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f^2 - 2*((a^4*b^2 +
 a^2*b^4 - 2*b^6)*d*f^2*x + (a^4*b^2 + a^2*b^4 - 2*b^6)*c*f^2)*cos(d*x + c)
^2 + 4*((a^5*b + a^3*b^3 - 2*a*b^5)*d*f^2*x + (a^5*b + a^3*b^3 - 2*a*b^5)*c
*f^2)*sin(d*x + c) + 3*((a^3*b^3 + a*b^5)*d^2*f^2*x^2 + 2*(a^3*b^3 + a*b^5)
*d^2*e*f*x + 2*(a^3*b^3 + a*b^5)*c*d*e*f - (a^3*b^3 + a*b^5)*c^2*f^2 - (a*b
^5*d^2*f^2*x^2 + 2*a*b^5*d^2*e*f*x + 2*a*b^5*c*d*e*f - a*b^5*c^2*f^2)*cos(d
*x + c)^2 + 2*(a^2*b^4*d^2*f^2*x^2 + 2*a^2*b^4*d^2*e*f*x + 2*a^2*b^4*c*d*e*
f - a^2*b^4*c^2*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(2*I*a*c
os(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f
^2*x + 2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f^2 - 2*((a^4*b^2 + a^2*b^4
- 2*b^6)*d*f^2*x + (a^4*b^2 + a^2*b^4 - 2*b^6)*c*f^2)*cos(d*x + c)^2 + 4*((
a^5*b + a^3*b^3 - 2*a*b^5)*d*f^2*x + (a^5*b + a^3*b^3 - 2*a*b^5)*c*f^2)*sin
(d*x + c) - 3*((a^3*b^3 + a*b^5)*d^2*f^2*x^2 + 2*(a^3*b^3 + a*b^5)*d^2*e*f*
x + 2*(a^3*b^3 + a*b^5)*c*d*e*f - (a^3*b^3 + a*b^5)*c^2*f^2 - (a*b^5*d^2*f^
2*x^2 + 2*a*b^5*d^2*e*f*x + 2*a*b^5*c*d*e*f - a*b^5*c^2*f^2)*cos(d*x + c)^2
 + 2*(a^2*b^4*d^2*f^2*x^2 + 2*a^2*b^4*d^2*e*f*x + 2*a^2*b^4*c*d*e*f - a^2*b
^4*c^2*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(-2*I*a*cos(d*x +
 c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2) + 2*b)/b) - 2*(2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f^2*x + 2
*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f^2 - 2*((a^4*b^2 + a^2*b^4 - 2*b^6)
*d*f^2*x + (a^4*b^2 + a^2*b^4 - 2*b^6)*c*f^2)*cos(d*x + c)^2 + 4*((a^5*b +
a^3*b^3 - 2*a*b^5)*d*f^2*x + (a^5*b + a^3*b^3 - 2*a*b^5)*c*f^2)*sin(d*x + c
) + 3*((a^3*b^3 + a*b^5)*d^2*f^2*x^2 + 2*(a^3*b^3 + a*b^5)*d^2*e*f*x + 2*(a
^3*b^3 + a*b^5)*c*d*e*f - (a^3*b^3 + a*b^5)*c^2*f^2 - (a*b^5*d^2*f^2*x^2 +
2*a*b^5*d^2*e*f*x + 2*a*b^5*c*d*e*f - a*b^5*c^2*f^2)*cos(d*x + c)^2 + 2*(a^
2*b^4*d^2*f^2*x^2 + 2*a^2*b^4*d^2*e*f*x + 2*a^2*b^4*c*d*e*f - a^2*b^4*c^2*f
^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(-2*I*a*cos(d*x + c) + 2*
a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2) + 2*b)/b) + 4*(2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*f^2*x + 2*(a^5*b - 2*a^3*
b^3 + a*b^5)*d*e*f + ((a^4*b^2 + a^2*b^4 - 2*b^6)*d^2*f^2*x^2 + 2*(a^4*b^2
+ a^2*b^4 - 2*b^6)*d^2*e*f*x + (a^4*b^2 + a^2*b^4 - 2*b^6)*d^2*e^2)*cos(d*x
 + c))*sin(d*x + c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d^3*cos(d*x +
 c)^2 - 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d^3*sin(d*x + c) - (a^8
*b - 2*a^6*b^3 + 2*a^2*b^7 - b^9)*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sin(dx+c)}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

3.250 $\int \frac{(e+fx)^3 \sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=2348

result too large to display

```
[Out] (((-3*I)/2)*a^2*(e + f*x)^3)/(b*(a^2 - b^2)^2*d) + (I*(e + f*x)^3)/(b*(a^2
- b^2)*d) - ((3*I)*a*f^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[
a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + (9*a^2*f*(e + f*x)^2*Log[1 - (I*b
*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^2*d^2) - (3*f*(e
 + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b
^2)*d^2) + (((3*I)/2)*a^3*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d) - (((3*I)/2)*a*(e + f*x)^3*Log[1 -
 (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + ((
3*I)*a*f^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/
(b*(a^2 - b^2)^(3/2)*d^3) + (9*a^2*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x
)))/(a + Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^2*d^2) - (3*f*(e + f*x)^2*Log[
1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) - (((
3*I)/2)*a^3*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])
])/(b*(a^2 - b^2)^(5/2)*d) + (((3*I)/2)*a*(e + f*x)^3*Log[1 - (I*b*E^(I*(c
+ d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) - (3*a*f^3*PolyLog
[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^4)
 - ((9*I)*a^2*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
- b^2])])/(b*(a^2 - b^2)^2*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I
*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) + (9*a^3*f*(e + f*
x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(2*b*(a^2 - b
^2)^(5/2)*d^2) - (9*a*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^(3/2)*d^2) + (3*a*f^3*PolyLog[2, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^4) - ((9*I)*a
^2*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(
b*(a^2 - b^2)^2*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x))
)/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) - (9*a^3*f*(e + f*x)^2*PolyLo
g[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^(5/2)*d
^2) + (9*a*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b
^2])])/(2*b*(a^2 - b^2)^(3/2)*d^2) + (9*a^2*f^3*PolyLog[3, (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^4) - (6*f^3*PolyLog[3, (I*
b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^4) + ((9*I)*a^3
*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*
(a^2 - b^2)^(5/2)*d^3) - ((9*I)*a*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + (9*a^2*f^3*PolyLo
g[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^4) -
(6*f^3*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b
^2)*d^4) - ((9*I)*a^3*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + S
qrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d^3) + ((9*I)*a*f^2*(e + f*x)*PolyLo
```

```
g[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3
) - (9*a^3*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*
(a^2 - b^2)^(5/2)*d^4) + (9*a*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqr
t[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^4) + (9*a^3*f^3*PolyLog[4, (I*b*E^(I
*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d^4) - (9*a*f^3*P
olyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2
)*d^4) - (a*(e + f*x)^3*Cos[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])
^2) - (3*a*f*(e + f*x)^2)/(2*b*(a^2 - b^2)*d^2*(a + b*Sin[c + d*x])) - (3*a
^2*(e + f*x)^3*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) + ((e
 + f*x)^3*Cos[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 8.37404, antiderivative size = 2348, normalized size of antiderivative = 1., number of steps used = 92, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {6742, 3325, 3324, 3323, 2264, 2190, 2531, 6609, 2282, 6589, 4519, 4422, 2279, 2391}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (((-3*I)/2)*a^2*(e + f*x)^3)/(b*(a^2 - b^2)^2*d) + (I*(e + f*x)^3)/(b*(a^2
- b^2)*d) - ((3*I)*a*f^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[
a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + (9*a^2*f*(e + f*x)^2*Log[1 - (I*b
*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^2*d^2) - (3*f*(e
 + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b
^2)*d^2) + (((3*I)/2)*a^3*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d) - (((3*I)/2)*a*(e + f*x)^3*Log[1 -
 (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) + ((
3*I)*a*f^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/
(b*(a^2 - b^2)^(3/2)*d^3) + (9*a^2*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x
)))/(a + Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^2*d^2) - (3*f*(e + f*x)^2*Log[
1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^2) - (((
3*I)/2)*a^3*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])
])/(b*(a^2 - b^2)^(5/2)*d) + (((3*I)/2)*a*(e + f*x)^3*Log[1 - (I*b*E^(I*(c
+ d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d) - (3*a*f^3*PolyLog
[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^4)
 - ((9*I)*a^2*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
- b^2])])/(b*(a^2 - b^2)^2*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I
*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) + (9*a^3*f*(e + f*
x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(2*b*(a^2 - b
```

```
^2)^(5/2)*d^2) - (9*a*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^(3/2)*d^2) + (3*a*f^3*PolyLog[2, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^4) - ((9*I)*a
^2*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(
b*(a^2 - b^2)^2*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x))
)/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) - (9*a^3*f*(e + f*x)^2*PolyLo
g[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(2*b*(a^2 - b^2)^(5/2)*d
^2) + (9*a*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b
^2])])/(2*b*(a^2 - b^2)^(3/2)*d^2) + (9*a^2*f^3*PolyLog[3, (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^4) - (6*f^3*PolyLog[3, (I*
b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^4) + ((9*I)*a^3
*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*
(a^2 - b^2)^(5/2)*d^3) - ((9*I)*a*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + (9*a^2*f^3*PolyLo
g[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^2*d^4) -
(6*f^3*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b
^2)*d^4) - ((9*I)*a^3*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + S
qrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d^3) + ((9*I)*a*f^2*(e + f*x)*PolyLo
g[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3
) - (9*a^3*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*
(a^2 - b^2)^(5/2)*d^4) + (9*a*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqr
t[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^4) + (9*a^3*f^3*PolyLog[4, (I*b*E^(I
*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(5/2)*d^4) - (9*a*f^3*P
olyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2
)*d^4) - (a*(e + f*x)^3*Cos[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])
^2) - (3*a*f*(e + f*x)^2)/(2*b*(a^2 - b^2)*d^2*(a + b*Sin[c + d*x])) - (3*a
^2*(e + f*x)^3*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) + ((e
 + f*x)^3*Cos[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3325

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> -Simp[(b*(c + d*x)^m*Cos[e + f*x]*(a + b*Sin[e + f*x])^(n + 1
))/(f*(n + 1)*(a^2 - b^2)), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m*(a +
b*Sin[e + f*x])^(n + 1), x], x] - Dist[(b*(n + 2))/((n + 1)*(a^2 - b^2)), I
nt[(c + d*x)^m*Sin[e + f*x]*(a + b*Sin[e + f*x])^(n + 1), x], x] + Dist[(b*
d*m)/(f*(n + 1)*(a^2 - b^2)), Int[(c + d*x)^(m - 1)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && ILtQ[n, -2] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
```

```
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(-\frac{a(e+fx)^3}{b(a+b\sin(c+dx))^3}+\frac{(e+fx)^3}{b(a+b\sin(c+dx))^2}\right)dx$$

$$= \frac{\int \frac{(e+fx)^3}{(a+b\sin(c+dx))^2}\,dx}{b} - \frac{a\int \frac{(e+fx)^3}{(a+b\sin(c+dx))^3}\,dx}{b}$$

$$= -\frac{a(e+fx)^3\cos(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2}+\frac{(e+fx)^3\cos(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))}+\frac{a\int \frac{(e+fx)^3\sin(c+dx)}{(a+b\sin(c+dx))^2}\,d\cdots}{2\left(a^2-b^2\right)}$$

$$= \frac{i(e+fx)^3}{b\left(a^2-b^2\right)d}-\frac{a(e+fx)^3\cos(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2}-\frac{3af(e+fx)^2}{2b\left(a^2-b^2\right)d^2(a+b\sin(c+dx))}-\cdots$$

$$= -\frac{ia^2(e+fx)^3}{b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^3}{b\left(a^2-b^2\right)d}-\frac{3f(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^2}-\frac{3f(e+fx)^2\log\left(1-\cdots\right.}{b\left(a^2-b^2\right)d^2}\cdots$$

$$= -\frac{ia^2(e+fx)^3}{b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^3}{b\left(a^2-b^2\right)d}+\frac{3a^2f(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^2d^2}-\frac{3f(e+fx)^2\log\left(1-\cdots\right.}{b\left(a^2-b^2\right)\cdots}\cdots$$

$$= -\frac{3ia^2(e+fx)^3}{2b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^3}{b\left(a^2-b^2\right)d}-\frac{3iaf^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^3}+\frac{3a^2f(e+fx)^2\log\left(\cdots\right.}{b\left(a^2-b^2\cdots\right.}\cdots$$

$$= -\frac{3ia^2(e+fx)^3}{2b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^3}{b\left(a^2-b^2\right)d}-\frac{3iaf^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^3}+\frac{9a^2f(e+fx)^2\log\left(\cdots\right.}{2b\left(a^2-b^2\cdots\right.}\cdots$$

$$= -\frac{3ia^2(e+fx)^3}{2b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^3}{b\left(a^2-b^2\right)d}-\frac{3iaf^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^3}+\frac{9a^2f(e+fx)^2\log\left(\cdots\right.}{2b\left(a^2-b^2\cdots\right.}\cdots$$

$$= -\frac{3ia^2(e+fx)^3}{2b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^3}{b\left(a^2-b^2\right)d}-\frac{3iaf^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^3}+\frac{9a^2f(e+fx)^2\log\left(\cdots\right.}{2b\left(a^2-b^2\cdots\right.}\cdots$$

$$= -\frac{3ia^2(e+fx)^3}{2b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^3}{b\left(a^2-b^2\right)d}-\frac{3iaf^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^3}+\frac{9a^2f(e+fx)^2\log\left(\cdots\right.}{2b\left(a^2-b^2\cdots\right.}\cdots$$

$$= -\frac{3ia^2(e+fx)^3}{2b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^3}{b\left(a^2-b^2\right)d}-\frac{3iaf^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^3}+\frac{9a^2f(e+fx)^2\log\left(\cdots\right.}{2b\left(a^2-b^2\cdots\right.}\cdots$$

$$= -\frac{3ia^2(e+fx)^3}{2b\left(a^2-b^2\right)^2d}+\frac{i(e+fx)^3}{b\left(a^2-b^2\right)d}-\frac{3iaf^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^3}+\frac{9a^2f(e+fx)^2\log\left(\cdots\right.}{2b\left(a^2-b^2\cdots\right.}\cdots$$

Mathematica [B] time = 22.235, size = 11204, normalized size = 4.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 1.711, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \sin(dx+c)}{(a+b\sin(dx+c))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 13.0213, size = 22152, normalized size = 9.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/8*(12*(a^6 - 2*a^4*b^2 + a^2*b^4)*d^2*f^3*x^2 + 24*(a^6 - 2*a^4*b^2 + a^2*b^4)*d^2*e*f^2*x + 12*(a^6 - 2*a^4*b^2 + a^2*b^4)*d^2*e^2*f + 2*(18*I*a*b^5*f^3*cos(d*x + c)^2 - 36*I*a^2*b^4*f^3*sin(d*x + c) - 18*I*(a^3*b^3 + a*b^5)*f^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(-18*I*a*b^5*f^3*cos(d*x + c)^2 + 36*I*a^2*b^4*f^3*sin(d*x + c) + 18*I*(a^3*b^3 + a*b^5)*f^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(18*I*a*b^5*f^3*cos(d*x + c)^2 - 36*I*a^2*b^4*f^3*sin(d*x + c) - 18*I*(a^3*b^3 + a*b^5)*f^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(-18*I*a*b^5*f^3*cos(d*x + c)^2 + 36*I*a^2*b^4*f^3*sin(d*x + c) + 18*I*(a^3*b^3 + a*b^5)*f^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 4*((2*a^5*b - a^3*b^3 - a*b^5)*d^3*f^3*x^3 + 3*(2*a^5*b - a^3*b^3 - a*b^5)*d^3*e*f^2*x^2 + 3*(2*a^5*b - a^3*b^3 - a*b^5)*d^3*e^2*f*x + (2*a^5*b - a^3*b^3 - a*b^5)*d^3*e^3)*cos(d*x + c) + (-12*I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f^3*x - 12*I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e*f^2 + (12*I*(a^4*b^2 + a^2*b^4 - 2*b^6)*d*f^3*x + 12*I*(a^4*b^2 + a^2*b^4 - 2*b^6)*d*e*f^2)*cos(d*x + c)^2 + (-24*I*(a^5*b + a^3*b^3 - 2*a*b^5)*d*f^3*x - 24*I*(a^5*b + a^3*b^3 - 2*a*b^5)*d*e*f^2)*sin(d*x + c) + 2*(9*I*(a^3*b^3 + a*b^5)*d^2*f^3*x^2 + 18*I*(a^3*b^3 + a*b^5)*d^2*e*f^2*x + 9*I*(a^3*b^3 + a*b^5)*d^2*e^2*f - 6*I*(a^5*b - a*b^5)*f^3 + (-9*I*a*b^5*d^2*f^3*x^2 - 18*I*a*b^5*d^2*e*f^2*x - 9*I*a*b^5*d^2*e^2*f + 6*I*(a^3*b^3 - a*b^5)*f^3)*cos(d*x + c)^2 + (18*I*a^2*b^4*d^2*f^3*x^2 + 36*I*a^2*b^4*d^2*e*f^2*x + 18*I*a^2*b^4*d^2*e^2*f - 12*I*(a^4*b^2 - a^2*b^4)*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-12*I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f^3*x - 12*I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e*f^2 + (12*I*(a^4*b^2 + a^2*b^4 - 2*b^6)*d*f^3*x + 12*I*(a^4*b^2 + a^2*b^4 - 2*b^6)*d*e*f^2)*cos(d*x + c)^2 + (-24*I*(a^5*b + a^3*b^3 - 2*a*b^5)*d*f^3*x - 24*I*(a^5*b + a^3*b^3 - 2*a*b^5)*d*e*f^2)*sin(d*x + c) + 2*(-9*I*(a^3*b^3 + a*b^5)*d^2*f^3*x^2 - 18*I*(a^3*b^3 + a*b^5)*d^2*e*f^2*x - 9*I*(a^3*b^3 + a*b^5)*d^2*e^2*f + 6*I*(a^5*b - a*b^5)*f^3 + (9*I*a*b^5*d^2*f^3*x^2 + 18*I*a*b^5*d^2*e*f^2*x + 9*I*a*b^5*d^2*e^2*f - 6*I*(a^3*b^3 - a*b^5)*f^3)*cos(d*x + c)^2 + (-18*I*a^2*b^4*d^2*f^3*x^2 - 36*I*a^2*b^4*d^2*e*f^2*x - 18*I*a^2*b^4*d^2*e^2*f + 12*I*(a^4*b^2 - a^2*b^4)*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (12*I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f^3*x + 12*I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e*f^2 + (-12*I*(a^4*b^2 + a^2*b^4 - 2*b^6)*d*f^3*x - 12*I*(a^4*b^2 + a^2*b^4 - 2*b^6)*d*e*f^2)*cos(d*x

```
+ c)^2 + (24*I*(a^5*b + a^3*b^3 - 2*a*b^5)*d*f^3*x + 24*I*(a^5*b + a^3*b^3
- 2*a*b^5)*d*e*f^2)*sin(d*x + c) + 2*(-9*I*(a^3*b^3 + a*b^5)*d^2*f^3*x^2 -
18*I*(a^3*b^3 + a*b^5)*d^2*e*f^2*x - 9*I*(a^3*b^3 + a*b^5)*d^2*e^2*f + 6*I*
(a^5*b - a*b^5)*f^3 + (9*I*a*b^5*d^2*f^3*x^2 + 18*I*a*b^5*d^2*e*f^2*x + 9*I
*a*b^5*d^2*e^2*f - 6*I*(a^3*b^3 - a*b^5)*f^3)*cos(d*x + c)^2 + (-18*I*a^2*b
^4*d^2*f^3*x^2 - 36*I*a^2*b^4*d^2*e*f^2*x - 18*I*a^2*b^4*d^2*e^2*f + 12*I*(
a^4*b^2 - a^2*b^4)*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog(-1/2*(-
2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (12*I*(a^6 + 2*a^4*b^2 - a^2*b^4
- 2*b^6)*d*f^3*x + 12*I*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e*f^2 + (-12*
I*(a^4*b^2 + a^2*b^4 - 2*b^6)*d*f^3*x - 12*I*(a^4*b^2 + a^2*b^4 - 2*b^6)*d*
e*f^2)*cos(d*x + c)^2 + (24*I*(a^5*b + a^3*b^3 - 2*a*b^5)*d*f^3*x + 24*I*(a
^5*b + a^3*b^3 - 2*a*b^5)*d*e*f^2)*sin(d*x + c) + 2*(9*I*(a^3*b^3 + a*b^5)*
d^2*f^3*x^2 + 18*I*(a^3*b^3 + a*b^5)*d^2*e*f^2*x + 9*I*(a^3*b^3 + a*b^5)*d^
2*e^2*f - 6*I*(a^5*b - a*b^5)*f^3 + (-9*I*a*b^5*d^2*f^3*x^2 - 18*I*a*b^5*d^
2*e*f^2*x - 9*I*a*b^5*d^2*e^2*f + 6*I*(a^3*b^3 - a*b^5)*f^3)*cos(d*x + c)^2
 + (18*I*a^2*b^4*d^2*f^3*x^2 + 36*I*a^2*b^4*d^2*e*f^2*x + 18*I*a^2*b^4*d^2*
e^2*f - 12*I*(a^4*b^2 - a^2*b^4)*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))
*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 6*((a^6 + 2*a^4*b^2
 - a^2*b^4 - 2*b^6)*d^2*e^2*f - 2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*d*e
*f^2 + (a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c^2*f^3 - ((a^4*b^2 + a^2*b^4 -
2*b^6)*d^2*e^2*f - 2*(a^4*b^2 + a^2*b^4 - 2*b^6)*c*d*e*f^2 + (a^4*b^2 + a^2
*b^4 - 2*b^6)*c^2*f^3)*cos(d*x + c)^2 + 2*((a^5*b + a^3*b^3 - 2*a*b^5)*d^2*
e^2*f - 2*(a^5*b + a^3*b^3 - 2*a*b^5)*c*d*e*f^2 + (a^5*b + a^3*b^3 - 2*a*b^
5)*c^2*f^3)*sin(d*x + c) - ((a^3*b^3 + a*b^5)*d^3*e^3 - 3*(a^3*b^3 + a*b^5)
*c*d^2*e^2*f - (2*a^5*b - 2*a*b^5 - 3*(a^3*b^3 + a*b^5)*c^2)*d*e*f^2 - ((a^
3*b^3 + a*b^5)*c^3 - 2*(a^5*b - a*b^5)*c)*f^3 - (a*b^5*d^3*e^3 - 3*a*b^5*c*
d^2*e^2*f + (3*a*b^5*c^2 - 2*a^3*b^3 + 2*a*b^5)*d*e*f^2 - (a*b^5*c^3 - 2*(a
^3*b^3 - a*b^5)*c)*f^3)*cos(d*x + c)^2 + 2*(a^2*b^4*d^3*e^3 - 3*a^2*b^4*c*d
^2*e^2*f + (3*a^2*b^4*c^2 - 2*a^4*b^2 + 2*a^2*b^4)*d*e*f^2 - (a^2*b^4*c^3 -
 2*(a^4*b^2 - a^2*b^4)*c)*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*
b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) -
 6*((a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d^2*e^2*f - 2*(a^6 + 2*a^4*b^2 - a^
2*b^4 - 2*b^6)*c*d*e*f^2 + (a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c^2*f^3 - ((
a^4*b^2 + a^2*b^4 - 2*b^6)*d^2*e^2*f - 2*(a^4*b^2 + a^2*b^4 - 2*b^6)*c*d*e*
f^2 + (a^4*b^2 + a^2*b^4 - 2*b^6)*c^2*f^3)*cos(d*x + c)^2 + 2*((a^5*b + a^3
*b^3 - 2*a*b^5)*d^2*e^2*f - 2*(a^5*b + a^3*b^3 - 2*a*b^5)*c*d*e*f^2 + (a^5*
b + a^3*b^3 - 2*a*b^5)*c^2*f^3)*sin(d*x + c) - ((a^3*b^3 + a*b^5)*d^3*e^3 -
 3*(a^3*b^3 + a*b^5)*c*d^2*e^2*f - (2*a^5*b - 2*a*b^5 - 3*(a^3*b^3 + a*b^5)
*c^2)*d*e*f^2 - ((a^3*b^3 + a*b^5)*c^3 - 2*(a^5*b - a*b^5)*c)*f^3 - (a*b^5*
d^3*e^3 - 3*a*b^5*c*d^2*e^2*f + (3*a*b^5*c^2 - 2*a^3*b^3 + 2*a*b^5)*d*e*f^2
 - (a*b^5*c^3 - 2*(a^3*b^3 - a*b^5)*c)*f^3)*cos(d*x + c)^2 + 2*(a^2*b^4*d^3
*e^3 - 3*a^2*b^4*c*d^2*e^2*f + (3*a^2*b^4*c^2 - 2*a^4*b^2 + 2*a^2*b^4)*d*e*
f^2 - (a^2*b^4*c^3 - 2*(a^4*b^2 - a^2*b^4)*c)*f^3)*sin(d*x + c))*sqrt(-(a^2
```

- b^2)/b^2))*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 6*((a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d^2*e^2*f - 2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*d*e*f^2 + (a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c^2*f^3 - ((a^4*b^2 + a^2*b^4 - 2*b^6)*d^2*e^2*f - 2*(a^4*b^2 + a^2 *b^4 - 2*b^6)*c*d*e*f^2 + (a^4*b^2 + a^2*b^4 - 2*b^6)*c^2*f^3)*cos(d*x + c) ^2 + 2*((a^5*b + a^3*b^3 - 2*a*b^5)*d^2*e^2*f - 2*(a^5*b + a^3*b^3 - 2*a*b^ 5)*c*d*e*f^2 + (a^5*b + a^3*b^3 - 2*a*b^5)*c^2*f^3)*sin(d*x + c) + ((a^3*b^ 3 + a*b^5)*d^3*e^3 - 3*(a^3*b^3 + a*b^5)*c*d^2*e^2*f - (2*a^5*b - 2*a*b^5 - 3*(a^3*b^3 + a*b^5)*c^2)*d*e*f^2 - ((a^3*b^3 + a*b^5)*c^3 - 2*(a^5*b - a*b ^5)*c)*f^3 - (a*b^5*d^3*e^3 - 3*a*b^5*c*d^2*e^2*f + (3*a*b^5*c^2 - 2*a^3*b^ 3 + 2*a*b^5)*d*e*f^2 - (a*b^5*c^3 - 2*(a^3*b^3 - a*b^5)*c)*f^3)*cos(d*x + c)^2 + 2*(a^2*b^4*d^3*e^3 - 3*a^2*b^4*c*d^2*e^2*f + (3*a^2*b^4*c^2 - 2*a^4*b ^2 + 2*a^2*b^4)*d*e*f^2 - (a^2*b^4*c^3 - 2*(a^4*b^2 - a^2*b^4)*c)*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 6*((a^6 + 2*a^4*b^2 - a^2*b^4 - 2 *b^6)*d^2*e^2*f - 2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*d*e*f^2 + (a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c^2*f^3 - ((a^4*b^2 + a^2*b^4 - 2*b^6)*d^2*e^2 *f - 2*(a^4*b^2 + a^2*b^4 - 2*b^6)*c*d*e*f^2 + (a^4*b^2 + a^2*b^4 - 2*b^6)* c^2*f^3)*cos(d*x + c)^2 + 2*((a^5*b + a^3*b^3 - 2*a*b^5)*d^2*e^2*f - 2*(a^5 *b + a^3*b^3 - 2*a*b^5)*c*d*e*f^2 + (a^5*b + a^3*b^3 - 2*a*b^5)*c^2*f^3)*si n(d*x + c) + ((a^3*b^3 + a*b^5)*d^3*e^3 - 3*(a^3*b^3 + a*b^5)*c*d^2*e^2*f - (2*a^5*b - 2*a*b^5 - 3*(a^3*b^3 + a*b^5)*c^2)*d*e*f^2 - ((a^3*b^3 + a*b^5) *c^3 - 2*(a^5*b - a*b^5)*c)*f^3 - (a*b^5*d^3*e^3 - 3*a*b^5*c*d^2*e^2*f + (3 *a*b^5*c^2 - 2*a^3*b^3 + 2*a*b^5)*d*e*f^2 - (a*b^5*c^3 - 2*(a^3*b^3 - a*b^5)*c)*f^3)*cos(d*x + c)^2 + 2*(a^2*b^4*d^3*e^3 - 3*a^2*b^4*c*d^2*e^2*f + (3* a^2*b^4*c^2 - 2*a^4*b^2 + 2*a^2*b^4)*d*e*f^2 - (a^2*b^4*c^3 - 2*(a^4*b^2 - a^2*b^4)*c)*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 6*((a^6 + 2* a^4*b^2 - a^2*b^4 - 2*b^6)*d^2*f^3*x^2 + 2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b ^6)*d^2*e*f^2*x + 2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*d*e*f^2 - (a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c^2*f^3 - ((a^4*b^2 + a^2*b^4 - 2*b^6)*d^2*f^3 *x^2 + 2*(a^4*b^2 + a^2*b^4 - 2*b^6)*d^2*e*f^2*x + 2*(a^4*b^2 + a^2*b^4 - 2 *b^6)*c*d*e*f^2 - (a^4*b^2 + a^2*b^4 - 2*b^6)*c^2*f^3)*cos(d*x + c)^2 + 2*((a^5*b + a^3*b^3 - 2*a*b^5)*d^2*f^3*x^2 + 2*(a^5*b + a^3*b^3 - 2*a*b^5)*d^2 *e*f^2*x + 2*(a^5*b + a^3*b^3 - 2*a*b^5)*c*d*e*f^2 - (a^5*b + a^3*b^3 - 2*a *b^5)*c^2*f^3)*sin(d*x + c) - ((a^3*b^3 + a*b^5)*d^3*f^3*x^3 + 3*(a^3*b^3 + a*b^5)*d^3*e*f^2*x^2 + 3*(a^3*b^3 + a*b^5)*c*d^2*e^2*f - 3*(a^3*b^3 + a*b^ 5)*c^2*d*e*f^2 + ((a^3*b^3 + a*b^5)*c^3 - 2*(a^5*b - a*b^5)*c)*f^3 - (a*b^5 *d^3*f^3*x^3 + 3*a*b^5*d^3*e*f^2*x^2 + 3*a*b^5*c*d^2*e^2*f - 3*a*b^5*c^2*d* e*f^2 + (a*b^5*c^3 - 2*(a^3*b^3 - a*b^5)*c)*f^3 + (3*a*b^5*d^3*e^2*f - 2*(a ^3*b^3 - a*b^5)*d*f^3)*x)*cos(d*x + c)^2 + (3*(a^3*b^3 + a*b^5)*d^3*e^2*f - 2*(a^5*b - a*b^5)*d*f^3)*x + 2*(a^2*b^4*d^3*f^3*x^3 + 3*a^2*b^4*d^3*e*f^2* x^2 + 3*a^2*b^4*c*d^2*e^2*f - 3*a^2*b^4*c^2*d*e*f^2 + (a^2*b^4*c^3 - 2*(a^4 *b^2 - a^2*b^4)*c)*f^3 + (3*a^2*b^4*d^3*e^2*f - 2*(a^4*b^2 - a^2*b^4)*d*f^3)*x)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(2*I*a*cos(d*x + c) + 2*

a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2) + 2*b)/b) - 6*((a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d^2*f^3*x^2 + 2*(a^6
+ 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d^2*e*f^2*x + 2*(a^6 + 2*a^4*b^2 - a^2*b^4 -
2*b^6)*c*d*e*f^2 - (a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c^2*f^3 - ((a^4*b^2
+ a^2*b^4 - 2*b^6)*d^2*f^3*x^2 + 2*(a^4*b^2 + a^2*b^4 - 2*b^6)*d^2*e*f^2*x
+ 2*(a^4*b^2 + a^2*b^4 - 2*b^6)*c*d*e*f^2 - (a^4*b^2 + a^2*b^4 - 2*b^6)*c^
2*f^3)*cos(d*x + c)^2 + 2*((a^5*b + a^3*b^3 - 2*a*b^5)*d^2*f^3*x^2 + 2*(a^5
*b + a^3*b^3 - 2*a*b^5)*d^2*e*f^2*x + 2*(a^5*b + a^3*b^3 - 2*a*b^5)*c*d*e*f
^2 - (a^5*b + a^3*b^3 - 2*a*b^5)*c^2*f^3)*sin(d*x + c) + ((a^3*b^3 + a*b^5)
*d^3*f^3*x^3 + 3*(a^3*b^3 + a*b^5)*d^3*e*f^2*x^2 + 3*(a^3*b^3 + a*b^5)*c*d^
2*e^2*f - 3*(a^3*b^3 + a*b^5)*c^2*d*e*f^2 + ((a^3*b^3 + a*b^5)*c^3 - 2*(a^5
*b - a*b^5)*c)*f^3 - (a*b^5*d^3*f^3*x^3 + 3*a*b^5*d^3*e*f^2*x^2 + 3*a*b^5*c
*d^2*e^2*f - 3*a*b^5*c^2*d*e*f^2 + (a*b^5*c^3 - 2*(a^3*b^3 - a*b^5)*c)*f^3
+ (3*a*b^5*d^3*e^2*f - 2*(a^3*b^3 - a*b^5)*d*f^3)*x)*cos(d*x + c)^2 + (3*(a
^3*b^3 + a*b^5)*d^3*e^2*f - 2*(a^5*b - a*b^5)*d*f^3)*x + 2*(a^2*b^4*d^3*f^3
*x^3 + 3*a^2*b^4*d^3*e*f^2*x^2 + 3*a^2*b^4*c*d^2*e^2*f - 3*a^2*b^4*c^2*d*e*
f^2 + (a^2*b^4*c^3 - 2*(a^4*b^2 - a^2*b^4)*c)*f^3 + (3*a^2*b^4*d^3*e^2*f -
2*(a^4*b^2 - a^2*b^4)*d*f^3)*x)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(1
/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 6*((a^6 + 2*a^4*b^2 - a^2*b^4 - 2
*b^6)*d^2*f^3*x^2 + 2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d^2*e*f^2*x + 2*(
a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*d*e*f^2 - (a^6 + 2*a^4*b^2 - a^2*b^4 -
2*b^6)*c^2*f^3 - ((a^4*b^2 + a^2*b^4 - 2*b^6)*d^2*f^3*x^2 + 2*(a^4*b^2 + a
^2*b^4 - 2*b^6)*d^2*e*f^2*x + 2*(a^4*b^2 + a^2*b^4 - 2*b^6)*c*d*e*f^2 - (a^
4*b^2 + a^2*b^4 - 2*b^6)*c^2*f^3)*cos(d*x + c)^2 + 2*((a^5*b + a^3*b^3 - 2*
a*b^5)*d^2*f^3*x^2 + 2*(a^5*b + a^3*b^3 - 2*a*b^5)*d^2*e*f^2*x + 2*(a^5*b +
a^3*b^3 - 2*a*b^5)*c*d*e*f^2 - (a^5*b + a^3*b^3 - 2*a*b^5)*c^2*f^3)*sin(d*
x + c) - ((a^3*b^3 + a*b^5)*d^3*f^3*x^3 + 3*(a^3*b^3 + a*b^5)*d^3*e*f^2*x^2
+ 3*(a^3*b^3 + a*b^5)*c*d^2*e^2*f - 3*(a^3*b^3 + a*b^5)*c^2*d*e*f^2 + ((a^
3*b^3 + a*b^5)*c^3 - 2*(a^5*b - a*b^5)*c)*f^3 - (a*b^5*d^3*f^3*x^3 + 3*a*b^
5*d^3*e*f^2*x^2 + 3*a*b^5*c*d^2*e^2*f - 3*a*b^5*c^2*d*e*f^2 + (a*b^5*c^3 -
2*(a^3*b^3 - a*b^5)*c)*f^3 + (3*a*b^5*d^3*e^2*f - 2*(a^3*b^3 - a*b^5)*d*f^3
)*x)*cos(d*x + c)^2 + (3*(a^3*b^3 + a*b^5)*d^3*e^2*f - 2*(a^5*b - a*b^5)*d*
f^3)*x + 2*(a^2*b^4*d^3*f^3*x^3 + 3*a^2*b^4*d^3*e*f^2*x^2 + 3*a^2*b^4*c*d^2
*e^2*f - 3*a^2*b^4*c^2*d*e*f^2 + (a^2*b^4*c^3 - 2*(a^4*b^2 - a^2*b^4)*c)*f^
3 + (3*a^2*b^4*d^3*e^2*f - 2*(a^4*b^2 - a^2*b^4)*d*f^3)*x)*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2))*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(
b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 6*((a
^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*d^2*f^3*x^2 + 2*(a^6 + 2*a^4*b^2 - a^2*b^
4 - 2*b^6)*d^2*e*f^2*x + 2*(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c*d*e*f^2 -
(a^6 + 2*a^4*b^2 - a^2*b^4 - 2*b^6)*c^2*f^3 - ((a^4*b^2 + a^2*b^4 - 2*b^6)*
d^2*f^3*x^2 + 2*(a^4*b^2 + a^2*b^4 - 2*b^6)*d^2*e*f^2*x + 2*(a^4*b^2 + a^2*
b^4 - 2*b^6)*c*d*e*f^2 - (a^4*b^2 + a^2*b^4 - 2*b^6)*c^2*f^3)*cos(d*x + c)^
2 + 2*((a^5*b + a^3*b^3 - 2*a*b^5)*d^2*f^3*x^2 + 2*(a^5*b + a^3*b^3 - 2*a*b
^5)*d^2*e*f^2*x + 2*(a^5*b + a^3*b^3 - 2*a*b^5)*c*d*e*f^2 - (a^5*b + a^3*b^

```
3 - 2*a*b^5)*c^2*f^3)*sin(d*x + c) + ((a^3*b^3 + a*b^5)*d^3*f^3*x^3 + 3*(a^
3*b^3 + a*b^5)*d^3*e*f^2*x^2 + 3*(a^3*b^3 + a*b^5)*c*d^2*e^2*f - 3*(a^3*b^3
 + a*b^5)*c^2*d*e*f^2 + ((a^3*b^3 + a*b^5)*c^3 - 2*(a^5*b - a*b^5)*c)*f^3 -
 (a*b^5*d^3*f^3*x^3 + 3*a*b^5*d^3*e*f^2*x^2 + 3*a*b^5*c*d^2*e^2*f - 3*a*b^5
*c^2*d*e*f^2 + (a*b^5*c^3 - 2*(a^3*b^3 - a*b^5)*c)*f^3 + (3*a*b^5*d^3*e^2*f
 - 2*(a^3*b^3 - a*b^5)*d*f^3)*x)*cos(d*x + c)^2 + (3*(a^3*b^3 + a*b^5)*d^3*
e^2*f - 2*(a^5*b - a*b^5)*d*f^3)*x + 2*(a^2*b^4*d^3*f^3*x^3 + 3*a^2*b^4*d^3
*e*f^2*x^2 + 3*a^2*b^4*c*d^2*e^2*f - 3*a^2*b^4*c^2*d*e*f^2 + (a^2*b^4*c^3 -
 2*(a^4*b^2 - a^2*b^4)*c)*f^3 + (3*a^2*b^4*d^3*e^2*f - 2*(a^4*b^2 - a^2*b^4
)*d*f^3)*x)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(-2*I*a*cos(d*x +
 c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2) + 2*b)/b) + 12*((a^4*b^2 + a^2*b^4 - 2*b^6)*f^3*cos(d*x + c)^2 -
 2*(a^5*b + a^3*b^3 - 2*a*b^5)*f^3*sin(d*x + c) - (a^6 + 2*a^4*b^2 - a^2*b^
4 - 2*b^6)*f^3 - 3*((a^3*b^3 + a*b^5)*d*f^3*x + (a^3*b^3 + a*b^5)*d*e*f^2 -
 (a*b^5*d*f^3*x + a*b^5*d*e*f^2)*cos(d*x + c)^2 + 2*(a^2*b^4*d*f^3*x + a^2*
b^4*d*e*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*polylog(3, 1/2*(2*I*a*co
s(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2))/b) + 12*((a^4*b^2 + a^2*b^4 - 2*b^6)*f^3*cos(d*x + c)^2
 - 2*(a^5*b + a^3*b^3 - 2*a*b^5)*f^3*sin(d*x + c) - (a^6 + 2*a^4*b^2 - a^2*b
^4 - 2*b^6)*f^3 + 3*((a^3*b^3 + a*b^5)*d*f^3*x + (a^3*b^3 + a*b^5)*d*e*f^2
 - (a*b^5*d*f^3*x + a*b^5*d*e*f^2)*cos(d*x + c)^2 + 2*(a^2*b^4*d*f^3*x + a^2
*b^4*d*e*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*polylog(3, 1/2*(2*I*a*c
os(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2))/b) + 12*((a^4*b^2 + a^2*b^4 - 2*b^6)*f^3*cos(d*x + c)^2
 - 2*(a^5*b + a^3*b^3 - 2*a*b^5)*f^3*sin(d*x + c) - (a^6 + 2*a^4*b^2 - a^2*
b^4 - 2*b^6)*f^3 + 3*((a^3*b^3 + a*b^5)*d*f^3*x + (a^3*b^3 + a*b^5)*d*e*f^2
 - (a*b^5*d*f^3*x + a*b^5*d*e*f^2)*cos(d*x + c)^2 + 2*(a^2*b^4*d*f^3*x + a^
2*b^4*d*e*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*polylog(3, -(I*a*cos(d
*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2))/b) + 12*((a^4*b^2 + a^2*b^4 - 2*b^6)*f^3*cos(d*x + c)^2 - 2*(a^
5*b + a^3*b^3 - 2*a*b^5)*f^3*sin(d*x + c) - (a^6 + 2*a^4*b^2 - a^2*b^4 - 2*
b^6)*f^3 - 3*((a^3*b^3 + a*b^5)*d*f^3*x + (a^3*b^3 + a*b^5)*d*e*f^2 - (a*b^
5*d*f^3*x + a*b^5*d*e*f^2)*cos(d*x + c)^2 + 2*(a^2*b^4*d*f^3*x + a^2*b^4*d*
e*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*polylog(3, -(I*a*cos(d*x + c)
+ a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2))/b) + 4*(3*(a^5*b - 2*a^3*b^3 + a*b^5)*d^2*f^3*x^2 + 6*(a^5*b - 2*a^3*b^
3 + a*b^5)*d^2*e*f^2*x + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*d^2*e^2*f + ((a^4*b^
2 + a^2*b^4 - 2*b^6)*d^3*f^3*x^3 + 3*(a^4*b^2 + a^2*b^4 - 2*b^6)*d^3*e*f^2*
x^2 + 3*(a^4*b^2 + a^2*b^4 - 2*b^6)*d^3*e^2*f*x + (a^4*b^2 + a^2*b^4 - 2*b^
6)*d^3*e^3)*cos(d*x + c))*sin(d*x + c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 -
 b^9)*d^4*cos(d*x + c)^2 - 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d^4*
sin(d*x + c) - (a^8*b - 2*a^6*b^3 + 2*a^2*b^7 - b^9)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \sin(dx+c)}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

3.251 $\int \frac{(e+fx)^3\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=151

$$\frac{12f^2(e+fx)\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^3}-\frac{6if(e+fx)^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^2}+\frac{12if^3\text{PolyLog}\left(4,ie^{i(c+dx)}\right)}{ad^4}+\frac{2(e+fx)^3\log}{ad}$$

```
[Out] ((-I/4)*(e + f*x)^4)/(a*f) + (2*(e + f*x)^3*Log[1 - I*E^(I*(c + d*x))])/(a*
d) - ((6*I)*f*(e + f*x)^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2) + (12*f^2*
(e + f*x)*PolyLog[3, I*E^(I*(c + d*x))])/(a*d^3) + ((12*I)*f^3*PolyLog[4, I
*E^(I*(c + d*x))])/(a*d^4)
```

Rubi [A] time = 0.233964, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4517, 2190, 2531, 6609, 2282, 6589}

$$\frac{12f^2(e+fx)\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^3}-\frac{6if(e+fx)^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^2}+\frac{12if^3\text{PolyLog}\left(4,ie^{i(c+dx)}\right)}{ad^4}+\frac{2(e+fx)^3\log}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```


```
[Out] ((-I/4)*(e + f*x)^4)/(a*f) + (2*(e + f*x)^3*Log[1 - I*E^(I*(c + d*x))])/(a*
d) - ((6*I)*f*(e + f*x)^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2) + (12*f^2*
(e + f*x)*PolyLog[3, I*E^(I*(c + d*x))])/(a*d^3) + ((12*I)*f^3*PolyLog[4, I
*E^(I*(c + d*x))])/(a*d^4)
```


Rule 4517

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + Dist[2, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - I*b*E^(I*(c + d*x)))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{i(e+fx)^4}{4af} + 2\int \frac{e^{i(c+dx)}(e+fx)^3}{a-iae^{i(c+dx)}}\,dx$$

$$= -\frac{i(e+fx)^4}{4af} + \frac{2(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad} - \frac{(6f)\int(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)dx}{ad}$$

$$= -\frac{i(e+fx)^4}{4af} + \frac{2(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad} - \frac{6if(e+fx)^2\text{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2} + \frac{\left(12if^2\right)\int(e+\ldots}{}$$

$$= -\frac{i(e+fx)^4}{4af} + \frac{2(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad} - \frac{6if(e+fx)^2\text{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2} + \frac{12f^2(e+fx)\text{L}\ldots}{ad^3}$$

$$= -\frac{i(e+fx)^4}{4af} + \frac{2(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad} - \frac{6if(e+fx)^2\text{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2} + \frac{12f^2(e+fx)\text{L}\ldots}{ad^3}$$

$$= -\frac{i(e+fx)^4}{4af} + \frac{2(e+fx)^3\log\left(1-ie^{i(c+dx)}\right)}{ad} - \frac{6if(e+fx)^2\text{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2} + \frac{12f^2(e+fx)\text{L}\ldots}{ad^3}$$

Mathematica [A] time = 1.40661, size = 276, normalized size = 1.83

$$\frac{x\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(6e^2fx+4e^3+4ef^2x^2+f^3x^3\right)}{4a\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)} - \frac{2(\cos(c)+i\sin(c))\left(\frac{3f(\cos(c)-i\sin(c))(\sin(c)-i\cos(c)+1)\left(d^2(e+fx)^2\text{PolyLog}\ldots\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*(Cos[c/2] - Sin[c/2]))/(4*a*
(Cos[c/2] + Sin[c/2])) - (2*(Cos[c] + I*Sin[c])*(((e + f*x)^4*(Cos[c] - I*S
in[c]))/(4*f) + (3*f*(d^2*(e + f*x)^2*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c
+ d*x]] - (2*I)*d*f*(e + f*x)*PolyLog[3, (-I)*Cos[c + d*x] - Sin[c + d*x]]
- 2*f^2*PolyLog[4, (-I)*Cos[c + d*x] - Sin[c + d*x]])*(Cos[c] - I*Sin[c])*(
1 - I*Cos[c] + Sin[c]))/d^4 - ((e + f*x)^3*Log[1 + I*Cos[c + d*x] + Sin[c +
 d*x]]*(1 + I*Cos[c] + Sin[c]))/d))/(a*(Cos[c] + I*(1 + Sin[c])))
```

Maple [B] time = 0.184, size = 679, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] I/a*e^3*x-1/4*I/a*f^3*x^4-2/a/d*ln(exp(I*(d*x+c)))*e^3+2/a/d*ln(exp(I*(d*x+
c))+I)*e^3+2/a/d^4*f^3*c^3*ln(exp(I*(d*x+c)))-I/a*e*f^2*x^3-2/a/d^4*f^3*c^3
*ln(exp(I*(d*x+c))+I)+12/a/d^3*e*f^2*polylog(3,I*exp(I*(d*x+c)))+12/a/d^3*f
^3*polylog(3,I*exp(I*(d*x+c)))*x-3/2*I/a/d^4*f^3*c^4-3/2*I/a*e^2*f*x^2+12*I
*f^3*polylog(4,I*exp(I*(d*x+c)))/a/d^4+2/a/d^4*f^3*c^3*ln(1-I*exp(I*(d*x+c)
))+2/a/d*f^3*ln(1-I*exp(I*(d*x+c)))*x^3+4*I/a/d^3*e*f^2*c^3-3*I/a/d^2*e^2*f
*c^2-6*I/a/d^2*e^2*f*polylog(2,I*exp(I*(d*x+c)))-6*I/a/d^2*f^3*polylog(2,I*
exp(I*(d*x+c)))*x^2-2*I/a/d^3*f^3*c^3*x+6*I/a/d^2*e*f^2*c^2*x+6/a/d*e*f^2*l
n(1-I*exp(I*(d*x+c)))*x^2-12*I/a/d^2*e*f^2*polylog(2,I*exp(I*(d*x+c)))*x-6*
I/a/d*e^2*f*c*x-6/a/d^3*e*f^2*c^2*ln(1-I*exp(I*(d*x+c)))+6/a/d*e^2*f*ln(1-I
*exp(I*(d*x+c)))*x+6/a/d^2*e^2*f*ln(1-I*exp(I*(d*x+c)))*c-6/a/d^2*e^2*f*c*l
n(exp(I*(d*x+c))+I)+6/a/d^3*e*f^2*c^2*ln(exp(I*(d*x+c))+I)+6/a/d^2*e^2*f*c*
ln(exp(I*(d*x+c)))-6/a/d^3*e*f^2*c^2*ln(exp(I*(d*x+c)))
```

Maxima [B] time = 1.39536, size = 689, normalized size = 4.56

$$-\frac{\frac{12\,ce^2f\log(ad\sin(dx+c)+ad)}{ad}-\frac{4\,e^3\log(a\sin(dx+c)+a)}{a}-\frac{-i\,(dx+c)^4f^3+\left(-4i\,def^2+4i\,cf^3\right)(dx+c)^3+48i\,f^3\mathrm{Li}_4(i\,e^{(i\,dx+i\,c)})+\left(-6i\,d^2e^2f+12i\,cdef^2-6i\,c^2f^3\right)(dx}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(12*c*e^2*f*log(a*d*sin(d*x + c) + a*d)/(a*d) - 4*e^3*log(a*sin(d*x +
c) + a)/a - (-I*(d*x + c)^4*f^3 + (-4*I*d*e*f^2 + 4*I*c*f^3)*(d*x + c)^3 +
48*I*f^3*polylog(4, I*e^(I*d*x + I*c)) + (-6*I*d^2*e^2*f + 12*I*c*d*e*f^2 -
 6*I*c^2*f^3)*(d*x + c)^2 + (-12*I*c^2*d*e*f^2 + 4*I*c^3*f^3)*(d*x + c) + (
24*I*c^2*d*e*f^2 - 8*I*c^3*f^3)*arctan2(sin(d*x + c) + 1, cos(d*x + c)) + (
-8*I*(d*x + c)^3*f^3 + (-24*I*d*e*f^2 + 24*I*c*f^3)*(d*x + c)^2 + (-24*I*d^
2*e^2*f + 48*I*c*d*e*f^2 - 24*I*c^2*f^3)*(d*x + c))*arctan2(cos(d*x + c), s
in(d*x + c) + 1) + (-24*I*d^2*e^2*f + 48*I*c*d*e*f^2 - 24*I*(d*x + c)^2*f^3
 - 24*I*c^2*f^3 + (-48*I*d*e*f^2 + 48*I*c*f^3)*(d*x + c))*dilog(I*e^(I*d*x
+ I*c)) + 4*(3*c^2*d*e*f^2 + (d*x + c)^3*f^3 - c^3*f^3 + 3*(d*e*f^2 - c*f^3
)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*(d*x + c))*log(cos(d*
x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 48*(d*e*f^2 + (d*x + c)*f
^3 - c*f^3)*polylog(3, I*e^(I*d*x + I*c)))/(a*d^3))/d
```

Fricas [C] time = 2.03033, size = 1189, normalized size = 7.87

$$\frac{6i f^3\text{polylog}(4, i\cos(dx+c) - \sin(dx+c)) - 6i f^3\text{polylog}(4, -i\cos(dx+c) - \sin(dx+c)) + \left(-3i d^2 f^3 x^2 - 6i d^2 e f^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (6*I*f^3*polylog(4, I*cos(d*x + c) - sin(d*x + c)) - 6*I*f^3*polylog(4, -I*
cos(d*x + c) - sin(d*x + c)) + (-3*I*d^2*f^3*x^2 - 6*I*d^2*e*f^2*x - 3*I*d^
2*e^2*f)*dilog(I*cos(d*x + c) - sin(d*x + c)) + (3*I*d^2*f^3*x^2 + 6*I*d^2*
e*f^2*x + 3*I*d^2*e^2*f)*dilog(-I*cos(d*x + c) - sin(d*x + c)) + (d^3*e^3 -
 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(cos(d*x + c) + I*sin(d*x + c)
 + I) + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*
c^2*d*e*f^2 + c^3*f^3)*log(I*cos(d*x + c) + sin(d*x + c) + 1) + (d^3*f^3*x^
3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f
^3)*log(-I*cos(d*x + c) + sin(d*x + c) + 1) + (d^3*e^3 - 3*c*d^2*e^2*f + 3*
c^2*d*e*f^2 - c^3*f^3)*log(-cos(d*x + c) + I*sin(d*x + c) + I) + 6*(d*f^3*x
 + d*e*f^2)*polylog(3, I*cos(d*x + c) - sin(d*x + c)) + 6*(d*f^3*x + d*e*f^
2)*polylog(3, -I*cos(d*x + c) - sin(d*x + c)))/(a*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^3\cos(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^3x^3\cos(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3ef^2x^2\cos(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{3e^2fx\cos(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**3*cos(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*cos
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*cos(c + d*x)/(sin
(c + d*x) + 1), x) + Integral(3*e**2*f*x*cos(c + d*x)/(sin(c + d*x) + 1), x
))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3\cos(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.252 $\int \frac{(e+fx)^2\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=114

$$-\frac{4if(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^2}+\frac{4f^2\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^3}+\frac{2(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{i(e+fx)^3}{3af}$$

```
[Out] ((-I/3)*(e + f*x)^3)/(a*f) + (2*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*
d) - ((4*I)*f*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2) + (4*f^2*Pol
yLog[3, I*E^(I*(c + d*x))])/(a*d^3)
```

Rubi [A] time = 0.211358, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4517, 2190, 2531, 2282, 6589}

$$-\frac{4if(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^2}+\frac{4f^2\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^3}+\frac{2(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{i(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-I/3)*(e + f*x)^3)/(a*f) + (2*(e + f*x)^2*Log[1 - I*E^(I*(c + d*x))])/(a*
d) - ((4*I)*f*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2) + (4*f^2*Pol
yLog[3, I*E^(I*(c + d*x))])/(a*d^3)
```

Rule 4517

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + Dist[2, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - I*b*E^(I*(c + d*x)))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)^2\cos(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{i(e+fx)^3}{3af}+2\int \frac{e^{i(c+dx)}(e+fx)^2}{a-iae^{i(c+dx)}}\,dx\\
&= -\frac{i(e+fx)^3}{3af}+\frac{2(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{(4f)\int(e+fx)\log\left(1-ie^{i(c+dx)}\right)dx}{ad}\\
&= -\frac{i(e+fx)^3}{3af}+\frac{2(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{4if(e+fx)\mathrm{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2}+\frac{\left(4if^2\right)\int \mathrm{Li}_2\left(ie^{i(c}\right.}{ad^2}\\
&= -\frac{i(e+fx)^3}{3af}+\frac{2(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{4if(e+fx)\mathrm{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2}+\frac{\left(4f^2\right)\mathrm{Subst}\left(\int \right.}{a}\\
&= -\frac{i(e+fx)^3}{3af}+\frac{2(e+fx)^2\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{4if(e+fx)\mathrm{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2}+\frac{4f^2\mathrm{Li}_3\left(ie^{i(c+dx)}\right)}{ad^3}
\end{aligned}
$$

Mathematica [A] time = 0.990181, size = 221, normalized size = 1.94

$$
\frac{x\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(3e^2+3efx+f^2x^2\right)}{3a\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)}-\frac{2(\cos(c)+i\sin(c))\left(\frac{2f(\cos(c)-i(\sin(c)+1))(d(e+fx)\mathrm{PolyLog}(2,-\sin(c+dx)-i\cos(c+dx))-if\mathrm{P}}{d^3}\right.}{a(}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*(Cos[c/2] - Sin[c/2]))/(3*a*(Cos[c/2] + Sin[
c/2])) - (2*(Cos[c] + I*Sin[c])*(((e + f*x)^3*(Cos[c] - I*Sin[c]))/(3*f) -
((e + f*x)^2*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(1 + I*Cos[c] + Sin[c])
)/d + (2*f*(d*(e + f*x)*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]] - I*f*
PolyLog[3, (-I)*Cos[c + d*x] - Sin[c + d*x]])*(Cos[c] - I*(1 + Sin[c])))/d^
3))/(a*(Cos[c] + I*(1 + Sin[c])))
```

Maple [B] time = 0.131, size = 421, normalized size = 3.7

$$\frac{2\,if^2c^2x}{ad^2}-\frac{ifex^2}{a}-\frac{2\,ifec^2}{ad^2}+4\,\frac{ef\ln\left(1-ie^{i(dx+c)}\right)x}{da}+4\,\frac{ef\ln\left(1-ie^{i(dx+c)}\right)c}{ad^2}+4\,\frac{efc\ln\left(e^{i(dx+c)}\right)}{ad^2}-\frac{4\,ife\mathrm{polylog}\left(2,ie^{i\ldots}\right.}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 2*I/a/d^2*f^2*c^2*x-I/a*f*e*x^2-2*I/a/d^2*e*f*c^2+4/a/d*f*e*ln(1-I*exp(I*(d
*x+c)))*x+4/a/d^2*f*e*ln(1-I*exp(I*(d*x+c)))*c+4/a/d^2*f*e*c*ln(exp(I*(d*x+
c)))-4*I/a/d^2*f*e*polylog(2,I*exp(I*(d*x+c)))-4/a/d^2*f*e*c*ln(exp(I*(d*x+
c))+I)-1/3*I/a*f^2*x^3-4*I/a/d*e*f*c*x+4*f^2*polylog(3,I*exp(I*(d*x+c)))/a/
d^3-2/a/d^3*f^2*c^2*ln(exp(I*(d*x+c)))+2/a/d*f^2*ln(1-I*exp(I*(d*x+c)))*x^2
-2/a/d^3*f^2*ln(1-I*exp(I*(d*x+c)))*c^2+2/a/d^3*f^2*c^2*ln(exp(I*(d*x+c))+I
)-4*I/a/d^2*f^2*polylog(2,I*exp(I*(d*x+c)))*x+4/3*I/a/d^3*f^2*c^3+I/a*e^2*x
+2/a/d*ln(exp(I*(d*x+c))+I)*e^2-2/a/d*ln(exp(I*(d*x+c)))*e^2
```

Maxima [B] time = 1.67445, size = 396, normalized size = 3.47

$$-\frac{\frac{6\,cef\log(ad\sin(dx+c)+ad)}{ad}-\frac{3\,e^2\log(a\sin(dx+c)+a)}{a}-\frac{-i\,(dx+c)^3f^2-3i\,(dx+c)c^2f^2+6i\,c^2f^2\arctan(\sin(dx+c)+1,\cos(dx+c))+\left(-3i\,def+3i\,cf^2\right)(dx+c)^2+\ldots}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(6*c*e*f*log(a*d*sin(d*x + c) + a*d)/(a*d) - 3*e^2*log(a*sin(d*x + c)
+ a)/a - (-I*(d*x + c)^3*f^2 - 3*I*(d*x + c)*c^2*f^2 + 6*I*c^2*f^2*arctan2(
sin(d*x + c) + 1, cos(d*x + c)) + (-3*I*d*e*f + 3*I*c*f^2)*(d*x + c)^2 + 12
*f^2*polylog(3, I*e^(I*d*x + I*c)) + (-6*I*(d*x + c)^2*f^2 + (-12*I*d*e*f +
 12*I*c*f^2)*(d*x + c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) + (-12*I*d*
e*f - 12*I*(d*x + c)*f^2 + 12*I*c*f^2)*dilog(I*e^(I*d*x + I*c)) + 3*((d*x +
 c)^2*f^2 + c^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*log(cos(d*x + c)^2 + sin
(d*x + c)^2 + 2*sin(d*x + c) + 1))/(a*d^2))/d
```

Fricas [C] time = 1.79578, size = 772, normalized size = 6.77

$$\frac{2 f^2 \text{polylog}(3, i \cos(dx+c) - \sin(dx+c)) + 2 f^2 \text{polylog}(3, -i \cos(dx+c) - \sin(dx+c)) + \left(-2i d f^2 x - 2i d e f\right) \text{Li}_2(i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*f^2*polylog(3, I*cos(d*x + c) - sin(d*x + c)) + 2*f^2*polylog(3, -I*cos(
d*x + c) - sin(d*x + c)) + (-2*I*d*f^2*x - 2*I*d*e*f)*dilog(I*cos(d*x + c)
- sin(d*x + c)) + (2*I*d*f^2*x + 2*I*d*e*f)*dilog(-I*cos(d*x + c) - sin(d*x
 + c)) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(cos(d*x + c) + I*sin(d*x + c)
+ I) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(I*cos(d*x + c)
 + sin(d*x + c) + 1) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*lo
g(-I*cos(d*x + c) + sin(d*x + c) + 1) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log
(-cos(d*x + c) + I*sin(d*x + c) + I))/(a*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^2 \cos(c+dx)}{\sin(c+dx)+1}\, dx + \int \frac{f^2 x^2 \cos(c+dx)}{\sin(c+dx)+1}\, dx + \int \frac{2efx \cos(c+dx)}{\sin(c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*cos(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*cos
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*cos(c + d*x)/(sin(c + d
*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2\cos(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.253 $\int \frac{(e+fx)\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=79

$$-\frac{2ifPolyLog\left(2,ie^{i(c+dx)}\right)}{ad^2}+\frac{2(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{i(e+fx)^2}{2af}$$

```
[Out] ((-I/2)*(e + f*x)^2)/(a*f) + (2*(e + f*x)*Log[1 - I*E^(I*(c + d*x))])/(a*d)
 - ((2*I)*f*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2)
```

Rubi [A] time = 0.125063, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4517, 2190, 2279, 2391}

$$-\frac{2ifPolyLog\left(2,ie^{i(c+dx)}\right)}{ad^2}+\frac{2(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{i(e+fx)^2}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-I/2)*(e + f*x)^2)/(a*f) + (2*(e + f*x)*Log[1 - I*E^(I*(c + d*x))])/(a*d)
 - ((2*I)*f*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2)
```

Rule 4517

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + Dist[2, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - I*b*E^(I*(c + d*x)))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)\cos(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{i(e+fx)^2}{2af}+2\int \frac{e^{i(c+dx)}(e+fx)}{a-iae^{i(c+dx)}}\,dx\\ &= -\frac{i(e+fx)^2}{2af}+\frac{2(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{(2f)\int \log\left(1-ie^{i(c+dx)}\right)dx}{ad}\\ &= -\frac{i(e+fx)^2}{2af}+\frac{2(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad}+\frac{(2if)\,\mathrm{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{i(c+dx)}\right)}{ad^2}\\ &= -\frac{i(e+fx)^2}{2af}+\frac{2(e+fx)\log\left(1-ie^{i(c+dx)}\right)}{ad}-\frac{2if\,\mathrm{Li}_2\left(ie^{i(c+dx)}\right)}{ad^2}\end{aligned}$$

Mathematica [B] time = 0.506804, size = 246, normalized size = 3.11

$$-4if\,\mathrm{PolyLog}\left(2,ie^{i(c+dx)}\right)-ic^2f+4de\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)-2icdfx+4cf\log\left(1-ie^{i(c+dx)}\right)+4\pi f$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-I)*c^2*f + I*c*f*Pi - (2*I)*c*d*f*x + I*d*f*Pi*x - I*d^2*f*x^2 + 4*f*Pi*
Log[1 + E^((-I)*(c + d*x))] + 4*c*f*Log[1 - I*E^(I*(c + d*x))] + 2*f*Pi*Log
[1 - I*E^(I*(c + d*x))] + 4*d*f*x*Log[1 - I*E^(I*(c + d*x))] - 4*f*Pi*Log[C
os[(c + d*x)/2]] + 4*d*e*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 4*c*f*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*f*Pi*Log[Sin[(2*c + Pi + 2*d*x)
/4]] - (4*I)*f*PolyLog[2, I*E^(I*(c + d*x))])/(2*a*d^2)
```

Maple [B] time = 0.148, size = 203, normalized size = 2.6

$$\frac{-\frac{i}{2}fx^2}{a}+\frac{iex}{a}-2\frac{\ln\left(e^{i(dx+c)}\right)e}{da}+2\frac{\ln\left(e^{i(dx+c)}+i\right)e}{da}-\frac{2ifcx}{da}-\frac{ifc^2}{ad^2}+2\frac{f\ln\left(1-ie^{i(dx+c)}\right)x}{da}+2\frac{f\ln\left(1-ie^{i(dx+c)}\right)c}{ad^2}-\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2*I/a*f*x^2+I/a*e*x-2/a/d*ln(exp(I*(d*x+c)))*e+2/a/d*ln(exp(I*(d*x+c))+I
)*e-2*I/a/d*f*c*x-I/a/d^2*f*c^2+2/a/d*f*ln(1-I*exp(I*(d*x+c)))*x+2/a/d^2*f*
ln(1-I*exp(I*(d*x+c)))*c-2*I*f*polylog(2,I*exp(I*(d*x+c)))/a/d^2+2/a/d^2*f*
c*ln(exp(I*(d*x+c)))-2/a/d^2*f*c*ln(exp(I*(d*x+c))+I)
```

Maxima [A] time = 1.36503, size = 157, normalized size = 1.99

$$\frac{-i\,d^2fx^2-2i\,d^2ex-4i\,dfx\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)+4i\,de\arctan\left(\sin\left(dx+c\right)+1,\cos\left(dx+c\right)\right)-4i\,f\mathrm{Li}_2}{2\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*d^2*f*x^2 - 2*I*d^2*e*x - 4*I*d*f*x*arctan2(cos(d*x + c), sin(d*x +
 c) + 1) + 4*I*d*e*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - 4*I*f*dilog(I*
e^(I*d*x + I*c)) + 2*(d*f*x + d*e)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*
sin(d*x + c) + 1))/(a*d^2)
```

Fricas [B] time = 1.94904, size = 425, normalized size = 5.38

$$\frac{-i\,f\mathrm{Li}_2\left(i\,\cos\left(dx+c\right)-\sin\left(dx+c\right)\right)+i\,f\mathrm{Li}_2\left(-i\,\cos\left(dx+c\right)-\sin\left(dx+c\right)\right)+\left(de-cf\right)\log\left(\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (-I*f*dilog(I*cos(d*x + c) - sin(d*x + c)) + I*f*dilog(-I*cos(d*x + c) - si
n(d*x + c)) + (d*e - c*f)*log(cos(d*x + c) + I*sin(d*x + c) + I) + (d*f*x +
```

```
 c*f)*log(I*cos(d*x + c) + sin(d*x + c) + 1) + (d*f*x + c*f)*log(-I*cos(d*x
 + c) + sin(d*x + c) + 1) + (d*e - c*f)*log(-cos(d*x + c) + I*sin(d*x + c)
+ I))/(a*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e\cos(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{fx\cos(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e*cos(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f*x*cos(c + d*x)
/(sin(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cos(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.254 $\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=16

$$\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] Log[1 + Sin[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0253777, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 31}

$$\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\log(1+\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.0112043, size = 16, normalized size = 1.

$$\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a*d)
```

Maple [A] time = 0.012, size = 19, normalized size = 1.2

$$\frac{\ln(a+a\sin(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*ln(a+a*sin(d*x+c))/a
```

Maxima [A] time = 0.995789, size = 24, normalized size = 1.5

$$\frac{\log(a\sin(dx+c)+a)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(a*sin(d*x + c) + a)/(a*d)
```

Fricas [A] time = 1.71757, size = 39, normalized size = 2.44

$$\frac{\log\left(\sin\left(dx+c\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] log(sin(d*x + c) + 1)/(a*d)
```

Sympy [A] time = 0.490675, size = 24, normalized size = 1.5

$$\begin{cases} \frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad} & \text{for}\ d \neq 0 \\ \frac{x\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((log(sin(c + d*x) + 1)/(a*d), Ne(d, 0)), (x*cos(c)/(a*sin(c) + a)
, True))
```

Giac [A] time = 1.14857, size = 26, normalized size = 1.62

$$\frac{\log\left(|a\sin\left(dx+c\right)+a|\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(a*sin(d*x + c) + a))/(a*d)
```

3.255 $\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\cos(c+dx)}{(e+fx)(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Cos[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0468099, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Cos[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 3.06412, size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Cos[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 0.181, size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(fx+e)(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)}{afx+ae+(afx+ae)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos (c+dx)}{e\sin (c+dx)+e+fx\sin (c+dx)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx+c)}{(fx+e)(a\sin (dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

3.256 $\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\cos(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Cos[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.047282, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Cos[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx=\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 3.91048, size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Cos[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 0.217, size = 0, normalized size = 0.

$$\int \frac{\cos (dx+c)}{\left(fx+e\right)^{2} \left(a+a\sin (dx+c)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx+c)}{\left(fx+e\right)^{2}\left(a\sin (dx+c)+a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos (dx+c)}{af^{2}x^{2}+2\,aefx+ae^{2}+\left(af^{2}x^{2}+2\,aefx+ae^{2}\right)\sin (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e*f
*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{\left(fx+e\right)^2 (a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

3.257 $\int \frac{(e+fx)^3\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=99

$$-\frac{6f^2(e+fx)\cos(c+dx)}{ad^3}-\frac{3f(e+fx)^2\sin(c+dx)}{ad^2}+\frac{6f^3\sin(c+dx)}{ad^4}+\frac{(e+fx)^3\cos(c+dx)}{ad}+\frac{(e+fx)^4}{4af}$$

```
[Out] (e + f*x)^4/(4*a*f) - (6*f^2*(e + f*x)*Cos[c + d*x])/(a*d^3) + ((e + f*x)^3
*Cos[c + d*x])/(a*d) + (6*f^3*Sin[c + d*x])/(a*d^4) - (3*f*(e + f*x)^2*Sin[
c + d*x])/(a*d^2)
```

Rubi [A] time = 0.144176, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4523, 32, 3296, 2637}

$$-\frac{6f^2(e+fx)\cos(c+dx)}{ad^3}-\frac{3f(e+fx)^2\sin(c+dx)}{ad^2}+\frac{6f^3\sin(c+dx)}{ad^4}+\frac{(e+fx)^3\cos(c+dx)}{ad}+\frac{(e+fx)^4}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (e + f*x)^4/(4*a*f) - (6*f^2*(e + f*x)*Cos[c + d*x])/(a*d^3) + ((e + f*x)^3
*Cos[c + d*x])/(a*d) + (6*f^3*Sin[c + d*x])/(a*d^4) - (3*f*(e + f*x)^2*Sin[
c + d*x])/(a*d^2)
```

Rule 4523

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\,dx}{a} - \frac{\int (e+fx)^3\sin(c+dx)\,dx}{a}$$
$$= \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{(3f)\int (e+fx)^2\cos(c+dx)\,dx}{ad}$$
$$= \frac{(e+fx)^4}{4af} + \frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{3f(e+fx)^2\sin(c+dx)}{ad^2} + \frac{\left(6f^2\right)\int (e+fx)\sin(c+dx$$
$$= \frac{(e+fx)^4}{4af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} + \frac{(e+fx)^3\cos(c+dx)}{ad} - \frac{3f(e+fx)^2\sin(c+dx)}{ad^2} +$$
$$= \frac{(e+fx)^4}{4af} - \frac{6f^2(e+fx)\cos(c+dx)}{ad^3} + \frac{(e+fx)^3\cos(c+dx)}{ad} + \frac{6f^3\sin(c+dx)}{ad^4} - \frac{3f(e+$$

Mathematica [A] time = 0.642866, size = 102, normalized size = 1.03

$$\frac{-12f\sin(c+dx)\left(d^2(e+fx)^2-2f^2\right)+4d(e+fx)\cos(c+dx)\left(d^2(e+fx)^2-6f^2\right)+d^4x\left(6e^2fx+4e^3+4ef^2x^2+f^3x^3\right)}{4ad^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) + 4*d*(e + f*x)*(-6*f^2
+ d^2*(e + f*x)^2)*Cos[c + d*x] - 12*f*(-2*f^2 + d^2*(e + f*x)^2)*Sin[c + d
*x])/(4*a*d^4)
```

Maple [B] time = 0.066, size = 436, normalized size = 4.4

$$-\frac{1}{d^4a}\left(f^3\left(-(dx+c)^3\cos(dx+c)+3(dx+c)^2\sin(dx+c)-6\sin(dx+c)+6(dx+c)\cos(dx+c)\right)-3cf^3\left(-(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d^4/a*(f^3*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*
(d*x+c)*cos(d*x+c))-3*c*f^3*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*s
in(d*x+c))+3*f^2*e*d*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+
c))+3*c^2*f^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-6*c*d*e*f^2*(sin(d*x+c)-(d*x+
c)*cos(d*x+c))+3*d^2*e^2*f*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+c^3*f^3*cos(d*x+
c)-3*c^2*d*e*f^2*cos(d*x+c)+3*c*d^2*e^2*f*cos(d*x+c)-d^3*e^3*cos(d*x+c)-1/4
*f^3*(d*x+c)^4+c*f^3*(d*x+c)^3-f^2*e*d*(d*x+c)^3-3/2*c^2*f^3*(d*x+c)^2+3*c*
d*e*f^2*(d*x+c)^2-3/2*d^2*e^2*f*(d*x+c)^2+c^3*f^3*(d*x+c)-3*c^2*d*e*f^2*(d*
x+c)+3*c*d^2*e^2*f*(d*x+c)-d^3*e^3*(d*x+c))
```

Maxima [B] time = 1.63407, size = 721, normalized size = 7.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(8*c^3*f^3*(1/(a*d^3 + a*d^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + ar
ctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d^3)) - 24*c^2*e*f^2*(1/(a*d^2 + a
*d^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/(a*d^2)) + 24*c*e^2*f*(1/(a*d + a*d*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d)) - 8*e^3*(arctan(sin
(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)) - 6*((d*x + c)^2 + 2*(d*x + c)*cos(d*x + c) - 2*sin(d*x + c))*e^2*f/(
a*d) + 12*((d*x + c)^2 + 2*(d*x + c)*cos(d*x + c) - 2*sin(d*x + c))*c*e*f^2
/(a*d^2) - 6*((d*x + c)^2 + 2*(d*x + c)*cos(d*x + c) - 2*sin(d*x + c))*c^2*
f^3/(a*d^3) - 4*((d*x + c)^3 + 3*((d*x + c)^2 - 2)*cos(d*x + c) - 6*(d*x +
c)*sin(d*x + c))*e*f^2/(a*d^2) + 4*((d*x + c)^3 + 3*((d*x + c)^2 - 2)*cos(d
*x + c) - 6*(d*x + c)*sin(d*x + c))*c*f^3/(a*d^3) - ((d*x + c)^4 + 4*((d*x
+ c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 12*((d*x + c)^2 - 2)*sin(d*x + c))*f^3
/(a*d^3))/d
```

Fricas [A] time = 1.6683, size = 329, normalized size = 3.32

$$\frac{d^4 f^3 x^4 + 4\,d^4 e f^2 x^3 + 6\,d^4 e^2 f x^2 + 4\,d^4 e^3 x + 4\left(d^3 f^3 x^3 + 3\,d^3 e f^2 x^2 + d^3 e^3 - 6\,d e f^2 + 3\left(d^3 e^2 f - 2\,d f^3\right)x\right)\cos\left(dx + c\right) - }{4\,a d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2 + 4*d^4*e^3*x + 4*(d^3
*f^3*x^3 + 3*d^3*e*f^2*x^2 + d^3*e^3 - 6*d*e*f^2 + 3*(d^3*e^2*f - 2*d*f^3)*
x)*cos(d*x + c) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f - 2*f^3)*sin(
d*x + c))/(a*d^4)
```

Sympy [A] time = 12.7856, size = 1232, normalized size = 12.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((4*d**4*e**3*x*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2
+ 4*a*d**4) + 4*d**4*e**3*x/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 6*d
**4*e**2*f*x**2*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**
4) + 6*d**4*e**2*f*x**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 4*d**4*
e*f**2*x**3*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) +
 4*d**4*e*f**2*x**3/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + d**4*f**3*x
**4*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + d**4*f*
*3*x**4/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) - 4*d**3*e**3*tan(c/2 + d
*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 4*d**3*e**3/(4*a*d**4*
tan(c/2 + d*x/2)**2 + 4*a*d**4) - 12*d**3*e**2*f*x*tan(c/2 + d*x/2)**2/(4*a
*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 12*d**3*e**2*f*x/(4*a*d**4*tan(c/2
+ d*x/2)**2 + 4*a*d**4) - 12*d**3*e*f**2*x**2*tan(c/2 + d*x/2)**2/(4*a*d**4
*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 12*d**3*e*f**2*x**2/(4*a*d**4*tan(c/2 +
d*x/2)**2 + 4*a*d**4) - 4*d**3*f**3*x**3*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(
c/2 + d*x/2)**2 + 4*a*d**4) + 4*d**3*f**3*x**3/(4*a*d**4*tan(c/2 + d*x/2)**
2 + 4*a*d**4) + 12*d**2*e**2*f*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/
2)**2 + 4*a*d**4) - 24*d**2*e**2*f*tan(c/2 + d*x/2)/(4*a*d**4*tan(c/2 + d*x
/2)**2 + 4*a*d**4) + 12*d**2*e**2*f/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**
4) - 48*d**2*e*f**2*x*tan(c/2 + d*x/2)/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*
d**4) - 24*d**2*f**3*x**2*tan(c/2 + d*x/2)/(4*a*d**4*tan(c/2 + d*x/2)**2 +
4*a*d**4) + 24*d*e*f**2*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2 +
 4*a*d**4) - 24*d*e*f**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 24*d*f
**3*x*tan(c/2 + d*x/2)**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) - 24*d*
f**3*x/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) - 24*f**3*tan(c/2 + d*x/2)
**2/(4*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) + 48*f**3*tan(c/2 + d*x/2)/(4
```

```
*a*d**4*tan(c/2 + d*x/2)**2 + 4*a*d**4) - 24*f**3/(4*a*d**4*tan(c/2 + d*x/2
)**2 + 4*a*d**4), Ne(d, 0)), ((e**3*x + 3*e**2*f*x**2/2 + e*f**2*x**3 + f**
3*x**4/4)*cos(c)**2/(a*sin(c) + a), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.258 $\int \frac{(e+fx)^2 \cos^2(c+dx)}{a+a\sin(c+dx)} dx$

Optimal. Leaf size=75

$$-\frac{2f(e+fx)\sin(c+dx)}{ad^2}-\frac{2f^2\cos(c+dx)}{ad^3}+\frac{(e+fx)^2\cos(c+dx)}{ad}+\frac{(e+fx)^3}{3af}$$

```
[Out] (e + f*x)^3/(3*a*f) - (2*f^2*Cos[c + d*x])/(a*d^3) + ((e + f*x)^2*Cos[c + d
*x])/(a*d) - (2*f*(e + f*x)*Sin[c + d*x])/(a*d^2)
```

Rubi [A] time = 0.114828, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4523, 32, 3296, 2638}

$$-\frac{2f(e+fx)\sin(c+dx)}{ad^2}-\frac{2f^2\cos(c+dx)}{ad^3}+\frac{(e+fx)^2\cos(c+dx)}{ad}+\frac{(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (e + f*x)^3/(3*a*f) - (2*f^2*Cos[c + d*x])/(a*d^3) + ((e + f*x)^2*Cos[c + d
*x])/(a*d) - (2*f*(e + f*x)*Sin[c + d*x])/(a*d^2)
```

Rule 4523

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)^2\,dx}{a}-\frac{\int (e+fx)^2\sin(c+dx)\,dx}{a}\\
&= \frac{(e+fx)^3}{3af}+\frac{(e+fx)^2\cos(c+dx)}{ad}-\frac{(2f)\int (e+fx)\cos(c+dx)\,dx}{ad}\\
&= \frac{(e+fx)^3}{3af}+\frac{(e+fx)^2\cos(c+dx)}{ad}-\frac{2f(e+fx)\sin(c+dx)}{ad^2}+\frac{\left(2f^2\right)\int \sin(c+dx)\,dx}{ad^2}\\
&= \frac{(e+fx)^3}{3af}-\frac{2f^2\cos(c+dx)}{ad^3}+\frac{(e+fx)^2\cos(c+dx)}{ad}-\frac{2f(e+fx)\sin(c+dx)}{ad^2}
\end{aligned}$$

Mathematica [A] time = 0.423374, size = 74, normalized size = 0.99

$$\frac{3\cos(c+dx)\left(d^2(e+fx)^2-2f^2\right)-6df(e+fx)\sin(c+dx)+d^3x\left(3e^2+3efx+f^2x^2\right)}{3ad^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 3*(-2*f^2 + d^2*(e + f*x)^2)*Cos[c + d
*x] - 6*d*f*(e + f*x)*Sin[c + d*x])/(3*a*d^3)
```

Maple [B] time = 0.059, size = 215, normalized size = 2.9

$$-\frac{1}{ad^3}\left(f^2\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)-2cf^2\left(\sin(dx+c)-(dx+c)\cos(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d^3/a*(f^2*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-2*c
*f^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+2*d*e*f*(sin(d*x+c)-(d*x+c)*cos(d*x+c)
)-c^2*f^2*cos(d*x+c)+2*c*d*e*f*cos(d*x+c)-d^2*e^2*cos(d*x+c)-1/3*f^2*(d*x+c
)^3+c*f^2*(d*x+c)^2-d*e*f*(d*x+c)^2-c^2*f^2*(d*x+c)+2*c*d*e*f*(d*x+c)-d^2*e
^2*(d*x+c))
```

Maxima [B] time = 1.56341, size = 417, normalized size = 5.56

$$\frac{6\,c^2 f^2\left(\frac{1}{ad^2+\frac{ad^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{ad^2}\right)-12\,cef\left(\frac{1}{ad+\frac{ad\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{ad}\right)+6\,e^2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin\ldots}{(\cos(d\ldots}}\right.\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(6*c^2*f^2*(1/(a*d^2 + a*d^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + arc
tan(sin(d*x + c)/(cos(d*x + c) + 1))/(a*d^2)) - 12*c*e*f*(1/(a*d + a*d*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/
(a*d)) + 6*e^2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)) + 3*((d*x + c)^2 + 2*(d*x + c)*cos(d*x + c)
 - 2*sin(d*x + c))*e*f/(a*d) - 3*((d*x + c)^2 + 2*(d*x + c)*cos(d*x + c) -
2*sin(d*x + c))*c*f^2/(a*d^2) + ((d*x + c)^3 + 3*((d*x + c)^2 - 2)*cos(d*x
+ c) - 6*(d*x + c)*sin(d*x + c))*f^2/(a*d^2))/d
```

Fricas [A] time = 1.75602, size = 209, normalized size = 2.79

$$\frac{d^3f^2x^3+3\,d^3efx^2+3\,d^3e^2x+3\left(d^2f^2x^2+2\,d^2efx+d^2e^2-2\,f^2\right)\cos\left(dx+c\right)-6\left(df^2x+def\right)\sin\left(dx+c\right)}{3\,ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 3*(d^2*f^2*x^2 + 2*d^2*e*f
*x + d^2*e^2 - 2*f^2)*cos(d*x + c) - 6*(d*f^2*x + d*e*f)*sin(d*x + c))/(a*d
```

```
^3)
```

Sympy [A] time = 8.52252, size = 770, normalized size = 10.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*d**3*e**2*x*tan(c/2 + d*x/2)**2/(3*a*d**3*tan(c/2 + d*x/2)**2
+ 3*a*d**3) + 3*d**3*e**2*x/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) + 3*d
**3*e*f*x**2*tan(c/2 + d*x/2)**2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3)
+ 3*d**3*e*f*x**2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) + d**3*f**2*x**
3*tan(c/2 + d*x/2)**2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) + d**3*f**2
*x**3/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) - 3*d**2*e**2*tan(c/2 + d*x
/2)**2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) + 3*d**2*e**2/(3*a*d**3*ta
n(c/2 + d*x/2)**2 + 3*a*d**3) - 6*d**2*e*f*x*tan(c/2 + d*x/2)**2/(3*a*d**3*
tan(c/2 + d*x/2)**2 + 3*a*d**3) + 6*d**2*e*f*x/(3*a*d**3*tan(c/2 + d*x/2)**
2 + 3*a*d**3) - 3*d**2*f**2*x**2*tan(c/2 + d*x/2)**2/(3*a*d**3*tan(c/2 + d*
x/2)**2 + 3*a*d**3) + 3*d**2*f**2*x**2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*
d**3) + 6*d*e*f*tan(c/2 + d*x/2)**2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**
3) - 12*d*e*f*tan(c/2 + d*x/2)/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) +
6*d*e*f/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) - 12*d*f**2*x*tan(c/2 + d
*x/2)/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) + 6*f**2*tan(c/2 + d*x/2)**
2/(3*a*d**3*tan(c/2 + d*x/2)**2 + 3*a*d**3) - 6*f**2/(3*a*d**3*tan(c/2 + d*
x/2)**2 + 3*a*d**3), Ne(d, 0)), ((e**2*x + e*f*x**2 + f**2*x**3/3)*cos(c)**
2/(a*sin(c) + a), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.259 $\int \frac{(e+fx)\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=51

$$-\frac{f\sin(c+dx)}{ad^2}+\frac{(e+fx)\cos(c+dx)}{ad}+\frac{ex}{a}+\frac{fx^2}{2a}$$

```
[Out] (e*x)/a + (f*x^2)/(2*a) + ((e + f*x)*Cos[c + d*x])/(a*d) - (f*Sin[c + d*x])
/(a*d^2)
```

Rubi [A] time = 0.0641802, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {4523, 3296, 2637}

$$-\frac{f\sin(c+dx)}{ad^2}+\frac{(e+fx)\cos(c+dx)}{ad}+\frac{ex}{a}+\frac{fx^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (e*x)/a + (f*x^2)/(2*a) + ((e + f*x)*Cos[c + d*x])/(a*d) - (f*Sin[c + d*x])
/(a*d^2)
```

Rule 4523

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)\,dx}{a}-\frac{\int (e+fx)\sin(c+dx)\,dx}{a}\\ &= \frac{ex}{a}+\frac{fx^2}{2a}+\frac{(e+fx)\cos(c+dx)}{ad}-\frac{f\int \cos(c+dx)\,dx}{ad}\\ &= \frac{ex}{a}+\frac{fx^2}{2a}+\frac{(e+fx)\cos(c+dx)}{ad}-\frac{f\sin(c+dx)}{ad^2}\end{aligned}$$

Mathematica [A] time = 0.505234, size = 53, normalized size = 1.04

$$-\frac{(c+dx)(cf-2de-dfx)-2d(e+fx)\cos(c+dx)+2f\sin(c+dx)}{2ad^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((c + d*x)*(-2*d*e + c*f - d*f*x) - 2*d*(e + f*x)*Cos[c + d*x] + 2*f*Sin[c
 + d*x])/(2*a*d^2)
```

Maple [A] time = 0.058, size = 78, normalized size = 1.5

$$-\frac{1}{ad^2}\left(f\left(\sin\left(dx+c\right)-\left(dx+c\right)\cos\left(dx+c\right)\right)+cf\cos\left(dx+c\right)-de\cos\left(dx+c\right)-\frac{f\left(dx+c\right)^2}{2}+cf\left(dx+c\right)-de\left(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d^2/a*(f*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+c*f*cos(d*x+c)-d*e*cos(d*x+c)-1
/2*f*(d*x+c)^2+c*f*(d*x+c)-d*e*(d*x+c))
```

Maxima [B] time = 1.52251, size = 204, normalized size = 4.

$$-\frac{4\,cf\left(\frac{1}{ad+\frac{ad\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{ad}\right)-4\,e\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}\right)-\frac{\left((dx+c)^2+2\,(dx+c)\cos(dx+c)-2\,\sin(dx+c)\right)f}{ad}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(4*c*f*(1/(a*d + a*d*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + arctan(sin
(d*x + c)/(cos(d*x + c) + 1))/(a*d)) - 4*e*(arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a + 1/(a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)) - ((d*x + c)^2
+ 2*(d*x + c)*cos(d*x + c) - 2*sin(d*x + c))*f/(a*d))/d
```

Fricas [A] time = 1.60205, size = 117, normalized size = 2.29

$$\frac{d^2fx^2 + 2\,d^2ex + 2\left(dfx + de\right)\cos\left(dx + c\right) - 2\,f\sin\left(dx + c\right)}{2\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(d^2*f*x^2 + 2*d^2*e*x + 2*(d*f*x + d*e)*cos(d*x + c) - 2*f*sin(d*x + c
))/(a*d^2)
```

Sympy [A] time = 5.37991, size = 439, normalized size = 8.61

$$\begin{cases} \frac{2d^2ex\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2} + \frac{2d^2ex}{2ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2} + \frac{d^2fx^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2} + \frac{d^2fx^2}{2ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2} - \frac{2de\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad^2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad^2} + \frac{2d}{2ad^2\tan^2\left(\frac{c}{2}\right.} \\ \frac{\left(ex+\frac{fx^2}{2}\right)\cos^2(c)}{a\sin(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*d**2*e*x*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2
*a*d**2) + 2*d**2*e*x/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2) + d**2*f*x*
*2*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2) + d**2*f*x
**2/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2) - 2*d*e*tan(c/2 + d*x/2)**2/(
2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2) + 2*d*e/(2*a*d**2*tan(c/2 + d*x/2)
**2 + 2*a*d**2) - 2*d*f*x*tan(c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**2
```

```
 + 2*a*d**2) + 2*d*f*x/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2) + 2*f*tan(
c/2 + d*x/2)**2/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2) - 4*f*tan(c/2 + d
*x/2)/(2*a*d**2*tan(c/2 + d*x/2)**2 + 2*a*d**2) + 2*f/(2*a*d**2*tan(c/2 + d
*x/2)**2 + 2*a*d**2), Ne(d, 0)), ((e*x + f*x**2/2)*cos(c)**2/(a*sin(c) + a)
, True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.260 $\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=19

$$\frac{\cos(c+dx)}{ad}+\frac{x}{a}$$

```
[Out] x/a + Cos[c + d*x]/(a*d)
```

Rubi [A] time = 0.0421265, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2682, 8}

$$\frac{\cos(c+dx)}{ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + Cos[c + d*x]/(a*d)
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\cos(c+dx)}{ad}+\frac{\int 1\,dx}{a}\\ &= \frac{x}{a}+\frac{\cos(c+dx)}{ad}\end{aligned}$$

Mathematica [B] time = 0.138564, size = 97, normalized size = 5.11

$$-\frac{\left(2\sqrt{1-\sin(c+dx)}\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+(\sin(c+dx)-1)\sqrt{\sin(c+dx)+1}\right)\cos^3(c+dx)}{ad(\sin(c+dx)-1)^2(\sin(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]^3*(2*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c
+ d*x]] + (-1 + Sin[c + d*x])*Sqrt[1 + Sin[c + d*x]]))/(a*d*(-1 + Sin[c + d
*x])^2*(1 + Sin[c + d*x])^(3/2)))
```

Maple [B] time = 0.048, size = 43, normalized size = 2.3

$$2\,\frac{1}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)}+2\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 2/a/d/(1+tan(1/2*d*x+1/2*c)^2)+2/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.59223, size = 70, normalized size = 3.68

$$\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2))/d
```

Fricas [A] time = 1.59846, size = 38, normalized size = 2.

$$\frac{dx + \cos(dx + c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x + cos(d*x + c))/(a*d)
```

Sympy [A] time = 3.26248, size = 119, normalized size = 6.26

$$\begin{cases} \frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{dx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{1}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) + d*x/(a
*d*tan(c/2 + d*x/2)**2 + a*d) - tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**
2 + a*d) + 1/(a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*cos(c)**2/(a*si
n(c) + a), True))
```

Giac [A] time = 1.11696, size = 46, normalized size = 2.42

$$\frac{\frac{dx+c}{a} + \frac{2}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a + 2/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

3.261 $\int \frac{\cos^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=72

$$-\frac{\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{af}-\frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af}+\frac{\log(e+fx)}{af}$$

```
[Out] Log[e + f*x]/(a*f) - (CosIntegral[(d*e)/f + d*x]*Sin[c - (d*e)/f])/(a*f) -
(Cos[c - (d*e)/f]*SinIntegral[(d*e)/f + d*x])/(a*f)
```

Rubi [A] time = 0.20148, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4523, 31, 3303, 3299, 3302}

$$-\frac{\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{af}-\frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af}+\frac{\log(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Log[e + f*x]/(a*f) - (CosIntegral[(d*e)/f + d*x]*Sin[c - (d*e)/f])/(a*f) -
(Cos[c - (d*e)/f]*SinIntegral[(d*e)/f + d*x])/(a*f)
```

Rule 4523

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx &= \frac{\int \frac{1}{e+fx}\,dx}{a} - \frac{\int \frac{\sin(c+dx)}{e+fx}\,dx}{a}\\ &= \frac{\log(e+fx)}{af} - \frac{\cos\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{a} - \frac{\sin\left(c-\frac{de}{f}\right)\int \frac{\cos\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{a}\\ &= \frac{\log(e+fx)}{af} - \frac{\text{Ci}\left(\frac{de}{f}+dx\right)\sin\left(c-\frac{de}{f}\right)}{af} - \frac{\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af}\end{aligned}$$

Mathematica [A] time = 0.275474, size = 58, normalized size = 0.81

$$\frac{-\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(d\left(\frac{e}{f}+x\right)\right)-\cos\left(c-\frac{de}{f}\right)\text{Si}\left(d\left(\frac{e}{f}+x\right)\right)+\log(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (Log[e + f*x] - CosIntegral[d*(e/f + x)]*Sin[c - (d*e)/f] - Cos[c - (d*e)/f
]*SinIntegral[d*(e/f + x)])/(a*f)
```

Maple [A] time = 0.054, size = 102, normalized size = 1.4

$$-\frac{1}{af}\mathrm{Si}\left(dx+c+\frac{-cf+de}{f}\right)\cos\left(\frac{-cf+de}{f}\right)+\frac{1}{af}\mathrm{Ci}\left(dx+c+\frac{-cf+de}{f}\right)\sin\left(\frac{-cf+de}{f}\right)+\frac{\ln\left((dx+c)f-cf+de\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/a*Si(d*x+c+(-c*f+d*e)/f)*cos((-c*f+d*e)/f)/f+1/a*Ci(d*x+c+(-c*f+d*e)/f)*
sin((-c*f+d*e)/f)/f+1/a*ln((d*x+c)*f-c*f+d*e)/f
```

Maxima [C] time = 1.32096, size = 220, normalized size = 3.06

$$\frac{d\left(iE_1\left(\frac{ide+i(dx+c)f-icf}{f}\right)-iE_1\left(-\frac{ide+i(dx+c)f-icf}{f}\right)\right)\cos\left(-\frac{de-cf}{f}\right)+d\left(E_1\left(\frac{ide+i(dx+c)f-icf}{f}\right)+E_1\left(-\frac{ide+i(dx+c)f-icf}{f}\right)\right)\sin\left(-\right.}{2adf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(d*(I*exp_integral_e(1, (I*d*e + I*(d*x + c)*f - I*c*f)/f) - I*exp_inte
gral_e(1, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*cos(-(d*e - c*f)/f) + d*(exp
_integral_e(1, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + exp_integral_e(1, -(I*d
*e + I*(d*x + c)*f - I*c*f)/f))*sin(-(d*e - c*f)/f) + 2*d*log(d*e + (d*x +
c)*f - c*f))/(a*d*f)
```

Fricas [A] time = 1.73619, size = 230, normalized size = 3.19

$$-\frac{\left(\mathrm{Ci}\left(\frac{dfx+de}{f}\right)+\mathrm{Ci}\left(-\frac{dfx+de}{f}\right)\right)\sin\left(-\frac{de-cf}{f}\right)+2\cos\left(-\frac{de-cf}{f}\right)\mathrm{Si}\left(\frac{dfx+de}{f}\right)-2\log\left(fx+e\right)}{2af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*((cos_integral((d*f*x + d*e)/f) + cos_integral(-(d*f*x + d*e)/f))*sin(
-(d*e - c*f)/f) + 2*cos(-(d*e - c*f)/f)*sin_integral((d*f*x + d*e)/f) - 2*l
og(f*x + e))/(a*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.3586, size = 967, normalized size = 13.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f)^2 -
imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f)^2 - 2*log
(abs(f*x + e))*tan(1/2*c)^2*tan(1/2*d*e/f)^2 + 2*sin_integral((d*f*x + d*e)
/f)*tan(1/2*c)^2*tan(1/2*d*e/f)^2 + 2*real_part(cos_integral(d*x + d*e/f))*
tan(1/2*c)^2*tan(1/2*d*e/f) + 2*real_part(cos_integral(-d*x - d*e/f))*tan(1
/2*c)^2*tan(1/2*d*e/f) - 2*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c)*
tan(1/2*d*e/f)^2 - 2*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)*tan(1
/2*d*e/f)^2 - imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^2 + imag_part
(cos_integral(-d*x - d*e/f))*tan(1/2*c)^2 - 2*log(abs(f*x + e))*tan(1/2*c)^
2 - 2*sin_integral((d*f*x + d*e)/f)*tan(1/2*c)^2 + 4*imag_part(cos_integral
(d*x + d*e/f))*tan(1/2*c)*tan(1/2*d*e/f) - 4*imag_part(cos_integral(-d*x -
d*e/f))*tan(1/2*c)*tan(1/2*d*e/f) + 8*sin_integral((d*f*x + d*e)/f)*tan(1/2
*c)*tan(1/2*d*e/f) - imag_part(cos_integral(d*x + d*e/f))*tan(1/2*d*e/f)^2
+ imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*e/f)^2 - 2*log(abs(f*x +
e))*tan(1/2*d*e/f)^2 - 2*sin_integral((d*f*x + d*e)/f)*tan(1/2*d*e/f)^2 + 2
*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c) + 2*real_part(cos_integral
(-d*x - d*e/f))*tan(1/2*c) - 2*real_part(cos_integral(d*x + d*e/f))*tan(1/2
```

```
*d*e/f) - 2*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*e/f) + imag_par
t(cos_integral(d*x + d*e/f)) - imag_part(cos_integral(-d*x - d*e/f)) - 2*lo
g(abs(f*x + e)) + 2*sin_integral((d*f*x + d*e)/f))/(a*f*tan(1/2*c)^2*tan(1/
2*d*e/f)^2 + a*f*tan(1/2*c)^2 + a*f*tan(1/2*d*e/f)^2 + a*f)
```

3.262 $\int \frac{\cos^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=95

$$-\frac{d\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{d\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{\sin(c+dx)}{af(e+fx)}-\frac{1}{af(e+fx)}$$

```
[Out] -(1/(a*f*(e + f*x))) - (d*Cos[c - (d*e)/f]*CosIntegral[(d*e)/f + d*x])/(a*f
^2) + Sin[c + d*x]/(a*f*(e + f*x)) + (d*Sin[c - (d*e)/f]*SinIntegral[(d*e)/
f + d*x])/(a*f^2)
```

Rubi [A] time = 0.200108, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4523, 32, 3297, 3303, 3299, 3302}

$$-\frac{d\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{d\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{\sin(c+dx)}{af(e+fx)}-\frac{1}{af(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] -(1/(a*f*(e + f*x))) - (d*Cos[c - (d*e)/f]*CosIntegral[(d*e)/f + d*x])/(a*f
^2) + Sin[c + d*x]/(a*f*(e + f*x)) + (d*Sin[c - (d*e)/f]*SinIntegral[(d*e)/
f + d*x])/(a*f^2)
```

Rule 4523

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx &= \frac{\int \frac{1}{(e+fx)^2}\,dx}{a} - \frac{\int \frac{\sin(c+dx)}{(e+fx)^2}\,dx}{a} \\
&= -\frac{1}{af(e+fx)} + \frac{\sin(c+dx)}{af(e+fx)} - \frac{d\int \frac{\cos(c+dx)}{e+fx}\,dx}{af} \\
&= -\frac{1}{af(e+fx)} + \frac{\sin(c+dx)}{af(e+fx)} - \frac{\left(d\cos\left(c-\frac{de}{f}\right)\right)\int \frac{\cos\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{af} + \frac{\left(d\sin\left(c-\frac{de}{f}\right)\right)\int}{af} \\
&= -\frac{1}{af(e+fx)} - \frac{d\cos\left(c-\frac{de}{f}\right)\mathrm{Ci}\left(\frac{de}{f}+dx\right)}{af^2} + \frac{\sin(c+dx)}{af(e+fx)} + \frac{d\sin\left(c-\frac{de}{f}\right)\mathrm{Si}\left(\frac{de}{f}+\right.}{af^2}
\end{aligned}
$$

Mathematica [A] time = 0.415589, size = 80, normalized size = 0.84

$$\frac{-d(e+fx)\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(d\left(\frac{e}{f}+x\right)\right)+d(e+fx)\sin\left(c-\frac{de}{f}\right)\text{Si}\left(d\left(\frac{e}{f}+x\right)\right)+f(\sin(c+dx)-1)}{af^2(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] (-(d*(e + f*x)*Cos[c - (d*e)/f]*CosIntegral[d*(e/f + x)]) + f*(-1 + Sin[c +
 d*x]) + d*(e + f*x)*Sin[c - (d*e)/f]*SinIntegral[d*(e/f + x)])/(a*f^2*(e +
 f*x))
```

Maple [A] time = 0.053, size = 132, normalized size = 1.4

$$\frac{d}{a}\left(\frac{\sin(dx+c)}{\left((dx+c)f-cf+de\right)f}-\frac{1}{f}\left(\frac{1}{f}\text{Si}\left(dx+c+\frac{-cf+de}{f}\right)\sin\left(\frac{-cf+de}{f}\right)+\frac{1}{f}\text{Ci}\left(dx+c+\frac{-cf+de}{f}\right)\cos\left(\frac{-cf+de}{f}\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] d/a*(sin(d*x+c)/((d*x+c)*f-c*f+d*e)/f-(Si(d*x+c+(-c*f+d*e)/f)*sin((-c*f+d*e
)/f)/f+Ci(d*x+c+(-c*f+d*e)/f)*cos((-c*f+d*e)/f)/f)/f-1/((d*x+c)*f-c*f+d*e)/
f)
```

Maxima [C] time = 1.41665, size = 232, normalized size = 2.44

$$\frac{d^2\left(iE_2\left(\frac{ide+i(dx+c)f-icf}{f}\right)-iE_2\left(-\frac{ide+i(dx+c)f-icf}{f}\right)\right)\cos\left(-\frac{de-cf}{f}\right)+d^2\left(E_2\left(\frac{ide+i(dx+c)f-icf}{f}\right)+E_2\left(-\frac{ide+i(dx+c)f-icf}{f}\right)\right)\sin\left(-\right.}{2\left(adef+(dx+c)af^2-acf^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(d^2*(I*exp_integral_e(2, (I*d*e + I*(d*x + c)*f - I*c*f)/f) - I*exp_in
tegral_e(2, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*cos(-(d*e - c*f)/f) + d^2*
```

```
(exp_integral_e(2, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + exp_integral_e(2, -
(I*d*e + I*(d*x + c)*f - I*c*f)/f))*sin(-(d*e - c*f)/f) - 2*d^2)/((a*d*e*f
+ (d*x + c)*a*f^2 - a*c*f^2)*d)
```

Fricas [A] time = 1.61146, size = 315, normalized size = 3.32

$$\frac{2\left(dfx+de\right)\sin\left(-\frac{de-cf}{f}\right)\text{Si}\left(\frac{dfx+de}{f}\right)-\left(\left(dfx+de\right)\text{Ci}\left(\frac{dfx+de}{f}\right)+\left(dfx+de\right)\text{Ci}\left(-\frac{dfx+de}{f}\right)\right)\cos\left(-\frac{de-cf}{f}\right)+2\,f\sin\left(dx+\right.}{2\left(af^3x+aef^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(d*f*x + d*e)*sin(-(d*e - c*f)/f)*sin_integral((d*f*x + d*e)/f) - ((
d*f*x + d*e)*cos_integral((d*f*x + d*e)/f) + (d*f*x + d*e)*cos_integral(-(d
*f*x + d*e)/f))*cos(-(d*e - c*f)/f) + 2*f*sin(d*x + c) - 2*f)/(a*f^3*x + a*
e*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^2}{\left(fx+e\right)^2\left(a\sin\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

3.263 $\int \frac{(e+fx)^3\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=219

$$\frac{3f^2(e+fx)\sin^2(c+dx)}{4ad^3}-\frac{6f^2(e+fx)\sin(c+dx)}{ad^3}+\frac{3f(e+fx)^2\cos(c+dx)}{ad^2}-\frac{3f(e+fx)^2\sin(c+dx)\cos(c+dx)}{4ad^2}-$$

```
[Out] (-3*f^3*x)/(8*a*d^3) + (e + f*x)^3/(4*a*d) - (6*f^3*Cos[c + d*x])/(a*d^4) +
 (3*f*(e + f*x)^2*Cos[c + d*x])/(a*d^2) - (6*f^2*(e + f*x)*Sin[c + d*x])/(a
*d^3) + ((e + f*x)^3*Sin[c + d*x])/(a*d) + (3*f^3*Cos[c + d*x]*Sin[c + d*x]
)/(8*a*d^4) - (3*f*(e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(4*a*d^2) + (3*f^
2*(e + f*x)*Sin[c + d*x]^2)/(4*a*d^3) - ((e + f*x)^3*Sin[c + d*x]^2)/(2*a*d
)
```

Rubi [A] time = 0.242949, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4523, 3296, 2638, 4404, 3311, 32, 2635, 8}

$$\frac{3f^2(e+fx)\sin^2(c+dx)}{4ad^3}-\frac{6f^2(e+fx)\sin(c+dx)}{ad^3}+\frac{3f(e+fx)^2\cos(c+dx)}{ad^2}-\frac{3f(e+fx)^2\sin(c+dx)\cos(c+dx)}{4ad^2}-$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-3*f^3*x)/(8*a*d^3) + (e + f*x)^3/(4*a*d) - (6*f^3*Cos[c + d*x])/(a*d^4) +
 (3*f*(e + f*x)^2*Cos[c + d*x])/(a*d^2) - (6*f^2*(e + f*x)*Sin[c + d*x])/(a
*d^3) + ((e + f*x)^3*Sin[c + d*x])/(a*d) + (3*f^3*Cos[c + d*x]*Sin[c + d*x]
)/(8*a*d^4) - (3*f*(e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(4*a*d^2) + (3*f^
2*(e + f*x)*Sin[c + d*x]^2)/(4*a*d^3) - ((e + f*x)^3*Sin[c + d*x]^2)/(2*a*d
)
```


Rule 4523

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\cos(c+dx)\,dx}{a} - \frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)\,dx}{a}$$
$$= \frac{(e+fx)^3\sin(c+dx)}{ad} - \frac{(e+fx)^3\sin^2(c+dx)}{2ad} + \frac{(3f)\int (e+fx)^2\sin^2(c+dx)\,dx}{2ad} - \frac{(3f)$$
$$= \frac{3f(e+fx)^2\cos(c+dx)}{ad^2} + \frac{(e+fx)^3\sin(c+dx)}{ad} - \frac{3f(e+fx)^2\cos(c+dx)\sin(c+dx)}{4ad^2} +$$
$$= \frac{(e+fx)^3}{4ad} + \frac{3f(e+fx)^2\cos(c+dx)}{ad^2} - \frac{6f^2(e+fx)\sin(c+dx)}{ad^3} + \frac{(e+fx)^3\sin(c+dx)}{ad}$$
$$= -\frac{3f^3x}{8ad^3} + \frac{(e+fx)^3}{4ad} - \frac{6f^3\cos(c+dx)}{ad^4} + \frac{3f(e+fx)^2\cos(c+dx)}{ad^2} - \frac{6f^2(e+fx)\sin(c+}{ad^3}$$

Mathematica [A] time = 1.14536, size = 132, normalized size = 0.6

$$\frac{96f\cos(c+dx)\left(d^2(e+fx)^2-2f^2\right)+4d(e+fx)\cos(2(c+dx))\left(2d^2(e+fx)^2-3f^2\right)+4\sin(c+dx)\left(8d(e+fx)\left(d^2(e\right.\right.}{32ad^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (96*f*(-2*f^2 + d^2*(e + f*x)^2)*Cos[c + d*x] + 4*d*(e + f*x)*(-3*f^2 + 2*d
^2*(e + f*x)^2)*Cos[2*(c + d*x)] + 4*(8*d*(e + f*x)*(-6*f^2 + d^2*(e + f*x)
^2) - 3*f*(-f^2 + 2*d^2*(e + f*x)^2)*Cos[c + d*x])*Sin[c + d*x])/(32*a*d^4)
```

Maple [B] time = 0.063, size = 737, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d^4/a*(f^3*(-1/2*(d*x+c)^3*cos(d*x+c)^2+3/2*(d*x+c)^2*(1/2*cos(d*x+c)*si
n(d*x+c)+1/2*d*x+1/2*c)+3/4*(d*x+c)*cos(d*x+c)^2-3/8*cos(d*x+c)*sin(d*x+c)-
3/8*d*x-3/8*c-1/2*(d*x+c)^3)-3*c*f^3*(-1/2*(d*x+c)^2*cos(d*x+c)^2+(d*x+c)*(
1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-1/4*(d*x+c)^2-1/4*sin(d*x+c)^2)+3*
f^2*e*d*(-1/2*(d*x+c)^2*cos(d*x+c)^2+(d*x+c)*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
```

```
*d*x+1/2*c)-1/4*(d*x+c)^2-1/4*sin(d*x+c)^2)+3*c^2*f^3*(-1/2*(d*x+c)*cos(d*x
+c)^2+1/4*cos(d*x+c)*sin(d*x+c)+1/4*d*x+1/4*c)-6*c*d*e*f^2*(-1/2*(d*x+c)*co
s(d*x+c)^2+1/4*cos(d*x+c)*sin(d*x+c)+1/4*d*x+1/4*c)+3*d^2*e^2*f*(-1/2*(d*x+
c)*cos(d*x+c)^2+1/4*cos(d*x+c)*sin(d*x+c)+1/4*d*x+1/4*c)+1/2*c^3*f^3*cos(d*
x+c)^2-3/2*c^2*d*e*f^2*cos(d*x+c)^2+3/2*c*d^2*e^2*f*cos(d*x+c)^2-1/2*d^3*e^
3*cos(d*x+c)^2-f^3*((d*x+c)^3*sin(d*x+c)+3*(d*x+c)^2*cos(d*x+c)-6*cos(d*x+c
)-6*(d*x+c)*sin(d*x+c))+3*c*f^3*((d*x+c)^2*sin(d*x+c)-2*sin(d*x+c)+2*(d*x+c
)*cos(d*x+c))-3*f^2*e*d*((d*x+c)^2*sin(d*x+c)-2*sin(d*x+c)+2*(d*x+c)*cos(d*
x+c))-3*c^2*f^3*(cos(d*x+c)+(d*x+c)*sin(d*x+c))+6*c*d*e*f^2*(cos(d*x+c)+(d*
x+c)*sin(d*x+c))-3*d^2*e^2*f*(cos(d*x+c)+(d*x+c)*sin(d*x+c))+sin(d*x+c)*c^3
*f^3-3*sin(d*x+c)*c^2*d*e*f^2+3*sin(d*x+c)*c*d^2*e^2*f-sin(d*x+c)*d^3*e^3)
```

Maxima [B] time = 1.19585, size = 772, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(8*(sin(d*x + c)^2 - 2*sin(d*x + c))*e^3/a - 24*(sin(d*x + c)^2 - 2*s
in(d*x + c))*c*e^2*f/(a*d) + 24*(sin(d*x + c)^2 - 2*sin(d*x + c))*c^2*e*f^2
/(a*d^2) - 8*(sin(d*x + c)^2 - 2*sin(d*x + c))*c^3*f^3/(a*d^3) - 6*(2*(d*x
+ c)*cos(2*d*x + 2*c) + 8*(d*x + c)*sin(d*x + c) + 8*cos(d*x + c) - sin(2*d
*x + 2*c))*e^2*f/(a*d) + 12*(2*(d*x + c)*cos(2*d*x + 2*c) + 8*(d*x + c)*sin
(d*x + c) + 8*cos(d*x + c) - sin(2*d*x + 2*c))*c*e*f^2/(a*d^2) - 6*(2*(d*x
+ c)*cos(2*d*x + 2*c) + 8*(d*x + c)*sin(d*x + c) + 8*cos(d*x + c) - sin(2*d
*x + 2*c))*c^2*f^3/(a*d^3) - 6*((2*(d*x + c)^2 - 1)*cos(2*d*x + 2*c) + 16*(
d*x + c)*cos(d*x + c) - 2*(d*x + c)*sin(2*d*x + 2*c) + 8*((d*x + c)^2 - 2)*
sin(d*x + c))*e*f^2/(a*d^2) + 6*((2*(d*x + c)^2 - 1)*cos(2*d*x + 2*c) + 16*
(d*x + c)*cos(d*x + c) - 2*(d*x + c)*sin(2*d*x + 2*c) + 8*((d*x + c)^2 - 2)
*sin(d*x + c))*c*f^3/(a*d^3) - (2*(2*(d*x + c)^3 - 3*d*x - 3*c)*cos(2*d*x +
 2*c) + 48*((d*x + c)^2 - 2)*cos(d*x + c) - 3*(2*(d*x + c)^2 - 1)*sin(2*d*x
 + 2*c) + 16*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*f^3/(a*d^3))/d
```

Fricas [A] time = 1.69721, size = 570, normalized size = 2.6

$$-\frac{2\,d^3f^3x^3+6\,d^3ef^2x^2-2\left(2\,d^3f^3x^3+6\,d^3ef^2x^2+2\,d^3e^3-3\,def^2+3\left(2\,d^3e^2f-df^3\right)x\right)\cos\left(dx+c\right)^2+3\left(2\,d^3e^2f-df^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*d^3*f^3*x^3 + 6*d^3*e*f^2*x^2 - 2*(2*d^3*f^3*x^3 + 6*d^3*e*f^2*x^2
+ 2*d^3*e^3 - 3*d*e*f^2 + 3*(2*d^3*e^2*f - d*f^3)*x)*cos(d*x + c)^2 + 3*(2*
d^3*e^2*f - d*f^3)*x - 24*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f - 2*f^3)
*cos(d*x + c) - (8*d^3*f^3*x^3 + 24*d^3*e*f^2*x^2 + 8*d^3*e^3 - 48*d*e*f^2
+ 24*(d^3*e^2*f - 2*d*f^3)*x - 3*(2*d^2*f^3*x^2 + 4*d^2*e*f^2*x + 2*d^2*e^2
*f - f^3)*cos(d*x + c))*sin(d*x + c))/(a*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.264 $\int \frac{(e+fx)^2\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{2f(e+fx)\cos(c+dx)}{ad^2}-\frac{f(e+fx)\sin(c+dx)\cos(c+dx)}{2ad^2}+\frac{f^2\sin^2(c+dx)}{4ad^3}-\frac{2f^2\sin(c+dx)}{ad^3}-\frac{(e+fx)^2\sin^2(c+dx)}{2ad}$$

```
[Out] (e*f*x)/(2*a*d) + (f^2*x^2)/(4*a*d) + (2*f*(e + f*x)*Cos[c + d*x])/(a*d^2)
- (2*f^2*Sin[c + d*x])/(a*d^3) + ((e + f*x)^2*Sin[c + d*x])/(a*d) - (f*(e +
 f*x)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d^2) + (f^2*Sin[c + d*x]^2)/(4*a*d^3)
 - ((e + f*x)^2*Sin[c + d*x]^2)/(2*a*d)
```

Rubi [A] time = 0.172781, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4523, 3296, 2637, 4404, 3310}

$$\frac{2f(e+fx)\cos(c+dx)}{ad^2}-\frac{f(e+fx)\sin(c+dx)\cos(c+dx)}{2ad^2}+\frac{f^2\sin^2(c+dx)}{4ad^3}-\frac{2f^2\sin(c+dx)}{ad^3}-\frac{(e+fx)^2\sin^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (e*f*x)/(2*a*d) + (f^2*x^2)/(4*a*d) + (2*f*(e + f*x)*Cos[c + d*x])/(a*d^2)
- (2*f^2*Sin[c + d*x])/(a*d^3) + ((e + f*x)^2*Sin[c + d*x])/(a*d) - (f*(e +
 f*x)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d^2) + (f^2*Sin[c + d*x]^2)/(4*a*d^3)
 - ((e + f*x)^2*Sin[c + d*x]^2)/(2*a*d)
```


Rule 4523

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)^2\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)^2\cos(c+dx)\,dx}{a}-\frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)\,dx}{a}\\ &= \frac{(e+fx)^2\sin(c+dx)}{ad}-\frac{(e+fx)^2\sin^2(c+dx)}{2ad}+\frac{f\int (e+fx)\sin^2(c+dx)\,dx}{ad}-\frac{(2f)\int (e\\ &= \frac{2f(e+fx)\cos(c+dx)}{ad^2}+\frac{(e+fx)^2\sin(c+dx)}{ad}-\frac{f(e+fx)\cos(c+dx)\sin(c+dx)}{2ad^2}+\frac{f\\ &= \frac{efx}{2ad}+\frac{f^2x^2}{4ad}+\frac{2f(e+fx)\cos(c+dx)}{ad^2}-\frac{2f^2\sin(c+dx)}{ad^3}+\frac{(e+fx)^2\sin(c+dx)}{ad}-\frac{f(e\end{aligned}$$

Mathematica [A] time = 0.795724, size = 95, normalized size = 0.59

$$\frac{\cos(2(c+dx))\left(2d^2(e+fx)^2-f^2\right)-4\sin(c+dx)\left(df(e+fx)\cos(c+dx)-2\left(d^2(e+fx)^2-2f^2\right)\right)+16df(e+fx)\cos(}{8ad^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (16*d*f*(e + f*x)*Cos[c + d*x] + (-f^2 + 2*d^2*(e + f*x)^2)*Cos[2*(c + d*x)
] - 4*(-2*(-2*f^2 + d^2*(e + f*x)^2) + d*f*(e + f*x)*Cos[c + d*x])*Sin[c +
```

d*x])/(8*a*d^3)

Maple [B] time = 0.062, size = 339, normalized size = 2.1

$$-\frac{1}{ad^3}\left(f^2\left(-\frac{(dx+c)^2(\cos(dx+c))^2}{2}+(dx+c)\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{(dx+c)^2}{4}-\frac{(\sin(dx+c))^2}{4}\right)-2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d^3/a*(f^2*(-1/2*(d*x+c)^2*cos(d*x+c)^2+(d*x+c)*(1/2*cos(d*x+c)*sin(d*x+
c)+1/2*d*x+1/2*c)-1/4*(d*x+c)^2-1/4*sin(d*x+c)^2)-2*c*f^2*(-1/2*(d*x+c)*cos
(d*x+c)^2+1/4*cos(d*x+c)*sin(d*x+c)+1/4*d*x+1/4*c)+2*d*e*f*(-1/2*(d*x+c)*co
s(d*x+c)^2+1/4*cos(d*x+c)*sin(d*x+c)+1/4*d*x+1/4*c)-1/2*c^2*f^2*cos(d*x+c)^
2+c*d*e*f*cos(d*x+c)^2-1/2*d^2*e^2*cos(d*x+c)^2-f^2*((d*x+c)^2*sin(d*x+c)-2
*sin(d*x+c)+2*(d*x+c)*cos(d*x+c))+2*c*f^2*(cos(d*x+c)+(d*x+c)*sin(d*x+c))-2
*d*e*f*(cos(d*x+c)+(d*x+c)*sin(d*x+c))-sin(d*x+c)*c^2*f^2+2*sin(d*x+c)*c*d*
e*f-sin(d*x+c)*d^2*e^2)
```

Maxima [A] time = 1.09502, size = 390, normalized size = 2.42

$$-\frac{\frac{4\left(\sin(dx+c)^2-2\sin(dx+c)\right)e^2}{a}-\frac{8\left(\sin(dx+c)^2-2\sin(dx+c)\right)cef}{ad}+\frac{4\left(\sin(dx+c)^2-2\sin(dx+c)\right)c^2f^2}{ad^2}-\frac{2(2(dx+c)\cos(2dx+2c)+8(dx+c)\sin(dx+c)+8\cos}{ad}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(sin(d*x + c)^2 - 2*sin(d*x + c))*e^2/a - 8*(sin(d*x + c)^2 - 2*sin
(d*x + c))*c*e*f/(a*d) + 4*(sin(d*x + c)^2 - 2*sin(d*x + c))*c^2*f^2/(a*d^2
) - 2*(2*(d*x + c)*cos(2*d*x + 2*c) + 8*(d*x + c)*sin(d*x + c) + 8*cos(d*x
+ c) - sin(2*d*x + 2*c))*e*f/(a*d) + 2*(2*(d*x + c)*cos(2*d*x + 2*c) + 8*(d
*x + c)*sin(d*x + c) + 8*cos(d*x + c) - sin(2*d*x + 2*c))*c*f^2/(a*d^2) - (
(2*(d*x + c)^2 - 1)*cos(2*d*x + 2*c) + 16*(d*x + c)*cos(d*x + c) - 2*(d*x +
 c)*sin(2*d*x + 2*c) + 8*((d*x + c)^2 - 2)*sin(d*x + c))*f^2/(a*d^2))/d
```

Fricas [A] time = 1.72647, size = 327, normalized size = 2.03

$$-\frac{d^2f^2x^2+2\,d^2efx-\left(2\,d^2f^2x^2+4\,d^2efx+2\,d^2e^2-f^2\right)\cos\left(dx+c\right)^2-8\left(df^2x+def\right)\cos\left(dx+c\right)-2\left(2\,d^2f^2x^2+4\,d\right.}{4\,ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(d^2*f^2*x^2 + 2*d^2*e*f*x - (2*d^2*f^2*x^2 + 4*d^2*e*f*x + 2*d^2*e^2
- f^2)*cos(d*x + c)^2 - 8*(d*f^2*x + d*e*f)*cos(d*x + c) - 2*(2*d^2*f^2*x^2
 + 4*d^2*e*f*x + 2*d^2*e^2 - 4*f^2 - (d*f^2*x + d*e*f)*cos(d*x + c))*sin(d*
x + c))/(a*d^3)
```

Sympy [A] time = 17.4589, size = 1705, normalized size = 10.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((24*d**2*e**2*tan(c/2 + d*x/2)**3/(12*a*d**3*tan(c/2 + d*x/2)**4
+ 24*a*d**3*tan(c/2 + d*x/2)**2 + 12*a*d**3) - 24*d**2*e**2*tan(c/2 + d*x/2
)**2/(12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d**3*tan(c/2 + d*x/2)**2 + 12*a*
d**3) + 24*d**2*e**2*tan(c/2 + d*x/2)/(12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a
*d**3*tan(c/2 + d*x/2)**2 + 12*a*d**3) + 6*d**2*e*f*x*tan(c/2 + d*x/2)**4/(
12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d**3*tan(c/2 + d*x/2)**2 + 12*a*d**3)
+ 48*d**2*e*f*x*tan(c/2 + d*x/2)**3/(12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d
**3*tan(c/2 + d*x/2)**2 + 12*a*d**3) - 36*d**2*e*f*x*tan(c/2 + d*x/2)**2/(1
2*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d**3*tan(c/2 + d*x/2)**2 + 12*a*d**3) +
 48*d**2*e*f*x*tan(c/2 + d*x/2)/(12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d**3*
tan(c/2 + d*x/2)**2 + 12*a*d**3) + 6*d**2*e*f*x/(12*a*d**3*tan(c/2 + d*x/2)
**4 + 24*a*d**3*tan(c/2 + d*x/2)**2 + 12*a*d**3) + 3*d**2*f**2*x**2*tan(c/2
 + d*x/2)**4/(12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d**3*tan(c/2 + d*x/2)**2
 + 12*a*d**3) + 24*d**2*f**2*x**2*tan(c/2 + d*x/2)**3/(12*a*d**3*tan(c/2 +
d*x/2)**4 + 24*a*d**3*tan(c/2 + d*x/2)**2 + 12*a*d**3) - 18*d**2*f**2*x**2*
tan(c/2 + d*x/2)**2/(12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d**3*tan(c/2 + d*
x/2)**2 + 12*a*d**3) + 24*d**2*f**2*x**2*tan(c/2 + d*x/2)/(12*a*d**3*tan(c/
2 + d*x/2)**4 + 24*a*d**3*tan(c/2 + d*x/2)**2 + 12*a*d**3) + 3*d**2*f**2*x*
*2/(12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d**3*tan(c/2 + d*x/2)**2 + 12*a*d*
*3) - 28*d*e*f*tan(c/2 + d*x/2)**4/(12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d*
```

```
*3*tan(c/2 + d*x/2)**2 + 12*a*d**3) + 12*d*e*f*tan(c/2 + d*x/2)**3/(12*a*d*
*3*tan(c/2 + d*x/2)**4 + 24*a*d**3*tan(c/2 + d*x/2)**2 + 12*a*d**3) - 8*d*e
*f*tan(c/2 + d*x/2)**2/(12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d**3*tan(c/2 +
 d*x/2)**2 + 12*a*d**3) - 12*d*e*f*tan(c/2 + d*x/2)/(12*a*d**3*tan(c/2 + d*
x/2)**4 + 24*a*d**3*tan(c/2 + d*x/2)**2 + 12*a*d**3) + 20*d*e*f/(12*a*d**3*
tan(c/2 + d*x/2)**4 + 24*a*d**3*tan(c/2 + d*x/2)**2 + 12*a*d**3) - 24*d*f**
2*x*tan(c/2 + d*x/2)**4/(12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d**3*tan(c/2
+ d*x/2)**2 + 12*a*d**3) + 12*d*f**2*x*tan(c/2 + d*x/2)**3/(12*a*d**3*tan(c
/2 + d*x/2)**4 + 24*a*d**3*tan(c/2 + d*x/2)**2 + 12*a*d**3) - 12*d*f**2*x*t
an(c/2 + d*x/2)/(12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d**3*tan(c/2 + d*x/2)
**2 + 12*a*d**3) + 24*d*f**2*x/(12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d**3*t
an(c/2 + d*x/2)**2 + 12*a*d**3) + 12*f**2*tan(c/2 + d*x/2)**4/(12*a*d**3*ta
n(c/2 + d*x/2)**4 + 24*a*d**3*tan(c/2 + d*x/2)**2 + 12*a*d**3) - 48*f**2*ta
n(c/2 + d*x/2)**3/(12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d**3*tan(c/2 + d*x/
2)**2 + 12*a*d**3) + 36*f**2*tan(c/2 + d*x/2)**2/(12*a*d**3*tan(c/2 + d*x/2
)**4 + 24*a*d**3*tan(c/2 + d*x/2)**2 + 12*a*d**3) - 48*f**2*tan(c/2 + d*x/2
)/(12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d**3*tan(c/2 + d*x/2)**2 + 12*a*d**
3) + 12*f**2/(12*a*d**3*tan(c/2 + d*x/2)**4 + 24*a*d**3*tan(c/2 + d*x/2)**2
 + 12*a*d**3), Ne(d, 0)), ((e**2*x + e*f*x**2 + f**2*x**3/3)*cos(c)**3/(a*s
in(c) + a), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.265 $\int \frac{(e+fx)\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{f\cos(c+dx)}{ad^2}-\frac{f\sin(c+dx)\cos(c+dx)}{4ad^2}-\frac{(e+fx)\sin^2(c+dx)}{2ad}+\frac{(e+fx)\sin(c+dx)}{ad}+\frac{fx}{4ad}$$

```
[Out] (f*x)/(4*a*d) + (f*Cos[c + d*x])/(a*d^2) + ((e + f*x)*Sin[c + d*x])/(a*d) -
 (f*Cos[c + d*x]*Sin[c + d*x])/(4*a*d^2) - ((e + f*x)*Sin[c + d*x]^2)/(2*a*
d)
```

Rubi [A] time = 0.0910735, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4523, 3296, 2638, 4404, 2635, 8}

$$\frac{f\cos(c+dx)}{ad^2}-\frac{f\sin(c+dx)\cos(c+dx)}{4ad^2}-\frac{(e+fx)\sin^2(c+dx)}{2ad}+\frac{(e+fx)\sin(c+dx)}{ad}+\frac{fx}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (f*x)/(4*a*d) + (f*Cos[c + d*x])/(a*d^2) + ((e + f*x)*Sin[c + d*x])/(a*d) -
 (f*Cos[c + d*x]*Sin[c + d*x])/(4*a*d^2) - ((e + f*x)*Sin[c + d*x]^2)/(2*a*
d)
```

Rule 4523

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int (e+fx)\cos(c+dx)\,dx}{a}-\frac{\int (e+fx)\cos(c+dx)\sin(c+dx)\,dx}{a}\\ &= \frac{(e+fx)\sin(c+dx)}{ad}-\frac{(e+fx)\sin^2(c+dx)}{2ad}+\frac{f\int \sin^2(c+dx)\,dx}{2ad}-\frac{f\int \sin(c+dx)\,dx}{ad}\\ &= \frac{f\cos(c+dx)}{ad^2}+\frac{(e+fx)\sin(c+dx)}{ad}-\frac{f\cos(c+dx)\sin(c+dx)}{4ad^2}-\frac{(e+fx)\sin^2(c+dx)}{2ad}+\\ &= \frac{fx}{4ad}+\frac{f\cos(c+dx)}{ad^2}+\frac{(e+fx)\sin(c+dx)}{ad}-\frac{f\cos(c+dx)\sin(c+dx)}{4ad^2}-\frac{(e+fx)\sin^2(c+\end{aligned}$$

Mathematica [A] time = 0.886143, size = 52, normalized size = 0.57

$$\frac{d(e+fx)(4\sin(c+dx)+\cos(2(c+dx)))-f(\sin(c+dx)-4)\cos(c+dx)}{4ad^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-(f*Cos[c + d*x]*(-4 + Sin[c + d*x])) + d*(e + f*x)*(Cos[2*(c + d*x)] + 4*
Sin[c + d*x]))/(4*a*d^2)
```

Maple [A] time = 0.054, size = 114, normalized size = 1.3

$$-\frac{1}{ad^2}\left(f\left(-\frac{(dx+c)(\cos(dx+c))^2}{2}+\frac{\cos(dx+c)\sin(dx+c)}{4}+\frac{dx}{4}+\frac{c}{4}\right)+\frac{cf(\cos(dx+c))^2}{2}-\frac{(\cos(dx+c))^2 de}{2}-f(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d^2/a*(f*(-1/2*(d*x+c)*cos(d*x+c)^2+1/4*cos(d*x+c)*sin(d*x+c)+1/4*d*x+1/
4*c)+1/2*c*f*cos(d*x+c)^2-1/2*cos(d*x+c)^2*d*e-f*(cos(d*x+c)+(d*x+c)*sin(d*
x+c))+sin(d*x+c)*c*f-sin(d*x+c)*d*e)
```

Maxima [A] time = 1.03606, size = 154, normalized size = 1.69

$$-\frac{\frac{4\left(\sin(dx+c)^2-2\sin(dx+c)\right)e}{a}-\frac{4\left(\sin(dx+c)^2-2\sin(dx+c)\right)cf}{ad}-\frac{(2(dx+c)\cos(2dx+2c)+8(dx+c)\sin(dx+c)+8\cos(dx+c)-\sin(2dx+2c))f}{ad}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(sin(d*x + c)^2 - 2*sin(d*x + c))*e/a - 4*(sin(d*x + c)^2 - 2*sin(d
*x + c))*c*f/(a*d) - (2*(d*x + c)*cos(2*d*x + 2*c) + 8*(d*x + c)*sin(d*x +
c) + 8*cos(d*x + c) - sin(2*d*x + 2*c))*f/(a*d))/d
```

Fricas [A] time = 1.6327, size = 167, normalized size = 1.84

$$-\frac{dfx-2\left(dfx+de\right)\cos(dx+c)^2-4f\cos(dx+c)-\left(4dfx+4de-f\cos(dx+c)\right)\sin(dx+c)}{4ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(d*f*x - 2*(d*f*x + d*e)*cos(d*x + c)^2 - 4*f*cos(d*x + c) - (4*d*f*x
+ 4*d*e - f*cos(d*x + c))*sin(d*x + c))/(a*d^2)
```

Sympy [A] time = 10.7832, size = 787, normalized size = 8.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((24*d*e*tan(c/2 + d*x/2)**3/(12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a
*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2) - 24*d*e*tan(c/2 + d*x/2)**2/(12*a*d
**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2) + 24*d
*e*tan(c/2 + d*x/2)/(12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*
x/2)**2 + 12*a*d**2) + 3*d*f*x*tan(c/2 + d*x/2)**4/(12*a*d**2*tan(c/2 + d*x
/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2) + 24*d*f*x*tan(c/2 + d*
x/2)**3/(12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12
*a*d**2) - 18*d*f*x*tan(c/2 + d*x/2)**2/(12*a*d**2*tan(c/2 + d*x/2)**4 + 24
*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2) + 24*d*f*x*tan(c/2 + d*x/2)/(12*a*
d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2) + 3*d
*f*x/(12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*
d**2) - 14*f*tan(c/2 + d*x/2)**4/(12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2
*tan(c/2 + d*x/2)**2 + 12*a*d**2) + 6*f*tan(c/2 + d*x/2)**3/(12*a*d**2*tan(
c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2) - 4*f*tan(c/2
+ d*x/2)**2/(12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d**2*tan(c/2 + d*x/2)**2
+ 12*a*d**2) - 6*f*tan(c/2 + d*x/2)/(12*a*d**2*tan(c/2 + d*x/2)**4 + 24*a*d
**2*tan(c/2 + d*x/2)**2 + 12*a*d**2) + 10*f/(12*a*d**2*tan(c/2 + d*x/2)**4
+ 24*a*d**2*tan(c/2 + d*x/2)**2 + 12*a*d**2), Ne(d, 0)), ((e*x + f*x**2/2)*
cos(c)**3/(a*sin(c) + a), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.266 $\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

```
[Out] Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d)
```

Rubi [A] time = 0.045625, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2667}

$$\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)\,dx, x, a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.0423879, size = 24, normalized size = 0.75

$$-\frac{(\sin(c+dx)-2)\sin(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((-2 + Sin[c + d*x])*Sin[c + d*x])/(2*a*d)
```

Maple [A] time = 0.016, size = 28, normalized size = 0.9

$$-\frac{1}{da}\left(\frac{(\sin(dx+c))^2}{2}-\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d/a*(1/2*sin(d*x+c)^2-sin(d*x+c))
```

Maxima [A] time = 1.01049, size = 34, normalized size = 1.06

$$-\frac{\sin(dx+c)^2-2\sin(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

Fricas [A] time = 1.63596, size = 61, normalized size = 1.91

$$\frac{\cos(dx+c)^2+2\sin(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(d*x + c)^2 + 2*sin(d*x + c))/(a*d)
```

Sympy [A] time = 7.35931, size = 158, normalized size = 4.94

$$\begin{cases} \frac{2\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{2\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for}\ d \neq 0 \\ \frac{x\cos^{3}(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*tan(c/2 +
 d*x/2)**2 + a*d) - 2*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*
tan(c/2 + d*x/2)**2 + a*d) + 2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**4 +
2*a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*cos(c)**3/(a*sin(c) + a), T
rue))
```

Giac [A] time = 1.14697, size = 34, normalized size = 1.06

$$-\frac{\sin(dx+c)^{2} - 2\sin(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

3.267 $\int \frac{\cos^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=128

$$-\frac{\sin\left(2c-\frac{2de}{f}\right)\text{CosIntegral}\left(\frac{2de}{f}+2dx\right)}{2af}+\frac{\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{af}-\frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af}-\frac{\cos\left(2c-\right.}{}$$

```
[Out] (Cos[c - (d*e)/f]*CosIntegral[(d*e)/f + d*x])/(a*f) - (CosIntegral[(2*d*e)/
f + 2*d*x]*Sin[2*c - (2*d*e)/f])/(2*a*f) - (Sin[c - (d*e)/f]*SinIntegral[(d
*e)/f + d*x])/(a*f) - (Cos[2*c - (2*d*e)/f]*SinIntegral[(2*d*e)/f + 2*d*x])
/(2*a*f)
```

Rubi [A] time = 0.296219, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4523, 3303, 3299, 3302, 4406, 12}

$$-\frac{\sin\left(2c-\frac{2de}{f}\right)\text{CosIntegral}\left(\frac{2de}{f}+2dx\right)}{2af}+\frac{\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{af}-\frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af}-\frac{\cos\left(2c-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (Cos[c - (d*e)/f]*CosIntegral[(d*e)/f + d*x])/(a*f) - (CosIntegral[(2*d*e)/
f + 2*d*x]*Sin[2*c - (2*d*e)/f])/(2*a*f) - (Sin[c - (d*e)/f]*SinIntegral[(d
*e)/f + d*x])/(a*f) - (Cos[2*c - (2*d*e)/f]*SinIntegral[(2*d*e)/f + 2*d*x])
/(2*a*f)
```

Rule 4523

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\cos(c+dx)}{e+fx}\,dx}{a} - \frac{\int \frac{\cos(c+dx)\sin(c+dx)}{e+fx}\,dx}{a}$$

$$= -\frac{\int \frac{\sin(2c+2dx)}{2(e+fx)}\,dx}{a} + \frac{\cos\left(c-\frac{de}{f}\right)\int \frac{\cos\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{a} - \frac{\sin\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{a}$$

$$= \frac{\cos\left(c-\frac{de}{f}\right)\text{Ci}\left(\frac{de}{f}+dx\right)}{af} - \frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af} - \frac{\int \frac{\sin(2c+2dx)}{e+fx}\,dx}{2a}$$

$$= \frac{\cos\left(c-\frac{de}{f}\right)\text{Ci}\left(\frac{de}{f}+dx\right)}{af} - \frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af} - \frac{\cos\left(2c-\frac{2de}{f}\right)\int \frac{\sin\left(\frac{2de}{f}+2dx\right)}{e+fx}\,d}{2a}$$

$$= \frac{\cos\left(c-\frac{de}{f}\right)\text{Ci}\left(\frac{de}{f}+dx\right)}{af} - \frac{\text{Ci}\left(\frac{2de}{f}+2dx\right)\sin\left(2c-\frac{2de}{f}\right)}{2af} - \frac{\sin\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+dx\right)}{af}$$

Mathematica [A] time = 0.386467, size = 105, normalized size = 0.82

$$-\frac{\sin\left(2c-\frac{2de}{f}\right)\text{CosIntegral}\left(\frac{2d(e+fx)}{f}\right) - 2\cos\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(d\left(\frac{e}{f}+x\right)\right) + 2\sin\left(c-\frac{de}{f}\right)\text{Si}\left(d\left(\frac{e}{f}+x\right)\right) + \cos\left(2c\right.}{2af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] -(-2*Cos[c - (d*e)/f]*CosIntegral[d*(e/f + x)] + CosIntegral[(2*d*(e + f*x)
)/f]*Sin[2*c - (2*d*e)/f] + 2*Sin[c - (d*e)/f]*SinIntegral[d*(e/f + x)] + C
os[2*c - (2*d*e)/f]*SinIntegral[(2*d*(e + f*x))/f])/(2*a*f)
```

Maple [A] time = 0.051, size = 161, normalized size = 1.3

$$-\frac{1}{a}\left(\frac{1}{2f}\text{Si}\left(2\,dx+2\,c+2\,\frac{-cf+de}{f}\right)\cos\left(2\,\frac{-cf+de}{f}\right) - \frac{1}{2f}\text{Ci}\left(2\,dx+2\,c+2\,\frac{-cf+de}{f}\right)\sin\left(2\,\frac{-cf+de}{f}\right) - \frac{1}{f}\text{Si}\left(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/a*(1/2*Si(2*d*x+2*c+2*(-c*f+d*e)/f)*cos(2*(-c*f+d*e)/f)/f-1/2*Ci(2*d*x+2
*c+2*(-c*f+d*e)/f)*sin(2*(-c*f+d*e)/f)/f-Si(d*x+c+(-c*f+d*e)/f)*sin((-c*f+d
*e)/f)/f-Ci(d*x+c+(-c*f+d*e)/f)*cos((-c*f+d*e)/f)/f)
```

Maxima [C] time = 1.40728, size = 378, normalized size = 2.95

$$-\frac{2\,d\left(E_1\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)+E_1\left(-\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)\right)\cos\left(-\frac{de-cf}{f}\right)-d\left(i\,E_1\left(\frac{2i\,de+2i\,(dx+c)f-2i\,cf}{f}\right)-i\,E_1\left(-\frac{2i\,de+2i\,(dx+c)f-2i\,cf}{f}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*d*(exp_integral_e(1, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + exp_integ
ral_e(1, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*cos(-(d*e - c*f)/f) - d*(I*ex
p_integral_e(1, (2*I*d*e + 2*I*(d*x + c)*f - 2*I*c*f)/f) - I*exp_integral_e
(1, -(2*I*d*e + 2*I*(d*x + c)*f - 2*I*c*f)/f))*cos(-2*(d*e - c*f)/f) - d*(2
*I*exp_integral_e(1, (I*d*e + I*(d*x + c)*f - I*c*f)/f) - 2*I*exp_integral_
e(1, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*sin(-(d*e - c*f)/f) - d*(exp_inte
gral_e(1, (2*I*d*e + 2*I*(d*x + c)*f - 2*I*c*f)/f) + exp_integral_e(1, -(2*
I*d*e + 2*I*(d*x + c)*f - 2*I*c*f)/f))*sin(-2*(d*e - c*f)/f))/(a*d*f)
```

Fricas [A] time = 1.72022, size = 412, normalized size = 3.22

$$\frac{2\left(\mathrm{Ci}\left(\frac{dfx+de}{f}\right)+\mathrm{Ci}\left(-\frac{dfx+de}{f}\right)\right)\cos\left(-\frac{de-cf}{f}\right)-\left(\mathrm{Ci}\left(\frac{2\left(dfx+de\right)}{f}\right)+\mathrm{Ci}\left(-\frac{2\left(dfx+de\right)}{f}\right)\right)\sin\left(-\frac{2\left(de-cf\right)}{f}\right)-2\cos\left(-\frac{2\left(de-cf\right)}{f}\right)\mathrm{Si}}{4\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(cos_integral((d*f*x + d*e)/f) + cos_integral(-(d*f*x + d*e)/f))*cos
(-(d*e - c*f)/f) - (cos_integral(2*(d*f*x + d*e)/f) + cos_integral(-2*(d*f*
x + d*e)/f))*sin(-2*(d*e - c*f)/f) - 2*cos(-2*(d*e - c*f)/f)*sin_integral(2
*(d*f*x + d*e)/f) - 4*sin(-(d*e - c*f)/f)*sin_integral((d*f*x + d*e)/f))/(a
*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(f*x+e)/(a+a*sin(d*x+c)),x)

[Out] Timed out
```

Giac [C] time = 2.10492, size = 6518, normalized size = 50.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/8*(3*pi + 3*pi*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 2*imag_part(
cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 +
 2*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2*tan(
1/2*d*e/f)^2 - 4*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^4*tan(d*e/
f)^2*tan(1/2*d*e/f)^2 - 4*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^
4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 4*sin_integral(2*(d*f*x + d*e)/f)*tan(1/2
*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 8*imag_part(cos_integral(d*x + d*e/f)
)*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f) - 8*imag_part(cos_integral(-d*x
- d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f) + 16*sin_integral((d*f*x
 + d*e)/f)*tan(1/2*c)^4*tan(d*e/f)^2*tan(1/2*d*e/f) - 4*real_part(cos_integ
ral(2*d*x + 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)*tan(1/2*d*e/f)^2 - 4*real_par
t(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)*tan(1/2*d*e/f)^2
- 8*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*
d*e/f)^2 + 8*imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^3*tan(d*e/f)^
2*tan(1/2*d*e/f)^2 + 8*real_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)^
3*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 8*real_part(cos_integral(-2*d*x - 2*d*e/f
))*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 16*sin_integral((d*f*x + d*
e)/f)*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 3*pi*tan(1/2*c)^4*tan(d*
e/f)^2 - 2*imag_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)
^2 + 2*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2
+ 4*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2 + 4*real
_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^4*tan(d*e/f)^2 - 4*sin_integra
```

l(2*(d*f*x + d*e)/f)*tan(1/2*c)^4*tan(d*e/f)^2 - 16*real_part(cos_integral(
d*x + d*e/f))*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f) - 16*real_part(cos_i
ntegral(-d*x - d*e/f))*tan(1/2*c)^3*tan(d*e/f)^2*tan(1/2*d*e/f) + 3*pi*tan(
1/2*c)^4*tan(1/2*d*e/f)^2 + 2*imag_part(cos_integral(2*d*x + 2*d*e/f))*tan(
1/2*c)^4*tan(1/2*d*e/f)^2 - 2*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan
(1/2*c)^4*tan(1/2*d*e/f)^2 - 4*real_part(cos_integral(d*x + d*e/f))*tan(1/2
*c)^4*tan(1/2*d*e/f)^2 - 4*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)
^4*tan(1/2*d*e/f)^2 + 4*sin_integral(2*(d*f*x + d*e)/f)*tan(1/2*c)^4*tan(1/
2*d*e/f)^2 - 16*imag_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)^3*tan(d
*e/f)*tan(1/2*d*e/f)^2 + 16*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan(1
/2*c)^3*tan(d*e/f)*tan(1/2*d*e/f)^2 - 32*sin_integral(2*(d*f*x + d*e)/f)*ta
n(1/2*c)^3*tan(d*e/f)*tan(1/2*d*e/f)^2 + 6*pi*tan(1/2*c)^2*tan(d*e/f)^2*tan
(1/2*d*e/f)^2 + 12*imag_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)^2*ta
n(d*e/f)^2*tan(1/2*d*e/f)^2 - 12*imag_part(cos_integral(-2*d*x - 2*d*e/f))*
tan(1/2*c)^2*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 24*sin_integral(2*(d*f*x + d*e
)/f)*tan(1/2*c)^2*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 4*real_part(cos_integral(
2*d*x + 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f) - 4*real_part(cos_integral(-2*d*x
 - 2*d*e/f))*tan(1/2*c)^4*tan(d*e/f) + 8*imag_part(cos_integral(d*x + d*e/f
))*tan(1/2*c)^3*tan(d*e/f)^2 - 8*imag_part(cos_integral(-d*x - d*e/f))*tan(
1/2*c)^3*tan(d*e/f)^2 + 8*real_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*
c)^3*tan(d*e/f)^2 + 8*real_part(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)^
3*tan(d*e/f)^2 + 16*sin_integral((d*f*x + d*e)/f)*tan(1/2*c)^3*tan(d*e/f)^2
 + 8*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^4*tan(1/2*d*e/f) - 8*i
mag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^4*tan(1/2*d*e/f) + 16*sin_i
ntegral((d*f*x + d*e)/f)*tan(1/2*c)^4*tan(1/2*d*e/f) - 8*imag_part(cos_inte
gral(d*x + d*e/f))*tan(1/2*c)^3*tan(1/2*d*e/f)^2 + 8*imag_part(cos_integral
(-d*x - d*e/f))*tan(1/2*c)^3*tan(1/2*d*e/f)^2 - 8*real_part(cos_integral(2*
d*x + 2*d*e/f))*tan(1/2*c)^3*tan(1/2*d*e/f)^2 - 8*real_part(cos_integral(-2
*d*x - 2*d*e/f))*tan(1/2*c)^3*tan(1/2*d*e/f)^2 - 16*sin_integral((d*f*x + d
*e)/f)*tan(1/2*c)^3*tan(1/2*d*e/f)^2 + 24*real_part(cos_integral(2*d*x + 2*
d*e/f))*tan(1/2*c)^2*tan(d*e/f)*tan(1/2*d*e/f)^2 + 24*real_part(cos_integra
l(-2*d*x - 2*d*e/f))*tan(1/2*c)^2*tan(d*e/f)*tan(1/2*d*e/f)^2 - 8*imag_part
(cos_integral(d*x + d*e/f))*tan(1/2*c)*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 8*im
ag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)*tan(d*e/f)^2*tan(1/2*d*e/f)^
2 - 8*real_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)*tan(d*e/f)^2*tan(
1/2*d*e/f)^2 - 8*real_part(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)*tan(d
*e/f)^2*tan(1/2*d*e/f)^2 - 16*sin_integral((d*f*x + d*e)/f)*tan(1/2*c)*tan(
d*e/f)^2*tan(1/2*d*e/f)^2 + 3*pi*tan(1/2*c)^4 + 2*imag_part(cos_integral(2*
d*x + 2*d*e/f))*tan(1/2*c)^4 - 2*imag_part(cos_integral(-2*d*x - 2*d*e/f))*
tan(1/2*c)^4 + 4*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^4 + 4*real
_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^4 + 4*sin_integral(2*(d*f*x +
d*e)/f)*tan(1/2*c)^4 - 16*imag_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*
c)^3*tan(d*e/f) + 16*imag_part(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)^3
*tan(d*e/f) - 32*sin_integral(2*(d*f*x + d*e)/f)*tan(1/2*c)^3*tan(d*e/f) +
6*pi*tan(1/2*c)^2*tan(d*e/f)^2 + 12*imag_part(cos_integral(2*d*x + 2*d*e/f)

)*tan(1/2*c)^2*tan(d*e/f)^2 - 12*imag_part(cos_integral(-2*d*x - 2*d*e/f))*
tan(1/2*c)^2*tan(d*e/f)^2 + 24*sin_integral(2*(d*f*x + d*e)/f)*tan(1/2*c)^2
*tan(d*e/f)^2 - 16*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^3*tan(1/
2*d*e/f) - 16*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^3*tan(1/2*d*
e/f) - 16*real_part(cos_integral(d*x + d*e/f))*tan(1/2*c)*tan(d*e/f)^2*tan(
1/2*d*e/f) - 16*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)*tan(d*e/f)
^2*tan(1/2*d*e/f) + 6*pi*tan(1/2*c)^2*tan(1/2*d*e/f)^2 - 12*imag_part(cos_i
ntegral(2*d*x + 2*d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f)^2 + 12*imag_part(cos_
integral(-2*d*x - 2*d*e/f))*tan(1/2*c)^2*tan(1/2*d*e/f)^2 - 24*sin_integral
(2*(d*f*x + d*e)/f)*tan(1/2*c)^2*tan(1/2*d*e/f)^2 + 16*imag_part(cos_integr
al(2*d*x + 2*d*e/f))*tan(1/2*c)*tan(d*e/f)*tan(1/2*d*e/f)^2 - 16*imag_part(
cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)*tan(d*e/f)*tan(1/2*d*e/f)^2 + 32
sin_integral(2(d*f*x + d*e)/f)*tan(1/2*c)*tan(d*e/f)*tan(1/2*d*e/f)^2 + 3
*pi*tan(d*e/f)^2*tan(1/2*d*e/f)^2 - 2*imag_part(cos_integral(2*d*x + 2*d*e/
f))*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 2*imag_part(cos_integral(-2*d*x - 2*d*e
/f))*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 4*real_part(cos_integral(d*x + d*e/f))
*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 4*real_part(cos_integral(-d*x - d*e/f))*ta
n(d*e/f)^2*tan(1/2*d*e/f)^2 - 4*sin_integral(2*(d*f*x + d*e)/f)*tan(d*e/f)^
2*tan(1/2*d*e/f)^2 + 8*imag_part(cos_integral(d*x + d*e/f))*tan(1/2*c)^3 -
8*imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c)^3 - 8*real_part(cos_inte
gral(2*d*x + 2*d*e/f))*tan(1/2*c)^3 - 8*real_part(cos_integral(-2*d*x - 2*d
*e/f))*tan(1/2*c)^3 + 16*sin_integral((d*f*x + d*e)/f)*tan(1/2*c)^3 + 24*re
al_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)^2*tan(d*e/f) + 24*real_pa
rt(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)^2*tan(d*e/f) + 8*imag_part(co
s_integral(d*x + d*e/f))*tan(1/2*c)*tan(d*e/f)^2 - 8*imag_part(cos_integral
(-d*x - d*e/f))*tan(1/2*c)*tan(d*e/f)^2 - 8*real_part(cos_integral(2*d*x +
2*d*e/f))*tan(1/2*c)*tan(d*e/f)^2 - 8*real_part(cos_integral(-2*d*x - 2*d*e
/f))*tan(1/2*c)*tan(d*e/f)^2 + 16*sin_integral((d*f*x + d*e)/f)*tan(1/2*c)*
tan(d*e/f)^2 - 8*imag_part(cos_integral(d*x + d*e/f))*tan(d*e/f)^2*tan(1/2*
d*e/f) + 8*imag_part(cos_integral(-d*x - d*e/f))*tan(d*e/f)^2*tan(1/2*d*e/f
) - 16*sin_integral((d*f*x + d*e)/f)*tan(d*e/f)^2*tan(1/2*d*e/f) - 8*imag_p
art(cos_integral(d*x + d*e/f))*tan(1/2*c)*tan(1/2*d*e/f)^2 + 8*imag_part(co
s_integral(-d*x - d*e/f))*tan(1/2*c)*tan(1/2*d*e/f)^2 + 8*real_part(cos_int
egral(2*d*x + 2*d*e/f))*tan(1/2*c)*tan(1/2*d*e/f)^2 + 8*real_part(cos_integ
ral(-2*d*x - 2*d*e/f))*tan(1/2*c)*tan(1/2*d*e/f)^2 - 16*sin_integral((d*f*x
 + d*e)/f)*tan(1/2*c)*tan(1/2*d*e/f)^2 - 4*real_part(cos_integral(2*d*x + 2
*d*e/f))*tan(d*e/f)*tan(1/2*d*e/f)^2 - 4*real_part(cos_integral(-2*d*x - 2*
d*e/f))*tan(d*e/f)*tan(1/2*d*e/f)^2 + 6*pi*tan(1/2*c)^2 - 12*imag_part(cos_
integral(2*d*x + 2*d*e/f))*tan(1/2*c)^2 + 12*imag_part(cos_integral(-2*d*x
- 2*d*e/f))*tan(1/2*c)^2 - 24*sin_integral(2*(d*f*x + d*e)/f)*tan(1/2*c)^2
+ 16*imag_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c)*tan(d*e/f) - 16*im
ag_part(cos_integral(-2*d*x - 2*d*e/f))*tan(1/2*c)*tan(d*e/f) + 32*sin_inte
gral(2*(d*f*x + d*e)/f)*tan(1/2*c)*tan(d*e/f) + 3*pi*tan(d*e/f)^2 - 2*imag_
part(cos_integral(2*d*x + 2*d*e/f))*tan(d*e/f)^2 + 2*imag_part(cos_integral
(-2*d*x - 2*d*e/f))*tan(d*e/f)^2 - 4*real_part(cos_integral(d*x + d*e/f))*t

```
an(d*e/f)^2 - 4*real_part(cos_integral(-d*x - d*e/f))*tan(d*e/f)^2 - 4*sin_
integral(2*(d*f*x + d*e)/f)*tan(d*e/f)^2 - 16*real_part(cos_integral(d*x +
d*e/f))*tan(1/2*c)*tan(1/2*d*e/f) - 16*real_part(cos_integral(-d*x - d*e/f)
)*tan(1/2*c)*tan(1/2*d*e/f) + 3*pi*tan(1/2*d*e/f)^2 + 2*imag_part(cos_integ
ral(2*d*x + 2*d*e/f))*tan(1/2*d*e/f)^2 - 2*imag_part(cos_integral(-2*d*x -
2*d*e/f))*tan(1/2*d*e/f)^2 + 4*real_part(cos_integral(d*x + d*e/f))*tan(1/2
*d*e/f)^2 + 4*real_part(cos_integral(-d*x - d*e/f))*tan(1/2*d*e/f)^2 + 4*si
n_integral(2*(d*f*x + d*e)/f)*tan(1/2*d*e/f)^2 + 8*imag_part(cos_integral(d
*x + d*e/f))*tan(1/2*c) - 8*imag_part(cos_integral(-d*x - d*e/f))*tan(1/2*c
) + 8*real_part(cos_integral(2*d*x + 2*d*e/f))*tan(1/2*c) + 8*real_part(cos
_integral(-2*d*x - 2*d*e/f))*tan(1/2*c) + 16*sin_integral((d*f*x + d*e)/f)*
tan(1/2*c) - 4*real_part(cos_integral(2*d*x + 2*d*e/f))*tan(d*e/f) - 4*real
_part(cos_integral(-2*d*x - 2*d*e/f))*tan(d*e/f) - 8*imag_part(cos_integral
(d*x + d*e/f))*tan(1/2*d*e/f) + 8*imag_part(cos_integral(-d*x - d*e/f))*tan
(1/2*d*e/f) - 16*sin_integral((d*f*x + d*e)/f)*tan(1/2*d*e/f) + 2*imag_part
(cos_integral(2*d*x + 2*d*e/f)) - 2*imag_part(cos_integral(-2*d*x - 2*d*e/f
)) - 4*real_part(cos_integral(d*x + d*e/f)) - 4*real_part(cos_integral(-d*x
 - d*e/f)) + 4*sin_integral(2*(d*f*x + d*e)/f))/(a*f*tan(1/2*c)^4*tan(d*e/f
)^2*tan(1/2*d*e/f)^2 + a*f*tan(1/2*c)^4*tan(d*e/f)^2 + a*f*tan(1/2*c)^4*tan
(1/2*d*e/f)^2 + 2*a*f*tan(1/2*c)^2*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + a*f*tan(
1/2*c)^4 + 2*a*f*tan(1/2*c)^2*tan(d*e/f)^2 + 2*a*f*tan(1/2*c)^2*tan(1/2*d*e
/f)^2 + a*f*tan(d*e/f)^2*tan(1/2*d*e/f)^2 + 2*a*f*tan(1/2*c)^2 + a*f*tan(d*
e/f)^2 + a*f*tan(1/2*d*e/f)^2 + a*f)
```

3.268 $\int \frac{\cos^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=175

$$-\frac{d\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{af^2}-\frac{d\cos\left(2c-\frac{2de}{f}\right)\text{CosIntegral}\left(\frac{2de}{f}+2dx\right)}{af^2}+\frac{d\sin\left(2c-\frac{2de}{f}\right)\text{Si}\left(\frac{2de}{f}+2dx\right)}{af^2}-$$

```
[Out] -(Cos[c + d*x]/(a*f*(e + f*x))) - (d*Cos[2*c - (2*d*e)/f]*CosIntegral[(2*d*
e)/f + 2*d*x])/(a*f^2) - (d*CosIntegral[(d*e)/f + d*x]*Sin[c - (d*e)/f])/(a
*f^2) + Sin[2*c + 2*d*x]/(2*a*f*(e + f*x)) - (d*Cos[c - (d*e)/f]*SinIntegra
l[(d*e)/f + d*x])/(a*f^2) + (d*Sin[2*c - (2*d*e)/f]*SinIntegral[(2*d*e)/f +
 2*d*x])/(a*f^2)
```

Rubi [A] time = 0.334109, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4523, 3297, 3303, 3299, 3302, 4406, 12}

$$-\frac{d\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(\frac{de}{f}+dx\right)}{af^2}-\frac{d\cos\left(2c-\frac{2de}{f}\right)\text{CosIntegral}\left(\frac{2de}{f}+2dx\right)}{af^2}+\frac{d\sin\left(2c-\frac{2de}{f}\right)\text{Si}\left(\frac{2de}{f}+2dx\right)}{af^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] -(Cos[c + d*x]/(a*f*(e + f*x))) - (d*Cos[2*c - (2*d*e)/f]*CosIntegral[(2*d*
e)/f + 2*d*x])/(a*f^2) - (d*CosIntegral[(d*e)/f + d*x]*Sin[c - (d*e)/f])/(a
*f^2) + Sin[2*c + 2*d*x]/(2*a*f*(e + f*x)) - (d*Cos[c - (d*e)/f]*SinIntegra
l[(d*e)/f + d*x])/(a*f^2) + (d*Sin[2*c - (2*d*e)/f]*SinIntegral[(2*d*e)/f +
 2*d*x])/(a*f^2)
```

Rule 4523

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \frac{\int \frac{\cos(c+dx)}{(e+fx)^2}\,dx}{a} - \frac{\int \frac{\cos(c+dx)\sin(c+dx)}{(e+fx)^2}\,dx}{a}$$

$$= -\frac{\cos(c+dx)}{af(e+fx)} - \frac{\int \frac{\sin(2c+2dx)}{2(e+fx)^2}\,dx}{a} - \frac{d\int \frac{\sin(c+dx)}{e+fx}\,dx}{af}$$

$$= -\frac{\cos(c+dx)}{af(e+fx)} - \frac{\int \frac{\sin(2c+2dx)}{(e+fx)^2}\,dx}{2a} - \frac{\left(d\cos\left(c-\frac{de}{f}\right)\right)\int \frac{\sin\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{af} - \frac{\left(d\sin\left(c-\frac{de}{f}\right)\right)}{a}$$

$$= -\frac{\cos(c+dx)}{af(e+fx)} - \frac{d\text{Ci}\left(\frac{de}{f}+dx\right)\sin\left(c-\frac{de}{f}\right)}{af^2} + \frac{\sin(2c+2dx)}{2af(e+fx)} - \frac{d\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+\right.}{af^2}$$

$$= -\frac{\cos(c+dx)}{af(e+fx)} - \frac{d\text{Ci}\left(\frac{de}{f}+dx\right)\sin\left(c-\frac{de}{f}\right)}{af^2} + \frac{\sin(2c+2dx)}{2af(e+fx)} - \frac{d\cos\left(c-\frac{de}{f}\right)\text{Si}\left(\frac{de}{f}+\right.}{af^2}$$

$$= -\frac{\cos(c+dx)}{af(e+fx)} - \frac{d\cos\left(2c-\frac{2de}{f}\right)\text{Ci}\left(\frac{2de}{f}+2dx\right)}{af^2} - \frac{d\text{Ci}\left(\frac{de}{f}+dx\right)\sin\left(c-\frac{de}{f}\right)}{af^2} + \frac{\sin(2}{2af}$$

Mathematica [A] time = 0.571508, size = 203, normalized size = 1.16

$$\frac{-2d(e+fx)\sin\left(c-\frac{de}{f}\right)\text{CosIntegral}\left(d\left(\frac{e}{f}+x\right)\right) - 2d(e+fx)\cos\left(2c-\frac{2de}{f}\right)\text{CosIntegral}\left(\frac{2d(e+fx)}{f}\right) + 2de\sin\left(2c-\frac{2de}{f}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] (-2*f*Cos[c + d*x] - 2*d*(e + f*x)*Cos[2*c - (2*d*e)/f]*CosIntegral[(2*d*(e
 + f*x))/f] - 2*d*(e + f*x)*CosIntegral[d*(e/f + x)]*Sin[c - (d*e)/f] + f*S
in[2*(c + d*x)] - 2*d*e*Cos[c - (d*e)/f]*SinIntegral[d*(e/f + x)] - 2*d*f*x
*Cos[c - (d*e)/f]*SinIntegral[d*(e/f + x)] + 2*d*e*Sin[2*c - (2*d*e)/f]*Sin
Integral[(2*d*(e + f*x))/f] + 2*d*f*x*Sin[2*c - (2*d*e)/f]*SinIntegral[(2*d
*(e + f*x))/f])/(2*a*f^2*(e + f*x))
```

Maple [A] time = 0.049, size = 230, normalized size = 1.3

$$-\frac{d}{a}\left(-\frac{\sin(2dx+2c)}{\left(2(dx+c)f-2cf+2de\right)f} + \frac{1}{2f}\left(2\frac{1}{f}\text{Si}\left(2dx+2c+2\frac{-cf+de}{f}\right)\sin\left(2\frac{-cf+de}{f}\right) + 2\frac{1}{f}\text{Ci}\left(2dx+2c+2\frac{-c}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -d/a*(-1/2*sin(2*d*x+2*c)/((d*x+c)*f-c*f+d*e)/f+1/2*(2*Si(2*d*x+2*c+2*(-c*f
+d*e)/f)*sin(2*(-c*f+d*e)/f)/f+2*Ci(2*d*x+2*c+2*(-c*f+d*e)/f)*cos(2*(-c*f+d
*e)/f)/f)/f+cos(d*x+c)/((d*x+c)*f-c*f+d*e)/f+(Si(d*x+c+(-c*f+d*e)/f)*cos((-
c*f+d*e)/f)/f-Ci(d*x+c+(-c*f+d*e)/f)*sin((-c*f+d*e)/f)/f)/f)
```

Maxima [C] time = 1.61909, size = 414, normalized size = 2.37

$$-\frac{2\,d^2\left(E_2\left(\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)+E_2\left(-\frac{i\,de+i\,(dx+c)f-i\,cf}{f}\right)\right)\cos\left(-\frac{de-cf}{f}\right)-d^2\left(i\,E_2\left(\frac{2i\,de+2i\,(dx+c)f-2i\,cf}{f}\right)-i\,E_2\left(-\frac{2i\,de+2i\,(dx+c)f-2i\,c}{f}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*d^2*(exp_integral_e(2, (I*d*e + I*(d*x + c)*f - I*c*f)/f) + exp_int
egral_e(2, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*cos(-(d*e - c*f)/f) - d^2*(
I*exp_integral_e(2, (2*I*d*e + 2*I*(d*x + c)*f - 2*I*c*f)/f) - I*exp_integr
al_e(2, -(2*I*d*e + 2*I*(d*x + c)*f - 2*I*c*f)/f))*cos(-2*(d*e - c*f)/f) -
d^2*(2*I*exp_integral_e(2, (I*d*e + I*(d*x + c)*f - I*c*f)/f) - 2*I*exp_int
egral_e(2, -(I*d*e + I*(d*x + c)*f - I*c*f)/f))*sin(-(d*e - c*f)/f) - d^2*(
exp_integral_e(2, (2*I*d*e + 2*I*(d*x + c)*f - 2*I*c*f)/f) + exp_integral_e
(2, -(2*I*d*e + 2*I*(d*x + c)*f - 2*I*c*f)/f))*sin(-2*(d*e - c*f)/f))/((a*d
*e*f + (d*x + c)*a*f^2 - a*c*f^2)*d)
```

Fricas [A] time = 1.92626, size = 610, normalized size = 3.49

$$\frac{2\,f\cos\left(dx+c\right)\sin\left(dx+c\right)+2\left(dfx+de\right)\sin\left(-\frac{2\left(de-cf\right)}{f}\right)\mathrm{Si}\left(\frac{2\left(dfx+de\right)}{f}\right)-2\left(dfx+de\right)\cos\left(-\frac{de-cf}{f}\right)\mathrm{Si}\left(\frac{dfx+de}{f}\right)-2\,f\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*f*cos(d*x + c)*sin(d*x + c) + 2*(d*f*x + d*e)*sin(-2*(d*e - c*f)/f)*
sin_integral(2*(d*f*x + d*e)/f) - 2*(d*f*x + d*e)*cos(-(d*e - c*f)/f)*sin_i
ntegral((d*f*x + d*e)/f) - 2*f*cos(d*x + c) - ((d*f*x + d*e)*cos_integral(2
*(d*f*x + d*e)/f) + (d*f*x + d*e)*cos_integral(-2*(d*f*x + d*e)/f))*cos(-2*
(d*e - c*f)/f) - ((d*f*x + d*e)*cos_integral((d*f*x + d*e)/f) + (d*f*x + d*
e)*cos_integral(-(d*f*x + d*e)/f))*sin(-(d*e - c*f)/f))/(a*f^3*x + a*e*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

3.269 $\int \frac{(e+fx)^3 \sec(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=502

$$-\frac{3f^2(e+fx)\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{ad^3}+\frac{3f^2(e+fx)\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^3}+\frac{3if(e+fx)^2\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{2ad^2}-\frac{3if(e$$

```
[Out] (((-3*I)/2)*f*(e + f*x)^2)/(a*d^2) - ((6*I)*f^2*(e + f*x)*ArcTan[E^(I*(c +
d*x))])/(a*d^3) - (I*(e + f*x)^3*ArcTan[E^(I*(c + d*x))])/(a*d) + (3*f^2*(e
 + f*x)*Log[1 + E^((2*I)*(c + d*x))])/(a*d^3) + ((3*I)*f^3*PolyLog[2, (-I)*
E^(I*(c + d*x))])/(a*d^4) + (((3*I)/2)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(I*(
c + d*x))])/(a*d^2) - ((3*I)*f^3*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^4) - (
((3*I)/2)*f*(e + f*x)^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2) - (((3*I)/2)
*f^3*PolyLog[2, -E^((2*I)*(c + d*x))])/(a*d^4) - (3*f^2*(e + f*x)*PolyLog[3
, (-I)*E^(I*(c + d*x))])/(a*d^3) + (3*f^2*(e + f*x)*PolyLog[3, I*E^(I*(c +
d*x))])/(a*d^3) - ((3*I)*f^3*PolyLog[4, (-I)*E^(I*(c + d*x))])/(a*d^4) + ((
3*I)*f^3*PolyLog[4, I*E^(I*(c + d*x))])/(a*d^4) - (3*f*(e + f*x)^2*Sec[c +
d*x])/(2*a*d^2) - ((e + f*x)^3*Sec[c + d*x]^2)/(2*a*d) + (3*f*(e + f*x)^2*T
an[c + d*x])/(2*a*d^2) + ((e + f*x)^3*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.488311, antiderivative size = 502, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4531, 4186, 4181, 2279, 2391, 2531, 6609, 2282, 6589, 4409, 4184, 3719, 2190}

$$-\frac{3f^2(e+fx)\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{ad^3}+\frac{3f^2(e+fx)\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{ad^3}+\frac{3if(e+fx)^2\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{2ad^2}-\frac{3if(e$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```


```
[Out] (((-3*I)/2)*f*(e + f*x)^2)/(a*d^2) - ((6*I)*f^2*(e + f*x)*ArcTan[E^(I*(c +
d*x))])/(a*d^3) - (I*(e + f*x)^3*ArcTan[E^(I*(c + d*x))])/(a*d) + (3*f^2*(e
 + f*x)*Log[1 + E^((2*I)*(c + d*x))])/(a*d^3) + ((3*I)*f^3*PolyLog[2, (-I)*
E^(I*(c + d*x))])/(a*d^4) + (((3*I)/2)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(I*(
c + d*x))])/(a*d^2) - ((3*I)*f^3*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^4) - (
((3*I)/2)*f*(e + f*x)^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2) - (((3*I)/2)
*f^3*PolyLog[2, -E^((2*I)*(c + d*x))])/(a*d^4) - (3*f^2*(e + f*x)*PolyLog[3
, (-I)*E^(I*(c + d*x))])/(a*d^3) + (3*f^2*(e + f*x)*PolyLog[3, I*E^(I*(c +
d*x))])/(a*d^3) - ((3*I)*f^3*PolyLog[4, (-I)*E^(I*(c + d*x))])/(a*d^4) + ((
```

```
3*I)*f^3*PolyLog[4, I*E^(I*(c + d*x))])/(a*d^4) - (3*f*(e + f*x)^2*Sec[c +
d*x])/(2*a*d^2) - ((e + f*x)^3*Sec[c + d*x]^2)/(2*a*d) + (3*f*(e + f*x)^2*T
an[c + d*x])/(2*a*d^2) + ((e + f*x)^3*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rule 4531

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
```

```
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\sec^3(c+dx)\,dx}{a} - \frac{\int (e+fx)^3\sec^2(c+dx)\tan(c+dx)\,dx}{a}$$

$$= -\frac{3f(e+fx)^2\sec(c+dx)}{2ad^2} - \frac{(e+fx)^3\sec^2(c+dx)}{2ad} + \frac{(e+fx)^3\sec(c+dx)\tan(c+dx)}{2ad} + \cdots$$

$$= -\frac{6if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{3f(e+fx)^2\sec(c+dx)}{2ad^2} - \cdots$$

$$= -\frac{3if(e+fx)^2}{2ad^2} - \frac{6if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{3if(e+fx)^2\mathrm{L}\cdots}{2a\cdots}$$

$$= -\frac{3if(e+fx)^2}{2ad^2} - \frac{6if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{3f^2(e+fx)\log\cdots}{a\cdots}$$

$$= -\frac{3if(e+fx)^2}{2ad^2} - \frac{6if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{3f^2(e+fx)\log\cdots}{a\cdots}$$

$$= -\frac{3if(e+fx)^2}{2ad^2} - \frac{6if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{3f^2(e+fx)\log\cdots}{a\cdots}$$

Mathematica [A] time = 8.75896, size = 865, normalized size = 1.72

$$-\frac{(e+fx)^3}{2ad\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2} - \frac{(\cos(c)+i\sin(c))\left(\frac{(\cos(c)-i\sin(c))(e+fx)^4}{4f} + \frac{\log(-i\cos(c+dx)-\sin(c+dx)+1)(-i\cos(c)-\sin(c)+1)(\cdots}{d}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(8*a*(Cos[c/2] - Sin[c/2])*
(Cos[c/2] + Sin[c/2])) - ((Cos[c] + I*Sin[c])*(((e + f*x)^3*Log[1 - I*Cos[c
 + d*x] - Sin[c + d*x]]*(1 - I*Cos[c] - Sin[c]))/d + ((e + f*x)^4*(Cos[c] -
 I*Sin[c]))/(4*f) + (3*f*(d^2*(e + f*x)^2*PolyLog[2, I*Cos[c + d*x] + Sin[c
 + d*x]] - (2*I)*d*f*(e + f*x)*PolyLog[3, I*Cos[c + d*x] + Sin[c + d*x]] -
2*f^2*PolyLog[4, I*Cos[c + d*x] + Sin[c + d*x]])*(Cos[c] + I*(-1 + Sin[c]))
*(I*Cos[c] + Sin[c]))/d^4))/(2*a*(Cos[c] + I*(-1 + Sin[c]))) - ((12*f^2 + d
^2*(e + f*x)^2)^2 + 12*f^2*(d^2*e^2 + 4*f^2)*PolyLog[2, (-I)*Cos[c + d*x] -
 Sin[c + d*x]]*(1 - I*Cos[c] + Sin[c]) + 24*d*e*f^3*(d*x*PolyLog[2, (-I)*Co
s[c + d*x] - Sin[c + d*x]] - I*PolyLog[3, (-I)*Cos[c + d*x] - Sin[c + d*x]]
)*(1 - I*Cos[c] + Sin[c]) + 12*f^4*(d^2*x^2*PolyLog[2, (-I)*Cos[c + d*x] -
Sin[c + d*x]] - (2*I)*d*x*PolyLog[3, (-I)*Cos[c + d*x] - Sin[c + d*x]] - 2*
PolyLog[4, (-I)*Cos[c + d*x] - Sin[c + d*x]])*(1 - I*Cos[c] + Sin[c]) - 12*
d*f^2*(d^2*e^2 + 4*f^2)*x*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] +
I*(1 + Sin[c])) - 12*d^3*e*f^3*x^2*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(
Cos[c] + I*(1 + Sin[c])) - 4*d^3*f^4*x^3*Log[1 + I*Cos[c + d*x] + Sin[c + d
*x]]*(Cos[c] + I*(1 + Sin[c])) + (4*I)*d*e*f*(d^2*e^2 + 12*f^2)*(d*x + I*Lo
g[Cos[c + d*x] + I*(1 + Sin[c + d*x])])*(Cos[c] + I*(1 + Sin[c]))))/(8*a*d^4
*f*(Cos[c] + I*(1 + Sin[c]))) - (e + f*x)^3/(2*a*d*(Cos[c/2 + (d*x)/2] + Si
n[c/2 + (d*x)/2])^2) + (3*(e^2*f*Sin[(d*x)/2] + 2*e*f^2*x*Sin[(d*x)/2] + f^
3*x^2*Sin[(d*x)/2]))/(a*d^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin
[c/2 + (d*x)/2]))
```

Maple [B] time = 0.26, size = 1265, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -3/2/d/a*ln(1+I*exp(I*(d*x+c)))*e^2*f*x-3/2/d^2/a*ln(1+I*exp(I*(d*x+c)))*c*
e^2*f-6*I/d^3/a*f^3*c*x+3/2*I/d^2/a*e^2*f*polylog(2,-I*exp(I*(d*x+c)))-3/2*
I/d^2/a*e^2*f*polylog(2,I*exp(I*(d*x+c)))-3/2*I/d^2/a*f^3*polylog(2,I*exp(I
*(d*x+c)))*x^2+3/2*I/d^2/a*f^3*polylog(2,-I*exp(I*(d*x+c)))*x^2-1/2/d^4/a*f
^3*ln(1+I*exp(I*(d*x+c)))*c^3-3/2/d^3/a*e*f^2*c^2*ln(exp(I*(d*x+c))-I)+3/2/
d^2/a*e^2*f*c*ln(exp(I*(d*x+c))-I)-1/2/d/a*f^3*ln(1+I*exp(I*(d*x+c)))*x^3-1
/2/d/a*e^3*ln(exp(I*(d*x+c))-I)+6/d^3/a*e*f^2*ln(exp(I*(d*x+c))+I)-3/d^3/a*
e*f^2*polylog(3,-I*exp(I*(d*x+c)))-6/d^3/a*e*f^2*ln(exp(I*(d*x+c)))+1/2/d^4
/a*f^3*c^3*ln(exp(I*(d*x+c))-I)+6/d^4/a*f^3*c*ln(exp(I*(d*x+c)))-6/d^4/a*f^
3*c*ln(exp(I*(d*x+c))+I)-3/d^3/a*f^3*polylog(3,-I*exp(I*(d*x+c)))*x+6/d^3/a
*f^3*ln(1-I*exp(I*(d*x+c)))*x+6/d^4/a*f^3*ln(1-I*exp(I*(d*x+c)))*c-3*I/d^2/
```

```
a*f^3*x^2-6*I/d^4/a*f^3*polylog(2,I*exp(I*(d*x+c)))-3*I/d^4/a*f^3*c^2+3/2/d
^3/a*ln(1+I*exp(I*(d*x+c)))*c^2*e*f^2-3/2/d/a*ln(1+I*exp(I*(d*x+c)))*e*f^2*
x^2-3*I/d^2/a*polylog(2,I*exp(I*(d*x+c)))*e*f^2*x+1/2/a/d*ln(exp(I*(d*x+c))
+I)*e^3-3*I*f^3*polylog(4,-I*exp(I*(d*x+c)))/a/d^4-1/2/a/d^4*f^3*c^3*ln(exp
(I*(d*x+c))+I)+3/a/d^3*e*f^2*polylog(3,I*exp(I*(d*x+c)))+3/a/d^3*f^3*polylo
g(3,I*exp(I*(d*x+c)))*x+3*I/d^2/a*polylog(2,-I*exp(I*(d*x+c)))*e*f^2*x+3*I*
f^3*polylog(4,I*exp(I*(d*x+c)))/a/d^4+1/2/a/d^4*f^3*c^3*ln(1-I*exp(I*(d*x+c
)))+1/2/a/d*f^3*ln(1-I*exp(I*(d*x+c)))*x^3-I*(d*f^3*x^3*exp(I*(d*x+c))+3*d*
e*f^2*x^2*exp(I*(d*x+c))+3*d*e^2*f*x*exp(I*(d*x+c))+d*e^3*exp(I*(d*x+c))+3*
f^3*x^2-3*I*f^3*x^2*exp(I*(d*x+c))+6*e*f^2*x-6*I*e*f^2*x*exp(I*(d*x+c))+3*e
^2*f-3*I*e^2*f*exp(I*(d*x+c)))/d^2/(exp(I*(d*x+c))+I)^2/a+3/2/a/d*e*f^2*ln(
1-I*exp(I*(d*x+c)))*x^2-3/2/a/d^3*e*f^2*c^2*ln(1-I*exp(I*(d*x+c)))+3/2/a/d*
e^2*f*ln(1-I*exp(I*(d*x+c)))*x+3/2/a/d^2*e^2*f*ln(1-I*exp(I*(d*x+c)))*c-3/2
/a/d^2*e^2*f*c*ln(exp(I*(d*x+c))+I)+3/2/a/d^3*e*f^2*c^2*ln(exp(I*(d*x+c))+I
)
```

Maxima [B] time = 5.55909, size = 5164, normalized size = 10.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(3*c*e^2*f*(2/(a*d*sin(d*x + c) + a*d) - log(sin(d*x + c) + 1)/(a*d) +
log(sin(d*x + c) - 1)/(a*d)) + e^3*(log(sin(d*x + c) + 1)/a - log(sin(d*x +
 c) - 1)/a - 2/(a*sin(d*x + c) + a)) - 4*(12*d^2*e^2*f - 24*c*d*e*f^2 + 12*
c^2*f^3 + (6*(c^2 + 4)*d*e*f^2 - 2*(c^3 + 12*c)*f^3 - 2*(3*(c^2 + 4)*d*e*f^
2 - (c^3 + 12*c)*f^3)*cos(2*d*x + 2*c) - ((12*I*c^2 + 48*I)*d*e*f^2 + (-4*I
*c^3 - 48*I*c)*f^3)*cos(d*x + c) - ((6*I*c^2 + 24*I)*d*e*f^2 + (-2*I*c^3 -
24*I*c)*f^3)*sin(2*d*x + 2*c) + 4*(3*(c^2 + 4)*d*e*f^2 - (c^3 + 12*c)*f^3)*
sin(d*x + c))*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - (6*c^2*d*e*f^2 - 2*
c^3*f^3 - 2*(3*c^2*d*e*f^2 - c^3*f^3)*cos(2*d*x + 2*c) + (-12*I*c^2*d*e*f^2
 + 4*I*c^3*f^3)*cos(d*x + c) + (-6*I*c^2*d*e*f^2 + 2*I*c^3*f^3)*sin(2*d*x +
 2*c) + 4*(3*c^2*d*e*f^2 - c^3*f^3)*sin(d*x + c))*arctan2(sin(d*x + c) - 1,
 cos(d*x + c)) - (2*(d*x + c)^3*f^3 + 6*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 6*(
d^2*e^2*f - 2*c*d*e*f^2 + (c^2 + 4)*f^3)*(d*x + c) - 2*((d*x + c)^3*f^3 + 3
*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + (c^2 + 4)*f^3
)*(d*x + c))*cos(2*d*x + 2*c) + (-4*I*(d*x + c)^3*f^3 + (-12*I*d*e*f^2 + 12
*I*c*f^3)*(d*x + c)^2 + (-12*I*d^2*e^2*f + 24*I*c*d*e*f^2 + (-12*I*c^2 - 48
*I)*f^3)*(d*x + c))*cos(d*x + c) + (-2*I*(d*x + c)^3*f^3 + (-6*I*d*e*f^2 +
6*I*c*f^3)*(d*x + c)^2 + (-6*I*d^2*e^2*f + 12*I*c*d*e*f^2 + (-6*I*c^2 - 24*
```

```
I)*f^3)*(d*x + c))*sin(2*d*x + 2*c) + 4*((d*x + c)^3*f^3 + 3*(d*e*f^2 - c*f
^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + (c^2 + 4)*f^3)*(d*x + c))*si
n(d*x + c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) - (2*(d*x + c)^3*f^3 +
6*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 6*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*(d*
x + c) - 2*((d*x + c)^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*
f - 2*c*d*e*f^2 + c^2*f^3)*(d*x + c))*cos(2*d*x + 2*c) + (-4*I*(d*x + c)^3*
f^3 + (-12*I*d*e*f^2 + 12*I*c*f^3)*(d*x + c)^2 + (-12*I*d^2*e^2*f + 24*I*c*
d*e*f^2 - 12*I*c^2*f^3)*(d*x + c))*cos(d*x + c) + (-2*I*(d*x + c)^3*f^3 + (
-6*I*d*e*f^2 + 6*I*c*f^3)*(d*x + c)^2 + (-6*I*d^2*e^2*f + 12*I*c*d*e*f^2 -
6*I*c^2*f^3)*(d*x + c))*sin(2*d*x + 2*c) + 4*((d*x + c)^3*f^3 + 3*(d*e*f^2
- c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*(d*x + c))*sin
(d*x + c))*arctan2(cos(d*x + c), -sin(d*x + c) + 1) + 12*((d*x + c)^2*f^3 +
 2*(d*e*f^2 - c*f^3)*(d*x + c))*cos(2*d*x + 2*c) + (4*(d*x + c)^3*f^3 - 12*
I*d^2*e^2*f + 12*(c^2 + 2*I*c)*d*e*f^2 - 4*(c^3 + 3*I*c^2)*f^3 + (12*d*e*f^
2 - (12*c - 12*I)*f^3)*(d*x + c)^2 + (12*d^2*e^2*f - (24*c - 24*I)*d*e*f^2
+ 12*(c^2 - 2*I*c)*f^3)*(d*x + c))*cos(d*x + c) - (6*d^2*e^2*f - 12*c*d*e*f
^2 + 6*(d*x + c)^2*f^3 + 6*(c^2 + 4)*f^3 + 12*(d*e*f^2 - c*f^3)*(d*x + c) -
 6*(d^2*e^2*f - 2*c*d*e*f^2 + (d*x + c)^2*f^3 + (c^2 + 4)*f^3 + 2*(d*e*f^2
- c*f^3)*(d*x + c))*cos(2*d*x + 2*c) + (-12*I*d^2*e^2*f + 24*I*c*d*e*f^2 -
12*I*(d*x + c)^2*f^3 + (-12*I*c^2 - 48*I)*f^3 + (-24*I*d*e*f^2 + 24*I*c*f^3
)*(d*x + c))*cos(d*x + c) + (-6*I*d^2*e^2*f + 12*I*c*d*e*f^2 - 6*I*(d*x + c
)^2*f^3 + (-6*I*c^2 - 24*I)*f^3 + (-12*I*d*e*f^2 + 12*I*c*f^3)*(d*x + c))*s
in(2*d*x + 2*c) + 12*(d^2*e^2*f - 2*c*d*e*f^2 + (d*x + c)^2*f^3 + (c^2 + 4)
*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*sin(d*x + c))*dilog(I*e^(I*d*x + I*c)
) + (6*d^2*e^2*f - 12*c*d*e*f^2 + 6*(d*x + c)^2*f^3 + 6*c^2*f^3 + 12*(d*e*f
^2 - c*f^3)*(d*x + c) - 6*(d^2*e^2*f - 2*c*d*e*f^2 + (d*x + c)^2*f^3 + c^2*
f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*cos(2*d*x + 2*c) - (12*I*d^2*e^2*f - 2
4*I*c*d*e*f^2 + 12*I*(d*x + c)^2*f^3 + 12*I*c^2*f^3 + (24*I*d*e*f^2 - 24*I*
c*f^3)*(d*x + c))*cos(d*x + c) - (6*I*d^2*e^2*f - 12*I*c*d*e*f^2 + 6*I*(d*x
 + c)^2*f^3 + 6*I*c^2*f^3 + (12*I*d*e*f^2 - 12*I*c*f^3)*(d*x + c))*sin(2*d*
x + 2*c) + 12*(d^2*e^2*f - 2*c*d*e*f^2 + (d*x + c)^2*f^3 + c^2*f^3 + 2*(d*e
*f^2 - c*f^3)*(d*x + c))*sin(d*x + c))*dilog(-I*e^(I*d*x + I*c)) - (I*(d*x
+ c)^3*f^3 + (3*I*c^2 + 12*I)*d*e*f^2 + (-I*c^3 - 12*I*c)*f^3 + (3*I*d*e*f^
2 - 3*I*c*f^3)*(d*x + c)^2 + (3*I*d^2*e^2*f - 6*I*c*d*e*f^2 + (3*I*c^2 + 12
*I)*f^3)*(d*x + c) + (-I*(d*x + c)^3*f^3 + (-3*I*c^2 - 12*I)*d*e*f^2 + (I*c
^3 + 12*I*c)*f^3 + (-3*I*d*e*f^2 + 3*I*c*f^3)*(d*x + c)^2 + (-3*I*d^2*e^2*f
 + 6*I*c*d*e*f^2 + (-3*I*c^2 - 12*I)*f^3)*(d*x + c))*cos(2*d*x + 2*c) + 2*(
(d*x + c)^3*f^3 + 3*(c^2 + 4)*d*e*f^2 - (c^3 + 12*c)*f^3 + 3*(d*e*f^2 - c*f
^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + (c^2 + 4)*f^3)*(d*x + c))*co
s(d*x + c) + ((d*x + c)^3*f^3 + 3*(c^2 + 4)*d*e*f^2 - (c^3 + 12*c)*f^3 + 3*
(d*e*f^2 - c*f^3)*(d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + (c^2 + 4)*f^3)
*(d*x + c))*sin(2*d*x + 2*c) + (2*I*(d*x + c)^3*f^3 + (6*I*c^2 + 24*I)*d*e*
f^2 + (-2*I*c^3 - 24*I*c)*f^3 + (6*I*d*e*f^2 - 6*I*c*f^3)*(d*x + c)^2 + (6*
I*d^2*e^2*f - 12*I*c*d*e*f^2 + (6*I*c^2 + 24*I)*f^3)*(d*x + c))*sin(d*x + c
))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - (-3*I*c^2*d*
```

```
e*f^2 - I*(d*x + c)^3*f^3 + I*c^3*f^3 + (-3*I*d*e*f^2 + 3*I*c*f^3)*(d*x + c
)^2 + (-3*I*d^2*e^2*f + 6*I*c*d*e*f^2 - 3*I*c^2*f^3)*(d*x + c) + (3*I*c^2*d
*e*f^2 + I*(d*x + c)^3*f^3 - I*c^3*f^3 + (3*I*d*e*f^2 - 3*I*c*f^3)*(d*x + c
)^2 + (3*I*d^2*e^2*f - 6*I*c*d*e*f^2 + 3*I*c^2*f^3)*(d*x + c))*cos(2*d*x +
2*c) - 2*(3*c^2*d*e*f^2 + (d*x + c)^3*f^3 - c^3*f^3 + 3*(d*e*f^2 - c*f^3)*(
d*x + c)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*(d*x + c))*cos(d*x + c)
- (3*c^2*d*e*f^2 + (d*x + c)^3*f^3 - c^3*f^3 + 3*(d*e*f^2 - c*f^3)*(d*x + c
)^2 + 3*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*(d*x + c))*sin(2*d*x + 2*c) + (
-6*I*c^2*d*e*f^2 - 2*I*(d*x + c)^3*f^3 + 2*I*c^3*f^3 + (-6*I*d*e*f^2 + 6*I*
c*f^3)*(d*x + c)^2 + (-6*I*d^2*e^2*f + 12*I*c*d*e*f^2 - 6*I*c^2*f^3)*(d*x +
 c))*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1
) - (12*f^3*cos(2*d*x + 2*c) + 24*I*f^3*cos(d*x + c) + 12*I*f^3*sin(2*d*x +
 2*c) - 24*f^3*sin(d*x + c) - 12*f^3)*polylog(4, I*e^(I*d*x + I*c)) + (12*f
^3*cos(2*d*x + 2*c) + 24*I*f^3*cos(d*x + c) + 12*I*f^3*sin(2*d*x + 2*c) - 2
4*f^3*sin(d*x + c) - 12*f^3)*polylog(4, -I*e^(I*d*x + I*c)) - (12*I*d*e*f^2
 + 12*I*(d*x + c)*f^3 - 12*I*c*f^3 + (-12*I*d*e*f^2 - 12*I*(d*x + c)*f^3 +
12*I*c*f^3)*cos(2*d*x + 2*c) + 24*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*cos(d*x
 + c) + 12*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*sin(2*d*x + 2*c) + (24*I*d*e*f
^2 + 24*I*(d*x + c)*f^3 - 24*I*c*f^3)*sin(d*x + c))*polylog(3, I*e^(I*d*x +
 I*c)) - (-12*I*d*e*f^2 - 12*I*(d*x + c)*f^3 + 12*I*c*f^3 + (12*I*d*e*f^2 +
 12*I*(d*x + c)*f^3 - 12*I*c*f^3)*cos(2*d*x + 2*c) - 24*(d*e*f^2 + (d*x + c
)*f^3 - c*f^3)*cos(d*x + c) - 12*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*sin(2*d*
x + 2*c) + (-24*I*d*e*f^2 - 24*I*(d*x + c)*f^3 + 24*I*c*f^3)*sin(d*x + c))*
polylog(3, -I*e^(I*d*x + I*c)) - (-12*I*(d*x + c)^2*f^3 + (-24*I*d*e*f^2 +
24*I*c*f^3)*(d*x + c))*sin(2*d*x + 2*c) - (-4*I*(d*x + c)^3*f^3 - 12*d^2*e^
2*f + (-12*I*c^2 + 24*c)*d*e*f^2 + (4*I*c^3 - 12*c^2)*f^3 - 12*(I*d*e*f^2 +
 (-I*c - 1)*f^3)*(d*x + c)^2 + (-12*I*d^2*e^2*f - 24*(-I*c - 1)*d*e*f^2 + (
-12*I*c^2 - 24*c)*f^3)*(d*x + c))*sin(d*x + c))/(-4*I*a*d^3*cos(2*d*x + 2*c
) + 8*a*d^3*cos(d*x + c) + 4*a*d^3*sin(2*d*x + 2*c) + 8*I*a*d^3*sin(d*x + c
) + 4*I*a*d^3))/d
```

Fricas [C] time = 3.15735, size = 4439, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^3*f^3*x^3 + 6*d^3*e*f^2*x^2 + 6*d^3*e^2*f*x + 2*d^3*e^3 + 6*(d^2*
f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*cos(d*x + c) - (-3*I*d^2*f^3*x^2 - 6*I
*d^2*e*f^2*x - 3*I*d^2*e^2*f + (-3*I*d^2*f^3*x^2 - 6*I*d^2*e*f^2*x - 3*I*d^
```

```
2*e^2*f)*sin(d*x + c))*dilog(I*cos(d*x + c) + sin(d*x + c)) - (-3*I*d^2*f^3
*x^2 - 6*I*d^2*e*f^2*x - 3*I*d^2*e^2*f - 12*I*f^3 + (-3*I*d^2*f^3*x^2 - 6*I
*d^2*e*f^2*x - 3*I*d^2*e^2*f - 12*I*f^3)*sin(d*x + c))*dilog(I*cos(d*x + c)
 - sin(d*x + c)) - (3*I*d^2*f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^2*f + (3*
I*d^2*f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^2*f)*sin(d*x + c))*dilog(-I*cos
(d*x + c) + sin(d*x + c)) - (3*I*d^2*f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^
2*f + 12*I*f^3 + (3*I*d^2*f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^2*f + 12*I*
f^3)*sin(d*x + c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) - (d^3*e^3 - 3*c*d
^2*e^2*f + 3*(c^2 + 4)*d*e*f^2 - (c^3 + 12*c)*f^3 + (d^3*e^3 - 3*c*d^2*e^2*
f + 3*(c^2 + 4)*d*e*f^2 - (c^3 + 12*c)*f^3)*sin(d*x + c))*log(cos(d*x + c)
+ I*sin(d*x + c) + I) + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3
+ (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*sin(d*x + c))*log(cos
(d*x + c) - I*sin(d*x + c) + I) - (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*c*d^2*
e^2*f - 3*c^2*d*e*f^2 + (c^3 + 12*c)*f^3 + 3*(d^3*e^2*f + 4*d*f^3)*x + (d^3
*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + (c^3 + 12*c)*f
^3 + 3*(d^3*e^2*f + 4*d*f^3)*x)*sin(d*x + c))*log(I*cos(d*x + c) + sin(d*x
+ c) + 1) + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f
- 3*c^2*d*e*f^2 + c^3*f^3 + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x
+ 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*sin(d*x + c))*log(I*cos(d*x + c)
 - sin(d*x + c) + 1) - (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*c
^2*d*e*f^2 + (c^3 + 12*c)*f^3 + 3*(d^3*e^2*f + 4*d*f^3)*x + (d^3*f^3*x^3 +
3*d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + (c^3 + 12*c)*f^3 + 3*(d^3
*e^2*f + 4*d*f^3)*x)*sin(d*x + c))*log(-I*cos(d*x + c) + sin(d*x + c) + 1)
+ (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*
e*f^2 + c^3*f^3 + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*
e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*sin(d*x + c))*log(-I*cos(d*x + c) - sin(d*
x + c) + 1) - (d^3*e^3 - 3*c*d^2*e^2*f + 3*(c^2 + 4)*d*e*f^2 - (c^3 + 12*c)
*f^3 + (d^3*e^3 - 3*c*d^2*e^2*f + 3*(c^2 + 4)*d*e*f^2 - (c^3 + 12*c)*f^3)*s
in(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) + (d^3*e^3 - 3*c*d^2*e
^2*f + 3*c^2*d*e*f^2 - c^3*f^3 + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 -
 c^3*f^3)*sin(d*x + c))*log(-cos(d*x + c) - I*sin(d*x + c) + I) - (6*I*f^3*
sin(d*x + c) + 6*I*f^3)*polylog(4, I*cos(d*x + c) + sin(d*x + c)) - (6*I*f^
3*sin(d*x + c) + 6*I*f^3)*polylog(4, I*cos(d*x + c) - sin(d*x + c)) - (-6*I
*f^3*sin(d*x + c) - 6*I*f^3)*polylog(4, -I*cos(d*x + c) + sin(d*x + c)) - (
-6*I*f^3*sin(d*x + c) - 6*I*f^3)*polylog(4, -I*cos(d*x + c) - sin(d*x + c))
 + 6*(d*f^3*x + d*e*f^2 + (d*f^3*x + d*e*f^2)*sin(d*x + c))*polylog(3, I*co
s(d*x + c) + sin(d*x + c)) - 6*(d*f^3*x + d*e*f^2 + (d*f^3*x + d*e*f^2)*sin
(d*x + c))*polylog(3, I*cos(d*x + c) - sin(d*x + c)) + 6*(d*f^3*x + d*e*f^2
 + (d*f^3*x + d*e*f^2)*sin(d*x + c))*polylog(3, -I*cos(d*x + c) + sin(d*x +
 c)) - 6*(d*f^3*x + d*e*f^2 + (d*f^3*x + d*e*f^2)*sin(d*x + c))*polylog(3,
-I*cos(d*x + c) - sin(d*x + c)))/(a*d^4*sin(d*x + c) + a*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^3 \sec (c+dx)}{\sin (c+dx)+1}\, dx + \int \frac{f^3x^3 \sec (c+dx)}{\sin (c+dx)+1}\, dx + \int \frac{3ef^2x^2 \sec (c+dx)}{\sin (c+dx)+1}\, dx + \int \frac{3e^2fx \sec (c+dx)}{\sin (c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**3*sec(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**3*x**3*sec
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(3*e*f**2*x**2*sec(c + d*x)/(sin
(c + d*x) + 1), x) + Integral(3*e**2*f*x*sec(c + d*x)/(sin(c + d*x) + 1), x
))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \sec (dx+c)}{a \sin (dx+c)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sec(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.270 $\int \frac{(e+fx)^2 \sec(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=278

$$\frac{if(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{ad^2}-\frac{if(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^2}-\frac{f^2\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{ad^3}+\frac{f^2\text{PolyLog}\left(3,ie^{i(c+\cdots}\right.}{ad^3}$$

```
[Out] ((-I)*(e + f*x)^2*ArcTan[E^(I*(c + d*x))])/(a*d) + (f^2*ArcTanh[Sin[c + d*x
]])/(a*d^3) + (f^2*Log[Cos[c + d*x]])/(a*d^3) + (I*f*(e + f*x)*PolyLog[2, (
-I)*E^(I*(c + d*x))])/(a*d^2) - (I*f*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))
])/(a*d^2) - (f^2*PolyLog[3, (-I)*E^(I*(c + d*x))])/(a*d^3) + (f^2*PolyLog[
3, I*E^(I*(c + d*x))])/(a*d^3) - (f*(e + f*x)*Sec[c + d*x])/(a*d^2) - ((e +
 f*x)^2*Sec[c + d*x]^2)/(2*a*d) + (f*(e + f*x)*Tan[c + d*x])/(a*d^2) + ((e
+ f*x)^2*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.266679, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {4531, 4186, 3770, 4181, 2531, 2282, 6589, 4409, 4184, 3475}

$$\frac{if(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{ad^2}-\frac{if(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^2}-\frac{f^2\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{ad^3}+\frac{f^2\text{PolyLog}\left(3,ie^{i(c+\cdots}\right.}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```


```
[Out] ((-I)*(e + f*x)^2*ArcTan[E^(I*(c + d*x))])/(a*d) + (f^2*ArcTanh[Sin[c + d*x
]])/(a*d^3) + (f^2*Log[Cos[c + d*x]])/(a*d^3) + (I*f*(e + f*x)*PolyLog[2, (
-I)*E^(I*(c + d*x))])/(a*d^2) - (I*f*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))
])/(a*d^2) - (f^2*PolyLog[3, (-I)*E^(I*(c + d*x))])/(a*d^3) + (f^2*PolyLog[
3, I*E^(I*(c + d*x))])/(a*d^3) - (f*(e + f*x)*Sec[c + d*x])/(a*d^2) - ((e +
 f*x)^2*Sec[c + d*x]^2)/(2*a*d) + (f*(e + f*x)*Tan[c + d*x])/(a*d^2) + ((e
+ f*x)^2*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```


Rule 4531

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
```

```
 - b^2, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\sec^3(c+dx)\,dx}{a} - \frac{\int (e+fx)^2\sec^2(c+dx)\tan(c+dx)\,dx}{a}$$

$$= -\frac{f(e+fx)\sec(c+dx)}{ad^2} - \frac{(e+fx)^2\sec^2(c+dx)}{2ad} + \frac{(e+fx)^2\sec(c+dx)\tan(c+dx)}{2ad} + \int$$

$$= -\frac{i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{f^2\tanh^{-1}(\sin(c+dx))}{ad^3} - \frac{f(e+fx)\sec(c+dx)}{ad^2} - \frac{(e+fx)^2}{}$$

$$= -\frac{i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{f^2\tanh^{-1}(\sin(c+dx))}{ad^3} + \frac{f^2\log(\cos(c+dx))}{ad^3} + \frac{if(e+fx)}{}$$

$$= -\frac{i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{f^2\tanh^{-1}(\sin(c+dx))}{ad^3} + \frac{f^2\log(\cos(c+dx))}{ad^3} + \frac{if(e+fx)}{}$$

$$= -\frac{i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{f^2\tanh^{-1}(\sin(c+dx))}{ad^3} + \frac{f^2\log(\cos(c+dx))}{ad^3} + \frac{if(e+fx)}{}$$

Mathematica [B] time = 8.01837, size = 670, normalized size = 2.41

$$-\frac{(\cos(c)+i\sin(c))\Big(2ef(\cos(c)-i(\sin(c)+1))\text{PolyLog}(2,-\sin(c+dx)-i\cos(c+dx)) + \frac{2f^2(\cos(c)-i\sin(c))(\sin(c)-i\cos(c)+}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((e + f*x)^3/((-I + E^(I*c))*f) + (3*(e + f*x)^2*Log[1 - I/E^(I*(c + d*x))
])/d + (6*f*(I*d*(e + f*x)*PolyLog[2, I/E^(I*(c + d*x))] + f*PolyLog[3, I/E
^(I*(c + d*x))]))/d^3)/(6*a) + (x*(3*e^2 + 3*e*f*x + f^2*x^2))/(6*a*(Cos[c/
2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])) - ((Cos[c] + I*Sin[c])*(d^2*e*f*x^2*C
os[c] + (d^2*e^2 + 4*f^2)*x*(Cos[c] - I*Sin[c]) + (d^2*f^2*x^3*(Cos[c] - I*
Sin[c]))/3 - I*d^2*e*f*x^2*Sin[c] + (2*f^2*(d*x*PolyLog[2, (-I)*Cos[c + d*x
] - Sin[c + d*x]] - I*PolyLog[3, (-I)*Cos[c + d*x] - Sin[c + d*x]])*(Cos[c]
 - I*Sin[c])*(1 - I*Cos[c] + Sin[c]))/d + 2*e*f*PolyLog[2, (-I)*Cos[c + d*x
] - Sin[c + d*x]]*(Cos[c] - I*(1 + Sin[c])) - 2*d*e*f*x*Log[1 + I*Cos[c + d
*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(Cos[c] + I*(1 + Sin[c])) - d*f^2*x
^2*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(Cos[c] + I*(
1 + Sin[c])) + ((d^2*e^2 + 4*f^2)*(d*x + I*Log[Cos[c + d*x] + I*(1 + Sin[c
+ d*x])])*(I*Cos[c] + Sin[c])*(Cos[c] + I*(1 + Sin[c])))/d))/(2*a*d^2*(Cos[
c] + I*(1 + Sin[c]))) - (e + f*x)^2/(2*a*d*(Cos[c/2 + (d*x)/2] + Sin[c/2 +
(d*x)/2])^2) + (2*(e*f*Sin[(d*x)/2] + f^2*x*Sin[(d*x)/2]))/(a*d^2*(Cos[c/2]
 + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] time = 0.181, size = 677, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -I/d^2/a*e*f*polylog(2,I*exp(I*(d*x+c)))-f^2*polylog(3,-I*exp(I*(d*x+c)))/a
/d^3+1/a/d*f*e*ln(1-I*exp(I*(d*x+c)))*x+1/a/d^2*f*e*ln(1-I*exp(I*(d*x+c)))*
c-1/a/d^2*f*e*c*ln(exp(I*(d*x+c))+I)+1/2/a/d*f^2*ln(1-I*exp(I*(d*x+c)))*x^2
-1/2/a/d^3*f^2*ln(1-I*exp(I*(d*x+c)))*c^2+1/d^2/a*e*f*c*ln(exp(I*(d*x+c))-I
)-I*(d*f^2*x^2*exp(I*(d*x+c))+2*d*e*f*x*exp(I*(d*x+c))+d*e^2*exp(I*(d*x+c))
+2*f^2*x-2*I*f^2*x*exp(I*(d*x+c))+2*e*f-2*I*e*f*exp(I*(d*x+c)))/d^2/(exp(I*
(d*x+c))+I)^2/a-I/d^2/a*polylog(2,I*exp(I*(d*x+c)))*f^2*x+1/2/a/d*ln(exp(I*
(d*x+c))+I)*e^2+1/2/d^3/a*ln(1+I*exp(I*(d*x+c)))*c^2*f^2+I/d^2/a*polylog(2,
-I*exp(I*(d*x+c)))*f^2*x+I/d^2/a*e*f*polylog(2,-I*exp(I*(d*x+c)))-1/2/d/a*l
n(1+I*exp(I*(d*x+c)))*f^2*x^2-1/d/a*ln(1+I*exp(I*(d*x+c)))*e*f*x-1/d^2/a*ln
(1+I*exp(I*(d*x+c)))*c*e*f+1/2/a/d^3*f^2*c^2*ln(exp(I*(d*x+c))+I)+f^2*polyl
og(3,I*exp(I*(d*x+c)))/a/d^3-1/2/d^3/a*f^2*c^2*ln(exp(I*(d*x+c))-I)-2/d^3/a
*f^2*ln(exp(I*(d*x+c)))+2/d^3/a*f^2*ln(exp(I*(d*x+c))+I)-1/2/d/a*e^2*ln(exp
```

```
(I*(d*x+c))-I)
```

Maxima [B] time = 2.28636, size = 2596, normalized size = 9.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*c*e*f*(2/(a*d*sin(d*x + c) + a*d) - log(sin(d*x + c) + 1)/(a*d) + lo
g(sin(d*x + c) - 1)/(a*d)) + e^2*(log(sin(d*x + c) + 1)/a - log(sin(d*x + c
) - 1)/a - 2/(a*sin(d*x + c) + a)) - 4*(8*(d*x + c)*f^2*cos(2*d*x + 2*c) +
8*I*(d*x + c)*f^2*sin(2*d*x + 2*c) + 8*d*e*f - 8*c*f^2 - (2*(c^2 + 4)*f^2*c
os(2*d*x + 2*c) + (4*I*c^2 + 16*I)*f^2*cos(d*x + c) + (2*I*c^2 + 8*I)*f^2*s
in(2*d*x + 2*c) - 4*(c^2 + 4)*f^2*sin(d*x + c) - 2*(c^2 + 4)*f^2)*arctan2(s
in(d*x + c) + 1, cos(d*x + c)) + (2*c^2*f^2*cos(2*d*x + 2*c) + 4*I*c^2*f^2*
cos(d*x + c) + 2*I*c^2*f^2*sin(2*d*x + 2*c) - 4*c^2*f^2*sin(d*x + c) - 2*c^
2*f^2)*arctan2(sin(d*x + c) - 1, cos(d*x + c)) - (2*(d*x + c)^2*f^2 + 4*(d*
e*f - c*f^2)*(d*x + c) - 2*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*
cos(2*d*x + 2*c) + (-4*I*(d*x + c)^2*f^2 + (-8*I*d*e*f + 8*I*c*f^2)*(d*x +
c))*cos(d*x + c) + (-2*I*(d*x + c)^2*f^2 + (-4*I*d*e*f + 4*I*c*f^2)*(d*x +
c))*sin(2*d*x + 2*c) + 4*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*si
n(d*x + c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) - (2*(d*x + c)^2*f^2 +
4*(d*e*f - c*f^2)*(d*x + c) - 2*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x +
 c))*cos(2*d*x + 2*c) + (-4*I*(d*x + c)^2*f^2 + (-8*I*d*e*f + 8*I*c*f^2)*(d
*x + c))*cos(d*x + c) + (-2*I*(d*x + c)^2*f^2 + (-4*I*d*e*f + 4*I*c*f^2)*(d
*x + c))*sin(2*d*x + 2*c) + 4*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c
))*sin(d*x + c))*arctan2(cos(d*x + c), -sin(d*x + c) + 1) + (4*(d*x + c)^2*
f^2 - 8*I*d*e*f + 4*(c^2 + 2*I*c)*f^2 + (8*d*e*f - (8*c - 8*I)*f^2)*(d*x +
c))*cos(d*x + c) - (4*d*e*f + 4*(d*x + c)*f^2 - 4*c*f^2 - 4*(d*e*f + (d*x +
 c)*f^2 - c*f^2)*cos(2*d*x + 2*c) + (-8*I*d*e*f - 8*I*(d*x + c)*f^2 + 8*I*c
*f^2)*cos(d*x + c) + (-4*I*d*e*f - 4*I*(d*x + c)*f^2 + 4*I*c*f^2)*sin(2*d*x
 + 2*c) + 8*(d*e*f + (d*x + c)*f^2 - c*f^2)*sin(d*x + c))*dilog(I*e^(I*d*x
+ I*c)) + (4*d*e*f + 4*(d*x + c)*f^2 - 4*c*f^2 - 4*(d*e*f + (d*x + c)*f^2 -
 c*f^2)*cos(2*d*x + 2*c) - (8*I*d*e*f + 8*I*(d*x + c)*f^2 - 8*I*c*f^2)*cos(
d*x + c) - (4*I*d*e*f + 4*I*(d*x + c)*f^2 - 4*I*c*f^2)*sin(2*d*x + 2*c) + 8
*(d*e*f + (d*x + c)*f^2 - c*f^2)*sin(d*x + c))*dilog(-I*e^(I*d*x + I*c)) -
(I*(d*x + c)^2*f^2 + (I*c^2 + 4*I)*f^2 + (2*I*d*e*f - 2*I*c*f^2)*(d*x + c)
+ (-I*(d*x + c)^2*f^2 + (-I*c^2 - 4*I)*f^2 + (-2*I*d*e*f + 2*I*c*f^2)*(d*x
+ c))*cos(2*d*x + 2*c) + 2*((d*x + c)^2*f^2 + (c^2 + 4)*f^2 + 2*(d*e*f - c*
f^2)*(d*x + c))*cos(d*x + c) + ((d*x + c)^2*f^2 + (c^2 + 4)*f^2 + 2*(d*e*f
```

```
- c*f^2)*(d*x + c))*sin(2*d*x + 2*c) + (2*I*(d*x + c)^2*f^2 + (2*I*c^2 + 8*
I)*f^2 + (4*I*d*e*f - 4*I*c*f^2)*(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^
2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - (-I*(d*x + c)^2*f^2 - I*c^2*f^2
+ (-2*I*d*e*f + 2*I*c*f^2)*(d*x + c) + (I*(d*x + c)^2*f^2 + I*c^2*f^2 + (2*
I*d*e*f - 2*I*c*f^2)*(d*x + c))*cos(2*d*x + 2*c) - 2*((d*x + c)^2*f^2 + c^2
*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*cos(d*x + c) - ((d*x + c)^2*f^2 + c^2*f
^2 + 2*(d*e*f - c*f^2)*(d*x + c))*sin(2*d*x + 2*c) + (-2*I*(d*x + c)^2*f^2
- 2*I*c^2*f^2 + (-4*I*d*e*f + 4*I*c*f^2)*(d*x + c))*sin(d*x + c))*log(cos(d
*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) - (-4*I*f^2*cos(2*d*x + 2*
c) + 8*f^2*cos(d*x + c) + 4*f^2*sin(2*d*x + 2*c) + 8*I*f^2*sin(d*x + c) + 4
*I*f^2)*polylog(3, I*e^(I*d*x + I*c)) - (4*I*f^2*cos(2*d*x + 2*c) - 8*f^2*c
os(d*x + c) - 4*f^2*sin(2*d*x + 2*c) - 8*I*f^2*sin(d*x + c) - 4*I*f^2)*poly
log(3, -I*e^(I*d*x + I*c)) - (-4*I*(d*x + c)^2*f^2 - 8*d*e*f + (-4*I*c^2 +
8*c)*f^2 - 8*(I*d*e*f + (-I*c - 1)*f^2)*(d*x + c))*sin(d*x + c))/(-4*I*a*d^
2*cos(2*d*x + 2*c) + 8*a*d^2*cos(d*x + c) + 4*a*d^2*sin(2*d*x + 2*c) + 8*I*
a*d^2*sin(d*x + c) + 4*I*a*d^2))/d
```

Fricas [C] time = 2.6464, size = 2646, normalized size = 9.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*f^2*x^2 + 4*d^2*e*f*x + 2*d^2*e^2 + 4*(d*f^2*x + d*e*f)*cos(d*x
 + c) - (-2*I*d*f^2*x - 2*I*d*e*f + (-2*I*d*f^2*x - 2*I*d*e*f)*sin(d*x + c)
)*dilog(I*cos(d*x + c) + sin(d*x + c)) - (-2*I*d*f^2*x - 2*I*d*e*f + (-2*I*
d*f^2*x - 2*I*d*e*f)*sin(d*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) - (
2*I*d*f^2*x + 2*I*d*e*f + (2*I*d*f^2*x + 2*I*d*e*f)*sin(d*x + c))*dilog(-I*
cos(d*x + c) + sin(d*x + c)) - (2*I*d*f^2*x + 2*I*d*e*f + (2*I*d*f^2*x + 2*
I*d*e*f)*sin(d*x + c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) - (d^2*e^2 - 2
*c*d*e*f + (c^2 + 4)*f^2 + (d^2*e^2 - 2*c*d*e*f + (c^2 + 4)*f^2)*sin(d*x +
c))*log(cos(d*x + c) + I*sin(d*x + c) + I) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2
 + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*sin(d*x + c))*log(cos(d*x + c) - I*sin(d
*x + c) + I) - (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2 + (d^2*f^2*
x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*sin(d*x + c))*log(I*cos(d*x + c) +
 sin(d*x + c) + 1) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2 + (d^
2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*sin(d*x + c))*log(I*cos(d*x
+ c) - sin(d*x + c) + 1) - (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2
 + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*sin(d*x + c))*log(-I*c
os(d*x + c) + sin(d*x + c) + 1) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f -
```

```
c^2*f^2 + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*sin(d*x + c))*l
og(-I*cos(d*x + c) - sin(d*x + c) + 1) - (d^2*e^2 - 2*c*d*e*f + (c^2 + 4)*f
^2 + (d^2*e^2 - 2*c*d*e*f + (c^2 + 4)*f^2)*sin(d*x + c))*log(-cos(d*x + c)
+ I*sin(d*x + c) + I) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2 + (d^2*e^2 - 2*c*d*e
*f + c^2*f^2)*sin(d*x + c))*log(-cos(d*x + c) - I*sin(d*x + c) + I) + 2*(f^
2*sin(d*x + c) + f^2)*polylog(3, I*cos(d*x + c) + sin(d*x + c)) - 2*(f^2*si
n(d*x + c) + f^2)*polylog(3, I*cos(d*x + c) - sin(d*x + c)) + 2*(f^2*sin(d*
x + c) + f^2)*polylog(3, -I*cos(d*x + c) + sin(d*x + c)) - 2*(f^2*sin(d*x +
 c) + f^2)*polylog(3, -I*cos(d*x + c) - sin(d*x + c)))/(a*d^3*sin(d*x + c)
+ a*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^2 \sec(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2 \sec(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx \sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*sec(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*sec
(c + d*x)/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sec(c + d*x)/(sin(c + d
*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sec(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sec(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.271 $\int \frac{(e+fx)\sec(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{if\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{2ad^2}-\frac{if\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{2ad^2}+\frac{f\tan(c+dx)}{2ad^2}-\frac{f\sec(c+dx)}{2ad^2}-\frac{i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{(e+f$$

```
[Out] ((-I)*(e + f*x)*ArcTan[E^(I*(c + d*x))])/(a*d) + ((I/2)*f*PolyLog[2, (-I)*E
^(I*(c + d*x))])/(a*d^2) - ((I/2)*f*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2)
- (f*Sec[c + d*x])/(2*a*d^2) - ((e + f*x)*Sec[c + d*x]^2)/(2*a*d) + (f*Tan[
c + d*x])/(2*a*d^2) + ((e + f*x)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.138839, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4531, 4185, 4181, 2279, 2391, 4409, 3767, 8}

$$\frac{if\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{2ad^2}-\frac{if\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{2ad^2}+\frac{f\tan(c+dx)}{2ad^2}-\frac{f\sec(c+dx)}{2ad^2}-\frac{i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad}-\frac{(e+f$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```



```
[Out] ((-I)*(e + f*x)*ArcTan[E^(I*(c + d*x))])/(a*d) + ((I/2)*f*PolyLog[2, (-I)*E
^(I*(c + d*x))])/(a*d^2) - ((I/2)*f*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2)
- (f*Sec[c + d*x])/(2*a*d^2) - ((e + f*x)*Sec[c + d*x]^2)/(2*a*d) + (f*Tan[
c + d*x])/(2*a*d^2) + ((e + f*x)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```


Rule 4531

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
```

```
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(e+fx)\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)\sec^3(c+dx)\,dx}{a} - \frac{\int (e+fx)\sec^2(c+dx)\tan(c+dx)\,dx}{a}$$

$$= -\frac{f\sec(c+dx)}{2ad^2} - \frac{(e+fx)\sec^2(c+dx)}{2ad} + \frac{(e+fx)\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int (e+fx)\sec(}{2a}$$

$$= -\frac{i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f\sec(c+dx)}{2ad^2} - \frac{(e+fx)\sec^2(c+dx)}{2ad} + \frac{(e+fx)\sec(c+dx)\,\mathrm{ta}}{2ad}$$

$$= -\frac{i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{f\sec(c+dx)}{2ad^2} - \frac{(e+fx)\sec^2(c+dx)}{2ad} + \frac{f\tan(c+dx)}{2ad^2} + \frac{(e+f}{}$$

$$= -\frac{i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{if\,\mathrm{Li}_2\left(-ie^{i(c+dx)}\right)}{2ad^2} - \frac{if\,\mathrm{Li}_2\left(ie^{i(c+dx)}\right)}{2ad^2} - \frac{f\sec(c+dx)}{2ad^2} - \frac{(e+fx)}{}$$

Mathematica [B] time = 2.93253, size = 655, normalized size = 3.81

$$-\frac{-\frac{f\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left((-1)^{3/4}(c+dx)^2+\frac{4i\mathrm{PolyLog}\left(2,-ie^{i(c+dx)}\right)-3i\pi(c+dx)-4\pi\log\left(1+e^{-i(c+dx)}\right)+2(-2c-2dx+\pi)\log\left(1+ie^{i(c+dx)}\right)-2\pi\log\left(\sin\left(\frac{1}{4}(2c+2dx-\pi)\right)\right)+4}{\sqrt{2}}\right.}{\sqrt{2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(2*d*(e + f*x) - 4*f*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
) + (c + d*x)*(c*f - d*(2*e + f*x))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2
 + d*e*(c + d*x + 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^2 - c*f*(c + d*x + 2*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + d*e*(c + d*x - 2*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^2 - c*f*(c + d*x - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^2 - (f*((-1)^(3/4)*(c + d*x)^2 + ((-3*I)*Pi*(c
+ d*x) - 4*Pi*Log[1 + E^((-I)*(c + d*x))] + 2*(-2*c + Pi - 2*d*x)*Log[1 + I
*E^(I*(c + d*x))] + 4*Pi*Log[Cos[(c + d*x)/2]] - 2*Pi*Log[Sin[(2*c - Pi + 2
*d*x)/4]] + (4*I)*PolyLog[2, (-I)*E^(I*(c + d*x))])/Sqrt[2])*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^2)/Sqrt[2] + (f*((-1)^(1/4)*(c + d*x)^2 + ((-I)*Pi*
(c + d*x) - 4*Pi*Log[1 + E^((-I)*(c + d*x))] - 2*(2*c + Pi + 2*d*x)*Log[1 -
 I*E^(I*(c + d*x))] + 4*Pi*Log[Cos[(c + d*x)/2]] + 2*Pi*Log[Sin[(2*c + Pi +
 2*d*x)/4]] + (4*I)*PolyLog[2, I*E^(I*(c + d*x))])/Sqrt[2])*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^2)/Sqrt[2])/(4*a*d^2*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.214, size = 303, normalized size = 1.8

$$\frac{-i\left(dfx\,\mathrm{e}^{i(dx+c)}+de\,\mathrm{e}^{i(dx+c)}+f-if\,\mathrm{e}^{i(dx+c)}\right)}{d^{2}\left(\mathrm{e}^{i(dx+c)}+i\right)^{2}a}-\frac{e\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2\,da}+\frac{e\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2\,da}-\frac{f\ln\left(1+i\mathrm{e}^{i(dx+c)}\right)x}{2\,da}-\frac{f\ln\left(1+i\mathrm{e}^{i\ldots}\right.}{2\,ad^{2}\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -I*(d*f*x*exp(I*(d*x+c))+d*e*exp(I*(d*x+c))+f-I*f*exp(I*(d*x+c)))/d^2/(exp(
I*(d*x+c))+I)^2/a-1/2/a/d*e*ln(exp(I*(d*x+c))-I)+1/2/a/d*ln(exp(I*(d*x+c))+
I)*e-1/2/a/d*f*ln(1+I*exp(I*(d*x+c)))*x-1/2/a/d^2*f*ln(1+I*exp(I*(d*x+c)))*
c+1/2*I*f*polylog(2,-I*exp(I*(d*x+c)))/a/d^2+1/2/a/d*f*ln(1-I*exp(I*(d*x+c)
))*x+1/2/a/d^2*f*ln(1-I*exp(I*(d*x+c)))*c-1/2*I*f*polylog(2,I*exp(I*(d*x+c)
))/a/d^2+1/2/a/d^2*f*c*ln(exp(I*(d*x+c))-I)-1/2/a/d^2*f*c*ln(exp(I*(d*x+c))
+I)
```

Maxima [B] time = 1.62525, size = 986, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((2*d*e*cos(2*d*x + 2*c) + 4*I*d*e*cos(d*x + c) + 2*I*d*e*sin(2*d*x + 2*c)
- 4*d*e*sin(d*x + c) - 2*d*e)*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - (2*
d*e*cos(2*d*x + 2*c) + 4*I*d*e*cos(d*x + c) + 2*I*d*e*sin(2*d*x + 2*c) - 4*
d*e*sin(d*x + c) - 2*d*e)*arctan2(sin(d*x + c) - 1, cos(d*x + c)) - (2*d*f*
x*cos(2*d*x + 2*c) + 4*I*d*f*x*cos(d*x + c) + 2*I*d*f*x*sin(2*d*x + 2*c) -
4*d*f*x*sin(d*x + c) - 2*d*f*x)*arctan2(cos(d*x + c), sin(d*x + c) + 1) - (
2*d*f*x*cos(2*d*x + 2*c) + 4*I*d*f*x*cos(d*x + c) + 2*I*d*f*x*sin(2*d*x + 2
*c) - 4*d*f*x*sin(d*x + c) - 2*d*f*x)*arctan2(cos(d*x + c), -sin(d*x + c) +
 1) - (4*d*f*x + 4*d*e - 4*I*f)*cos(d*x + c) - (2*f*cos(2*d*x + 2*c) + 4*I*
f*cos(d*x + c) + 2*I*f*sin(2*d*x + 2*c) - 4*f*sin(d*x + c) - 2*f)*dilog(I*e
^(I*d*x + I*c)) + (2*f*cos(2*d*x + 2*c) + 4*I*f*cos(d*x + c) + 2*I*f*sin(2*
d*x + 2*c) - 4*f*sin(d*x + c) - 2*f)*dilog(-I*e^(I*d*x + I*c)) + (I*d*f*x +
 I*d*e + (-I*d*f*x - I*d*e)*cos(2*d*x + 2*c) + 2*(d*f*x + d*e)*cos(d*x + c)
 + (d*f*x + d*e)*sin(2*d*x + 2*c) + (2*I*d*f*x + 2*I*d*e)*sin(d*x + c))*log
(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + (-I*d*f*x - I*d*e
+ (I*d*f*x + I*d*e)*cos(2*d*x + 2*c) - 2*(d*f*x + d*e)*cos(d*x + c) - (d*f*
```

```
x + d*e)*sin(2*d*x + 2*c) + (-2*I*d*f*x - 2*I*d*e)*sin(d*x + c))*log(cos(d*
x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + (-4*I*d*f*x - 4*I*d*e - 4
*f)*sin(d*x + c) - 4*f)/(-4*I*a*d^2*cos(2*d*x + 2*c) + 8*a*d^2*cos(d*x + c)
 + 4*a*d^2*sin(2*d*x + 2*c) + 8*I*a*d^2*sin(d*x + c) + 4*I*a*d^2)
```

Fricas [B] time = 2.11148, size = 1339, normalized size = 7.78

$$-\frac{2\,dfx + 2\,de + 2\,f\cos\left(dx + c\right) - \left(-i\,f\sin\left(dx + c\right) - i\,f\right)\mathrm{Li}_2\left(i\,\cos\left(dx + c\right) + \sin\left(dx + c\right)\right) - \left(-i\,f\sin\left(dx + c\right) - i\,f\right)\mathrm{Li}_2\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*d*f*x + 2*d*e + 2*f*cos(d*x + c) - (-I*f*sin(d*x + c) - I*f)*dilog(
I*cos(d*x + c) + sin(d*x + c)) - (-I*f*sin(d*x + c) - I*f)*dilog(I*cos(d*x
+ c) - sin(d*x + c)) - (I*f*sin(d*x + c) + I*f)*dilog(-I*cos(d*x + c) + sin
(d*x + c)) - (I*f*sin(d*x + c) + I*f)*dilog(-I*cos(d*x + c) - sin(d*x + c))
 - (d*e - c*f + (d*e - c*f)*sin(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c)
 + I) + (d*e - c*f + (d*e - c*f)*sin(d*x + c))*log(cos(d*x + c) - I*sin(d*x
 + c) + I) - (d*f*x + c*f + (d*f*x + c*f)*sin(d*x + c))*log(I*cos(d*x + c)
+ sin(d*x + c) + 1) + (d*f*x + c*f + (d*f*x + c*f)*sin(d*x + c))*log(I*cos(
d*x + c) - sin(d*x + c) + 1) - (d*f*x + c*f + (d*f*x + c*f)*sin(d*x + c))*l
og(-I*cos(d*x + c) + sin(d*x + c) + 1) + (d*f*x + c*f + (d*f*x + c*f)*sin(d
*x + c))*log(-I*cos(d*x + c) - sin(d*x + c) + 1) - (d*e - c*f + (d*e - c*f)
*sin(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c) + I) + (d*e - c*f + (d*e
- c*f)*sin(d*x + c))*log(-cos(d*x + c) - I*sin(d*x + c) + I))/(a*d^2*sin(d*
x + c) + a*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e\sec\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx + \int \frac{fx\sec\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e*sec(c + d*x)/(sin(c + d*x) + 1), x) + Integral(f*x*sec(c + d*x)
/(sin(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\sec(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sec(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.272 $\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{1}{2d(a\sin(c+dx)+a)}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) - 1/(2*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.051378, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2667, 44, 206}

$$\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{1}{2d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) - 1/(2*d*(a + a*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^2}+\frac{1}{2a\left(a^2-x^2\right)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{1}{2d(a+a\sin(c+dx))}+\frac{\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\sin(c+dx)\right)}{2d}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{1}{2d(a+a\sin(c+dx))}$$

Mathematica [A] time = 0.0374701, size = 30, normalized size = 0.81

$$\frac{\tanh^{-1}(\sin(c+dx))-\frac{1}{\sin(c+dx)+1}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]] - (1 + Sin[c + d*x])^(-1))/(2*a*d)
```

Maple [A] time = 0.052, size = 54, normalized size = 1.5

$$-\frac{\ln(\sin(dx+c)-1)}{4\,da}-\frac{1}{2\,da\,(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{4\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/a/d*ln(sin(d*x+c)-1)-1/2/a/d/(1+sin(d*x+c))+1/4*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 0.981991, size = 63, normalized size = 1.7

$$\frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c)-1)}{a} - \frac{2}{a\sin(dx+c)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(log(sin(d*x + c) + 1)/a - log(sin(d*x + c) - 1)/a - 2/(a*sin(d*x + c)
+ a))/d
```

Fricas [A] time = 1.69868, size = 163, normalized size = 4.41

$$\frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1) - (\sin(dx+c)+1)\log(-\sin(dx+c)+1) - 2}{4\,(ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (sin(d*x + c) + 1)*log(-sin
(d*x + c) + 1) - 2)/(a*d*sin(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.20812, size = 78, normalized size = 2.11

$$\frac{\frac{\log(|\sin(dx+c)+1|)}{a} - \frac{\log(|\sin(dx+c)-1|)}{a} - \frac{\sin(dx+c)+3}{a(\sin(dx+c)+1)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c) - 1))/a - (sin(d*x
 + c) + 3)/(a*(sin(d*x + c) + 1)))/d
```

3.273 $\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sec(c+dx)}{(e+fx)(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0453772, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Sec[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 13.8387, size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Sec[c + d*x]/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 1.774, size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(fx+e)(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*(d*f*x + d*e)*cos(d*x + c)^2 + 2*(d*f*x + d*e)*sin(d*x + c)^2 - (f*cos(
d*x + c) + (d*f*x + d*e)*sin(d*x + c))*cos(2*d*x + 2*c) - f*cos(d*x + c) -
(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x
 + a*d^2*e^2)*cos(2*d*x + 2*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2
*e^2)*cos(d*x + c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*
x + c)*sin(2*d*x + 2*c) + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(2
*d*x + 2*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c)^
2 - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2 + 2*(a*d^2*f^2*x^2 + 2*a*d
^2*e*f*x + a*d^2*e^2)*sin(d*x + c))*cos(2*d*x + 2*c) + 4*(a*d^2*f^2*x^2 + 2
*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c))*integrate(1/2*(d^2*f^2*x^2 + 2*d^2*
e*f*x + d^2*e^2 + 4*f^2)*cos(d*x + c)/(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 +
3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*
e^2*f*x + a*d^2*e^3)*cos(d*x + c)^2 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 +
3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c)^2 + 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*
f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c)), x) - (a*d^2*f^2*x^2 +
 2*a*d^2*e*f*x + a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*co
s(2*d*x + 2*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*x +
c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*x + c)*sin(2*d*x
 + 2*c) + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(2*d*x + 2*c)^2 +
4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c)^2 - 2*(a*d^2*f^2
*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2 + 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2
```

```
*e^2)*sin(d*x + c))*cos(2*d*x + 2*c) + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a
*d^2*e^2)*sin(d*x + c))*integrate(1/2*cos(d*x + c)/(a*f*x + (a*f*x + a*e)*c
os(d*x + c)^2 + (a*f*x + a*e)*sin(d*x + c)^2 + a*e - 2*(a*f*x + a*e)*sin(d*
x + c)), x) + ((d*f*x + d*e)*cos(d*x + c) - f*sin(d*x + c) - f)*sin(2*d*x +
 2*c) + (d*f*x + d*e)*sin(d*x + c))/(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*
e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(2*d*x + 2*c)^2 + 4*(a
*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*x + c)^2 + 4*(a*d^2*f^2*x^2
 + 2*a*d^2*e*f*x + a*d^2*e^2)*cos(d*x + c)*sin(2*d*x + 2*c) + (a*d^2*f^2*x^
2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(2*d*x + 2*c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*
d^2*e*f*x + a*d^2*e^2)*sin(d*x + c)^2 - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x +
a*d^2*e^2 + 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c))*cos
(2*d*x + 2*c) + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*sin(d*x + c))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(dx+c\right)}{afx+ae+\left(afx+ae\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec\left(c+dx\right)}{e\sin\left(c+dx\right)+e+fx\sin\left(c+dx\right)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(fx+e)(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/((f*x + e)*(a*sin(d*x + c) + a)), x)
```

3.274 $\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sec(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0456112, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Sec[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx = \int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 22.1101, size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Sec[c + d*x]/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 3.25, size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(fx+e)^2 (a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*(d*f*x + d*e)*cos(d*x + c)^2 + 2*(d*f*x + d*e)*sin(d*x + c)^2 - (2*f*co
s(d*x + c) + (d*f*x + d*e)*sin(d*x + c))*cos(2*d*x + 2*c) - 2*f*cos(d*x + c
) - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d
^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(2*d*x + 2
*c)^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)
*cos(d*x + c)^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x +
a*d^2*e^3)*cos(d*x + c)*sin(2*d*x + 2*c) + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x
^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(2*d*x + 2*c)^2 + 4*(a*d^2*f^3*x^3 + 3
*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c)^2 - 2*(a*d^2*f
^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + 2*(a*d^2*f^3*x^3
 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c))*cos(2*d*x
 + 2*c) + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^
3)*sin(d*x + c))*integrate(1/2*(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 + 12*f^
2)*cos(d*x + c)/(a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 +
4*a*d^2*e^3*f*x + a*d^2*e^4 + (a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*
e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4)*cos(d*x + c)^2 + (a*d^2*f^4*x^4
+ 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4)*si
n(d*x + c)^2 + 2*(a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 +
 4*a*d^2*e^3*f*x + a*d^2*e^4)*sin(d*x + c)), x) - (a*d^2*f^3*x^3 + 3*a*d^2*
e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^
2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(2*d*x + 2*c)^2 + 4*(a*d^2*f^3*x^3 + 3*
```

```
a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(d*x + c)^2 + 4*(a*d^2*f^
3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(d*x + c)*sin(2
*d*x + 2*c) + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*
e^3)*sin(2*d*x + 2*c)^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^
2*f*x + a*d^2*e^3)*sin(d*x + c)^2 - 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 +
3*a*d^2*e^2*f*x + a*d^2*e^3 + 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^
2*e^2*f*x + a*d^2*e^3)*sin(d*x + c))*cos(2*d*x + 2*c) + 4*(a*d^2*f^3*x^3 +
3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c))*integrate(1/
2*cos(d*x + c)/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e*f*x + a*
e^2)*cos(d*x + c)^2 + (a*f^2*x^2 + 2*a*e*f*x + a*e^2)*sin(d*x + c)^2 - 2*(a
*f^2*x^2 + 2*a*e*f*x + a*e^2)*sin(d*x + c)), x) + ((d*f*x + d*e)*cos(d*x +
c) - 2*f*sin(d*x + c) - 2*f)*sin(2*d*x + 2*c) + (d*f*x + d*e)*sin(d*x + c))
/(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*
f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*cos(2*d*x + 2*c)
^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*co
s(d*x + c)^2 + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d
^2*e^3)*cos(d*x + c)*sin(2*d*x + 2*c) + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2
+ 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(2*d*x + 2*c)^2 + 4*(a*d^2*f^3*x^3 + 3*a*
d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c)^2 - 2*(a*d^2*f^3*
x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + 2*(a*d^2*f^3*x^3 +
3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*sin(d*x + c))*cos(2*d*x +
2*c) + 4*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*
sin(d*x + c))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(dx+c\right)}{af^2x^2+2\,aefx+ae^2+\left(af^2x^2+2\,aefx+ae^2\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e*f
*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec\left(c+dx\right)}{e^2\sin\left(c+dx\right)+e^2+2efx\sin\left(c+dx\right)+2efx+f^2x^2\sin\left(c+dx\right)+f^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) + 2*
e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

3.275 $\int \frac{(e+fx)^3\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=475

$$\frac{if^2(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{ad^3}-\frac{if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{2if^2(e+fx)\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{ad^3}-f^3\text{PolyL}$$

```
[Out] (((-2*I)/3)*(e + f*x)^3)/(a*d) - (I*f*(e + f*x)^2*ArcTan[E^(I*(c + d*x))])/
(a*d^2) + (f^3*ArcTanh[Sin[c + d*x]])/(a*d^4) + (2*f*(e + f*x)^2*Log[1 + E^
((2*I)*(c + d*x))])/(a*d^2) + (f^3*Log[Cos[c + d*x]])/(a*d^4) + (I*f^2*(e +
 f*x)*PolyLog[2, (-I)*E^(I*(c + d*x))])/(a*d^3) - (I*f^2*(e + f*x)*PolyLog[
2, I*E^(I*(c + d*x))])/(a*d^3) - ((2*I)*f^2*(e + f*x)*PolyLog[2, -E^((2*I)*
(c + d*x))])/(a*d^3) - (f^3*PolyLog[3, (-I)*E^(I*(c + d*x))])/(a*d^4) + (f^
3*PolyLog[3, I*E^(I*(c + d*x))])/(a*d^4) + (f^3*PolyLog[3, -E^((2*I)*(c + d
*x))])/(a*d^4) - (f^2*(e + f*x)*Sec[c + d*x])/(a*d^3) - (f*(e + f*x)^2*Sec[
c + d*x]^2)/(2*a*d^2) - ((e + f*x)^3*Sec[c + d*x]^3)/(3*a*d) + (f^2*(e + f*
x)*Tan[c + d*x])/(a*d^3) + (2*(e + f*x)^3*Tan[c + d*x])/(3*a*d) + (f*(e + f
*x)^2*Sec[c + d*x]*Tan[c + d*x])/(2*a*d^2) + ((e + f*x)^3*Sec[c + d*x]^2*Ta
n[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.593911, antiderivative size = 475, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4531, 4186, 4184, 3475, 3719, 2190, 2531, 2282, 6589, 4409, 3770, 4181}

$$\frac{if^2(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{ad^3}-\frac{if^2(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{ad^3}-\frac{2if^2(e+fx)\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{ad^3}-f^3\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (((-2*I)/3)*(e + f*x)^3)/(a*d) - (I*f*(e + f*x)^2*ArcTan[E^(I*(c + d*x))])/
(a*d^2) + (f^3*ArcTanh[Sin[c + d*x]])/(a*d^4) + (2*f*(e + f*x)^2*Log[1 + E^
((2*I)*(c + d*x))])/(a*d^2) + (f^3*Log[Cos[c + d*x]])/(a*d^4) + (I*f^2*(e +
 f*x)*PolyLog[2, (-I)*E^(I*(c + d*x))])/(a*d^3) - (I*f^2*(e + f*x)*PolyLog[
2, I*E^(I*(c + d*x))])/(a*d^3) - ((2*I)*f^2*(e + f*x)*PolyLog[2, -E^((2*I)*
(c + d*x))])/(a*d^3) - (f^3*PolyLog[3, (-I)*E^(I*(c + d*x))])/(a*d^4) + (f^
3*PolyLog[3, I*E^(I*(c + d*x))])/(a*d^4) + (f^3*PolyLog[3, -E^((2*I)*(c + d
*x))])/(a*d^4) - (f^2*(e + f*x)*Sec[c + d*x])/(a*d^3) - (f*(e + f*x)^2*Sec[
c + d*x]^2)/(2*a*d^2) - ((e + f*x)^3*Sec[c + d*x]^3)/(3*a*d) + (f^2*(e + f*
x)*Tan[c + d*x])/(a*d^3) + (2*(e + f*x)^3*Tan[c + d*x])/(3*a*d) + (f*(e + f
```

```
*x)^2*Sec[c + d*x]*Tan[c + d*x])/(2*a*d^2) + ((e + f*x)^3*Sec[c + d*x]^2*Ta
n[c + d*x])/(3*a*d)
```

Rule 4531

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int(e+fx)^3\sec^4(c+dx)\,dx}{a} - \frac{\int(e+fx)^3\sec^3(c+dx)\tan(c+dx)\,dx}{a}$$

$$= -\frac{f(e+fx)^2\sec^2(c+dx)}{2ad^2} - \frac{(e+fx)^3\sec^3(c+dx)}{3ad} + \frac{(e+fx)^3\sec^2(c+dx)\tan(c+dx)}{3ad}$$

$$= -\frac{f^2(e+fx)\sec(c+dx)}{ad^3} - \frac{f(e+fx)^2\sec^2(c+dx)}{2ad^2} - \frac{(e+fx)^3\sec^3(c+dx)}{3ad} + \frac{f^2(e+fx}{}$$

$$= -\frac{2i(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^2} + \frac{f^3\tanh^{-1}(\sin(c+dx))}{ad^4} + \frac{f^3\log(\cos(c+d}{ad^4}$$

$$= -\frac{2i(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^2} + \frac{f^3\tanh^{-1}(\sin(c+dx))}{ad^4} + \frac{2f(e+fx)^2\log}{ad^2}$$

$$= -\frac{2i(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^2} + \frac{f^3\tanh^{-1}(\sin(c+dx))}{ad^4} + \frac{2f(e+fx)^2\log}{ad^2}$$

$$= -\frac{2i(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^2} + \frac{f^3\tanh^{-1}(\sin(c+dx))}{ad^4} + \frac{2f(e+fx)^2\log}{ad^2}$$

$$= -\frac{2i(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^2} + \frac{f^3\tanh^{-1}(\sin(c+dx))}{ad^4} + \frac{2f(e+fx)^2\log}{ad^2}$$

Mathematica [B] time = 8.75923, size = 1117, normalized size = 2.35

$$\frac{\frac{d^3(e+fx)^3}{-i+e^{ic}} + 3d^2f\log\left(1-ie^{-i(c+dx)}\right)(e+fx)^2 + 6f^2\left(id(e+fx)\text{PolyLog}\left(2,ie^{-i(c+dx)}\right) + f\text{PolyLog}\left(3,ie^{-i(c+dx)}\right)\right)}{2ad^4} - \frac{f(\text{co}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((d^3*(e + f*x)^3)/(-I + E^(I*c)) + 3*d^2*f*(e + f*x)^2*Log[1 - I/E^(I*(c +
 d*x))] + 6*f^2*(I*d*(e + f*x)*PolyLog[2, I/E^(I*(c + d*x))] + f*PolyLog[3,
 I/E^(I*(c + d*x))]))/(2*a*d^4) - (f*(Cos[c] + I*Sin[c])*(5*d^2*e*f*x^2*Cos
[c] + (5*d^2*e^2 + 4*f^2)*x*(Cos[c] - I*Sin[c]) + (5*d^2*f^2*x^3*(Cos[c] -
I*Sin[c]))/3 - (5*I)*d^2*e*f*x^2*Sin[c] + (10*f^2*(d*x*PolyLog[2, (-I)*Cos[
c + d*x] - Sin[c + d*x]] - I*PolyLog[3, (-I)*Cos[c + d*x] - Sin[c + d*x]])*
(Cos[c] - I*Sin[c])*(1 - I*Cos[c] + Sin[c]))/d + 10*e*f*PolyLog[2, (-I)*Cos
[c + d*x] - Sin[c + d*x]]*(Cos[c] - I*(1 + Sin[c])) - 10*d*e*f*x*Log[1 + I*
Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(Cos[c] + I*(1 + Sin[c]))
```

```
- 5*d*f^2*x^2*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(C
os[c] + I*(1 + Sin[c])) + ((5*d^2*e^2 + 4*f^2)*(d*x + I*Log[Cos[c + d*x] +
I*(1 + Sin[c + d*x])])*(I*Cos[c] + Sin[c])*(Cos[c] + I*(1 + Sin[c])))/d))/(
2*a*d^3*(Cos[c] + I*(1 + Sin[c]))) + (e^3*Sin[(d*x)/2] + 3*e^2*f*x*Sin[(d*x
)/2] + 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/2])/(2*a*d*(Cos[c/2] -
Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (e^3*Sin[(d*x)/2] +
3*e^2*f*x*Sin[(d*x)/2] + 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/2])/(
3*a*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) +
(-(d*e^3*Cos[c/2]) - 3*e^2*f*Cos[c/2] - 3*d*e^2*f*x*Cos[c/2] - 6*e*f^2*x*Co
s[c/2] - 3*d*e*f^2*x^2*Cos[c/2] - 3*f^3*x^2*Cos[c/2] - d*f^3*x^3*Cos[c/2] +
 d*e^3*Sin[c/2] - 3*e^2*f*Sin[c/2] + 3*d*e^2*f*x*Sin[c/2] - 6*e*f^2*x*Sin[c
/2] + 3*d*e*f^2*x^2*Sin[c/2] - 3*f^3*x^2*Sin[c/2] + d*f^3*x^3*Sin[c/2])/(6*
a*d^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) +
(5*d^2*e^3*Sin[(d*x)/2] + 12*e*f^2*Sin[(d*x)/2] + 15*d^2*e^2*f*x*Sin[(d*x)/
2] + 12*f^3*x*Sin[(d*x)/2] + 15*d^2*e*f^2*x^2*Sin[(d*x)/2] + 5*d^2*f^3*x^3*
Sin[(d*x)/2])/(6*a*d^3*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2]))
```

Maple [B] time = 0.338, size = 1124, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 3*f^3*polylog(3,-I*exp(I*(d*x+c)))/a/d^4+5*f^2/d^2/a*e*ln(1-I*exp(I*(d*x+c)
))*x+3/2/d^4/a*f^3*c^2*ln(exp(I*(d*x+c))-I)+3/2/d^2/a*e^2*f*ln(exp(I*(d*x+c
))-I)-3/2/d^4/a*ln(1+I*exp(I*(d*x+c)))*c^2*f^3+8/3*I/d^4/a*c^3*f^3-4/3*I/d/
a*f^3*x^3-4*f^3/d^4/a*c^2*ln(exp(I*(d*x+c)))+5/2*f/d^2/a*ln(exp(I*(d*x+c))+
I)*e^2+5/2*f^3/d^4/a*c^2*ln(exp(I*(d*x+c))+I)-4*f/d^2/a*ln(exp(I*(d*x+c)))*
e^2+5*f^2/d^3/a*e*ln(1-I*exp(I*(d*x+c)))*c+5/2*f^3/d^2/a*ln(1-I*exp(I*(d*x+
c)))*x^2-5/2*f^3/d^4/a*ln(1-I*exp(I*(d*x+c)))*c^2+8*f^2/d^3/a*e*c*ln(exp(I*
(d*x+c)))-5*f^2/d^3/a*e*c*ln(exp(I*(d*x+c))+I)-1/3*(12*I*d^2*e^2*f*x+6*I*d*
e*f^2*x*exp(I*(d*x+c))+6*f^3*x*exp(I*(d*x+c))+6*e*f^2*exp(I*(d*x+c))+3*I*d*
f^3*x^2*exp(3*I*(d*x+c))+3*I*d*e^2*f*exp(3*I*(d*x+c))+6*I*e*f^2*exp(2*I*(d*
x+c))+12*I*d^2*e*f^2*x^2+6*I*e*f^2+6*I*d*e*f^2*x*exp(3*I*(d*x+c))+24*d^2*e*
f^2*x^2*exp(I*(d*x+c))+24*d^2*e^2*f*x*exp(I*(d*x+c))+3*I*d*e^2*f*exp(I*(d*x
+c))+6*I*f^3*x*exp(2*I*(d*x+c))+8*d^2*f^3*x^3*exp(I*(d*x+c))+3*I*d*f^3*x^2*
exp(I*(d*x+c))+4*I*d^2*f^3*x^3+8*d^2*e^3*exp(I*(d*x+c))+4*I*d^2*e^3+6*f^3*x
*exp(3*I*(d*x+c))+6*e*f^2*exp(3*I*(d*x+c))+6*I*f^3*x)/(exp(I*(d*x+c))-I)/(e
xp(I*(d*x+c))+I)^3/d^3/a-8*I/d^2/a*c*e*f^2*x+2/d^4/a*f^3*ln(exp(I*(d*x+c))+
```

```
I)-2/d^4/a*f^3*ln(exp(I*(d*x+c)))-3/d^3/a*e*f^2*c*ln(exp(I*(d*x+c))-I)+3/d^
2/a*ln(1+I*exp(I*(d*x+c)))*e*f^2*x+3/d^3/a*ln(1+I*exp(I*(d*x+c)))*c*e*f^2+3
/2/d^2/a*ln(1+I*exp(I*(d*x+c)))*f^3*x^2-4*I/d^3/a*c^2*e*f^2-3*I/d^3/a*polyl
og(2,-I*exp(I*(d*x+c)))*f^3*x-5*I/d^3/a*polylog(2,I*exp(I*(d*x+c)))*f^3*x-3
*I/d^3/a*e*f^2*polylog(2,-I*exp(I*(d*x+c)))-5*I/d^3/a*e*f^2*polylog(2,I*exp
(I*(d*x+c)))+4*I/d^3/a*c^2*f^3*x-4*I/d/a*e*f^2*x^2+5*f^3*polylog(3,I*exp(I*
(d*x+c)))/a/d^4
```

Maxima [B] time = 6.6732, size = 6893, normalized size = 14.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(24*c^2*e*f^2*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a*d^2 + 2*a*
d^2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*d^2*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 - a*d^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + 6*(4*(8*(d*x + c)*cos
(d*x + c) - sin(3*d*x + 3*c) - sin(d*x + c))*cos(4*d*x + 4*c) + 16*(2*d*x +
 4*(d*x + c)*sin(d*x + c) + 2*c + cos(d*x + c))*cos(3*d*x + 3*c) + 8*cos(3*
d*x + 3*c)^2 + 8*cos(d*x + c)^2 + 5*(2*(2*sin(3*d*x + 3*c) + 2*sin(d*x + c)
 + 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4*c)^2 - 4*cos(3*d*x + 3*c)^2 - 8*cos(
3*d*x + 3*c)*cos(d*x + c) - 4*cos(d*x + c)^2 - 4*(cos(3*d*x + 3*c) + cos(d*
x + c))*sin(4*d*x + 4*c) - sin(4*d*x + 4*c)^2 - 4*(2*sin(d*x + c) + 1)*sin(
3*d*x + 3*c) - 4*sin(3*d*x + 3*c)^2 - 4*sin(d*x + c)^2 - 4*sin(d*x + c) - 1
)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3*(2*(2*sin(3
*d*x + 3*c) + 2*sin(d*x + c) + 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4*c)^2 - 4
*cos(3*d*x + 3*c)^2 - 8*cos(3*d*x + 3*c)*cos(d*x + c) - 4*cos(d*x + c)^2 -
4*(cos(3*d*x + 3*c) + cos(d*x + c))*sin(4*d*x + 4*c) - sin(4*d*x + 4*c)^2 -
 4*(2*sin(d*x + c) + 1)*sin(3*d*x + 3*c) - 4*sin(3*d*x + 3*c)^2 - 4*sin(d*x
 + c)^2 - 4*sin(d*x + c) - 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d
*x + c) + 1) + 4*(4*d*x + 8*(d*x + c)*sin(d*x + c) + 4*c + cos(3*d*x + 3*c)
 + cos(d*x + c))*sin(4*d*x + 4*c) - 4*(16*(d*x + c)*cos(d*x + c) - 4*sin(d*
x + c) - 1)*sin(3*d*x + 3*c) + 8*sin(3*d*x + 3*c)^2 + 8*sin(d*x + c)^2 + 4*
sin(d*x + c))*c*e*f^2/(a*d^2*cos(4*d*x + 4*c)^2 + 4*a*d^2*cos(3*d*x + 3*c)^
2 + 8*a*d^2*cos(3*d*x + 3*c)*cos(d*x + c) + 4*a*d^2*cos(d*x + c)^2 + a*d^2*
sin(4*d*x + 4*c)^2 + 4*a*d^2*sin(3*d*x + 3*c)^2 + 4*a*d^2*sin(d*x + c)^2 +
4*a*d^2*sin(d*x + c) + a*d^2 - 2*(2*a*d^2*sin(3*d*x + 3*c) + 2*a*d^2*sin(d*
x + c) + a*d^2)*cos(4*d*x + 4*c) + 4*(a*d^2*cos(3*d*x + 3*c) + a*d^2*cos(d*
x + c))*sin(4*d*x + 4*c) + 4*(2*a*d^2*sin(d*x + c) + a*d^2)*sin(3*d*x + 3*c
```

)) - 24*c*e^2*f*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a*d + 2*a*d*sin
(d*x + c)/(cos(d*x + c) + 1) - 2*a*d*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 -
a*d*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*(4*(8*(d*x + c)*cos(d*x + c) -
 sin(3*d*x + 3*c) - sin(d*x + c))*cos(4*d*x + 4*c) + 16*(2*d*x + 4*(d*x + c
)*sin(d*x + c) + 2*c + cos(d*x + c))*cos(3*d*x + 3*c) + 8*cos(3*d*x + 3*c)^
2 + 8*cos(d*x + c)^2 + 5*(2*(2*sin(3*d*x + 3*c) + 2*sin(d*x + c) + 1)*cos(4
*d*x + 4*c) - cos(4*d*x + 4*c)^2 - 4*cos(3*d*x + 3*c)^2 - 8*cos(3*d*x + 3*c
)*cos(d*x + c) - 4*cos(d*x + c)^2 - 4*(cos(3*d*x + 3*c) + cos(d*x + c))*sin
(4*d*x + 4*c) - sin(4*d*x + 4*c)^2 - 4*(2*sin(d*x + c) + 1)*sin(3*d*x + 3*c
) - 4*sin(3*d*x + 3*c)^2 - 4*sin(d*x + c)^2 - 4*sin(d*x + c) - 1)*log(cos(d
*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3*(2*(2*sin(3*d*x + 3*c)
 + 2*sin(d*x + c) + 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4*c)^2 - 4*cos(3*d*x
+ 3*c)^2 - 8*cos(3*d*x + 3*c)*cos(d*x + c) - 4*cos(d*x + c)^2 - 4*(cos(3*d*
x + 3*c) + cos(d*x + c))*sin(4*d*x + 4*c) - sin(4*d*x + 4*c)^2 - 4*(2*sin(d
*x + c) + 1)*sin(3*d*x + 3*c) - 4*sin(3*d*x + 3*c)^2 - 4*sin(d*x + c)^2 - 4
*sin(d*x + c) - 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1
) + 4*(4*d*x + 8*(d*x + c)*sin(d*x + c) + 4*c + cos(3*d*x + 3*c) + cos(d*x
+ c))*sin(4*d*x + 4*c) - 4*(16*(d*x + c)*cos(d*x + c) - 4*sin(d*x + c) - 1)
*sin(3*d*x + 3*c) + 8*sin(3*d*x + 3*c)^2 + 8*sin(d*x + c)^2 + 4*sin(d*x + c
))*e^2*f/(a*d*cos(4*d*x + 4*c)^2 + 4*a*d*cos(3*d*x + 3*c)^2 + 8*a*d*cos(3*d
*x + 3*c)*cos(d*x + c) + 4*a*d*cos(d*x + c)^2 + a*d*sin(4*d*x + 4*c)^2 + 4*
a*d*sin(3*d*x + 3*c)^2 + 4*a*d*sin(d*x + c)^2 + 4*a*d*sin(d*x + c) + a*d -
2*(2*a*d*sin(3*d*x + 3*c) + 2*a*d*sin(d*x + c) + a*d)*cos(4*d*x + 4*c) + 4*
(a*d*cos(3*d*x + 3*c) + a*d*cos(d*x + c))*sin(4*d*x + 4*c) + 4*(2*a*d*sin(d
*x + c) + a*d)*sin(3*d*x + 3*c)) + 8*e^3*(sin(d*x + c)/(cos(d*x + c) + 1) +
 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 - 1)/(a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 12*(24*d*e*f^2
- 8*(2*c^3 + 3*c)*f^3 - (6*(5*c^2 + 4)*f^3*cos(4*d*x + 4*c) + (60*I*c^2 + 4
8*I)*f^3*cos(3*d*x + 3*c) + (60*I*c^2 + 48*I)*f^3*cos(d*x + c) + (30*I*c^2
+ 24*I)*f^3*sin(4*d*x + 4*c) - 12*(5*c^2 + 4)*f^3*sin(3*d*x + 3*c) - 12*(5*
c^2 + 4)*f^3*sin(d*x + c) - 6*(5*c^2 + 4)*f^3)*arctan2(sin(d*x + c) + 1, co
s(d*x + c)) - (18*c^2*f^3*cos(4*d*x + 4*c) + 36*I*c^2*f^3*cos(3*d*x + 3*c)
+ 36*I*c^2*f^3*cos(d*x + c) + 18*I*c^2*f^3*sin(4*d*x + 4*c) - 36*c^2*f^3*si
n(3*d*x + 3*c) - 36*c^2*f^3*sin(d*x + c) - 18*c^2*f^3)*arctan2(sin(d*x + c)
 - 1, cos(d*x + c)) - (30*(d*x + c)^2*f^3 + 60*(d*e*f^2 - c*f^3)*(d*x + c)
- 30*((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))*cos(4*d*x + 4*c) + (
-60*I*(d*x + c)^2*f^3 + (-120*I*d*e*f^2 + 120*I*c*f^3)*(d*x + c))*cos(3*d*x
 + 3*c) + (-60*I*(d*x + c)^2*f^3 + (-120*I*d*e*f^2 + 120*I*c*f^3)*(d*x + c)
)*cos(d*x + c) + (-30*I*(d*x + c)^2*f^3 + (-60*I*d*e*f^2 + 60*I*c*f^3)*(d*x
 + c))*sin(4*d*x + 4*c) + 60*((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x +
c))*sin(3*d*x + 3*c) + 60*((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f^3)*(d*x + c))
*sin(d*x + c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) + (18*(d*x + c)^2*f^
3 + 36*(d*e*f^2 - c*f^3)*(d*x + c) - 18*((d*x + c)^2*f^3 + 2*(d*e*f^2 - c*f

^3)*(d*x + c))*cos(4*d*x + 4*c) - (36*I*(d*x + c)^2*f^3 + (72*I*d*e*f^2 - 7
2*I*c*f^3)*(d*x + c))*cos(3*d*x + 3*c) - (36*I*(d*x + c)^2*f^3 + (72*I*d*e*
f^2 - 72*I*c*f^3)*(d*x + c))*cos(d*x + c) - (18*I*(d*x + c)^2*f^3 + (36*I*d
*e*f^2 - 36*I*c*f^3)*(d*x + c))*sin(4*d*x + 4*c) + 36*((d*x + c)^2*f^3 + 2*
(d*e*f^2 - c*f^3)*(d*x + c))*sin(3*d*x + 3*c) + 36*((d*x + c)^2*f^3 + 2*(d*
e*f^2 - c*f^3)*(d*x + c))*sin(d*x + c))*arctan2(cos(d*x + c), -sin(d*x + c)
 + 1) + 8*(2*(d*x + c)^3*f^3 + 3*(2*c^2 + 1)*(d*x + c)*f^3 + 6*(d*e*f^2 - c
f^3)(d*x + c)^2)*cos(4*d*x + 4*c) - (-32*I*(d*x + c)^3*f^3 + 24*I*d*e*f^2
 - 12*(c^2 + 2*I*c)*f^3 - 12*(8*I*d*e*f^2 + (-8*I*c + 1)*f^3)*(d*x + c)^2 -
 (24*d*e*f^2 - (-96*I*c^2 + 24*c - 24*I)*f^3)*(d*x + c))*cos(3*d*x + 3*c) +
 24*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*cos(2*d*x + 2*c) + (12*(d*x + c)^2*f^
3 - 24*I*d*e*f^2 - (-32*I*c^3 - 12*c^2 - 24*I*c)*f^3 + (24*d*e*f^2 - (24*c
- 24*I)*f^3)*(d*x + c))*cos(d*x + c) - (60*d*e*f^2 + 60*(d*x + c)*f^3 - 60*
c*f^3 - 60*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*cos(4*d*x + 4*c) + (-120*I*d*e
*f^2 - 120*I*(d*x + c)*f^3 + 120*I*c*f^3)*cos(3*d*x + 3*c) + (-120*I*d*e*f^
2 - 120*I*(d*x + c)*f^3 + 120*I*c*f^3)*cos(d*x + c) + (-60*I*d*e*f^2 - 60*I
*(d*x + c)*f^3 + 60*I*c*f^3)*sin(4*d*x + 4*c) + 120*(d*e*f^2 + (d*x + c)*f^
3 - c*f^3)*sin(3*d*x + 3*c) + 120*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*sin(d*x
 + c))*dilog(I*e^(I*d*x + I*c)) - (36*d*e*f^2 + 36*(d*x + c)*f^3 - 36*c*f^3
 - 36*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*cos(4*d*x + 4*c) + (-72*I*d*e*f^2 -
 72*I*(d*x + c)*f^3 + 72*I*c*f^3)*cos(3*d*x + 3*c) + (-72*I*d*e*f^2 - 72*I*
(d*x + c)*f^3 + 72*I*c*f^3)*cos(d*x + c) + (-36*I*d*e*f^2 - 36*I*(d*x + c)*
f^3 + 36*I*c*f^3)*sin(4*d*x + 4*c) + 72*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*s
in(3*d*x + 3*c) + 72*(d*e*f^2 + (d*x + c)*f^3 - c*f^3)*sin(d*x + c))*dilog(
-I*e^(I*d*x + I*c)) - (15*I*(d*x + c)^2*f^3 + (15*I*c^2 + 12*I)*f^3 + (30*I
*d*e*f^2 - 30*I*c*f^3)*(d*x + c) + (-15*I*(d*x + c)^2*f^3 + (-15*I*c^2 - 12
*I)*f^3 + (-30*I*d*e*f^2 + 30*I*c*f^3)*(d*x + c))*cos(4*d*x + 4*c) + 6*(5*(
d*x + c)^2*f^3 + (5*c^2 + 4)*f^3 + 10*(d*e*f^2 - c*f^3)*(d*x + c))*cos(3*d*
x + 3*c) + 6*(5*(d*x + c)^2*f^3 + (5*c^2 + 4)*f^3 + 10*(d*e*f^2 - c*f^3)*(d
*x + c))*cos(d*x + c) + 3*(5*(d*x + c)^2*f^3 + (5*c^2 + 4)*f^3 + 10*(d*e*f^
2 - c*f^3)*(d*x + c))*sin(4*d*x + 4*c) + (30*I*(d*x + c)^2*f^3 + (30*I*c^2
+ 24*I)*f^3 + (60*I*d*e*f^2 - 60*I*c*f^3)*(d*x + c))*sin(3*d*x + 3*c) + (30
I(d*x + c)^2*f^3 + (30*I*c^2 + 24*I)*f^3 + (60*I*d*e*f^2 - 60*I*c*f^3)*(d
*x + c))*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c)
 + 1) - (9*I*(d*x + c)^2*f^3 + 9*I*c^2*f^3 + (18*I*d*e*f^2 - 18*I*c*f^3)*(d
*x + c) + (-9*I*(d*x + c)^2*f^3 - 9*I*c^2*f^3 + (-18*I*d*e*f^2 + 18*I*c*f^3
)*(d*x + c))*cos(4*d*x + 4*c) + 18*((d*x + c)^2*f^3 + c^2*f^3 + 2*(d*e*f^2
- c*f^3)*(d*x + c))*cos(3*d*x + 3*c) + 18*((d*x + c)^2*f^3 + c^2*f^3 + 2*(d
*e*f^2 - c*f^3)*(d*x + c))*cos(d*x + c) + 9*((d*x + c)^2*f^3 + c^2*f^3 + 2*
(d*e*f^2 - c*f^3)*(d*x + c))*sin(4*d*x + 4*c) + (18*I*(d*x + c)^2*f^3 + 18*
I*c^2*f^3 + (36*I*d*e*f^2 - 36*I*c*f^3)*(d*x + c))*sin(3*d*x + 3*c) + (18*I
*(d*x + c)^2*f^3 + 18*I*c^2*f^3 + (36*I*d*e*f^2 - 36*I*c*f^3)*(d*x + c))*si
n(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) - (-6
0*I*f^3*cos(4*d*x + 4*c) + 120*f^3*cos(3*d*x + 3*c) + 120*f^3*cos(d*x + c)
+ 60*f^3*sin(4*d*x + 4*c) + 120*I*f^3*sin(3*d*x + 3*c) + 120*I*f^3*sin(d*x

```
+ c) + 60*I*f^3)*polylog(3, I*e^(I*d*x + I*c)) - (-36*I*f^3*cos(4*d*x + 4*c
) + 72*f^3*cos(3*d*x + 3*c) + 72*f^3*cos(d*x + c) + 36*f^3*sin(4*d*x + 4*c)
 + 72*I*f^3*sin(3*d*x + 3*c) + 72*I*f^3*sin(d*x + c) + 36*I*f^3)*polylog(3,
 -I*e^(I*d*x + I*c)) - (-16*I*(d*x + c)^3*f^3 + (-48*I*c^2 - 24*I)*(d*x + c
)*f^3 + (-48*I*d*e*f^2 + 48*I*c*f^3)*(d*x + c)^2)*sin(4*d*x + 4*c) - (32*(d
*x + c)^3*f^3 - 24*d*e*f^2 + (-12*I*c^2 + 24*c)*f^3 + (96*d*e*f^2 - (96*c +
 12*I)*f^3)*(d*x + c)^2 + (-24*I*d*e*f^2 + 24*(4*c^2 + I*c + 1)*f^3)*(d*x +
 c))*sin(3*d*x + 3*c) - (-24*I*d*e*f^2 - 24*I*(d*x + c)*f^3 + 24*I*c*f^3)*s
in(2*d*x + 2*c) - (-12*I*(d*x + c)^2*f^3 - 24*d*e*f^2 + (32*c^3 - 12*I*c^2
+ 24*c)*f^3 - 24*(I*d*e*f^2 + (-I*c - 1)*f^3)*(d*x + c))*sin(d*x + c))/(-12
*I*a*d^3*cos(4*d*x + 4*c) + 24*a*d^3*cos(3*d*x + 3*c) + 24*a*d^3*cos(d*x +
c) + 12*a*d^3*sin(4*d*x + 4*c) + 24*I*a*d^3*sin(3*d*x + 3*c) + 24*I*a*d^3*s
in(d*x + c) + 12*I*a*d^3))/d
```

Fricas [C] time = 3.11652, size = 3767, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*d^3*f^3*x^3 + 12*d^3*e*f^2*x^2 + 12*d^3*e^2*f*x + 4*d^3*e^3 - 4*(2*
d^3*f^3*x^3 + 6*d^3*e*f^2*x^2 + 2*d^3*e^3 + 3*d*e*f^2 + 3*(2*d^3*e^2*f + d*
f^3)*x)*cos(d*x + c)^2 - 6*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*cos(d*
x + c) + ((18*I*d*f^3*x + 18*I*d*e*f^2)*cos(d*x + c)*sin(d*x + c) + (18*I*d
*f^3*x + 18*I*d*e*f^2)*cos(d*x + c))*dilog(I*cos(d*x + c) + sin(d*x + c)) +
 ((-30*I*d*f^3*x - 30*I*d*e*f^2)*cos(d*x + c)*sin(d*x + c) + (-30*I*d*f^3*x
 - 30*I*d*e*f^2)*cos(d*x + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) + ((-18
*I*d*f^3*x - 18*I*d*e*f^2)*cos(d*x + c)*sin(d*x + c) + (-18*I*d*f^3*x - 18*
I*d*e*f^2)*cos(d*x + c))*dilog(-I*cos(d*x + c) + sin(d*x + c)) + ((30*I*d*f
^3*x + 30*I*d*e*f^2)*cos(d*x + c)*sin(d*x + c) + (30*I*d*f^3*x + 30*I*d*e*f
^2)*cos(d*x + c))*dilog(-I*cos(d*x + c) - sin(d*x + c)) + 3*((5*d^2*e^2*f -
 10*c*d*e*f^2 + (5*c^2 + 4)*f^3)*cos(d*x + c)*sin(d*x + c) + (5*d^2*e^2*f -
 10*c*d*e*f^2 + (5*c^2 + 4)*f^3)*cos(d*x + c))*log(cos(d*x + c) + I*sin(d*x
 + c) + I) + 9*((d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c)*sin(d*x +
c) + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c))*log(cos(d*x + c) - I
*sin(d*x + c) + I) + 15*((d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f
^3)*cos(d*x + c)*sin(d*x + c) + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2
- c^2*f^3)*cos(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c) + 1) + 9*((d^2*f
^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c)*sin(d*x + c) +
 (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c))*log(I*
```

```
cos(d*x + c) - sin(d*x + c) + 1) + 15*((d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d
*e*f^2 - c^2*f^3)*cos(d*x + c)*sin(d*x + c) + (d^2*f^3*x^2 + 2*d^2*e*f^2*x
+ 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c))*log(-I*cos(d*x + c) + sin(d*x + c) +
 1) + 9*((d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d*x + c)
*sin(d*x + c) + (d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*cos(d
*x + c))*log(-I*cos(d*x + c) - sin(d*x + c) + 1) + 3*((5*d^2*e^2*f - 10*c*d
*e*f^2 + (5*c^2 + 4)*f^3)*cos(d*x + c)*sin(d*x + c) + (5*d^2*e^2*f - 10*c*d
*e*f^2 + (5*c^2 + 4)*f^3)*cos(d*x + c))*log(-cos(d*x + c) + I*sin(d*x + c)
+ I) + 9*((d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c)*sin(d*x + c) + (
d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*cos(d*x + c))*log(-cos(d*x + c) - I*sin(
d*x + c) + I) + 18*(f^3*cos(d*x + c)*sin(d*x + c) + f^3*cos(d*x + c))*polyl
og(3, I*cos(d*x + c) + sin(d*x + c)) + 30*(f^3*cos(d*x + c)*sin(d*x + c) +
f^3*cos(d*x + c))*polylog(3, I*cos(d*x + c) - sin(d*x + c)) + 18*(f^3*cos(d
*x + c)*sin(d*x + c) + f^3*cos(d*x + c))*polylog(3, -I*cos(d*x + c) + sin(d
*x + c)) + 30*(f^3*cos(d*x + c)*sin(d*x + c) + f^3*cos(d*x + c))*polylog(3,
 -I*cos(d*x + c) - sin(d*x + c)) + 8*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3
*e^2*f*x + d^3*e^3)*sin(d*x + c))/(a*d^4*cos(d*x + c)*sin(d*x + c) + a*d^4*
cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \sec(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sec(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

3.276 $\int \frac{(e+fx)^2 \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=343

$$\frac{if^2\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{3ad^3}-\frac{if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{3ad^3}-\frac{2if^2\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{3ad^3}+\frac{4f(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{3ad^2}-\frac{2i}{}$$

```
[Out] (((-2*I)/3)*(e + f*x)^2)/(a*d) - (((2*I)/3)*f*(e + f*x)*ArcTan[E^(I*(c + d*
x))])/(a*d^2) + (4*f*(e + f*x)*Log[1 + E^((2*I)*(c + d*x))])/(3*a*d^2) + ((
I/3)*f^2*PolyLog[2, (-I)*E^(I*(c + d*x))])/(a*d^3) - ((I/3)*f^2*PolyLog[2,
I*E^(I*(c + d*x))])/(a*d^3) - (((2*I)/3)*f^2*PolyLog[2, -E^((2*I)*(c + d*x)
)])/(a*d^3) - (f^2*Sec[c + d*x])/(3*a*d^3) - (f*(e + f*x)*Sec[c + d*x]^2)/(
3*a*d^2) - ((e + f*x)^2*Sec[c + d*x]^3)/(3*a*d) + (f^2*Tan[c + d*x])/(3*a*d
^3) + (2*(e + f*x)^2*Tan[c + d*x])/(3*a*d) + (f*(e + f*x)*Sec[c + d*x]*Tan[
c + d*x])/(3*a*d^2) + ((e + f*x)^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.37765, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4531, 4186, 3767, 8, 4184, 3719, 2190, 2279, 2391, 4409, 4185, 4181}

$$\frac{if^2\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{3ad^3}-\frac{if^2\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{3ad^3}-\frac{2if^2\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)}{3ad^3}+\frac{4f(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{3ad^2}-\frac{2i}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```



```
[Out] (((-2*I)/3)*(e + f*x)^2)/(a*d) - (((2*I)/3)*f*(e + f*x)*ArcTan[E^(I*(c + d*
x))])/(a*d^2) + (4*f*(e + f*x)*Log[1 + E^((2*I)*(c + d*x))])/(3*a*d^2) + ((
I/3)*f^2*PolyLog[2, (-I)*E^(I*(c + d*x))])/(a*d^3) - ((I/3)*f^2*PolyLog[2,
I*E^(I*(c + d*x))])/(a*d^3) - (((2*I)/3)*f^2*PolyLog[2, -E^((2*I)*(c + d*x)
)])/(a*d^3) - (f^2*Sec[c + d*x])/(3*a*d^3) - (f*(e + f*x)*Sec[c + d*x]^2)/(
3*a*d^2) - ((e + f*x)^2*Sec[c + d*x]^3)/(3*a*d) + (f^2*Tan[c + d*x])/(3*a*d
^3) + (2*(e + f*x)^2*Tan[c + d*x])/(3*a*d) + (f*(e + f*x)*Sec[c + d*x]*Tan[
c + d*x])/(3*a*d^2) + ((e + f*x)^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```


Rule 4531

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
```

```
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\sec^4(c+dx)\,dx}{a} - \frac{\int (e+fx)^2\sec^3(c+dx)\tan(c+dx)\,dx}{a}$$

$$= -\frac{f(e+fx)\sec^2(c+dx)}{3ad^2} - \frac{(e+fx)^2\sec^3(c+dx)}{3ad} + \frac{(e+fx)^2\sec^2(c+dx)\tan(c+dx)}{3ad} +$$

$$= -\frac{f^2\sec(c+dx)}{3ad^3} - \frac{f(e+fx)\sec^2(c+dx)}{3ad^2} - \frac{(e+fx)^2\sec^3(c+dx)}{3ad} + \frac{2(e+fx)^2\tan(c+}{3ad}$$

$$= -\frac{2i(e+fx)^2}{3ad} - \frac{2if(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{3ad^2} - \frac{f^2\sec(c+dx)}{3ad^3} - \frac{f(e+fx)\sec^2(c+dx)}{3ad^2} -$$

$$= -\frac{2i(e+fx)^2}{3ad} - \frac{2if(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{3ad^2} + \frac{4f(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{3ad^2} - \frac{f^2\sec(c+}{3ad^3}$$

$$= -\frac{2i(e+fx)^2}{3ad} - \frac{2if(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{3ad^2} + \frac{4f(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{3ad^2} + \frac{if^2\text{Li}_2\left(-i\right.}{3ad^3}$$

$$= -\frac{2i(e+fx)^2}{3ad} - \frac{2if(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{3ad^2} + \frac{4f(e+fx)\log\left(1+e^{2i(c+dx)}\right)}{3ad^2} + \frac{if^2\text{Li}_2\left(-i\right.}{3ad^3}$$

Mathematica [A] time = 6.60974, size = 637, normalized size = 1.86

$$\frac{\frac{12d^2f(\cos(c)+i\sin(c))\left(\frac{f(\cos(c)-i(\sin(c)-1))\text{PolyLog}(2,\sin(c+dx)+i\cos(c+dx))}{d^2}+\frac{(-\sin(c)-i\cos(c)+1)(e+fx)\log(-\sin(c+dx)-i\cos(c+dx)+1)}{d}+\frac{(\cos(c)-i\sin(c))(e+fx)^2}{2f}\right)}{\cos(c)+i(\sin(c)-1)} - \frac{20d^2}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((12*d^2*f*((f*PolyLog[2, I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] - I*(-1 +
Sin[c])))/d^2 + ((e + f*x)*Log[1 - I*Cos[c + d*x] - Sin[c + d*x]]*(1 - I*Co
s[c] - Sin[c]))/d + ((e + f*x)^2*(Cos[c] - I*Sin[c]))/(2*f))*(Cos[c] + I*Si
n[c]))/(Cos[c] + I*(-1 + Sin[c])) - (20*d^2*f*(Cos[c] + I*Sin[c])*(((e + f*
x)^2*(Cos[c] - I*Sin[c]))/(2*f) - ((e + f*x)*Log[1 + I*Cos[c + d*x] + Sin[c
 + d*x]]*(1 + I*Cos[c] + Sin[c]))/d + (f*PolyLog[2, (-I)*Cos[c + d*x] - Sin
[c + d*x]]*(Cos[c] - I*(1 + Sin[c])))/d^2))/(Cos[c] + I*(1 + Sin[c])) + (-2
*f^2*Cos[c] - 2*d*f*(e + f*x)*Cos[d*x] + 2*d^2*e^2*Cos[c + d*x] + 4*f^2*Cos
[c + d*x] + 4*d^2*e*f*x*Cos[c + d*x] + 2*d^2*f^2*x^2*Cos[c + d*x] - 2*d*e*f
*Cos[2*c + d*x] - 2*d*f^2*x*Cos[2*c + d*x] - 4*d^2*e^2*Cos[c + 2*d*x] - 2*f
^2*Cos[c + 2*d*x] - 8*d^2*e*f*x*Cos[c + 2*d*x] - 4*d^2*f^2*x^2*Cos[c + 2*d*
x] + 8*d^2*e^2*Sin[d*x] + 2*f^2*Sin[d*x] + 16*d^2*e*f*x*Sin[d*x] + 8*d^2*f^
2*x^2*Sin[d*x] + d^2*e^2*Sin[2*(c + d*x)] + 2*f^2*Sin[2*(c + d*x)] + 2*d^2*
```

```
e*f*x*Sin[2*(c + d*x)] + d^2*f^2*x^2*Sin[2*(c + d*x)] - 2*f^2*Sin[2*c + d*x
])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3))/(12*a*d^3)
```

Maple [A] time = 0.236, size = 573, normalized size = 1.7

$$-\frac{2 f^2 e^{i(dx+c)} + 2 i f^2 + 16 d^2 e f x e^{i(dx+c)} + 2 i d f^2 x e^{i(dx+c)} + 2 i d e f e^{i(dx+c)} + 2 i d f^2 x e^{3 i(dx+c)} + 2 i d e f e^{3 i(dx+c)} + 8 i d^2 e f x + 2}{\left(3 e^{i(dx+c)} - 3 i\right)\left(e^{i(dx+c)} + i\right)^3 d^3 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/3*(f^2*exp(I*(d*x+c))+I*f^2+8*d^2*e*f*x*exp(I*(d*x+c))+I*d*f^2*x*exp(I*(
d*x+c))+I*d*e*f*exp(I*(d*x+c))+I*d*f^2*x*exp(3*I*(d*x+c))+I*d*e*f*exp(3*I*(
d*x+c))+4*I*d^2*e*f*x+f^2*exp(3*I*(d*x+c))+4*d^2*e^2*exp(I*(d*x+c))+2*I*d^2
*e^2+I*f^2*exp(2*I*(d*x+c))+4*d^2*f^2*x^2*exp(I*(d*x+c))+2*I*d^2*f^2*x^2)/(
exp(I*(d*x+c))-I)/(exp(I*(d*x+c))+I)^3/d^3/a+1/d^2/a*e*f*ln(exp(I*(d*x+c))-
I)+5/3*f/d^2/a*ln(exp(I*(d*x+c))+I)*e-8/3*f/d^2/a*ln(exp(I*(d*x+c)))*e-1/d^
3/a*f^2*c*ln(exp(I*(d*x+c))-I)+8/3*f^2/d^3/a*c*ln(exp(I*(d*x+c)))-5/3*f^2/d
^3/a*c*ln(exp(I*(d*x+c))+I)-4/3*I/d^3/a*f^2*c^2-I/d^3/a*f^2*polylog(2,-I*ex
p(I*(d*x+c)))-5/3*I/d^3/a*f^2*polylog(2,I*exp(I*(d*x+c)))+1/d^2/a*f^2*ln(1+
I*exp(I*(d*x+c)))*x+1/d^3/a*f^2*ln(1+I*exp(I*(d*x+c)))*c-4/3*I/d/a*f^2*x^2+
5/3*f^2/d^2/a*ln(1-I*exp(I*(d*x+c)))*x+5/3*f^2/d^3/a*ln(1-I*exp(I*(d*x+c)))
*c-8/3*I/d^2/a*f^2*c*x
```

Maxima [B] time = 2.92584, size = 1800, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(8*d^2*e^2 + 4*f^2*cos(2*d*x + 2*c) + 4*I*f^2*sin(2*d*x + 2*c) + 4*f^2 - (
10*d*e*f*cos(4*d*x + 4*c) + 20*I*d*e*f*cos(3*d*x + 3*c) + 20*I*d*e*f*cos(d*
x + c) + 10*I*d*e*f*sin(4*d*x + 4*c) - 20*d*e*f*sin(3*d*x + 3*c) - 20*d*e*f
*sin(d*x + c) - 10*d*e*f)*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - (6*d*e*
f*cos(4*d*x + 4*c) + 12*I*d*e*f*cos(3*d*x + 3*c) + 12*I*d*e*f*cos(d*x + c)
```

```
+ 6*I*d*e*f*sin(4*d*x + 4*c) - 12*d*e*f*sin(3*d*x + 3*c) - 12*d*e*f*sin(d*x
 + c) - 6*d*e*f)*arctan2(sin(d*x + c) - 1, cos(d*x + c)) + (10*d*f^2*x*cos(
4*d*x + 4*c) + 20*I*d*f^2*x*cos(3*d*x + 3*c) + 20*I*d*f^2*x*cos(d*x + c) +
10*I*d*f^2*x*sin(4*d*x + 4*c) - 20*d*f^2*x*sin(3*d*x + 3*c) - 20*d*f^2*x*si
n(d*x + c) - 10*d*f^2*x)*arctan2(cos(d*x + c), sin(d*x + c) + 1) - (6*d*f^2
*x*cos(4*d*x + 4*c) + 12*I*d*f^2*x*cos(3*d*x + 3*c) + 12*I*d*f^2*x*cos(d*x
+ c) + 6*I*d*f^2*x*sin(4*d*x + 4*c) - 12*d*f^2*x*sin(3*d*x + 3*c) - 12*d*f^
2*x*sin(d*x + c) - 6*d*f^2*x)*arctan2(cos(d*x + c), -sin(d*x + c) + 1) + 8*
(d^2*f^2*x^2 + 2*d^2*e*f*x)*cos(4*d*x + 4*c) - (-16*I*d^2*f^2*x^2 - 4*d*e*f
 + 4*I*f^2 + (-32*I*d^2*e*f - 4*d*f^2)*x)*cos(3*d*x + 3*c) - (16*I*d^2*e^2
- 4*d*f^2*x - 4*d*e*f + 4*I*f^2)*cos(d*x + c) + (10*f^2*cos(4*d*x + 4*c) +
20*I*f^2*cos(3*d*x + 3*c) + 20*I*f^2*cos(d*x + c) + 10*I*f^2*sin(4*d*x + 4*
c) - 20*f^2*sin(3*d*x + 3*c) - 20*f^2*sin(d*x + c) - 10*f^2)*dilog(I*e^(I*d
*x + I*c)) + (6*f^2*cos(4*d*x + 4*c) + 12*I*f^2*cos(3*d*x + 3*c) + 12*I*f^2
*cos(d*x + c) + 6*I*f^2*sin(4*d*x + 4*c) - 12*f^2*sin(3*d*x + 3*c) - 12*f^2
*sin(d*x + c) - 6*f^2)*dilog(-I*e^(I*d*x + I*c)) - (5*I*d*f^2*x + 5*I*d*e*f
 + (-5*I*d*f^2*x - 5*I*d*e*f)*cos(4*d*x + 4*c) + 10*(d*f^2*x + d*e*f)*cos(3
*d*x + 3*c) + 10*(d*f^2*x + d*e*f)*cos(d*x + c) + 5*(d*f^2*x + d*e*f)*sin(4
*d*x + 4*c) + (10*I*d*f^2*x + 10*I*d*e*f)*sin(3*d*x + 3*c) + (10*I*d*f^2*x
+ 10*I*d*e*f)*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x
 + c) + 1) - (3*I*d*f^2*x + 3*I*d*e*f + (-3*I*d*f^2*x - 3*I*d*e*f)*cos(4*d*
x + 4*c) + 6*(d*f^2*x + d*e*f)*cos(3*d*x + 3*c) + 6*(d*f^2*x + d*e*f)*cos(d
*x + c) + 3*(d*f^2*x + d*e*f)*sin(4*d*x + 4*c) + (6*I*d*f^2*x + 6*I*d*e*f)*
sin(3*d*x + 3*c) + (6*I*d*f^2*x + 6*I*d*e*f)*sin(d*x + c))*log(cos(d*x + c)
^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) - (-8*I*d^2*f^2*x^2 - 16*I*d^2*e*
f*x)*sin(4*d*x + 4*c) - (16*d^2*f^2*x^2 - 4*I*d*e*f - 4*f^2 + 4*(8*d^2*e*f
- I*d*f^2)*x)*sin(3*d*x + 3*c) + (16*d^2*e^2 + 4*I*d*f^2*x + 4*I*d*e*f + 4*
f^2)*sin(d*x + c))/(-6*I*a*d^3*cos(4*d*x + 4*c) + 12*a*d^3*cos(3*d*x + 3*c)
 + 12*a*d^3*cos(d*x + c) + 6*a*d^3*sin(4*d*x + 4*c) + 12*I*a*d^3*sin(3*d*x
+ 3*c) + 12*I*a*d^3*sin(d*x + c) + 6*I*a*d^3)
```

Fricas [B] time = 2.35719, size = 2186, normalized size = 6.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*d^2*f^2*x^2 + 4*d^2*e*f*x + 2*d^2*e^2 - 2*(2*d^2*f^2*x^2 + 4*d^2*e*f
*x + 2*d^2*e^2 + f^2)*cos(d*x + c)^2 - 2*(d*f^2*x + d*e*f)*cos(d*x + c) + (
3*I*f^2*cos(d*x + c)*sin(d*x + c) + 3*I*f^2*cos(d*x + c))*dilog(I*cos(d*x +
```

```
 c) + sin(d*x + c)) + (-5*I*f^2*cos(d*x + c)*sin(d*x + c) - 5*I*f^2*cos(d*x
 + c))*dilog(I*cos(d*x + c) - sin(d*x + c)) + (-3*I*f^2*cos(d*x + c)*sin(d*
x + c) - 3*I*f^2*cos(d*x + c))*dilog(-I*cos(d*x + c) + sin(d*x + c)) + (5*I
*f^2*cos(d*x + c)*sin(d*x + c) + 5*I*f^2*cos(d*x + c))*dilog(-I*cos(d*x + c
) - sin(d*x + c)) + 5*((d*e*f - c*f^2)*cos(d*x + c)*sin(d*x + c) + (d*e*f -
 c*f^2)*cos(d*x + c))*log(cos(d*x + c) + I*sin(d*x + c) + I) + 3*((d*e*f -
c*f^2)*cos(d*x + c)*sin(d*x + c) + (d*e*f - c*f^2)*cos(d*x + c))*log(cos(d*
x + c) - I*sin(d*x + c) + I) + 5*((d*f^2*x + c*f^2)*cos(d*x + c)*sin(d*x +
c) + (d*f^2*x + c*f^2)*cos(d*x + c))*log(I*cos(d*x + c) + sin(d*x + c) + 1)
 + 3*((d*f^2*x + c*f^2)*cos(d*x + c)*sin(d*x + c) + (d*f^2*x + c*f^2)*cos(d
*x + c))*log(I*cos(d*x + c) - sin(d*x + c) + 1) + 5*((d*f^2*x + c*f^2)*cos(
d*x + c)*sin(d*x + c) + (d*f^2*x + c*f^2)*cos(d*x + c))*log(-I*cos(d*x + c)
 + sin(d*x + c) + 1) + 3*((d*f^2*x + c*f^2)*cos(d*x + c)*sin(d*x + c) + (d*
f^2*x + c*f^2)*cos(d*x + c))*log(-I*cos(d*x + c) - sin(d*x + c) + 1) + 5*((
d*e*f - c*f^2)*cos(d*x + c)*sin(d*x + c) + (d*e*f - c*f^2)*cos(d*x + c))*lo
g(-cos(d*x + c) + I*sin(d*x + c) + I) + 3*((d*e*f - c*f^2)*cos(d*x + c)*sin
(d*x + c) + (d*e*f - c*f^2)*cos(d*x + c))*log(-cos(d*x + c) - I*sin(d*x + c
) + I) + 4*(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*sin(d*x + c))/(a*d^3*cos(d
*x + c)*sin(d*x + c) + a*d^3*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^2 \sec^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{f^2x^2\sec^2(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{2efx\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e**2*sec(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f**2*x**2*
sec(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(2*e*f*x*sec(c + d*x)**2/(
sin(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sec(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sec(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

3.277 $\int \frac{(e+fx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=152

$$-\frac{f\sec^2(c+dx)}{6ad^2}+\frac{f\tanh^{-1}(\sin(c+dx))}{6ad^2}+\frac{2f\log(\cos(c+dx))}{3ad^2}+\frac{f\tan(c+dx)\sec(c+dx)}{6ad^2}+\frac{2(e+fx)\tan(c+dx)}{3ad}-$$

```
[Out] (f*ArcTanh[Sin[c + d*x]])/(6*a*d^2) + (2*f*Log[Cos[c + d*x]])/(3*a*d^2) - (
f*Sec[c + d*x]^2)/(6*a*d^2) - ((e + f*x)*Sec[c + d*x]^3)/(3*a*d) + (2*(e +
f*x)*Tan[c + d*x])/(3*a*d) + (f*Sec[c + d*x]*Tan[c + d*x])/(6*a*d^2) + ((e
+ f*x)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.145067, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4531, 4185, 4184, 3475, 4409, 3768, 3770}

$$-\frac{f\sec^2(c+dx)}{6ad^2}+\frac{f\tanh^{-1}(\sin(c+dx))}{6ad^2}+\frac{2f\log(\cos(c+dx))}{3ad^2}+\frac{f\tan(c+dx)\sec(c+dx)}{6ad^2}+\frac{2(e+fx)\tan(c+dx)}{3ad}-$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (f*ArcTanh[Sin[c + d*x]])/(6*a*d^2) + (2*f*Log[Cos[c + d*x]])/(3*a*d^2) - (
f*Sec[c + d*x]^2)/(6*a*d^2) - ((e + f*x)*Sec[c + d*x]^3)/(3*a*d) + (2*(e +
f*x)*Tan[c + d*x])/(3*a*d) + (f*Sec[c + d*x]*Tan[c + d*x])/(6*a*d^2) + ((e
+ f*x)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```


Rule 4531

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
```

```
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)\sec^4(c+dx)\,dx}{a} - \frac{\int (e+fx)\sec^3(c+dx)\tan(c+dx)\,dx}{a}$$
$$= -\frac{f\sec^2(c+dx)}{6ad^2} - \frac{(e+fx)\sec^3(c+dx)}{3ad} + \frac{(e+fx)\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{2\int (e+fx)}{}$$
$$= -\frac{f\sec^2(c+dx)}{6ad^2} - \frac{(e+fx)\sec^3(c+dx)}{3ad} + \frac{2(e+fx)\tan(c+dx)}{3ad} + \frac{f\sec(c+dx)\tan(c+}{6ad^2}$$
$$= \frac{f\tanh^{-1}(\sin(c+dx))}{6ad^2} + \frac{2f\log(\cos(c+dx))}{3ad^2} - \frac{f\sec^2(c+dx)}{6ad^2} - \frac{(e+fx)\sec^3(c+dx)}{3ad} + \frac{2}{}$$

Mathematica [A] time = 1.05244, size = 231, normalized size = 1.52

$$\frac{\cos(c+dx)\left(\sin(c+dx)\left(3f\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+5f\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)-cf+\right.\right.}{6ad^2(\sin(c+dx)+1)\left(\cos\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*d*(e + f*x)*(Cos[2*(c + d*x)] - 2*Sin[c + d*x]) + Cos[c + d*x]*(d*e - f
 - c*f + 3*f*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 5*f*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]] + (d*e - c*f + 3*f*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 5*f*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[c + d*x]))/(
6*a*d^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.172, size = 466, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/a*e/d/(tan(1/2*d*x+1/2*c)-1)-2/3/a*e/d/(tan(1/2*d*x+1/2*c)+1)^3+1/a*e/
d/(tan(1/2*d*x+1/2*c)+1)^2-3/2/a*e/d/(tan(1/2*d*x+1/2*c)+1)+1/3/a*f/(tan(1/
2*d*x+1/2*c)-1)/(tan(1/2*d*x+1/2*c)+1)^3*x/d-4/3/a*f/(tan(1/2*d*x+1/2*c)-1)
```

```
/(tan(1/2*d*x+1/2*c)+1)^3*x/d*tan(1/2*d*x+1/2*c)-2/a*f/(tan(1/2*d*x+1/2*c)-
1)/(tan(1/2*d*x+1/2*c)+1)^3*x/d*tan(1/2*d*x+1/2*c)^2-4/3/a*f/(tan(1/2*d*x+1
/2*c)-1)/(tan(1/2*d*x+1/2*c)+1)^3*x/d*tan(1/2*d*x+1/2*c)^3+1/3/a*f/(tan(1/2
*d*x+1/2*c)-1)/(tan(1/2*d*x+1/2*c)+1)^3*x/d*tan(1/2*d*x+1/2*c)^4-1/3/a*f/(t
an(1/2*d*x+1/2*c)-1)/(tan(1/2*d*x+1/2*c)+1)^3/d^2*tan(1/2*d*x+1/2*c)+1/3/a*
f/(tan(1/2*d*x+1/2*c)-1)/(tan(1/2*d*x+1/2*c)+1)^3/d^2*tan(1/2*d*x+1/2*c)^3+
1/2/a*f/d^2*ln(tan(1/2*d*x+1/2*c)-1)+5/6/a*f/d^2*ln(tan(1/2*d*x+1/2*c)+1)-2
/3/a*f/d^2*ln(1+tan(1/2*d*x+1/2*c)^2)
```

Maxima [B] time = 1.13416, size = 1505, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(8*c*f*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a*d + 2*a*d*sin(d*
x + c)/(cos(d*x + c) + 1) - 2*a*d*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*d
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (4*(8*(d*x + c)*cos(d*x + c) - sin(
3*d*x + 3*c) - sin(d*x + c))*cos(4*d*x + 4*c) + 16*(2*d*x + 4*(d*x + c)*sin
(d*x + c) + 2*c + cos(d*x + c))*cos(3*d*x + 3*c) + 8*cos(3*d*x + 3*c)^2 + 8
*cos(d*x + c)^2 + 5*(2*(2*sin(3*d*x + 3*c) + 2*sin(d*x + c) + 1)*cos(4*d*x
+ 4*c) - cos(4*d*x + 4*c)^2 - 4*cos(3*d*x + 3*c)^2 - 8*cos(3*d*x + 3*c)*cos
(d*x + c) - 4*cos(d*x + c)^2 - 4*(cos(3*d*x + 3*c) + cos(d*x + c))*sin(4*d*
x + 4*c) - sin(4*d*x + 4*c)^2 - 4*(2*sin(d*x + c) + 1)*sin(3*d*x + 3*c) - 4
*sin(3*d*x + 3*c)^2 - 4*sin(d*x + c)^2 - 4*sin(d*x + c) - 1)*log(cos(d*x +
c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3*(2*(2*sin(3*d*x + 3*c) + 2*
sin(d*x + c) + 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4*c)^2 - 4*cos(3*d*x + 3*c
)^2 - 8*cos(3*d*x + 3*c)*cos(d*x + c) - 4*cos(d*x + c)^2 - 4*(cos(3*d*x + 3
*c) + cos(d*x + c))*sin(4*d*x + 4*c) - sin(4*d*x + 4*c)^2 - 4*(2*sin(d*x +
c) + 1)*sin(3*d*x + 3*c) - 4*sin(3*d*x + 3*c)^2 - 4*sin(d*x + c)^2 - 4*sin(
d*x + c) - 1)*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 4
*(4*d*x + 8*(d*x + c)*sin(d*x + c) + 4*c + cos(3*d*x + 3*c) + cos(d*x + c))
*sin(4*d*x + 4*c) - 4*(16*(d*x + c)*cos(d*x + c) - 4*sin(d*x + c) - 1)*sin(
3*d*x + 3*c) + 8*sin(3*d*x + 3*c)^2 + 8*sin(d*x + c)^2 + 4*sin(d*x + c))*f/
(a*d*cos(4*d*x + 4*c)^2 + 4*a*d*cos(3*d*x + 3*c)^2 + 8*a*d*cos(3*d*x + 3*c)
*cos(d*x + c) + 4*a*d*cos(d*x + c)^2 + a*d*sin(4*d*x + 4*c)^2 + 4*a*d*sin(3
*d*x + 3*c)^2 + 4*a*d*sin(d*x + c)^2 + 4*a*d*sin(d*x + c) + a*d - 2*(2*a*d*
sin(3*d*x + 3*c) + 2*a*d*sin(d*x + c) + a*d)*cos(4*d*x + 4*c) + 4*(a*d*cos(
3*d*x + 3*c) + a*d*cos(d*x + c))*sin(4*d*x + 4*c) + 4*(2*a*d*sin(d*x + c) +
```

```
a*d)*sin(3*d*x + 3*c)) - 8*e*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a
 + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 - a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

Fricas [A] time = 1.73828, size = 417, normalized size = 2.74

$$\frac{4\,dfx - 8\left(dfx + de\right)\cos\left(dx + c\right)^{2} + 4\,de - 2\,f\cos\left(dx + c\right) + 5\left(f\cos\left(dx + c\right)\sin\left(dx + c\right) + f\cos\left(dx + c\right)\right)\log\left(\sin\left(dx +\right.\right.}{12\left(ad^{2}\cos\left(dx + c\right)\sin\left(dx + c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*d*f*x - 8*(d*f*x + d*e)*cos(d*x + c)^2 + 4*d*e - 2*f*cos(d*x + c) +
 5*(f*cos(d*x + c)*sin(d*x + c) + f*cos(d*x + c))*log(sin(d*x + c) + 1) + 3
*(f*cos(d*x + c)*sin(d*x + c) + f*cos(d*x + c))*log(-sin(d*x + c) + 1) + 8*
(d*f*x + d*e)*sin(d*x + c))/(a*d^2*cos(d*x + c)*sin(d*x + c) + a*d^2*cos(d*
x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e\sec^{2}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx + \int \frac{fx\sec^{2}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e*sec(c + d*x)**2/(sin(c + d*x) + 1), x) + Integral(f*x*sec(c + d
*x)**2/(sin(c + d*x) + 1), x))/a
```

Giac [B] time = 3.82079, size = 8986, normalized size = 59.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/12*(4*d*f*x*tan(1/2*d*x)^4*tan(1/2*c)^4 + 16*d*f*x*tan(1/2*d*x)^4*tan(1/
2*c)^3 + 16*d*f*x*tan(1/2*d*x)^3*tan(1/2*c)^4 + 4*d*e*tan(1/2*d*x)^4*tan(1/
2*c)^4 - 3*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*t
an(1/2*d*x)^4*tan(1/2*c)^4 - 5*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
 tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
2*tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 24*d*f*x*tan(1/2*d*x)^4*ta
n(1/2*c)^2 - 64*d*f*x*tan(1/2*d*x)^3*tan(1/2*c)^3 + 16*d*e*tan(1/2*d*x)^4*t
an(1/2*c)^3 + 6*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4
 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) +
1))*tan(1/2*d*x)^4*tan(1/2*c)^3 + 10*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*
x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*
c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 -
2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^3 - 24*d*f*x*tan(1/2*d*x
)^2*tan(1/2*c)^4 + 16*d*e*tan(1/2*d*x)^3*tan(1/2*c)^4 + 6*f*log(2*(tan(1/2*
c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*ta
n(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^3*tan(1/2*c)^4 +
 10*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*
d*x)^3*tan(1/2*c)^4 + 2*f*tan(1/2*d*x)^4*tan(1/2*c)^4 + 16*d*f*x*tan(1/2*d*
x)^4*tan(1/2*c) - 24*d*e*tan(1/2*d*x)^4*tan(1/2*c)^2 - 64*d*e*tan(1/2*d*x)^
3*tan(1/2*c)^3 + 12*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*
x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c
) + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 20*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/
2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(
1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 16*d*f*x*tan(1/2
*d*x)*tan(1/2*c)^4 - 24*d*e*tan(1/2*d*x)^2*tan(1/2*c)^4 + 4*d*f*x*tan(1/2*d
```

*x)^4 + 64*d*f*x*tan(1/2*d*x)^3*tan(1/2*c) + 16*d*e*tan(1/2*d*x)^4*tan(1/2*
c) - 6*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1
/2*d*x)^4*tan(1/2*c) - 10*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(
1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d
*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan
(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c) + 144*d*f*x*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 36*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*ta
n(1/2*d*x)^3*tan(1/2*c)^2 - 60*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
 tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
2*tan(1/2*c) + 1))*tan(1/2*d*x)^3*tan(1/2*c)^2 + 64*d*f*x*tan(1/2*d*x)*tan(
1/2*c)^3 - 36*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 +
 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)
)*tan(1/2*d*x)^2*tan(1/2*c)^3 - 60*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)
^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)
^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^3 - 8*f*tan(1/2*d*x)^3*tan
(1/2*c)^3 + 4*d*f*x*tan(1/2*c)^4 + 16*d*e*tan(1/2*d*x)*tan(1/2*c)^4 - 6*f*l
og(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan
(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)*ta
n(1/2*c)^4 - 10*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4
 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) +
1))*tan(1/2*d*x)*tan(1/2*c)^4 - 16*d*f*x*tan(1/2*d*x)^3 + 4*d*e*tan(1/2*d*x
)^4 + 3*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(
1/2*d*x)^4 + 5*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*
tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4
+ 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2

*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1
))*tan(1/2*d*x)^4 + 64*d*e*tan(1/2*d*x)^3*tan(1/2*c) + 12*f*log(2*(tan(1/2*
c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*ta
n(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^3*tan(1/2*c) + 2
0*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^
4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*
x)^3*tan(1/2*c) + 144*d*e*tan(1/2*d*x)^2*tan(1/2*c)^2 - 16*d*f*x*tan(1/2*c)
^3 + 64*d*e*tan(1/2*d*x)*tan(1/2*c)^3 + 12*f*log(2*(tan(1/2*c)^2 + 1)/(tan(
1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*ta
n(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c)^3 + 20*f*log(2*(tan(
1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c)^3
 + 4*d*e*tan(1/2*c)^4 + 3*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(
1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d
*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan
(1/2*c) + 1))*tan(1/2*c)^4 + 5*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
 tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
2*tan(1/2*c) + 1))*tan(1/2*c)^4 - 24*d*f*x*tan(1/2*d*x)^2 - 16*d*e*tan(1/2*
d*x)^3 + 6*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*t
an(1/2*d*x)^3 + 10*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2
- 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x
)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
 + 1))*tan(1/2*d*x)^3 - 2*f*tan(1/2*d*x)^4 - 64*d*f*x*tan(1/2*d*x)*tan(1/2*
c) + 36*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(
1/2*d*x)^2*tan(1/2*c) + 60*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan
(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*

d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c) - 8*f*tan(1/2*d*x)^3*tan(1/2*c) -
24*d*f*x*tan(1/2*c)^2 + 36*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan
(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*
d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*ta
n(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c)^2 + 60*f*log(2*(tan(1/2*c)^2 + 1)/(t
an(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3
*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c)^2 - 16*d*e*tan(1/
2*c)^3 + 6*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*t
an(1/2*c)^3 + 10*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 -
2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^
4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) +
1))*tan(1/2*c)^3 - 8*f*tan(1/2*d*x)*tan(1/2*c)^3 - 2*f*tan(1/2*c)^4 - 16*d
*f*x*tan(1/2*d*x) - 24*d*e*tan(1/2*d*x)^2 - 16*d*f*x*tan(1/2*c) - 64*d*e*ta
n(1/2*d*x)*tan(1/2*c) + 12*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan
(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*
d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*ta
n(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c) + 20*f*log(2*(tan(1/2*c)^2 + 1)/(tan
(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*t
an(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*ta
n(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c) - 24*d*e*tan(1/2*c)
^2 + 4*d*f*x - 16*d*e*tan(1/2*d*x) - 6*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*
d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/
2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x) - 10*f*log(2*(tan(1/2*c)^2 + 1)/(ta
n(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*
tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*t
an(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x) - 16*d*e*tan(1/2*c) - 6*f*log
(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1
/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*c) - 10*f
log(2(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*t
an(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)

```
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*c) -
8*f*tan(1/2*d*x)*tan(1/2*c) + 4*d*e - 3*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2
*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1
/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3
 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 2*tan(1/2*c) + 1)) - 5*f*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2
+ tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan
(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
 2*tan(1/2*c) + 1)) + 2*f)/(a*d^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 2*a*d^2*tan
(1/2*d*x)^4*tan(1/2*c)^3 - 2*a*d^2*tan(1/2*d*x)^3*tan(1/2*c)^4 - 4*a*d^2*ta
n(1/2*d*x)^3*tan(1/2*c)^3 + 2*a*d^2*tan(1/2*d*x)^4*tan(1/2*c) + 12*a*d^2*ta
n(1/2*d*x)^3*tan(1/2*c)^2 + 12*a*d^2*tan(1/2*d*x)^2*tan(1/2*c)^3 + 2*a*d^2*
tan(1/2*d*x)*tan(1/2*c)^4 - a*d^2*tan(1/2*d*x)^4 - 4*a*d^2*tan(1/2*d*x)^3*t
an(1/2*c) - 4*a*d^2*tan(1/2*d*x)*tan(1/2*c)^3 - a*d^2*tan(1/2*c)^4 - 2*a*d^
2*tan(1/2*d*x)^3 - 12*a*d^2*tan(1/2*d*x)^2*tan(1/2*c) - 12*a*d^2*tan(1/2*d*
x)*tan(1/2*c)^2 - 2*a*d^2*tan(1/2*c)^3 - 4*a*d^2*tan(1/2*d*x)*tan(1/2*c) +
2*a*d^2*tan(1/2*d*x) + 2*a*d^2*tan(1/2*c) + a*d^2)
```

3.278 $\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=42

$$\frac{2\tan(c+dx)}{3ad}-\frac{\sec(c+dx)}{3d(a\sin(c+dx)+a)}$$

```
[Out] -Sec[c + d*x]/(3*d*(a + a*Sin[c + d*x])) + (2*Tan[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.0514837, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2672, 3767, 8}

$$\frac{2\tan(c+dx)}{3ad}-\frac{\sec(c+dx)}{3d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Sec[c + d*x]/(3*d*(a + a*Sin[c + d*x])) + (2*Tan[c + d*x])/(3*a*d)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))}+\frac{2\int \sec^2(c+dx)\,dx}{3a}\\ &= -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))}-\frac{2\,\text{Subst}(\int 1\,dx,x,-\tan(c+dx))}{3ad}\\ &= -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))}+\frac{2\tan(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 0.0520047, size = 45, normalized size = 1.07

$$\frac{2\tan(c+dx)-\cos(2(c+dx))\sec(c+dx)}{3ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-(Cos[2*(c + d*x)]*Sec[c + d*x]) + 2*Tan[c + d*x])/(3*a*d*(1 + Sin[c + d*x
]))
```

Maple [A] time = 0.05, size = 70, normalized size = 1.7

$$2\,\frac{1}{da}\left(-1/4\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{-1}-1/3\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-3}+1/2\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-2}-3/4\,\left(\tan\left(1/2\,dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 2/d/a*(-1/4/(tan(1/2*d*x+1/2*c)-1)-1/3/(tan(1/2*d*x+1/2*c)+1)^3+1/2/(tan(1/
2*d*x+1/2*c)+1)^2-3/4/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.00448, size = 174, normalized size = 4.14

$$\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1\right)}{3\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/((a + 2*a*sin(d*x + c)/(cos(
d*x + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4)*d)
```

Fricas [A] time = 1.60337, size = 131, normalized size = 3.12

$$-\frac{2\cos(dx+c)^2-2\sin(dx+c)-1}{3\left(ad\cos(dx+c)\sin(dx+c)+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(d*x + c)^2 - 2*sin(d*x + c) - 1)/(a*d*cos(d*x + c)*sin(d*x + c)
 + a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.17055, size = 90, normalized size = 2.14

$$-\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) + (9*tan(1/2*d*x + 1/2*c)^2 + 12*tan
(1/2*d*x + 1/2*c) + 7)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

3.279 $\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sec^2(c+dx)}{(e+fx)(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0662987, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 18.8599, size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Sec[c + d*x]^2/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 3.027, size = 0, normalized size = 0.

$$\int \frac{(\sec(dx+c))^2}{(fx+e)(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(4*f^2*cos(2*d*x + 2*c)*cos(d*x + c) - 2*(d*f^2*x + d*e*f)*cos(3*d*x +
 3*c)^2 + 2*f^2*cos(d*x + c) - 2*(d*f^2*x + d*e*f)*cos(d*x + c)^2 - 2*(d*f^
2*x + d*e*f)*sin(3*d*x + 3*c)^2 - 2*(d*f^2*x + d*e*f)*sin(d*x + c)^2 + (2*f
^2*cos(3*d*x + 3*c) - 2*f^2*sin(2*d*x + 2*c) + 2*(4*d^2*f^2*x^2 + 8*d^2*e*f
*x + 4*d^2*e^2 + f^2)*cos(d*x + c) + (d*f^2*x + d*e*f)*sin(3*d*x + 3*c) + (
d*f^2*x + d*e*f)*sin(d*x + c))*cos(4*d*x + 4*c) + 2*(4*d^2*f^2*x^2 + 8*d^2*
e*f*x + 4*d^2*e^2 + 2*f^2*cos(2*d*x + 2*c) + f^2 - 2*(d*f^2*x + d*e*f)*cos(
d*x + c) + 8*(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*sin(d*x + c))*cos(3*d*x
+ 3*c) + 3*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3
 + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cos(4*
d*x + 4*c)^2 + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d
^3*e^3)*cos(3*d*x + 3*c)^2 + 8*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3
*e^2*f*x + a*d^3*e^3)*cos(3*d*x + 3*c)*cos(d*x + c) + 4*(a*d^3*f^3*x^3 + 3*
a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cos(d*x + c)^2 + (a*d^3*f^3*
x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*sin(4*d*x + 4*c)^2 +
 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*sin(3*
d*x + 3*c)^2 + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d
^3*e^3)*sin(d*x + c)^2 - 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2
*f*x + a*d^3*e^3 + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x +
 a*d^3*e^3)*sin(3*d*x + 3*c) + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d
^3*e^2*f*x + a*d^3*e^3)*sin(d*x + c))*cos(4*d*x + 4*c) + 4*((a*d^3*f^3*x^3
```

```
+ 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cos(3*d*x + 3*c) + (a*d^
3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cos(d*x + c))*
sin(4*d*x + 4*c) + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x +
 a*d^3*e^3 + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3
*e^3)*sin(d*x + c))*sin(3*d*x + 3*c) + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2
 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*sin(d*x + c))*integrate(1/6*(5*d^2*f^3*x^2
+ 10*d^2*e*f^2*x + 5*d^2*e^2*f + 12*f^3)*cos(d*x + c)/(a*d^3*f^4*x^4 + 4*a*
d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4 + (a*d^3*
f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3
*e^4)*cos(d*x + c)^2 + (a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2
*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*sin(d*x + c)^2 + 2*(a*d^3*f^4*x^4 + 4*a
*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*sin(d*x
 + c)), x) - 3*(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d
^3*e^3*f + (a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e
^3*f)*cos(4*d*x + 4*c)^2 + 4*(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e
^2*f^2*x + a*d^3*e^3*f)*cos(3*d*x + 3*c)^2 + 8*(a*d^3*f^4*x^3 + 3*a*d^3*e*f
^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f)*cos(3*d*x + 3*c)*cos(d*x + c) + 4
*(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f)*cos(
d*x + c)^2 + (a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3
*e^3*f)*sin(4*d*x + 4*c)^2 + 4*(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3
*e^2*f^2*x + a*d^3*e^3*f)*sin(3*d*x + 3*c)^2 + 4*(a*d^3*f^4*x^3 + 3*a*d^3*e
*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f)*sin(d*x + c)^2 - 2*(a*d^3*f^4*x
^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f + 2*(a*d^3*f^4*x^3
 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f)*sin(3*d*x + 3*c) +
2*(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f)*sin
(d*x + c))*cos(4*d*x + 4*c) + 4*((a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d
^3*e^2*f^2*x + a*d^3*e^3*f)*cos(3*d*x + 3*c) + (a*d^3*f^4*x^3 + 3*a*d^3*e*f
^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f)*cos(d*x + c))*sin(4*d*x + 4*c) +
4*(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f + 2*
(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*e^2*f^2*x + a*d^3*e^3*f)*sin(d
*x + c))*sin(3*d*x + 3*c) + 4*(a*d^3*f^4*x^3 + 3*a*d^3*e*f^3*x^2 + 3*a*d^3*
e^2*f^2*x + a*d^3*e^3*f)*sin(d*x + c))*integrate(1/2*cos(d*x + c)/(a*d*f^2*
x^2 + 2*a*d*e*f*x + a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*cos(d*x
 + c)^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*sin(d*x + c)^2 - 2*(a*d*f^2
*x^2 + 2*a*d*e*f*x + a*d*e^2)*sin(d*x + c)), x) + (4*d^2*f^2*x^2 + 8*d^2*e*
f*x + 4*d^2*e^2 + 2*f^2*cos(2*d*x + 2*c) + 2*f^2*sin(3*d*x + 3*c) + 2*f^2 -
 (d*f^2*x + d*e*f)*cos(3*d*x + 3*c) - (d*f^2*x + d*e*f)*cos(d*x + c) + 2*(4
*d^2*f^2*x^2 + 8*d^2*e*f*x + 4*d^2*e^2 + f^2)*sin(d*x + c))*sin(4*d*x + 4*c
) - (d*f^2*x + d*e*f - 4*f^2*sin(2*d*x + 2*c) + 16*(d^2*f^2*x^2 + 2*d^2*e*f
*x + d^2*e^2)*cos(d*x + c) + 4*(d*f^2*x + d*e*f)*sin(d*x + c))*sin(3*d*x +
3*c) + 2*(2*f^2*sin(d*x + c) + f^2)*sin(2*d*x + 2*c) - (d*f^2*x + d*e*f)*si
n(d*x + c))/(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^
3 + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cos(4
*d*x + 4*c)^2 + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*
d^3*e^3)*cos(3*d*x + 3*c)^2 + 8*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^
```

```
3*e^2*f*x + a*d^3*e^3)*cos(3*d*x + 3*c)*cos(d*x + c) + 4*(a*d^3*f^3*x^3 + 3
*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cos(d*x + c)^2 + (a*d^3*f^3
*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*sin(4*d*x + 4*c)^2
+ 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*sin(3
*d*x + 3*c)^2 + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*
d^3*e^3)*sin(d*x + c)^2 - 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^
2*f*x + a*d^3*e^3 + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x
+ a*d^3*e^3)*sin(3*d*x + 3*c) + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*
d^3*e^2*f*x + a*d^3*e^3)*sin(d*x + c))*cos(4*d*x + 4*c) + 4*((a*d^3*f^3*x^3
 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cos(3*d*x + 3*c) + (a*d
^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cos(d*x + c))
*sin(4*d*x + 4*c) + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x
+ a*d^3*e^3 + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^
3*e^3)*sin(d*x + c))*sin(3*d*x + 3*c) + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^
2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*sin(d*x + c))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{2}}{afx+ae+\left(afx+ae\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^2/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sec^{2}\left(c+dx\right)}{e\sin\left(c+dx\right)+e+fx\sin\left(c+dx\right)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/
a
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.280 $\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sec^2(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0670309, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx=\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 24.8459, size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Sec[c + d*x]^2/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 5.532, size = 0, normalized size = 0.

$$\int \frac{\left(\sec \left(dx+c\right)\right)^{2}}{\left(fx+e\right)^{2}\left(a+a\sin \left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec \left(dx+c\right)^{2}}{af^{2}x^{2}+2\,aefx+ae^{2}+\left(af^{2}x^{2}+2\,aefx+ae^{2}\right)\sin \left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^2/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e
*f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^2(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) +
 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

3.281 $\int \frac{(e+fx)^3 \sec^3(c+dx)}{a+a\sin(c+dx)} dx$

Optimal. Leaf size=698

$$-\frac{9f^2(e+fx)\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{4ad^3}+\frac{9f^2(e+fx)\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{4ad^3}+\frac{9if(e+fx)^2\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{8ad^2}-\frac{9if(e-$$

```
[Out] ((-I/2)*f*(e + f*x)^2)/(a*d^2) - ((5*I)*f^2*(e + f*x)*ArcTan[E^(I*(c + d*x)
)])/(a*d^3) - (((3*I)/4)*(e + f*x)^3*ArcTan[E^(I*(c + d*x))])/(a*d) + (f^2*
(e + f*x)*Log[1 + E^((2*I)*(c + d*x))])/(a*d^3) + (((5*I)/2)*f^3*PolyLog[2,
 (-I)*E^(I*(c + d*x))])/(a*d^4) + (((9*I)/8)*f*(e + f*x)^2*PolyLog[2, (-I)*
E^(I*(c + d*x))])/(a*d^2) - (((5*I)/2)*f^3*PolyLog[2, I*E^(I*(c + d*x))])/(
a*d^4) - (((9*I)/8)*f*(e + f*x)^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2) -
((I/2)*f^3*PolyLog[2, -E^((2*I)*(c + d*x))])/(a*d^4) - (9*f^2*(e + f*x)*Pol
yLog[3, (-I)*E^(I*(c + d*x))])/(4*a*d^3) + (9*f^2*(e + f*x)*PolyLog[3, I*E^
(I*(c + d*x))])/(4*a*d^3) - (((9*I)/4)*f^3*PolyLog[4, (-I)*E^(I*(c + d*x))]
)/(a*d^4) + (((9*I)/4)*f^3*PolyLog[4, I*E^(I*(c + d*x))])/(a*d^4) - (f^3*Se
c[c + d*x])/(4*a*d^4) - (9*f*(e + f*x)^2*Sec[c + d*x])/(8*a*d^2) - (f^2*(e
+ f*x)*Sec[c + d*x]^2)/(4*a*d^3) - (f*(e + f*x)^2*Sec[c + d*x]^3)/(4*a*d^2)
 - ((e + f*x)^3*Sec[c + d*x]^4)/(4*a*d) + (f^3*Tan[c + d*x])/(4*a*d^4) + (f
*(e + f*x)^2*Tan[c + d*x])/(2*a*d^2) + (f^2*(e + f*x)*Sec[c + d*x]*Tan[c +
d*x])/(4*a*d^3) + (3*(e + f*x)^3*Sec[c + d*x]*Tan[c + d*x])/(8*a*d) + (f*(e
 + f*x)^2*Sec[c + d*x]^2*Tan[c + d*x])/(4*a*d^2) + ((e + f*x)^3*Sec[c + d*x
]^3*Tan[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.73537, antiderivative size = 698, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 16, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4531, 4186, 4185, 4181, 2279, 2391, 2531, 6609, 2282, 6589, 4409, 3767, 8, 4184, 3719, 2190}

$$-\frac{9f^2(e+fx)\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{4ad^3}+\frac{9f^2(e+fx)\text{PolyLog}\left(3,ie^{i(c+dx)}\right)}{4ad^3}+\frac{9if(e+fx)^2\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{8ad^2}-\frac{9if(e-$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] ((-I/2)*f*(e + f*x)^2)/(a*d^2) - ((5*I)*f^2*(e + f*x)*ArcTan[E^(I*(c + d*x)
)])/(a*d^3) - (((3*I)/4)*(e + f*x)^3*ArcTan[E^(I*(c + d*x))])/(a*d) + (f^2*
(e + f*x)*Log[1 + E^((2*I)*(c + d*x))])/(a*d^3) + (((5*I)/2)*f^3*PolyLog[2,
 (-I)*E^(I*(c + d*x))])/(a*d^4) + (((9*I)/8)*f*(e + f*x)^2*PolyLog[2, (-I)*
```

```
E^(I*(c + d*x))])/(a*d^2) - (((5*I)/2)*f^3*PolyLog[2, I*E^(I*(c + d*x))])/(
a*d^4) - (((9*I)/8)*f*(e + f*x)^2*PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2) -
((I/2)*f^3*PolyLog[2, -E^((2*I)*(c + d*x))])/(a*d^4) - (9*f^2*(e + f*x)*Pol
yLog[3, (-I)*E^(I*(c + d*x))])/(4*a*d^3) + (9*f^2*(e + f*x)*PolyLog[3, I*E^
(I*(c + d*x))])/(4*a*d^3) - (((9*I)/4)*f^3*PolyLog[4, (-I)*E^(I*(c + d*x))]
)/(a*d^4) + (((9*I)/4)*f^3*PolyLog[4, I*E^(I*(c + d*x))])/(a*d^4) - (f^3*Se
c[c + d*x])/(4*a*d^4) - (9*f*(e + f*x)^2*Sec[c + d*x])/(8*a*d^2) - (f^2*(e
+ f*x)*Sec[c + d*x]^2)/(4*a*d^3) - (f*(e + f*x)^2*Sec[c + d*x]^3)/(4*a*d^2)
 - ((e + f*x)^3*Sec[c + d*x]^4)/(4*a*d) + (f^3*Tan[c + d*x])/(4*a*d^4) + (f
*(e + f*x)^2*Tan[c + d*x])/(2*a*d^2) + (f^2*(e + f*x)*Sec[c + d*x]*Tan[c +
d*x])/(4*a*d^3) + (3*(e + f*x)^3*Sec[c + d*x]*Tan[c + d*x])/(8*a*d) + (f*(e
 + f*x)^2*Sec[c + d*x]^2*Tan[c + d*x])/(4*a*d^2) + ((e + f*x)^3*Sec[c + d*x
]^3*Tan[c + d*x])/(4*a*d)
```

Rule 4531

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
```

```
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3 \sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^3 \sec^5(c+dx)\,dx}{a} - \frac{\int (e+fx)^3 \sec^4(c+dx)\tan(c+dx)\,dx}{a}$$

$$= -\frac{f(e+fx)^2\sec^3(c+dx)}{4ad^2} - \frac{(e+fx)^3\sec^4(c+dx)}{4ad} + \frac{(e+fx)^3\sec^3(c+dx)\tan(c+dx)}{4ad} +$$

$$= -\frac{f^3\sec(c+dx)}{4ad^4} - \frac{9f(e+fx)^2\sec(c+dx)}{8ad^2} - \frac{f^2(e+fx)\sec^2(c+dx)}{4ad^3} - \frac{f(e+fx)^2\sec^3(c}{4ad^2}$$

$$= -\frac{5if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} - \frac{f^3\sec(c+dx)}{4ad^4} - \frac{9f(e+fx}{8}$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} + \frac{9if(e+fx)^2}{8a}$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} + \frac{f^2(e+fx)\log}{a}$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} + \frac{f^2(e+fx)\log}{a}$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5if^2(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{ad^3} - \frac{3i(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} + \frac{f^2(e+fx)\log}{a}$$

Mathematica [B] time = 9.98643, size = 1901, normalized size = 2.72

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*(4*d^2*e^3*x + 16*e*f^2*x + 6*d^2*e^2*f*x^2 + 8*f^3*x^2 + 4*d^2*e*f^2*x
^3 + d^2*f^3*x^4 + (4*e*(d^2*e^2 + 4*f^2)*((-I)*d*x + Log[-Cos[c + d*x] - I
*(-1 + Sin[c + d*x])])*(Cos[c] + I*(-1 + Sin[c])))/d + (4*f*(3*d^2*e^2 + 4*
f^2)*x*Log[1 - I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] + I*(-1 + Sin[c])))/d
 + 12*d*e*f^2*x^2*Log[1 - I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] + I*(-1 +
Sin[c])) + 4*d*f^3*x^3*Log[1 - I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] + I*(
-1 + Sin[c])) + (24*e*f^2*(I*d*x*PolyLog[2, I*Cos[c + d*x] + Sin[c + d*x]]
+ PolyLog[3, I*Cos[c + d*x] + Sin[c + d*x]])*(Cos[c] + I*(-1 + Sin[c])))/d
+ (12*f^3*(I*d^2*x^2*PolyLog[2, I*Cos[c + d*x] + Sin[c + d*x]] + 2*d*x*Poly
Log[3, I*Cos[c + d*x] + Sin[c + d*x]] - (2*I)*PolyLog[4, I*Cos[c + d*x] + S
in[c + d*x]])*(Cos[c] + I*(-1 + Sin[c])))/d^2 + (4*f*(3*d^2*e^2 + 4*f^2)*Po
lyLog[2, I*Cos[c + d*x] + Sin[c + d*x]]*(1 + I*Cos[c] - Sin[c]))/d^2))/(32*
a*d^2*(Cos[c] + I*(-1 + Sin[c]))) - ((28*f^2 + 3*d^2*(e + f*x)^2)^2/f + 12*
```

```
f*(9*d^2*e^2 + 28*f^2)*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]]*(1 - I*
Cos[c] + Sin[c]) + 216*d*e*f^2*(d*x*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c +
d*x]] - I*PolyLog[3, (-I)*Cos[c + d*x] - Sin[c + d*x]])*(1 - I*Cos[c] + Sin
[c]) + 108*f^3*(d^2*x^2*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]] - (2*I
)*d*x*PolyLog[3, (-I)*Cos[c + d*x] - Sin[c + d*x]] - 2*PolyLog[4, (-I)*Cos[
c + d*x] - Sin[c + d*x]])*(1 - I*Cos[c] + Sin[c]) - 12*d*f*(9*d^2*e^2 + 28*
f^2)*x*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] + I*(1 + Sin[c])) - 1
08*d^3*e*f^2*x^2*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] + I*(1 + Si
n[c])) - 36*d^3*f^3*x^3*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(Cos[c] + I*
(1 + Sin[c])) + (12*I)*d*e*(3*d^2*e^2 + 28*f^2)*(d*x + I*Log[Cos[c + d*x] +
 I*(1 + Sin[c + d*x])])*(Cos[c] + I*(1 + Sin[c]))))/(96*a*d^4*(Cos[c] + I*(1
 + Sin[c]))) + ((3*e^3*x*Cos[c])/(4*a) + (((3*I)/4)*e^3*x*Sin[c])/a)/(1 + C
os[2*c] + I*Sin[2*c]) + ((9*e^2*f*x^2*Cos[c])/(8*a) + (((9*I)/8)*e^2*f*x^2*
Sin[c])/a)/(1 + Cos[2*c] + I*Sin[2*c]) + ((3*e*f^2*x^3*Cos[c])/(4*a) + (((3
*I)/4)*e*f^2*x^3*Sin[c])/a)/(1 + Cos[2*c] + I*Sin[2*c]) + ((3*f^3*x^4*Cos[c
])/(16*a) + (((3*I)/16)*f^3*x^4*Sin[c])/a)/(1 + Cos[2*c] + I*Sin[2*c]) + (e
^3 + 3*e^2*f*x + 3*e*f^2*x^2 + f^3*x^3)/(8*a*d*(Cos[c/2 + (d*x)/2] - Sin[c/
2 + (d*x)/2])^2) - (3*(e^2*f*Sin[(d*x)/2] + 2*e*f^2*x*Sin[(d*x)/2] + f^3*x^
2*Sin[(d*x)/2]))/(4*a*d^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c
/2 + (d*x)/2])) + (-e^3 - 3*e^2*f*x - 3*e*f^2*x^2 - f^3*x^3)/(8*a*d*(Cos[c/
2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) + (e^2*f*Sin[(d*x)/2] + 2*e*f^2*x*Sin
[(d*x)/2] + f^3*x^2*Sin[(d*x)/2])/(4*a*d^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 +
 (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (-2*d^2*e^3*Cos[c/2] - d*e^2*f*Cos[c/2
] - 2*e*f^2*Cos[c/2] - 6*d^2*e^2*f*x*Cos[c/2] - 2*d*e*f^2*x*Cos[c/2] - 2*f^
3*x*Cos[c/2] - 6*d^2*e*f^2*x^2*Cos[c/2] - d*f^3*x^2*Cos[c/2] - 2*d^2*f^3*x^
3*Cos[c/2] - 2*d^2*e^3*Sin[c/2] + d*e^2*f*Sin[c/2] - 2*e*f^2*Sin[c/2] - 6*d
^2*e^2*f*x*Sin[c/2] + 2*d*e*f^2*x*Sin[c/2] - 2*f^3*x*Sin[c/2] - 6*d^2*e*f^2
*x^2*Sin[c/2] + d*f^3*x^2*Sin[c/2] - 2*d^2*f^3*x^3*Sin[c/2])/(8*a*d^3*(Cos[
c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (7*d^2*e^2*
f*Sin[(d*x)/2] + 2*f^3*Sin[(d*x)/2] + 14*d^2*e*f^2*x*Sin[(d*x)/2] + 7*d^2*f
^3*x^2*Sin[(d*x)/2])/(4*a*d^4*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + S
in[c/2 + (d*x)/2]))
```

Maple [B] time = 0.325, size = 2161, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sec(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -9/8/d/a*ln(1+I*exp(I*(d*x+c)))*e^2*f*x-9/8/d^2/a*ln(1+I*exp(I*(d*x+c)))*c*
e^2*f-3/8/d^4/a*f^3*ln(1+I*exp(I*(d*x+c)))*c^3-9/8/d^3/a*e*f^2*c^2*ln(exp(I
```

*(d*x+c))-I)+9/8/d^2/a*e^2*f*c*ln(exp(I*(d*x+c))-I)-3/8/d/a*f^3*ln(1+I*exp(
I*(d*x+c)))*x^3-9/4*I*f^3*polylog(4,-I*exp(I*(d*x+c)))/a/d^4-3/8/d/a*e^3*ln
(exp(I*(d*x+c))-I)+7/2/d^3/a*e*f^2*ln(exp(I*(d*x+c))+I)-9/4/d^3/a*e*f^2*pol
ylog(3,-I*exp(I*(d*x+c)))-2/d^3/a*e*f^2*ln(exp(I*(d*x+c)))+3/8/d^4/a*f^3*c^
3*ln(exp(I*(d*x+c))-I)+2/d^4/a*f^3*c*ln(exp(I*(d*x+c)))-7/2/d^4/a*f^3*c*ln(
exp(I*(d*x+c))+I)-9/4/d^3/a*f^3*polylog(3,-I*exp(I*(d*x+c)))*x+7/2/d^3/a*f^
3*ln(1-I*exp(I*(d*x+c)))*x+7/2/d^4/a*f^3*ln(1-I*exp(I*(d*x+c)))*c+9/8/d^3/a
*ln(1+I*exp(I*(d*x+c)))*c^2*e*f^2-9/8/d/a*ln(1+I*exp(I*(d*x+c)))*e*f^2*x^2+
3/8/a/d*ln(exp(I*(d*x+c))+I)*e^3+3/2/d^4/a*f^3*c*ln(exp(I*(d*x+c))-I)-3/2/d
^3/a*f^3*ln(1+I*exp(I*(d*x+c)))*x-3/2/d^3/a*e*f^2*ln(exp(I*(d*x+c))-I)-3/2/
d^4/a*f^3*c*ln(1+I*exp(I*(d*x+c)))-I/d^4/a*f^3*c^2-I/d^2/a*f^3*x^2+3/2*I/d^
4/a*f^3*polylog(2,-I*exp(I*(d*x+c)))-7/2*I/d^4/a*f^3*polylog(2,I*exp(I*(d*x
+c)))-3/8/a/d^4*f^3*c^3*ln(exp(I*(d*x+c))+I)+9/4/a/d^3*e*f^2*polylog(3,I*ex
p(I*(d*x+c)))+9/4/a/d^3*f^3*polylog(3,I*exp(I*(d*x+c)))*x+3/8/a/d^4*f^3*c^3
*ln(1-I*exp(I*(d*x+c)))+3/8/a/d*f^3*ln(1-I*exp(I*(d*x+c)))*x^3-1/4*I*(-9*I*
d^2*e^2*f*exp(5*I*(d*x+c))-6*I*d^3*f^3*x^3*exp(2*I*(d*x+c))-8*I*d^2*f^3*x^2
*exp(3*I*(d*x+c))-8*I*d^2*e^2*f*exp(3*I*(d*x+c))+6*d^3*e^2*f*x*exp(3*I*(d*x
+c))+44*d^2*e*f^2*x*exp(2*I*(d*x+c))+36*d^2*e*f^2*x*exp(4*I*(d*x+c))+9*d^3*
e*f^2*x^2*exp(5*I*(d*x+c))+9*d^3*e^2*f*x*exp(5*I*(d*x+c))+2*f^3+2*d*f^3*x*e
xp(I*(d*x+c))+2*d*e*f^2*exp(I*(d*x+c))+3*d^3*f^3*x^3*exp(I*(d*x+c))-4*I*f^3
*exp(3*I*(d*x+c))-2*I*f^3*exp(5*I*(d*x+c))+2*d^3*f^3*x^3*exp(3*I*(d*x+c))+4
*d^2*e^2*f+3*d^3*e^3*exp(5*I*(d*x+c))+2*d^3*e^3*exp(3*I*(d*x+c))+3*d^3*e^3*
exp(I*(d*x+c))-2*I*f^3*exp(I*(d*x+c))-18*I*d^3*e*f^2*x^2*exp(2*I*(d*x+c))-1
8*I*d^3*e^2*f*x*exp(2*I*(d*x+c))-16*I*d^2*e*f^2*x*exp(3*I*(d*x+c))-18*I*d^2
*e*f^2*x*exp(5*I*(d*x+c))+4*d^2*f^3*x^2+8*d^2*e*f^2*x+6*I*d^3*f^3*x^3*exp(4
I(d*x+c))-9*I*d^2*f^3*x^2*exp(5*I*(d*x+c))+2*f^3*exp(4*I*(d*x+c))+4*f^3*e
xp(2*I*(d*x+c))+2*I*d^2*e*f^2*x*exp(I*(d*x+c))+4*d*f^3*x*exp(3*I*(d*x+c))+4
*d*e*f^2*exp(3*I*(d*x+c))+18*I*d^3*e*f^2*x^2*exp(4*I*(d*x+c))+18*I*d^3*e^2*
f*x*exp(4*I*(d*x+c))+22*d^2*e^2*f*exp(2*I*(d*x+c))+22*d^2*f^3*x^2*exp(2*I*(
d*x+c))+18*d^2*f^3*x^2*exp(4*I*(d*x+c))+18*d^2*e^2*f*exp(4*I*(d*x+c))+2*d*f
^3*x*exp(5*I*(d*x+c))+2*d*e*f^2*exp(5*I*(d*x+c))+3*d^3*f^3*x^3*exp(5*I*(d*x
+c))-6*I*d^3*e^3*exp(2*I*(d*x+c))+6*I*d^3*e^3*exp(4*I*(d*x+c))+6*d^3*e*f^2*
x^2*exp(3*I*(d*x+c))+9*d^3*e^2*f*x*exp(I*(d*x+c))+9*d^3*e*f^2*x^2*exp(I*(d*
x+c))+I*d^2*f^3*x^2*exp(I*(d*x+c))+I*d^2*e^2*f*exp(I*(d*x+c)))/(exp(I*(d*x+
c))+I)^4/d^4/(exp(I*(d*x+c))-I)^2/a+9/4*I/d^2/a*polylog(2,-I*exp(I*(d*x+c))
)*e*f^2*x-9/4*I/d^2/a*polylog(2,I*exp(I*(d*x+c)))*e*f^2*x+9/4*I*f^3*polylog
(4,I*exp(I*(d*x+c)))/a/d^4+9/8/a/d*e*f^2*ln(1-I*exp(I*(d*x+c)))*x^2-9/8/a/d
^3*e*f^2*c^2*ln(1-I*exp(I*(d*x+c)))+9/8/a/d*e^2*f*ln(1-I*exp(I*(d*x+c)))*x+
9/8/a/d^2*e^2*f*ln(1-I*exp(I*(d*x+c)))*c-9/8/a/d^2*e^2*f*c*ln(exp(I*(d*x+c)
)+I)+9/8/a/d^3*e*f^2*c^2*ln(exp(I*(d*x+c))+I)-9/8*I/d^2/a*f^3*polylog(2,I*e
xp(I*(d*x+c)))*x^2+9/8*I/d^2/a*f^3*polylog(2,-I*exp(I*(d*x+c)))*x^2+9/8*I/d
^2/a*e^2*f*polylog(2,-I*exp(I*(d*x+c)))-9/8*I/d^2/a*e^2*f*polylog(2,I*exp(I
*(d*x+c)))-2*I/d^3/a*f^3*c*x

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 4.69846, size = 6093, normalized size = 8.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*d^3*f^3*x^3 + 6*d^3*e*f^2*x^2 + 6*d^3*e^2*f*x + 2*d^3*e^3 - 4*(2*d^
2*f^3*x^2 + 4*d^2*e*f^2*x + 2*d^2*e^2*f + f^3)*cos(d*x + c)^3 - 2*(3*d^3*f^
3*x^3 + 9*d^3*e*f^2*x^2 + 3*d^3*e^3 + 2*d*e*f^2 + (9*d^3*e^2*f + 2*d*f^3)*x
)*cos(d*x + c)^2 - 14*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*cos(d*x + c
) + ((-9*I*d^2*f^3*x^2 - 18*I*d^2*e*f^2*x - 9*I*d^2*e^2*f - 12*I*f^3)*cos(d
*x + c)^2*sin(d*x + c) + (-9*I*d^2*f^3*x^2 - 18*I*d^2*e*f^2*x - 9*I*d^2*e^2
*f - 12*I*f^3)*cos(d*x + c)^2)*dilog(I*cos(d*x + c) + sin(d*x + c)) + ((-9*
I*d^2*f^3*x^2 - 18*I*d^2*e*f^2*x - 9*I*d^2*e^2*f - 28*I*f^3)*cos(d*x + c)^2
*sin(d*x + c) + (-9*I*d^2*f^3*x^2 - 18*I*d^2*e*f^2*x - 9*I*d^2*e^2*f - 28*I
*f^3)*cos(d*x + c)^2)*dilog(I*cos(d*x + c) - sin(d*x + c)) + ((9*I*d^2*f^3*
x^2 + 18*I*d^2*e*f^2*x + 9*I*d^2*e^2*f + 12*I*f^3)*cos(d*x + c)^2*sin(d*x +
 c) + (9*I*d^2*f^3*x^2 + 18*I*d^2*e*f^2*x + 9*I*d^2*e^2*f + 12*I*f^3)*cos(d
*x + c)^2)*dilog(-I*cos(d*x + c) + sin(d*x + c)) + ((9*I*d^2*f^3*x^2 + 18*I
*d^2*e*f^2*x + 9*I*d^2*e^2*f + 28*I*f^3)*cos(d*x + c)^2*sin(d*x + c) + (9*I
*d^2*f^3*x^2 + 18*I*d^2*e*f^2*x + 9*I*d^2*e^2*f + 28*I*f^3)*cos(d*x + c)^2)
*dilog(-I*cos(d*x + c) - sin(d*x + c)) + ((3*d^3*e^3 - 9*c*d^2*e^2*f + (9*c
^2 + 28)*d*e*f^2 - (3*c^3 + 28*c)*f^3)*cos(d*x + c)^2*sin(d*x + c) + (3*d^3
*e^3 - 9*c*d^2*e^2*f + (9*c^2 + 28)*d*e*f^2 - (3*c^3 + 28*c)*f^3)*cos(d*x +
 c)^2)*log(cos(d*x + c) + I*sin(d*x + c) + I) - 3*((d^3*e^3 - 3*c*d^2*e^2*f
 + (3*c^2 + 4)*d*e*f^2 - (c^3 + 4*c)*f^3)*cos(d*x + c)^2*sin(d*x + c) + (d^
3*e^3 - 3*c*d^2*e^2*f + (3*c^2 + 4)*d*e*f^2 - (c^3 + 4*c)*f^3)*cos(d*x + c)
^2)*log(cos(d*x + c) - I*sin(d*x + c) + I) + ((3*d^3*f^3*x^3 + 9*d^3*e*f^2*
```

```
x^2 + 9*c*d^2*e^2*f - 9*c^2*d*e*f^2 + (3*c^3 + 28*c)*f^3 + (9*d^3*e^2*f + 2
8*d*f^3)*x)*cos(d*x + c)^2*sin(d*x + c) + (3*d^3*f^3*x^3 + 9*d^3*e*f^2*x^2
+ 9*c*d^2*e^2*f - 9*c^2*d*e*f^2 + (3*c^3 + 28*c)*f^3 + (9*d^3*e^2*f + 28*d*
f^3)*x)*cos(d*x + c)^2)*log(I*cos(d*x + c) + sin(d*x + c) + 1) - 3*((d^3*f^
3*x^3 + 3*d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + (c^3 + 4*c)*f^3 +
 (3*d^3*e^2*f + 4*d*f^3)*x)*cos(d*x + c)^2*sin(d*x + c) + (d^3*f^3*x^3 + 3*
d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + (c^3 + 4*c)*f^3 + (3*d^3*e^
2*f + 4*d*f^3)*x)*cos(d*x + c)^2)*log(I*cos(d*x + c) - sin(d*x + c) + 1) +
((3*d^3*f^3*x^3 + 9*d^3*e*f^2*x^2 + 9*c*d^2*e^2*f - 9*c^2*d*e*f^2 + (3*c^3
+ 28*c)*f^3 + (9*d^3*e^2*f + 28*d*f^3)*x)*cos(d*x + c)^2*sin(d*x + c) + (3*
d^3*f^3*x^3 + 9*d^3*e*f^2*x^2 + 9*c*d^2*e^2*f - 9*c^2*d*e*f^2 + (3*c^3 + 28
*c)*f^3 + (9*d^3*e^2*f + 28*d*f^3)*x)*cos(d*x + c)^2)*log(-I*cos(d*x + c) +
 sin(d*x + c) + 1) - 3*((d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*
c^2*d*e*f^2 + (c^3 + 4*c)*f^3 + (3*d^3*e^2*f + 4*d*f^3)*x)*cos(d*x + c)^2*s
in(d*x + c) + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*c^2*d*e*f^
2 + (c^3 + 4*c)*f^3 + (3*d^3*e^2*f + 4*d*f^3)*x)*cos(d*x + c)^2)*log(-I*cos
(d*x + c) - sin(d*x + c) + 1) + ((3*d^3*e^3 - 9*c*d^2*e^2*f + (9*c^2 + 28)*
d*e*f^2 - (3*c^3 + 28*c)*f^3)*cos(d*x + c)^2*sin(d*x + c) + (3*d^3*e^3 - 9*
c*d^2*e^2*f + (9*c^2 + 28)*d*e*f^2 - (3*c^3 + 28*c)*f^3)*cos(d*x + c)^2)*lo
g(-cos(d*x + c) + I*sin(d*x + c) + I) - 3*((d^3*e^3 - 3*c*d^2*e^2*f + (3*c^
2 + 4)*d*e*f^2 - (c^3 + 4*c)*f^3)*cos(d*x + c)^2*sin(d*x + c) + (d^3*e^3 -
3*c*d^2*e^2*f + (3*c^2 + 4)*d*e*f^2 - (c^3 + 4*c)*f^3)*cos(d*x + c)^2)*log(
-cos(d*x + c) - I*sin(d*x + c) + I) + (18*I*f^3*cos(d*x + c)^2*sin(d*x + c)
 + 18*I*f^3*cos(d*x + c)^2)*polylog(4, I*cos(d*x + c) + sin(d*x + c)) + (18
*I*f^3*cos(d*x + c)^2*sin(d*x + c) + 18*I*f^3*cos(d*x + c)^2)*polylog(4, I*
cos(d*x + c) - sin(d*x + c)) + (-18*I*f^3*cos(d*x + c)^2*sin(d*x + c) - 18*
I*f^3*cos(d*x + c)^2)*polylog(4, -I*cos(d*x + c) + sin(d*x + c)) + (-18*I*f
^3*cos(d*x + c)^2*sin(d*x + c) - 18*I*f^3*cos(d*x + c)^2)*polylog(4, -I*cos
(d*x + c) - sin(d*x + c)) - 18*((d*f^3*x + d*e*f^2)*cos(d*x + c)^2*sin(d*x
+ c) + (d*f^3*x + d*e*f^2)*cos(d*x + c)^2)*polylog(3, I*cos(d*x + c) + sin(
d*x + c)) + 18*((d*f^3*x + d*e*f^2)*cos(d*x + c)^2*sin(d*x + c) + (d*f^3*x
+ d*e*f^2)*cos(d*x + c)^2)*polylog(3, I*cos(d*x + c) - sin(d*x + c)) - 18*(
(d*f^3*x + d*e*f^2)*cos(d*x + c)^2*sin(d*x + c) + (d*f^3*x + d*e*f^2)*cos(d
*x + c)^2)*polylog(3, -I*cos(d*x + c) + sin(d*x + c)) + 18*((d*f^3*x + d*e*
f^2)*cos(d*x + c)^2*sin(d*x + c) + (d*f^3*x + d*e*f^2)*cos(d*x + c)^2)*poly
log(3, -I*cos(d*x + c) - sin(d*x + c)) + 2*(3*d^3*f^3*x^3 + 9*d^3*e*f^2*x^2
 + 9*d^3*e^2*f*x + 3*d^3*e^3 - 5*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*
cos(d*x + c))*sin(d*x + c))/(a*d^4*cos(d*x + c)^2*sin(d*x + c) + a*d^4*cos(
d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sec(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \sec(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sec(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

3.282 $\int \frac{(e+fx)^2 \sec^3(c+dx)}{a+a\sin(c+dx)} dx$

Optimal. Leaf size=431

$$\frac{3if(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{4ad^2}-\frac{3if(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{4ad^2}-\frac{3f^2\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{4ad^3}+\frac{3f^2\text{PolyLog}\left(3,i\right.}{4ad^3}$$

```
[Out] (((-3*I)/4)*(e + f*x)^2*ArcTan[E^(I*(c + d*x))])/(a*d) + (5*f^2*ArcTanh[Sin
[c + d*x]])/(6*a*d^3) + (f^2*Log[Cos[c + d*x]])/(3*a*d^3) + (((3*I)/4)*f*(e
 + f*x)*PolyLog[2, (-I)*E^(I*(c + d*x))])/(a*d^2) - (((3*I)/4)*f*(e + f*x)*
PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2) - (3*f^2*PolyLog[3, (-I)*E^(I*(c + d
*x))])/(4*a*d^3) + (3*f^2*PolyLog[3, I*E^(I*(c + d*x))])/(4*a*d^3) - (3*f*(
e + f*x)*Sec[c + d*x])/(4*a*d^2) - (f^2*Sec[c + d*x]^2)/(12*a*d^3) - (f*(e
+ f*x)*Sec[c + d*x]^3)/(6*a*d^2) - ((e + f*x)^2*Sec[c + d*x]^4)/(4*a*d) + (
f*(e + f*x)*Tan[c + d*x])/(3*a*d^2) + (f^2*Sec[c + d*x]*Tan[c + d*x])/(12*a
*d^3) + (3*(e + f*x)^2*Sec[c + d*x]*Tan[c + d*x])/(8*a*d) + (f*(e + f*x)*Se
c[c + d*x]^2*Tan[c + d*x])/(6*a*d^2) + ((e + f*x)^2*Sec[c + d*x]^3*Tan[c +
d*x])/(4*a*d)
```

Rubi [A] time = 0.397982, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4531, 4186, 3768, 3770, 4181, 2531, 2282, 6589, 4409, 4185, 4184, 3475}

$$\frac{3if(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{4ad^2}-\frac{3if(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{4ad^2}-\frac{3f^2\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)}{4ad^3}+\frac{3f^2\text{PolyLog}\left(3,i\right.}{4ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (((-3*I)/4)*(e + f*x)^2*ArcTan[E^(I*(c + d*x))])/(a*d) + (5*f^2*ArcTanh[Sin
[c + d*x]])/(6*a*d^3) + (f^2*Log[Cos[c + d*x]])/(3*a*d^3) + (((3*I)/4)*f*(e
 + f*x)*PolyLog[2, (-I)*E^(I*(c + d*x))])/(a*d^2) - (((3*I)/4)*f*(e + f*x)*
PolyLog[2, I*E^(I*(c + d*x))])/(a*d^2) - (3*f^2*PolyLog[3, (-I)*E^(I*(c + d
*x))])/(4*a*d^3) + (3*f^2*PolyLog[3, I*E^(I*(c + d*x))])/(4*a*d^3) - (3*f*(
e + f*x)*Sec[c + d*x])/(4*a*d^2) - (f^2*Sec[c + d*x]^2)/(12*a*d^3) - (f*(e
+ f*x)*Sec[c + d*x]^3)/(6*a*d^2) - ((e + f*x)^2*Sec[c + d*x]^4)/(4*a*d) + (
f*(e + f*x)*Tan[c + d*x])/(3*a*d^2) + (f^2*Sec[c + d*x]*Tan[c + d*x])/(12*a
*d^3) + (3*(e + f*x)^2*Sec[c + d*x]*Tan[c + d*x])/(8*a*d) + (f*(e + f*x)*Se
c[c + d*x]^2*Tan[c + d*x])/(6*a*d^2) + ((e + f*x)^2*Sec[c + d*x]^3*Tan[c +
d*x])/(4*a*d)
```

Rule 4531

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)^2\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\sec^5(c+dx)\,dx}{a} - \frac{\int (e+fx)^2\sec^4(c+dx)\tan(c+dx)\,dx}{a}$$

$$= -\frac{f(e+fx)\sec^3(c+dx)}{6ad^2} - \frac{(e+fx)^2\sec^4(c+dx)}{4ad} + \frac{(e+fx)^2\sec^3(c+dx)\tan(c+dx)}{4ad} +$$

$$= -\frac{3f(e+fx)\sec(c+dx)}{4ad^2} - \frac{f^2\sec^2(c+dx)}{12ad^3} - \frac{f(e+fx)\sec^3(c+dx)}{6ad^2} - \frac{(e+fx)^2\sec^4(c}{4ad}$$

$$= -\frac{3i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} + \frac{5f^2\tanh^{-1}(\sin(c+dx))}{6ad^3} - \frac{3f(e+fx)\sec(c+dx)}{4ad^2} - \frac{f^2 s}{}$$

$$= -\frac{3i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} + \frac{5f^2\tanh^{-1}(\sin(c+dx))}{6ad^3} + \frac{f^2\log(\cos(c+dx))}{3ad^3} + \frac{3if(e}{}$$

$$= -\frac{3i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} + \frac{5f^2\tanh^{-1}(\sin(c+dx))}{6ad^3} + \frac{f^2\log(\cos(c+dx))}{3ad^3} + \frac{3if(e}{}$$

$$= -\frac{3i(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} + \frac{5f^2\tanh^{-1}(\sin(c+dx))}{6ad^3} + \frac{f^2\log(\cos(c+dx))}{3ad^3} + \frac{3if(e}{}$$

Mathematica [B] time = 8.92612, size = 1468, normalized size = 3.41

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((Cos[c] + I*Sin[c])*(3*d^2*e*f*x^2*Cos[c] + 6*e*f*PolyLog[2, I*Cos[c + d*
x] + Sin[c + d*x]]*(Cos[c] - I*(-1 + Sin[c])) + (3*d^2*e^2 + 4*f^2)*x*(Cos[
c] - I*Sin[c]) + d^2*f^2*x^3*(Cos[c] - I*Sin[c]) + 6*d*e*f*x*Log[1 - I*Cos[
c + d*x] - Sin[c + d*x]]*(Cos[c] + I*(-1 + Sin[c]))*(Cos[c] - I*Sin[c]) + 3
*d*f^2*x^2*Log[1 - I*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c] + I*(-1 + Sin[c])
)*(Cos[c] - I*Sin[c]) + (6*f^2*(I*d*x*PolyLog[2, I*Cos[c + d*x] + Sin[c + d
*x]] + PolyLog[3, I*Cos[c + d*x] + Sin[c + d*x]])*(Cos[c] + I*(-1 + Sin[c])
)*(Cos[c] - I*Sin[c]))/d - (3*I)*d^2*e*f*x^2*Sin[c] + ((3*d^2*e^2 + 4*f^2)*
(d*x + I*Log[-Cos[c + d*x] - I*(-1 + Sin[c + d*x])])*(Cos[c] - I*Sin[c])*(-
1 - I*Cos[c] + Sin[c]))/d))/(8*a*d^2*(Cos[c] + I*(-1 + Sin[c]))) - ((Cos[c]
 + I*Sin[c])*(9*d^2*e*f*x^2*Cos[c] + 3*d^2*f^2*x^3*Cos[c] + (9*d^2*e^2 + 28
*f^2)*x*(Cos[c] - I*Sin[c]) - (9*I)*d^2*e*f*x^2*Sin[c] - (3*I)*d^2*f^2*x^3*
Sin[c] + (18*f^2*(d*x*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]] - I*Poly
Log[3, (-I)*Cos[c + d*x] - Sin[c + d*x]])*(Cos[c] - I*Sin[c])*(1 - I*Cos[c]
 + Sin[c]))/d + 18*e*f*PolyLog[2, (-I)*Cos[c + d*x] - Sin[c + d*x]]*(Cos[c]
 - I*(1 + Sin[c])) - 18*d*e*f*x*Log[1 + I*Cos[c + d*x] + Sin[c + d*x]]*(Cos
```

```
[c] - I*Sin[c])*(Cos[c] + I*(1 + Sin[c])) - 9*d*f^2*x^2*Log[1 + I*Cos[c + d
*x] + Sin[c + d*x]]*(Cos[c] - I*Sin[c])*(Cos[c] + I*(1 + Sin[c])) + ((9*d^2
*e^2 + 28*f^2)*(d*x + I*Log[Cos[c + d*x] + I*(1 + Sin[c + d*x])])*(I*Cos[c]
 + Sin[c])*(Cos[c] + I*(1 + Sin[c]))))/d))/(24*a*d^2*(Cos[c] + I*(1 + Sin[c]
))) + ((3*e^2*x*Cos[c])/(4*a) + (((3*I)/4)*e^2*x*Sin[c])/a)/(1 + Cos[2*c] +
 I*Sin[2*c]) + ((3*e*f*x^2*Cos[c])/(4*a) + (((3*I)/4)*e*f*x^2*Sin[c])/a)/(1
 + Cos[2*c] + I*Sin[2*c]) + ((f^2*x^3*Cos[c])/(4*a) + ((I/4)*f^2*x^3*Sin[c]
)/a)/(1 + Cos[2*c] + I*Sin[2*c]) + (e^2 + 2*e*f*x + f^2*x^2)/(8*a*d*(Cos[c/
2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (-(e*f*Sin[(d*x)/2]) - f^2*x*Sin[(d
*x)/2])/(2*a*d^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x
)/2])) + (-e^2 - 2*e*f*x - f^2*x^2)/(8*a*d*(Cos[c/2 + (d*x)/2] + Sin[c/2 +
(d*x)/2])^4) + (e*f*Sin[(d*x)/2] + f^2*x*Sin[(d*x)/2])/(6*a*d^2*(Cos[c/2] +
 Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (-3*d^2*e^2*Cos[c
/2] - d*e*f*Cos[c/2] - f^2*Cos[c/2] - 6*d^2*e*f*x*Cos[c/2] - d*f^2*x*Cos[c/
2] - 3*d^2*f^2*x^2*Cos[c/2] - 3*d^2*e^2*Sin[c/2] + d*e*f*Sin[c/2] - f^2*Sin
[c/2] - 6*d^2*e*f*x*Sin[c/2] + d*f^2*x*Sin[c/2] - 3*d^2*f^2*x^2*Sin[c/2])/(
12*a*d^3*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2)
 + (7*(e*f*Sin[(d*x)/2] + f^2*x*Sin[(d*x)/2]))/(6*a*d^2*(Cos[c/2] + Sin[c/2
])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] time = 0.361, size = 1119, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sec(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 3/4*I/d^2/a*polylog(2,-I*exp(I*(d*x+c)))*f^2*x-1/2/d^3/a*f^2*ln(exp(I*(d*x+
c))-I)-3/4*f^2*polylog(3,-I*exp(I*(d*x+c)))/a/d^3+3/4/a/d*f*e*ln(1-I*exp(I*
(d*x+c)))*x+3/4/a/d^2*f*e*ln(1-I*exp(I*(d*x+c)))*c-3/4/a/d^2*f*e*c*ln(exp(I
*(d*x+c))+I)+3/8/a/d*f^2*ln(1-I*exp(I*(d*x+c)))*x^2-3/8/a/d^3*f^2*ln(1-I*ex
p(I*(d*x+c)))*c^2+3/4/d^2/a*e*f*c*ln(exp(I*(d*x+c))-I)+3/8/a/d*ln(exp(I*(d*
x+c))+I)*e^2+3/8/d^3/a*ln(1+I*exp(I*(d*x+c)))*c^2*f^2-3/8/d/a*ln(1+I*exp(I*
(d*x+c)))*f^2*x^2-3/4/d/a*ln(1+I*exp(I*(d*x+c)))*e*f*x-3/4/d^2/a*ln(1+I*exp
(I*(d*x+c)))*c*e*f+3/8/a/d^3*f^2*c^2*ln(exp(I*(d*x+c))+I)+3/4*f^2*polylog(3
,I*exp(I*(d*x+c)))/a/d^3-3/8/d^3/a*f^2*c^2*ln(exp(I*(d*x+c))-I)-2/3/d^3/a*f
^2*ln(exp(I*(d*x+c)))+7/6/d^3/a*f^2*ln(exp(I*(d*x+c))+I)-3/8/d/a*e^2*ln(exp
(I*(d*x+c))-I)-1/12*I*(-18*I*d^2*f^2*x^2*exp(2*I*(d*x+c))-16*I*d*f^2*x*exp(
3*I*(d*x+c))-16*I*d*e*f*exp(3*I*(d*x+c))+6*d^2*f^2*x^2*exp(3*I*(d*x+c))+2*f
^2*exp(I*(d*x+c))+18*I*d^2*e^2*exp(4*I*(d*x+c))+18*d^2*e*f*x*exp(I*(d*x+c))
+2*I*d*e*f*exp(I*(d*x+c))+2*I*d*f^2*x*exp(I*(d*x+c))+9*d^2*e^2*exp(I*(d*x+c
```

```
))+6*d^2*e^2*exp(3*I*(d*x+c))+9*d^2*e^2*exp(5*I*(d*x+c))+44*d*f^2*x*exp(2*I
*(d*x+c))+36*d*e*f*exp(4*I*(d*x+c))+36*I*d^2*e*f*x*exp(4*I*(d*x+c))-36*I*d^
2*e*f*x*exp(2*I*(d*x+c))+8*d*f^2*x+2*f^2*exp(5*I*(d*x+c))+18*I*d^2*f^2*x^2*
exp(4*I*(d*x+c))+18*d^2*e*f*x*exp(5*I*(d*x+c))+12*d^2*e*f*x*exp(3*I*(d*x+c)
)-18*I*d*f^2*x*exp(5*I*(d*x+c))-18*I*d*e*f*exp(5*I*(d*x+c))+9*d^2*f^2*x^2*e
xp(I*(d*x+c))+8*d*e*f+4*f^2*exp(3*I*(d*x+c))+44*d*e*f*exp(2*I*(d*x+c))+9*d^
2*f^2*x^2*exp(5*I*(d*x+c))+36*d*f^2*x*exp(4*I*(d*x+c))-18*I*d^2*e^2*exp(2*I
*(d*x+c)))/(exp(I*(d*x+c))+I)^4/d^3/(exp(I*(d*x+c))-I)^2/a-3/4*I/d^2/a*e*f*
polylog(2,I*exp(I*(d*x+c)))-3/4*I/d^2/a*polylog(2,I*exp(I*(d*x+c)))*f^2*x+3
/4*I/d^2/a*e*f*polylog(2,-I*exp(I*(d*x+c)))
```

Maxima [B] time = 40.6915, size = 7104, normalized size = 16.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(2*c*e*f*(2*(3*sin(d*x + c)^2 + 3*sin(d*x + c) - 2)/(a*d*sin(d*x + c)^
3 + a*d*sin(d*x + c)^2 - a*d*sin(d*x + c) - a*d) - 3*log(sin(d*x + c) + 1)/
(a*d) + 3*log(sin(d*x + c) - 1)/(a*d)) - e^2*(2*(3*sin(d*x + c)^2 + 3*sin(d
*x + c) - 2)/(a*sin(d*x + c)^3 + a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 3
*log(sin(d*x + c) + 1)/a + 3*log(sin(d*x + c) - 1)/a) - 16*(32*(d*x + c)*f^
2*cos(6*d*x + 6*c) + 32*I*(d*x + c)*f^2*sin(6*d*x + 6*c) + 32*d*e*f - 32*c*
f^2 - (2*(9*c^2 + 28)*f^2*cos(6*d*x + 6*c) + (36*I*c^2 + 112*I)*f^2*cos(5*d
*x + 5*c) + 2*(9*c^2 + 28)*f^2*cos(4*d*x + 4*c) + (72*I*c^2 + 224*I)*f^2*co
s(3*d*x + 3*c) - 2*(9*c^2 + 28)*f^2*cos(2*d*x + 2*c) + (36*I*c^2 + 112*I)*f
^2*cos(d*x + c) + (18*I*c^2 + 56*I)*f^2*sin(6*d*x + 6*c) - 4*(9*c^2 + 28)*f
^2*sin(5*d*x + 5*c) + (18*I*c^2 + 56*I)*f^2*sin(4*d*x + 4*c) - 8*(9*c^2 + 2
8)*f^2*sin(3*d*x + 3*c) + (-18*I*c^2 - 56*I)*f^2*sin(2*d*x + 2*c) - 4*(9*c^
2 + 28)*f^2*sin(d*x + c) - 2*(9*c^2 + 28)*f^2)*arctan2(sin(d*x + c) + 1, co
s(d*x + c)) + (6*(3*c^2 + 4)*f^2*cos(6*d*x + 6*c) - (-36*I*c^2 - 48*I)*f^2*
cos(5*d*x + 5*c) + 6*(3*c^2 + 4)*f^2*cos(4*d*x + 4*c) - (-72*I*c^2 - 96*I)*
f^2*cos(3*d*x + 3*c) - 6*(3*c^2 + 4)*f^2*cos(2*d*x + 2*c) - (-36*I*c^2 - 48
*I)*f^2*cos(d*x + c) - (-18*I*c^2 - 24*I)*f^2*sin(6*d*x + 6*c) - 12*(3*c^2
+ 4)*f^2*sin(5*d*x + 5*c) - (-18*I*c^2 - 24*I)*f^2*sin(4*d*x + 4*c) - 24*(3
*c^2 + 4)*f^2*sin(3*d*x + 3*c) - (18*I*c^2 + 24*I)*f^2*sin(2*d*x + 2*c) - 1
2*(3*c^2 + 4)*f^2*sin(d*x + c) - 6*(3*c^2 + 4)*f^2)*arctan2(sin(d*x + c) -
1, cos(d*x + c)) - (18*(d*x + c)^2*f^2 + 36*(d*e*f - c*f^2)*(d*x + c) - 18*
((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*cos(6*d*x + 6*c) + (-36*I*(
d*x + c)^2*f^2 + (-72*I*d*e*f + 72*I*c*f^2)*(d*x + c))*cos(5*d*x + 5*c) - 1
```

8*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*cos(4*d*x + 4*c) + (-72*I*(d*x + c)^2*f^2 + (-144*I*d*e*f + 144*I*c*f^2)*(d*x + c))*cos(3*d*x + 3*c) + 18*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*cos(2*d*x + 2*c) + (-36*I*(d*x + c)^2*f^2 + (-72*I*d*e*f + 72*I*c*f^2)*(d*x + c))*cos(d*x + c) + (-18*I*(d*x + c)^2*f^2 + (-36*I*d*e*f + 36*I*c*f^2)*(d*x + c))*sin(6*d*x + 6*c) + 36*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*sin(5*d*x + 5*c) + (-18*I*(d*x + c)^2*f^2 + (-36*I*d*e*f + 36*I*c*f^2)*(d*x + c))*sin(4*d*x + 4*c) + 72*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*sin(3*d*x + 3*c) + (18*I*(d*x + c)^2*f^2 + (36*I*d*e*f - 36*I*c*f^2)*(d*x + c))*sin(2*d*x + 2*c) + 36*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*sin(d*x + c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) - (18*(d*x + c)^2*f^2 + 36*(d*e*f - c*f^2)*(d*x + c) - 18*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*cos(6*d*x + 6*c) + (-36*I*(d*x + c)^2*f^2 + (-72*I*d*e*f + 72*I*c*f^2)*(d*x + c))*cos(5*d*x + 5*c) - 18*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*cos(4*d*x + 4*c) + (-72*I*(d*x + c)^2*f^2 + (-144*I*d*e*f + 144*I*c*f^2)*(d*x + c))*cos(3*d*x + 3*c) + 18*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*cos(2*d*x + 2*c) + (-36*I*(d*x + c)^2*f^2 + (-72*I*d*e*f + 72*I*c*f^2)*(d*x + c))*cos(d*x + c) + (-18*I*(d*x + c)^2*f^2 + (-36*I*d*e*f + 36*I*c*f^2)*(d*x + c))*sin(6*d*x + 6*c) + 36*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*sin(5*d*x + 5*c) + (-18*I*(d*x + c)^2*f^2 + (-36*I*d*e*f + 36*I*c*f^2)*(d*x + c))*sin(4*d*x + 4*c) + 72*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*sin(3*d*x + 3*c) + (18*I*(d*x + c)^2*f^2 + (36*I*d*e*f - 36*I*c*f^2)*(d*x + c))*sin(2*d*x + 2*c) + 36*((d*x + c)^2*f^2 + 2*(d*e*f - c*f^2)*(d*x + c))*sin(d*x + c))*arctan2(cos(d*x + c), -sin(d*x + c) + 1) + (36*(d*x + c)^2*f^2 - 72*I*d*e*f + 4*(9*c^2 + 18*I*c + 2)*f^2 + (72*d*e*f - (72*c + 8*I)*f^2)*(d*x + c))*cos(5*d*x + 5*c) - (-72*I*(d*x + c)^2*f^2 - 144*d*e*f + (-72*I*c^2 + 144*c)*f^2 - 16*(9*I*d*e*f + (-9*I*c + 11)*f^2)*(d*x + c))*cos(4*d*x + 4*c) + (24*(d*x + c)^2*f^2 - 64*I*d*e*f + 8*(3*c^2 + 8*I*c + 2)*f^2 + (48*d*e*f - (48*c - 64*I)*f^2)*(d*x + c))*cos(3*d*x + 3*c) - (72*I*(d*x + c)^2*f^2 - 176*d*e*f + (72*I*c^2 + 176*c)*f^2 - 144*(-I*d*e*f + (I*c + 1)*f^2)*(d*x + c))*cos(2*d*x + 2*c) + (36*(d*x + c)^2*f^2 + 8*I*d*e*f + 4*(9*c^2 - 2*I*c + 2)*f^2 + (72*d*e*f - (72*c - 72*I)*f^2)*(d*x + c))*cos(d*x + c) - (36*d*e*f + 36*(d*x + c)*f^2 - 36*c*f^2 - 36*(d*e*f + (d*x + c)*f^2 - c*f^2)*cos(6*d*x + 6*c) + (-72*I*d*e*f - 72*I*(d*x + c)*f^2 + 72*I*c*f^2)*cos(5*d*x + 5*c) - 36*(d*e*f + (d*x + c)*f^2 - c*f^2)*cos(4*d*x + 4*c) + (-144*I*d*e*f - 144*I*(d*x + c)*f^2 + 144*I*c*f^2)*cos(3*d*x + 3*c) + 36*(d*e*f + (d*x + c)*f^2 - c*f^2)*cos(2*d*x + 2*c) + (-72*I*d*e*f - 72*I*(d*x + c)*f^2 + 72*I*c*f^2)*cos(d*x + c) + (-36*I*d*e*f - 36*I*(d*x + c)*f^2 + 36*I*c*f^2)*sin(6*d*x + 6*c) + 72*(d*e*f + (d*x + c)*f^2 - c*f^2)*sin(5*d*x + 5*c) + (-36*I*d*e*f - 36*I*(d*x + c)*f^2 + 36*I*c*f^2)*sin(4*d*x + 4*c) + 144*(d*e*f + (d*x + c)*f^2 - c*f^2)*sin(3*d*x + 3*c) + (36*I*d*e*f + 36*I*(d*x + c)*f^2 - 36*I*c*f^2)*sin(2*d*x + 2*c) + 72*(d*e*f + (d*x + c)*f^2 - c*f^2)*sin(d*x + c))*dilog(I*e^(I*d*x + I*c)) + (36*d*e*f + 36*(d*x + c)*f^2 - 36*c*f^2 - 36*(d*e*f + (d*x + c)*f^2 - c*f^2)*cos(6*d*x + 6*c) - (72*I*d*e*f + 72*I*(d*x + c)*f^2 - 72*I*c*f^2)*cos(5*d*x + 5*c) - 36*(d*e*f + (d*x

+ c)*f^2 - c*f^2)*cos(4*d*x + 4*c) - (144*I*d*e*f + 144*I*(d*x + c)*f^2 - 144*I*c*f^2)*cos(3*d*x + 3*c) + 36*(d*e*f + (d*x + c)*f^2 - c*f^2)*cos(2*d*x + 2*c) - (72*I*d*e*f + 72*I*(d*x + c)*f^2 - 72*I*c*f^2)*cos(d*x + c) - (36*I*d*e*f + 36*I*(d*x + c)*f^2 - 36*I*c*f^2)*sin(6*d*x + 6*c) + 72*(d*e*f + (d*x + c)*f^2 - c*f^2)*sin(5*d*x + 5*c) - (36*I*d*e*f + 36*I*(d*x + c)*f^2 - 36*I*c*f^2)*sin(4*d*x + 4*c) + 144*(d*e*f + (d*x + c)*f^2 - c*f^2)*sin(3*d*x + 3*c) - (-36*I*d*e*f - 36*I*(d*x + c)*f^2 + 36*I*c*f^2)*sin(2*d*x + 2*c) + 72*(d*e*f + (d*x + c)*f^2 - c*f^2)*sin(d*x + c))*dilog(-I*e^(I*d*x + I*c)) - (9*I*(d*x + c)^2*f^2 + (9*I*c^2 + 28*I)*f^2 + (18*I*d*e*f - 18*I*c*f^2)*(d*x + c) + (-9*I*(d*x + c)^2*f^2 + (-9*I*c^2 - 28*I)*f^2 + (-18*I*d*e*f + 18*I*c*f^2)*(d*x + c))*cos(6*d*x + 6*c) + 2*(9*(d*x + c)^2*f^2 + (9*c^2 + 28)*f^2 + 18*(d*e*f - c*f^2)*(d*x + c))*cos(5*d*x + 5*c) + (-9*I*(d*x + c)^2*f^2 + (-9*I*c^2 - 28*I)*f^2 + (-18*I*d*e*f + 18*I*c*f^2)*(d*x + c))*cos(4*d*x + 4*c) + 4*(9*(d*x + c)^2*f^2 + (9*c^2 + 28)*f^2 + 18*(d*e*f - c*f^2)*(d*x + c))*cos(3*d*x + 3*c) + (9*I*(d*x + c)^2*f^2 + (9*I*c^2 + 28*I)*f^2 + (18*I*d*e*f - 18*I*c*f^2)*(d*x + c))*cos(2*d*x + 2*c) + 2*(9*(d*x + c)^2*f^2 + (9*c^2 + 28)*f^2 + 18*(d*e*f - c*f^2)*(d*x + c))*cos(d*x + c) + (9*(d*x + c)^2*f^2 + (9*c^2 + 28)*f^2 + 18*(d*e*f - c*f^2)*(d*x + c))*sin(6*d*x + 6*c) + (18*I*(d*x + c)^2*f^2 + (18*I*c^2 + 56*I)*f^2 + (36*I*d*e*f - 36*I*c*f^2)*(d*x + c))*sin(5*d*x + 5*c) + (9*(d*x + c)^2*f^2 + (9*c^2 + 28)*f^2 + 18*(d*e*f - c*f^2)*(d*x + c))*sin(4*d*x + 4*c) + (36*I*(d*x + c)^2*f^2 + (36*I*c^2 + 112*I)*f^2 + (72*I*d*e*f - 72*I*c*f^2)*(d*x + c))*sin(3*d*x + 3*c) - (9*(d*x + c)^2*f^2 + (9*c^2 + 28)*f^2 + 18*(d*e*f - c*f^2)*(d*x + c))*sin(2*d*x + 2*c) + (18*I*(d*x + c)^2*f^2 + (18*I*c^2 + 56*I)*f^2 + (36*I*d*e*f - 36*I*c*f^2)*(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - (-9*I*(d*x + c)^2*f^2 + (-9*I*c^2 - 12*I)*f^2 + (-18*I*d*e*f + 18*I*c*f^2)*(d*x + c) + (9*I*(d*x + c)^2*f^2 + (9*I*c^2 + 12*I)*f^2 + (18*I*d*e*f - 18*I*c*f^2)*(d*x + c))*cos(6*d*x + 6*c) - 6*(3*(d*x + c)^2*f^2 + (3*c^2 + 4)*f^2 + 6*(d*e*f - c*f^2)*(d*x + c))*cos(5*d*x + 5*c) + (9*I*(d*x + c)^2*f^2 + (9*I*c^2 + 12*I)*f^2 + (18*I*d*e*f - 18*I*c*f^2)*(d*x + c))*cos(4*d*x + 4*c) - 12*(3*(d*x + c)^2*f^2 + (3*c^2 + 4)*f^2 + 6*(d*e*f - c*f^2)*(d*x + c))*cos(3*d*x + 3*c) + (-9*I*(d*x + c)^2*f^2 + (-9*I*c^2 - 12*I)*f^2 + (-18*I*d*e*f + 18*I*c*f^2)*(d*x + c))*cos(2*d*x + 2*c) - 6*(3*(d*x + c)^2*f^2 + (3*c^2 + 4)*f^2 + 6*(d*e*f - c*f^2)*(d*x + c))*cos(d*x + c) - 3*(3*(d*x + c)^2*f^2 + (3*c^2 + 4)*f^2 + 6*(d*e*f - c*f^2)*(d*x + c))*sin(6*d*x + 6*c) + (-18*I*(d*x + c)^2*f^2 + (-18*I*c^2 - 24*I)*f^2 + (-36*I*d*e*f + 36*I*c*f^2)*(d*x + c))*sin(5*d*x + 5*c) - 3*(3*(d*x + c)^2*f^2 + (3*c^2 + 4)*f^2 + 6*(d*e*f - c*f^2)*(d*x + c))*sin(4*d*x + 4*c) + (-36*I*(d*x + c)^2*f^2 + (-36*I*c^2 - 48*I)*f^2 + (-72*I*d*e*f + 72*I*c*f^2)*(d*x + c))*sin(3*d*x + 3*c) + 3*(3*(d*x + c)^2*f^2 + (3*c^2 + 4)*f^2 + 6*(d*e*f - c*f^2)*(d*x + c))*sin(2*d*x + 2*c) + (-18*I*(d*x + c)^2*f^2 + (-18*I*c^2 - 24*I)*f^2 + (-36*I*d*e*f + 36*I*c*f^2)*(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) - (-36*I*f^2*cos(6*d*x + 6*c) + 72*f^2*cos(5*d*x + 5*c) - 36*I*f^2*cos(4*d*x + 4*c) + 144*f^2*cos(3*d*x + 3*c) + 36*I*f^2*cos(2*d*x + 2*c) + 72*f^2*cos(d*x + c) + 36*

```
f^2*sin(6*d*x + 6*c) + 72*I*f^2*sin(5*d*x + 5*c) + 36*f^2*sin(4*d*x + 4*c)
+ 144*I*f^2*sin(3*d*x + 3*c) - 36*f^2*sin(2*d*x + 2*c) + 72*I*f^2*sin(d*x +
 c) + 36*I*f^2)*polylog(3, I*e^(I*d*x + I*c)) - (36*I*f^2*cos(6*d*x + 6*c)
- 72*f^2*cos(5*d*x + 5*c) + 36*I*f^2*cos(4*d*x + 4*c) - 144*f^2*cos(3*d*x +
 3*c) - 36*I*f^2*cos(2*d*x + 2*c) - 72*f^2*cos(d*x + c) - 36*f^2*sin(6*d*x
+ 6*c) - 72*I*f^2*sin(5*d*x + 5*c) - 36*f^2*sin(4*d*x + 4*c) - 144*I*f^2*si
n(3*d*x + 3*c) + 36*f^2*sin(2*d*x + 2*c) - 72*I*f^2*sin(d*x + c) - 36*I*f^2
)*polylog(3, -I*e^(I*d*x + I*c)) - (-36*I*(d*x + c)^2*f^2 - 72*d*e*f + (-36
*I*c^2 + 72*c - 8*I)*f^2 - 8*(9*I*d*e*f + (-9*I*c + 1)*f^2)*(d*x + c))*sin(
5*d*x + 5*c) - (72*(d*x + c)^2*f^2 - 144*I*d*e*f + 72*(c^2 + 2*I*c)*f^2 + (
144*d*e*f - (144*c + 176*I)*f^2)*(d*x + c))*sin(4*d*x + 4*c) - (-24*I*(d*x
+ c)^2*f^2 - 64*d*e*f + (-24*I*c^2 + 64*c - 16*I)*f^2 - 16*(3*I*d*e*f + (-3
*I*c - 4)*f^2)*(d*x + c))*sin(3*d*x + 3*c) + (72*(d*x + c)^2*f^2 + 176*I*d*
e*f + 8*(9*c^2 - 22*I*c)*f^2 + (144*d*e*f - (144*c - 144*I)*f^2)*(d*x + c))
*sin(2*d*x + 2*c) - (-36*I*(d*x + c)^2*f^2 + 8*d*e*f + (-36*I*c^2 - 8*c - 8
*I)*f^2 - 72*(I*d*e*f + (-I*c - 1)*f^2)*(d*x + c))*sin(d*x + c))/(-48*I*a*d
^2*cos(6*d*x + 6*c) + 96*a*d^2*cos(5*d*x + 5*c) - 48*I*a*d^2*cos(4*d*x + 4*
c) + 192*a*d^2*cos(3*d*x + 3*c) + 48*I*a*d^2*cos(2*d*x + 2*c) + 96*a*d^2*co
s(d*x + c) + 48*a*d^2*sin(6*d*x + 6*c) + 96*I*a*d^2*sin(5*d*x + 5*c) + 48*a
*d^2*sin(4*d*x + 4*c) + 192*I*a*d^2*sin(3*d*x + 3*c) - 48*a*d^2*sin(2*d*x +
 2*c) + 96*I*a*d^2*sin(d*x + c) + 48*I*a*d^2))/d
```

Fricas [C] time = 3.18991, size = 3775, normalized size = 8.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(6*d^2*f^2*x^2 + 12*d^2*e*f*x + 6*d^2*e^2 - 16*(d*f^2*x + d*e*f)*cos(d
*x + c)^3 - 2*(9*d^2*f^2*x^2 + 18*d^2*e*f*x + 9*d^2*e^2 + 2*f^2)*cos(d*x +
c)^2 - 28*(d*f^2*x + d*e*f)*cos(d*x + c) + ((-18*I*d*f^2*x - 18*I*d*e*f)*co
s(d*x + c)^2*sin(d*x + c) + (-18*I*d*f^2*x - 18*I*d*e*f)*cos(d*x + c)^2)*di
log(I*cos(d*x + c) + sin(d*x + c)) + ((-18*I*d*f^2*x - 18*I*d*e*f)*cos(d*x
+ c)^2*sin(d*x + c) + (-18*I*d*f^2*x - 18*I*d*e*f)*cos(d*x + c)^2)*dilog(I*
cos(d*x + c) - sin(d*x + c)) + ((18*I*d*f^2*x + 18*I*d*e*f)*cos(d*x + c)^2*
sin(d*x + c) + (18*I*d*f^2*x + 18*I*d*e*f)*cos(d*x + c)^2)*dilog(-I*cos(d*x
 + c) + sin(d*x + c)) + ((18*I*d*f^2*x + 18*I*d*e*f)*cos(d*x + c)^2*sin(d*x
 + c) + (18*I*d*f^2*x + 18*I*d*e*f)*cos(d*x + c)^2)*dilog(-I*cos(d*x + c) -
 sin(d*x + c)) + ((9*d^2*e^2 - 18*c*d*e*f + (9*c^2 + 28)*f^2)*cos(d*x + c)^
2*sin(d*x + c) + (9*d^2*e^2 - 18*c*d*e*f + (9*c^2 + 28)*f^2)*cos(d*x + c)^2
```

```
)*log(cos(d*x + c) + I*sin(d*x + c) + I) - 3*((3*d^2*e^2 - 6*c*d*e*f + (3*c
^2 + 4)*f^2)*cos(d*x + c)^2*sin(d*x + c) + (3*d^2*e^2 - 6*c*d*e*f + (3*c^2
+ 4)*f^2)*cos(d*x + c)^2)*log(cos(d*x + c) - I*sin(d*x + c) + I) + 9*((d^2*
f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*cos(d*x + c)^2*sin(d*x + c) +
(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*cos(d*x + c)^2)*log(I*cos
(d*x + c) + sin(d*x + c) + 1) - 9*((d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f -
 c^2*f^2)*cos(d*x + c)^2*sin(d*x + c) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*
e*f - c^2*f^2)*cos(d*x + c)^2)*log(I*cos(d*x + c) - sin(d*x + c) + 1) + 9*(
(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*cos(d*x + c)^2*sin(d*x +
c) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*cos(d*x + c)^2)*log(
-I*cos(d*x + c) + sin(d*x + c) + 1) - 9*((d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d
*e*f - c^2*f^2)*cos(d*x + c)^2*sin(d*x + c) + (d^2*f^2*x^2 + 2*d^2*e*f*x +
2*c*d*e*f - c^2*f^2)*cos(d*x + c)^2)*log(-I*cos(d*x + c) - sin(d*x + c) + 1
) + ((9*d^2*e^2 - 18*c*d*e*f + (9*c^2 + 28)*f^2)*cos(d*x + c)^2*sin(d*x + c
) + (9*d^2*e^2 - 18*c*d*e*f + (9*c^2 + 28)*f^2)*cos(d*x + c)^2)*log(-cos(d*
x + c) + I*sin(d*x + c) + I) - 3*((3*d^2*e^2 - 6*c*d*e*f + (3*c^2 + 4)*f^2)
*cos(d*x + c)^2*sin(d*x + c) + (3*d^2*e^2 - 6*c*d*e*f + (3*c^2 + 4)*f^2)*co
s(d*x + c)^2)*log(-cos(d*x + c) - I*sin(d*x + c) + I) - 18*(f^2*cos(d*x + c
)^2*sin(d*x + c) + f^2*cos(d*x + c)^2)*polylog(3, I*cos(d*x + c) + sin(d*x
+ c)) + 18*(f^2*cos(d*x + c)^2*sin(d*x + c) + f^2*cos(d*x + c)^2)*polylog(3
, I*cos(d*x + c) - sin(d*x + c)) - 18*(f^2*cos(d*x + c)^2*sin(d*x + c) + f^
2*cos(d*x + c)^2)*polylog(3, -I*cos(d*x + c) + sin(d*x + c)) + 18*(f^2*cos(
d*x + c)^2*sin(d*x + c) + f^2*cos(d*x + c)^2)*polylog(3, -I*cos(d*x + c) -
sin(d*x + c)) + 2*(9*d^2*f^2*x^2 + 18*d^2*e*f*x + 9*d^2*e^2 - 10*(d*f^2*x +
 d*e*f)*cos(d*x + c))*sin(d*x + c))/(a*d^3*cos(d*x + c)^2*sin(d*x + c) + a*
d^3*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sec(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sec(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sec(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

3.283 $\int \frac{(e+fx)\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=241

$$\frac{3ifPolyLog\left(2,-ie^{i(c+dx)}\right)}{8ad^2}-\frac{3ifPolyLog\left(2,ie^{i(c+dx)}\right)}{8ad^2}+\frac{f\tan^3(c+dx)}{12ad^2}+\frac{f\tan(c+dx)}{4ad^2}-\frac{f\sec^3(c+dx)}{12ad^2}-\frac{3f\sec(c+}{8ad^2}$$

```
[Out] (((-3*I)/4)*(e + f*x)*ArcTan[E^(I*(c + d*x))])/(a*d) + (((3*I)/8)*f*PolyLog
[2, (-I)*E^(I*(c + d*x))])/(a*d^2) - (((3*I)/8)*f*PolyLog[2, I*E^(I*(c + d*
x))])/(a*d^2) - (3*f*Sec[c + d*x])/(8*a*d^2) - (f*Sec[c + d*x]^3)/(12*a*d^2
) - ((e + f*x)*Sec[c + d*x]^4)/(4*a*d) + (f*Tan[c + d*x])/(4*a*d^2) + (3*(e
 + f*x)*Sec[c + d*x]*Tan[c + d*x])/(8*a*d) + ((e + f*x)*Sec[c + d*x]^3*Tan[
c + d*x])/(4*a*d) + (f*Tan[c + d*x]^3)/(12*a*d^2)
```

Rubi [A] time = 0.191473, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4531, 4185, 4181, 2279, 2391, 4409, 3767}

$$\frac{3ifPolyLog\left(2,-ie^{i(c+dx)}\right)}{8ad^2}-\frac{3ifPolyLog\left(2,ie^{i(c+dx)}\right)}{8ad^2}+\frac{f\tan^3(c+dx)}{12ad^2}+\frac{f\tan(c+dx)}{4ad^2}-\frac{f\sec^3(c+dx)}{12ad^2}-\frac{3f\sec(c+}{8ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (((-3*I)/4)*(e + f*x)*ArcTan[E^(I*(c + d*x))])/(a*d) + (((3*I)/8)*f*PolyLog
[2, (-I)*E^(I*(c + d*x))])/(a*d^2) - (((3*I)/8)*f*PolyLog[2, I*E^(I*(c + d*
x))])/(a*d^2) - (3*f*Sec[c + d*x])/(8*a*d^2) - (f*Sec[c + d*x]^3)/(12*a*d^2
) - ((e + f*x)*Sec[c + d*x]^4)/(4*a*d) + (f*Tan[c + d*x])/(4*a*d^2) + (3*(e
 + f*x)*Sec[c + d*x]*Tan[c + d*x])/(8*a*d) + ((e + f*x)*Sec[c + d*x]^3*Tan[
c + d*x])/(4*a*d) + (f*Tan[c + d*x]^3)/(12*a*d^2)
```


Rule 4531

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sec[c +
d*x]^(n + 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Sec[c + d*x]^(n + 1)*Tan[c
 + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2
 - b^2, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)\sec^5(c+dx)\,dx}{a} - \frac{\int (e+fx)\sec^4(c+dx)\tan(c+dx)\,dx}{a}$$

$$= -\frac{f\sec^3(c+dx)}{12ad^2} - \frac{(e+fx)\sec^4(c+dx)}{4ad} + \frac{(e+fx)\sec^3(c+dx)\tan(c+dx)}{4ad} + \frac{3\int (e+f\ldots}{}$$

$$= -\frac{3f\sec(c+dx)}{8ad^2} - \frac{f\sec^3(c+dx)}{12ad^2} - \frac{(e+fx)\sec^4(c+dx)}{4ad} + \frac{3(e+fx)\sec(c+dx)\tan(c\ldots}{8ad}$$

$$= -\frac{3i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} - \frac{3f\sec(c+dx)}{8ad^2} - \frac{f\sec^3(c+dx)}{12ad^2} - \frac{(e+fx)\sec^4(c+dx)}{4ad} + \ldots$$

$$= -\frac{3i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} - \frac{3f\sec(c+dx)}{8ad^2} - \frac{f\sec^3(c+dx)}{12ad^2} - \frac{(e+fx)\sec^4(c+dx)}{4ad} + \ldots$$

$$= -\frac{3i(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{4ad} + \frac{3if\,\mathrm{Li}_2\left(-ie^{i(c+dx)}\right)}{8ad^2} - \frac{3if\,\mathrm{Li}_2\left(ie^{i(c+dx)}\right)}{8ad^2} - \frac{3f\sec(c+dx)}{8ad^2} - \ldots$$

Mathematica [B] time = 6.59522, size = 1171, normalized size = 4.86

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sec[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-6*d*e - f + 6*c*f - 6*f*(c + d*x))/(24*d^2*(a + a*Sin[c + d*x])) + (-(d*e
) + c*f - f*(c + d*x))/(8*d^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a +
a*Sin[c + d*x])) + (f*Sin[(c + d*x)/2])/(12*d^2*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])*(a + a*Sin[c + d*x])) + (7*f*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]))/(12*d^2*(a + a*Sin[c + d*x])) + (3*(c + d*x)*(2*d*e -
2*c*f + f*(c + d*x))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(16*d^2*(a +
a*Sin[c + d*x])) + (3*e*((-c - d*x)/2 - Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(8*d*(a + a*Sin[c + d*x]))
- (3*c*f*((-c - d*x)/2 - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^2)/(8*d^2*(a + a*Sin[c + d*x])) - (3*e*((c +
d*x)/2 - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2)/(8*d*(a + a*Sin[c + d*x])) + (3*c*f*((c + d*x)/2 - Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)
/(8*d^2*(a + a*Sin[c + d*x])) - (3*f*((c + d*x)^2/(4*E^((I/4)*Pi)) - (((-3*
I)/4)*Pi*(c + d*x) - Pi*Log[1 + E^((-I)*(c + d*x))] - 2*(-Pi/4 + (c + d*x)/
2)*Log[1 - E^((2*I)*(-Pi/4 + (c + d*x)/2))] + Pi*Log[Cos[(c + d*x)/2]] - (P
i*Log[-Sin[Pi/4 + (-c - d*x)/2]])/2 + I*PolyLog[2, E^((2*I)*(-Pi/4 + (c + d
*x)/2))])/Sqrt[2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(4*Sqrt[2]*d^2*
```

```
(a + a*Sin[c + d*x])) - (3*f*((E^((I/4)*Pi)*(c + d*x)^2)/4 + ((-I/4)*Pi*(c
+ d*x) - Pi*Log[1 + E^((-I)*(c + d*x))] - 2*(Pi/4 + (c + d*x)/2)*Log[1 - E^
((2*I)*(Pi/4 + (c + d*x)/2))] + Pi*Log[Cos[(c + d*x)/2]] + (Pi*Log[Sin[Pi/4
 + (c + d*x)/2]])/2 + I*PolyLog[2, E^((2*I)*(Pi/4 + (c + d*x)/2))])/Sqrt[2]
)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(4*Sqrt[2]*d^2*(a + a*Sin[c + d*
x])) + ((d*e - c*f + f*(c + d*x))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/
(8*d^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a + a*Sin[c + d*x])) - (f*S
in[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(4*d^2*(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2])*(a + a*Sin[c + d*x]))
```

Maple [B] time = 0.378, size = 483, normalized size = 2.

$$-\frac{\frac{i}{12}\left(18 idfx\,\mathrm{e}^{4 i(dx+c)}+9 dfx\,\mathrm{e}^{5 i(dx+c)}-8 if\,\mathrm{e}^{3 i(dx+c)}+18 ide\,\mathrm{e}^{4 i(dx+c)}+9 de\,\mathrm{e}^{5 i(dx+c)}-18 ide\,\mathrm{e}^{2 i(dx+c)}+6 dfx\,\mathrm{e}^{3 i(dx+c)}-18 i\cdots\right.}{\left(\mathrm{e}^{i(dx+c)}+i\right)^{4} d^{2}\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sec(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/12*I*(18*I*d*f*x*exp(4*I*(d*x+c))+9*d*f*x*exp(5*I*(d*x+c))-8*I*f*exp(3*I
*(d*x+c))+18*I*d*e*exp(4*I*(d*x+c))+9*d*e*exp(5*I*(d*x+c))-18*I*d*e*exp(2*I
*(d*x+c))+6*d*f*x*exp(3*I*(d*x+c))-18*I*d*f*x*exp(2*I*(d*x+c))+I*f*exp(I*(d
*x+c))+6*d*e*exp(3*I*(d*x+c))+18*f*exp(4*I*(d*x+c))+9*d*f*x*exp(I*(d*x+c))-
9*I*f*exp(5*I*(d*x+c))+9*d*e*exp(I*(d*x+c))+22*f*exp(2*I*(d*x+c))+4*f)/(exp
(I*(d*x+c))+I)^4/d^2/(exp(I*(d*x+c))-I)^2/a-3/8/a/d*e*ln(exp(I*(d*x+c))-I)+
3/8/a/d*ln(exp(I*(d*x+c))+I)*e-3/8/a/d*f*ln(1+I*exp(I*(d*x+c)))*x-3/8/a/d^2
*f*ln(1+I*exp(I*(d*x+c)))*c+3/8*I*f*polylog(2,-I*exp(I*(d*x+c)))/a/d^2+3/8/
a/d*f*ln(1-I*exp(I*(d*x+c)))*x+3/8/a/d^2*f*ln(1-I*exp(I*(d*x+c)))*c-3/8*I*f
*polylog(2,I*exp(I*(d*x+c)))/a/d^2+3/8/a/d^2*f*c*ln(exp(I*(d*x+c))-I)-3/8/a
/d^2*f*c*ln(exp(I*(d*x+c))+I)
```

Maxima [B] time = 4.78787, size = 2665, normalized size = 11.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((18*d*e*cos(6*d*x + 6*c) + 36*I*d*e*cos(5*d*x + 5*c) + 18*d*e*cos(4*d*x +
4*c) + 72*I*d*e*cos(3*d*x + 3*c) - 18*d*e*cos(2*d*x + 2*c) + 36*I*d*e*cos(d
*x + c) + 18*I*d*e*sin(6*d*x + 6*c) - 36*d*e*sin(5*d*x + 5*c) + 18*I*d*e*si
n(4*d*x + 4*c) - 72*d*e*sin(3*d*x + 3*c) - 18*I*d*e*sin(2*d*x + 2*c) - 36*d
*e*sin(d*x + c) - 18*d*e)*arctan2(sin(d*x + c) + 1, cos(d*x + c)) - (18*d*e
*cos(6*d*x + 6*c) + 36*I*d*e*cos(5*d*x + 5*c) + 18*d*e*cos(4*d*x + 4*c) + 7
2*I*d*e*cos(3*d*x + 3*c) - 18*d*e*cos(2*d*x + 2*c) + 36*I*d*e*cos(d*x + c)
+ 18*I*d*e*sin(6*d*x + 6*c) - 36*d*e*sin(5*d*x + 5*c) + 18*I*d*e*sin(4*d*x
+ 4*c) - 72*d*e*sin(3*d*x + 3*c) - 18*I*d*e*sin(2*d*x + 2*c) - 36*d*e*sin(d
*x + c) - 18*d*e)*arctan2(sin(d*x + c) - 1, cos(d*x + c)) - (18*d*f*x*cos(6
*d*x + 6*c) + 36*I*d*f*x*cos(5*d*x + 5*c) + 18*d*f*x*cos(4*d*x + 4*c) + 72*
I*d*f*x*cos(3*d*x + 3*c) - 18*d*f*x*cos(2*d*x + 2*c) + 36*I*d*f*x*cos(d*x +
 c) + 18*I*d*f*x*sin(6*d*x + 6*c) - 36*d*f*x*sin(5*d*x + 5*c) + 18*I*d*f*x*
sin(4*d*x + 4*c) - 72*d*f*x*sin(3*d*x + 3*c) - 18*I*d*f*x*sin(2*d*x + 2*c)
- 36*d*f*x*sin(d*x + c) - 18*d*f*x)*arctan2(cos(d*x + c), sin(d*x + c) + 1)
 - (18*d*f*x*cos(6*d*x + 6*c) + 36*I*d*f*x*cos(5*d*x + 5*c) + 18*d*f*x*cos(
4*d*x + 4*c) + 72*I*d*f*x*cos(3*d*x + 3*c) - 18*d*f*x*cos(2*d*x + 2*c) + 36
*I*d*f*x*cos(d*x + c) + 18*I*d*f*x*sin(6*d*x + 6*c) - 36*d*f*x*sin(5*d*x +
5*c) + 18*I*d*f*x*sin(4*d*x + 4*c) - 72*d*f*x*sin(3*d*x + 3*c) - 18*I*d*f*x
*sin(2*d*x + 2*c) - 36*d*f*x*sin(d*x + c) - 18*d*f*x)*arctan2(cos(d*x + c),
 -sin(d*x + c) + 1) - (36*d*f*x + 36*d*e - 36*I*f)*cos(5*d*x + 5*c) + (-72*
I*d*f*x - 72*I*d*e - 72*f)*cos(4*d*x + 4*c) - (24*d*f*x + 24*d*e - 32*I*f)*
cos(3*d*x + 3*c) + (72*I*d*f*x + 72*I*d*e - 88*f)*cos(2*d*x + 2*c) - (36*d*
f*x + 36*d*e + 4*I*f)*cos(d*x + c) - (18*f*cos(6*d*x + 6*c) + 36*I*f*cos(5*
d*x + 5*c) + 18*f*cos(4*d*x + 4*c) + 72*I*f*cos(3*d*x + 3*c) - 18*f*cos(2*d
*x + 2*c) + 36*I*f*cos(d*x + c) + 18*I*f*sin(6*d*x + 6*c) - 36*f*sin(5*d*x
+ 5*c) + 18*I*f*sin(4*d*x + 4*c) - 72*f*sin(3*d*x + 3*c) - 18*I*f*sin(2*d*x
 + 2*c) - 36*f*sin(d*x + c) - 18*f)*dilog(I*e^(I*d*x + I*c)) + (18*f*cos(6*
d*x + 6*c) + 36*I*f*cos(5*d*x + 5*c) + 18*f*cos(4*d*x + 4*c) + 72*I*f*cos(3
*d*x + 3*c) - 18*f*cos(2*d*x + 2*c) + 36*I*f*cos(d*x + c) + 18*I*f*sin(6*d*
x + 6*c) - 36*f*sin(5*d*x + 5*c) + 18*I*f*sin(4*d*x + 4*c) - 72*f*sin(3*d*x
 + 3*c) - 18*I*f*sin(2*d*x + 2*c) - 36*f*sin(d*x + c) - 18*f)*dilog(-I*e^(I
*d*x + I*c)) + (9*I*d*f*x + 9*I*d*e + (-9*I*d*f*x - 9*I*d*e)*cos(6*d*x + 6*
c) + 18*(d*f*x + d*e)*cos(5*d*x + 5*c) + (-9*I*d*f*x - 9*I*d*e)*cos(4*d*x +
 4*c) + 36*(d*f*x + d*e)*cos(3*d*x + 3*c) + (9*I*d*f*x + 9*I*d*e)*cos(2*d*x
 + 2*c) + 18*(d*f*x + d*e)*cos(d*x + c) + 9*(d*f*x + d*e)*sin(6*d*x + 6*c)
+ (18*I*d*f*x + 18*I*d*e)*sin(5*d*x + 5*c) + 9*(d*f*x + d*e)*sin(4*d*x + 4*
c) + (36*I*d*f*x + 36*I*d*e)*sin(3*d*x + 3*c) - 9*(d*f*x + d*e)*sin(2*d*x +
 2*c) + (18*I*d*f*x + 18*I*d*e)*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x
+ c)^2 + 2*sin(d*x + c) + 1) + (-9*I*d*f*x - 9*I*d*e + (9*I*d*f*x + 9*I*d*e
)*cos(6*d*x + 6*c) - 18*(d*f*x + d*e)*cos(5*d*x + 5*c) + (9*I*d*f*x + 9*I*d
*e)*cos(4*d*x + 4*c) - 36*(d*f*x + d*e)*cos(3*d*x + 3*c) + (-9*I*d*f*x - 9*
I*d*e)*cos(2*d*x + 2*c) - 18*(d*f*x + d*e)*cos(d*x + c) - 9*(d*f*x + d*e)*s
in(6*d*x + 6*c) + (-18*I*d*f*x - 18*I*d*e)*sin(5*d*x + 5*c) - 9*(d*f*x + d*
e)*sin(4*d*x + 4*c) + (-36*I*d*f*x - 36*I*d*e)*sin(3*d*x + 3*c) + 9*(d*f*x
```

```
+ d*e)*sin(2*d*x + 2*c) + (-18*I*d*f*x - 18*I*d*e)*sin(d*x + c))*log(cos(d*
x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + (-36*I*d*f*x - 36*I*d*e -
 36*f)*sin(5*d*x + 5*c) + (72*d*f*x + 72*d*e - 72*I*f)*sin(4*d*x + 4*c) + (
-24*I*d*f*x - 24*I*d*e - 32*f)*sin(3*d*x + 3*c) - (72*d*f*x + 72*d*e + 88*I
*f)*sin(2*d*x + 2*c) + (-36*I*d*f*x - 36*I*d*e + 4*f)*sin(d*x + c) - 16*f)/
(-48*I*a*d^2*cos(6*d*x + 6*c) + 96*a*d^2*cos(5*d*x + 5*c) - 48*I*a*d^2*cos(
4*d*x + 4*c) + 192*a*d^2*cos(3*d*x + 3*c) + 48*I*a*d^2*cos(2*d*x + 2*c) + 9
6*a*d^2*cos(d*x + c) + 48*a*d^2*sin(6*d*x + 6*c) + 96*I*a*d^2*sin(5*d*x + 5
*c) + 48*a*d^2*sin(4*d*x + 4*c) + 192*I*a*d^2*sin(3*d*x + 3*c) - 48*a*d^2*s
in(2*d*x + 2*c) + 96*I*a*d^2*sin(d*x + c) + 48*I*a*d^2)
```

Fricas [B] time = 2.48275, size = 2080, normalized size = 8.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(8*f*cos(d*x + c)^3 - 6*d*f*x + 18*(d*f*x + d*e)*cos(d*x + c)^2 - 6*d
*e + 14*f*cos(d*x + c) - (-9*I*f*cos(d*x + c)^2*sin(d*x + c) - 9*I*f*cos(d*
x + c)^2)*dilog(I*cos(d*x + c) + sin(d*x + c)) - (-9*I*f*cos(d*x + c)^2*sin
(d*x + c) - 9*I*f*cos(d*x + c)^2)*dilog(I*cos(d*x + c) - sin(d*x + c)) - (9
*I*f*cos(d*x + c)^2*sin(d*x + c) + 9*I*f*cos(d*x + c)^2)*dilog(-I*cos(d*x +
 c) + sin(d*x + c)) - (9*I*f*cos(d*x + c)^2*sin(d*x + c) + 9*I*f*cos(d*x +
c)^2)*dilog(-I*cos(d*x + c) - sin(d*x + c)) - 9*((d*e - c*f)*cos(d*x + c)^2
*sin(d*x + c) + (d*e - c*f)*cos(d*x + c)^2)*log(cos(d*x + c) + I*sin(d*x +
c) + I) + 9*((d*e - c*f)*cos(d*x + c)^2*sin(d*x + c) + (d*e - c*f)*cos(d*x
+ c)^2)*log(cos(d*x + c) - I*sin(d*x + c) + I) - 9*((d*f*x + c*f)*cos(d*x +
 c)^2*sin(d*x + c) + (d*f*x + c*f)*cos(d*x + c)^2)*log(I*cos(d*x + c) + sin
(d*x + c) + 1) + 9*((d*f*x + c*f)*cos(d*x + c)^2*sin(d*x + c) + (d*f*x + c*
f)*cos(d*x + c)^2)*log(I*cos(d*x + c) - sin(d*x + c) + 1) - 9*((d*f*x + c*f
)*cos(d*x + c)^2*sin(d*x + c) + (d*f*x + c*f)*cos(d*x + c)^2)*log(-I*cos(d*
x + c) + sin(d*x + c) + 1) + 9*((d*f*x + c*f)*cos(d*x + c)^2*sin(d*x + c) +
 (d*f*x + c*f)*cos(d*x + c)^2)*log(-I*cos(d*x + c) - sin(d*x + c) + 1) - 9*
((d*e - c*f)*cos(d*x + c)^2*sin(d*x + c) + (d*e - c*f)*cos(d*x + c)^2)*log(
-cos(d*x + c) + I*sin(d*x + c) + I) + 9*((d*e - c*f)*cos(d*x + c)^2*sin(d*x
 + c) + (d*e - c*f)*cos(d*x + c)^2)*log(-cos(d*x + c) - I*sin(d*x + c) + I)
 - 2*(9*d*f*x + 9*d*e - 5*f*cos(d*x + c))*sin(d*x + c))/(a*d^2*cos(d*x + c)
^2*sin(d*x + c) + a*d^2*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e\sec^3(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{fx\sec^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(e*sec(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(f*x*sec(c + d
*x)**3/(sin(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\sec(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sec(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

3.284 $\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=77

$$-\frac{a}{8d(a\sin(c+dx)+a)^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{1}{4d(a\sin(c+dx)+a)}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(8*a*d) + 1/(8*d*(a - a*Sin[c + d*x])) - a/(8*d*(
a + a*Sin[c + d*x])^2) - 1/(4*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.0798467, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$-\frac{a}{8d(a\sin(c+dx)+a)^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{1}{4d(a\sin(c+dx)+a)}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(8*a*d) + 1/(8*d*(a - a*Sin[c + d*x])) - a/(8*d*(
a + a*Sin[c + d*x])^2) - 1/(4*d*(a + a*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^3\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^2}+\frac{1}{4a^2(a+x)^3}+\frac{1}{4a^3(a+x)^2}+\frac{3}{8a^3(a^2-x^2)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{1}{8d(a-a\sin(c+dx))}-\frac{a}{8d(a+a\sin(c+dx))^2}-\frac{1}{4d(a+a\sin(c+dx))}+\frac{3\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx\right.}{8d}$$
$$= \frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{a}{8d(a+a\sin(c+dx))^2}-\frac{1}{4d(a+a\sin(c+dx}$$

Mathematica [A] time = 0.102742, size = 75, normalized size = 0.97

$$-\frac{\sec^2(c+dx)\left(-3\sin^2(c+dx)-3\sin(c+dx)+3(\sin(c+dx)-1)(\sin(c+dx)+1)^2\tanh^{-1}(\sin(c+dx))+2\right)}{8ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]^2*(2 - 3*Sin[c + d*x] - 3*Sin[c + d*x]^2 + 3*ArcTanh[Sin[c +
 d*x]]*(-1 + Sin[c + d*x])*(1 + Sin[c + d*x])^2))/(8*a*d*(1 + Sin[c + d*x])
)
```

Maple [A] time = 0.058, size = 90, normalized size = 1.2

$$-\frac{1}{8\,da\,(\sin(dx+c)-1)}-\frac{3\,\ln(\sin(dx+c)-1)}{16\,da}-\frac{1}{8\,da\,(1+\sin(dx+c))^2}-\frac{1}{4\,da\,(1+\sin(dx+c))}+\frac{3\,\ln(1+\sin(dx}{16\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/8/a/d/(sin(d*x+c)-1)-3/16/a/d*ln(sin(d*x+c)-1)-1/8/a/d/(1+sin(d*x+c))^2-
1/4/a/d/(1+sin(d*x+c))+3/16*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 0.966444, size = 123, normalized size = 1.6

$$-\frac{\frac{2\left(3 \sin(dx+c)^2+3 \sin(dx+c)-2\right)}{a \sin(dx+c)^3+a \sin(dx+c)^2-a \sin(dx+c)-a}-\frac{3 \log(\sin(dx+c)+1)}{a}+\frac{3 \log(\sin(dx+c)-1)}{a}}{16\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*sin(d*x + c)^2 + 3*sin(d*x + c) - 2)/(a*sin(d*x + c)^3 + a*sin(
d*x + c)^2 - a*sin(d*x + c) - a) - 3*log(sin(d*x + c) + 1)/a + 3*log(sin(d*
x + c) - 1)/a)/d
```

Fricas [A] time = 1.7482, size = 336, normalized size = 4.36

$$-\frac{6 \cos(dx+c)^2 - 3\left(\cos(dx+c)^2 \sin(dx+c) + \cos(dx+c)^2\right)\log(\sin(dx+c)+1) + 3\left(\cos(dx+c)^2 \sin(dx+c) + \cos\right.}{16\left(ad\cos(dx+c)^2 \sin(dx+c) + ad\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(6*cos(d*x + c)^2 - 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*
log(sin(d*x + c) + 1) + 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*lo
g(-sin(d*x + c) + 1) - 6*sin(d*x + c) - 2)/(a*d*cos(d*x + c)^2*sin(d*x + c)
 + a*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^3(c+dx)}{\sin(c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.23488, size = 130, normalized size = 1.69

$$\frac{\frac{6\log(|\sin(dx+c)+1|)}{a}-\frac{6\log(|\sin(dx+c)-1|)}{a}+\frac{2(3\sin(dx+c)-5)}{a(\sin(dx+c)-1)}-\frac{9\sin(dx+c)^2+26\sin(dx+c)+21}{a(\sin(dx+c)+1)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(6*log(abs(sin(d*x + c) + 1))/a - 6*log(abs(sin(d*x + c) - 1))/a + 2*(
3*sin(d*x + c) - 5)/(a*(sin(d*x + c) - 1)) - (9*sin(d*x + c)^2 + 26*sin(d*x
 + c) + 21)/(a*(sin(d*x + c) + 1)^2))/d
```

3.285 $\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sec^3(c+dx)}{(e+fx)(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0721693, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]

[Out] Defer[Int][Sec[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx = \int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 34.3505, size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(e+fx)(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])),x]

[Out] Integrate[Sec[c + d*x]^3/((e + f*x)*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 5.279, size = 0, normalized size = 0.

$$\int \frac{(\sec(dx+c))^3}{(fx+e)(a+a\sin(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(dx+c)^3}{afx+ae+(afx+ae)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^3/(a*f*x + a*e + (a*f*x + a*e)*sin(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^3(c+dx)}{e\sin(c+dx)+e+fx\sin(c+dx)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(f*x+e)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(e*sin(c + d*x) + e + f*x*sin(c + d*x) + f*x), x)/
a
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(f*x+e)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.286 $\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sec^3(c+dx)}{(e+fx)^2(a\sin(c+dx)+a)},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi [A] time = 0.0721515, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx=\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Mathematica [A] time = 51.9916, size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(e+fx)^2(a+a\sin(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])),x]
```

```
[Out] Integrate[Sec[c + d*x]^3/((e + f*x)^2*(a + a*Sin[c + d*x])), x]
```

Maple [A] time = 1.749, size = 0, normalized size = 0.

$$\int \frac{\left(\sec \left(dx+c\right)\right)^{3}}{\left(fx+e\right)^{2}\left(a+a\sin \left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec \left(dx+c\right)^{3}}{af^{2}x^{2}+2\,aefx+ae^{2}+\left(af^{2}x^{2}+2\,aefx+ae^{2}\right)\sin \left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^3/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2 + 2*a*e
*f*x + a*e^2)*sin(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^3(c+dx)}{e^2\sin(c+dx)+e^2+2efx\sin(c+dx)+2efx+f^2x^2\sin(c+dx)+f^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(f*x+e)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(e**2*sin(c + d*x) + e**2 + 2*e*f*x*sin(c + d*x) +
 2*e*f*x + f**2*x**2*sin(c + d*x) + f**2*x**2), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(fx+e)^2(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/((f*x + e)^2*(a*sin(d*x + c) + a)), x)
```

3.287 $\int \frac{(e+fx)^m \cos^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=449

$$\frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\frac{id(e+fx)}{f}\right)}{8ad}-\frac{i2^{-m-3}e^{2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2id(}{}\right.}{ad}$$

```
[Out] (e + f*x)^(1 + m)/(2*a*f*(1 + m)) + (E^(I*(c - (d*e)/f))*(e + f*x)^m*Gamma[
1 + m, ((-I)*d*(e + f*x))/f])/(8*a*d*(((-I)*d*(e + f*x))/f)^m) + ((e + f*x)
^m*Gamma[1 + m, (I*d*(e + f*x))/f])/(8*a*d*E^(I*(c - (d*e)/f))*((I*d*(e + f
*x))/f)^m) - (I*2^(-3 - m)*E^((2*I)*(c - (d*e)/f))*(e + f*x)^m*Gamma[1 + m,
 ((-2*I)*d*(e + f*x))/f])/(a*d*(((-I)*d*(e + f*x))/f)^m) + (I*2^(-3 - m)*(e
 + f*x)^m*Gamma[1 + m, ((2*I)*d*(e + f*x))/f])/(a*d*E^((2*I)*(c - (d*e)/f))
*((I*d*(e + f*x))/f)^m) + (3^(-1 - m)*E^((3*I)*(c - (d*e)/f))*(e + f*x)^m*G
amma[1 + m, ((-3*I)*d*(e + f*x))/f])/(8*a*d*(((-I)*d*(e + f*x))/f)^m) + (3^
(-1 - m)*(e + f*x)^m*Gamma[1 + m, ((3*I)*d*(e + f*x))/f])/(8*a*d*E^((3*I)*(
c - (d*e)/f))*((I*d*(e + f*x))/f)^m)
```

Rubi [A] time = 0.642543, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4523, 3312, 3307, 2181, 4406, 3308}

$$\frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\frac{id(e+fx)}{f}\right)}{8ad}-\frac{i2^{-m-3}e^{2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2id(}{}\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^m*Cos[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (e + f*x)^(1 + m)/(2*a*f*(1 + m)) + (E^(I*(c - (d*e)/f))*(e + f*x)^m*Gamma[
1 + m, ((-I)*d*(e + f*x))/f])/(8*a*d*(((-I)*d*(e + f*x))/f)^m) + ((e + f*x)
^m*Gamma[1 + m, (I*d*(e + f*x))/f])/(8*a*d*E^(I*(c - (d*e)/f))*((I*d*(e + f
*x))/f)^m) - (I*2^(-3 - m)*E^((2*I)*(c - (d*e)/f))*(e + f*x)^m*Gamma[1 + m,
 ((-2*I)*d*(e + f*x))/f])/(a*d*(((-I)*d*(e + f*x))/f)^m) + (I*2^(-3 - m)*(e
 + f*x)^m*Gamma[1 + m, ((2*I)*d*(e + f*x))/f])/(a*d*E^((2*I)*(c - (d*e)/f))
*((I*d*(e + f*x))/f)^m) + (3^(-1 - m)*E^((3*I)*(c - (d*e)/f))*(e + f*x)^m*G
amma[1 + m, ((-3*I)*d*(e + f*x))/f])/(8*a*d*(((-I)*d*(e + f*x))/f)^m) + (3^
(-1 - m)*(e + f*x)^m*Gamma[1 + m, ((3*I)*d*(e + f*x))/f])/(8*a*d*E^((3*I)*(
c - (d*e)/f))*((I*d*(e + f*x))/f)^m)
```

Rule 4523

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{(e+fx)^m \cos^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^m \cos^2(c+dx)\,dx}{a} - \frac{\int (e+fx)^m \cos^2(c+dx)\sin(c+dx)\,dx}{a}$$
$$= \frac{\int \left(\frac{1}{2}(e+fx)^m + \frac{1}{2}(e+fx)^m \cos(2c+2dx)\right) dx}{a} - \frac{\int \left(\frac{1}{4}(e+fx)^m \sin(c+dx) + \frac{1}{4}(e+fx)^{m}\right.}{a}$$
$$= \frac{(e+fx)^{1+m}}{2af(1+m)} - \frac{\int (e+fx)^m \sin(c+dx)\,dx}{4a} - \frac{\int (e+fx)^m \sin(3c+3dx)\,dx}{4a} + \frac{\int (e+fx)^m \mathrm{c}}{}$$
$$= \frac{(e+fx)^{1+m}}{2af(1+m)} - \frac{i\int e^{-i(c+dx)}(e+fx)^m\,dx}{8a} + \frac{i\int e^{i(c+dx)}(e+fx)^m\,dx}{8a} - \frac{i\int e^{-i(3c+3dx)}(e+fx)^{m}}{8a}$$
$$= \frac{(e+fx)^{1+m}}{2af(1+m)} + \frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m \left(-\frac{id(e+fx)}{f}\right)^{-m} \Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{8ad} + \frac{e^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m \left(\frac{id}{}\right.}{}$$

Mathematica [A] time = 4.71712, size = 405, normalized size = 0.9

$$\frac{i(e+fx)^m \left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2 \left(-3ie^{i\left(c-\frac{de}{f}\right)}\left(-\frac{id(e+fx)}{f}\right)^{-m} \mathrm{Gamma}\left(m+1,-\frac{id(e+fx)}{f}\right) - 3\, 2^{-m} e^{2i\left(c-\frac{de}{f}\right)}\left(-\frac{i}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^m*Cos[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((I/24)*(e + f*x)^m*(((-12*I)*d*(e + f*x))/(f*(1 + m)) - ((3*I)*E^(I*(c - (
d*e)/f))*Gamma[1 + m, ((-I)*d*(e + f*x))/f])/(((-I)*d*(e + f*x))/f)^m - ((3
*I)*Gamma[1 + m, (I*d*(e + f*x))/f])/(E^(I*(c - (d*e)/f))*((I*d*(e + f*x))/
f)^m) - (3*E^((2*I)*(c - (d*e)/f))*Gamma[1 + m, ((-2*I)*d*(e + f*x))/f])/(2
^m*(((-I)*d*(e + f*x))/f)^m) + (3*Gamma[1 + m, ((2*I)*d*(e + f*x))/f])/(2^m
*E^((2*I)*(c - (d*e)/f))*((I*d*(e + f*x))/f)^m) - (I*E^((3*I)*(c - (d*e)/f)
)*Gamma[1 + m, ((-3*I)*d*(e + f*x))/f])/(3^m*(((-I)*d*(e + f*x))/f)^m) - (I
*Gamma[1 + m, ((3*I)*d*(e + f*x))/f])/(3^m*E^((3*I)*(c - (d*e)/f))*((I*d*(e
 + f*x))/f)^m))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(a*d*(1 + Sin[c +
d*x]))
```

Maple [F] time = 0.199, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^m (\cos(dx+c))^4}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \cos(dx+c)^4}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 1.99043, size = 861, normalized size = 1.92

$$\frac{(fm+f)e^{\left(-\frac{fm\log\left(\frac{3id}{f}\right)-3ide+3icf}{f}\right)}\Gamma\left(m+1,\frac{3idfx+3ide}{f}\right)+(3ifm+3if)e^{\left(-\frac{fm\log\left(\frac{2id}{f}\right)-2ide+2icf}{f}\right)}\Gamma\left(m+1,\frac{2idfx+2ide}{f}\right)+3\,(fm+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*((f*m + f)*e^(-(f*m*log(3*I*d/f) - 3*I*d*e + 3*I*c*f)/f)*gamma(m + 1,
(3*I*d*f*x + 3*I*d*e)/f) + (3*I*f*m + 3*I*f)*e^(-(f*m*log(2*I*d/f) - 2*I*d*
e + 2*I*c*f)/f)*gamma(m + 1, (2*I*d*f*x + 2*I*d*e)/f) + 3*(f*m + f)*e^(-(f*
m*log(I*d/f) - I*d*e + I*c*f)/f)*gamma(m + 1, (I*d*f*x + I*d*e)/f) + 3*(f*m
 + f)*e^(-(f*m*log(-I*d/f) + I*d*e - I*c*f)/f)*gamma(m + 1, (-I*d*f*x - I*d
*e)/f) + (-3*I*f*m - 3*I*f)*e^(-(f*m*log(-2*I*d/f) + 2*I*d*e - 2*I*c*f)/f)*
gamma(m + 1, (-2*I*d*f*x - 2*I*d*e)/f) + (f*m + f)*e^(-(f*m*log(-3*I*d/f) +
 3*I*d*e - 3*I*c*f)/f)*gamma(m + 1, (-3*I*d*f*x - 3*I*d*e)/f) + 12*(d*f*x +
 d*e)*(f*x + e)^m)/(a*d*f*m + a*d*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*cos(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \cos(dx+c)^4}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

3.288 $\int \frac{(e+fx)^m \cos^3(c+dx)}{a+a\sin(c+dx)} dx$

Optimal. Leaf size=277

$$-\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\frac{id(e+fx)}{f}\right)}{2ad}+\frac{2^{-m-3}e^{2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\right.}{ad}$$

```
[Out] ((-I/2)*E^(I*(c - (d*e)/f))*(e + f*x)^m*Gamma[1 + m, ((-I)*d*(e + f*x))/f])
/(a*d*(((-I)*d*(e + f*x))/f)^m) + ((I/2)*(e + f*x)^m*Gamma[1 + m, (I*d*(e +
 f*x))/f])/(a*d*E^(I*(c - (d*e)/f))*((I*d*(e + f*x))/f)^m) + (2^(-3 - m)*E^
((2*I)*(c - (d*e)/f))*(e + f*x)^m*Gamma[1 + m, ((-2*I)*d*(e + f*x))/f])/(a*
d*(((-I)*d*(e + f*x))/f)^m) + (2^(-3 - m)*(e + f*x)^m*Gamma[1 + m, ((2*I)*d
*(e + f*x))/f])/(a*d*E^((2*I)*(c - (d*e)/f))*((I*d*(e + f*x))/f)^m)
```

Rubi [A] time = 0.319053, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4523, 3307, 2181, 4406, 12, 3308}

$$-\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\frac{id(e+fx)}{f}\right)}{2ad}+\frac{2^{-m-3}e^{2i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^m*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] ((-I/2)*E^(I*(c - (d*e)/f))*(e + f*x)^m*Gamma[1 + m, ((-I)*d*(e + f*x))/f])
/(a*d*(((-I)*d*(e + f*x))/f)^m) + ((I/2)*(e + f*x)^m*Gamma[1 + m, (I*d*(e +
 f*x))/f])/(a*d*E^(I*(c - (d*e)/f))*((I*d*(e + f*x))/f)^m) + (2^(-3 - m)*E^
((2*I)*(c - (d*e)/f))*(e + f*x)^m*Gamma[1 + m, ((-2*I)*d*(e + f*x))/f])/(a*
d*(((-I)*d*(e + f*x))/f)^m) + (2^(-3 - m)*(e + f*x)^m*Gamma[1 + m, ((2*I)*d
*(e + f*x))/f])/(a*d*E^((2*I)*(c - (d*e)/f))*((I*d*(e + f*x))/f)^m)
```


Rule 4523

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{(e+fx)^m\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^m\cos(c+dx)\,dx}{a} - \frac{\int (e+fx)^m\cos(c+dx)\sin(c+dx)\,dx}{a}$$
$$= \frac{\int e^{-i(c+dx)}(e+fx)^m\,dx}{2a} + \frac{\int e^{i(c+dx)}(e+fx)^m\,dx}{2a} - \frac{\int \frac{1}{2}(e+fx)^m\sin(2c+2dx)\,dx}{a}$$
$$= -\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{2ad} + \frac{ie^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma}{2ad}$$
$$= -\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{2ad} + \frac{ie^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma}{2ad}$$
$$= -\frac{ie^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{2ad} + \frac{ie^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\Gamma}{2ad}$$

Mathematica [A] time = 2.51116, size = 253, normalized size = 0.91

$$\frac{2^{-m-3}e^{-\frac{2i(cf+de)}{f}}(e+fx)^m\left(\frac{d^2(e+fx)^2}{f^2}\right)^{-m}\left(i2^{m+2}e^{i\left(c+\frac{3de}{f}\right)}\left(-\frac{id(e+fx)}{f}\right)^m\text{Gamma}\left(m+1,\frac{id(e+fx)}{f}\right)-i2^{m+2}e^{i\left(3c+\frac{de}{f}\right)}\left(\frac{id(e+fx)}{f}\right)^m\text{G}\right.}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^m*Cos[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2^(-3 - m)*(e + f*x)^m*((-I)*2^(2 + m)*E^(I*(3*c + (d*e)/f))*((I*d*(e + f*
x))/f)^m*Gamma[1 + m, ((-I)*d*(e + f*x))/f] + I*2^(2 + m)*E^(I*(c + (3*d*e)
/f))*(((-I)*d*(e + f*x))/f)^m*Gamma[1 + m, (I*d*(e + f*x))/f] + E^((4*I)*c)
*((I*d*(e + f*x))/f)^m*Gamma[1 + m, ((-2*I)*d*(e + f*x))/f] + E^(((4*I)*d*e
)/f)*(((-I)*d*(e + f*x))/f)^m*Gamma[1 + m, ((2*I)*d*(e + f*x))/f]))/(a*d*E^
(((2*I)*(d*e + c*f))/f)*((d^2*(e + f*x)^2)/f^2)^m)
```

Maple [F] time = 0.19, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m(\cos(dx+c))^3}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \cos(dx+c)^3}{a \sin(dx+c)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 1.9995, size = 468, normalized size = 1.69

$$\frac{e^{\left(-\frac{fm\log\left(\frac{2id}{f}\right)-2ide+2icf}{f}\right)}\Gamma\left(m+1,\frac{2idfx+2ide}{f}\right)+4ie^{\left(-\frac{fm\log\left(\frac{id}{f}\right)-ide+icf}{f}\right)}\Gamma\left(m+1,\frac{idfx+ide}{f}\right)-4ie^{\left(-\frac{fm\log\left(-\frac{id}{f}\right)+ide-icf}{f}\right)}\Gamma\left(m+1,\frac{-idf}{}\right.}{8ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(e^(-(f*m*log(2*I*d/f) - 2*I*d*e + 2*I*c*f)/f)*gamma(m + 1, (2*I*d*f*x
+ 2*I*d*e)/f) + 4*I*e^(-(f*m*log(I*d/f) - I*d*e + I*c*f)/f)*gamma(m + 1, (I
*d*f*x + I*d*e)/f) - 4*I*e^(-(f*m*log(-I*d/f) + I*d*e - I*c*f)/f)*gamma(m +
 1, (-I*d*f*x - I*d*e)/f) + e^(-(f*m*log(-2*I*d/f) + 2*I*d*e - 2*I*c*f)/f)*
gamma(m + 1, (-2*I*d*f*x - 2*I*d*e)/f))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \cos(dx+c)^3}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

3.289 $\int \frac{(e+fx)^m \cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\frac{id(e+fx)}{f}\right)}{2ad}+\frac{e^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,\frac{id(e+fx)}{f}\right)}{2ad}+$$

```
[Out] (e + f*x)^(1 + m)/(a*f*(1 + m)) + (E^(I*(c - (d*e)/f))*(e + f*x)^m*Gamma[1
+ m, ((-I)*d*(e + f*x))/f])/(2*a*d*(((-I)*d*(e + f*x))/f)^m) + ((e + f*x)^m
*Gamma[1 + m, (I*d*(e + f*x))/f])/(2*a*d*E^(I*(c - (d*e)/f))*((I*d*(e + f*x
))/f)^m)
```

Rubi [A] time = 0.176591, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4523, 32, 3308, 2181}

$$\frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,-\frac{id(e+fx)}{f}\right)}{2ad}+\frac{e^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\frac{id(e+fx)}{f}\right)^{-m}\text{Gamma}\left(m+1,\frac{id(e+fx)}{f}\right)}{2ad}+$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^m*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (e + f*x)^(1 + m)/(a*f*(1 + m)) + (E^(I*(c - (d*e)/f))*(e + f*x)^m*Gamma[1
+ m, ((-I)*d*(e + f*x))/f])/(2*a*d*(((-I)*d*(e + f*x))/f)^m) + ((e + f*x)^m
*Gamma[1 + m, (I*d*(e + f*x))/f])/(2*a*d*E^(I*(c - (d*e)/f))*((I*d*(e + f*x
))/f)^m)
```


Rule 4523

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cos[c + d
*x]^(n - 2), x], x] - Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Sin[c
+ d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2
- b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int (e+fx)^m\,dx}{a} - \frac{\int (e+fx)^m \sin(c+dx)\,dx}{a}$$
$$= \frac{(e+fx)^{1+m}}{af(1+m)} - \frac{i\int e^{-i(c+dx)}(e+fx)^m\,dx}{2a} + \frac{i\int e^{i(c+dx)}(e+fx)^m\,dx}{2a}$$
$$= \frac{(e+fx)^{1+m}}{af(1+m)} + \frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(-\frac{id(e+fx)}{f}\right)^{-m}\Gamma\left(1+m,-\frac{id(e+fx)}{f}\right)}{2ad} + \frac{e^{-i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\right.}{}$$

Mathematica [A] time = 0.983611, size = 220, normalized size = 1.43

$$\frac{e^{i\left(c-\frac{de}{f}\right)}(e+fx)^m\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(\frac{d^2(e+fx)^2}{f^2}\right)^{-m}\left(f(m+1)e^{-2i\left(c-\frac{de}{f}\right)}\left(-\frac{id(e+fx)}{f}\right)^m \text{Gamma}\left(m+1,\frac{id(\right.\right.}{2adf(m+1)(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^m*Cos[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (E^(I*(c - (d*e)/f))*(e + f*x)^m*((2*d*(e + f*x)*((d^2*(e + f*x)^2)/f^2)^m)
/E^(I*(c - (d*e)/f)) + f*(1 + m)*((I*d*(e + f*x))/f)^m*Gamma[1 + m, ((-I)*d
*(e + f*x))/f] + (f*(1 + m)*(((-I)*d*(e + f*x))/f)^m*Gamma[1 + m, (I*d*(e +
```

```
 f*x))/f])/E^((2*I)*(c - (d*e)/f)))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2
)/(2*a*d*f*(1 + m)*((d^2*(e + f*x)^2)/f^2)^m*(1 + Sin[c + d*x]))
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \left(\cos \left(dx+c\right)\right)^{2}}{a+a\sin \left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \cos \left(dx+c\right)^{2}}{a\sin \left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 1.8099, size = 306, normalized size = 1.99

$$\frac{\left(fm+f\right)e^{\left(-\frac{fm\log\left(\frac{id}{f}\right)-ide+icf}{f}\right)}\Gamma\left(m+1,\frac{idfx+ide}{f}\right)+\left(fm+f\right)e^{\left(-\frac{fm\log\left(-\frac{id}{f}\right)+ide-icf}{f}\right)}\Gamma\left(m+1,\frac{-idfx-ide}{f}\right)+2\left(dfx+de\right)\left(fx+e\right)}{2\left(adfm+adf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((f*m + f)*e^(-(f*m*log(I*d/f) - I*d*e + I*c*f)/f)*gamma(m + 1, (I*d*f*
x + I*d*e)/f) + (f*m + f)*e^(-(f*m*log(-I*d/f) + I*d*e - I*c*f)/f)*gamma(m
+ 1, (-I*d*f*x - I*d*e)/f) + 2*(d*f*x + d*e)*(f*x + e)^m)/(a*d*f*m + a*d*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \cos(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

3.290 $\int \frac{(e+fx)^m\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\cos(c+dx)(e+fx)^m}{a\sin(c+dx)+a},x\right)$$

```
[Out] Unintegrable[((e + f*x)^m*Cos[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0437905, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(e+fx)^m\cos(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((e + f*x)^m*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Defer[Int][((e + f*x)^m*Cos[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m\cos(c+dx)}{a+a\sin(c+dx)}\,dx$$

Mathematica [A] time = 7.7043, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m\cos(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Cos[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Integrate[((e + f*x)^m*Cos[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Maple [A] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \cos\left(dx+c\right)}{a+a\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \cos\left(dx+c\right)}{a\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx+e\right)^{m} \cos\left(dx+c\right)}{a\sin\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(e+fx)^m \cos(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m*cos(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \cos(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.291 $\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(e+fx)^m}{a\sin(c+dx)+a},x\right)$$

```
[Out] Unintegrable[(e + f*x)^m/(a + a*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0636119, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)^m/(a + a*Sin[c + d*x]),x]
```

```
[Out] Defer[Int][(e + f*x)^m/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx = \int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$$

Mathematica [A] time = 0.332647, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^m/(a + a*Sin[c + d*x]),x]
```

```
[Out] Integrate[(e + f*x)^m/(a + a*Sin[c + d*x]), x]
```

Maple [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m/(a+a*sin(d*x+c)),x)

[Out] int((f*x+e)^m/(a+a*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] integrate((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^m}{a\sin(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] integral((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(e+fx)^m}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m/(a*sin(d*x + c) + a), x)
```

3.292 $\int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sec(c+dx)(e+fx)^m}{a\sin(c+dx)+a},x\right)$$

```
[Out] Unintegrable[((e + f*x)^m*Sec[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi [A] time = 0.044138, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((e + f*x)^m*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Defer[Int][((e + f*x)^m*Sec[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx$$

Mathematica [A] time = 9.26743, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Sec[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Integrate[((e + f*x)^m*Sec[c + d*x])/(a + a*Sin[c + d*x]), x]
```

Maple [A] time = 0.126, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \sec \left(dx+c\right)}{a+a\sin \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx+e\right)^{m} \sec \left(dx+c\right)}{a\sin \left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*sec(d*x + c)/(a*sin(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\left(e+fx\right)^{m} \sec \left(c+dx\right)}{\sin \left(c+dx\right)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m*sec(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \sec(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*sec(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.293 $\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sec^2(c+dx)(e+fx)^m}{a\sin(c+dx)+a},x\right)$$

```
[Out] Unintegrable[((e + f*x)^m*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0735751, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((e + f*x)^m*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Defer[Int][((e + f*x)^m*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx=\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

Mathematica [A] time = 13.5437, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Integrate[((e + f*x)^m*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]), x]
```

Maple [A] time = 0.165, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \left(\sec \left(dx+c\right)\right)^{2}}{a+a \sin \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^{m} \sec \left(dx+c\right)^{2}}{a \sin \left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*sec(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \sec(dx+c)^2}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*sec(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

3.294 $\int \frac{(e+fx)^3 \cos(c+dx)}{a+b\sin(c+dx)}\, dx$

Optimal. Leaf size=432

$$\frac{6f^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}+\frac{6f^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3}-\frac{3if(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{3if(e+}{}$$

```
[Out] ((-I/4)*(e + f*x)^4)/(b*f) + ((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a
+ Sqrt[a^2 - b^2])])/(b*d) - ((3*I)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c +
 d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^2) - ((3*I)*f*(e + f*x)^2*PolyLog[2, (
I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^2) + (6*f^2*(e + f*x)*Pol
yLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^3) + (6*f^2*(e +
 f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^3) + ((
6*I)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^4) +
 ((6*I)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^4
)
```

Rubi [A] time = 0.607751, antiderivative size = 432, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4519, 2190, 2531, 6609, 2282, 6589}

$$\frac{6f^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}+\frac{6f^2(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3}-\frac{3if(e+fx)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{3if(e+}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```


```
[Out] ((-I/4)*(e + f*x)^4)/(b*f) + ((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a
+ Sqrt[a^2 - b^2])])/(b*d) - ((3*I)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c +
 d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^2) - ((3*I)*f*(e + f*x)^2*PolyLog[2, (
I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^2) + (6*f^2*(e + f*x)*Pol
yLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^3) + (6*f^2*(e +
 f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^3) + ((
6*I)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^4) +
 ((6*I)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^4
)
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{i(e+fx)^4}{4bf} + \int \frac{e^{i(c+dx)}(e+fx)^3}{a-\sqrt{a^2-b^2}-ibe^{i(c+dx)}}\,dx + \int \frac{e^{i(c+dx)}(e+fx)^3}{a+\sqrt{a^2-b^2}-ibe^{i(c+dx)}}\,dx$$

$$= -\frac{i(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{(3f)\int(e+fx)}{}$$

$$= -\frac{i(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{3if(e+fx)^2\mathrm{L}}{bd}$$

$$= -\frac{i(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{3if(e+fx)^2\mathrm{L}}{bd}$$

$$= -\frac{i(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{3if(e+fx)^2\mathrm{L}}{bd}$$

$$= -\frac{i(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{3if(e+fx)^2\mathrm{L}}{bd}$$

Mathematica [A] time = 0.186359, size = 410, normalized size = 0.95

$$\frac{\frac{12f\left(2f\left(d(e+fx)\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)+if\,\mathrm{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)\right)-id^2(e+fx)^2\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)\right)}{d^4} + \frac{12f\left(2f\left(d(e+fx)\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)+if\,\mathrm{PolyLog}\left(\right.\right.\right.}{d^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((-I)*(e + f*x)^4)/f + (4*(e + f*x)^3*Log[1 + (I*b*E^(I*(c + d*x)))/(-a +
Sqrt[a^2 - b^2])])/d + (4*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sq
rt[a^2 - b^2])])/d + (12*f*((-I)*d^2*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c +
d*x)))/(a - Sqrt[a^2 - b^2])] + 2*f*(d*(e + f*x)*PolyLog[3, (I*b*E^(I*(c +
d*x)))/(a - Sqrt[a^2 - b^2])] + I*f*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])))/d^4 + (12*f*((-I)*d^2*(e + f*x)^2*PolyLog[2, (I*b*E^(I*
(c + d*x)))/(a + Sqrt[a^2 - b^2])] + 2*f*(d*(e + f*x)*PolyLog[3, (I*b*E^(I*
(c + d*x)))/(a + Sqrt[a^2 - b^2])] + I*f*PolyLog[4, (I*b*E^(I*(c + d*x)))/(
```

```
a + Sqrt[a^2 - b^2])])))/d^4)/(4*b)
```

Maple [F] time = 0.921, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cos(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.98501, size = 4316, normalized size = 9.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*f^3*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*c
os(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*I*f^3*polylo
g(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*I*f^3*polylog(4, -(I*a*cos(d*x
+ c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^
2)/b^2))/b) - 6*I*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*c
```

```
os(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + (3*I*d^2*f^3*x
^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^2*f)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*
sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
 + 2*b)/b + 1) + (3*I*d^2*f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^2*f)*dilog(
-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-3*I*d^2*f^3*x^2 - 6*I*d^2
*e*f^2*x - 3*I*d^2*e^2*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c
) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b +
 1) + (-3*I*d^2*f^3*x^2 - 6*I*d^2*e*f^2*x - 3*I*d^2*e^2*f)*dilog(-1/2*(-2*I
*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e
*f^2 - c^3*f^3)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2
- b^2)/b^2) + 2*I*a) + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*
log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*
I*a) + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(-2*b*cos(d*x
 + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (d^3*e^3
 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(-2*b*cos(d*x + c) - 2*I*b*s
in(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (d^3*f^3*x^3 + 3*d^3*e*
f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*log(1/2*
(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d
^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*log(1/2*(2*I*a*cos(d*
x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2) + 2*b)/b) + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x +
3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a
*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
) + 2*b)/b) + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*
f - 3*c^2*d*e*f^2 + c^3*f^3)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c
) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)
+ 6*(d*f^3*x + d*e*f^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x +
c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(
d*f^3*x + d*e*f^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) -
2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(d*f^3
*x + d*e*f^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x +
 c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(d*f^3*x + d*e*f^2)*
polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b))/(b*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3\cos(dx+c)}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

3.295 $\int \frac{(e+fx)^2 \cos(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=320

$$-\frac{2if(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{2if(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}+\frac{2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}+\frac{2f^2\text{PolyLog}\left(3\right.}{bd^3}$$

```
[Out] ((-I/3)*(e + f*x)^3)/(b*f) + ((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a
+ Sqrt[a^2 - b^2])])/(b*d) - ((2*I)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^2) - ((2*I)*f*(e + f*x)*PolyLog[2, (I*b*
E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^2) + (2*f^2*PolyLog[3, (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^3) + (2*f^2*PolyLog[3, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^3)
```

Rubi [A] time = 0.512569, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4519, 2190, 2531, 2282, 6589}

$$-\frac{2if(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{2if(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}+\frac{2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}+\frac{2f^2\text{PolyLog}\left(3\right.}{bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-I/3)*(e + f*x)^3)/(b*f) + ((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a
+ Sqrt[a^2 - b^2])])/(b*d) - ((2*I)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^2) - ((2*I)*f*(e + f*x)*PolyLog[2, (I*b*
E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^2) + (2*f^2*PolyLog[3, (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d^3) + (2*f^2*PolyLog[3, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*d^3)
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
```

```
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{i(e+fx)^3}{3bf} + \int \frac{e^{i(c+dx)}(e+fx)^2}{a-\sqrt{a^2-b^2}-ibe^{i(c+dx)}}\,dx + \int \frac{e^{i(c+dx)}(e+fx)^2}{a+\sqrt{a^2-b^2}-ibe^{i(c+dx)}}\,dx$$

$$= -\frac{i(e+fx)^3}{3bf} + \frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{(2f)\int(e+fx}{}$$

$$= -\frac{i(e+fx)^3}{3bf} + \frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{2if(e+fx)\mathrm{Li}}{bd^2}$$

$$= -\frac{i(e+fx)^3}{3bf} + \frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{2if(e+fx)\mathrm{Li}}{bd^2}$$

$$= -\frac{i(e+fx)^3}{3bf} + \frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{2if(e+fx)\mathrm{Li}}{bd^2}$$

Mathematica [A] time = 0.172019, size = 302, normalized size = 0.94

$$\frac{\frac{6f\left(f\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)-id(e+fx)\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)\right)}{d^3} + \frac{6f\left(f\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)-id(e+fx)\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)\right)}{d^3} + \frac{3(e+fx)^2\log\left(1+\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}-a}\right)}{d}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((-I)*(e + f*x)^3)/f + (3*(e + f*x)^2*Log[1 + (I*b*E^(I*(c + d*x)))/(-a +
Sqrt[a^2 - b^2])])/d + (3*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sq
rt[a^2 - b^2])])/d + (6*f*((-I)*d*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x))
)/(a - Sqrt[a^2 - b^2])] + f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
 - b^2])]))/d^3 + (6*f*((-I)*d*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])] + f*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
b^2])]))/d^3)/(3*b)
```

Maple [F] time = 0.716, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2\cos(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.6733, size = 3082, normalized size = 9.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*f^2*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos
(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*f^2*polylog(3,
 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*f^2*polylog(3, -(I*a*cos(d*x + c) +
 a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
))/b) + 2*f^2*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x +
 c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + (2*I*d*f^2*x + 2*I*d*e
*f)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) -
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (2*I*d*f^2*x + 2*
I*d*e*f)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x +
 c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-2*I*d*f^2*
x - 2*I*d*e*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*co
s(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-2*I
*d*f^2*x - 2*I*d*e*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) -
2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)
```

```
+ (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c)
 + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*lo
g(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*
a) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x
+ c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2
)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
 2*I*a) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(1/2*(2*I*a*
cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) + 2*b)/b) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^
2*f^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) -
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (d^2*f^2*x^2 + 2*d^2*
e*f*x + 2*c*d*e*f - c^2*f^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c
) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)
+ (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(1/2*(-2*I*a*cos(d*x
 + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) + 2*b)/b))/(b*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cos(dx+c)}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

3.296 $\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=212

$$-\frac{if\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{if\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e$$

```
[Out] ((-I/2)*(e + f*x)^2)/(b*f) + ((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a -
Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sq
rt[a^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^
2 - b^2])])/(b*d^2) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
 b^2])])/(b*d^2)
```

Rubi [A] time = 0.284708, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4519, 2190, 2279, 2391}

$$-\frac{if\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{if\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}-\frac{i(e$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-I/2)*(e + f*x)^2)/(b*f) + ((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a -
Sqrt[a^2 - b^2])])/(b*d) + ((e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sq
rt[a^2 - b^2])])/(b*d) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^
2 - b^2])])/(b*d^2) - (I*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
 b^2])])/(b*d^2)
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{i(e+fx)^2}{2bf} + \int \frac{e^{i(c+dx)}(e+fx)}{a-\sqrt{a^2-b^2}-ibe^{i(c+dx)}}\,dx + \int \frac{e^{i(c+dx)}(e+fx)}{a+\sqrt{a^2-b^2}-ibe^{i(c+dx)}}\,dx$$

$$= -\frac{i(e+fx)^2}{2bf} + \frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{f\int \log\left(1-\frac{ibe^{i(c}}{a-\sqrt{a}}\right.}{bd}$$

$$= -\frac{i(e+fx)^2}{2bf} + \frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{(if)\,\mathrm{Subst}\left(\int \frac{\log}{}\right.}{}$$

$$= -\frac{i(e+fx)^2}{2bf} + \frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{if\,\mathrm{Li}_2\left(\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}$$

Mathematica [A] time = 0.0485715, size = 197, normalized size = 0.93

$$-\frac{i\left(2f^2\mathrm{PolyLog}\left(2,-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}-a}\right)+2f^2\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)+d(e+fx)\left(2if\log\left(1+\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}-a}\right)+2if\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)\right.\right.}{2bd^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-I/2)*(d*(e + f*x)*(d*e + d*f*x + (2*I)*f*Log[1 + (I*b*E^(I*(c + d*x)))/(
-a + Sqrt[a^2 - b^2])] + (2*I)*f*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^
2 - b^2])]) + 2*f^2*PolyLog[2, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^
2])] + 2*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]))/(b*d
^2*f)
```

Maple [B] time = 0.162, size = 1006, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] I/b*e*x+I/b/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/
(I*a+(-a^2+b^2)^(1/2)))*a^2-1/b/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a
^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2*x-1/b/d^2*f/(-a^2+b^2)*ln((I*a+b
*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2*c-1/b/d*f/(-a
^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*
a^2*x-1/b/d^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-
(-a^2+b^2)^(1/2)))*a^2*c-1/b/d^2*f*c*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x
+c))-I*b)-1/2*I/b*f*x^2-I*b/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-
a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2*I/b/d*f*c*x+b/d*f/(-a^2+b^2)*ln((
I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+b/d^2*f/(-
a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))
*c+b/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b
^2)^(1/2)))*x+b/d^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))
/(I*a+(-a^2+b^2)^(1/2)))*c-I/b/d^2*f*c^2-I*b/d^2*f/(-a^2+b^2)*dilog((I*a+b*
exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))+2/b/d^2*f*c*ln(exp
(I*(d*x+c)))+I/b/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1
/2))/(I*a-(-a^2+b^2)^(1/2)))*a^2+1/b/d*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*
(d*x+c))-I*b)-2/b/d*ln(exp(I*(d*x+c)))*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.87564, size = 1986, normalized size = 9.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*f*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x +
 c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + I*f*dilog(-1
/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - I*f*dilog(-1/2*(-2*I*a*cos(d*
x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2) + 2*b)/b + 1) - I*f*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin
(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) +
2*b)/b + 1) + (d*e - c*f)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*s
qrt(-(a^2 - b^2)/b^2) + 2*I*a) + (d*e - c*f)*log(2*b*cos(d*x + c) - 2*I*b*s
in(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (d*e - c*f)*log(-2*b*co
s(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (d*
e - c*f)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)
/b^2) - 2*I*a) + (d*f*x + c*f)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x +
c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)
 + (d*f*x + c*f)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(
d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (d*f*x + c*
f)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*
b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (d*f*x + c*f)*log(1/2*(-
2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b))/(b*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cos(dx+c)}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

3.297 $\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(a+b\sin(c+dx))}{bd}$$

```
[Out] Log[a + b*Sin[c + d*x]]/(b*d)
```

Rubi [A] time = 0.0263503, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 31}

$$\frac{\log(a+b\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[a + b*Sin[c + d*x]]/(b*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{\log(a+b\sin(c+dx))}{bd}\end{aligned}$$

Mathematica [A] time = 0.0067264, size = 18, normalized size = 1.

$$\frac{\log(a+b\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[a + b*Sin[c + d*x]]/(b*d)
```

Maple [A] time = 0., size = 19, normalized size = 1.1

$$\frac{\ln\left(a+b\sin\left(dx+c\right)\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] ln(a+b*sin(d*x+c))/b/d
```

Maxima [A] time = 0.952665, size = 24, normalized size = 1.33

$$\frac{\log\left(b\sin\left(dx+c\right)+a\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(b*sin(d*x + c) + a)/(b*d)
```

Fricas [A] time = 1.57402, size = 42, normalized size = 2.33

$$\frac{\log\left(b\sin\left(dx+c\right)+a\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] log(b*sin(d*x + c) + a)/(b*d)
```

Sympy [A] time = 0.579853, size = 41, normalized size = 2.28

$$\begin{cases} \frac{x\cos(c)}{a} & \text{for } b = 0 \wedge d = 0 \\ \frac{x\cos(c)}{a+b\sin(c)} & \text{for } d = 0 \\ \frac{\sin(c+dx)}{ad} & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b}+\sin(c+dx)\right)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*cos(c)/a, Eq(b, 0) & Eq(d, 0)), (x*cos(c)/(a + b*sin(c)), Eq(d
, 0)), (sin(c + d*x)/(a*d), Eq(b, 0)), (log(a/b + sin(c + d*x))/(b*d), True
))
```

Giac [A] time = 1.21605, size = 26, normalized size = 1.44

$$\frac{\log(|b\sin(dx+c)+a|)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(b*sin(d*x + c) + a))/(b*d)
```

3.298 $\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=618

$$\frac{6if^2\sqrt{a^2-b^2}(e+fx)\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}-\frac{6if^2\sqrt{a^2-b^2}(e+fx)\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^3}+\frac{3f\sqrt{a^2-b^2}(e+fx)^2\mathrm{Poly}}{b^2d^2}$$

```
[Out] (a*(e + f*x)^4)/(4*b^2*f) - (6*f^2*(e + f*x)*Cos[c + d*x])/(b*d^3) + ((e +
f*x)^3*Cos[c + d*x])/(b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)^3*Log[1 - (I*b*E^
(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d) - (I*Sqrt[a^2 - b^2]*(e + f*
x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d) + (3*Sqr
t[a^2 - b^2]*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
 b^2])])/(b^2*d^2) - (3*Sqrt[a^2 - b^2]*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*
(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d^2) + ((6*I)*Sqrt[a^2 - b^2]*f^2*
(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d^3
) - ((6*I)*Sqrt[a^2 - b^2]*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b^2*d^3) - (6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d^4) + (6*Sqrt[a^2 - b^2]*f^3*
PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d^4) + (6*f^3
*Sin[c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Sin[c + d*x])/(b*d^2)
```

Rubi [A] time = 1.06764, antiderivative size = 618, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {4525, 32, 3296, 2637, 3323, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{6if^2\sqrt{a^2-b^2}(e+fx)\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}-\frac{6if^2\sqrt{a^2-b^2}(e+fx)\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^3}+\frac{3f\sqrt{a^2-b^2}(e+fx)^2\mathrm{Poly}}{b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] (a*(e + f*x)^4)/(4*b^2*f) - (6*f^2*(e + f*x)*Cos[c + d*x])/(b*d^3) + ((e +
f*x)^3*Cos[c + d*x])/(b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)^3*Log[1 - (I*b*E^
(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d) - (I*Sqrt[a^2 - b^2]*(e + f*
x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d) + (3*Sqr
t[a^2 - b^2]*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
 b^2])])/(b^2*d^2) - (3*Sqrt[a^2 - b^2]*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*
(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d^2) + ((6*I)*Sqrt[a^2 - b^2]*f^2*
(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d^3
```

```
) - ((6*I)*Sqrt[a^2 - b^2]*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b^2*d^3) - (6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (I*b*E
^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d^4) + (6*Sqrt[a^2 - b^2]*f^3*
PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d^4) + (6*f^3
*Sin[c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Sin[c + d*x])/(b*d^2)
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int (e+fx)^3\,dx}{b^2} - \frac{\int (e+fx)^3\sin(c+dx)\,dx}{b} - \frac{\left(a^2-b^2\right)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{\left(2\left(a^2-b^2\right)\right)\int \frac{e^{i(c+dx)}(e+fx)^3}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b^2} - (3f)\int(e+$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{(e+fx)^3\cos(c+dx)}{bd} - \frac{3f(e+fx)^2\sin(c+dx)}{bd^2} + \frac{\left(2i\sqrt{a^2-b^2}\right)\int \frac{e^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}}{b}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} + \frac{(e+fx)^3\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}(e+fx)^3\log\left(1\right.}{b^2d}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} + \frac{(e+fx)^3\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}(e+fx)^3\log\left(1\right.}{b^2d}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} + \frac{(e+fx)^3\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}(e+fx)^3\log\left(1\right.}{b^2d}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} + \frac{(e+fx)^3\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}(e+fx)^3\log\left(1\right.}{b^2d}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^2(e+fx)\cos(c+dx)}{bd^3} + \frac{(e+fx)^3\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}(e+fx)^3\log\left(1\right.}{b^2d}$$

Mathematica [A] time = 3.47397, size = 1025, normalized size = 1.66

$$\frac{ax\left(4e^3+6fxe^2+4f^2x^2e+f^3x^3\right)d^4+4b(e+fx)\left(d^2(e+fx)^2-6f^2\right)\cos(c+dx)d+\frac{4\left(b^2-a^2\right)\left(2\sqrt{b^2-a^2}e^3\tan^{-1}\left(\frac{ia+be^{i(c+dx)}}{\sqrt{a^2-b^2}}\right)d^3+\cdots\right.}{}}{}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f*x)^3*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]

[Out] (a*d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) + 4*b*d*(e + f*x)*(-6*
f^2 + d^2*(e + f*x)^2)*Cos[c + d*x] + (4*(-a^2 + b^2)*(2*Sqrt[-a^2 + b^2]*d
^3*e^3*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + 3*Sqrt[a^2 - b^2
]*d^3*e^2*f*x*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 3*
Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a

```
^2 + b^2])] + Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 - (b*E^(I*(c + d*x)))/((-I)
*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 + (b*E^(I*(c
+ d*x)))/(I*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1
+ (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - Sqrt[a^2 - b^2]*d^3*f^3*x
^3*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - (3*I)*Sqrt[a^2 -
 b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2
+ b^2])] + (3*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(I*(c
+ d*x)))/(I*a + Sqrt[-a^2 + b^2]))] + 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3,
(b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 6*Sqrt[a^2 - b^2]*d*f^3*
x*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 6*Sqrt[a^2
- b^2]*d*e*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))]
- 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a
^2 + b^2]))] + (6*I)*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*E^(I*(c + d*x)))/((-
I)*a + Sqrt[-a^2 + b^2])] - (6*I)*Sqrt[a^2 - b^2]*f^3*PolyLog[4, -((b*E^(I*
(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))]))/Sqrt[-(a^2 - b^2)^2] - 12*b*f*(-2*
f^2 + d^2*(e + f*x)^2)*Sin[c + d*x])/(4*b^2*d^4)
```

Maple [F] time = 1.177, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\cos(dx+c))^2}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.82456, size = 5505, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^4*f^3*x^4 + 4*a*d^4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e^3*x
+ 12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*
a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2))/b) - 12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(d*x +
c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2))/b) + 12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x
+ c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^
2)/b^2))/b) - 12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x +
c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2))/b) + 2*(-3*I*b*d^2*f^3*x^2 - 6*I*b*d^2*e*f^2*x - 3*I*b*d^2*e^2*f)*sq
rt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*
(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) +
2*(3*I*b*d^2*f^3*x^2 + 6*I*b*d^2*e*f^2*x + 3*I*b*d^2*e^2*f)*sqrt(-(a^2 - b^
2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x +
c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(3*I*b*d^2*
f^3*x^2 + 6*I*b*d^2*e*f^2*x + 3*I*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog
(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin
(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(-3*I*b*d^2*f^3*x^2 - 6
*I*b*d^2*e*f^2*x - 3*I*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I
*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b
*c^2*d*e*f^2 - b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I
*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(b*d^3*e^3 - 3*b*
c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*c
os(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*
(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt(-(a^2 - b^
2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/
b^2) + 2*I*a) + 2*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^
3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*
sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*
b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*sqrt(-(a^2 -
 b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x +
c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b*d^3*f^3*x^3
+ 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 +
 b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*
x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b
```

```
)/b) - 2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e
^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*
cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*
d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*sqrt(-(a^2 - b
^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c
) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 12*(b*d*f^3*x + b*
d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*si
n(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/
b) - 12*(b*d*f^3*x + b*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*
a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2))/b) - 12*(b*d*f^3*x + b*d*e*f^2)*sqrt(-(a^2 - b^2)/b^
2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(b*d*f^3*x + b*d*e*f^2)*sqrt(-
(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*
x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 4*(b*d^3*f^3*x^3 +
3*b*d^3*e*f^2*x^2 + b*d^3*e^3 - 6*b*d*e*f^2 + 3*(b*d^3*e^2*f - 2*b*d*f^3)*x
)*cos(d*x + c) - 12*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f - 2*b*f^
3)*sin(d*x + c))/(b^2*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cos(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

3.299 $\int \frac{(e+fx)^2 \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=460

$$\frac{2f\sqrt{a^2-b^2}(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{2f\sqrt{a^2-b^2}(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}+\frac{2if^2\sqrt{a^2-b^2}\text{PolyLog}\left(3,\frac{ibe^{i}}{a-\sqrt{}}\right.}{b^2d^3}$$

```
[Out] (a*(e + f*x)^3)/(3*b^2*f) - (2*f^2*Cos[c + d*x])/(b*d^3) + ((e + f*x)^2*Cos
[c + d*x])/(b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*
x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d) - (I*Sqrt[a^2 - b^2]*(e + f*x)^2*Log[1
 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d) + (2*Sqrt[a^2 - b^
2]*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^
2*d^2) - (2*Sqrt[a^2 - b^2]*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a
 + Sqrt[a^2 - b^2])])/(b^2*d^2) + ((2*I)*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (I*
b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d^3) - ((2*I)*Sqrt[a^2 - b^
2]*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d^3) -
 (2*f*(e + f*x)*Sin[c + d*x])/(b*d^2)
```

Rubi [A] time = 0.928734, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4525, 32, 3296, 2638, 3323, 2264, 2190, 2531, 2282, 6589}

$$\frac{2f\sqrt{a^2-b^2}(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{2f\sqrt{a^2-b^2}(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}+\frac{2if^2\sqrt{a^2-b^2}\text{PolyLog}\left(3,\frac{ibe^{i}}{a-\sqrt{}}\right.}{b^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^3)/(3*b^2*f) - (2*f^2*Cos[c + d*x])/(b*d^3) + ((e + f*x)^2*Cos
[c + d*x])/(b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*
x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d) - (I*Sqrt[a^2 - b^2]*(e + f*x)^2*Log[1
 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d) + (2*Sqrt[a^2 - b^
2]*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^
2*d^2) - (2*Sqrt[a^2 - b^2]*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a
 + Sqrt[a^2 - b^2])])/(b^2*d^2) + ((2*I)*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (I*
b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d^3) - ((2*I)*Sqrt[a^2 - b^
2]*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d^3) -
 (2*f*(e + f*x)*Sin[c + d*x])/(b*d^2)
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int (e+fx)^2\,dx}{b^2} - \frac{\int (e+fx)^2\sin(c+dx)\,dx}{b} - \frac{\left(a^2-b^2\right)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{a(e+fx)^3}{3b^2f} + \frac{(e+fx)^2\cos(c+dx)}{bd} - \frac{\left(2\left(a^2-b^2\right)\right)\int \frac{e^{i(c+dx)}(e+fx)^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b^2} - \frac{(2f)\int(e+$$

$$= \frac{a(e+fx)^3}{3b^2f} + \frac{(e+fx)^2\cos(c+dx)}{bd} - \frac{2f(e+fx)\sin(c+dx)}{bd^2} + \frac{\left(2i\sqrt{a^2-b^2}\right)\int \frac{e^{i(c+dx)}}{2a-2\sqrt{a^2-b}}}{b}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f^2\cos(c+dx)}{bd^3} + \frac{(e+fx)^2\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}(e+fx)^2\log\left(1-\frac{ibe^{i}}{a-\sqrt{}}\right.}{b^2d}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f^2\cos(c+dx)}{bd^3} + \frac{(e+fx)^2\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}(e+fx)^2\log\left(1-\frac{ibe^{i}}{a-\sqrt{}}\right.}{b^2d}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f^2\cos(c+dx)}{bd^3} + \frac{(e+fx)^2\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}(e+fx)^2\log\left(1-\frac{ibe^{i}}{a-\sqrt{}}\right.}{b^2d}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f^2\cos(c+dx)}{bd^3} + \frac{(e+fx)^2\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}(e+fx)^2\log\left(1-\frac{ibe^{i}}{a-\sqrt{}}\right.}{b^2d}$$

Mathematica [A] time = 2.62308, size = 536, normalized size = 1.17

$$\frac{3i\left(b^2-a^2\right)\left(-i\left(2f^2\sqrt{a^2-b^2}\,\text{PolyLog}\left(3,\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}-ia}\right)-2f^2\sqrt{a^2-b^2}\,\text{PolyLog}\left(3,-\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}+ia}\right)+d^2\left(2e^2\sqrt{b^2-a^2}\tan^{-1}\left(\frac{ia+be^{i(c+dx)}}{\sqrt{a^2-b^2}}\right)+fx\sqrt{a^2-b^2}(2e+fx)\left(\log\left(1-\frac{be^{i(c+d}}{\sqrt{b^2-a^2}}\right.\right.\right.\right.\right.}{d^3\sqrt{-\left(a^2-b^2\right)^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x*(3*e^2 + 3*e*f*x + f^2*x^2) + ((3*I)*(-a^2 + b^2)*(-2*Sqrt[a^2 - b^2]*
d*f*(e + f*x)*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] +
 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sq
rt[-a^2 + b^2]))] - I*(d^2*(2*Sqrt[-a^2 + b^2]*e^2*ArcTan[(I*a + b*E^(I*(c
+ d*x)))/Sqrt[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(Log[1 - (b*E^(
I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - Log[1 + (b*E^(I*(c + d*x)))/(I
*a + Sqrt[-a^2 + b^2])])) + 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*E^(I*(c + d
*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, -((b*
```

```
E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))]))))/(Sqrt[-(a^2 - b^2)^2]*d^3) +
 (3*b*Cos[d*x]*((-2*f^2 + d^2*(e + f*x)^2)*Cos[c] - 2*d*f*(e + f*x)*Sin[c])
)/d^3 - (3*b*(2*d*f*(e + f*x)*Cos[c] + (-2*f^2 + d^2*(e + f*x)^2)*Sin[c])*S
in[d*x])/d^3)/(3*b^2)
```

Maple [F] time = 0.956, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \left(\cos \left(dx+c\right)\right)^{2}}{a+b\sin \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.1454, size = 3931, normalized size = 8.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*f^2*x^3 + 6*a*d^3*e*f*x^2 + 6*a*d^3*e^2*x + 6*b*f^2*sqrt(-(a^2
 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*c
os(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*b*f^2*sqrt(-
```

```
(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*
(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*b*f^2*sq
rt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*co
s(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*b*f^2*sqrt(-(
a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x
 + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + (-6*I*b*d*f^2*x - 6*
I*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(
d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2
*b)/b + 1) + (6*I*b*d*f^2*x + 6*I*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/
2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (6*I*b*d*f^2*x + 6*I*b*d*e*f)*
sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) +
 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)
 + (-6*I*b*d*f^2*x - 6*I*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a
*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2
)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sq
rt(-(a^2 - b^2)/b^2) + 2*I*a) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqr
t(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(
a^2 - b^2)/b^2) - 2*I*a) + 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt(-(a
^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2
- b^2)/b^2) + 2*I*a) + 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt(-(a^2 -
 b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^
2)/b^2) - 2*I*a) - 3*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f
^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) +
 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 3
*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*sqrt(-(a^2 - b^2
)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) -
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 3*(b*d^2*f^2*x^2 + 2*
b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I
*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 3*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c
*d*e*f - b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2
*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) + 2*b)/b) + 6*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + b*d^2*e^2 - 2*b*f^2)*cos
(d*x + c) - 12*(b*d*f^2*x + b*d*e*f)*sin(d*x + c))/(b^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cos(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

3.300 $\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=298

$$\frac{f\sqrt{a^2-b^2}\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{f\sqrt{a^2-b^2}\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}+\frac{i\sqrt{a^2-b^2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{i\sqrt{a^2-}}{}$$

```
[Out] (a*e*x)/b^2 + (a*f*x^2)/(2*b^2) + ((e + f*x)*Cos[c + d*x])/(b*d) + (I*Sqrt[
a^2 - b^2]*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/
(b^2*d) - (I*Sqrt[a^2 - b^2]*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + S
qrt[a^2 - b^2])])/(b^2*d) + (Sqrt[a^2 - b^2]*f*PolyLog[2, (I*b*E^(I*(c + d*
x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d^2) - (Sqrt[a^2 - b^2]*f*PolyLog[2, (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d^2) - (f*Sin[c + d*x])/(b*d
^2)
```

Rubi [A] time = 0.535025, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4525, 3296, 2637, 3323, 2264, 2190, 2279, 2391}

$$\frac{f\sqrt{a^2-b^2}\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{f\sqrt{a^2-b^2}\,\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}+\frac{i\sqrt{a^2-b^2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{i\sqrt{a^2-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] (a*e*x)/b^2 + (a*f*x^2)/(2*b^2) + ((e + f*x)*Cos[c + d*x])/(b*d) + (I*Sqrt[
a^2 - b^2]*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/
(b^2*d) - (I*Sqrt[a^2 - b^2]*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + S
qrt[a^2 - b^2])])/(b^2*d) + (Sqrt[a^2 - b^2]*f*PolyLog[2, (I*b*E^(I*(c + d*
x)))/(a - Sqrt[a^2 - b^2])])/(b^2*d^2) - (Sqrt[a^2 - b^2]*f*PolyLog[2, (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^2*d^2) - (f*Sin[c + d*x])/(b*d
^2)
```


Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
```

```
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int(e+fx)\,dx}{b^2} - \frac{\int(e+fx)\sin(c+dx)\,dx}{b} - \frac{\left(a^2-b^2\right)\int\frac{e+fx}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{aex}{b^2} + \frac{afx^2}{2b^2} + \frac{(e+fx)\cos(c+dx)}{bd} - \frac{\left(2\left(a^2-b^2\right)\right)\int\frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{b^2} - \frac{f\int\cos(c+}{bd}$$

$$= \frac{aex}{b^2} + \frac{afx^2}{2b^2} + \frac{(e+fx)\cos(c+dx)}{bd} - \frac{f\sin(c+dx)}{bd^2} + \frac{\left(2i\sqrt{a^2-b^2}\right)\int\frac{e^{i(c+dx)}(e+fx)}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}}{b}$$

$$= \frac{aex}{b^2} + \frac{afx^2}{2b^2} + \frac{(e+fx)\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{i\sqrt{a^2-b^2}(e}{}$$

$$= \frac{aex}{b^2} + \frac{afx^2}{2b^2} + \frac{(e+fx)\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{i\sqrt{a^2-b^2}(e}{}$$

$$= \frac{aex}{b^2} + \frac{afx^2}{2b^2} + \frac{(e+fx)\cos(c+dx)}{bd} + \frac{i\sqrt{a^2-b^2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{i\sqrt{a^2-b^2}(e}{}$$

Mathematica [B] time = 6.91867, size = 716, normalized size = 2.4

$$\frac{2d\left(b^2-a^2\right)(e+fx)\left(-\frac{if\left(\text{PolyLog}\left(2,\frac{a\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a+i\left(\sqrt{b^2-a^2}+b\right)}\right)+\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{\sqrt{b^2-a^2}+a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{b^2-a^2}-ia+b}\right)\right)}{\sqrt{b^2-a^2}}+\frac{if\left(\text{PolyLog}\left(2,\frac{a\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a-i\left(\sqrt{b^2-a^2}+b\right)}\right)+\log\left(1+i\tan\left(\frac{1}{2}(c+dx\right.\right.\right.}{\sqrt{b^2-a^2}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(a*(c + d*x)*(c*f - d*(2*e + f*x))) + 2*b*d*(e + f*x)*Cos[c + d*x] + (2*(
-a^2 + b^2)*d*(e + f*x)*((2*(d*e - c*f)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(b +
 Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] +
PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]))/S
```

```
qrt[-a^2 + b^2] + (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^
2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 +
 I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] +
(I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(-b + Sqrt[-a^2 + b^2] - a*Tan[(c + d
*x)/2])/(I*a - b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(I + Tan[(c + d*x)/2]
))/(I*a - b + Sqrt[-a^2 + b^2])]))/Sqrt[-a^2 + b^2] - (I*f*(Log[1 + I*Tan[(
c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqr
t[-a^2 + b^2])] + PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-
a^2 + b^2]))]))/Sqrt[-a^2 + b^2]))/(d*e - c*f + I*f*Log[1 - I*Tan[(c + d*x)
/2]] - I*f*Log[1 + I*Tan[(c + d*x)/2]]) - 2*b*f*Sin[c + d*x])/(2*b^2*d^2)
```

Maple [B] time = 0.337, size = 1123, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2*a*f*x^2/b^2+a*e*x/b^2+1/2*(d*f*x+I*f+d*e)/b/d^2*exp(I*(d*x+c))+1/2*(d*f
*x-I*f+d*e)/b/d^2*exp(-I*(d*x+c))-a^2/b^2/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*ex
p(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-a^2/b^2/d^2*f/(-a^
2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/
2)))*c+a^2/b^2/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/
2))/(I*a+(-a^2+b^2)^(1/2)))*x+a^2/b^2/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(
I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-2*I/d^2*f*c/(-a^2+b^
2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+1/d*f/(-a^
2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/
2)))*x+1/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/
(I*a-(-a^2+b^2)^(1/2)))*c-1/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(
-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-1/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a
+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+I/b^2/d^2*a^2
*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^
2+b^2)^(1/2)))+I/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b
^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+2*I/d*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I
*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))-I/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*
a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))+2*I/b^2/d^2*a^
2*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/
2))-I*a^2/b^2/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)
^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2*I/b^2/d*a^2*e/(-a^2+b^2)^(1/2)*arctan(1/2
*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.22178, size = 2564, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d^2*f*x^2 + 4*a*d^2*e*x - 2*I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/
2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*b*f*sqrt(-(a^2 - b^2)/b^2)
*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*
b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*b*f*sqrt(-(a^2 -
 b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*
x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*I*b*f*s
qrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) -
2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)
- 4*b*f*sin(d*x + c) - 2*(b*d*e - b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos
(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(b
*d*e - b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x +
 c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(b*d*e - b*c*f)*sqrt(-(a^2 -
b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) + 2*I*a) + 2*(b*d*e - b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x
 + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(b*d*f
*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*
x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b
)/b) + 2*(b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x +
c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2) + 2*b)/b) - 2*(b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-
2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c
```

```
))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)
/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) +
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 4*(b*d*f*x + b*d*e)*c
os(d*x + c))/(b^2*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cos(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

3.301 $\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=70

$$-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}$$

```
[Out] (a*x)/b^2 - (2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(b^2*d) + Cos[c + d*x]/(b*d)
```

Rubi [A] time = 0.115843, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2695, 2735, 2660, 618, 204}

$$-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/b^2 - (2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(b^2*d) + Cos[c + d*x]/(b*d)
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\cos(c+dx)}{bd}+\frac{\int \frac{b+a\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}\\
&= \frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}-\frac{\left(a^2-b^2\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^2}\\
&= \frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}-\frac{\left(2\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}\\
&= \frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}+\frac{\left(4\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}\\
&= \frac{ax}{b^2}-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{\cos(c+dx)}{bd}
\end{aligned}
$$

Mathematica [B] time = 2.12276, size = 398, normalized size = 5.69

$$
\frac{b\cos(c+dx)\left(\sqrt{a+b}\left(2\sqrt{-b^2}\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sinh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(\sin(c+dx)+1)}{a-b}}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{a-b}\sqrt{1-\sin(c+dx)}\left(\sqrt{-b}\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\right.\right.\right.}{(-b)^{5/2}d\sqrt{a-b}\sqrt{a+b}\sqrt{1-\sin(c+dx)}\sqrt{}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Cos[c + d*x]*(-2*Sqrt[-b]*(-a + b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + S
in[c + d*x]))/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b
))])]*Sqrt[1 - Sin[c + d*x]] + Sqrt[a + b]*(2*Sqrt[-b^2]*ArcSinh[(Sqrt[a -
b]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(Sqrt[2]*Sqrt[b])]*Sqrt[-((b*(-
1 + Sin[c + d*x]))/(a + b))] + Sqrt[a - b]*Sqrt[1 - Sin[c + d*x]]*(2*Sqrt[b
]*ArcTan[(Sqrt[b]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(Sqrt[-b]*Sqrt[-((
b*(-1 + Sin[c + d*x]))/(a + b))])] + Sqrt[-b]*Sqrt[-((b*(-1 + Sin[c + d*x])
)/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]))))/(Sqrt[a - b]*(-b)^(5/
2)*Sqrt[a + b]*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a +
 b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])
```

Maple [B] time = 0.001, size = 142, normalized size = 2.

$$2\,\frac{1}{bd\left(1+(\tan(1/2\,dx+c/2))^2\right)}+2\,\frac{a\arctan(\tan(1/2\,dx+c/2))}{b^2d}-2\,\frac{a^2}{b^2d\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(1/2\,dx+c/2)+2}{\sqrt{a^2-b^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 2/d/b/(1+tan(1/2*d*x+1/2*c)^2)+2/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))-2/d/b^2
/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a
^2+2/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85215, size = 498, normalized size = 7.11

$$\left[\frac{2\,adx+2\,b\cos\left(dx+c\right)+\sqrt{-a^2+b^2}\log\left(\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)}{2\,b^2d},\ \frac{adx}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*d*x + 2*b*cos(d*x + c) + sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c
) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2)))/(b^2*d), (a*d*x + b*cos(d*x + c) + sqrt(a^2 - b^2)*arcta
n(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))))/(b^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14904, size = 128, normalized size = 1.83

$$\frac{\frac{(dx+c)a}{b^2}-\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{b^2}+\frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*a/b^2 - 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*t
an(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2 + 2/((tan(1/
2*d*x + 1/2*c)^2 + 1)*b))/d
```

3.302 $\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=737

$$-\frac{6f^2\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3}-\frac{6f^2\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^3}+\frac{3if\left(a^2-b^2\right)(e+fx)^2\text{Po}}{b^3d^2}$$

```
[Out] (-3*f^3*x)/(8*b*d^3) + (e + f*x)^3/(4*b*d) + ((I/4)*(a^2 - b^2)*(e + f*x)^4
)/(b^3*f) - (6*a*f^3*Cos[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Cos[c + d
*x])/(b^2*d^2) - ((a^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(b^3*d) - ((a^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(
c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d) + ((3*I)*(a^2 - b^2)*f*(e + f*x)
^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*d^2) + ((3
*I)*(a^2 - b^2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^
2 - b^2])])/(b^3*d^2) - (6*(a^2 - b^2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(
c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*d^3) - (6*(a^2 - b^2)*f^2*(e + f*x)
*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d^3) - ((6*I
)*(a^2 - b^2)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/
(b^3*d^4) - ((6*I)*(a^2 - b^2)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sq
rt[a^2 - b^2])])/(b^3*d^4) - (6*a*f^2*(e + f*x)*Sin[c + d*x])/(b^2*d^3) + (
a*(e + f*x)^3*Sin[c + d*x])/(b^2*d) + (3*f^3*Cos[c + d*x]*Sin[c + d*x])/(8*
b*d^4) - (3*f*(e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(4*b*d^2) + (3*f^2*(e
+ f*x)*Sin[c + d*x]^2)/(4*b*d^3) - ((e + f*x)^3*Sin[c + d*x]^2)/(2*b*d)
```

Rubi [A] time = 0.880909, antiderivative size = 737, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4525, 3296, 2638, 4404, 3311, 32, 2635, 8, 4519, 2190, 2531, 6609, 2282, 6589}

$$-\frac{6f^2\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3}-\frac{6f^2\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^3}+\frac{3if\left(a^2-b^2\right)(e+fx)^2\text{Po}}{b^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```


```
[Out] (-3*f^3*x)/(8*b*d^3) + (e + f*x)^3/(4*b*d) + ((I/4)*(a^2 - b^2)*(e + f*x)^4
)/(b^3*f) - (6*a*f^3*Cos[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Cos[c + d
*x])/(b^2*d^2) - ((a^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(b^3*d) - ((a^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(
```

```
c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d) + ((3*I)*(a^2 - b^2)*f*(e + f*x)
^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*d^2) + ((3
*I)*(a^2 - b^2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^
2 - b^2])])/(b^3*d^2) - (6*(a^2 - b^2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(
c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*d^3) - (6*(a^2 - b^2)*f^2*(e + f*x)
*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d^3) - ((6*I
)*(a^2 - b^2)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/
(b^3*d^4) - ((6*I)*(a^2 - b^2)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sq
rt[a^2 - b^2])])/(b^3*d^4) - (6*a*f^2*(e + f*x)*Sin[c + d*x])/(b^2*d^3) + (
a*(e + f*x)^3*Sin[c + d*x])/(b^2*d) + (3*f^3*Cos[c + d*x]*Sin[c + d*x])/(8*
b*d^4) - (3*f*(e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(4*b*d^2) + (3*f^2*(e
+ f*x)*Sin[c + d*x]^2)/(4*b*d^3) - ((e + f*x)^3*Sin[c + d*x]^2)/(2*b*d)
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
```

```
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int (e+fx)^3\cos(c+dx)\,dx}{b^2} - \frac{\int (e+fx)^3\cos(c+dx)\sin(c+dx)\,dx}{b} - \frac{\left(a^2-b^2\right)\int \frac{(e+fx)^3 c}{a+b\sin}}{b^2}$$

$$= \frac{i\left(a^2-b^2\right)(e+fx)^4}{4b^3f} + \frac{a(e+fx)^3\sin(c+dx)}{b^2d} - \frac{(e+fx)^3\sin^2(c+dx)}{2bd} - \frac{\left(a^2-b^2\right)\int \frac{e^{i(c}}{a-\sqrt{a^2}}}{b^2}$$

$$= \frac{i\left(a^2-b^2\right)(e+fx)^4}{4b^3f} + \frac{3af(e+fx)^2\cos(c+dx)}{b^2d^2} - \frac{\left(a^2-b^2\right)(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}$$

$$= \frac{(e+fx)^3}{4bd} + \frac{i\left(a^2-b^2\right)(e+fx)^4}{4b^3f} + \frac{3af(e+fx)^2\cos(c+dx)}{b^2d^2} - \frac{\left(a^2-b^2\right)(e+fx)^3\log\left(1\right.}{b^3d}$$

$$= -\frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} + \frac{i\left(a^2-b^2\right)(e+fx)^4}{4b^3f} - \frac{6af^3\cos(c+dx)}{b^2d^4} + \frac{3af(e+fx)^2\cos(c+dx)}{b^2d^2}$$

$$= -\frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} + \frac{i\left(a^2-b^2\right)(e+fx)^4}{4b^3f} - \frac{6af^3\cos(c+dx)}{b^2d^4} + \frac{3af(e+fx)^2\cos(c+dx)}{b^2d^2}$$

$$= -\frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} + \frac{i\left(a^2-b^2\right)(e+fx)^4}{4b^3f} - \frac{6af^3\cos(c+dx)}{b^2d^4} + \frac{3af(e+fx)^2\cos(c+dx)}{b^2d^2}$$

Mathematica [B] time = 10.1211, size = 2452, normalized size = 3.33

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-32*(a^2 - b^2)*e^3*x*Cot[c] - 48*(a^2 - b^2)*e^2*f*x^2*Cot[c] - 32*(a^2 -
 b^2)*e*f^2*x^3*Cot[c] - 8*(a^2 - b^2)*f^3*x^4*Cot[c] + (16*(a^2 - b^2)*((4
*I)*d^4*e^3*E^((2*I)*c)*x + (6*I)*d^4*e^2*E^((2*I)*c)*f*x^2 + (4*I)*d^4*e*E
^((2*I)*c)*f^2*x^3 + I*d^4*E^((2*I)*c)*f^3*x^4 + (2*I)*d^3*e^3*ArcTan[(2*a*
E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] - (2*I)*d^3*e^3*E^((2*I)*c
)*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] + d^3*e^3*Lo
g[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] - d^3*e^3*E
^((2*I)*c)*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2
] + 6*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 +
 b^2)*E^((2*I)*c)])] - 6*d^3*e^2*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x
)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e*f^2*x^2*Log[1
```

+ (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 2*d^3*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 2*d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e^2*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 2*d^3*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 2*d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (6*I)*d^2*(-1 + E^((2*I)*c))*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (6*I)*d^2*(-1 + E^((2*I)*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 12*d*e*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d*e*E^((2*I)*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*d*f^3*x*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d*E^((2*I)*c)*f^3*x*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*d*e*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 12*d*e*E^((2*I)*c)*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 12*d*f^3*x*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 12*d*E^((2*I)*c)*f^3*x*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (12*I)*f^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (12*I)*E^((2*I)*c)*f^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (12*I)*f^3*PolyLog[4, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (12*I)*E^((2*I)*c)*f^3*PolyLog[4, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]))/(d^4*(-1 + E^((2*I)*c))) + (16*a*b*(-6*f^3 - (6*I)*d*f^2*(e + f*x) + 3*d^2*f*(e + f*x)^2 + I*d^3*(e + f*x)^3)*(Cos[c + d*x] - I*Sin[c + d*x]))/d^4 + (16*a*b*(-6*f^3 + (6*I)*d*f^2*(e + f*x) + 3*d^2*f*(e + f*x)^2 - I*d^3*(e + f*x)^3)*(Cos[c + d*x] + I*Sin[c + d*x]))/d^4 + (b^2*((3*I)*f^3 - 6*d*f^2*(e + f*x) - (6*I)*d^2*f*(e + f*x)^2 + 4*d^3*(e + f*x)^3)*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]))/d^4 + (b^2*((-3*I)*f^3 - 6*d*f^2*(e + f*x) + (6*I)*d^2*f*(e + f*x)^2 + 4*d^3*(e + f*x)^3)*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]))/d^4)/(32*b^3)

Maple [F] time = 1.418, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3} \left(\cos \left(dx+c\right)\right)^{3}}{a+b\sin \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.34691, size = 6086, normalized size = 8.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b^2*d^3*f^3*x^3 + 6*b^2*d^3*e*f^2*x^2 + 24*I*(a^2 - b^2)*f^3*polylo
g(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 24*I*(a^2 - b^2)*f^3*polylog(4, 1
/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 24*I*(a^2 - b^2)*f^3*polylog(4, -(I*a*c
os(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2))/b) - 24*I*(a^2 - b^2)*f^3*polylog(4, -(I*a*cos(d*x + c) + a
*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))
/b) - 2*(2*b^2*d^3*f^3*x^3 + 6*b^2*d^3*e*f^2*x^2 + 2*b^2*d^3*e^3 - 3*b^2*d*
e*f^2 + 3*(2*b^2*d^3*e^2*f - b^2*d*f^3)*x)*cos(d*x + c)^2 + 3*(2*b^2*d^3*e^
```

```
2*f - b^2*d*f^3)*x - 24*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + a*b*d^2*e^2*
f - 2*a*b*f^3)*cos(d*x + c) - (-12*I*(a^2 - b^2)*d^2*f^3*x^2 - 24*I*(a^2 -
b^2)*d^2*e*f^2*x - 12*I*(a^2 - b^2)*d^2*e^2*f)*dilog(-1/2*(2*I*a*cos(d*x +
c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2) + 2*b)/b + 1) - (-12*I*(a^2 - b^2)*d^2*f^3*x^2 - 24*I*(a^2 - b^2)
*d^2*e*f^2*x - 12*I*(a^2 - b^2)*d^2*e^2*f)*dilog(-1/2*(2*I*a*cos(d*x + c) +
 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2) + 2*b)/b + 1) - (12*I*(a^2 - b^2)*d^2*f^3*x^2 + 24*I*(a^2 - b^2)*d^2*
e*f^2*x + 12*I*(a^2 - b^2)*d^2*e^2*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a
*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
) + 2*b)/b + 1) - (12*I*(a^2 - b^2)*d^2*f^3*x^2 + 24*I*(a^2 - b^2)*d^2*e*f^
2*x + 12*I*(a^2 - b^2)*d^2*e^2*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin
(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) +
2*b)/b + 1) + 4*((a^2 - b^2)*d^3*e^3 - 3*(a^2 - b^2)*c*d^2*e^2*f + 3*(a^2 -
 b^2)*c^2*d*e*f^2 - (a^2 - b^2)*c^3*f^3)*log(2*b*cos(d*x + c) + 2*I*b*sin(d
*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 4*((a^2 - b^2)*d^3*e^3 - 3*
(a^2 - b^2)*c*d^2*e^2*f + 3*(a^2 - b^2)*c^2*d*e*f^2 - (a^2 - b^2)*c^3*f^3)*
log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*
I*a) + 4*((a^2 - b^2)*d^3*e^3 - 3*(a^2 - b^2)*c*d^2*e^2*f + 3*(a^2 - b^2)*c
^2*d*e*f^2 - (a^2 - b^2)*c^3*f^3)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c
) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 4*((a^2 - b^2)*d^3*e^3 - 3*(a^2 -
 b^2)*c*d^2*e^2*f + 3*(a^2 - b^2)*c^2*d*e*f^2 - (a^2 - b^2)*c^3*f^3)*log(-2
*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)
 + 4*((a^2 - b^2)*d^3*f^3*x^3 + 3*(a^2 - b^2)*d^3*e*f^2*x^2 + 3*(a^2 - b^2)*
d^3*e^2*f*x + 3*(a^2 - b^2)*c*d^2*e^2*f - 3*(a^2 - b^2)*c^2*d*e*f^2 + (a^2
- b^2)*c^3*f^3)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d
*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 4*((a^2 - b^
2)*d^3*f^3*x^3 + 3*(a^2 - b^2)*d^3*e*f^2*x^2 + 3*(a^2 - b^2)*d^3*e^2*f*x +
3*(a^2 - b^2)*c*d^2*e^2*f - 3*(a^2 - b^2)*c^2*d*e*f^2 + (a^2 - b^2)*c^3*f^3
)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 4*((a^2 - b^2)*d^3*f^3*x^3
 + 3*(a^2 - b^2)*d^3*e*f^2*x^2 + 3*(a^2 - b^2)*d^3*e^2*f*x + 3*(a^2 - b^2)*
c*d^2*e^2*f - 3*(a^2 - b^2)*c^2*d*e*f^2 + (a^2 - b^2)*c^3*f^3)*log(1/2*(-2*
I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 4*((a^2 - b^2)*d^3*f^3*x^3 + 3*(a^2 - b
^2)*d^3*e*f^2*x^2 + 3*(a^2 - b^2)*d^3*e^2*f*x + 3*(a^2 - b^2)*c*d^2*e^2*f -
 3*(a^2 - b^2)*c^2*d*e*f^2 + (a^2 - b^2)*c^3*f^3)*log(1/2*(-2*I*a*cos(d*x +
 c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2) + 2*b)/b) + 24*((a^2 - b^2)*d*f^3*x + (a^2 - b^2)*d*e*f^2)*polyl
og(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 24*((a^2 - b^2)*d*f^3*x + (a^2 -
 b^2)*d*e*f^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b
*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 24*((a^2 - b
^2)*d*f^3*x + (a^2 - b^2)*d*e*f^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*
x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2
```

```
4*((a^2 - b^2)*d*f^3*x + (a^2 - b^2)*d*e*f^2)*polylog(3, -(I*a*cos(d*x + c)
 + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2))/b) - (8*a*b*d^3*f^3*x^3 + 24*a*b*d^3*e*f^2*x^2 + 8*a*b*d^3*e^3 - 48*a*
b*d*e*f^2 + 24*(a*b*d^3*e^2*f - 2*a*b*d*f^3)*x - 3*(2*b^2*d^2*f^3*x^2 + 4*b
^2*d^2*e*f^2*x + 2*b^2*d^2*e^2*f - b^2*f^3)*cos(d*x + c))*sin(d*x + c))/(b^
3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**3/(a+b*sin(d*x+c)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cos(dx+c)^3}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] integrate((f*x + e)^3*cos(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

3.303 $\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=548

$$\frac{2if\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{2if\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2}-\frac{2f^2\left(a^2-b^2\right)\text{PolyLog}\left(3\right.}{b^3d^3}$$

```
[Out] (e*f*x)/(2*b*d) + (f^2*x^2)/(4*b*d) + ((I/3)*(a^2 - b^2)*(e + f*x)^3)/(b^3*
f) + (2*a*f*(e + f*x)*Cos[c + d*x])/(b^2*d^2) - ((a^2 - b^2)*(e + f*x)^2*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*d) - ((a^2 - b^2)*
(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d) +
 ((2*I)*(a^2 - b^2)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[
a^2 - b^2])])/(b^3*d^2) + ((2*I)*(a^2 - b^2)*f*(e + f*x)*PolyLog[2, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d^2) - (2*(a^2 - b^2)*f^2*PolyL
og[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*d^3) - (2*(a^2 - b
^2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d^3)
- (2*a*f^2*Sin[c + d*x])/(b^2*d^3) + (a*(e + f*x)^2*Sin[c + d*x])/(b^2*d) -
 (f*(e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(2*b*d^2) + (f^2*Sin[c + d*x]^2)/(
4*b*d^3) - ((e + f*x)^2*Sin[c + d*x]^2)/(2*b*d)
```

Rubi [A] time = 0.733307, antiderivative size = 548, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4525, 3296, 2637, 4404, 3310, 4519, 2190, 2531, 2282, 6589}

$$\frac{2if\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{2if\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2}-\frac{2f^2\left(a^2-b^2\right)\text{PolyLog}\left(3\right.}{b^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```


```
[Out] (e*f*x)/(2*b*d) + (f^2*x^2)/(4*b*d) + ((I/3)*(a^2 - b^2)*(e + f*x)^3)/(b^3*
f) + (2*a*f*(e + f*x)*Cos[c + d*x])/(b^2*d^2) - ((a^2 - b^2)*(e + f*x)^2*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*d) - ((a^2 - b^2)*
(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d) +
 ((2*I)*(a^2 - b^2)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[
a^2 - b^2])])/(b^3*d^2) + ((2*I)*(a^2 - b^2)*f*(e + f*x)*PolyLog[2, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d^2) - (2*(a^2 - b^2)*f^2*PolyL
og[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*d^3) - (2*(a^2 - b
^2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d^3)
```

```
- (2*a*f^2*Sin[c + d*x])/(b^2*d^3) + (a*(e + f*x)^2*Sin[c + d*x])/(b^2*d) -
 (f*(e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(2*b*d^2) + (f^2*Sin[c + d*x]^2)/(
4*b*d^3) - ((e + f*x)^2*Sin[c + d*x]^2)/(2*b*d)
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
```

```
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int (e+fx)^2\cos(c+dx)\,dx}{b^2} - \frac{\int (e+fx)^2\cos(c+dx)\sin(c+dx)\,dx}{b} - \frac{\left(a^2-b^2\right)\int \frac{(e+fx)^2 \mathrm{c}}{a+b\sin}}{b^2}$$

$$= \frac{i\left(a^2-b^2\right)(e+fx)^3}{3b^3f} + \frac{a(e+fx)^2\sin(c+dx)}{b^2d} - \frac{(e+fx)^2\sin^2(c+dx)}{2bd} - \frac{\left(a^2-b^2\right)\int \frac{e^{i(c\cdot}}{a-\sqrt{a^2}}}{b^2}$$

$$= \frac{i\left(a^2-b^2\right)(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cos(c+dx)}{b^2d^2} - \frac{\left(a^2-b^2\right)(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} -$$

$$= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} + \frac{i\left(a^2-b^2\right)(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cos(c+dx)}{b^2d^2} - \frac{\left(a^2-b^2\right)(e+fx)^2\log\left(1\right.}{b^3d}$$

$$= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} + \frac{i\left(a^2-b^2\right)(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cos(c+dx)}{b^2d^2} - \frac{\left(a^2-b^2\right)(e+fx)^2\log\left(1\right.}{b^3d}$$

$$= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} + \frac{i\left(a^2-b^2\right)(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cos(c+dx)}{b^2d^2} - \frac{\left(a^2-b^2\right)(e+fx)^2\log\left(1\right.}{b^3d}$$

Mathematica [B] time = 5.01989, size = 2397, normalized size = 4.37

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((48*I)*a^2*d^3*e^2*E^((2*I)*c)*x - (48*I)*b^2*d^3*e^2*E^((2*I)*c)*x + (48*
I)*a^2*d^3*e*E^((2*I)*c)*f*x^2 - (48*I)*b^2*d^3*e*E^((2*I)*c)*f*x^2 + (16*I
)*a^2*d^3*E^((2*I)*c)*f^2*x^3 - (16*I)*b^2*d^3*E^((2*I)*c)*f^2*x^3 - (48*I)
*a^2*d^2*e^2*E^((2*I)*c)*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c
+ d*x))))] + (48*I)*b^2*d^2*e^2*E^((2*I)*c)*ArcTan[(2*a*E^(I*(c + d*x)))/(b
*(-1 + E^((2*I)*(c + d*x))))] + (24*I)*a*b*d^2*e^2*E^(I*c)*Cos[d*x] - (24*I
)*a*b*d^2*e^2*E^((3*I)*c)*Cos[d*x] + 48*a*b*d*e*E^(I*c)*f*Cos[d*x] + 48*a*b
*d*e*E^((3*I)*c)*f*Cos[d*x] - (48*I)*a*b*E^(I*c)*f^2*Cos[d*x] + (48*I)*a*b*
E^((3*I)*c)*f^2*Cos[d*x] + (48*I)*a*b*d^2*e*E^(I*c)*f*x*Cos[d*x] - (48*I)*a
*b*d^2*e*E^((3*I)*c)*f*x*Cos[d*x] + 48*a*b*d*E^(I*c)*f^2*x*Cos[d*x] + 48*a*
b*d*E^((3*I)*c)*f^2*x*Cos[d*x] + (24*I)*a*b*d^2*E^(I*c)*f^2*x^2*Cos[d*x] -
(24*I)*a*b*d^2*E^((3*I)*c)*f^2*x^2*Cos[d*x] + 6*b^2*d^2*e^2*Cos[2*d*x] + 6*
b^2*d^2*e^2*E^((4*I)*c)*Cos[2*d*x] - (6*I)*b^2*d*e*f*Cos[2*d*x] + (6*I)*b^2
*d*e*E^((4*I)*c)*f*Cos[2*d*x] - 3*b^2*f^2*Cos[2*d*x] - 3*b^2*E^((4*I)*c)*f^
```

```
2*Cos[2*d*x] + 12*b^2*d^2*e*f*x*Cos[2*d*x] + 12*b^2*d^2*e*E^((4*I)*c)*f*x*C
os[2*d*x] - (6*I)*b^2*d*f^2*x*Cos[2*d*x] + (6*I)*b^2*d*E^((4*I)*c)*f^2*x*Co
s[2*d*x] + 6*b^2*d^2*f^2*x^2*Cos[2*d*x] + 6*b^2*d^2*E^((4*I)*c)*f^2*x^2*Cos
[2*d*x] - 24*a^2*d^2*e^2*E^((2*I)*c)*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-
1 + E^((2*I)*(c + d*x)))^2] + 24*b^2*d^2*e^2*E^((2*I)*c)*Log[4*a^2*E^((2*I)
*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] - 96*a^2*d^2*e*E^((2*I)*c)*
f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)
*c)])] + 96*b^2*d^2*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^
(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 48*a^2*d^2*E^((2*I)*c)*f^2*x^2*L
og[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])
] + 48*b^2*d^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*
c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 96*a^2*d^2*e*E^((2*I)*c)*f*x*Log[1
+ (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 9
6*b^2*d^2*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sq
rt[(-a^2 + b^2)*E^((2*I)*c)])] - 48*a^2*d^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*
E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 48*b^2
*d^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[
(-a^2 + b^2)*E^((2*I)*c)])] + (96*I)*(a^2 - b^2)*d*E^((2*I)*c)*f*(e + f*x)*
PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I
)*c)])] + (96*I)*(a^2 - b^2)*d*E^((2*I)*c)*f*(e + f*x)*PolyLog[2, -((b*E^(I
*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 96*a^2*E^
((2*I)*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2
+ b^2)*E^((2*I)*c)])] + 96*b^2*E^((2*I)*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c +
d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 96*a^2*E^((2*I)*c)
*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^
((2*I)*c)]))] + 96*b^2*E^((2*I)*c)*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(
I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 24*a*b*d^2*e^2*E^(I*c)*Si
n[d*x] + 24*a*b*d^2*e^2*E^((3*I)*c)*Sin[d*x] - (48*I)*a*b*d*e*E^(I*c)*f*Sin
[d*x] + (48*I)*a*b*d*e*E^((3*I)*c)*f*Sin[d*x] - 48*a*b*E^(I*c)*f^2*Sin[d*x]
 - 48*a*b*E^((3*I)*c)*f^2*Sin[d*x] + 48*a*b*d^2*e*E^(I*c)*f*x*Sin[d*x] + 48
*a*b*d^2*e*E^((3*I)*c)*f*x*Sin[d*x] - (48*I)*a*b*d*E^(I*c)*f^2*x*Sin[d*x] +
 (48*I)*a*b*d*E^((3*I)*c)*f^2*x*Sin[d*x] + 24*a*b*d^2*E^(I*c)*f^2*x^2*Sin[d
*x] + 24*a*b*d^2*E^((3*I)*c)*f^2*x^2*Sin[d*x] - (6*I)*b^2*d^2*e^2*Sin[2*d*x
] + (6*I)*b^2*d^2*e^2*E^((4*I)*c)*Sin[2*d*x] - 6*b^2*d*e*f*Sin[2*d*x] - 6*b
^2*d*e*E^((4*I)*c)*f*Sin[2*d*x] + (3*I)*b^2*f^2*Sin[2*d*x] - (3*I)*b^2*E^((
4*I)*c)*f^2*Sin[2*d*x] - (12*I)*b^2*d^2*e*f*x*Sin[2*d*x] + (12*I)*b^2*d^2*e
*E^((4*I)*c)*f*x*Sin[2*d*x] - 6*b^2*d*f^2*x*Sin[2*d*x] - 6*b^2*d*E^((4*I)*c
)*f^2*x*Sin[2*d*x] - (6*I)*b^2*d^2*f^2*x^2*Sin[2*d*x] + (6*I)*b^2*d^2*E^((4
*I)*c)*f^2*x^2*Sin[2*d*x])/(48*b^3*d^3*E^((2*I)*c))
```

Maple [F] time = 1.442, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 (\cos(dx+c))^3}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.33455, size = 4139, normalized size = 7.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 4*(a^2 - b^2)*f^2*polylog(3, 1/2*
(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2))/b) + 4*(a^2 - b^2)*f^2*polylog(3, 1/2*(2*I*a*co
s(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2))/b) + 4*(a^2 - b^2)*f^2*polylog(3, -(I*a*cos(d*x + c) + a
*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))
/b) + 4*(a^2 - b^2)*f^2*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b
*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - (2*b^2*d^2*f
^2*x^2 + 4*b^2*d^2*e*f*x + 2*b^2*d^2*e^2 - b^2*f^2)*cos(d*x + c)^2 - 8*(a*b
*d*f^2*x + a*b*d*e*f)*cos(d*x + c) - (-4*I*(a^2 - b^2)*d*f^2*x - 4*I*(a^2 -
```

```
 b^2)*d*e*f)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d
*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - (-4*I*(a
^2 - b^2)*d*f^2*x - 4*I*(a^2 - b^2)*d*e*f)*dilog(-1/2*(2*I*a*cos(d*x + c) +
 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2) + 2*b)/b + 1) - (4*I*(a^2 - b^2)*d*f^2*x + 4*I*(a^2 - b^2)*d*e*f)*dil
og(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - (4*I*(a^2 - b^2)*d*f^2*
x + 4*I*(a^2 - b^2)*d*e*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x +
c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b
+ 1) + 2*((a^2 - b^2)*d^2*e^2 - 2*(a^2 - b^2)*c*d*e*f + (a^2 - b^2)*c^2*f^2
)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) +
2*I*a) + 2*((a^2 - b^2)*d^2*e^2 - 2*(a^2 - b^2)*c*d*e*f + (a^2 - b^2)*c^2*f
^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
- 2*I*a) + 2*((a^2 - b^2)*d^2*e^2 - 2*(a^2 - b^2)*c*d*e*f + (a^2 - b^2)*c^2
*f^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^
2) + 2*I*a) + 2*((a^2 - b^2)*d^2*e^2 - 2*(a^2 - b^2)*c*d*e*f + (a^2 - b^2)*
c^2*f^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)
/b^2) - 2*I*a) + 2*((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x + 2*(
a^2 - b^2)*c*d*e*f - (a^2 - b^2)*c^2*f^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a
*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
) + 2*b)/b) + 2*((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x + 2*(a^2
 - b^2)*c*d*e*f - (a^2 - b^2)*c^2*f^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*si
n(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) +
 2*b)/b) + 2*((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x + 2*(a^2 -
b^2)*c*d*e*f - (a^2 - b^2)*c^2*f^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(
d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2
*b)/b) + 2*((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x + 2*(a^2 - b^
2)*c*d*e*f - (a^2 - b^2)*c^2*f^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*
x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b
)/b) - 2*(2*a*b*d^2*f^2*x^2 + 4*a*b*d^2*e*f*x + 2*a*b*d^2*e^2 - 4*a*b*f^2 -
 (b^2*d*f^2*x + b^2*d*e*f)*cos(d*x + c))*sin(d*x + c))/(b^3*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^2\cos\left(dx+c\right)^3}{b\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cos(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

3.304 $\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=351

$$\frac{if\left(a^2-b^2\right)\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{if\left(a^2-b^2\right)\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2}-\frac{\left(a^2-b^2\right)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}-\frac{(a^2-\ldots}{}$$

```
[Out] (f*x)/(4*b*d) + ((I/2)*(a^2 - b^2)*(e + f*x)^2)/(b^3*f) + (a*f*Cos[c + d*x]
)/(b^2*d^2) - ((a^2 - b^2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqr
t[a^2 - b^2])])/(b^3*d) - ((a^2 - b^2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x
)))/(a + Sqrt[a^2 - b^2])])/(b^3*d) + (I*(a^2 - b^2)*f*PolyLog[2, (I*b*E^(I
*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*d^2) + (I*(a^2 - b^2)*f*PolyLog[2
, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d^2) + (a*(e + f*x)*Si
n[c + d*x])/(b^2*d) - (f*Cos[c + d*x]*Sin[c + d*x])/(4*b*d^2) - ((e + f*x)*
Sin[c + d*x]^2)/(2*b*d)
```

Rubi [A] time = 0.409249, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {4525, 3296, 2638, 4404, 2635, 8, 4519, 2190, 2279, 2391}

$$\frac{if\left(a^2-b^2\right)\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{if\left(a^2-b^2\right)\mathrm{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2}-\frac{\left(a^2-b^2\right)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}-\frac{(a^2-\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (f*x)/(4*b*d) + ((I/2)*(a^2 - b^2)*(e + f*x)^2)/(b^3*f) + (a*f*Cos[c + d*x]
)/(b^2*d^2) - ((a^2 - b^2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqr
t[a^2 - b^2])])/(b^3*d) - ((a^2 - b^2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x
)))/(a + Sqrt[a^2 - b^2])])/(b^3*d) + (I*(a^2 - b^2)*f*PolyLog[2, (I*b*E^(I
*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b^3*d^2) + (I*(a^2 - b^2)*f*PolyLog[2
, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b^3*d^2) + (a*(e + f*x)*Si
n[c + d*x])/(b^2*d) - (f*Cos[c + d*x]*Sin[c + d*x])/(4*b*d^2) - ((e + f*x)*
Sin[c + d*x]^2)/(2*b*d)
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
```

```
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int(e+fx)\cos(c+dx)\,dx}{b^2} - \frac{\int(e+fx)\cos(c+dx)\sin(c+dx)\,dx}{b} - \frac{\left(a^2-b^2\right)\int\frac{(e+fx)\cos}{a+b\sin(c}}{b^2}$$

$$= \frac{i\left(a^2-b^2\right)(e+fx)^2}{2b^3f} + \frac{a(e+fx)\sin(c+dx)}{b^2d} - \frac{(e+fx)\sin^2(c+dx)}{2bd} - \frac{\left(a^2-b^2\right)\int\frac{e^{i(c+}}{a-\sqrt{a^2-}}}{b^2}$$

$$= \frac{i\left(a^2-b^2\right)(e+fx)^2}{2b^3f} + \frac{af\cos(c+dx)}{b^2d^2} - \frac{\left(a^2-b^2\right)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} - \frac{\left(a^2-b^2\right)}{}$$

$$= \frac{fx}{4bd} + \frac{i\left(a^2-b^2\right)(e+fx)^2}{2b^3f} + \frac{af\cos(c+dx)}{b^2d^2} - \frac{\left(a^2-b^2\right)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} - \frac{(a}{}$$

$$= \frac{fx}{4bd} + \frac{i\left(a^2-b^2\right)(e+fx)^2}{2b^3f} + \frac{af\cos(c+dx)}{b^2d^2} - \frac{\left(a^2-b^2\right)(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} - \frac{(a}{}$$

Mathematica [B] time = 14.4568, size = 2165, normalized size = 6.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^3)/(a + b*Sin[c + d*x]),x]

[Out] (a*f*Cos[c + d*x])/(b^2*d^2) + ((d*e - c*f + f*(c + d*x))*Cos[2*(c + d*x)])
/(4*b*d^2) + (a*(d*e - c*f + f*(c + d*x))*Sin[c + d*x])/(b^2*d^2) - (f*Sin[
2*(c + d*x)])/(8*b*d^2) + ((f*(c + d*x)^2 + (2*I)*d*e*Log[Sec[(c + d*x)/2]^
2] - (2*I)*c*f*Log[Sec[(c + d*x)/2]^2] - (2*I)*d*e*Log[Sec[(c + d*x)/2]^2*(
a + b*Sin[c + d*x])] + (2*I)*c*f*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x]
)] - (4*I)*f*(c + d*x)*Log[(-2*I)/(-I + Tan[(c + d*x)/2])] - 2*f*Log[1 + I*
Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b
- Sqrt[-a^2 + b^2])] + 2*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2
 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))] + 2*f*Log[1 -
I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a
 + b + Sqrt[-a^2 + b^2])] - 2*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-
a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + 4*f*PolyLo
g[2, -Cos[c + d*x] + I*Sin[c + d*x]] + 2*f*PolyLog[2, (a*(1 - I*Tan[(c + d*
x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))] - 2*f*PolyLog[2, (a*(1 + I*Tan[(c +
 d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))] + 2*f*PolyLog[2, (a*(I + Tan[(c
+ d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])] - 2*f*PolyLog[2, (a + I*a*Tan[(c
+ d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))])*((e*Cos[c + d*x])/(a + b*Sin[c
 + d*x]) - (a^2*e*Cos[c + d*x])/(b^2*(a + b*Sin[c + d*x])) - (c*f*Cos[c + d
*x])/(d*(a + b*Sin[c + d*x])) + (a^2*c*f*Cos[c + d*x])/(b^2*d*(a + b*Sin[c
+ d*x])) + (f*(c + d*x)*Cos[c + d*x])/(d*(a + b*Sin[c + d*x])) - (a^2*f*(c
+ d*x)*Cos[c + d*x])/(b^2*d*(a + b*Sin[c + d*x]))))/(d*(2*f*(c + d*x) - (4*
I)*f*Log[(-2*I)/(-I + Tan[(c + d*x)/2])] - (4*f*Log[1 + Cos[c + d*x] - I*Si
n[c + d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))/(-Cos[c + d*x] + I*Sin[c + d*x
]) + (I*f*Log[1 - (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]
))]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) - (I*f*Log[-((b - Sqrt[-a^
2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)
/2]^2)/(1 - I*Tan[(c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c
 + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 - I*Tan
[(c + d*x)/2]) + (I*f*Log[1 - (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt
[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*Log[(b
- Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])]*Sec[
(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] +
 a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 +
 I*Tan[(c + d*x)/2]) + (2*I)*d*e*Tan[(c + d*x)/2] - (2*I)*c*f*Tan[(c + d*x)
/2] + ((2*I)*f*(c + d*x)*Sec[(c + d*x)/2]^2)/(-I + Tan[(c + d*x)/2]) - (f*L
og[1 - (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]*Sec[(c + d*
x)/2]^2)/(I + Tan[(c + d*x)/2]) + (I*a*f*Log[1 - (a + I*a*Tan[(c + d*x)/2])
/(a + I*(-b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(a + I*a*Tan[(c + d*x
)/2]) + (a*f*Log[1 - I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b - Sqrt[-a^2
 + b^2] + a*Tan[(c + d*x)/2]) - (a*f*Log[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d
*x)/2]^2)/(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) + (a*f*Log[1 - I*Tan[
(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2
```

```
]) - (a*f*Log[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[-a^2 +
b^2] + a*Tan[(c + d*x)/2]) - ((2*I)*d*e*Cos[(c + d*x)/2]^2*(b*Cos[c + d*x]*
Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c + d*x)/
2]))/(a + b*Sin[c + d*x]) + ((2*I)*c*f*Cos[(c + d*x)/2]^2*(b*Cos[c + d*x]*S
ec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c + d*x)/2
]))/(a + b*Sin[c + d*x])))
```

Maple [B] time = 0.773, size = 1750, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/b^3/d*a^2*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+2/b^3/d*a^2*
e*ln(exp(I*(d*x+c)))+1/2*I/b^3*a^2*f*x^2+1/b^3/d*a^4*f/(-a^2+b^2)*ln((I*a+b
*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+1/b^3/d^2*a^4*f
/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2
)))*c+1/b^3/d*a^4*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(
I*a+(-a^2+b^2)^(1/2)))*x+1/b^3/d^2*a^4*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c)
)+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-I/b^3/d^2*a^4*f/(-a^2+b^2)*di
log((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-I/b^3/d
^2*a^4*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^
2+b^2)^(1/2)))+2*I/b^3/d*a^2*f*c*x+2*I/b/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(
I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*a^2+2*I/b/d^2*f/(-a^2+
b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*
a^2-1/b/d^2*f*c*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+2/b/d^2*f*c
*ln(exp(I*(d*x+c)))-I/b/d^2*f*c^2+b/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))
-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+b/d^2*f/(-a^2+b^2)*ln((I*a+b*e
xp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+b/d*f/(-a^2+b^2)*
ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+b/d^2*
f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/
2)))*c-2*I/b/d*f*c*x+I/b*e*x+1/b/d*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x
+c))-I*b)-2/b/d*ln(exp(I*(d*x+c)))*e-1/2*I/b*f*x^2-1/2*I*a*(d*f*x+I*f+d*e)/
d^2/b^2*exp(I*(d*x+c))-I*b/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a
^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-I*b/d^2*f/(-a^2+b^2)*dilog((I*a+b*ex
p(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2/b/d*f/(-a^2+b^2)*l
n((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2*x-2/b
/d^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2
)^(1/2)))*a^2*c-2/b/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2
))/(I*a-(-a^2+b^2)^(1/2)))*a^2*x-2/b/d^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+
```

```
c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*a^2*c+I/b^3/d^2*a^2*f*c^2-2/b
^3/d^2*a^2*f*c*ln(exp(I*(d*x+c)))+1/b^3/d^2*a^2*f*c*ln(I*b*exp(2*I*(d*x+c))
-2*a*exp(I*(d*x+c))-I*b)-I/b^3*a^2*e*x+1/2*I*a*(d*f*x-I*f+d*e)/d^2/b^2*exp(
-I*(d*x+c))+1/16*(2*d*f*x+I*f+2*d*e)/b/d^2*exp(2*I*(d*x+c))+1/16*(2*d*f*x-I
*f+2*d*e)/b/d^2*exp(-2*I*(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.21873, size = 2533, normalized size = 7.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d*f*x - 4*a*b*f*cos(d*x + c) - 2*(b^2*d*f*x + b^2*d*e)*cos(d*x +
c)^2 + 2*I*(a^2 - b^2)*f*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c)
+ 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1
) + 2*I*(a^2 - b^2)*f*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2
*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) -
 2*I*(a^2 - b^2)*f*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(
b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2
*I*(a^2 - b^2)*f*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*
cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(
(a^2 - b^2)*d*e - (a^2 - b^2)*c*f)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c
) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*((a^2 - b^2)*d*e - (a^2 - b^2)*
c*f)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
 - 2*I*a) + 2*((a^2 - b^2)*d*e - (a^2 - b^2)*c*f)*log(-2*b*cos(d*x + c) + 2
*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*((a^2 - b^2)*d*
e - (a^2 - b^2)*c*f)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(
-(a^2 - b^2)/b^2) - 2*I*a) + 2*((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(1/
```

```
2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*((a^2 - b^2)*d*f*x + (a^2 - b^2)
*c*f)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) -
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*((a^2 - b^2)*d*f*x +
 (a^2 - b^2)*c*f)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*co
s(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*((a^2 -
 b^2)*d*f*x + (a^2 - b^2)*c*f)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x +
 c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b
) - (4*a*b*d*f*x + 4*a*b*d*e - b^2*f*cos(d*x + c))*sin(d*x + c))/(b^3*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cos(dx+c)^3}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cos(d*x + c)^3/(b*sin(d*x + c) + a), x)
```

3.305 $\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=61

$$-\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{b^3d}+\frac{a\sin(c+dx)}{b^2d}-\frac{\sin^2(c+dx)}{2bd}$$

```
[Out] -(((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^3*d)) + (a*Sin[c + d*x])/(b^2*d)
 - Sin[c + d*x]^2/(2*b*d)
```

Rubi [A] time = 0.0677086, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{b^3d}+\frac{a\sin(c+dx)}{b^2d}-\frac{\sin^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^3*d)) + (a*Sin[c + d*x])/(b^2*d)
 - Sin[c + d*x]^2/(2*b*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2+b^2}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= -\frac{\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^3d} + \frac{a\sin(c+dx)}{b^2d} - \frac{\sin^2(c+dx)}{2bd}$$

Mathematica [A] time = 0.0766312, size = 54, normalized size = 0.89

$$\frac{-\left(a^2-b^2\right)\log(a+b\sin(c+dx))+ab\sin(c+dx)-\frac{1}{2}b^2\sin^2(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-((a^2 - b^2)*Log[a + b*Sin[c + d*x]]) + a*b*Sin[c + d*x] - (b^2*Sin[c + d
*x]^2)/2)/(b^3*d)
```

Maple [A] time = 0.001, size = 72, normalized size = 1.2

$$-\frac{(\sin(dx+c))^2}{2\,bd} + \frac{a\sin(dx+c)}{b^2d} - \frac{\ln(a+b\sin(dx+c))\,a^2}{db^3} + \frac{\ln(a+b\sin(dx+c))}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2*sin(d*x+c)^2/b/d+a*sin(d*x+c)/b^2/d-1/d/b^3*ln(a+b*sin(d*x+c))*a^2+ln(
a+b*sin(d*x+c))/b/d
```

Maxima [A] time = 0.945538, size = 74, normalized size = 1.21

$$-\frac{\frac{b\sin(dx+c)^2-2\,a\sin(dx+c)}{b^2} + \frac{2\left(a^2-b^2\right)\log(b\sin(dx+c)+a)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2 + 2*(a^2 - b^2)*log(b*sin(d
*x + c) + a)/b^3)/d
```

Fricas [A] time = 1.73674, size = 128, normalized size = 2.1

$$\frac{b^2\cos(dx+c)^2+2\,ab\sin(dx+c)-2\left(a^2-b^2\right)\log(b\sin(dx+c)+a)}{2\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cos(d*x + c)^2 + 2*a*b*sin(d*x + c) - 2*(a^2 - b^2)*log(b*sin(d*x
+ c) + a))/(b^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18506, size = 76, normalized size = 1.25

$$-\frac{\frac{b\sin(dx+c)^2-2\,a\sin(dx+c)}{b^2}+\frac{2\left(a^2-b^2\right)\log(|b\sin(dx+c)+a|)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2 + 2*(a^2 - b^2)*log(abs(b*s
in(d*x + c) + a))/b^3)/d
```

3.306 $\int \frac{(e+fx)^3 \sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=937

$$-\frac{6ia\text{PolyLog}\left(4,-ie^{i(c+dx)}\right)f^3}{\left(a^2-b^2\right)d^4}+\frac{6ia\text{PolyLog}\left(4,ie^{i(c+dx)}\right)f^3}{\left(a^2-b^2\right)d^4}-\frac{6ib\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f^3}{\left(a^2-b^2\right)d^4}-\frac{6ib\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^4}$$

```
[Out] ((-2*I)*a*(e + f*x)^3*ArcTan[E^(I*(c + d*x))])/((a^2 - b^2)*d) - (b*(e + f*
x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*d)
- (b*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^
2 - b^2)*d) + (b*(e + f*x)^3*Log[1 + E^((2*I)*(c + d*x))])/((a^2 - b^2)*d)
+ ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^2
) - ((3*I)*a*f*(e + f*x)^2*PolyLog[2, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^2)
 + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
b^2])])/((a^2 - b^2)*d^2) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c
+ d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) - (((3*I)/2)*b*f*(e + f*
x)^2*PolyLog[2, -E^((2*I)*(c + d*x))])/((a^2 - b^2)*d^2) - (6*a*f^2*(e + f*
x)*PolyLog[3, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) + (6*a*f^2*(e + f*x)
*PolyLog[3, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) - (6*b*f^2*(e + f*x)*Poly
Log[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^3) - (6
*b*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(
(a^2 - b^2)*d^3) + (3*b*f^2*(e + f*x)*PolyLog[3, -E^((2*I)*(c + d*x))])/(2*
(a^2 - b^2)*d^3) - ((6*I)*a*f^3*PolyLog[4, (-I)*E^(I*(c + d*x))])/((a^2 - b
^2)*d^4) + ((6*I)*a*f^3*PolyLog[4, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^4) -
((6*I)*b*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2
 - b^2)*d^4) - ((6*I)*b*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
- b^2])])/((a^2 - b^2)*d^4) + (((3*I)/4)*b*f^3*PolyLog[4, -E^((2*I)*(c + d*
x))])/((a^2 - b^2)*d^4)
```

Rubi [A] time = 1.61759, antiderivative size = 937, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {4533, 4519, 2190, 2531, 6609, 2282, 6589, 6742, 4181, 3719}

$$-\frac{6ia\text{PolyLog}\left(4,-ie^{i(c+dx)}\right)f^3}{\left(a^2-b^2\right)d^4}+\frac{6ia\text{PolyLog}\left(4,ie^{i(c+dx)}\right)f^3}{\left(a^2-b^2\right)d^4}-\frac{6ib\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)f^3}{\left(a^2-b^2\right)d^4}-\frac{6ib\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*I)*a*(e + f*x)^3*ArcTan[E^(I*(c + d*x))])/((a^2 - b^2)*d) - (b*(e + f*
x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*d)
- (b*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^
2 - b^2)*d) + (b*(e + f*x)^3*Log[1 + E^((2*I)*(c + d*x))])/((a^2 - b^2)*d)
+ ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^2
) - ((3*I)*a*f*(e + f*x)^2*PolyLog[2, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^2)
 + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
b^2])])/((a^2 - b^2)*d^2) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c
+ d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) - (((3*I)/2)*b*f*(e + f*
x)^2*PolyLog[2, -E^((2*I)*(c + d*x))])/((a^2 - b^2)*d^2) - (6*a*f^2*(e + f*
x)*PolyLog[3, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) + (6*a*f^2*(e + f*x)
*PolyLog[3, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) - (6*b*f^2*(e + f*x)*Poly
Log[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^3) - (6
*b*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(
(a^2 - b^2)*d^3) + (3*b*f^2*(e + f*x)*PolyLog[3, -E^((2*I)*(c + d*x))])/(2*
(a^2 - b^2)*d^3) - ((6*I)*a*f^3*PolyLog[4, (-I)*E^(I*(c + d*x))])/((a^2 - b
^2)*d^4) + ((6*I)*a*f^3*PolyLog[4, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^4) -
((6*I)*b*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2
 - b^2)*d^4) - ((6*I)*b*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
- b^2])])/((a^2 - b^2)*d^4) + (((3*I)/4)*b*f^3*PolyLog[4, -E^((2*I)*(c + d*
x))])/((a^2 - b^2)*d^4)
```

Rule 4533

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[b^2/(a^2 - b^2), Int[((e + f
*x)^m*Sec[c + d*x]^(n - 2))/(a + b*Sin[c + d*x]), x], x] + Dist[1/(a^2 - b^
2), Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\sec(c+dx)(a-b\sin(c+dx))\,dx}{a^2-b^2} - \frac{b^2\int \frac{(e+fx)^3\cos(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{ib(e+fx)^4}{4\left(a^2-b^2\right)f} + \frac{\int \left(a(e+fx)^3\sec(c+dx) - b(e+fx)^3\tan(c+dx)\right)dx}{a^2-b^2} - \frac{b^2\int \frac{e^{i(c+dx)}(e+}{a-\sqrt{a^2-b^2}-ib}}{a^2-b^2}$$

$$= \frac{ib(e+fx)^4}{4\left(a^2-b^2\right)f} - \frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} + \frac{a\int (e+f}{}$$

$$= -\frac{2ia(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+}}{a+\sqrt{a^2}}\right)}{\left(a^2-b^2\right)d}$$

$$= -\frac{2ia(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+}}{a+\sqrt{a^2}}\right)}{\left(a^2-b^2\right)d}$$

$$= -\frac{2ia(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+}}{a+\sqrt{a^2}}\right)}{\left(a^2-b^2\right)d}$$

$$= -\frac{2ia(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+}}{a+\sqrt{a^2}}\right)}{\left(a^2-b^2\right)d}$$

$$= -\frac{2ia(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+}}{a+\sqrt{a^2}}\right)}{\left(a^2-b^2\right)d}$$

$$= -\frac{2ia(e+fx)^3\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^3\log\left(1-\frac{ibe^{i(c+}}{a+\sqrt{a^2}}\right)}{\left(a^2-b^2\right)d}$$

Mathematica [B] time = 9.83598, size = 2496, normalized size = 2.66

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((4*((I*b*(e + f*x)^4)/f - (2*(a - b)*(1 + E^((2*I)*c))*(e + f*x)^3*Log[1 -
 I/E^(I*(c + d*x))])/d + (2*(a + b)*(1 + E^((2*I)*c))*(e + f*x)^3*Log[1 + I
/E^(I*(c + d*x))])/d + (6*(a + b)*(1 + E^((2*I)*c))*f*(I*d^2*(e + f*x)^2*Po
lyLog[2, (-I)/E^(I*(c + d*x))] + 2*f*(d*(e + f*x)*PolyLog[3, (-I)/E^(I*(c +
 d*x))] - I*f*PolyLog[4, (-I)/E^(I*(c + d*x))])))/d^4 - ((6*I)*(a - b)*(1 +
 E^((2*I)*c))*f*(d^2*(e + f*x)^2*PolyLog[2, I/E^(I*(c + d*x))] - (2*I)*d*f*
(e + f*x)*PolyLog[3, I/E^(I*(c + d*x))] - 2*f^2*PolyLog[4, I/E^(I*(c + d*x)
)]))/d^4))/((a^2 - b^2)*(1 + E^((2*I)*c))) + (4*b*((-4*I)*d^4*e^3*E^((2*I)*
c)*x - (6*I)*d^4*e^2*E^((2*I)*c)*f*x^2 - (4*I)*d^4*e*E^((2*I)*c)*f^2*x^3 -
I*d^4*E^((2*I)*c)*f^3*x^4 - (2*I)*d^3*e^3*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(
-1 + E^((2*I)*(c + d*x))))] + (2*I)*d^3*e^3*E^((2*I)*c)*ArcTan[(2*a*E^(I*(c
 + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] - d^3*e^3*Log[4*a^2*E^((2*I)*(c +
 d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] + d^3*e^3*E^((2*I)*c)*Log[4*a^2*
E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] - 6*d^3*e^2*f*x*Log
[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
+ 6*d^3*e^2*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sq
rt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x
)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e*E^((2*I)*c)*f
^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2
*I)*c)])] - 2*d^3*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt
[(-a^2 + b^2)*E^((2*I)*c)])] + 2*d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(2
*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e^2*f*x
*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)
])] + 6*d^3*e^2*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c)
+ Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c +
 d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e*E^((2*I)*
c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E
^((2*I)*c)])] - 2*d^3*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) +
Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 2*d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(
I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (6*I)*d^2
*(-1 + E^((2*I)*c))*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(
I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (6*I)*d^2*(-1 + E^((2*I)*c))*f*
(e + f*x)^2*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 +
b^2)*E^((2*I)*c)]))] - 12*d*e*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(
I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*d*e*E^((2*I)*c)*f^2*PolyLog[
3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
- 12*d*f^3*x*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 +
 b^2)*E^((2*I)*c)])] + 12*d*E^((2*I)*c)*f^3*x*PolyLog[3, (I*b*E^(I*(2*c + d
*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d*e*f^2*PolyLog[
3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]
```

```
 + 12*d*e*E^((2*I)*c)*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) +
 Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 12*d*f^3*x*PolyLog[3, -((b*E^(I*(2*c +
 d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 12*d*E^((2*I)*c)
*f^3*x*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*
E^((2*I)*c)]))] - (12*I)*f^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c)
+ I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (12*I)*E^((2*I)*c)*f^3*PolyLog[4, (I
*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (12
*I)*f^3*PolyLog[4, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)
*E^((2*I)*c)]))] + (12*I)*E^((2*I)*c)*f^3*PolyLog[4, -((b*E^(I*(2*c + d*x))
)/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]))/((-a^2 + b^2)*d^4*(-1
+ E^((2*I)*c))) - (8*b*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*Csc[c]
^3)/((a - b)*(a + b)*(Csc[c/2] - Sec[c/2])*(Csc[c/2] + Sec[c/2])))/8
```

Maple [F] time = 0.914, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \sec(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.97027, size = 7549, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(6*I*b*f^3*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(
b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*I*b*f^3*p
olylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) +
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*I*b*f^3*polylog(4, -(I*a*c
os(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2))/b) - 6*I*b*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x +
c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*I*(
a - b)*f^3*polylog(4, I*cos(d*x + c) + sin(d*x + c)) - 6*I*(a + b)*f^3*poly
log(4, I*cos(d*x + c) - sin(d*x + c)) + 6*I*(a - b)*f^3*polylog(4, -I*cos(d
*x + c) + sin(d*x + c)) + 6*I*(a + b)*f^3*polylog(4, -I*cos(d*x + c) - sin(
d*x + c)) + (3*I*b*d^2*f^3*x^2 + 6*I*b*d^2*e*f^2*x + 3*I*b*d^2*e^2*f)*dilog
(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (3*I*b*d^2*f^3*x^2 + 6*I*b
*d^2*e*f^2*x + 3*I*b*d^2*e^2*f)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*
x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b
)/b + 1) + (-3*I*b*d^2*f^3*x^2 - 6*I*b*d^2*e*f^2*x - 3*I*b*d^2*e^2*f)*dilog
(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin
(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-3*I*b*d^2*f^3*x^2 - 6*I
*b*d^2*e*f^2*x - 3*I*b*d^2*e^2*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin
(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) +
2*b)/b + 1) + (3*I*(a - b)*d^2*f^3*x^2 + 6*I*(a - b)*d^2*e*f^2*x + 3*I*(a -
 b)*d^2*e^2*f)*dilog(I*cos(d*x + c) + sin(d*x + c)) + (3*I*(a + b)*d^2*f^3*
x^2 + 6*I*(a + b)*d^2*e*f^2*x + 3*I*(a + b)*d^2*e^2*f)*dilog(I*cos(d*x + c)
 - sin(d*x + c)) + (-3*I*(a - b)*d^2*f^3*x^2 - 6*I*(a - b)*d^2*e*f^2*x - 3*
I*(a - b)*d^2*e^2*f)*dilog(-I*cos(d*x + c) + sin(d*x + c)) + (-3*I*(a + b)*
d^2*f^3*x^2 - 6*I*(a + b)*d^2*e*f^2*x - 3*I*(a + b)*d^2*e^2*f)*dilog(-I*cos
(d*x + c) - sin(d*x + c)) + (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2
- b*c^3*f^3)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b
^2)/b^2) + 2*I*a) + (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*
f^3)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
 - 2*I*a) + (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log
(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*
a) + (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(-2*b*c
os(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (b
*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*
c^2*d*e*f^2 + b*c^3*f^3)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2
*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (b*
d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c
^2*d*e*f^2 + b*c^3*f^3)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*
(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (b*d
```

```
^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^
2*d*e*f^2 + b*c^3*f^3)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*
(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (b*d
^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^
2*d*e*f^2 + b*c^3*f^3)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*
(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - ((a
+ b)*d^3*e^3 - 3*(a + b)*c*d^2*e^2*f + 3*(a + b)*c^2*d*e*f^2 - (a + b)*c^3*
f^3)*log(cos(d*x + c) + I*sin(d*x + c) + I) + ((a - b)*d^3*e^3 - 3*(a - b)*
c*d^2*e^2*f + 3*(a - b)*c^2*d*e*f^2 - (a - b)*c^3*f^3)*log(cos(d*x + c) - I
*sin(d*x + c) + I) - ((a + b)*d^3*f^3*x^3 + 3*(a + b)*d^3*e*f^2*x^2 + 3*(a
+ b)*d^3*e^2*f*x + 3*(a + b)*c*d^2*e^2*f - 3*(a + b)*c^2*d*e*f^2 + (a + b)*
c^3*f^3)*log(I*cos(d*x + c) + sin(d*x + c) + 1) + ((a - b)*d^3*f^3*x^3 + 3*
(a - b)*d^3*e*f^2*x^2 + 3*(a - b)*d^3*e^2*f*x + 3*(a - b)*c*d^2*e^2*f - 3*(
a - b)*c^2*d*e*f^2 + (a - b)*c^3*f^3)*log(I*cos(d*x + c) - sin(d*x + c) + 1
) - ((a + b)*d^3*f^3*x^3 + 3*(a + b)*d^3*e*f^2*x^2 + 3*(a + b)*d^3*e^2*f*x
+ 3*(a + b)*c*d^2*e^2*f - 3*(a + b)*c^2*d*e*f^2 + (a + b)*c^3*f^3)*log(-I*c
os(d*x + c) + sin(d*x + c) + 1) + ((a - b)*d^3*f^3*x^3 + 3*(a - b)*d^3*e*f^
2*x^2 + 3*(a - b)*d^3*e^2*f*x + 3*(a - b)*c*d^2*e^2*f - 3*(a - b)*c^2*d*e*f
^2 + (a - b)*c^3*f^3)*log(-I*cos(d*x + c) - sin(d*x + c) + 1) - ((a + b)*d^
3*e^3 - 3*(a + b)*c*d^2*e^2*f + 3*(a + b)*c^2*d*e*f^2 - (a + b)*c^3*f^3)*lo
g(-cos(d*x + c) + I*sin(d*x + c) + I) + ((a - b)*d^3*e^3 - 3*(a - b)*c*d^2*
e^2*f + 3*(a - b)*c^2*d*e*f^2 - (a - b)*c^3*f^3)*log(-cos(d*x + c) - I*sin(
d*x + c) + I) + 6*(b*d*f^3*x + b*d*e*f^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c
) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2))/b) + 6*(b*d*f^3*x + b*d*e*f^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c)
 - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^
2)/b^2))/b) + 6*(b*d*f^3*x + b*d*e*f^2)*polylog(3, -(I*a*cos(d*x + c) + a*s
in(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b
) + 6*(b*d*f^3*x + b*d*e*f^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c
) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*((a
- b)*d*f^3*x + (a - b)*d*e*f^2)*polylog(3, I*cos(d*x + c) + sin(d*x + c)) -
 6*((a + b)*d*f^3*x + (a + b)*d*e*f^2)*polylog(3, I*cos(d*x + c) - sin(d*x
+ c)) + 6*((a - b)*d*f^3*x + (a - b)*d*e*f^2)*polylog(3, -I*cos(d*x + c) +
sin(d*x + c)) - 6*((a + b)*d*f^3*x + (a + b)*d*e*f^2)*polylog(3, -I*cos(d*x
 + c) - sin(d*x + c)))/((a^2 - b^2)*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^3 \sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^3\sec\left(dx+c\right)}{b\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

3.307 $\int \frac{(e+fx)^2 \sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=667

$$\frac{2iaf(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2\left(a^2-b^2\right)}-\frac{2iaf(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2\left(a^2-b^2\right)}+\frac{2ibf(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)}+\frac{2ibf(}{}$$

```
[Out] ((-2*I)*a*(e + f*x)^2*ArcTan[E^(I*(c + d*x))])/((a^2 - b^2)*d) - (b*(e + f*
x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*d)
- (b*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^
2 - b^2)*d) + (b*(e + f*x)^2*Log[1 + E^((2*I)*(c + d*x))])/((a^2 - b^2)*d)
+ ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^2)
- ((2*I)*a*f*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^2) + (
(2*I)*b*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])]
)/((a^2 - b^2)*d^2) + ((2*I)*b*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))
/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) - (I*b*f*(e + f*x)*PolyLog[2, -E
^((2*I)*(c + d*x))])/((a^2 - b^2)*d^2) - (2*a*f^2*PolyLog[3, (-I)*E^(I*(c +
 d*x))])/((a^2 - b^2)*d^3) + (2*a*f^2*PolyLog[3, I*E^(I*(c + d*x))])/((a^2
- b^2)*d^3) - (2*b*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2
])])/((a^2 - b^2)*d^3) - (2*b*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqr
t[a^2 - b^2])])/((a^2 - b^2)*d^3) + (b*f^2*PolyLog[3, -E^((2*I)*(c + d*x))]
)/(2*(a^2 - b^2)*d^3)
```

Rubi [A] time = 1.14299, antiderivative size = 667, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {4533, 4519, 2190, 2531, 2282, 6589, 6742, 4181, 3719}

$$\frac{2iaf(e+fx)\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)}{d^2\left(a^2-b^2\right)}-\frac{2iaf(e+fx)\text{PolyLog}\left(2,ie^{i(c+dx)}\right)}{d^2\left(a^2-b^2\right)}+\frac{2ibf(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)}+\frac{2ibf(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```



```
[Out] ((-2*I)*a*(e + f*x)^2*ArcTan[E^(I*(c + d*x))])/((a^2 - b^2)*d) - (b*(e + f*
x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*d)
- (b*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^
2 - b^2)*d) + (b*(e + f*x)^2*Log[1 + E^((2*I)*(c + d*x))])/((a^2 - b^2)*d)
+ ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^2)
```

```
- ((2*I)*a*f*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^2) + (
(2*I)*b*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])]
)/((a^2 - b^2)*d^2) + ((2*I)*b*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))
/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) - (I*b*f*(e + f*x)*PolyLog[2, -E
^((2*I)*(c + d*x))])/((a^2 - b^2)*d^2) - (2*a*f^2*PolyLog[3, (-I)*E^(I*(c +
 d*x))])/((a^2 - b^2)*d^3) + (2*a*f^2*PolyLog[3, I*E^(I*(c + d*x))])/((a^2
- b^2)*d^3) - (2*b*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2
])])/((a^2 - b^2)*d^3) - (2*b*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqr
t[a^2 - b^2])])/((a^2 - b^2)*d^3) + (b*f^2*PolyLog[3, -E^((2*I)*(c + d*x))]
)/(2*(a^2 - b^2)*d^3)
```

Rule 4533

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[b^2/(a^2 - b^2), Int[((e + f
*x)^m*Sec[c + d*x]^(n - 2))/(a + b*Sin[c + d*x]), x], x] + Dist[1/(a^2 - b^
2), Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\sec(c+dx)(a-b\sin(c+dx))\,dx}{a^2-b^2} - \frac{b^2\int \frac{(e+fx)^2\cos(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{ib(e+fx)^3}{3\left(a^2-b^2\right)f} + \frac{\int \left(a(e+fx)^2\sec(c+dx) - b(e+fx)^2\tan(c+dx)\right)dx}{a^2-b^2} - \frac{b^2\int \frac{e^{i(c+dx)}(e+fx}{a-\sqrt{a^2-b^2}-ibe^{i(}}}{a^2-b^2}$$

$$= \frac{ib(e+fx)^3}{3\left(a^2-b^2\right)f} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} + \frac{a\int (e+fx)}{a}$$

$$= -\frac{2ia(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b}}\right)}{\left(a^2-b^2\right)d}$$

$$= -\frac{2ia(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b}}\right)}{\left(a^2-b^2\right)d}$$

$$= -\frac{2ia(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b}}\right)}{\left(a^2-b^2\right)d}$$

$$= -\frac{2ia(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b}}\right)}{\left(a^2-b^2\right)d}$$

$$= -\frac{2ia(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b}}\right)}{\left(a^2-b^2\right)d}$$

Mathematica [B] time = 5.54789, size = 1561, normalized size = 2.34

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*(((2*I)*b*(e + f*x)^3)/f - (3*(a - b)*(1 + E^((2*I)*c))*(e + f*x)^2*Log
[1 - I/E^(I*(c + d*x))])/d + (3*(a + b)*(1 + E^((2*I)*c))*(e + f*x)^2*Log[1
 + I/E^(I*(c + d*x))])/d + (6*(a + b)*(1 + E^((2*I)*c))*f*(I*d*(e + f*x)*Po
lyLog[2, (-I)/E^(I*(c + d*x))] + f*PolyLog[3, (-I)/E^(I*(c + d*x))]))/d^3 -
 ((6*I)*(a - b)*(1 + E^((2*I)*c))*f*(d*(e + f*x)*PolyLog[2, I/E^(I*(c + d*x
))] - I*f*PolyLog[3, I/E^(I*(c + d*x))]))/d^3))/((a^2 - b^2)*(1 + E^((2*I)*
```

```
c))) + (b*((-12*I)*d^3*e^2*E^((2*I)*c)*x - (12*I)*d^3*e*E^((2*I)*c)*f*x^2 -
 (4*I)*d^3*E^((2*I)*c)*f^2*x^3 - (6*I)*d^2*e^2*ArcTan[(2*a*E^(I*(c + d*x)))
/(b*(-1 + E^((2*I)*(c + d*x))))] + (6*I)*d^2*e^2*E^((2*I)*c)*ArcTan[(2*a*E^
(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] - 3*d^2*e^2*Log[4*a^2*E^((2*
I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] + 3*d^2*e^2*E^((2*I)*c)*L
og[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2] - 12*d^2*e
*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I
)*c)])] + 12*d^2*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*
c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^2*f^2*x^2*Log[1 + (b*E^(I*(2*c
+ d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^2*E^((2*I)*c
)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^
((2*I)*c)])] - 12*d^2*e*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sq
rt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*d^2*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(
2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^2*f^2*x^
2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c
)])] + 6*d^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c)
 + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (12*I)*d*(-1 + E^((2*I)*c))*f*(e + f*
x)*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((
2*I)*c)])] - (12*I)*d*(-1 + E^((2*I)*c))*f*(e + f*x)*PolyLog[2, -((b*E^(I*(
2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 12*f^2*Poly
Log[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)
])] + 12*E^((2*I)*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*
Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x))
)/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 12*E^((2*I)*c)*f^2*Pol
yLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c
)]))]))/((-a^2 + b^2)*d^3*(-1 + E^((2*I)*c))) - (8*b*x*(3*e^2 + 3*e*f*x + f
^2*x^2)*Csc[c]^3)/((a - b)*(a + b)*(Csc[c/2] - Sec[c/2])*(Csc[c/2] + Sec[c/
2])))/6
```

Maple [F] time = 0.7, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sec(dx+c)}{a+b\sin(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.32858, size = 5157, normalized size = 7.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*f^2*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*
cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*b*f^2*polyl
og(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*b*f^2*polylog(3, -(I*a*cos(d*x
 + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2))/b) + 2*b*f^2*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*
cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(a - b)*f^2
*polylog(3, I*cos(d*x + c) + sin(d*x + c)) - 2*(a + b)*f^2*polylog(3, I*cos
(d*x + c) - sin(d*x + c)) + 2*(a - b)*f^2*polylog(3, -I*cos(d*x + c) + sin(
d*x + c)) - 2*(a + b)*f^2*polylog(3, -I*cos(d*x + c) - sin(d*x + c)) + (2*I
*b*d*f^2*x + 2*I*b*d*e*f)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c)
 + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b +
1) + (2*I*b*d*f^2*x + 2*I*b*d*e*f)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin
(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) +
2*b)/b + 1) + (-2*I*b*d*f^2*x - 2*I*b*d*e*f)*dilog(-1/2*(-2*I*a*cos(d*x + c
) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2) + 2*b)/b + 1) + (-2*I*b*d*f^2*x - 2*I*b*d*e*f)*dilog(-1/2*(-2*I*a*
cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (2*I*(a - b)*d*f^2*x + 2*I*(a - b)*d*e*
f)*dilog(I*cos(d*x + c) + sin(d*x + c)) + (2*I*(a + b)*d*f^2*x + 2*I*(a + b
)*d*e*f)*dilog(I*cos(d*x + c) - sin(d*x + c)) + (-2*I*(a - b)*d*f^2*x - 2*I
*(a - b)*d*e*f)*dilog(-I*cos(d*x + c) + sin(d*x + c)) + (-2*I*(a + b)*d*f^2
*x - 2*I*(a + b)*d*e*f)*dilog(-I*cos(d*x + c) - sin(d*x + c)) + (b*d^2*e^2
- 2*b*c*d*e*f + b*c^2*f^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*
```

```
sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log
(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a
) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin
(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (b*d^2*e^2 - 2*b*c*d*e*f
+ b*c^2*f^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 -
b^2)/b^2) - 2*I*a) + (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f
^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*
b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (b*d^2*f^2*x^2 + 2*b*d^2
*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x
 + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)
/b) + (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*log(1/2*(-2
*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c
*d*e*f - b*c^2*f^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*
cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - ((a + b
)*d^2*e^2 - 2*(a + b)*c*d*e*f + (a + b)*c^2*f^2)*log(cos(d*x + c) + I*sin(d
*x + c) + I) + ((a - b)*d^2*e^2 - 2*(a - b)*c*d*e*f + (a - b)*c^2*f^2)*log(
cos(d*x + c) - I*sin(d*x + c) + I) - ((a + b)*d^2*f^2*x^2 + 2*(a + b)*d^2*e
*f*x + 2*(a + b)*c*d*e*f - (a + b)*c^2*f^2)*log(I*cos(d*x + c) + sin(d*x +
c) + 1) + ((a - b)*d^2*f^2*x^2 + 2*(a - b)*d^2*e*f*x + 2*(a - b)*c*d*e*f -
(a - b)*c^2*f^2)*log(I*cos(d*x + c) - sin(d*x + c) + 1) - ((a + b)*d^2*f^2*
x^2 + 2*(a + b)*d^2*e*f*x + 2*(a + b)*c*d*e*f - (a + b)*c^2*f^2)*log(-I*cos
(d*x + c) + sin(d*x + c) + 1) + ((a - b)*d^2*f^2*x^2 + 2*(a - b)*d^2*e*f*x
+ 2*(a - b)*c*d*e*f - (a - b)*c^2*f^2)*log(-I*cos(d*x + c) - sin(d*x + c) +
 1) - ((a + b)*d^2*e^2 - 2*(a + b)*c*d*e*f + (a + b)*c^2*f^2)*log(-cos(d*x
+ c) + I*sin(d*x + c) + I) + ((a - b)*d^2*e^2 - 2*(a - b)*c*d*e*f + (a - b)
*c^2*f^2)*log(-cos(d*x + c) - I*sin(d*x + c) + I))/((a^2 - b^2)*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2 \sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^2 \sec\left(dx+c\right)}{b\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

3.308 $\int \frac{(e+fx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=413

$$\frac{iafPolyLog\left(2,-ie^{i(c+dx)}\right)}{d^2\left(a^2-b^2\right)}-\frac{iafPolyLog\left(2,ie^{i(c+dx)}\right)}{d^2\left(a^2-b^2\right)}+\frac{ibfPolyLog\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)}+\frac{ibfPolyLog\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d^2\left(a^2-b^2\right)}-\frac{ibf\mathrm{I}}{}$$

```
[Out] ((-2*I)*a*(e + f*x)*ArcTan[E^(I*(c + d*x))])/((a^2 - b^2)*d) - (b*(e + f*x)
*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*d) - (b
*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^
2)*d) + (b*(e + f*x)*Log[1 + E^((2*I)*(c + d*x))])/((a^2 - b^2)*d) + (I*a*f
*PolyLog[2, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^2) - (I*a*f*PolyLog[2, I*
E^(I*(c + d*x))])/((a^2 - b^2)*d^2) + (I*b*f*PolyLog[2, (I*b*E^(I*(c + d*x)
))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) + (I*b*f*PolyLog[2, (I*b*E^(I*
(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) - ((I/2)*b*f*PolyLog[
2, -E^((2*I)*(c + d*x))])/((a^2 - b^2)*d^2)
```

Rubi [A] time = 0.635319, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4533, 4519, 2190, 2279, 2391, 6742, 4181, 3719}

$$\frac{iafPolyLog\left(2,-ie^{i(c+dx)}\right)}{d^2\left(a^2-b^2\right)}-\frac{iafPolyLog\left(2,ie^{i(c+dx)}\right)}{d^2\left(a^2-b^2\right)}+\frac{ibfPolyLog\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)}+\frac{ibfPolyLog\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d^2\left(a^2-b^2\right)}-\frac{ibf\mathrm{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```


```
[Out] ((-2*I)*a*(e + f*x)*ArcTan[E^(I*(c + d*x))])/((a^2 - b^2)*d) - (b*(e + f*x)
*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*d) - (b
*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^
2)*d) + (b*(e + f*x)*Log[1 + E^((2*I)*(c + d*x))])/((a^2 - b^2)*d) + (I*a*f
*PolyLog[2, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^2) - (I*a*f*PolyLog[2, I*
E^(I*(c + d*x))])/((a^2 - b^2)*d^2) + (I*b*f*PolyLog[2, (I*b*E^(I*(c + d*x)
))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) + (I*b*f*PolyLog[2, (I*b*E^(I*
(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) - ((I/2)*b*f*PolyLog[
2, -E^((2*I)*(c + d*x))])/((a^2 - b^2)*d^2)
```


Rule 4533

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[b^2/(a^2 - b^2), Int[((e + f
*x)^m*Sec[c + d*x]^(n - 2))/(a + b*Sin[c + d*x]), x], x] + Dist[1/(a^2 - b^
2), Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
```

```
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\sec(c+dx)(a-b\sin(c+dx))\,dx}{a^2-b^2} - \frac{b^2\int \frac{(e+fx)\cos(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{ib(e+fx)^2}{2\left(a^2-b^2\right)f} + \frac{\int (a(e+fx)\sec(c+dx)-b(e+fx)\tan(c+dx))\,dx}{a^2-b^2} - \frac{b^2\int \frac{e^{i(c+dx)}(e+fx)}{a-\sqrt{a^2-b^2}-ibe^{i(c+dx}}}{a^2-b^2}$$

$$= \frac{ib(e+fx)^2}{2\left(a^2-b^2\right)f} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} + \frac{a\int (e+fx)\,\mathrm{s}}{a^2}$$

$$= -\frac{2ia(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d}$$

$$= -\frac{2ia(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d}$$

$$= -\frac{2ia(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d}$$

$$= -\frac{2ia(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d} - \frac{b(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d}$$

Mathematica [B] time = 16.59, size = 2743, normalized size = 6.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((d*e + d*f*x)*(((-I)*b*(d*e + d*f*x)^2)/f + 2*(a - b)*(d*e - c*f)*Log[1 -
Tan[(c + d*x)/2]] - 4*b*(d*e + d*f*x)*Log[(-2*I)/(-I + Tan[(c + d*x)/2])] -
 2*(a + b)*(d*e - c*f)*Log[1 + Tan[(c + d*x)/2]] - (4*I)*b*f*PolyLog[2, -Co
s[c + d*x] + I*Sin[c + d*x]] + (2*I)*(a + b)*f*(Log[1 + I*Tan[(c + d*x)/2]]
*Log[(1/2 - I/2)*(1 + Tan[(c + d*x)/2])] + PolyLog[2, ((1 + I) - (1 - I)*Ta
n[(c + d*x)/2])/2]) - (2*I)*(a + b)*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(1/2
 + I/2)*(1 + Tan[(c + d*x)/2])] + PolyLog[2, (-1/2 - I/2)*(I + Tan[(c + d*x
)/2])]) + (2*I)*(a - b)*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(-1/2 + I/2)*(-1
 + Tan[(c + d*x)/2])] + PolyLog[2, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/2])
 - (2*I)*(a - b)*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[
(c + d*x)/2])] + PolyLog[2, ((1 - I) + (1 + I)*Tan[(c + d*x)/2])/2]))*(a -
b*Sin[c + d*x]))/((a^2 - b^2)*d^2*(-2*a*d*e + 2*a*c*f - (2*I)*a*f*Log[1 - I
*Tan[(c + d*x)/2]] + (2*I)*a*f*Log[1 + I*Tan[(c + d*x)/2]] + 4*b*f*Cos[c +
d*x]*(Log[1 + Cos[c + d*x] - I*Sin[c + d*x]] - Log[(-2*I)/(-I + Tan[(c + d*
x)/2])]) + b*(d*e - c*f + f*(c + d*x))*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2
] + b*d*e*Tan[(c + d*x)/2] - b*c*f*Tan[(c + d*x)/2] - b*f*(c + d*x)*Tan[(c
+ d*x)/2] + (2*I)*b*f*Log[1 - I*Tan[(c + d*x)/2]]*Tan[(c + d*x)/2] - (2*I)*
b*f*Log[1 + I*Tan[(c + d*x)/2]]*Tan[(c + d*x)/2])) + ((f*(c + d*x)^2 + (2*I
)*d*e*Log[Sec[(c + d*x)/2]^2] - (2*I)*c*f*Log[Sec[(c + d*x)/2]^2] - (2*I)*d
*e*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])] + (2*I)*c*f*Log[Sec[(c + d*
x)/2]^2*(a + b*Sin[c + d*x])] - (4*I)*f*(c + d*x)*Log[(-2*I)/(-I + Tan[(c +
 d*x)/2])] - 2*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*
Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] + 2*f*Log[1 - I*Tan[(c + d*
x)/2]]*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a
^2 + b^2]))] + 2*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] +
a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] - 2*f*Log[1 + I*Tan[(c
 + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt
[-a^2 + b^2])] + 4*f*PolyLog[2, -Cos[c + d*x] + I*Sin[c + d*x]] + 2*f*PolyL
og[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))] - 2*f*Po
lyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))] + 2*f
*PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])] - 2*f*
PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))])*(-(
(b^2*e*Cos[c + d*x])/((a^2 - b^2)*(a + b*Sin[c + d*x]))) + (b^2*c*f*Cos[c +
 d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x])) - (b^2*f*(c + d*x)*Cos[c + d*x]
)/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))))/(d*(2*f*(c + d*x) - (4*I)*f*Log[(-
2*I)/(-I + Tan[(c + d*x)/2])] - (4*f*Log[1 + Cos[c + d*x] - I*Sin[c + d*x]]
*(I*Cos[c + d*x] + Sin[c + d*x]))/(-Cos[c + d*x] + I*Sin[c + d*x]) + (I*f*L
og[1 - (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]*Sec[(c
+ d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) - (I*f*Log[-((b - Sqrt[-a^2 + b^2] +
a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 -
 I*Tan[(c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])
/((-I)*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/
2]) + (I*f*Log[1 - (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2
]))]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*Log[(b - Sqrt[-a^2
 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2
```

```
]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c +
 d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c +
 d*x)/2]) + (2*I)*d*e*Tan[(c + d*x)/2] - (2*I)*c*f*Tan[(c + d*x)/2] + ((2*I
)*f*(c + d*x)*Sec[(c + d*x)/2]^2)/(-I + Tan[(c + d*x)/2]) - (f*Log[1 - (a*(
I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(I
 + Tan[(c + d*x)/2]) + (I*a*f*Log[1 - (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b
 + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(a + I*a*Tan[(c + d*x)/2]) + (a*
f*Log[1 - I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b - Sqrt[-a^2 + b^2] + a
*Tan[(c + d*x)/2]) - (a*f*Log[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(
b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) + (a*f*Log[1 - I*Tan[(c + d*x)/2
]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) - (a*f*L
og[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[-a^2 + b^2] + a*Ta
n[(c + d*x)/2]) - ((2*I)*d*e*Cos[(c + d*x)/2]^2*(b*Cos[c + d*x]*Sec[(c + d*
x)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c + d*x)/2]))/(a + b
*Sin[c + d*x]) + ((2*I)*c*f*Cos[(c + d*x)/2]^2*(b*Cos[c + d*x]*Sec[(c + d*x
)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c + d*x)/2]))/(a + b*
Sin[c + d*x])))
```

Maple [B] time = 0.254, size = 861, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 4/d*e/(4*a-4*b)*ln(exp(I*(d*x+c))+I)-1/d*e*b/(a-b)/(a+b)*ln(I*b*exp(2*I*(d*
x+c))-2*a*exp(I*(d*x+c))-I*b)-4/d*e/(4*a+4*b)*ln(exp(I*(d*x+c))-I)-1/d*f*b/
(a-b)/(a+b)*ln((I*b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))
*x-1/d^2*f*b/(a-b)/(a+b)*ln((I*b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2
-b^2)^(1/2)))*c-4*I/d^2*f/(4*a+4*b)*dilog(-I*exp(I*(d*x+c)))-4/d*f/(4*a+4*b
)*ln(-I*(I-exp(I*(d*x+c))))*x-4/d^2*f/(4*a+4*b)*ln(-I*(I-exp(I*(d*x+c))))*c
-4*I/d^2*f/(4*a-4*b)*dilog(-I*(exp(I*(d*x+c))+I))+4/d*f/(4*a-4*b)*ln(-I*(ex
p(I*(d*x+c))+I))*x+4/d^2*f/(4*a-4*b)*ln(-I*(exp(I*(d*x+c))+I))*c+I/d^2*f*b/
(a-b)/(a+b)*dilog((I*b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2
)))+I/d^2*f*b/(a-b)/(a+b)*dilog(-(I*b*exp(I*(d*x+c))-(a^2-b^2)^(1/2)-a)/(a+
(a^2-b^2)^(1/2)))-1/d*f*b/(a-b)/(a+b)*ln(-(I*b*exp(I*(d*x+c))-(a^2-b^2)^(1/
2)-a)/(a+(a^2-b^2)^(1/2)))*x-1/d^2*f*b/(a-b)/(a+b)*ln(-(I*b*exp(I*(d*x+c))-
(a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2)))*c-4*I/d^2*f/(4*a+4*b)*ln(-I*(I-exp(
I*(d*x+c))))*ln(-I*exp(I*(d*x+c)))-4/d^2*f*c/(4*a-4*b)*ln(exp(I*(d*x+c))+I)
+1/d^2*f*c*b/(a-b)/(a+b)*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+4/
d^2*f*c/(4*a+4*b)*ln(exp(I*(d*x+c))-I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 3.77881, size = 3094, normalized size = 7.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/2*(I*b*f*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*
x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + I*b*f*dil
og(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - I*b*f*dilog(-1/2*(-2*I*a
*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - I*b*f*dilog(-1/2*(-2*I*a*cos(d*x + c)
 + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) + 2*b)/b + 1) + I*(a - b)*f*dilog(I*cos(d*x + c) + sin(d*x + c)) + I
*(a + b)*f*dilog(I*cos(d*x + c) - sin(d*x + c)) - I*(a - b)*f*dilog(-I*cos(
d*x + c) + sin(d*x + c)) - I*(a + b)*f*dilog(-I*cos(d*x + c) - sin(d*x + c)
) + (b*d*e - b*c*f)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(
a^2 - b^2)/b^2) + 2*I*a) + (b*d*e - b*c*f)*log(2*b*cos(d*x + c) - 2*I*b*sin
(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (b*d*e - b*c*f)*log(-2*b*
cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (
b*d*e - b*c*f)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2
 - b^2)/b^2) - 2*I*a) + (b*d*f*x + b*c*f)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*
sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
 + 2*b)/b) + (b*d*f*x + b*c*f)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x +
c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)
 + (b*d*f*x + b*c*f)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b
*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (b*d*f
*x + b*c*f)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x
```

```
+ c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - ((a + b)*d*e -
(a + b)*c*f)*log(cos(d*x + c) + I*sin(d*x + c) + I) + ((a - b)*d*e - (a - b
)*c*f)*log(cos(d*x + c) - I*sin(d*x + c) + I) - ((a + b)*d*f*x + (a + b)*c*
f)*log(I*cos(d*x + c) + sin(d*x + c) + 1) + ((a - b)*d*f*x + (a - b)*c*f)*l
og(I*cos(d*x + c) - sin(d*x + c) + 1) - ((a + b)*d*f*x + (a + b)*c*f)*log(-
I*cos(d*x + c) + sin(d*x + c) + 1) + ((a - b)*d*f*x + (a - b)*c*f)*log(-I*c
os(d*x + c) - sin(d*x + c) + 1) - ((a + b)*d*e - (a + b)*c*f)*log(-cos(d*x
+ c) + I*sin(d*x + c) + I) + ((a - b)*d*e - (a - b)*c*f)*log(-cos(d*x + c)
- I*sin(d*x + c) + I))/((a^2 - b^2)*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)\sec(c + dx)}{a + b\sin(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)\sec(dx + c)}{b\sin(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

3.309 $\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$-\frac{b\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
- (b*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.080833, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2668, 706, 31, 633}

$$-\frac{b\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x]),x]
```


```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
- (b*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx)\right)}{\left(a^2-b^2\right)d} - \frac{b\,\text{Subst}\left(\int \frac{-a+x}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{\left(a^2-b^2\right)d}$$
$$= -\frac{b\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)d} - \frac{\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin(c+dx)\right)}{2(a-b)d} + \frac{\text{Subst}\left(\int \frac{1}{b-x}\,dx, x, b\sin(c\right.}{2(a+b)d}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{b\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.0609976, size = 64, normalized size = 0.85

$$\frac{(b-a)\log(1-\sin(c+dx)) + (a+b)\log(\sin(c+dx)+1) - 2b\log(a+b\sin(c+dx))}{2d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-a + b)*Log[1 - Sin[c + d*x]] + (a + b)*Log[1 + Sin[c + d*x]] - 2*b*Log[a
 + b*Sin[c + d*x]])/(2*(a - b)*(a + b)*d)
```

Maple [A] time = 0.002, size = 76, normalized size = 1.

$$-\frac{b\ln(a+b\sin(dx+c))}{d(a-b)(a+b)} - \frac{\ln(\sin(dx+c)-1)}{d(2a+2b)} + \frac{\ln(1+\sin(dx+c))}{d(2a-2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*b/(a-b)/(a+b)*ln(a+b*sin(d*x+c))-1/d/(2*a+2*b)*ln(sin(d*x+c)-1)+1/d/(2
*a-2*b)*ln(1+sin(d*x+c))
```

Maxima [A] time = 0.946195, size = 86, normalized size = 1.15

$$-\frac{\frac{2\,b\log\left(b\sin\left(dx+c\right)+a\right)}{a^2-b^2}-\frac{\log\left(\sin\left(dx+c\right)+1\right)}{a-b}+\frac{\log\left(\sin\left(dx+c\right)-1\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*log(b*sin(d*x + c) + a)/(a^2 - b^2) - log(sin(d*x + c) + 1)/(a -
b) + log(sin(d*x + c) - 1)/(a + b))/d
```

Fricas [A] time = 2.25565, size = 158, normalized size = 2.11

$$-\frac{2\,b\log\left(b\sin\left(dx+c\right)+a\right)-\left(a+b\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a-b\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*log(b*sin(d*x + c) + a) - (a + b)*log(sin(d*x + c) + 1) + (a - b)
*log(-sin(d*x + c) + 1))/((a^2 - b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.19167, size = 96, normalized size = 1.28

$$-\frac{\frac{2\,b^2\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^2b-b^3}-\frac{\log\left(|\sin\left(dx+c\right)+1|\right)}{a-b}+\frac{\log\left(|\sin\left(dx+c\right)-1|\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x +
c) + 1))/(a - b) + log(abs(sin(d*x + c) - 1))/(a + b))/d
```

3.310 $\int \frac{(e+fx)^3 \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=923

$$-\frac{6b\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)f^3}{\left(a^2-b^2\right)d^4}+\frac{6b\text{PolyLog}\left(3,ie^{i(c+dx)}\right)f^3}{\left(a^2-b^2\right)d^4}+\frac{3a\text{PolyLog}\left(3,-e^{2i(c+dx)}\right)f^3}{2\left(a^2-b^2\right)d^4}-\frac{6b^2\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d^4}$$

```
[Out] ((-I)*a*(e + f*x)^3)/((a^2 - b^2)*d) - ((6*I)*b*f*(e + f*x)^2*ArcTan[E^(I*(
c + d*x))])/((a^2 - b^2)*d^2) + (I*b^2*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - (I*b^2*(e + f*x)^3*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) +
 (3*a*f*(e + f*x)^2*Log[1 + E^((2*I)*(c + d*x))])/((a^2 - b^2)*d^2) + ((6*I
)*b*f^2*(e + f*x)*PolyLog[2, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) - ((6
*I)*b*f^2*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) + (3*b
^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(
(a^2 - b^2)^(3/2)*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x
)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^2) - ((3*I)*a*f^2*(e + f*x
)*PolyLog[2, -E^((2*I)*(c + d*x))])/((a^2 - b^2)*d^3) - (6*b*f^3*PolyLog[3,
 (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^4) + (6*b*f^3*PolyLog[3, I*E^(I*(c +
 d*x))])/((a^2 - b^2)*d^4) + ((6*I)*b^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*
(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^3) - ((6*I)*b^2*f^
2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2
- b^2)^(3/2)*d^3) + (3*a*f^3*PolyLog[3, -E^((2*I)*(c + d*x))])/(2*(a^2 - b^
2)*d^4) - (6*b^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])
])/((a^2 - b^2)^(3/2)*d^4) + (6*b^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a
 + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^4) - (b*(e + f*x)^3*Sec[c + d*x]
)/((a^2 - b^2)*d) + (a*(e + f*x)^3*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rubi [A] time = 1.93669, antiderivative size = 923, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {4533, 3323, 2264, 2190, 2531, 6609, 2282, 6589, 6742, 4184, 3719, 4409, 4181}

$$-\frac{6b\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)f^3}{\left(a^2-b^2\right)d^4}+\frac{6b\text{PolyLog}\left(3,ie^{i(c+dx)}\right)f^3}{\left(a^2-b^2\right)d^4}+\frac{3a\text{PolyLog}\left(3,-e^{2i(c+dx)}\right)f^3}{2\left(a^2-b^2\right)d^4}-\frac{6b^2\text{PolyLog}\left(4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-I)*a*(e + f*x)^3)/((a^2 - b^2)*d) - ((6*I)*b*f*(e + f*x)^2*ArcTan[E^(I*(
c + d*x))])/((a^2 - b^2)*d^2) + (I*b^2*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - (I*b^2*(e + f*x)^3*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) +
 (3*a*f*(e + f*x)^2*Log[1 + E^((2*I)*(c + d*x))])/((a^2 - b^2)*d^2) + ((6*I
)*b*f^2*(e + f*x)*PolyLog[2, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) - ((6
*I)*b*f^2*(e + f*x)*PolyLog[2, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) + (3*b
^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(
(a^2 - b^2)^(3/2)*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x
)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^2) - ((3*I)*a*f^2*(e + f*x
)*PolyLog[2, -E^((2*I)*(c + d*x))])/((a^2 - b^2)*d^3) - (6*b*f^3*PolyLog[3,
 (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^4) + (6*b*f^3*PolyLog[3, I*E^(I*(c +
 d*x))])/((a^2 - b^2)*d^4) + ((6*I)*b^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*
(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^3) - ((6*I)*b^2*f^
2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2
- b^2)^(3/2)*d^3) + (3*a*f^3*PolyLog[3, -E^((2*I)*(c + d*x))])/(2*(a^2 - b^
2)*d^4) - (6*b^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])
])/((a^2 - b^2)^(3/2)*d^4) + (6*b^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a
 + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^4) - (b*(e + f*x)^3*Sec[c + d*x]
)/((a^2 - b^2)*d) + (a*(e + f*x)^3*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rule 4533

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[b^2/(a^2 - b^2), Int[((e + f
*x)^m*Sec[c + d*x]^(n - 2))/(a + b*Sin[c + d*x]), x], x] + Dist[1/(a^2 - b^
2), Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int(e+fx)^3\sec^2(c+dx)(a-b\sin(c+dx))\,dx}{a^2-b^2} - \frac{b^2\int\frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{\int\left(a(e+fx)^3\sec^2(c+dx)-b(e+fx)^3\sec(c+dx)\tan(c+dx)\right)dx}{a^2-b^2} - \frac{\left(2b^2\right)\int\frac{e^{i(c+dx)}(e+}{ib+2ae^{i(c+dx)}-}}{a^2-b^2}$$

$$= \frac{\left(2ib^3\right)\int\frac{e^{i(c+dx)}(e+fx)^3}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\left(a^2-b^2\right)^{3/2}} - \frac{\left(2ib^3\right)\int\frac{e^{i(c+dx)}(e+fx)^3}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\left(a^2-b^2\right)^{3/2}} + \frac{a\int(e+fx)^3\sec^2(c+}{a^2-b^2}$$

$$= \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{b(e+fx)^3\sec(c+dx)}{\left(a^2-b^2\right)d}$$

$$= -\frac{ia(e+fx)^3}{\left(a^2-b^2\right)d} - \frac{6ibf(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d^2} + \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{ib^2(e+}{}$$

$$= -\frac{ia(e+fx)^3}{\left(a^2-b^2\right)d} - \frac{6ibf(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d^2} + \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{ib^2(e+}{}$$

$$= -\frac{ia(e+fx)^3}{\left(a^2-b^2\right)d} - \frac{6ibf(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d^2} + \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{ib^2(e+}{}$$

$$= -\frac{ia(e+fx)^3}{\left(a^2-b^2\right)d} - \frac{6ibf(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d^2} + \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{ib^2(e+}{}$$

$$= -\frac{ia(e+fx)^3}{\left(a^2-b^2\right)d} - \frac{6ibf(e+fx)^2\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d^2} + \frac{ib^2(e+fx)^3\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{ib^2(e+}{}$$

<u>Mathematica</u> [A] time = 9.37484, size = 1438, normalized size = 1.56

$$\frac{b\sec(c)(e+fx)^3}{\left(b^2-a^2\right)d} + \frac{f\left(\frac{2ia(e+fx)^3}{f} + \frac{3(a-b)\left(1+e^{2ic}\right)\log\left(1-ie^{-i(c+dx)}\right)(e+fx)^2}{d} + \frac{3(a+b)\left(1+e^{2ic}\right)\log\left(1+ie^{-i(c+dx)}\right)(e+fx)^2}{d} + \frac{6(a+b)\left(1+e^{2ic}\right)f\left(id(e+fx\right.}{}\right.}{\left(a^2-b^2\right)d\,(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (f*(((2*I)*a*(e + f*x)^3)/f + (3*(a - b)*(1 + E^((2*I)*c))*(e + f*x)^2*Log[
1 - I/E^(I*(c + d*x))])/d + (3*(a + b)*(1 + E^((2*I)*c))*(e + f*x)^2*Log[1
+ I/E^(I*(c + d*x))])/d + (6*(a + b)*(1 + E^((2*I)*c))*f*(I*d*(e + f*x)*Pol
yLog[2, (-I)/E^(I*(c + d*x))] + f*PolyLog[3, (-I)/E^(I*(c + d*x))]))/d^3 +
(6*(a - b)*(1 + E^((2*I)*c))*f*(I*d*(e + f*x)*PolyLog[2, I/E^(I*(c + d*x))]
 + f*PolyLog[3, I/E^(I*(c + d*x))]))/d^3))/((a^2 - b^2)*d*(1 + E^((2*I)*c))
) + (b^2*(2*Sqrt[-a^2 + b^2]*d^3*e^3*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[
a^2 - b^2]] + 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*E^(I*(c + d*x)))/((-
I)*a + Sqrt[-a^2 + b^2])] + 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*E^(I
*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log
[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d
^3*e^2*f*x*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a
^2 - b^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]
)] - Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a
^2 + b^2])] - (3*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(I*(c
 + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + (3*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f
*x)^2*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] + 6*Sqrt[
a^2 - b^2]*d*e*f^2*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2
])] + 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sq
rt[-a^2 + b^2])] - 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, -((b*E^(I*(c + d*x)
))/(I*a + Sqrt[-a^2 + b^2]))] - 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3, -((b*E
^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] + (6*I)*Sqrt[a^2 - b^2]*f^3*Poly
Log[4, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - (6*I)*Sqrt[a^2 -
b^2]*f^3*PolyLog[4, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))]))/(Sqr
t[-(a^2 - b^2)^2]*(-a^2 + b^2)*d^4) + (b*(e + f*x)^3*Sec[c])/((-a^2 + b^2)*
d) + (e^3*Sin[(d*x)/2] + 3*e^2*f*x*Sin[(d*x)/2] + 3*e*f^2*x^2*Sin[(d*x)/2]
+ f^3*x^3*Sin[(d*x)/2])/((a + b)*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2
] - Sin[c/2 + (d*x)/2])) + (e^3*Sin[(d*x)/2] + 3*e^2*f*x*Sin[(d*x)/2] + 3*e
*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/2])/((a - b)*d*(Cos[c/2] + Sin[c/
2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [F] time = 2.507, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3} \left(\sec\left(dx+c\right)\right)^{2}}{a+b\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.73611, size = 9469, normalized size = 10.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a^2*b - b^3)*d^3*f^3*x^3 + 12*(a^2*b - b^3)*d^3*e*f^2*x^2 - 12*I*b
^3*f^3*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*polylog(4, 1/2*(2*I*a*cos(d*x +
c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2))/b) + 12*I*b^3*f^3*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*polylog(4,
 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*I*b^3*f^3*sqrt(-(a^2 - b^2)/b^2)*c
os(d*x + c)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c
) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*I*b^3*f^3*sqrt(-(a^2
- b^2)/b^2)*cos(d*x + c)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (
b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(a^2*b -
 b^3)*d^3*e^2*f*x + 4*(a^2*b - b^3)*d^3*e^3 - 12*(a^3 - a^2*b - a*b^2 + b^3
)*f^3*cos(d*x + c)*polylog(3, I*cos(d*x + c) + sin(d*x + c)) - 12*(a^3 + a^
2*b - a*b^2 - b^3)*f^3*cos(d*x + c)*polylog(3, I*cos(d*x + c) - sin(d*x + c
)) - 12*(a^3 - a^2*b - a*b^2 + b^3)*f^3*cos(d*x + c)*polylog(3, -I*cos(d*x
+ c) + sin(d*x + c)) - 12*(a^3 + a^2*b - a*b^2 - b^3)*f^3*cos(d*x + c)*poly
log(3, -I*cos(d*x + c) - sin(d*x + c)) - 2*(-3*I*b^3*d^2*f^3*x^2 - 6*I*b^3*
d^2*e*f^2*x - 3*I*b^3*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*dilog(
-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(3*I*b^3*d^2*f^3*x^2 + 6*
I*b^3*d^2*e*f^2*x + 3*I*b^3*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*
dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(3*I*b^3*d^2*f^3*x^
```

```
2 + 6*I*b^3*d^2*e*f^2*x + 3*I*b^3*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x
 + c)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c
) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(-3*I*b^3*d^
2*f^3*x^2 - 6*I*b^3*d^2*e*f^2*x - 3*I*b^3*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)
*cos(d*x + c)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos
(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(b^3
*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*sqrt(-(a^2
- b^2)/b^2)*cos(d*x + c)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sq
rt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*
c^2*d*e*f^2 - b^3*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(2*b*cos(
d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(b^
3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*sqrt(-(a^2
 - b^2)/b^2)*cos(d*x + c)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*
sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^
3*c^2*d*e*f^2 - b^3*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(-2*b*c
os(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*
(b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^
2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*
log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(b^3*d^3*f^3*x^3 + 3*b^3*d
^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 +
b^3*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(1/2*(2*I*a*cos(d*x + c
) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2) + 2*b)/b) + 2*(b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e
^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*sqrt(-(a^2 -
b^2)/b^2)*cos(d*x + c)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*
(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(b
^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*
f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*lo
g(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin
(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 12*(b^3*d*f^3*x + b^3*d*e*f^2
)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*polylog(3, 1/2*(2*I*a*cos(d*x + c) -
2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2))/b) + 12*(b^3*d*f^3*x + b^3*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x +
c)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c
) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(b^3*d*f^3*x + b^3*d*
e*f^2)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*polylog(3, -(I*a*cos(d*x + c) +
a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
)/b) - 12*(b^3*d*f^3*x + b^3*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*p
olylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - (12*I*(a^3 - a^2*b - a*b^2 + b^3)*d*f
^3*x + 12*I*(a^3 - a^2*b - a*b^2 + b^3)*d*e*f^2)*cos(d*x + c)*dilog(I*cos(d
*x + c) + sin(d*x + c)) - (-12*I*(a^3 + a^2*b - a*b^2 - b^3)*d*f^3*x - 12*I
*(a^3 + a^2*b - a*b^2 - b^3)*d*e*f^2)*cos(d*x + c)*dilog(I*cos(d*x + c) - s
in(d*x + c)) - (-12*I*(a^3 - a^2*b - a*b^2 + b^3)*d*f^3*x - 12*I*(a^3 - a^2
```

```
*b - a*b^2 + b^3)*d*e*f^2)*cos(d*x + c)*dilog(-I*cos(d*x + c) + sin(d*x + c
)) - (12*I*(a^3 + a^2*b - a*b^2 - b^3)*d*f^3*x + 12*I*(a^3 + a^2*b - a*b^2
- b^3)*d*e*f^2)*cos(d*x + c)*dilog(-I*cos(d*x + c) - sin(d*x + c)) - 6*((a^
3 + a^2*b - a*b^2 - b^3)*d^2*e^2*f - 2*(a^3 + a^2*b - a*b^2 - b^3)*c*d*e*f^
2 + (a^3 + a^2*b - a*b^2 - b^3)*c^2*f^3)*cos(d*x + c)*log(cos(d*x + c) + I*
sin(d*x + c) + I) - 6*((a^3 - a^2*b - a*b^2 + b^3)*d^2*e^2*f - 2*(a^3 - a^2
*b - a*b^2 + b^3)*c*d*e*f^2 + (a^3 - a^2*b - a*b^2 + b^3)*c^2*f^3)*cos(d*x
+ c)*log(cos(d*x + c) - I*sin(d*x + c) + I) - 6*((a^3 + a^2*b - a*b^2 - b^3
)*d^2*f^3*x^2 + 2*(a^3 + a^2*b - a*b^2 - b^3)*d^2*e*f^2*x + 2*(a^3 + a^2*b
- a*b^2 - b^3)*c*d*e*f^2 - (a^3 + a^2*b - a*b^2 - b^3)*c^2*f^3)*cos(d*x + c
)*log(I*cos(d*x + c) + sin(d*x + c) + 1) - 6*((a^3 - a^2*b - a*b^2 + b^3)*d
^2*f^3*x^2 + 2*(a^3 - a^2*b - a*b^2 + b^3)*d^2*e*f^2*x + 2*(a^3 - a^2*b - a
*b^2 + b^3)*c*d*e*f^2 - (a^3 - a^2*b - a*b^2 + b^3)*c^2*f^3)*cos(d*x + c)*l
og(I*cos(d*x + c) - sin(d*x + c) + 1) - 6*((a^3 + a^2*b - a*b^2 - b^3)*d^2*
f^3*x^2 + 2*(a^3 + a^2*b - a*b^2 - b^3)*d^2*e*f^2*x + 2*(a^3 + a^2*b - a*b^
2 - b^3)*c*d*e*f^2 - (a^3 + a^2*b - a*b^2 - b^3)*c^2*f^3)*cos(d*x + c)*log(
-I*cos(d*x + c) + sin(d*x + c) + 1) - 6*((a^3 - a^2*b - a*b^2 + b^3)*d^2*f^
3*x^2 + 2*(a^3 - a^2*b - a*b^2 + b^3)*d^2*e*f^2*x + 2*(a^3 - a^2*b - a*b^2
+ b^3)*c*d*e*f^2 - (a^3 - a^2*b - a*b^2 + b^3)*c^2*f^3)*cos(d*x + c)*log(-I
*cos(d*x + c) - sin(d*x + c) + 1) - 6*((a^3 + a^2*b - a*b^2 - b^3)*d^2*e^2*
f - 2*(a^3 + a^2*b - a*b^2 - b^3)*c*d*e*f^2 + (a^3 + a^2*b - a*b^2 - b^3)*c
^2*f^3)*cos(d*x + c)*log(-cos(d*x + c) + I*sin(d*x + c) + I) - 6*((a^3 - a^
2*b - a*b^2 + b^3)*d^2*e^2*f - 2*(a^3 - a^2*b - a*b^2 + b^3)*c*d*e*f^2 + (a
^3 - a^2*b - a*b^2 + b^3)*c^2*f^3)*cos(d*x + c)*log(-cos(d*x + c) - I*sin(d
*x + c) + I) - 4*((a^3 - a*b^2)*d^3*f^3*x^3 + 3*(a^3 - a*b^2)*d^3*e*f^2*x^2
 + 3*(a^3 - a*b^2)*d^3*e^2*f*x + (a^3 - a*b^2)*d^3*e^3)*sin(d*x + c))/((a^4
 - 2*a^2*b^2 + b^4)*d^4*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.311 $\int \frac{(e+fx)^2\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=659

$$\frac{2b^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)^{3/2}}-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d^2\left(a^2-b^2\right)^{3/2}}+\frac{2ib^2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^3\left(a^2-b^2\right)^{3/2}}-\frac{2ib^2f^2\text{Poly}}{d^3(}$$

```
[Out] ((-I)*a*(e + f*x)^2)/((a^2 - b^2)*d) - ((4*I)*b*f*(e + f*x)*ArcTan[E^(I*(c
+ d*x))])/((a^2 - b^2)*d^2) + (I*b^2*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - (I*b^2*(e + f*x)^2*Log[
1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) + (
2*a*f*(e + f*x)*Log[1 + E^((2*I)*(c + d*x))])/((a^2 - b^2)*d^2) + ((2*I)*b*
f^2*PolyLog[2, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) - ((2*I)*b*f^2*Poly
Log[2, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) + (2*b^2*f*(e + f*x)*PolyLog[2
, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^2) - (
2*b^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/
((a^2 - b^2)^(3/2)*d^2) - (I*a*f^2*PolyLog[2, -E^((2*I)*(c + d*x))])/((a^2
- b^2)*d^3) + ((2*I)*b^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
 - b^2])])/((a^2 - b^2)^(3/2)*d^3) - ((2*I)*b^2*f^2*PolyLog[3, (I*b*E^(I*(c
 + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^3) - (b*(e + f*x)^2*
Sec[c + d*x])/((a^2 - b^2)*d) + (a*(e + f*x)^2*Tan[c + d*x])/((a^2 - b^2)*d
)
```

Rubi [A] time = 1.43355, antiderivative size = 659, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4533, 3323, 2264, 2190, 2531, 2282, 6589, 6742, 4184, 3719, 2279, 2391, 4409, 4181}

$$\frac{2b^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)^{3/2}}-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d^2\left(a^2-b^2\right)^{3/2}}+\frac{2ib^2f^2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^3\left(a^2-b^2\right)^{3/2}}-\frac{2ib^2f^2\text{Poly}}{d^3(}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] ((-I)*a*(e + f*x)^2)/((a^2 - b^2)*d) - ((4*I)*b*f*(e + f*x)*ArcTan[E^(I*(c
+ d*x))])/((a^2 - b^2)*d^2) + (I*b^2*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - (I*b^2*(e + f*x)^2*Log[
1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) + (
```

```
2*a*f*(e + f*x)*Log[1 + E^((2*I)*(c + d*x))])/((a^2 - b^2)*d^2) + ((2*I)*b*
f^2*PolyLog[2, (-I)*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) - ((2*I)*b*f^2*Poly
Log[2, I*E^(I*(c + d*x))])/((a^2 - b^2)*d^3) + (2*b^2*f*(e + f*x)*PolyLog[2
, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^2) - (
2*b^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/
((a^2 - b^2)^(3/2)*d^2) - (I*a*f^2*PolyLog[2, -E^((2*I)*(c + d*x))])/((a^2
- b^2)*d^3) + ((2*I)*b^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
 - b^2])])/((a^2 - b^2)^(3/2)*d^3) - ((2*I)*b^2*f^2*PolyLog[3, (I*b*E^(I*(c
 + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^3) - (b*(e + f*x)^2*
Sec[c + d*x])/((a^2 - b^2)*d) + (a*(e + f*x)^2*Tan[c + d*x])/((a^2 - b^2)*d
)
```

Rule 4533

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[b^2/(a^2 - b^2), Int[((e + f
*x)^m*Sec[c + d*x]^(n - 2))/(a + b*Sin[c + d*x]), x], x] + Dist[1/(a^2 - b^
2), Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\sec^2(c+dx)(a-b\sin(c+dx))\,dx}{a^2-b^2} - \frac{b^2\int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{\int \left(a(e+fx)^2\sec^2(c+dx) - b(e+fx)^2\sec(c+dx)\tan(c+dx)\right)\,dx}{a^2-b^2} - \frac{\left(2b^2\right)\int \frac{e^{i(c+dx)}(e+}{ib+2ae^{i(c+dx)}-}}{a^2-b^2}$$

$$= \frac{\left(2ib^3\right)\int \frac{e^{i(c+dx)}(e+fx)^2}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\left(a^2-b^2\right)^{3/2}} - \frac{\left(2ib^3\right)\int \frac{e^{i(c+dx)}(e+fx)^2}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\left(a^2-b^2\right)^{3/2}} + \frac{a\int (e+fx)^2\sec^2(c+}{a^2-b^2}$$

$$= \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{b(e+fx)^2\sec(c+dx)}{\left(a^2-b^2\right)d}$$

$$= -\frac{ia(e+fx)^2}{\left(a^2-b^2\right)d} - \frac{4ibf(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d^2} + \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{ib^2(e+f}{(}$$

$$= -\frac{ia(e+fx)^2}{\left(a^2-b^2\right)d} - \frac{4ibf(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d^2} + \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{ib^2(e+f}{(}$$

$$= -\frac{ia(e+fx)^2}{\left(a^2-b^2\right)d} - \frac{4ibf(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d^2} + \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{ib^2(e+f}{(}$$

$$= -\frac{ia(e+fx)^2}{\left(a^2-b^2\right)d} - \frac{4ibf(e+fx)\tan^{-1}\left(e^{i(c+dx)}\right)}{\left(a^2-b^2\right)d^2} + \frac{ib^2(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{ib^2(e+f}{(}$$

<u>Mathematica</u> [A] time = 7.9013, size = 1122, normalized size = 1.7

$$\frac{i\left(-2\sqrt{a^2-b^2}df(e+fx)\text{PolyLog}\left(2,\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}-ia}\right)+2\sqrt{a^2-b^2}df(e+fx)\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{ia+\sqrt{b^2-a^2}}\right)-i\left(\left(2\sqrt{b^2-a^2}\tan^{-1}\left(\frac{i}{}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (I*b^2*(-2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, (b*E^(I*(c + d*x)))/((-
I)*a + Sqrt[-a^2 + b^2])] + 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b
*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] - I*(d^2*(2*Sqrt[-a^2 + b^2]*e
^2*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*x*
(2*e + f*x)*(Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - Log
[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])])) + 2*Sqrt[a^2 - b^2]*f^
2*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 2*Sqrt[a^2
- b^2]*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))])))/(
Sqrt[-(a^2 - b^2)^2]*(-a^2 + b^2)*d^3) + (b*(e + f*x)^2*Sec[c])/((-a^2 + b^
2)*d) + (2*a*e*f*Sec[c]*(Cos[c]*Log[Cos[c]*Cos[d*x] - Sin[c]*Sin[d*x]] + d*
x*Sin[c]))/((a^2 - b^2)*d^2*(Cos[c]^2 + Sin[c]^2)) + ((4*I)*b*e*f*ArcTan[((
-I)*Sin[c] - I*Cos[c]*Tan[(d*x)/2])/Sqrt[Cos[c]^2 + Sin[c]^2]])/((a^2 - b^2
)*d^2*Sqrt[Cos[c]^2 + Sin[c]^2]) + (a*f^2*Csc[c]*((d^2*x^2)/E^(I*ArcTan[Cot
[c]]) - (Cot[c]*(I*d*x*(-Pi - 2*ArcTan[Cot[c]]) - Pi*Log[1 + E^((-2*I)*d*x)
] - 2*(d*x - ArcTan[Cot[c]])*Log[1 - E^((2*I)*(d*x - ArcTan[Cot[c]]))] + Pi
*Log[Cos[d*x]] - 2*ArcTan[Cot[c]]*Log[Sin[d*x - ArcTan[Cot[c]]]] + I*PolyLo
g[2, E^((2*I)*(d*x - ArcTan[Cot[c]]))]))/Sqrt[1 + Cot[c]^2])*Sec[c])/((a^2
- b^2)*d^3*Sqrt[Csc[c]^2*(Cos[c]^2 + Sin[c]^2)]) + (2*b*f^2*(-((Csc[c]*((d*
x - ArcTan[Cot[c]])*(Log[1 - E^(I*(d*x - ArcTan[Cot[c]]))] - Log[1 + E^(I*(
d*x - ArcTan[Cot[c]]))]) + I*(PolyLog[2, -E^(I*(d*x - ArcTan[Cot[c]]))] - P
olyLog[2, E^(I*(d*x - ArcTan[Cot[c]]))])))/Sqrt[1 + Cot[c]^2]) + (2*ArcTan[
Cot[c]]*ArcTanh[(Sin[c] + Cos[c]*Tan[(d*x)/2])/Sqrt[Cos[c]^2 + Sin[c]^2]])/
Sqrt[Cos[c]^2 + Sin[c]^2]))/((a^2 - b^2)*d^3) + (e^2*Sin[(d*x)/2] + 2*e*f*x
*Sin[(d*x)/2] + f^2*x^2*Sin[(d*x)/2])/((a + b)*d*(Cos[c/2] - Sin[c/2])*(Cos
[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (e^2*Sin[(d*x)/2] + 2*e*f*x*Sin[(d
*x)/2] + f^2*x^2*Sin[(d*x)/2])/((a - b)*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 +
(d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [F] time = 3.475, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \left(\sec \left(dx+c\right)\right)^{2}}{a+b\sin \left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 5.22192, size = 6276, normalized size = 9.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*polylog(3, 1/2*(2*I*a*co
s(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2))/b) - 4*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*polyl
og(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 4*b^3*f^2*sqrt(-(a^2 - b^2)/b^2)
*cos(d*x + c)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x +
 c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 4*b^3*f^2*sqrt(-(a^2 -
 b^2)/b^2)*cos(d*x + c)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b
*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 4*(a^2*b - b
^3)*d^2*f^2*x^2 - 8*(a^2*b - b^3)*d^2*e*f*x - 4*(a^2*b - b^3)*d^2*e^2 + 4*I
*(a^3 - a^2*b - a*b^2 + b^3)*f^2*cos(d*x + c)*dilog(I*cos(d*x + c) + sin(d*
x + c)) - 4*I*(a^3 + a^2*b - a*b^2 - b^3)*f^2*cos(d*x + c)*dilog(I*cos(d*x
+ c) - sin(d*x + c)) - 4*I*(a^3 - a^2*b - a*b^2 + b^3)*f^2*cos(d*x + c)*dil
og(-I*cos(d*x + c) + sin(d*x + c)) + 4*I*(a^3 + a^2*b - a*b^2 - b^3)*f^2*co
s(d*x + c)*dilog(-I*cos(d*x + c) - sin(d*x + c)) + 2*(-2*I*b^3*d*f^2*x - 2*
I*b^3*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*dilog(-1/2*(2*I*a*cos(d*x
+ c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) + 2*b)/b + 1) + 2*(2*I*b^3*d*f^2*x + 2*I*b^3*d*e*f)*sqrt(-(a^2
- b^2)/b^2)*cos(d*x + c)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c)
- 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1
) + 2*(2*I*b^3*d*f^2*x + 2*I*b^3*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)
*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(-2*I*b^3*d*f^2*x
 - 2*I*b^3*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*dilog(-1/2*(-2*I*a*co
```

```
s(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*
f^2)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(2*b*cos(d*x + c) + 2*I*b*sin(d
*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(b^3*d^2*e^2 - 2*b^3*c*d*
e*f + b^3*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(2*b*cos(d*x + c)
 - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(b^3*d^2*e^
2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(-2
*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)
+ 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*cos(
d*x + c)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)
/b^2) - 2*I*a) - 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3
*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(1/2*(2*I*a*cos(d*x + c) +
 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2) + 2*b)/b) + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^
3*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(1/2*(2*I*a*cos(d*x + c)
+ 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) + 2*b)/b) - 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b
^3*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(1/2*(-2*I*a*cos(d*x + c
) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2) + 2*b)/b) + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f -
 b^3*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(1/2*(-2*I*a*cos(d*x +
 c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2) + 2*b)/b) + 4*((a^3 + a^2*b - a*b^2 - b^3)*d*e*f - (a^3 + a^2*b
- a*b^2 - b^3)*c*f^2)*cos(d*x + c)*log(cos(d*x + c) + I*sin(d*x + c) + I) +
 4*((a^3 - a^2*b - a*b^2 + b^3)*d*e*f - (a^3 - a^2*b - a*b^2 + b^3)*c*f^2)*
cos(d*x + c)*log(cos(d*x + c) - I*sin(d*x + c) + I) + 4*((a^3 + a^2*b - a*b
^2 - b^3)*d*f^2*x + (a^3 + a^2*b - a*b^2 - b^3)*c*f^2)*cos(d*x + c)*log(I*c
os(d*x + c) + sin(d*x + c) + 1) + 4*((a^3 - a^2*b - a*b^2 + b^3)*d*f^2*x +
(a^3 - a^2*b - a*b^2 + b^3)*c*f^2)*cos(d*x + c)*log(I*cos(d*x + c) - sin(d*
x + c) + 1) + 4*((a^3 + a^2*b - a*b^2 - b^3)*d*f^2*x + (a^3 + a^2*b - a*b^2
 - b^3)*c*f^2)*cos(d*x + c)*log(-I*cos(d*x + c) + sin(d*x + c) + 1) + 4*((a
^3 - a^2*b - a*b^2 + b^3)*d*f^2*x + (a^3 - a^2*b - a*b^2 + b^3)*c*f^2)*cos(
d*x + c)*log(-I*cos(d*x + c) - sin(d*x + c) + 1) + 4*((a^3 + a^2*b - a*b^2
- b^3)*d*e*f - (a^3 + a^2*b - a*b^2 - b^3)*c*f^2)*cos(d*x + c)*log(-cos(d*x
 + c) + I*sin(d*x + c) + I) + 4*((a^3 - a^2*b - a*b^2 + b^3)*d*e*f - (a^3 -
 a^2*b - a*b^2 + b^3)*c*f^2)*cos(d*x + c)*log(-cos(d*x + c) - I*sin(d*x + c
) + I) + 4*((a^3 - a*b^2)*d^2*f^2*x^2 + 2*(a^3 - a*b^2)*d^2*e*f*x + (a^3 -
a*b^2)*d^2*e^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d^3*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e+fx\right)^{2}\sec^{2}\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.312 $\int \frac{(e+fx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=349

$$\frac{b^2 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)^{3/2}}-\frac{b^2 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d^2\left(a^2-b^2\right)^{3/2}}+\frac{bf\tanh^{-1}(\sin(c+dx))}{d^2\left(a^2-b^2\right)}+\frac{af\log(\cos(c+dx))}{d^2\left(a^2-b^2\right)}+\frac{ib^2(e+fx)\cdots}{d\left(\cdots\right.}$$

```
[Out] (b*f*ArcTanh[Sin[c + d*x]])/((a^2 - b^2)*d^2) + (I*b^2*(e + f*x)*Log[1 - (I
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - (I*b^2*
(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2
)^(3/2)*d) + (a*f*Log[Cos[c + d*x]])/((a^2 - b^2)*d^2) + (b^2*f*PolyLog[2,
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^2) - (b^
2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(
3/2)*d^2) - (b*(e + f*x)*Sec[c + d*x])/((a^2 - b^2)*d) + (a*(e + f*x)*Tan[c
 + d*x])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.794791, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {4533, 3323, 2264, 2190, 2279, 2391, 6742, 4184, 3475, 4409, 3770}

$$\frac{b^2 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)^{3/2}}-\frac{b^2 f \text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d^2\left(a^2-b^2\right)^{3/2}}+\frac{bf\tanh^{-1}(\sin(c+dx))}{d^2\left(a^2-b^2\right)}+\frac{af\log(\cos(c+dx))}{d^2\left(a^2-b^2\right)}+\frac{ib^2(e+fx)\cdots}{d\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*f*ArcTanh[Sin[c + d*x]])/((a^2 - b^2)*d^2) + (I*b^2*(e + f*x)*Log[1 - (I
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - (I*b^2*
(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2
)^(3/2)*d) + (a*f*Log[Cos[c + d*x]])/((a^2 - b^2)*d^2) + (b^2*f*PolyLog[2,
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^2) - (b^
2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(
3/2)*d^2) - (b*(e + f*x)*Sec[c + d*x])/((a^2 - b^2)*d) + (a*(e + f*x)*Tan[c
 + d*x])/((a^2 - b^2)*d)
```

Rule 4533

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[b^2/(a^2 - b^2), Int[((e + f
*x)^m*Sec[c + d*x]^(n - 2))/(a + b*Sin[c + d*x]), x], x] + Dist[1/(a^2 - b^
2), Int[(e + f*x)^m*Sec[c + d*x]^n*(a - b*Sin[c + d*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\sec^2(c+dx)(a-b\sin(c+dx))\,dx}{a^2-b^2} - \frac{b^2\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{\int \left(a(e+fx)\sec^2(c+dx) - b(e+fx)\sec(c+dx)\tan(c+dx)\right)dx}{a^2-b^2} - \frac{\left(2b^2\right)\int \frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i}\cdots}}{a^2-b^2}$$

$$= \frac{\left(2ib^3\right)\int \frac{e^{i(c+dx)}(e+fx)}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\left(a^2-b^2\right)^{3/2}} - \frac{\left(2ib^3\right)\int \frac{e^{i(c+dx)}(e+fx)}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\left(a^2-b^2\right)^{3/2}} + \frac{a\int (e+fx)\sec^2(c+dx\cdots}{a^2-b^2}$$

$$= \frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{b(e+fx)\sec(c+dx)}{\left(a^2-b^2\right)d} + \cdots$$

$$= \frac{bf\tanh^{-1}(\sin(c+dx))}{\left(a^2-b^2\right)d^2} + \frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} + \cdots$$

$$= \frac{bf\tanh^{-1}(\sin(c+dx))}{\left(a^2-b^2\right)d^2} + \frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{ib^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} + \cdots$$

Mathematica [B] time = 9.7094, size = 842, normalized size = 2.41

$$\frac{d(e+fx)\left(\frac{2(de-cf)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{if\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt{b^2-a^2}}{-ia+b+\sqrt{b^2-a^2}}\right)+\text{PolyLog}\left(2,\frac{a\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a+i\left(b+\sqrt{b^2-a^2}\right)}\right)\right)}{\sqrt{b^2-a^2}} + \frac{if\left(\log\left(i\tan\left(\frac{1}{2}(c+dx)\right)+1\right)\log\left(\frac{b+\cdots}{\cdots}\right)\cdots}{\cdots}\right)}{(\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((b*d*(e + f*x))/(-a^2 + b^2) + (f*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
)/(a + b) + (f*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a - b) + (b^2*d*(
e + f*x)*((2*(d*e - c*f)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
Sqrt[a^2 - b^2] - (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^
2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(
1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2]
 + (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c +
```

```
 d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(c + d*x
)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 - I
*Tan[(c + d*x)/2]]*Log[(-b + Sqrt[-a^2 + b^2] - a*Tan[(c + d*x)/2])/(I*a -
b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + S
qrt[-a^2 + b^2])]))/Sqrt[-a^2 + b^2] - (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Lo
g[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])]
 + PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]))
/Sqrt[-a^2 + b^2]))/((-a^2 + b^2)*(d*e - c*f + I*f*Log[1 - I*Tan[(c + d*x)/
2]] - I*f*Log[1 + I*Tan[(c + d*x)/2]])) + (d*(e + f*x)*Sin[(c + d*x)/2])/((
a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (d*(e + f*x)*Sin[(c + d*x)/
2])/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/d^2
```

Maple [B] time = 0.359, size = 1542, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 2*(f*x+e)*(-I*a+b*exp(I*(d*x+c)))/d/(-a^2+b^2)/(1+exp(2*I*(d*x+c)))-I/d/(a^
2-b^2)^(3/2)*b^2*f/(a-b)/(a+b)*ln(-(I*b*exp(I*(d*x+c))-(a^2-b^2)^(1/2)-a)/(
a+(a^2-b^2)^(1/2)))*a^2*x-2*I/d^2/(a^2-b^2)*b^4*c*f/(a-b)/(a+b)/(-a^2+b^2)^
(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+I/d/(a^2-b^2)
^(3/2)*b^4*f/(a-b)/(a+b)*ln(-(I*b*exp(I*(d*x+c))-(a^2-b^2)^(1/2)-a)/(a+(a^2
-b^2)^(1/2)))*x-I/d^2/(a^2-b^2)^(3/2)*b^4*f/(a-b)/(a+b)*ln((I*b*exp(I*(d*x+
c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c-I/d/(a^2-b^2)^(3/2)*b^4*f/(a
-b)/(a+b)*ln((I*b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*x
+2*I/d^2/(a^2-b^2)*b^2*c*f/(a-b)/(a+b)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*e
xp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))*a^2-4/d^2/(a^2-b^2)*b^2*f/(4*a-4*b)*ln
(exp(I*(d*x+c))+I)-4/d^2/(a^2-b^2)*b^2*f/(4*a+4*b)*ln(exp(I*(d*x+c))-I)+4/d
^2/(a^2-b^2)*a^2*f/(4*a-4*b)*ln(exp(I*(d*x+c))+I)+4/d^2/(a^2-b^2)*a^2*f/(4*
a+4*b)*ln(exp(I*(d*x+c))-I)-2/d^2/(a^2-b^2)*a*f*ln(exp(I*(d*x+c)))+1/d^2/(a
^2-b^2)^(3/2)*b^4*f/(a-b)/(a+b)*dilog(-(I*b*exp(I*(d*x+c))-(a^2-b^2)^(1/2)-
a)/(a+(a^2-b^2)^(1/2)))-1/d^2/(a^2-b^2)^(3/2)*b^4*f/(a-b)/(a+b)*dilog((I*b*
exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))+2*I/d/(a^2-b^2)*b^4
*e/(a-b)/(a+b)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2
+b^2)^(1/2))+I/d^2/(a^2-b^2)^(3/2)*b^4*f/(a-b)/(a+b)*ln(-(I*b*exp(I*(d*x+c)
)-(a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2)))*c-I/d^2/(a^2-b^2)^(3/2)*b^2*f/(a-
b)/(a+b)*ln(-(I*b*exp(I*(d*x+c))-(a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2)))*a^
2*c-2*I/d/(a^2-b^2)*b^2*e/(a-b)/(a+b)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*ex
p(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))*a^2+I/d/(a^2-b^2)^(3/2)*b^2*f/(a-b)/(a+
```

```
b)*ln((I*b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*a^2*x+I/
d^2/(a^2-b^2)^(3/2)*b^2*f/(a-b)/(a+b)*ln((I*b*exp(I*(d*x+c))+(a^2-b^2)^(1/2
)-a)/(-a+(a^2-b^2)^(1/2)))*a^2*c-1/d^2/(a^2-b^2)^(3/2)*b^2*f/(a-b)/(a+b)*di
log(-(I*b*exp(I*(d*x+c))-(a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2)))*a^2+1/d^2/
(a^2-b^2)^(3/2)*b^2*f/(a-b)/(a+b)*dilog((I*b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)
-a)/(-a+(a^2-b^2)^(1/2)))*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.95523, size = 3087, normalized size = 8.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(-2*I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*dilog(-1/2*(2*I*a*cos(d
*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c
)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*b^3*f*sqrt(-(a^
2 - b^2)/b^2)*cos(d*x + c)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x +
c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b
+ 1) - 2*I*b^3*f*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*dilog(-1/2*(-2*I*a*cos
(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) + 2*b)/b + 1) - 4*(a^2*b - b^3)*d*f*x + 2*(a^3 + a^2*b - a
*b^2 - b^3)*f*cos(d*x + c)*log(sin(d*x + c) + 1) + 2*(a^3 - a^2*b - a*b^2 +
 b^3)*f*cos(d*x + c)*log(-sin(d*x + c) + 1) - 2*(b^3*d*e - b^3*c*f)*sqrt(-(
a^2 - b^2)/b^2)*cos(d*x + c)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*
b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(b^3*d*e - b^3*c*f)*sqrt(-(a^2 - b^2)
/b^2)*cos(d*x + c)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a
```

```
^2 - b^2)/b^2) - 2*I*a) + 2*(b^3*d*e - b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*cos(
d*x + c)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)
/b^2) + 2*I*a) + 2*(b^3*d*e - b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*
log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2
*I*a) - 2*(b^3*d*f*x + b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(1/2
*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b^3*d*f*x + b^3*c*f)*sqrt(-(a^2
- b^2)/b^2)*cos(d*x + c)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2
*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(
b^3*d*f*x + b^3*c*f)*sqrt(-(a^2 - b^2)/b^2)*cos(d*x + c)*log(1/2*(-2*I*a*co
s(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b^3*d*f*x + b^3*c*f)*sqrt(-(a^2 - b^2)/b^2
)*cos(d*x + c)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d
*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 4*(a^2*b - b
^3)*d*e + 4*((a^3 - a*b^2)*d*f*x + (a^3 - a*b^2)*d*e)*sin(d*x + c))/((a^4 -
 2*a^2*b^2 + b^4)*d^2*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e + fx\right)\sec^2\left(c + dx\right)}{a + b\sin\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx + e\right)\sec\left(dx + c\right)^2}{b\sin\left(dx + c\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

3.313 $\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{d\left(a^2-b^2\right)}$$

```
[Out] (-2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2
)*d) - (Sec[c + d*x]*(b - a*Sin[c + d*x]))/((a^2 - b^2)*d)
```

Rubi [A] time = 0.100608, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2696, 12, 2660, 618, 204}

$$-\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```


```
[Out] (-2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2
)*d) - (Sec[c + d*x]*(b - a*Sin[c + d*x]))/((a^2 - b^2)*d)
```


Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx &= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d}+\frac{\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}\\
&= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d}-\frac{b^2\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2-b^2}\\
&= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d}-\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d}\\
&= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d}+\frac{\left(4b^2\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d}\\
&= -\frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d}
\end{aligned}
$$

Mathematica [A] time = 0.263402, size = 152, normalized size = 1.81

$$\frac{\sqrt{a^2-b^2}(-a\sin(c+dx)+b(-\cos(c+dx))+b)+2b^2\cos(c+dx)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(b-a)(a+b)\sqrt{a^2-b^2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqrt
[a^2 - b^2]*(b - b*Cos[c + d*x] - a*Sin[c + d*x]))/((-a + b)*(a + b)*Sqrt[a
^2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]))
```

Maple [A] time = 0.002, size = 117, normalized size = 1.4

$$-2\,\frac{1}{d\,(2\,a-2\,b)\,(\tan(1/2\,dx+c/2)+1)}-2\,\frac{1}{d\,(2\,a+2\,b)\,(\tan(1/2\,dx+c/2)-1)}-2\,\frac{b^2}{d\,(a-b)\,(a+b)\,\sqrt{a^2-b^2}}\arctan\Bigg(1/2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/d/(2*a-2*b)/(tan(1/2*d*x+1/2*c)+1)-2/d/(2*a+2*b)/(tan(1/2*d*x+1/2*c)-1)-
2/d*b^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.18245, size = 684, normalized size = 8.14

$$\left[\frac{\sqrt{-a^2+b^2}b^2\cos(dx+c)\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^2b+2b^3}{2(a^4-2a^2b^2+b^4)d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*b^2*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 +
 b^4)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*b^2*arctan(-(a*sin(d*x + c) + b)/(s
qrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*si
n(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.65691, size = 144, normalized size = 1.71

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^2}{(a^2-b^2)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b}{(a^2-b^2)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*b^2/(a^2 - b^2)^(3/2) + (a*tan(1/2*d*x + 1/2*c) -
 b)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.314 $\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\cos^2(c+dx)(e+fx)^m}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable[((e + f*x)^m*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0713742, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((e + f*x)^m*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] Defer[Int][((e + f*x)^m*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Mathematica [A] time = 4.55248, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m \cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] Integrate[((e + f*x)^m*Cos[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Maple [A] time = 0.243, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \left(\cos \left(dx+c\right)\right)^{2}}{a+b\sin \left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \cos \left(dx+c\right)^{2}}{b\sin \left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx+e\right)^{m} \cos \left(dx+c\right)^{2}}{b\sin \left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \cos(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

3.315 $\int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\cos(c+dx)(e+fx)^m}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable[((e + f*x)^m*Cos[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0457507, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((e + f*x)^m*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]

[Out] Defer[Int][((e + f*x)^m*Cos[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)}\,dx$$

Mathematica [A] time = 3.29973, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m \cos(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Cos[c + d*x])/(a + b*Sin[c + d*x]),x]

[Out] Integrate[((e + f*x)^m*Cos[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Maple [A] time = 0.151, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \cos(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \cos(dx+c)}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^m \cos(dx+c)}{b\sin(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e+fx\right)^{m}\cos\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m*cos(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m}\cos\left(dx+c\right)}{b\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*cos(d*x + c)/(b*sin(d*x + c) + a), x)
```

3.316 $\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(e+fx)^m}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable[(e + f*x)^m/(a + b*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0582688, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)^m/(a + b*Sin[c + d*x]),x]
```

```
[Out] Defer[Int][(e + f*x)^m/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

Mathematica [A] time = 0.10955, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^m/(a + b*Sin[c + d*x]),x]
```

```
[Out] Integrate[(e + f*x)^m/(a + b*Sin[c + d*x]), x]
```

Maple [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m}}{a+b\sin\left(dx+c\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m/(a+b*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m}}{b\sin\left(dx+c\right)+a}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^{m}}{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m/(b*sin(d*x + c) + a), x)
```

3.317 $\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sec(c+dx)(e+fx)^m}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable[((e + f*x)^m*Sec[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi [A] time = 0.044306, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((e + f*x)^m*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]

[Out] Defer[Int][((e + f*x)^m*Sec[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

Mathematica [A] time = 139.158, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]

[Out] Integrate[((e + f*x)^m*Sec[c + d*x])/(a + b*Sin[c + d*x]), x]
```

Maple [A] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \sec \left(dx+c\right)}{a+b \sin \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \sec \left(dx+c\right)}{b \sin \left(dx+c\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^{m} \sec \left(dx+c\right)}{b \sin \left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m \sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**m*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \sec(dx+c)}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*sec(d*x + c)/(b*sin(d*x + c) + a), x)
```

3.318 $\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sec^2(c+dx)(e+fx)^m}{a+b\sin(c+dx)},x\right)$$

```
[Out] Unintegrable[((e + f*x)^m*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi [A] time = 0.0672621, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((e + f*x)^m*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] Defer[Int][((e + f*x)^m*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Mathematica [A] time = 6.20772, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^m \sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((e + f*x)^m*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] Integrate[((e + f*x)^m*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]), x]
```

Maple [A] time = 0.129, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \left(\sec \left(dx+c\right)\right)^{2}}{a+b\sin \left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{m} \sec \left(dx+c\right)^{2}}{b\sin \left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^{m} \sec \left(dx+c\right)^{2}}{b\sin \left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^m \sec(dx+c)^2}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*sec(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

3.319 $\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=77

$$\frac{2f\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}-\frac{e+fx}{bd(a+b\sin(c+dx))}$$

```
[Out] (2*f*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*Sqrt[a^2 - b^2]*d
^2) - (e + f*x)/(b*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.0717487, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4422, 2660, 618, 204}

$$\frac{2f\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}-\frac{e+fx}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*f*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*Sqrt[a^2 - b^2]*d
^2) - (e + f*x)/(b*d*(a + b*Sin[c + d*x]))
```

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= -\frac{e+fx}{bd(a+b\sin(c+dx))}+\frac{f\int \frac{1}{a+b\sin(c+dx)}\,dx}{bd}\\
&= -\frac{e+fx}{bd(a+b\sin(c+dx))}+\frac{(2f)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd^2}\\
&= -\frac{e+fx}{bd(a+b\sin(c+dx))}-\frac{(4f)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd^2}\\
&= \frac{2f\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,d^2}-\frac{e+fx}{bd(a+b\sin(c+dx))}
\end{aligned}
$$

Mathematica [A] time = 0.445768, size = 73, normalized size = 0.95

$$
\frac{\frac{2f\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{d(e+fx)}{a+b\sin(c+dx)}}{bd^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*f*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (
d*(e + f*x))/(a + b*Sin[c + d*x]))/(b*d^2)
```

Maple [C] time = 0.889, size = 194, normalized size = 2.5

$$\frac{-2\,i\left(fx+e\right)\mathrm{e}^{i(dx+c)}}{bd\left(b\mathrm{e}^{2\,i(dx+c)}-b+2\,ia\mathrm{e}^{i(dx+c)}\right)}-\frac{f}{bd^2}\ln\left(\mathrm{e}^{i(dx+c)}+\frac{1}{b}\left(ia\sqrt{-a^2+b^2}-a^2+b^2\right)\frac{1}{\sqrt{-a^2+b^2}}\right)\frac{1}{\sqrt{-a^2+b^2}}+\frac{f}{bd^2}\ln\left(\mathrm{e}^{i(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -2*I*(f*x+e)*exp(I*(d*x+c))/b/d/(b*exp(2*I*(d*x+c))-b+2*I*a*exp(I*(d*x+c)))
-1/(-a^2+b^2)^(1/2)*f/d^2/b*ln(exp(I*(d*x+c))+(I*a*(-a^2+b^2)^(1/2)-a^2+b^2
)/(-a^2+b^2)^(1/2)/b)+1/(-a^2+b^2)^(1/2)*f/d^2/b*ln(exp(I*(d*x+c))+(I*a*(-a
^2+b^2)^(1/2)+a^2-b^2)/(-a^2+b^2)^(1/2)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07617, size = 736, normalized size = 9.56

$$\left[-\frac{2\left(a^2-b^2\right)dfx+2\left(a^2-b^2\right)de+\left(bf\sin\left(dx+c\right)+af\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2\,(a\cos(dx+c)\sin\cdots}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-\cdots}\right.}{2\left(\left(a^2b^2-b^4\right)d^2\sin\left(dx+c\right)+\left(a^3b-ab^3\right)d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2 - b^2)*d*f*x + 2*(a^2 - b^2)*d*e + (b*f*sin(d*x + c) + a*f)*s
qrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^
2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2)
)/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/((a^2*b^2 - b^4)*
d^2*sin(d*x + c) + (a^3*b - a*b^3)*d^2), -((a^2 - b^2)*d*f*x + (a^2 - b^2)*
d*e + (b*f*sin(d*x + c) + a*f)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)
/(sqrt(a^2 - b^2)*cos(d*x + c))))/((a^2*b^2 - b^4)*d^2*sin(d*x + c) + (a^3*
b - a*b^3)*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cos(dx+c)}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cos(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

3.320 $\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=280

$$-\frac{2f^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\sqrt{a^2-b^2}}+\frac{2f^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3\sqrt{a^2-b^2}}-\frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\sqrt{a^2-b^2}}$$

```
[Out] ((-2*I)*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(
b*Sqrt[a^2 - b^2]*d^2) + ((2*I)*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) - (2*f^2*PolyLog[2, (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3) + (2*f^2*Poly
Log[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3
) - (e + f*x)^2/(b*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.527671, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4422, 3323, 2264, 2190, 2279, 2391}

$$-\frac{2f^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\sqrt{a^2-b^2}}+\frac{2f^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3\sqrt{a^2-b^2}}-\frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\sqrt{a^2-b^2}}+\frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^2\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] ((-2*I)*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(
b*Sqrt[a^2 - b^2]*d^2) + ((2*I)*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) - (2*f^2*PolyLog[2, (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3) + (2*f^2*Poly
Log[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3
) - (e + f*x)^2/(b*d*(a + b*Sin[c + d*x]))
```


Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{(e+fx)^2}{bd(a+b\sin(c+dx))} + \frac{(2f)\int \frac{e+fx}{a+b\sin(c+dx)}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{bd(a+b\sin(c+dx))} + \frac{(4f)\int \frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{bd(a+b\sin(c+dx))} - \frac{(4if)\int \frac{e^{i(c+dx)}(e+fx)}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}d} + \frac{(4if)\int \frac{e^{i(c+dx)}(e+fx)}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}d}$$

$$= -\frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} + \frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} - \frac{(e+fx)^2}{bd(a+b\sin(c+dx))} +$$

$$= -\frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} + \frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} - \frac{(e+fx)^2}{bd(a+b\sin(c+dx))} +$$

$$= -\frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} + \frac{2if(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} - \frac{2f^2\text{Li}_2\left(\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^3} + \frac{2}{-}$$

Mathematica [A] time = 3.08874, size = 311, normalized size = 1.11

$$-\frac{(e+fx)^2}{bd(a+b\sin(c+dx))} + \frac{2if\left(-f\sqrt{a^2-b^2}\text{PolyLog}\left(2,\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}-ia}\right) + f\sqrt{a^2-b^2}\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}+ia}\right) - id\left(2e\sqrt{b^2-a^2}\right.\right.}{bd^3\sqrt{-\left(a^2-b^2\right)^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*I)*f*((-I)*d*(2*Sqrt[-a^2 + b^2]*e*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqr
t[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*x*(Log[1 - (b*E^(I*(c + d*x)))/((-I)*a +
Sqrt[-a^2 + b^2])] - Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])])
) - Sqrt[a^2 - b^2]*f*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 +
b^2])] + Sqrt[a^2 - b^2]*f*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^
2 + b^2]))]))/(b*Sqrt[-(a^2 - b^2)^2]*d^3) - (e + f*x)^2/(b*d*(a + b*Sin[c
+ d*x]))
```

Maple [B] time = 0.892, size = 606, normalized size = 2.2

$$\frac{-2i\left(f^2x^2+2fex+e^2\right)e^{i(dx+c)}}{bd\left(be^{2i(dx+c)}-b+2iae^{i(dx+c)}\right)}+\frac{4ife}{bd^2}\arctan\left(\frac{2ibe^{i(dx+c)}-2a}{2}\frac{1}{\sqrt{-a^2+b^2}}\right)\frac{1}{\sqrt{-a^2+b^2}}+2\frac{f^2x}{bd^2\sqrt{-a^2+b^2}}\ln\left(\frac{ia+be^{i}}{ia}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -2*I*(f^2*x^2+2*e*f*x+e^2)*exp(I*(d*x+c))/b/d/(b*exp(2*I*(d*x+c))-b+2*I*a*e
xp(I*(d*x+c)))+4*I*f/b/d^2*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+
c))-2*a)/(-a^2+b^2)^(1/2))+2*f^2/b/d^2/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*
x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+2*f^2/b/d^3/(-a^2+b^2)^(1
/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-2*
f^2/b/d^2/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+
(-a^2+b^2)^(1/2)))*x-2*f^2/b/d^3/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+
(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-2*I*f^2/b/d^3/(-a^2+b^2)^(1/2)*
dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))+2*I*f
^2/b/d^3/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*
a+(-a^2+b^2)^(1/2)))-4*I*f^2/b/d^3*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp
(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.8509, size = 3263, normalized size = 11.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x + (a^2 - b^2)*d^2*e^2 +
 (-I*b^2*f^2*sin(d*x + c) - I*a*b*f^2)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2
*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (I*b^2*f^2*sin(d*x + c) + I*a*b*f^
2)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c)
 - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b +
1) + (I*b^2*f^2*sin(d*x + c) + I*a*b*f^2)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2
*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-I*b^2*f^2*sin(d*x + c) - I*a
*b*f^2)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*
x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b
)/b + 1) - (a*b*d*e*f - a*b*c*f^2 + (b^2*d*e*f - b^2*c*f^2)*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(
-(a^2 - b^2)/b^2) + 2*I*a) - (a*b*d*e*f - a*b*c*f^2 + (b^2*d*e*f - b^2*c*f^
2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*
x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + (a*b*d*e*f - a*b*c*f^2 + (b^
2*d*e*f - b^2*c*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x
+ c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (a*b*d*e*
f - a*b*c*f^2 + (b^2*d*e*f - b^2*c*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
 2*I*a) - (a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x + b^2*c*f^2)*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2
*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (a*
b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x + b^2*c*f^2)*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x +
 c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - (a*b*d*f^2*x + a
*b*c*f^2 + (b^2*d*f^2*x + b^2*c*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*l
og(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (a*b*d*f^2*x + a*b*c*f^2 + (
b^2*d*f^2*x + b^2*c*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I
*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2) + 2*b)/b))/((a^2*b^2 - b^4)*d^3*sin(d*x + c) + (a^3*
b - a*b^3)*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cos(dx+c)}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cos(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

3.321 $\int \frac{(e+fx)^3 \cos(c+dx)}{(a+b\sin(c+dx))^2} \, dx$

Optimal. Leaf size=418

$$-\frac{6f^2(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\sqrt{a^2-b^2}}+\frac{6f^2(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3\sqrt{a^2-b^2}}-\frac{6if^3\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^4\sqrt{a^2-b^2}}+\frac{6if^3\text{PolyLog}}{bd^4\sqrt{a}}$$

```
[Out] ((-3*I)*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])
/(b*Sqrt[a^2 - b^2]*d^2) + ((3*I)*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) - (6*f^2*(e + f*x)*PolyL
og[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3)
 + (6*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]
)/(b*Sqrt[a^2 - b^2]*d^3) - ((6*I)*f^3*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^4) + ((6*I)*f^3*PolyLog[3, (I*b*E
^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^4) - (e + f*x)
^3/(b*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.885268, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4422, 3323, 2264, 2190, 2531, 2282, 6589}

$$-\frac{6f^2(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\sqrt{a^2-b^2}}+\frac{6f^2(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3\sqrt{a^2-b^2}}-\frac{6if^3\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^4\sqrt{a^2-b^2}}+\frac{6if^3\text{PolyLog}}{bd^4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] ((-3*I)*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])
/(b*Sqrt[a^2 - b^2]*d^2) + ((3*I)*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^2) - (6*f^2*(e + f*x)*PolyL
og[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^3)
 + (6*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]
)/(b*Sqrt[a^2 - b^2]*d^3) - ((6*I)*f^3*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^4) + ((6*I)*f^3*PolyLog[3, (I*b*E
^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]*d^4) - (e + f*x)
^3/(b*d*(a + b*Sin[c + d*x]))
```


Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{(e+fx)^3}{bd(a+b\sin(c+dx))} + \frac{(3f)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{bd}$$

$$= -\frac{(e+fx)^3}{bd(a+b\sin(c+dx))} + \frac{(6f)\int \frac{e^{i(c+dx)}(e+fx)^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx}{bd}$$

$$= -\frac{(e+fx)^3}{bd(a+b\sin(c+dx))} - \frac{(6if)\int \frac{e^{i(c+dx)}(e+fx)^2}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}d} + \frac{(6if)\int \frac{e^{i(c+dx)}(e+fx)^2}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}d}$$

$$= -\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} + \frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} - \frac{(e+fx)^3}{bd(a+b\sin(c+dx))}$$

$$= -\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} + \frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} - \frac{6f^2(e+fx)\mathrm{Li}_2\left(\frac{ibe^{i(\cdots}}{a-\sqrt{\cdots}}\right.}{b\sqrt{a^2-b^2}d^3}$$

$$= -\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} + \frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} - \frac{6f^2(e+fx)\mathrm{Li}_2\left(\frac{ibe^{i(\cdots}}{a-\sqrt{\cdots}}\right.}{b\sqrt{a^2-b^2}d^3}$$

$$= -\frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} + \frac{3if(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}d^2} - \frac{6f^2(e+fx)\mathrm{Li}_2\left(\frac{ibe^{i(\cdots}}{a-\sqrt{\cdots}}\right.}{b\sqrt{a^2-b^2}d^3}$$

Mathematica [A] time = 2.37336, size = 446, normalized size = 1.07

$$-\frac{(e+fx)^3}{bd(a+b\sin(c+dx))} + \frac{3if\left(-i\left(2f^2\sqrt{a^2-b^2}\mathrm{PolyLog}\left(3,\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}-ia}\right) - 2f^2\sqrt{a^2-b^2}\mathrm{PolyLog}\left(3,-\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}+ia}\right) + d^2\left(2e^2\sqrt{\cdots}\right.\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((3*I)*f*(-2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, (b*E^(I*(c + d*x)))/(
(-I)*a + Sqrt[-a^2 + b^2])] + 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -(
```

```
(b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] - I*(d^2*(2*Sqrt[-a^2 + b^2]
*e^2*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*
x*(2*e + f*x)*(Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - L
og[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])])) + 2*Sqrt[a^2 - b^2]*
f^2*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 2*Sqrt[a^
2 - b^2]*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))])))
/(b*Sqrt[-(a^2 - b^2)^2]*d^4) - (e + f*x)^3/(b*d*(a + b*Sin[c + d*x]))
```

Maple [F] time = 1.598, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3} \cos\left(dx+c\right)}{\left(a+b\sin\left(dx+c\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.70981, size = 5299, normalized size = 12.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2 - b^2)*d^3*f^3*x^3 + 6*(a^2 - b^2)*d^3*e*f^2*x^2 + 6*(a^2 - b^
2)*d^3*e^2*f*x + 2*(a^2 - b^2)*d^3*e^3 + (-6*I*a*b*d*f^3*x - 6*I*a*b*d*e*f^
2 + (-6*I*b^2*d*f^3*x - 6*I*b^2*d*e*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) -
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (6*I*a*b*d*f^3*x +
 6*I*a*b*d*e*f^2 + (6*I*b^2*d*f^3*x + 6*I*b^2*d*e*f^2)*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*c
os(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (6*I
*a*b*d*f^3*x + 6*I*a*b*d*e*f^2 + (6*I*b^2*d*f^3*x + 6*I*b^2*d*e*f^2)*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*
x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b
)/b + 1) + (-6*I*a*b*d*f^3*x - 6*I*a*b*d*e*f^2 + (-6*I*b^2*d*f^3*x - 6*I*b^
2*d*e*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x
+ c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
- b^2)/b^2) + 2*b)/b + 1) - 3*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^
3 + (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 -
b^2)/b^2) + 2*I*a) - 3*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (b^
2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^
2) - 2*I*a) + 3*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (b^2*d^2*e
^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*
log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2
*I*a) + 3*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (b^2*d^2*e^2*f -
 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(-2
*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)
- 3*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2*f^3 +
(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x +
 c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b
) + 3*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2*f^3
+ (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*sin
(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x
 + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)
/b) - 3*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2*f^
3 + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(
d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2
*b)/b) + 3*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2
*f^3 + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3
)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*s
in(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
+ 2*b)/b) + 6*(b^2*f^3*sin(d*x + c) + a*b*f^3)*sqrt(-(a^2 - b^2)/b^2)*polyl
og(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*(b^2*f^3*sin(d*x + c) + a*b*f^
```

```
3)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x
+ c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6
*(b^2*f^3*sin(d*x + c) + a*b*f^3)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*c
os(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2))/b) + 6*(b^2*f^3*sin(d*x + c) + a*b*f^3)*sqrt(-(a^2 - b^2)/b
^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b))/((a^2*b^2 - b^4)*d^4*sin(d*x + c)
 + (a^3*b - a*b^3)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cos(dx+c)}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cos(d*x + c)/(b*sin(d*x + c) + a)^2, x)
```

3.322 $\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=116

$$\frac{af\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd^2\left(a^2-b^2\right)^{3/2}}+\frac{f\cos(c+dx)}{2d^2\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{e+fx}{2bd(a+b\sin(c+dx))^2}$$

```
[Out] (a*f*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/2)
*d^2) - (e + f*x)/(2*b*d*(a + b*Sin[c + d*x])^2) + (f*Cos[c + d*x])/(2*(a^2
 - b^2)*d^2*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.0967485, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4422, 2664, 12, 2660, 618, 204}

$$\frac{af\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd^2\left(a^2-b^2\right)^{3/2}}+\frac{f\cos(c+dx)}{2d^2\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{e+fx}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*f*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/2)
*d^2) - (e + f*x)/(2*b*d*(a + b*Sin[c + d*x])^2) + (f*Cos[c + d*x])/(2*(a^2
 - b^2)*d^2*(a + b*Sin[c + d*x]))
```

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
```

```
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(e+fx)\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{e+fx}{2bd(a+b\sin(c+dx))^2} + \frac{f\int \frac{1}{(a+b\sin(c+dx))^2}\,dx}{2bd}$$

$$= -\frac{e+fx}{2bd(a+b\sin(c+dx))^2} + \frac{f\cos(c+dx)}{2\left(a^2-b^2\right)d^2(a+b\sin(c+dx))} + \frac{f\int \frac{a}{a+b\sin(c+dx)}\,dx}{2b\left(a^2-b^2\right)d}$$

$$= -\frac{e+fx}{2bd(a+b\sin(c+dx))^2} + \frac{f\cos(c+dx)}{2\left(a^2-b^2\right)d^2(a+b\sin(c+dx))} + \frac{(af)\int \frac{1}{a+b\sin(c+dx)}\,dx}{2b\left(a^2-b^2\right)d}$$

$$= -\frac{e+fx}{2bd(a+b\sin(c+dx))^2} + \frac{f\cos(c+dx)}{2\left(a^2-b^2\right)d^2(a+b\sin(c+dx))} + \frac{(af)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x\right.}{b\left(a^2-b^2\right)d}$$

$$= -\frac{e+fx}{2bd(a+b\sin(c+dx))^2} + \frac{f\cos(c+dx)}{2\left(a^2-b^2\right)d^2(a+b\sin(c+dx))} - \frac{(2af)\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,d\right.}{b\left(a^2-\right.}$$

$$= \frac{af\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^2} - \frac{e+fx}{2bd(a+b\sin(c+dx))^2} + \frac{f\cos(c+dx)}{2\left(a^2-b^2\right)d^2(a+b\sin(c+dx))}$$

Mathematica [A] time = 1.11504, size = 112, normalized size = 0.97

$$\frac{\frac{2af\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}} + \frac{\frac{f\cos(c+dx)(a+b\sin(c+dx))}{(a-b)(a+b)} - \frac{d(e+fx)}{b}}{(a+b\sin(c+dx))^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a*f*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3
/2)) + (-((d*(e + f*x))/b) + (f*Cos[c + d*x]*(a + b*Sin[c + d*x]))/((a - b)
*(a + b)))/(a + b*Sin[c + d*x])^2)/(2*d^2)
```

Maple [C] time = 1.747, size = 349, normalized size = 3.

$$\frac{2a^2dfxe^{2i(dx+c)} - 2b^2dfxe^{2i(dx+c)} + 2ia^2fe^{2i(dx+c)} + ib^2fe^{2i(dx+c)} + 2a^2dee^{2i(dx+c)} + bafe^{3i(dx+c)} - 2b^2dee^{2i(dx+c)} - ib^2f}{\left(be^{2i(dx+c)} - b + 2iae^{i(dx+c)}\right)^2 d^2\left(a^2-b^2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] (2*a^2*d*f*x*exp(2*I*(d*x+c))-2*b^2*d*f*x*exp(2*I*(d*x+c))+2*I*a^2*f*exp(2*
I*(d*x+c))+I*b^2*f*exp(2*I*(d*x+c))+2*a^2*d*e*exp(2*I*(d*x+c))+b*a*f*exp(3*
I*(d*x+c))-2*b^2*d*e*exp(2*I*(d*x+c))-I*b^2*f-3*a*b*f*exp(I*(d*x+c)))/(b*ex
p(2*I*(d*x+c))-b+2*I*a*exp(I*(d*x+c)))^2/d^2/(a^2-b^2)/b-1/2/(-a^2+b^2)^(1/
2)*f*a/(a+b)/(a-b)/d^2/b*ln(exp(I*(d*x+c))+(I*a*(-a^2+b^2)^(1/2)-a^2+b^2)/(
-a^2+b^2)^(1/2)/b)+1/2/(-a^2+b^2)^(1/2)*f*a/(a+b)/(a-b)/d^2/b*ln(exp(I*(d*x
+c))+(I*a*(-a^2+b^2)^(1/2)+a^2-b^2)/(-a^2+b^2)^(1/2)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.03261, size = 1364, normalized size = 11.76

$$\left[\frac{2\left(a^4-2a^2b^2+b^4\right)dfx-2\left(a^2b^2-b^4\right)f\cos(dx+c)\sin(dx+c)+2\left(a^4-2a^2b^2+b^4\right)de-2\left(a^3b-ab^3\right)f\cos(dx+c)}{4\left(\left(a^4b^3-2a^2b^5+b^7\right)d^2\cos(dx+c)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^4 - 2*a^2*b^2 + b^4)*d*f*x - 2*(a^2*b^2 - b^4)*f*cos(d*x + c)*si
n(d*x + c) + 2*(a^4 - 2*a^2*b^2 + b^4)*d*e - 2*(a^3*b - a*b^3)*f*cos(d*x +
c) + (a*b^2*f*cos(d*x + c)^2 - 2*a^2*b*f*sin(d*x + c) - (a^3 + a*b^2)*f)*sq
rt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^
2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2)
)/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/((a^4*b^3 - 2*a^2
```

```
*b^5 + b^7)*d^2*cos(d*x + c)^2 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d^2*sin(d*
x + c) - (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^2), 1/2*((a^4 - 2*a^2*b^2 + b^
4)*d*f*x - (a^2*b^2 - b^4)*f*cos(d*x + c)*sin(d*x + c) + (a^4 - 2*a^2*b^2 +
 b^4)*d*e - (a^3*b - a*b^3)*f*cos(d*x + c) - (a*b^2*f*cos(d*x + c)^2 - 2*a^
2*b*f*sin(d*x + c) - (a^3 + a*b^2)*f)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d^2*c
os(d*x + c)^2 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d^2*sin(d*x + c) - (a^6*b -
 a^4*b^3 - a^2*b^5 + b^7)*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cos(dx+c)}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cos(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

3.323 $\int \frac{(e+fx)^2 \cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=357

$$-\frac{af^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\left(a^2-b^2\right)^{3/2}}+\frac{af^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3\left(a^2-b^2\right)^{3/2}}-\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\left(a^2-b^2\right)^{3/2}}+\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+}}{\sqrt{a^2-b}}\right.}{bd^2\left(a^2-b^2\right)^{3/2}}$$

```
[Out] ((-I)*a*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(
b*(a^2 - b^2)^(3/2)*d^2) + (I*a*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) - (f^2*Log[a + b*Sin[c + d
*x]])/(b*(a^2 - b^2)*d^3) - (a*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + (a*f^2*PolyLog[2, (I*b*E^(I*(c
 + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) - (e + f*x)^2/(
2*b*d*(a + b*Sin[c + d*x])^2) + (f*(e + f*x)*Cos[c + d*x])/((a^2 - b^2)*d^2
*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.611193, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {4422, 3324, 3323, 2264, 2190, 2279, 2391, 2668, 31}

$$-\frac{af^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\left(a^2-b^2\right)^{3/2}}+\frac{af^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3\left(a^2-b^2\right)^{3/2}}-\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2\left(a^2-b^2\right)^{3/2}}+\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+}}{\sqrt{a^2-b}}\right.}{bd^2\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-I)*a*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(
b*(a^2 - b^2)^(3/2)*d^2) + (I*a*f*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2) - (f^2*Log[a + b*Sin[c + d
*x]])/(b*(a^2 - b^2)*d^3) - (a*f^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + (a*f^2*PolyLog[2, (I*b*E^(I*(c
 + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) - (e + f*x)^2/(
2*b*d*(a + b*Sin[c + d*x])^2) + (f*(e + f*x)*Cos[c + d*x])/((a^2 - b^2)*d^2
*(a + b*Sin[c + d*x]))
```

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}+\frac{f\int\frac{e+fx}{(a+b\sin(c+dx))^2}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}+\frac{f(e+fx)\cos(c+dx)}{\left(a^2-b^2\right)d^2(a+b\sin(c+dx))}+\frac{(af)\int\frac{e+fx}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)d}-\frac{f^2}{}$$

$$= -\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}+\frac{f(e+fx)\cos(c+dx)}{\left(a^2-b^2\right)d^2(a+b\sin(c+dx))}+\frac{(2af)\int\frac{e^{i(c+dx)}(e+fx)}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}}{b\left(a^2-b^2\right)d}$$

$$= -\frac{f^2\log(a+b\sin(c+dx))}{b\left(a^2-b^2\right)d^3}-\frac{(e+fx)^2}{2bd(a+b\sin(c+dx))^2}+\frac{f(e+fx)\cos(c+dx)}{\left(a^2-b^2\right)d^2(a+b\sin(c+dx))}-$$

$$= -\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^2}+\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^2}-\frac{f^2\log(a+b\sin(c+}{b\left(a^2-b^2\right)d^3}$$

$$= -\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^2}+\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^2}-\frac{f^2\log(a+b\sin(c+}{b\left(a^2-b^2\right)d^3}$$

$$= -\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^2}+\frac{iaf(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d^2}-\frac{f^2\log(a+b\sin(c+}{b\left(a^2-b^2\right)d^3}$$

Mathematica [B] time = 14.8127, size = 1104, normalized size = 3.09

$$\frac{x\cot(c)f^2}{b\left(b^2-a^2\right)d^2}-\frac{x\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)f^2}{2b(b-a)(a+b)d^2}-\frac{ie^{ic}\left(4e^{ic}fx-\frac{2iae^{2ic}f\log\left(\frac{e^{i(2c+dx)}b}{iae^{ic}-\sqrt{\left(b^2-a^2\right)e^{2ic}}}+1\right)x}{\sqrt{\left(b^2-a^2\right)e^{2ic}}}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (f^2*x*Cot[c])/(b*(-a^2 + b^2)*d^2) - ((I/2)*E^(I*c)*f*(4*E^(I*c)*f*x + ((4
*I)*a*e*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]
*E^(I*c)) - ((4*I)*a*e*E^(I*c)*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 -
b^2]])/Sqrt[a^2 - b^2] + (2*f*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I
)*(c + d*x))))])/(d*E^(I*c)) - (2*E^(I*c)*f*ArcTan[(2*a*E^(I*(c + d*x)))/(b
*(-1 + E^((2*I)*(c + d*x))))])/d - (I*f*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2
*(-1 + E^((2*I)*(c + d*x)))^2])/(d*E^(I*c)) + (I*E^(I*c)*f*Log[4*a^2*E^((2*
I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2])/d + ((2*I)*a*f*x*Log[1 +
 (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqr
t[(-a^2 + b^2)*E^((2*I)*c)] - ((2*I)*a*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c
 + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2
)*E^((2*I)*c)] - ((2*I)*a*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) +
Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] + ((2*I)*a
*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 +
b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - (2*a*(-1 + E^((2*I)*c
))*f*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^
((2*I)*c)])])/(d*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]) + (2*a*(-1 + E^((2*I)*c))*
f*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2
*I)*c)]))])/(d*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])))/(b*(-a^2 + b^2)*d^2*(-1 +
E^((2*I)*c))) - (f^2*x*Csc[c/2]*Sec[c/2]*(Cos[c/2] - Sin[c/2])*(Cos[c/2] +
Sin[c/2]))/(2*b*(-a + b)*(a + b)*d^2) - (e + f*x)^2/(2*b*d*(a + b*Sin[c + d
*x])^2) + (Csc[c/2]*Sec[c/2]*(-(a*e*f*Cos[c]) - a*f^2*x*Cos[c] - b*e*f*Sin[
d*x] - b*f^2*x*Sin[d*x]))/(2*(a - b)*b*(a + b)*d^2*(a + b*Sin[c + d*x]))
```

Maple [B] time = 1.967, size = 946, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 2*(a^2*d*f^2*x^2*exp(2*I*(d*x+c))-b^2*d*f^2*x^2*exp(2*I*(d*x+c))+I*b^2*e*f*
exp(2*I*(d*x+c))+2*I*a^2*e*f*exp(2*I*(d*x+c))+2*a^2*d*e*f*x*exp(2*I*(d*x+c)
)+b*a*f^2*x*exp(3*I*(d*x+c))-2*b^2*d*e*f*x*exp(2*I*(d*x+c))+I*b^2*f^2*x*exp
(2*I*(d*x+c))-I*b^2*e*f+a^2*d*e^2*exp(2*I*(d*x+c))+b*a*e*f*exp(3*I*(d*x+c))
-b^2*d*e^2*exp(2*I*(d*x+c))+2*I*a^2*f^2*x*exp(2*I*(d*x+c))-3*a*b*f^2*x*exp(
I*(d*x+c))-I*b^2*f^2*x-3*a*b*e*f*exp(I*(d*x+c)))/(b*exp(2*I*(d*x+c))-b+2*I*
a*exp(I*(d*x+c)))^2/d^2/(a^2-b^2)/b-2*f^2/d^3/(-a^2+b^2)/b*ln(exp(I*(d*x+c)
))+f^2/d^3/(-a^2+b^2)/b*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-I*f
^2/d^3/(-a^2+b^2)^(3/2)/b*a*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(
I*a+(-a^2+b^2)^(1/2)))-f^2/d^2/(-a^2+b^2)^(3/2)/b*a*ln((I*a+b*exp(I*(d*x+c)
)-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-f^2/d^3/(-a^2+b^2)^(3/2)/b*a*
ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+2*I*f^
2/d^3/(-a^2+b^2)^(3/2)/b*a*c*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^
2)^(1/2))-2*I*f/d^2/(-a^2+b^2)^(3/2)/b*a*e*arctan(1/2*(2*I*b*exp(I*(d*x+c))
-2*a)/(-a^2+b^2)^(1/2))+f^2/d^2/(-a^2+b^2)^(3/2)/b*a*ln((I*a+b*exp(I*(d*x+c
))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+f^2/d^3/(-a^2+b^2)^(3/2)/b*a
*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+I*f^2
/d^3/(-a^2+b^2)^(3/2)/b*a*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*
a-(-a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.33164, size = 5252, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*((a^4 - 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d^2*e*
f*x + (a^4 - 2*a^2*b^2 + b^4)*d^2*e^2 - 2*((a^2*b^2 - b^4)*d*f^2*x + (a^2*b
^2 - b^4)*d*e*f)*cos(d*x + c)*sin(d*x + c) - (-I*a*b^3*f^2*cos(d*x + c)^2 +
 2*I*a^2*b^2*f^2*sin(d*x + c) + I*(a^3*b + a*b^3)*f^2)*sqrt(-(a^2 - b^2)/b^
2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) -
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - (I*a*b^3*f^2*cos(d
*x + c)^2 - 2*I*a^2*b^2*f^2*sin(d*x + c) - I*(a^3*b + a*b^3)*f^2)*sqrt(-(a^
2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(
d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - (I*a*b^
3*f^2*cos(d*x + c)^2 - 2*I*a^2*b^2*f^2*sin(d*x + c) - I*(a^3*b + a*b^3)*f^2
)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c)
 + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b +
1) - (-I*a*b^3*f^2*cos(d*x + c)^2 + 2*I*a^2*b^2*f^2*sin(d*x + c) + I*(a^3*b
 + a*b^3)*f^2)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a
*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
) + 2*b)/b + 1) - ((a^3*b + a*b^3)*d*f^2*x + (a^3*b + a*b^3)*c*f^2 - (a*b^3
*d*f^2*x + a*b^3*c*f^2)*cos(d*x + c)^2 + 2*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2
)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*si
n(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) +
 2*b)/b) + ((a^3*b + a*b^3)*d*f^2*x + (a^3*b + a*b^3)*c*f^2 - (a*b^3*d*f^2*
x + a*b^3*c*f^2)*cos(d*x + c)^2 + 2*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x +
 c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b
) - ((a^3*b + a*b^3)*d*f^2*x + (a^3*b + a*b^3)*c*f^2 - (a*b^3*d*f^2*x + a*b
^3*c*f^2)*cos(d*x + c)^2 + 2*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*sin(d*x + c)
)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) +
2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + ((
a^3*b + a*b^3)*d*f^2*x + (a^3*b + a*b^3)*c*f^2 - (a*b^3*d*f^2*x + a*b^3*c*f
^2)*cos(d*x + c)^2 + 2*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*c
os(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*((a^3*
b - a*b^3)*d*f^2*x + (a^3*b - a*b^3)*d*e*f)*cos(d*x + c) - ((a^2*b^2 - b^4)
*f^2*cos(d*x + c)^2 - 2*(a^3*b - a*b^3)*f^2*sin(d*x + c) - (a^4 - b^4)*f^2
+ ((a^3*b + a*b^3)*d*e*f - (a^3*b + a*b^3)*c*f^2 - (a*b^3*d*e*f - a*b^3*c*f
^2)*cos(d*x + c)^2 + 2*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^
2 - b^2)/b^2) + 2*I*a) - ((a^2*b^2 - b^4)*f^2*cos(d*x + c)^2 - 2*(a^3*b - a
*b^3)*f^2*sin(d*x + c) - (a^4 - b^4)*f^2 + ((a^3*b + a*b^3)*d*e*f - (a^3*b
+ a*b^3)*c*f^2 - (a*b^3*d*e*f - a*b^3*c*f^2)*cos(d*x + c)^2 + 2*(a^2*b^2*d*
e*f - a^2*b^2*c*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x
+ c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - ((a^2*b^2
 - b^4)*f^2*cos(d*x + c)^2 - 2*(a^3*b - a*b^3)*f^2*sin(d*x + c) - (a^4 - b^
4)*f^2 - ((a^3*b + a*b^3)*d*e*f - (a^3*b + a*b^3)*c*f^2 - (a*b^3*d*e*f - a*
b^3*c*f^2)*cos(d*x + c)^2 + 2*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*s
```

```
qrt(-(a^2 - b^2)/b^2) + 2*I*a) - ((a^2*b^2 - b^4)*f^2*cos(d*x + c)^2 - 2*(a
^3*b - a*b^3)*f^2*sin(d*x + c) - (a^4 - b^4)*f^2 - ((a^3*b + a*b^3)*d*e*f -
 (a^3*b + a*b^3)*c*f^2 - (a*b^3*d*e*f - a*b^3*c*f^2)*cos(d*x + c)^2 + 2*(a^
2*b^2*d*e*f - a^2*b^2*c*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b
*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a))/(
(a^4*b^3 - 2*a^2*b^5 + b^7)*d^3*cos(d*x + c)^2 - 2*(a^5*b^2 - 2*a^3*b^4 + a
*b^6)*d^3*sin(d*x + c) - (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cos(dx+c)}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cos(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

3.324 $\int \frac{(e+fx)^3 \cos(c+dx)}{(a+b\sin(c+dx))^3} dx$

Optimal. Leaf size=753

$$-\frac{3af^2(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\left(a^2-b^2\right)^{3/2}}+\frac{3af^2(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3\left(a^2-b^2\right)^{3/2}}+\frac{3if^3\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^4\left(a^2-b^2\right)}+\frac{3if^3\text{PolyL}}{bd^4}$$

```
[Out] (((3*I)/2)*f*(e + f*x)^2)/(b*(a^2 - b^2)*d^2) - (3*f^2*(e + f*x)*Log[1 - (I
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) - (((3*I)/2
)*a*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*
(a^2 - b^2)^(3/2)*d^2) - (3*f^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a
+ Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) + (((3*I)/2)*a*f*(e + f*x)^2*Log[1
 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2)
+ ((3*I)*f^3*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a
^2 - b^2)*d^4) - (3*a*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + ((3*I)*f^3*PolyLog[2, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^4) + (3*a*f^2*(e +
f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2
)^(3/2)*d^3) - ((3*I)*a*f^3*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
- b^2])])/(b*(a^2 - b^2)^(3/2)*d^4) + ((3*I)*a*f^3*PolyLog[3, (I*b*E^(I*(c
+ d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^4) - (e + f*x)^3/(2
*b*d*(a + b*Sin[c + d*x])^2) + (3*f*(e + f*x)^2*Cos[c + d*x])/(2*(a^2 - b^2
)*d^2*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.2742, antiderivative size = 753, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {4422, 3324, 3323, 2264, 2190, 2531, 2282, 6589, 4519, 2279, 2391}

$$-\frac{3af^2(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^3\left(a^2-b^2\right)^{3/2}}+\frac{3af^2(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd^3\left(a^2-b^2\right)^{3/2}}+\frac{3if^3\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^4\left(a^2-b^2\right)}+\frac{3if^3\text{PolyL}}{bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```



```
[Out] (((3*I)/2)*f*(e + f*x)^2)/(b*(a^2 - b^2)*d^2) - (3*f^2*(e + f*x)*Log[1 - (I
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) - (((3*I)/2
)*a*f*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*
(a^2 - b^2)^(3/2)*d^2) - (3*f^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a
```

```
+ Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^3) + (((3*I)/2)*a*f*(e + f*x)^2*Log[1
 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^2)
+ ((3*I)*f^3*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*(a
^2 - b^2)*d^4) - (3*a*f^2*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^3) + ((3*I)*f^3*PolyLog[2, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)*d^4) + (3*a*f^2*(e +
f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2
)^(3/2)*d^3) - ((3*I)*a*f^3*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2
- b^2])])/(b*(a^2 - b^2)^(3/2)*d^4) + ((3*I)*a*f^3*PolyLog[3, (I*b*E^(I*(c
+ d*x)))/(a + Sqrt[a^2 - b^2])])/(b*(a^2 - b^2)^(3/2)*d^4) - (e + f*x)^3/(2
*b*d*(a + b*Sin[c + d*x])^2) + (3*f*(e + f*x)^2*Cos[c + d*x])/(2*(a^2 - b^2
)*d^2*(a + b*Sin[c + d*x]))
```

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2} + \frac{(3f)\int \frac{(e+fx)^2}{(a+b\sin(c+dx))^2}\,dx}{2bd}$$

$$= -\frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2} + \frac{3f(e+fx)^2\cos(c+dx)}{2\left(a^2-b^2\right)d^2(a+b\sin(c+dx))} + \frac{(3af)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx}{2b\left(a^2-b^2\right)d} - \cdots$$

$$= \frac{3if(e+fx)^2}{2b\left(a^2-b^2\right)d^2} - \frac{(e+fx)^3}{2bd(a+b\sin(c+dx))^2} + \frac{3f(e+fx)^2\cos(c+dx)}{2\left(a^2-b^2\right)d^2(a+b\sin(c+dx))} + \frac{(3af)\int \cdots}{b\cdots}$$

$$= \frac{3if(e+fx)^2}{2b\left(a^2-b^2\right)d^2} - \frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^3} - \frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^3} - \frac{\cdots}{2bd(\cdots}$$

$$= \frac{3if(e+fx)^2}{2b\left(a^2-b^2\right)d^2} - \frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^3} - \frac{3iaf(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{3/2}d^2} - \frac{3f\cdots}{\cdots}$$

$$= \frac{3if(e+fx)^2}{2b\left(a^2-b^2\right)d^2} - \frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^3} - \frac{3iaf(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{3/2}d^2} - \frac{3f\cdots}{\cdots}$$

$$= \frac{3if(e+fx)^2}{2b\left(a^2-b^2\right)d^2} - \frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^3} - \frac{3iaf(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{3/2}d^2} - \frac{3f\cdots}{\cdots}$$

$$= \frac{3if(e+fx)^2}{2b\left(a^2-b^2\right)d^2} - \frac{3f^2(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)d^3} - \frac{3iaf(e+fx)^2\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{2b\left(a^2-b^2\right)^{3/2}d^2} - \frac{3f\cdots}{\cdots}$$

Mathematica [B] time = 19.5174, size = 2311, normalized size = 3.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-3*I)*E^(I*c)*f*(2*e*E^(I*c)*f*x + E^(I*c)*f^2*x^2 + (I*a*e^2*ArcTan[(I*a
 + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*E^(I*c)) - (I*a*e^
2*E^(I*c)*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2
] + (2*a*e*f*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]])/(Sqrt[a^2 -
 b^2]*d*E^(I*c)) + (e*f*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c +
 d*x))))])/(d*E^(I*c)) - (e*E^(I*c)*f*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 +
 E^((2*I)*(c + d*x))))])/d + ((2*I)*a*e*f*ArcTanh[(-a + I*b*E^(I*(c + d*x))
)/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*d*E^(I*c)) - ((I/2)*e*f*Log[4*a^2*E^((
2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2])/(d*E^(I*c)) + ((I/2)*e
*E^(I*c)*f*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d*x)))^2
])/d + (I*a*e*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 +
 b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - (I*a*e*E^((2*I)*c)*f
*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*
c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - (I*f^2*x*Log[1 + (b*E^(I*(2*c + d*x
)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/(d*E^(I*c)) + (I*E^(I*
c)*f^2*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^(
(2*I)*c)])])/d + ((I/2)*a*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c
) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - ((I/
2)*a*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[
(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - (I*a*e*f*x*Lo
g[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
)/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] + (I*a*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(
2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/Sqrt[(-a^2 +
b^2)*E^((2*I)*c)] - (I*f^2*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + S
qrt[(-a^2 + b^2)*E^((2*I)*c)])])/(d*E^(I*c)) + (I*E^(I*c)*f^2*x*Log[1 + (b*
E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/d - ((I
/2)*a*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2
)*E^((2*I)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] + ((I/2)*a*E^((2*I)*c)*f^2
*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I
)*c)])])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - ((-1 + E^((2*I)*c))*f*(-(Sqrt[(-a
^2 + b^2)*E^((2*I)*c)]*f) + a*d*E^(I*c)*(e + f*x))*PolyLog[2, (I*b*E^(I*(2*
c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/(d^2*E^(I*c)*Sq
rt[(-a^2 + b^2)*E^((2*I)*c)]) + ((-1 + E^((2*I)*c))*f*(Sqrt[(-a^2 + b^2)*E^
((2*I)*c)]*f + a*d*E^(I*c)*(e + f*x))*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I
*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))])/(d^2*E^(I*c)*Sqrt[(-a^2 + b
^2)*E^((2*I)*c)]) + (I*a*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c)
+ I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])/(d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])
- (I*a*E^((2*I)*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sq
rt[(-a^2 + b^2)*E^((2*I)*c)])])/(d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]) - (I*a
*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^
((2*I)*c)]))])/(d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]) + (I*a*E^((2*I)*c)*f^2*
PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I
```

```
)*c)]))])/(d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])))/(b*(-a^2 + b^2)*d^2*(-1 +
E^((2*I)*c))) - (e + f*x)^3/(2*b*d*(a + b*Sin[c + d*x])^2) - (3*Csc[c/2]*Se
c[c/2]*(a*e^2*f*Cos[c] + 2*a*e*f^2*x*Cos[c] + a*f^3*x^2*Cos[c] + b*e^2*f*Si
n[d*x] + 2*b*e*f^2*x*Sin[d*x] + b*f^3*x^2*Sin[d*x]))/(4*(a - b)*b*(a + b)*d
^2*(a + b*Sin[c + d*x]))
```

Maple [F] time = 1.497, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3}\cos\left(dx+c\right)}{\left(a+b\sin\left(dx+c\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 6.01823, size = 10630, normalized size = 14.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(a^4 - 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 12*(a^4 - 2*a^2*b^2 + b^4)*d^3
*e*f^2*x^2 + 12*(a^4 - 2*a^2*b^2 + b^4)*d^3*e^2*f*x + 4*(a^4 - 2*a^2*b^2 +
```

```
b^4)*d^3*e^3 - 12*((a^2*b^2 - b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 - b^4)*d^2*e*f^
2*x + (a^2*b^2 - b^4)*d^2*e^2*f)*cos(d*x + c)*sin(d*x + c) - 12*(a*b^3*f^3*
cos(d*x + c)^2 - 2*a^2*b^2*f^3*sin(d*x + c) - (a^3*b + a*b^3)*f^3)*sqrt(-(a
^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b
*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(a*b^3*f^
3*cos(d*x + c)^2 - 2*a^2*b^2*f^3*sin(d*x + c) - (a^3*b + a*b^3)*f^3)*sqrt(-
(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*
(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(a*b^3*
f^3*cos(d*x + c)^2 - 2*a^2*b^2*f^3*sin(d*x + c) - (a^3*b + a*b^3)*f^3)*sqrt
(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(
d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*(a*b^3*f^3*cos
(d*x + c)^2 - 2*a^2*b^2*f^3*sin(d*x + c) - (a^3*b + a*b^3)*f^3)*sqrt(-(a^2
- b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c
) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*((a^3*b - a*b^3)*d^2*
f^3*x^2 + 2*(a^3*b - a*b^3)*d^2*e*f^2*x + (a^3*b - a*b^3)*d^2*e^2*f)*cos(d*
x + c) - (12*I*(a^2*b^2 - b^4)*f^3*cos(d*x + c)^2 - 24*I*(a^3*b - a*b^3)*f^
3*sin(d*x + c) - 12*I*(a^4 - b^4)*f^3 + 2*(6*I*(a^3*b + a*b^3)*d*f^3*x + 6*
I*(a^3*b + a*b^3)*d*e*f^2 + (-6*I*a*b^3*d*f^3*x - 6*I*a*b^3*d*e*f^2)*cos(d*
x + c)^2 + (12*I*a^2*b^2*d*f^3*x + 12*I*a^2*b^2*d*e*f^2)*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2))*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(
b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - (
12*I*(a^2*b^2 - b^4)*f^3*cos(d*x + c)^2 - 24*I*(a^3*b - a*b^3)*f^3*sin(d*x
+ c) - 12*I*(a^4 - b^4)*f^3 + 2*(-6*I*(a^3*b + a*b^3)*d*f^3*x - 6*I*(a^3*b
+ a*b^3)*d*e*f^2 + (6*I*a*b^3*d*f^3*x + 6*I*a*b^3*d*e*f^2)*cos(d*x + c)^2 +
 (-12*I*a^2*b^2*d*f^3*x - 12*I*a^2*b^2*d*e*f^2)*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2))*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x
 + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - (-12*I*(a^
2*b^2 - b^4)*f^3*cos(d*x + c)^2 + 24*I*(a^3*b - a*b^3)*f^3*sin(d*x + c) + 1
2*I*(a^4 - b^4)*f^3 + 2*(-6*I*(a^3*b + a*b^3)*d*f^3*x - 6*I*(a^3*b + a*b^3)
*d*e*f^2 + (6*I*a*b^3*d*f^3*x + 6*I*a*b^3*d*e*f^2)*cos(d*x + c)^2 + (-12*I*
a^2*b^2*d*f^3*x - 12*I*a^2*b^2*d*e*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
))*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) +
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - (-12*I*(a^2*b^2 -
 b^4)*f^3*cos(d*x + c)^2 + 24*I*(a^3*b - a*b^3)*f^3*sin(d*x + c) + 12*I*(a^
4 - b^4)*f^3 + 2*(6*I*(a^3*b + a*b^3)*d*f^3*x + 6*I*(a^3*b + a*b^3)*d*e*f^2
 + (-6*I*a*b^3*d*f^3*x - 6*I*a*b^3*d*e*f^2)*cos(d*x + c)^2 + (12*I*a^2*b^2*
d*f^3*x + 12*I*a^2*b^2*d*e*f^2)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*dilog
(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin
(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 6*(2*(a^4 - b^4)*d*e*f^2
- 2*(a^4 - b^4)*c*f^3 - 2*((a^2*b^2 - b^4)*d*e*f^2 - (a^2*b^2 - b^4)*c*f^3)
*cos(d*x + c)^2 + 4*((a^3*b - a*b^3)*d*e*f^2 - (a^3*b - a*b^3)*c*f^3)*sin(d
*x + c) - ((a^3*b + a*b^3)*d^2*e^2*f - 2*(a^3*b + a*b^3)*c*d*e*f^2 + (a^3*b
 + a*b^3)*c^2*f^3 - (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*c
os(d*x + c)^2 + 2*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^
3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) + 2*I*b*sin(d
```

```
*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 6*(2*(a^4 - b^4)*d*e*f^2 -
2*(a^4 - b^4)*c*f^3 - 2*((a^2*b^2 - b^4)*d*e*f^2 - (a^2*b^2 - b^4)*c*f^3)*c
os(d*x + c)^2 + 4*((a^3*b - a*b^3)*d*e*f^2 - (a^3*b - a*b^3)*c*f^3)*sin(d*x
 + c) - ((a^3*b + a*b^3)*d^2*e^2*f - 2*(a^3*b + a*b^3)*c*d*e*f^2 + (a^3*b +
 a*b^3)*c^2*f^3 - (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cos
(d*x + c)^2 + 2*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x
 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 6*(2*(a^4 - b^4)*d*e*f^2 - 2*
(a^4 - b^4)*c*f^3 - 2*((a^2*b^2 - b^4)*d*e*f^2 - (a^2*b^2 - b^4)*c*f^3)*cos
(d*x + c)^2 + 4*((a^3*b - a*b^3)*d*e*f^2 - (a^3*b - a*b^3)*c*f^3)*sin(d*x +
 c) + ((a^3*b + a*b^3)*d^2*e^2*f - 2*(a^3*b + a*b^3)*c*d*e*f^2 + (a^3*b + a
*b^3)*c^2*f^3 - (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cos(d
*x + c)^2 + 2*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x
+ c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 6*(2*(a^4 - b^4)*d*e*f^2 - 2*(
a^4 - b^4)*c*f^3 - 2*((a^2*b^2 - b^4)*d*e*f^2 - (a^2*b^2 - b^4)*c*f^3)*cos(
d*x + c)^2 + 4*((a^3*b - a*b^3)*d*e*f^2 - (a^3*b - a*b^3)*c*f^3)*sin(d*x +
c) + ((a^3*b + a*b^3)*d^2*e^2*f - 2*(a^3*b + a*b^3)*c*d*e*f^2 + (a^3*b + a*
b^3)*c^2*f^3 - (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cos(d*
x + c)^2 + 2*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x +
 c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 6*(2*(a^4 - b^4)*d*f^3*x + 2*(a
^4 - b^4)*c*f^3 - 2*((a^2*b^2 - b^4)*d*f^3*x + (a^2*b^2 - b^4)*c*f^3)*cos(d
*x + c)^2 + 4*((a^3*b - a*b^3)*d*f^3*x + (a^3*b - a*b^3)*c*f^3)*sin(d*x + c
) - ((a^3*b + a*b^3)*d^2*f^3*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f^2*x + 2*(a^3*b
 + a*b^3)*c*d*e*f^2 - (a^3*b + a*b^3)*c^2*f^3 - (a*b^3*d^2*f^3*x^2 + 2*a*b^
3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*cos(d*x + c)^2 + 2*(a^2*
b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2
*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(2*I*a*cos(d*x + c) + 2
*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) + 2*b)/b) + 6*(2*(a^4 - b^4)*d*f^3*x + 2*(a^4 - b^4)*c*f^3 - 2*((a^2*b^
2 - b^4)*d*f^3*x + (a^2*b^2 - b^4)*c*f^3)*cos(d*x + c)^2 + 4*((a^3*b - a*b^
3)*d*f^3*x + (a^3*b - a*b^3)*c*f^3)*sin(d*x + c) + ((a^3*b + a*b^3)*d^2*f^3
*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f^2*x + 2*(a^3*b + a*b^3)*c*d*e*f^2 - (a^3*b
 + a*b^3)*c^2*f^3 - (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*
e*f^2 - a*b^3*c^2*f^3)*cos(d*x + c)^2 + 2*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*
d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2))*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d
*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 6*(2*(a^4 -
b^4)*d*f^3*x + 2*(a^4 - b^4)*c*f^3 - 2*((a^2*b^2 - b^4)*d*f^3*x + (a^2*b^2
- b^4)*c*f^3)*cos(d*x + c)^2 + 4*((a^3*b - a*b^3)*d*f^3*x + (a^3*b - a*b^3)
*c*f^3)*sin(d*x + c) - ((a^3*b + a*b^3)*d^2*f^3*x^2 + 2*(a^3*b + a*b^3)*d^2
*e*f^2*x + 2*(a^3*b + a*b^3)*c*d*e*f^2 - (a^3*b + a*b^3)*c^2*f^3 - (a*b^3*d
^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*cos(d
*x + c)^2 + 2*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*
```

```
e*f^2 - a^2*b^2*c^2*f^3)*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(-2*
I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 6*(2*(a^4 - b^4)*d*f^3*x + 2*(a^4 - b^4
)*c*f^3 - 2*((a^2*b^2 - b^4)*d*f^3*x + (a^2*b^2 - b^4)*c*f^3)*cos(d*x + c)^
2 + 4*((a^3*b - a*b^3)*d*f^3*x + (a^3*b - a*b^3)*c*f^3)*sin(d*x + c) + ((a^
3*b + a*b^3)*d^2*f^3*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f^2*x + 2*(a^3*b + a*b^3
)*c*d*e*f^2 - (a^3*b + a*b^3)*c^2*f^3 - (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*
f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*cos(d*x + c)^2 + 2*(a^2*b^2*d^2*
f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(
d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2
*b)/b))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d^4*cos(d*x + c)^2 - 2*(a^5*b^2 - 2*a^
3*b^4 + a*b^6)*d^4*sin(d*x + c) - (a^6*b - a^4*b^3 - a^2*b^5 + b^7)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cos(dx+c)}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cos(d*x + c)/(b*sin(d*x + c) + a)^3, x)
```

3.325 $\int \frac{(e+fx)^3\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=765

$$-\frac{6if^2\sqrt{a^2-b^2}(e+fx)\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd^3}+\frac{6if^2\sqrt{a^2-b^2}(e+fx)\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{abd^3}-\frac{3f\sqrt{a^2-b^2}(e+fx)^2\mathrm{Po}}{abd^2}$$

```
[Out] -(e + f*x)^4/(4*b*f) - (2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a*d) - (I*
Sqrt[a^2 - b^2]*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b
^2])])/(a*b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a*b*d) + ((3*I)*f*(e + f*x)^2*PolyLog[2, -E^(I*
(c + d*x))])/(a*d^2) - ((3*I)*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a
*d^2) - (3*Sqrt[a^2 - b^2]*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(
a - Sqrt[a^2 - b^2])])/(a*b*d^2) + (3*Sqrt[a^2 - b^2]*f*(e + f*x)^2*PolyLog
[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b*d^2) - (6*f^2*(e + f
*x)*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[3, E^(
I*(c + d*x))])/(a*d^3) - ((6*I)*Sqrt[a^2 - b^2]*f^2*(e + f*x)*PolyLog[3, (I
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b*d^3) + ((6*I)*Sqrt[a^2 - b
^2]*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/
(a*b*d^3) - ((6*I)*f^3*PolyLog[4, -E^(I*(c + d*x))])/(a*d^4) + ((6*I)*f^3*P
olyLog[4, E^(I*(c + d*x))])/(a*d^4) + (6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (I*
b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b*d^4) - (6*Sqrt[a^2 - b^2]*f
^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b*d^4)
```

Rubi [A] time = 1.42568, antiderivative size = 765, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 14, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4543, 4408, 3296, 2637, 4183, 2531, 6609, 2282, 6589, 4525, 32, 3323, 2264, 2190}

$$-\frac{6if^2\sqrt{a^2-b^2}(e+fx)\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd^3}+\frac{6if^2\sqrt{a^2-b^2}(e+fx)\mathrm{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{abd^3}-\frac{3f\sqrt{a^2-b^2}(e+fx)^2\mathrm{Po}}{abd^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```


```
[Out] -(e + f*x)^4/(4*b*f) - (2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a*d) - (I*
Sqrt[a^2 - b^2]*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b
^2])])/(a*b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a*b*d) + ((3*I)*f*(e + f*x)^2*PolyLog[2, -E^(I*
```

```
(c + d*x))])/(a*d^2) - ((3*I)*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a
*d^2) - (3*Sqrt[a^2 - b^2]*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(
a - Sqrt[a^2 - b^2])])/(a*b*d^2) + (3*Sqrt[a^2 - b^2]*f*(e + f*x)^2*PolyLog
[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b*d^2) - (6*f^2*(e + f
*x)*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[3, E^(
I*(c + d*x))])/(a*d^3) - ((6*I)*Sqrt[a^2 - b^2]*f^2*(e + f*x)*PolyLog[3, (I
*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b*d^3) + ((6*I)*Sqrt[a^2 - b
^2]*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/
(a*b*d^3) - ((6*I)*f^3*PolyLog[4, -E^(I*(c + d*x))])/(a*d^4) + ((6*I)*f^3*P
olyLog[4, E^(I*(c + d*x))])/(a*d^4) + (6*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (I*
b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b*d^4) - (6*Sqrt[a^2 - b^2]*f
^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b*d^4)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\cos(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^3\csc(c+dx)\,dx}{a} - \frac{\int (e+fx)^3\,dx}{b} + \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{(e+fx)^3}{a+b\sin(c+dx)}\,dx$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\int \frac{e^{i(c+dx)}(e+f}{ib+2ae^{i(c+dx)}-i}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{3if(e+fx)^2\operatorname{Li}_2\left(-e^{i(c+dx)}\right)}{ad^2} - \frac{3if}{}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}(e+fx)^3\log\left(1-\frac{ibe^{i(c+}}{a-\sqrt{a^2}}\right.}{abd}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}(e+fx)^3\log\left(1-\frac{ibe^{i(c+}}{a-\sqrt{a^2}}\right.}{abd}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}(e+fx)^3\log\left(1-\frac{ibe^{i(c+}}{a-\sqrt{a^2}}\right.}{abd}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}(e+fx)^3\log\left(1-\frac{ibe^{i(c+}}{a-\sqrt{a^2}}\right.}{abd}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}(e+fx)^3\log\left(1-\frac{ibe^{i(c+}}{a-\sqrt{a^2}}\right.}{abd}$$

Mathematica [A] time = 2.24113, size = 1194, normalized size = 1.56

$$-\frac{x\left(4e^3+6fxe^2+4f^2x^2e+f^3x^3\right)}{4b} + \frac{\left(a^2-b^2\right)\left(2\sqrt{b^2-a^2}e^3\tan^{-1}\left(\frac{ia+be^{i(c+dx)}}{\sqrt{a^2-b^2}}\right)d^3+\sqrt{a^2-b^2}f^3x^3\log\left(1-\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}-ia}\right)d^3\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(4*b) + ((a^2 - b^2)*(2*Sq
rt[-a^2 + b^2]*d^3*e^3*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] +
3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a
```

```
^2 + b^2])] + 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*E^(I*(c + d*x)))/(
(-I)*a + Sqrt[-a^2 + b^2])] + Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 - (b*E^(I*(
c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log
[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - 3*Sqrt[a^2 - b^2]*d^3*
e*f^2*x^2*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] - Sqrt[a^2
- b^2]*d^3*f^3*x^3*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] -
(3*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(I*(c + d*x)))/((-I
)*a + Sqrt[-a^2 + b^2])] + (3*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[
2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] + 6*Sqrt[a^2 - b^2]*d*e
*f^2*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 6*Sqrt[a
^2 - b^2]*d*f^3*x*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2]
)] - 6*Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt
[-a^2 + b^2]))] - 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3, -((b*E^(I*(c + d*x))
)/(I*a + Sqrt[-a^2 + b^2]))] + (6*I)*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*E^(I
*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - (6*I)*Sqrt[a^2 - b^2]*f^3*PolyL
og[4, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))]))/(a*b*Sqrt[-(a^2 -
b^2)^2]*d^4) + (I*((2*I)*(e + f*x)^3*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]]
 + (3*f*(d^2*(e + f*x)^2*PolyLog[2, -Cos[c + d*x] - I*Sin[c + d*x]] + (2*I)
*d*f*(e + f*x)*PolyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] - 2*f^2*PolyLog[4
, -Cos[c + d*x] - I*Sin[c + d*x]]))/d^3 - (3*f*(d^2*(e + f*x)^2*PolyLog[2,
Cos[c + d*x] + I*Sin[c + d*x]] + (2*I)*d*f*(e + f*x)*PolyLog[3, Cos[c + d*x
] + I*Sin[c + d*x]] - 2*f^2*PolyLog[4, Cos[c + d*x] + I*Sin[c + d*x]]))/d^3
))/(a*d)
```

Maple [F] time = 2.293, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3} \cos \left(dx+c\right) \cot \left(dx+c\right)}{a+b \sin \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.96738, size = 7471, normalized size = 9.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/4*(a*d^4*f^3*x^4 + 4*a*d^4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e^3*x
 + 12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2
*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2))/b) - 12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(d*x +
 c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2))/b) + 12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x
 + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2))/b) - 12*I*b*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x +
 c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2))/b) - 12*I*b*f^3*polylog(4, cos(d*x + c) + I*sin(d*x + c)) + 12*I*b*
f^3*polylog(4, cos(d*x + c) - I*sin(d*x + c)) - 12*I*b*f^3*polylog(4, -cos(
d*x + c) + I*sin(d*x + c)) + 12*I*b*f^3*polylog(4, -cos(d*x + c) - I*sin(d*
x + c)) - 2*(3*I*b*d^2*f^3*x^2 + 6*I*b*d^2*e*f^2*x + 3*I*b*d^2*e^2*f)*sqrt(
-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*
cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(
-3*I*b*d^2*f^3*x^2 - 6*I*b*d^2*e*f^2*x - 3*I*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)
/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c)
 - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(-3*I*b*d^2*f
^3*x^2 - 6*I*b*d^2*e*f^2*x - 3*I*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(
-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(3*I*b*d^2*f^3*x^2 + 6*I
*b*d^2*e*f^2*x + 3*I*b*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a
*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c
^2*d*e*f^2 - b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b
*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*(b*d^3*e^3 - 3*b*c*
d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos
(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(b
```

```
*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt(-(a^2 - b^2)
/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^
2) + 2*I*a) + 2*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)
*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sq
rt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*
d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*sqrt(-(a^2 - b
^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c)
 - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b*d^3*f^3*x^3 +
3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b
*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x
+ c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/
b) - 2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2
*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*co
s(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(
-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^
3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*sqrt(-(a^2 - b^2
)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c)
+ I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 12*(b*d*f^3*x + b*d*
e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(
d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)
 - 12*(b*d*f^3*x + b*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*
cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2))/b) - 12*(b*d*f^3*x + b*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)
*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin
(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(b*d*f^3*x + b*d*e*f^2)*sqrt(-(a
^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x
+ c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - (-6*I*b*d^2*f^3*x^2 -
 12*I*b*d^2*e*f^2*x - 6*I*b*d^2*e^2*f)*dilog(cos(d*x + c) + I*sin(d*x + c))
 - (6*I*b*d^2*f^3*x^2 + 12*I*b*d^2*e*f^2*x + 6*I*b*d^2*e^2*f)*dilog(cos(d*x
 + c) - I*sin(d*x + c)) - (-6*I*b*d^2*f^3*x^2 - 12*I*b*d^2*e*f^2*x - 6*I*b*
d^2*e^2*f)*dilog(-cos(d*x + c) + I*sin(d*x + c)) - (6*I*b*d^2*f^3*x^2 + 12*
I*b*d^2*e*f^2*x + 6*I*b*d^2*e^2*f)*dilog(-cos(d*x + c) - I*sin(d*x + c)) +
2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + b*d^3*e^3)*log(cos
(d*x + c) + I*sin(d*x + c) + 1) + 2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*
b*d^3*e^2*f*x + b*d^3*e^3)*log(cos(d*x + c) - I*sin(d*x + c) + 1) - 2*(b*d^
3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(-1/2*cos(d*x + c
) + 1/2*I*sin(d*x + c) + 1/2) - 2*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*
e*f^2 - b*c^3*f^3)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) - 2*(b
*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*
c^2*d*e*f^2 + b*c^3*f^3)*log(-cos(d*x + c) + I*sin(d*x + c) + 1) - 2*(b*d^3
*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*
d*e*f^2 + b*c^3*f^3)*log(-cos(d*x + c) - I*sin(d*x + c) + 1) - 12*(b*d*f^3*
x + b*d*e*f^2)*polylog(3, cos(d*x + c) + I*sin(d*x + c)) - 12*(b*d*f^3*x +
b*d*e*f^2)*polylog(3, cos(d*x + c) - I*sin(d*x + c)) + 12*(b*d*f^3*x + b*d*
e*f^2)*polylog(3, -cos(d*x + c) + I*sin(d*x + c)) + 12*(b*d*f^3*x + b*d*e*f
```

```
^2)*polylog(3, -cos(d*x + c) - I*sin(d*x + c)))/(a*b*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^3 \cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*cos(c + d*x)*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.326 $\int \frac{(e+fx)^2\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=557

$$-\frac{2f\sqrt{a^2-b^2}(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd^2}+\frac{2f\sqrt{a^2-b^2}(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{abd^2}-\frac{2if^2\sqrt{a^2-b^2}\text{PolyLog}\left(3,\frac{ib}{a-}\right.}{abd^3}$$

```
[Out] -(e + f*x)^3/(3*b*f) - (2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d) - (I*
Sqrt[a^2 - b^2]*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b
^2])])/(a*b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a*b*d) + ((2*I)*f*(e + f*x)*PolyLog[2, -E^(I*(c
 + d*x))])/(a*d^2) - ((2*I)*f*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^2
) - (2*Sqrt[a^2 - b^2]*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(a*b*d^2) + (2*Sqrt[a^2 - b^2]*f*(e + f*x)*PolyLog[2, (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b*d^2) - (2*f^2*PolyLog[3, -E^
(I*(c + d*x))])/(a*d^3) + (2*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3) - ((2
*I)*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(a*b*d^3) + ((2*I)*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a*b*d^3)
```

Rubi [A] time = 1.1903, antiderivative size = 557, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {4543, 4408, 3296, 2638, 4183, 2531, 2282, 6589, 4525, 32, 3323, 2264, 2190}

$$-\frac{2f\sqrt{a^2-b^2}(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd^2}+\frac{2f\sqrt{a^2-b^2}(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{abd^2}-\frac{2if^2\sqrt{a^2-b^2}\text{PolyLog}\left(3,\frac{ib}{a-}\right.}{abd^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(e + f*x)^3/(3*b*f) - (2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d) - (I*
Sqrt[a^2 - b^2]*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b
^2])])/(a*b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a*b*d) + ((2*I)*f*(e + f*x)*PolyLog[2, -E^(I*(c
 + d*x))])/(a*d^2) - ((2*I)*f*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^2
) - (2*Sqrt[a^2 - b^2]*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(a*b*d^2) + (2*Sqrt[a^2 - b^2]*f*(e + f*x)*PolyLog[2, (I*b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b*d^2) - (2*f^2*PolyLog[3, -E^
```

```
(I*(c + d*x))])/(a*d^3) + (2*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3) - ((2
*I)*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(a*b*d^3) + ((2*I)*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a*b*d^3)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\cos(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^2\csc(c+dx)\,dx}{a} - \frac{\int (e+fx)^2\,dx}{b} + \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{(e+fx)^2}{a+b\sin(c+dx)}\,dx$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\int \frac{e^{i(c+dx)}(e+f}{ib+2ae^{i(c+dx)}-i}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{2if(e+fx)\text{Li}_2\left(-e^{i(c+dx)}\right)}{ad^2} - \frac{2if(}{}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}(e+fx)^2\log\left(1-\frac{ibe^{i(c+}}{a-\sqrt{a^2}}\right.}{abd}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}(e+fx)^2\log\left(1-\frac{ibe^{i(c+}}{a-\sqrt{a^2}}\right.}{abd}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}(e+fx)^2\log\left(1-\frac{ibe^{i(c+}}{a-\sqrt{a^2}}\right.}{abd}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}(e+fx)^2\log\left(1-\frac{ibe^{i(c+}}{a-\sqrt{a^2}}\right.}{abd}$$

Mathematica [A] time = 1.75015, size = 607, normalized size = 1.09

$$\frac{i\left(a^2-b^2\right)\left(-i\left(2f^2\sqrt{a^2-b^2}\text{PolyLog}\left(3,\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}-ia}\right)-2f^2\sqrt{a^2-b^2}\text{PolyLog}\left(3,-\frac{be^{i(c+dx)}}{\sqrt{b^2-a^2}+ia}\right)+d^2\left(2e^2\sqrt{b^2-a^2}\tan^{-1}\left(\frac{ia}{}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) + ((e + f*x)^2*Log[1 - E^(I*(c + d*x
))] - (e + f*x)^2*Log[1 + E^(I*(c + d*x))] + ((2*I)*f*(d*(e + f*x)*PolyLog[
2, -E^(I*(c + d*x))] + I*f*PolyLog[3, -E^(I*(c + d*x))]))/d^2 + (2*f*((-I)*
d*(e + f*x)*PolyLog[2, E^(I*(c + d*x))] + f*PolyLog[3, E^(I*(c + d*x))]))/d
^2)/(a*d) + (I*(a^2 - b^2)*(-2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, (b*
E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 2*Sqrt[a^2 - b^2]*d*f*(e +
f*x)*PolyLog[2, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] - I*(d^2*(
2*Sqrt[-a^2 + b^2]*e^2*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] +
Sqrt[a^2 - b^2]*f*x*(2*e + f*x)*(Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt
[-a^2 + b^2])] - Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])])) +
2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 +
b^2])] - 2*Sqrt[a^2 - b^2]*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt
[-a^2 + b^2]))])))/(a*b*Sqrt[-(a^2 - b^2)^2]*d^3)
```

Maple [F] time = 1.974, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cos(dx+c)\cot(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.30347, size = 5239, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/12*(4*a*d^3*f^2*x^3 + 12*a*d^3*e*f*x^2 + 12*a*d^3*e^2*x + 12*b*f^2*sqrt(
-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2
*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*b*f^2*
sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c
) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*b
*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c)
+ (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*b*f^2
*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b
*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 12*b*f^2*pol
ylog(3, cos(d*x + c) + I*sin(d*x + c)) - 12*b*f^2*polylog(3, cos(d*x + c) -
 I*sin(d*x + c)) + 12*b*f^2*polylog(3, -cos(d*x + c) + I*sin(d*x + c)) + 12
*b*f^2*polylog(3, -cos(d*x + c) - I*sin(d*x + c)) - 2*(6*I*b*d*f^2*x + 6*I*
b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*
x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b
)/b + 1) - 2*(-6*I*b*d*f^2*x - 6*I*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1
/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*(-6*I*b*d*f^2*x - 6*I*b*d*e
*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x +
c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b
+ 1) - 2*(6*I*b*d*f^2*x + 6*I*b*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-
2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 6*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^
2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2
*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 6*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2
)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sq
rt(-(a^2 - b^2)/b^2) - 2*I*a) + 6*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqr
t(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-
(a^2 - b^2)/b^2) + 2*I*a) + 6*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt(-(
a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2
 - b^2)/b^2) - 2*I*a) - 6*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*
c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x +
```

```
c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b
) + 6*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*sqrt(-(a^2
- b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x +
c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 6*(b*d^2*f^2*x^2
+ 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*
(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 6*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x +
2*b*c*d*e*f - b*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c
) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2) + 2*b)/b) - (-12*I*b*d*f^2*x - 12*I*b*d*e*f)*dilog(cos(d*x + c) +
I*sin(d*x + c)) - (12*I*b*d*f^2*x + 12*I*b*d*e*f)*dilog(cos(d*x + c) - I*si
n(d*x + c)) - (-12*I*b*d*f^2*x - 12*I*b*d*e*f)*dilog(-cos(d*x + c) + I*sin(
d*x + c)) - (12*I*b*d*f^2*x + 12*I*b*d*e*f)*dilog(-cos(d*x + c) - I*sin(d*x
+ c)) + 6*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + b*d^2*e^2)*log(cos(d*x + c) + I
*sin(d*x + c) + 1) + 6*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + b*d^2*e^2)*log(cos(
d*x + c) - I*sin(d*x + c) + 1) - 6*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*lo
g(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) - 6*(b*d^2*e^2 - 2*b*c*d*e*
f + b*c^2*f^2)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) - 6*(b*d^2
*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*log(-cos(d*x + c) + I*s
in(d*x + c) + 1) - 6*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f
^2)*log(-cos(d*x + c) - I*sin(d*x + c) + 1))/(a*b*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^2 \cos(c + dx) \cot(c + dx)}{a + b \sin(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)

[Out] Integral((e + f*x)**2*cos(c + d*x)*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.327 $\int \frac{(e+fx)\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=351

$$-\frac{f\sqrt{a^2-b^2}\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd^2}+\frac{f\sqrt{a^2-b^2}\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{abd^2}+\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{if\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}$$

```
[Out] -((e*x)/b) - (f*x^2)/(2*b) - (2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) -
 (I*Sqrt[a^2 - b^2]*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
 b^2])])/(a*b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a*b*d) + (I*f*PolyLog[2, -E^(I*(c + d*x))])/(a*
d^2) - (I*f*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) - (Sqrt[a^2 - b^2]*f*PolyL
og[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b*d^2) + (Sqrt[a^2 -
 b^2]*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b*d^2)
```

Rubi [A] time = 0.660472, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {4543, 4408, 3296, 2637, 4183, 2279, 2391, 4525, 3323, 2264, 2190}

$$-\frac{f\sqrt{a^2-b^2}\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{abd^2}+\frac{f\sqrt{a^2-b^2}\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{abd^2}+\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{if\text{PolyLog}\left(2,e^{i(c+dx)}\right)}{ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```


```
[Out] -((e*x)/b) - (f*x^2)/(2*b) - (2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) -
 (I*Sqrt[a^2 - b^2]*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
 b^2])])/(a*b*d) + (I*Sqrt[a^2 - b^2]*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a*b*d) + (I*f*PolyLog[2, -E^(I*(c + d*x))])/(a*
d^2) - (I*f*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) - (Sqrt[a^2 - b^2]*f*PolyL
og[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b*d^2) + (Sqrt[a^2 -
 b^2]*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b*d^2)
```


Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
```

```
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
```

```
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\cos(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)\csc(c+dx)\,dx}{a} - \frac{\int (e+fx)\,dx}{b} + \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{e+fx}{a+b\sin(c+dx)}\,dx$$

$$= -\frac{ex}{b} - \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\int \frac{e^{i(c+dx)}(e+fx}{ib+2ae^{i(c+dx)}-ibe}$$

$$= -\frac{ex}{b} - \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{\left(2i\sqrt{a^2-b^2}\right)\int \frac{e^{i(c+dx)}(e+fx)}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}}{a}$$

$$= -\frac{ex}{b} - \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right.}{abd}$$

$$= -\frac{ex}{b} - \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right.}{abd}$$

$$= -\frac{ex}{b} - \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{i\sqrt{a^2-b^2}(e+fx)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right.}{abd}$$

Mathematica [B] time = 6.79811, size = 812, normalized size = 2.31

$$\frac{\frac{(c+dx)(cf-d(2e+fx))}{b} + \frac{2de\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a} - \frac{2cf\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a} + \frac{2f\left((c+dx)\left(\log\left(1-e^{i(c+dx)}\right)-\log\left(1+e^{i(c+dx)}\right)\right)+i\left(\text{PolyLog}\left(2,-e^{i(c+dx)}\right)-\text{PolyL}\right.\right.}{a}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((c + d*x)*(c*f - d*(2*e + f*x)))/b + (2*d*e*Log[Tan[(c + d*x)/2]])/a - (2
*c*f*Log[Tan[(c + d*x)/2]])/a + (2*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] -
 Log[1 + E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E
^(I*(c + d*x))])))/a + (2*(a^2 - b^2)*d*(e + f*x)*((2*(d*e - c*f)*ArcTan[(b
 + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*(Log[1 - I*
Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a +
 b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b
```

```
 + Sqrt[-a^2 + b^2]))]]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 + I*Tan[(c + d*x)/2
]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b
^2])] + PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2
]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(-b + Sqrt[
-a^2 + b^2] - a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2])] + PolyLog[2
, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]))/Sqrt[-a^2 + b^
2] - (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c
 + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] + PolyLog[2, (a + I*a*Tan[(c + d*
x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2]))/(a*b*(d*e - c*
f + I*f*Log[1 - I*Tan[(c + d*x)/2]] - I*f*Log[1 + I*Tan[(c + d*x)/2]])))/(2
*d^2)
```

Maple [B] time = 0.306, size = 1207, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2*f*x^2/b-e*x/b+1/b*a/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^
2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+1/b*a/d^2*f/(-a^2+b^2)^(1/2)*ln((I*
a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+2*I/b/d*a*e/
(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+1/
d/a*e*ln(exp(I*(d*x+c))-1)-1/d/a*e*ln(exp(I*(d*x+c))+1)+I/b/d^2*a*f/(-a^2+b
^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/
2)))-1/d^2/a*f*c*ln(exp(I*(d*x+c))-1)-1/d*f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b*
exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-1/d^2*f*b/a/(-a^
2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/
2)))*c+2*I/d^2*f*c/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*
a)/(-a^2+b^2)^(1/2))+I/d^2*f*b/a/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c
))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-I/b*a/d^2*f/(-a^2+b^2)^(1/2)*d
ilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))+I/d^2*
f/a*dilog(exp(I*(d*x+c)))+I/d^2*f/a*dilog(exp(I*(d*x+c))+1)+1/d*f*b/a/(-a^2
+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2
)))*x+1/d^2*f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2
))/(I*a+(-a^2+b^2)^(1/2)))*c-1/b*a/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*
x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-1/b*a/d^2*f/(-a^2+b^2)^(1
/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-2*
I/d*e/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)
^(1/2))-2*I/b/d^2*a*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2
*a)/(-a^2+b^2)^(1/2))-1/d/a*ln(exp(I*(d*x+c))+1)*f*x-I/d^2*f*b/a/(-a^2+b^2)
```

```
^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2))
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.31358, size = 3267, normalized size = 9.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/4*(2*a*d^2*f*x^2 + 4*a*d^2*e*x - 2*I*b*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1
/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*b*f*sqrt(-(a^2 - b^2)/b^2
)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*b*f*sqrt(-(a^2
- b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d
*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*I*b*f*
sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) -
 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)
 + 2*I*b*f*dilog(cos(d*x + c) + I*sin(d*x + c)) - 2*I*b*f*dilog(cos(d*x + c
) - I*sin(d*x + c)) + 2*I*b*f*dilog(-cos(d*x + c) + I*sin(d*x + c)) - 2*I*b
*f*dilog(-cos(d*x + c) - I*sin(d*x + c)) - 2*(b*d*e - b*c*f)*sqrt(-(a^2 - b
^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/
b^2) + 2*I*a) - 2*(b*d*e - b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x +
c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(b*d*e -
b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) +
2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(b*d*e - b*c*f)*sqrt(-(a^2 - b^2)/b
```

```
^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
 - 2*I*a) - 2*(b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d
*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2) + 2*b)/b) + 2*(b*d*f*x + b*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1
/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(b*d*f*x + b*c*f)*sqrt(-(a^2 -
b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x +
c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b*d*f*x + b*c*
f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) -
 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2
*(b*d*f*x + b*d*e)*log(cos(d*x + c) + I*sin(d*x + c) + 1) + 2*(b*d*f*x + b*
d*e)*log(cos(d*x + c) - I*sin(d*x + c) + 1) - 2*(b*d*e - b*c*f)*log(-1/2*co
s(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) - 2*(b*d*e - b*c*f)*log(-1/2*cos(d*x
 + c) - 1/2*I*sin(d*x + c) + 1/2) - 2*(b*d*f*x + b*c*f)*log(-cos(d*x + c) +
 I*sin(d*x + c) + 1) - 2*(b*d*f*x + b*c*f)*log(-cos(d*x + c) - I*sin(d*x +
c) + 1))/(a*b*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cos(c + d*x)*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.328 $\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{b}$$

```
[Out] -(x/b) + (2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/(a*b*d) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Rubi [A] time = 0.184407, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2889, 3058, 2660, 618, 204, 3770}

$$\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(x/b) + (2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/(a*b*d) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3058

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(C*x
)/(b*d), x] + (Dist[(A*b^2 + a^2*C)/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f
*x]), x], x] - Dist[(c^2*C + A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f
*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx &= \int \frac{\csc(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx \\
&= -\frac{x}{b}+\frac{\int \csc(c+dx)\,dx}{a}-\left(-\frac{a}{b}+\frac{b}{a}\right)\int \frac{1}{a+b\sin(c+dx)}\,dx \\
&= -\frac{x}{b}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d} \\
&= -\frac{x}{b}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\left(4\left(\frac{a}{b}-\frac{b}{a}\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(c+\right.\right.}{d} \\
&= -\frac{x}{b}+\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}
\end{aligned}
$$

Mathematica [A] time = 0.111835, size = 90, normalized size = 1.2

$$-\frac{-2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)+ac+adx-b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*c + a*d*x - 2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]] + b*Log[Cos[(c + d*x)/2]] - b*Log[Sin[(c + d*x)/2]])/(a*b*d))
```

Maple [A] time = 0.003, size = 137, normalized size = 1.8

$$-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{bd}+\frac{1}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\,\frac{a}{bd\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan\left(1/2\,dx+c/2\right)+2\,b}{\sqrt{a^2-b^2}}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/d/b*arctan(tan(1/2*d*x+1/2*c))+1/a/d*ln(tan(1/2*d*x+1/2*c))+2/d/b*a/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a*
b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0534, size = 640, normalized size = 8.53

$$\left[-\frac{2\,adx + b\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right) - b\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right) - \sqrt{-a^2+b^2}\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-}{b^2\cos(dx+c)^2-2}\right.}{2\,abd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*d*x + b*log(1/2*cos(d*x + c) + 1/2) - b*log(-1/2*cos(d*x + c) +
1/2) - sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
+ c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a
^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/(a*b*d),
 -1/2*(2*a*d*x + b*log(1/2*cos(d*x + c) + 1/2) - b*log(-1/2*cos(d*x + c) +
1/2) + 2*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(
d*x + c))))/(a*b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(c+dx\right)\cot\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 2.16688, size = 127, normalized size = 1.69

$$-\frac{\frac{dx+c}{b} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a} - \frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/b - log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(pi*floor(1/2*(d*x + c
)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*
sqrt(a^2 - b^2)/(a*b))/d
```

3.329 $\int \frac{(e+fx)^3 \cos^2(c+dx) \cot(c+dx)}{a+b \sin(c+dx)} dx$

Optimal. Leaf size=763

$$\frac{6f^2(a^2-b^2)(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^3}+\frac{6f^2(a^2-b^2)(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^2d^3}-\frac{3if(a^2-b^2)(e+fx)^2\text{Pol}}{ab^2d^2}$$

```
[Out] ((-I/4)*(e + f*x)^4)/(a*f) - ((I/4)*(a^2 - b^2)*(e + f*x)^4)/(a*b^2*f) + (6
*f^3*Cos[c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Cos[c + d*x])/(b*d^2) + ((a^2
 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(
a*b^2*d) + ((a^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/(a*b^2*d) + ((e + f*x)^3*Log[1 - E^((2*I)*(c + d*x))])/(a*d)
 - ((3*I)*(a^2 - b^2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(a*b^2*d^2) - ((3*I)*(a^2 - b^2)*f*(e + f*x)^2*PolyLog[2,
 (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^2*d^2) - (((3*I)/2)*f*(
e + f*x)^2*PolyLog[2, E^((2*I)*(c + d*x))])/(a*d^2) + (6*(a^2 - b^2)*f^2*(e
 + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^2*d^3
) + (6*(a^2 - b^2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/(a*b^2*d^3) + (3*f^2*(e + f*x)*PolyLog[3, E^((2*I)*(c + d*x)
)])/(2*a*d^3) + ((6*I)*(a^2 - b^2)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(a*b^2*d^4) + ((6*I)*(a^2 - b^2)*f^3*PolyLog[4, (I*b*E
^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^2*d^4) + (((3*I)/4)*f^3*PolyLo
g[4, E^((2*I)*(c + d*x))])/(a*d^4) + (6*f^2*(e + f*x)*Sin[c + d*x])/(b*d^3)
 - ((e + f*x)^3*Sin[c + d*x])/(b*d)
```

Rubi [A] time = 1.35906, antiderivative size = 763, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 17, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4543, 4408, 4404, 3311, 32, 2635, 8, 3717, 2190, 2531, 6609, 2282, 6589, 4525, 3296, 2638, 4519}

$$\frac{6f^2(a^2-b^2)(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^3}+\frac{6f^2(a^2-b^2)(e+fx)\text{PolyLog}\left(3,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^2d^3}-\frac{3if(a^2-b^2)(e+fx)^2\text{Pol}}{ab^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-I/4)*(e + f*x)^4)/(a*f) - ((I/4)*(a^2 - b^2)*(e + f*x)^4)/(a*b^2*f) + (6
*f^3*Cos[c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Cos[c + d*x])/(b*d^2) + ((a^2
 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(
```

```
a*b^2*d) + ((a^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/(a*b^2*d) + ((e + f*x)^3*Log[1 - E^((2*I)*(c + d*x))])/(a*d)
 - ((3*I)*(a^2 - b^2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(a*b^2*d^2) - ((3*I)*(a^2 - b^2)*f*(e + f*x)^2*PolyLog[2,
 (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^2*d^2) - (((3*I)/2)*f*(
e + f*x)^2*PolyLog[2, E^((2*I)*(c + d*x))])/(a*d^2) + (6*(a^2 - b^2)*f^2*(e
 + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^2*d^3
) + (6*(a^2 - b^2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/(a*b^2*d^3) + (3*f^2*(e + f*x)*PolyLog[3, E^((2*I)*(c + d*x)
)])/(2*a*d^3) + ((6*I)*(a^2 - b^2)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a
- Sqrt[a^2 - b^2])])/(a*b^2*d^4) + ((6*I)*(a^2 - b^2)*f^3*PolyLog[4, (I*b*E
^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^2*d^4) + (((3*I)/4)*f^3*PolyLo
g[4, E^((2*I)*(c + d*x))])/(a*d^4) + (6*f^2*(e + f*x)*Sin[c + d*x])/(b*d^3)
 - ((e + f*x)^3*Sin[c + d*x])/(b*d)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
```

& PosQ[a^2 - b^2]

Rubi steps

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\cos^2(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^3\cot(c+dx)\,dx}{a} - \frac{\int (e+fx)^3\cos(c+dx)\,dx}{b} + \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{(e+fx)}{a+b\,\mathrm{s}}$$

$$= -\frac{i(e+fx)^4}{4af} - \frac{i\left(a^2-b^2\right)(e+fx)^4}{4ab^2f} - \frac{(e+fx)^3\sin(c+dx)}{bd} - \frac{(2i)\int \frac{e^{2i(c+dx)}(e+f}{1-e^{2i(c+dx)}}}{a}$$

$$= -\frac{i(e+fx)^4}{4af} - \frac{i\left(a^2-b^2\right)(e+fx)^4}{4ab^2f} - \frac{3f(e+fx)^2\cos(c+dx)}{bd^2} + \frac{\left(a^2-b^2\right)(e+}{}$$

$$= -\frac{i(e+fx)^4}{4af} - \frac{i\left(a^2-b^2\right)(e+fx)^4}{4ab^2f} - \frac{3f(e+fx)^2\cos(c+dx)}{bd^2} + \frac{\left(a^2-b^2\right)(e+}{}$$

$$= -\frac{i(e+fx)^4}{4af} - \frac{i\left(a^2-b^2\right)(e+fx)^4}{4ab^2f} + \frac{6f^3\cos(c+dx)}{bd^4} - \frac{3f(e+fx)^2\cos(c+d}{bd^2}$$

$$= -\frac{i(e+fx)^4}{4af} - \frac{i\left(a^2-b^2\right)(e+fx)^4}{4ab^2f} + \frac{6f^3\cos(c+dx)}{bd^4} - \frac{3f(e+fx)^2\cos(c+d}{bd^2}$$

$$= -\frac{i(e+fx)^4}{4af} - \frac{i\left(a^2-b^2\right)(e+fx)^4}{4ab^2f} + \frac{6f^3\cos(c+dx)}{bd^4} - \frac{3f(e+fx)^2\cos(c+d}{bd^2}$$

Mathematica [B] time = 10.6296, size = 4014, normalized size = 5.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(e*E^(I*c)*f^2*Csc[c]*((2*d^3*x^3)/E^((2*I)*c) + (3*I)*d^2*(1 - E^((-2*I)*
c))*x^2*Log[1 - E^((-I)*(c + d*x))] + (3*I)*d^2*(1 - E^((-2*I)*c))*x^2*Log[
1 + E^((-I)*(c + d*x))] - (6*(-1 + E^((2*I)*c))*(d*x*PolyLog[2, -E^((-I)*(c
 + d*x))] - I*PolyLog[3, -E^((-I)*(c + d*x))]))/E^((2*I)*c) - (6*(-1 + E^((
2*I)*c))*(d*x*PolyLog[2, E^((-I)*(c + d*x))] - I*PolyLog[3, E^((-I)*(c + d*
```

x))]))/E^((2*I)*c)))/(2*a*d^3) - (E^(I*c)*f^3*Csc[c]*((d^4*x^4)/E^((2*I)*c)
 + (2*I)*d^3*(1 - E^((-2*I)*c))*x^3*Log[1 - E^((-I)*(c + d*x))] + (2*I)*d^3
*(1 - E^((-2*I)*c))*x^3*Log[1 + E^((-I)*(c + d*x))] - (6*(-1 + E^((2*I)*c))
*(d^2*x^2*PolyLog[2, -E^((-I)*(c + d*x))] - (2*I)*d*x*PolyLog[3, -E^((-I)*(
c + d*x))] - 2*PolyLog[4, -E^((-I)*(c + d*x))]))/E^((2*I)*c) - (6*(-1 + E^(
(2*I)*c))*(d^2*x^2*PolyLog[2, E^((-I)*(c + d*x))] - (2*I)*d*x*PolyLog[3, E^
((-I)*(c + d*x))] - 2*PolyLog[4, E^((-I)*(c + d*x))]))/E^((2*I)*c)))/(4*a*d
^4) + ((a^2 - b^2)*((-4*I)*d^4*e^3*E^((2*I)*c)*x - (6*I)*d^4*e^2*E^((2*I)*c
)*f*x^2 - (4*I)*d^4*e*E^((2*I)*c)*f^2*x^3 - I*d^4*E^((2*I)*c)*f^3*x^4 - (2*
I)*d^3*e^3*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] + (
2*I)*d^3*e^3*E^((2*I)*c)*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c
+ d*x))))] - d^3*e^3*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c
+ d*x)))^2] + d^3*e^3*E^((2*I)*c)*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 +
 E^((2*I)*(c + d*x)))^2] - 6*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a
*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e^2*E^((2*I)*c)*f*x*Log
[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
- 6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 +
 b^2)*E^((2*I)*c)])] + 6*d^3*e*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d
*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 2*d^3*f^3*x^3*Log[1
 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] +
2*d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt
[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/
(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e^2*E^((2*I)*c)*f*x
*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)
])] - 6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a
^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c
 + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 2*d^3*f^3*x^3*L
og[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])
] + 2*d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) +
Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (6*I)*d^2*(-1 + E^((2*I)*c))*f*(e + f*x)
^2*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((
2*I)*c)])] - (6*I)*d^2*(-1 + E^((2*I)*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(
I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 12*d*e*f
^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((
2*I)*c)])] + 12*d*e*E^((2*I)*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E
^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d*f^3*x*PolyLog[3, (I*b*E^
(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*d*E^(
(2*I)*c)*f^3*x*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2
 + b^2)*E^((2*I)*c)])] - 12*d*e*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a
*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 12*d*e*E^((2*I)*c)*f^2*PolyL
og[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]
))] - 12*d*f^3*x*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a
^2 + b^2)*E^((2*I)*c)]))] + 12*d*E^((2*I)*c)*f^3*x*PolyLog[3, -((b*E^(I*(2*
c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (12*I)*f^3*Po
lyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*

```
c)])] + (12*I)*E^((2*I)*c)*f^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c
) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (12*I)*f^3*PolyLog[4, -((b*E^(I*(2
*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (12*I)*E^((2
*I)*c)*f^3*PolyLog[4, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b
^2)*E^((2*I)*c)]))]))/(2*a*b^2*d^4*(-1 + E^((2*I)*c))) + (e^3*Csc[c]*(-(d*x
*Cos[c]) + Log[Cos[d*x]*Sin[c] + Cos[c]*Sin[d*x]]*Sin[c]))/(a*d*(Cos[c]^2 +
 Sin[c]^2)) + Csc[c]*(Cos[c + d*x]/(8*b^2*d^4) - ((I/8)*Sin[c + d*x])/(b^2*
d^4))*(4*a*d^4*e^3*x*Cos[d*x] + 6*a*d^4*e^2*f*x^2*Cos[d*x] + 4*a*d^4*e*f^2*
x^3*Cos[d*x] + a*d^4*f^3*x^4*Cos[d*x] + 4*a*d^4*e^3*x*Cos[2*c + d*x] + 6*a*
d^4*e^2*f*x^2*Cos[2*c + d*x] + 4*a*d^4*e*f^2*x^3*Cos[2*c + d*x] + a*d^4*f^3
*x^4*Cos[2*c + d*x] - 2*b*d^3*e^3*Cos[c + 2*d*x] - (6*I)*b*d^2*e^2*f*Cos[c
+ 2*d*x] + 12*b*d*e*f^2*Cos[c + 2*d*x] + (12*I)*b*f^3*Cos[c + 2*d*x] - 6*b*
d^3*e^2*f*x*Cos[c + 2*d*x] - (12*I)*b*d^2*e*f^2*x*Cos[c + 2*d*x] + 12*b*d*f
^3*x*Cos[c + 2*d*x] - 6*b*d^3*e*f^2*x^2*Cos[c + 2*d*x] - (6*I)*b*d^2*f^3*x^
2*Cos[c + 2*d*x] - 2*b*d^3*f^3*x^3*Cos[c + 2*d*x] + 2*b*d^3*e^3*Cos[3*c + 2
*d*x] + (6*I)*b*d^2*e^2*f*Cos[3*c + 2*d*x] - 12*b*d*e*f^2*Cos[3*c + 2*d*x]
- (12*I)*b*f^3*Cos[3*c + 2*d*x] + 6*b*d^3*e^2*f*x*Cos[3*c + 2*d*x] + (12*I)
*b*d^2*e*f^2*x*Cos[3*c + 2*d*x] - 12*b*d*f^3*x*Cos[3*c + 2*d*x] + 6*b*d^3*e
*f^2*x^2*Cos[3*c + 2*d*x] + (6*I)*b*d^2*f^3*x^2*Cos[3*c + 2*d*x] + 2*b*d^3*
f^3*x^3*Cos[3*c + 2*d*x] - (4*I)*b*d^3*e^3*Sin[c] - 12*b*d^2*e^2*f*Sin[c] +
 (24*I)*b*d*e*f^2*Sin[c] + 24*b*f^3*Sin[c] - (12*I)*b*d^3*e^2*f*x*Sin[c] -
24*b*d^2*e*f^2*x*Sin[c] + (24*I)*b*d*f^3*x*Sin[c] - (12*I)*b*d^3*e*f^2*x^2*
Sin[c] - 12*b*d^2*f^3*x^2*Sin[c] - (4*I)*b*d^3*f^3*x^3*Sin[c] + (4*I)*a*d^4
*e^3*x*Sin[d*x] + (6*I)*a*d^4*e^2*f*x^2*Sin[d*x] + (4*I)*a*d^4*e*f^2*x^3*Si
n[d*x] + I*a*d^4*f^3*x^4*Sin[d*x] + (4*I)*a*d^4*e^3*x*Sin[2*c + d*x] + (6*I
)*a*d^4*e^2*f*x^2*Sin[2*c + d*x] + (4*I)*a*d^4*e*f^2*x^3*Sin[2*c + d*x] + I
*a*d^4*f^3*x^4*Sin[2*c + d*x] - (2*I)*b*d^3*e^3*Sin[c + 2*d*x] + 6*b*d^2*e^
2*f*Sin[c + 2*d*x] + (12*I)*b*d*e*f^2*Sin[c + 2*d*x] - 12*b*f^3*Sin[c + 2*d
*x] - (6*I)*b*d^3*e^2*f*x*Sin[c + 2*d*x] + 12*b*d^2*e*f^2*x*Sin[c + 2*d*x]
+ (12*I)*b*d*f^3*x*Sin[c + 2*d*x] - (6*I)*b*d^3*e*f^2*x^2*Sin[c + 2*d*x] +
6*b*d^2*f^3*x^2*Sin[c + 2*d*x] - (2*I)*b*d^3*f^3*x^3*Sin[c + 2*d*x] + (2*I)
*b*d^3*e^3*Sin[3*c + 2*d*x] - 6*b*d^2*e^2*f*Sin[3*c + 2*d*x] - (12*I)*b*d*e
*f^2*Sin[3*c + 2*d*x] + 12*b*f^3*Sin[3*c + 2*d*x] + (6*I)*b*d^3*e^2*f*x*Sin
[3*c + 2*d*x] - 12*b*d^2*e*f^2*x*Sin[3*c + 2*d*x] - (12*I)*b*d*f^3*x*Sin[3*
c + 2*d*x] + (6*I)*b*d^3*e*f^2*x^2*Sin[3*c + 2*d*x] - 6*b*d^2*f^3*x^2*Sin[3
*c + 2*d*x] + (2*I)*b*d^3*f^3*x^3*Sin[3*c + 2*d*x]) - (3*e^2*f*Csc[c]*Sec[c
]*(d^2*E^(I*ArcTan[Tan[c]])*x^2 + ((I*d*x*(-Pi + 2*ArcTan[Tan[c]]) - Pi*Log
[1 + E^((-2*I)*d*x)] - 2*(d*x + ArcTan[Tan[c]])*Log[1 - E^((2*I)*(d*x + Arc
Tan[Tan[c]]))] + Pi*Log[Cos[d*x]] + 2*ArcTan[Tan[c]]*Log[Sin[d*x + ArcTan[T
an[c]]]] + I*PolyLog[2, E^((2*I)*(d*x + ArcTan[Tan[c]]))])*Tan[c])/Sqrt[1 +
 Tan[c]^2]))/(2*a*d^2*Sqrt[Sec[c]^2*(Cos[c]^2 + Sin[c]^2)])
```

Maple [F] time = 4.033, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\cos (dx+c))^2 \cot (dx+c)}{a+b\sin (dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 5.83696, size = 7961, normalized size = 10.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/2*(6*I*b^2*f^3*polylog(4, cos(d*x + c) + I*sin(d*x + c)) - 6*I*b^2*f^3*po
lylog(4, cos(d*x + c) - I*sin(d*x + c)) - 6*I*b^2*f^3*polylog(4, -cos(d*x +
 c) + I*sin(d*x + c)) + 6*I*b^2*f^3*polylog(4, -cos(d*x + c) - I*sin(d*x +
c)) + 6*I*(a^2 - b^2)*f^3*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x
+ c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6
*I*(a^2 - b^2)*f^3*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) -
2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*I*(a^2
 - b^2)*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c
```

```
) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*I*(a^2 - b^2)*f^3*poly
log(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x +
 a*b*d^2*e^2*f - 2*a*b*f^3)*cos(d*x + c) + (3*I*(a^2 - b^2)*d^2*f^3*x^2 + 6
*I*(a^2 - b^2)*d^2*e*f^2*x + 3*I*(a^2 - b^2)*d^2*e^2*f)*dilog(-1/2*(2*I*a*c
os(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (3*I*(a^2 - b^2)*d^2*f^3*x^2 + 6*I*(a^2
- b^2)*d^2*e*f^2*x + 3*I*(a^2 - b^2)*d^2*e^2*f)*dilog(-1/2*(2*I*a*cos(d*x +
 c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2) + 2*b)/b + 1) + (-3*I*(a^2 - b^2)*d^2*f^3*x^2 - 6*I*(a^2 - b^2)*
d^2*e*f^2*x - 3*I*(a^2 - b^2)*d^2*e^2*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) +
2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2) + 2*b)/b + 1) + (-3*I*(a^2 - b^2)*d^2*f^3*x^2 - 6*I*(a^2 - b^2)*d^2*e*
f^2*x - 3*I*(a^2 - b^2)*d^2*e^2*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*si
n(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) +
 2*b)/b + 1) + (-3*I*b^2*d^2*f^3*x^2 - 6*I*b^2*d^2*e*f^2*x - 3*I*b^2*d^2*e^
2*f)*dilog(cos(d*x + c) + I*sin(d*x + c)) + (3*I*b^2*d^2*f^3*x^2 + 6*I*b^2*
d^2*e*f^2*x + 3*I*b^2*d^2*e^2*f)*dilog(cos(d*x + c) - I*sin(d*x + c)) + (3*
I*b^2*d^2*f^3*x^2 + 6*I*b^2*d^2*e*f^2*x + 3*I*b^2*d^2*e^2*f)*dilog(-cos(d*x
 + c) + I*sin(d*x + c)) + (-3*I*b^2*d^2*f^3*x^2 - 6*I*b^2*d^2*e*f^2*x - 3*I
*b^2*d^2*e^2*f)*dilog(-cos(d*x + c) - I*sin(d*x + c)) + ((a^2 - b^2)*d^3*e^
3 - 3*(a^2 - b^2)*c*d^2*e^2*f + 3*(a^2 - b^2)*c^2*d*e*f^2 - (a^2 - b^2)*c^3
*f^3)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2
) + 2*I*a) + ((a^2 - b^2)*d^3*e^3 - 3*(a^2 - b^2)*c*d^2*e^2*f + 3*(a^2 - b^
2)*c^2*d*e*f^2 - (a^2 - b^2)*c^3*f^3)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x
+ c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + ((a^2 - b^2)*d^3*e^3 - 3*(a^2
- b^2)*c*d^2*e^2*f + 3*(a^2 - b^2)*c^2*d*e*f^2 - (a^2 - b^2)*c^3*f^3)*log(-
2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)
 + ((a^2 - b^2)*d^3*e^3 - 3*(a^2 - b^2)*c*d^2*e^2*f + 3*(a^2 - b^2)*c^2*d*e
*f^2 - (a^2 - b^2)*c^3*f^3)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*
b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + ((a^2 - b^2)*d^3*f^3*x^3 + 3*(a^2 - b^2
)*d^3*e*f^2*x^2 + 3*(a^2 - b^2)*d^3*e^2*f*x + 3*(a^2 - b^2)*c*d^2*e^2*f - 3
*(a^2 - b^2)*c^2*d*e*f^2 + (a^2 - b^2)*c^3*f^3)*log(1/2*(2*I*a*cos(d*x + c)
 + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^
2)/b^2) + 2*b)/b) + ((a^2 - b^2)*d^3*f^3*x^3 + 3*(a^2 - b^2)*d^3*e*f^2*x^2
+ 3*(a^2 - b^2)*d^3*e^2*f*x + 3*(a^2 - b^2)*c*d^2*e^2*f - 3*(a^2 - b^2)*c^2
*d*e*f^2 + (a^2 - b^2)*c^3*f^3)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x +
 c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b
) + ((a^2 - b^2)*d^3*f^3*x^3 + 3*(a^2 - b^2)*d^3*e*f^2*x^2 + 3*(a^2 - b^2)*
d^3*e^2*f*x + 3*(a^2 - b^2)*c*d^2*e^2*f - 3*(a^2 - b^2)*c^2*d*e*f^2 + (a^2
- b^2)*c^3*f^3)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(
d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + ((a^2 - b^2
)*d^3*f^3*x^3 + 3*(a^2 - b^2)*d^3*e*f^2*x^2 + 3*(a^2 - b^2)*d^3*e^2*f*x + 3
*(a^2 - b^2)*c*d^2*e^2*f - 3*(a^2 - b^2)*c^2*d*e*f^2 + (a^2 - b^2)*c^3*f^3)
*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*
```

```
sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (b^2*d^3*f^3*x^3 + 3*b^2*d
^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + b^2*d^3*e^3)*log(cos(d*x + c) + I*sin(d*
x + c) + 1) + (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x +
b^2*d^3*e^3)*log(cos(d*x + c) - I*sin(d*x + c) + 1) + (b^2*d^3*e^3 - 3*b^2*
c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*log(-1/2*cos(d*x + c) + 1/2*
I*sin(d*x + c) + 1/2) + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^
2 - b^2*c^3*f^3)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) + (b^2*d
^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f -
3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*log(-cos(d*x + c) + I*sin(d*x + c) + 1) +
(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^
2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*log(-cos(d*x + c) - I*sin(d*x + c) +
 1) + 6*((a^2 - b^2)*d*f^3*x + (a^2 - b^2)*d*e*f^2)*polylog(3, 1/2*(2*I*a*c
os(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2))/b) + 6*((a^2 - b^2)*d*f^3*x + (a^2 - b^2)*d*e*f^2)*poly
log(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b
*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*((a^2 - b^2)*d*f^3*x + (a^2 -
 b^2)*d*e*f^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x
+ c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*((a^2 - b^2)*d*f^3*
x + (a^2 - b^2)*d*e*f^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (
b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 6*(b^2*d*f^
3*x + b^2*d*e*f^2)*polylog(3, cos(d*x + c) + I*sin(d*x + c)) + 6*(b^2*d*f^3
*x + b^2*d*e*f^2)*polylog(3, cos(d*x + c) - I*sin(d*x + c)) + 6*(b^2*d*f^3*
x + b^2*d*e*f^2)*polylog(3, -cos(d*x + c) + I*sin(d*x + c)) + 6*(b^2*d*f^3*
x + b^2*d*e*f^2)*polylog(3, -cos(d*x + c) - I*sin(d*x + c)) - 2*(a*b*d^3*f^
3*x^3 + 3*a*b*d^3*e*f^2*x^2 + a*b*d^3*e^3 - 6*a*b*d*e*f^2 + 3*(a*b*d^3*e^2*
f - 2*a*b*d*f^3)*x)*sin(d*x + c))/(a*b^2*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^3 \cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*cos(c + d*x)**2*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.330 $\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=566

$$-\frac{2if\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^2}-\frac{2if\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^2d^2}+\frac{2f^2\left(a^2-b^2\right)\text{PolyLog}}{ab^2d^3}$$

```
[Out] ((-I/3)*(e + f*x)^3)/(a*f) - ((I/3)*(a^2 - b^2)*(e + f*x)^3)/(a*b^2*f) - (2
*f*(e + f*x)*Cos[c + d*x])/(b*d^2) + ((a^2 - b^2)*(e + f*x)^2*Log[1 - (I*b*
E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^2*d) + ((a^2 - b^2)*(e + f*x)
^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^2*d) + ((e +
f*x)^2*Log[1 - E^((2*I)*(c + d*x))])/(a*d) - ((2*I)*(a^2 - b^2)*f*(e + f*x)
*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^2*d^2) - ((2
*I)*(a^2 - b^2)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
- b^2])])/(a*b^2*d^2) - (I*f*(e + f*x)*PolyLog[2, E^((2*I)*(c + d*x))])/(a*
d^2) + (2*(a^2 - b^2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
b^2])])/(a*b^2*d^3) + (2*(a^2 - b^2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(a*b^2*d^3) + (f^2*PolyLog[3, E^((2*I)*(c + d*x))])/
(2*a*d^3) + (2*f^2*Sin[c + d*x])/(b*d^3) - ((e + f*x)^2*Sin[c + d*x])/(b*d)
```

Rubi [A] time = 1.12232, antiderivative size = 566, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 13, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.382, Rules used = {4543, 4408, 4404, 3310, 3717, 2190, 2531, 2282, 6589, 4525, 3296, 2637, 4519}

$$-\frac{2if\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^2}-\frac{2if\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^2d^2}+\frac{2f^2\left(a^2-b^2\right)\text{PolyLog}}{ab^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```



```
[Out] ((-I/3)*(e + f*x)^3)/(a*f) - ((I/3)*(a^2 - b^2)*(e + f*x)^3)/(a*b^2*f) - (2
*f*(e + f*x)*Cos[c + d*x])/(b*d^2) + ((a^2 - b^2)*(e + f*x)^2*Log[1 - (I*b*
E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^2*d) + ((a^2 - b^2)*(e + f*x)
^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^2*d) + ((e +
f*x)^2*Log[1 - E^((2*I)*(c + d*x))])/(a*d) - ((2*I)*(a^2 - b^2)*f*(e + f*x)
*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^2*d^2) - ((2
*I)*(a^2 - b^2)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
- b^2])])/(a*b^2*d^2) - (I*f*(e + f*x)*PolyLog[2, E^((2*I)*(c + d*x))])/(a*
```

```
d^2) + (2*(a^2 - b^2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
b^2])])/(a*b^2*d^3) + (2*(a^2 - b^2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(a*b^2*d^3) + (f^2*PolyLog[3, E^((2*I)*(c + d*x))])/
(2*a*d^3) + (2*f^2*Sin[c + d*x])/(b*d^3) - ((e + f*x)^2*Sin[c + d*x])/(b*d)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\cos^2(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^2\cot(c+dx)\,dx}{a} - \frac{\int (e+fx)^2\cos(c+dx)\,dx}{b} + \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{(e+fx)\ldots}{a+b\,\mathrm{s}\ldots}$$

$$= -\frac{i(e+fx)^3}{3af} - \frac{i\left(a^2-b^2\right)(e+fx)^3}{3ab^2f} - \frac{(e+fx)^2\sin(c+dx)}{bd} - \frac{(2i)\int \frac{e^{2i(c+dx)}(e+f\ldots}{1-e^{2i(c+dx)}}}{a}\ldots$$

$$= -\frac{i(e+fx)^3}{3af} - \frac{i\left(a^2-b^2\right)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cos(c+dx)}{bd^2} + \frac{\left(a^2-b^2\right)(e+\ldots}{\ldots}$$

$$= -\frac{i(e+fx)^3}{3af} - \frac{i\left(a^2-b^2\right)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cos(c+dx)}{bd^2} + \frac{\left(a^2-b^2\right)(e+\ldots}{\ldots}$$

$$= -\frac{i(e+fx)^3}{3af} - \frac{i\left(a^2-b^2\right)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cos(c+dx)}{bd^2} + \frac{\left(a^2-b^2\right)(e+\ldots}{\ldots}$$

$$= -\frac{i(e+fx)^3}{3af} - \frac{i\left(a^2-b^2\right)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cos(c+dx)}{bd^2} + \frac{\left(a^2-b^2\right)(e+\ldots}{\ldots}$$

Mathematica [B] time = 9.36273, size = 1834, normalized size = 3.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(E^(I*c)*f^2*Csc[c]*((2*d^3*x^3)/E^((2*I)*c) + (3*I)*d^2*(1 - E^((-2*I)*c)
)*x^2*Log[1 - E^((-I)*(c + d*x))] + (3*I)*d^2*(1 - E^((-2*I)*c))*x^2*Log[1
+ E^((-I)*(c + d*x))] - (6*(-1 + E^((2*I)*c))*(d*x*PolyLog[2, -E^((-I)*(c +
 d*x))] - I*PolyLog[3, -E^((-I)*(c + d*x))]))/E^((2*I)*c) - (6*(-1 + E^((2*
I)*c))*(d*x*PolyLog[2, E^((-I)*(c + d*x))] - I*PolyLog[3, E^((-I)*(c + d*x)
)]))/E^((2*I)*c)))/(6*a*d^3) + ((a^2 - b^2)*((-12*I)*d^3*e^2*E^((2*I)*c)*x
- (12*I)*d^3*e*E^((2*I)*c)*f*x^2 - (4*I)*d^3*E^((2*I)*c)*f^2*x^3 - (6*I)*d^
2*e^2*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] + (6*I)*
d^2*e^2*E^((2*I)*c)*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x
))))] - 3*d^2*e^2*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c + d
*x)))^2] + 3*d^2*e^2*E^((2*I)*c)*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 +
E^((2*I)*(c + d*x)))^2] - 12*d^2*e*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E
^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*d^2*e*E^((2*I)*c)*f*x*Log[1
+ (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6
*d^2*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)
*E^((2*I)*c)])] + 6*d^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(
I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d^2*e*f*x*Log[1 + (b*E^
(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*d^2*e
*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 +
b^2)*E^((2*I)*c)])] - 6*d^2*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I
*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^2*E^((2*I)*c)*f^2*x^2*Log[1 +
(b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (12
*I)*d*(-1 + E^((2*I)*c))*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*
E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (12*I)*d*(-1 + E^((2*I)*c))*
f*(e + f*x)*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 +
b^2)*E^((2*I)*c)]))] - 12*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c)
 + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*E^((2*I)*c)*f^2*PolyLog[3, (I*b*
E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*f^2
*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*
I)*c)]))] + 12*E^((2*I)*c)*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I
*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]))/(6*a*b^2*d^3*(-1 + E^((2*I)*c)))
+ (a*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Cos[c]*Csc[c/2]*Sec[c/2])/(6*b^2) - (Cos
[d*x]*(2*d*e*f*Cos[c] + 2*d*f^2*x*Cos[c] + d^2*e^2*Sin[c] - 2*f^2*Sin[c] +
2*d^2*e*f*x*Sin[c] + d^2*f^2*x^2*Sin[c]))/(b*d^3) + (e^2*Csc[c]*(-(d*x*Cos[
c]) + Log[Cos[d*x]*Sin[c] + Cos[c]*Sin[d*x]]*Sin[c]))/(a*d*(Cos[c]^2 + Sin[
c]^2)) - ((d^2*e^2*Cos[c] - 2*f^2*Cos[c] + 2*d^2*e*f*x*Cos[c] + d^2*f^2*x^2
*Cos[c] - 2*d*e*f*Sin[c] - 2*d*f^2*x*Sin[c])*Sin[d*x])/(b*d^3) - (e*f*Csc[c
]*Sec[c]*(d^2*E^(I*ArcTan[Tan[c]])*x^2 + ((I*d*x*(-Pi + 2*ArcTan[Tan[c]]) -
 Pi*Log[1 + E^((-2*I)*d*x)] - 2*(d*x + ArcTan[Tan[c]])*Log[1 - E^((2*I)*(d*
x + ArcTan[Tan[c]]))] + Pi*Log[Cos[d*x]] + 2*ArcTan[Tan[c]]*Log[Sin[d*x + A
rcTan[Tan[c]]]] + I*PolyLog[2, E^((2*I)*(d*x + ArcTan[Tan[c]]))])*Tan[c])/S
qrt[1 + Tan[c]^2]))/(a*d^2*Sqrt[Sec[c]^2*(Cos[c]^2 + Sin[c]^2)])
```

Maple [F] time = 3.634, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 (\cos(dx+c))^2 \cot(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.79429, size = 5391, normalized size = 9.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/2*(2*b^2*f^2*polylog(3, cos(d*x + c) + I*sin(d*x + c)) + 2*b^2*f^2*polylo
g(3, cos(d*x + c) - I*sin(d*x + c)) + 2*b^2*f^2*polylog(3, -cos(d*x + c) +
I*sin(d*x + c)) + 2*b^2*f^2*polylog(3, -cos(d*x + c) - I*sin(d*x + c)) + 2*
(a^2 - b^2)*f^2*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(
b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(a^2 - b^
```

2)*f^2*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x
 + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(a^2 - b^2)*f^2*po
lylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 2*(a^2 - b^2)*f^2*polylog(3, -(I*a*cos
(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2))/b) - 4*(a*b*d*f^2*x + a*b*d*e*f)*cos(d*x + c) + (2*I*(a^2 - b
^2)*d*f^2*x + 2*I*(a^2 - b^2)*d*e*f)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*s
in(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
+ 2*b)/b + 1) + (2*I*(a^2 - b^2)*d*f^2*x + 2*I*(a^2 - b^2)*d*e*f)*dilog(-1/
2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x
+ c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-2*I*(a^2 - b^2)*d*f^2*x - 2*
I*(a^2 - b^2)*d*e*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2
*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) +
 (-2*I*(a^2 - b^2)*d*f^2*x - 2*I*(a^2 - b^2)*d*e*f)*dilog(-1/2*(-2*I*a*cos(
d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) + 2*b)/b + 1) + (-2*I*b^2*d*f^2*x - 2*I*b^2*d*e*f)*dilog(co
s(d*x + c) + I*sin(d*x + c)) + (2*I*b^2*d*f^2*x + 2*I*b^2*d*e*f)*dilog(cos(
d*x + c) - I*sin(d*x + c)) + (2*I*b^2*d*f^2*x + 2*I*b^2*d*e*f)*dilog(-cos(d
*x + c) + I*sin(d*x + c)) + (-2*I*b^2*d*f^2*x - 2*I*b^2*d*e*f)*dilog(-cos(d
*x + c) - I*sin(d*x + c)) + ((a^2 - b^2)*d^2*e^2 - 2*(a^2 - b^2)*c*d*e*f +
(a^2 - b^2)*c^2*f^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-
(a^2 - b^2)/b^2) + 2*I*a) + ((a^2 - b^2)*d^2*e^2 - 2*(a^2 - b^2)*c*d*e*f +
(a^2 - b^2)*c^2*f^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-
(a^2 - b^2)/b^2) - 2*I*a) + ((a^2 - b^2)*d^2*e^2 - 2*(a^2 - b^2)*c*d*e*f +
(a^2 - b^2)*c^2*f^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(
-(a^2 - b^2)/b^2) + 2*I*a) + ((a^2 - b^2)*d^2*e^2 - 2*(a^2 - b^2)*c*d*e*f +
 (a^2 - b^2)*c^2*f^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt
(-(a^2 - b^2)/b^2) - 2*I*a) + ((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*
e*f*x + 2*(a^2 - b^2)*c*d*e*f - (a^2 - b^2)*c^2*f^2)*log(1/2*(2*I*a*cos(d*x
 + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) + 2*b)/b) + ((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x
 + 2*(a^2 - b^2)*c*d*e*f - (a^2 - b^2)*c^2*f^2)*log(1/2*(2*I*a*cos(d*x + c)
 + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^
2)/b^2) + 2*b)/b) + ((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x + 2*
(a^2 - b^2)*c*d*e*f - (a^2 - b^2)*c^2*f^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2
*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) + 2*b)/b) + ((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x + 2*(a^2
 - b^2)*c*d*e*f - (a^2 - b^2)*c^2*f^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*s
in(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
+ 2*b)/b) + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + b^2*d^2*e^2)*log(cos(d*x +
 c) + I*sin(d*x + c) + 1) + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + b^2*d^2*e^
2)*log(cos(d*x + c) - I*sin(d*x + c) + 1) + (b^2*d^2*e^2 - 2*b^2*c*d*e*f +
b^2*c^2*f^2)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2) + (b^2*d^2*e
^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c
) + 1/2) + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2

```
)*log(-cos(d*x + c) + I*sin(d*x + c) + 1) + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*
f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*log(-cos(d*x + c) - I*sin(d*x + c) + 1)
- 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + a*b*d^2*e^2 - 2*a*b*f^2)*sin(d*x +
 c))/(a*b^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2 \cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*cos(c + d*x)**2*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.331 $\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=379

$$-\frac{if\left(a^2-b^2\right)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^2}-\frac{if\left(a^2-b^2\right)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^2d^2}-\frac{if\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{2ad^2}+\frac{\left(a^2-b^2\right)(e+f}{}$$

```
[Out] ((-I/2)*(e + f*x)^2)/(a*f) - ((I/2)*(a^2 - b^2)*(e + f*x)^2)/(a*b^2*f) - (f
*Cos[c + d*x])/(b*d^2) + ((a^2 - b^2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)
))/(a - Sqrt[a^2 - b^2])])/(a*b^2*d) + ((a^2 - b^2)*(e + f*x)*Log[1 - (I*b*
E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^2*d) + ((e + f*x)*Log[1 - E^(
(2*I)*(c + d*x))])/(a*d) - (I*(a^2 - b^2)*f*PolyLog[2, (I*b*E^(I*(c + d*x))
)/(a - Sqrt[a^2 - b^2])])/(a*b^2*d^2) - (I*(a^2 - b^2)*f*PolyLog[2, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^2*d^2) - ((I/2)*f*PolyLog[2, E^
((2*I)*(c + d*x))])/(a*d^2) - ((e + f*x)*Sin[c + d*x])/(b*d)
```

Rubi [A] time = 0.631195, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {4543, 4408, 4404, 2635, 8, 3717, 2190, 2279, 2391, 4525, 3296, 2638, 4519}

$$-\frac{if\left(a^2-b^2\right)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^2d^2}-\frac{if\left(a^2-b^2\right)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^2d^2}-\frac{if\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{2ad^2}+\frac{\left(a^2-b^2\right)(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```


```
[Out] ((-I/2)*(e + f*x)^2)/(a*f) - ((I/2)*(a^2 - b^2)*(e + f*x)^2)/(a*b^2*f) - (f
*Cos[c + d*x])/(b*d^2) + ((a^2 - b^2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)
))/(a - Sqrt[a^2 - b^2])])/(a*b^2*d) + ((a^2 - b^2)*(e + f*x)*Log[1 - (I*b*
E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^2*d) + ((e + f*x)*Log[1 - E^(
(2*I)*(c + d*x))])/(a*d) - (I*(a^2 - b^2)*f*PolyLog[2, (I*b*E^(I*(c + d*x))
)/(a - Sqrt[a^2 - b^2])])/(a*b^2*d^2) - (I*(a^2 - b^2)*f*PolyLog[2, (I*b*E^
(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^2*d^2) - ((I/2)*f*PolyLog[2, E^
((2*I)*(c + d*x))])/(a*d^2) - ((e + f*x)*Sin[c + d*x])/(b*d)
```


Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rubi steps

$$\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\cos^2(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)\cot(c+dx)\,dx}{a} - \frac{\int (e+fx)\cos(c+dx)\,dx}{b} + \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{(e+fx)\cos}{a+b\sin(}$$

$$= -\frac{i(e+fx)^2}{2af} - \frac{i\left(a^2-b^2\right)(e+fx)^2}{2ab^2f} - \frac{(e+fx)\sin(c+dx)}{bd} - \frac{(2i)\int \frac{e^{2i(c+dx)}(e+fx)}{1-e^{2i(c+dx)}}}{a}$$

$$= -\frac{i(e+fx)^2}{2af} - \frac{i\left(a^2-b^2\right)(e+fx)^2}{2ab^2f} - \frac{f\cos(c+dx)}{bd^2} + \frac{\left(a^2-b^2\right)(e+fx)\log\left(1\right.}{ab^2d}$$

$$= -\frac{i(e+fx)^2}{2af} - \frac{i\left(a^2-b^2\right)(e+fx)^2}{2ab^2f} - \frac{f\cos(c+dx)}{bd^2} + \frac{\left(a^2-b^2\right)(e+fx)\log\left(1\right.}{ab^2d}$$

$$= -\frac{i(e+fx)^2}{2af} - \frac{i\left(a^2-b^2\right)(e+fx)^2}{2ab^2f} - \frac{f\cos(c+dx)}{bd^2} + \frac{\left(a^2-b^2\right)(e+fx)\log\left(1\right.}{ab^2d}$$

Mathematica [B] time = 14.8322, size = 2209, normalized size = 5.83

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((f*Cos[c + d*x])/(b*d^2)) + (e*Log[Sin[c + d*x]])/(a*d) - (c*f*Log[Sin[c
+ d*x]])/(a*d^2) + (f*((c + d*x)*Log[1 - E^((2*I)*(c + d*x))] - (I/2)*((c +
 d*x)^2 + PolyLog[2, E^((2*I)*(c + d*x))])))/(a*d^2) - ((d*e - c*f + f*(c +
 d*x))*Sin[c + d*x])/(b*d^2) + ((f*(c + d*x)^2 + (2*I)*d*e*Log[Sec[(c + d*x
)/2]^2] - (2*I)*c*f*Log[Sec[(c + d*x)/2]^2] - (2*I)*d*e*Log[Sec[(c + d*x)/2
]^2*(a + b*Sin[c + d*x])] + (2*I)*c*f*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c +
 d*x])] - (4*I)*f*(c + d*x)*Log[(-2*I)/(-I + Tan[(c + d*x)/2])] - 2*f*Log[1
 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a
 + b - Sqrt[-a^2 + b^2])] + 2*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt
[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))] + 2*f*Log
[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((
-I)*a + b + Sqrt[-a^2 + b^2])] - 2*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + S
qrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + 4*f*P
olyLog[2, -Cos[c + d*x] + I*Sin[c + d*x]] + 2*f*PolyLog[2, (a*(1 - I*Tan[(c
 + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))] - 2*f*PolyLog[2, (a*(1 + I*Tan
```

```
[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))] + 2*f*PolyLog[2, (a*(I + Ta
n[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])] - 2*f*PolyLog[2, (a + I*a*Ta
n[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]])*((a*e*Cos[c + d*x])/(b*(a
 + b*Sin[c + d*x])) - (b*e*Cos[c + d*x])/(a*(a + b*Sin[c + d*x])) - (a*c*f*
Cos[c + d*x])/(b*d*(a + b*Sin[c + d*x])) + (b*c*f*Cos[c + d*x])/(a*d*(a + b
*Sin[c + d*x])) + (a*f*(c + d*x)*Cos[c + d*x])/(b*d*(a + b*Sin[c + d*x])) -
 (b*f*(c + d*x)*Cos[c + d*x])/(a*d*(a + b*Sin[c + d*x]))))/(d*(2*f*(c + d*x
) - (4*I)*f*Log[(-2*I)/(-I + Tan[(c + d*x)/2])] - (4*f*Log[1 + Cos[c + d*x]
 - I*Sin[c + d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))/(-Cos[c + d*x] + I*Sin[
c + d*x]) + (I*f*Log[1 - (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2
 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) - (I*f*Log[-((b - S
qrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))]*Sec[(c
 + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] + a
*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1
- I*Tan[(c + d*x)/2]) + (I*f*Log[1 - (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b
 + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*
Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2]
)]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 +
 b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^
2)/(1 + I*Tan[(c + d*x)/2]) + (2*I)*d*e*Tan[(c + d*x)/2] - (2*I)*c*f*Tan[(c
 + d*x)/2] + ((2*I)*f*(c + d*x)*Sec[(c + d*x)/2]^2)/(-I + Tan[(c + d*x)/2])
 - (f*Log[1 - (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]*Sec[
(c + d*x)/2]^2)/(I + Tan[(c + d*x)/2]) + (I*a*f*Log[1 - (a + I*a*Tan[(c + d
*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(a + I*a*Tan[(
c + d*x)/2]) + (a*f*Log[1 - I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b - Sq
rt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) - (a*f*Log[1 + I*Tan[(c + d*x)/2]]*Sec
[(c + d*x)/2]^2)/(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) + (a*f*Log[1 -
 I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[-a^2 + b^2] + a*Tan[(c +
 d*x)/2]) - (a*f*Log[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[
-a^2 + b^2] + a*Tan[(c + d*x)/2]) - ((2*I)*d*e*Cos[(c + d*x)/2]^2*(b*Cos[c
+ d*x]*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c
+ d*x)/2]))/(a + b*Sin[c + d*x]) + ((2*I)*c*f*Cos[(c + d*x)/2]^2*(b*Cos[c +
 d*x]*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c +
 d*x)/2]))/(a + b*Sin[c + d*x])))
```

Maple [B] time = 1.253, size = 1721, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] I*a/b^2*e*x+2/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*
a+(-a^2+b^2)^(1/2)))*a*x+2/d^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+
b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a*c+2/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d
*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*a*x+2/d^2*f/(-a^2+b^2)*ln(
(I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*a*c+2/b^2/d
^2*a*f*c*ln(exp(I*(d*x+c)))-1/b^2/d^2*a*f*c*ln(I*b*exp(2*I*(d*x+c))-2*a*exp
(I*(d*x+c))-I*b)-I/b^2/d^2*a*f*c^2-2*I/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*
(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*a-2*I/d^2*f/(-a^2+b^2)*d
ilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a+1/d^
2*f*c/a*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+1/b^2/d*a*e*ln(I*b*
exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-2/b^2/d*a*e*ln(exp(I*(d*x+c)))-I/d
^2*f/a*dilog(exp(I*(d*x+c))+1)+1/2*I*(d*f*x+I*f+d*e)/b/d^2*exp(I*(d*x+c))-1
/2*I*a/b^2*f*x^2+I/d^2*f/a*dilog(exp(I*(d*x+c)))-1/d^2/a*f*c*ln(exp(I*(d*x+
c))-1)+1/d/a*ln(exp(I*(d*x+c))+1)*f*x+1/d/a*e*ln(exp(I*(d*x+c))-1)+1/d/a*e*
ln(exp(I*(d*x+c))+1)-b^2/d*f/a/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^
2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-b^2/d^2*f/a/(-a^2+b^2)*ln((I*a+b*exp(I*
(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-b^2/d*f/a/(-a^2+b^2)*l
n((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-b^2/d^2
*f/a/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^
(1/2)))*c-1/b^2/d*a^3*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2
))/(I*a-(-a^2+b^2)^(1/2)))*x-1/b^2/d^2*a^3*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*
x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-1/b^2/d*a^3*f/(-a^2+b^2)*
ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-1/d*e/
a*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-1/2*I*(d*f*x-I*f+d*e)/b/d
^2*exp(-I*(d*x+c))-1/b^2/d^2*a^3*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^
2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+I/b^2/d^2*a^3*f/(-a^2+b^2)*dilog((I
*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))+I/b^2/d^2*a^3
*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)
^(1/2)))+I*b^2/d^2*f/a/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1
/2))/(I*a-(-a^2+b^2)^(1/2)))+I*b^2/d^2*f/a/(-a^2+b^2)*dilog((I*a+b*exp(I*(d
*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2*I/b^2/d*a*f*c*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.54769, size = 3194, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*a*b*f*cos(d*x + c) + I*b^2*f*dilog(cos(d*x + c) + I*sin(d*x + c)) -
 I*b^2*f*dilog(cos(d*x + c) - I*sin(d*x + c)) - I*b^2*f*dilog(-cos(d*x + c)
 + I*sin(d*x + c)) + I*b^2*f*dilog(-cos(d*x + c) - I*sin(d*x + c)) - I*(a^2
 - b^2)*f*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x
+ c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - I*(a^2 - b^
2)*f*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c)
- I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + I*(a^2 - b^2)*f*
dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*
b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + I*(a^2 - b^2)*f*dilo
g(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - ((a^2 - b^2)*d*e - (a^2
- b^2)*c*f)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^
2)/b^2) + 2*I*a) - ((a^2 - b^2)*d*e - (a^2 - b^2)*c*f)*log(2*b*cos(d*x + c)
 - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - ((a^2 - b^2)*
d*e - (a^2 - b^2)*c*f)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqr
t(-(a^2 - b^2)/b^2) + 2*I*a) - ((a^2 - b^2)*d*e - (a^2 - b^2)*c*f)*log(-2*b
*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) -
((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin
(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) +
2*b)/b) - ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(1/2*(2*I*a*cos(d*x + c)
 + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^
2)/b^2) + 2*b)/b) - ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(1/2*(-2*I*a*c
os(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2) + 2*b)/b) - ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(1/
2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - (b^2*d*f*x + b^2*d*e)*log(cos(d*x
 + c) + I*sin(d*x + c) + 1) - (b^2*d*f*x + b^2*d*e)*log(cos(d*x + c) - I*si
n(d*x + c) + 1) - (b^2*d*e - b^2*c*f)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x
 + c) + 1/2) - (b^2*d*e - b^2*c*f)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x +
c) + 1/2) - (b^2*d*f*x + b^2*c*f)*log(-cos(d*x + c) + I*sin(d*x + c) + 1) -
 (b^2*d*f*x + b^2*c*f)*log(-cos(d*x + c) - I*sin(d*x + c) + 1) + 2*(a*b*d*f
*x + a*b*d*e)*sin(d*x + c))/(a*b^2*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)**2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cos(c + d*x)**2*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cos(dx+c)^2\cot(dx+c)}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((f*x + e)*cos(d*x + c)^2*cot(d*x + c)/(b*sin(d*x + c) + a), x)
```

3.332 $\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{ab^2d}+\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{bd}$$

```
[Out] Log[Sin[c + d*x]]/(a*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a*b^2*d) -
 Sin[c + d*x]/(b*d)
```

Rubi [A] time = 0.107853, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{ab^2d}+\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a*b^2*d) -
 Sin[c + d*x]/(b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{b\left(b^2-x^2\right)}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3 d}\\
&= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^2 d}\\
&= \frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{ax}+\frac{a^2-b^2}{a(a+x)}\right)dx, x, b\sin(c+dx)\right)}{b^2 d}\\
&= \frac{\log(\sin(c+dx))}{ad}+\frac{\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{ab^2 d}-\frac{\sin(c+dx)}{bd}
\end{aligned}$$

Mathematica [A] time = 0.0751889, size = 53, normalized size = 0.9

$$\frac{\left(a^2-b^2\right)\log(a+b\sin(c+dx))-ab\sin(c+dx)+b^2\log(\sin(c+dx))}{ab^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^2*Log[Sin[c + d*x]] + (a^2 - b^2)*Log[a + b*Sin[c + d*x]] - a*b*Sin[c +
d*x])/(a*b^2*d)
```

Maple [A] time = 0.073, size = 68, normalized size = 1.2

$$-\frac{\sin(dx+c)}{bd}+\frac{a\ln(a+b\sin(dx+c))}{b^2 d}-\frac{\ln(a+b\sin(dx+c))}{da}+\frac{\ln(\sin(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -sin(d*x+c)/b/d+1/d/b^2*a*ln(a+b*sin(d*x+c))-1/d/a*ln(a+b*sin(d*x+c))+ln(si
n(d*x+c))/a/d
```

Maxima [A] time = 0.96684, size = 73, normalized size = 1.24

$$\frac{\frac{\log(\sin(dx+c))}{a} - \frac{\sin(dx+c)}{b} + \frac{(a^2-b^2)\log(b\sin(dx+c)+a)}{ab^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c))/a - sin(d*x + c)/b + (a^2 - b^2)*log(b*sin(d*x + c) + a)
/(a*b^2))/d
```

Fricas [A] time = 1.96669, size = 131, normalized size = 2.22

$$\frac{b^2\log\left(-\frac{1}{2}\sin(dx+c)\right) - ab\sin(dx+c) + \left(a^2-b^2\right)\log(b\sin(dx+c)+a)}{ab^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (b^2*log(-1/2*sin(d*x + c)) - a*b*sin(d*x + c) + (a^2 - b^2)*log(b*sin(d*x
+ c) + a))/(a*b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.25766, size = 76, normalized size = 1.29

$$\frac{\frac{\log(|\sin(dx+c)|)}{a} - \frac{\sin(dx+c)}{b} + \frac{\left(a^2-b^2\right)\log(|b\sin(dx+c)+a|)}{ab^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(sin(d*x + c)))/a - sin(d*x + c)/b + (a^2 - b^2)*log(abs(b*sin(d*x
+ c) + a))/(a*b^2))/d
```

3.333 $\int \frac{(e+fx)^3 \cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=1138

result too large to display

```
[Out] (3*e*f^2*x)/(4*b*d^2) + (3*f^3*x^2)/(8*b*d^2) - (e + f*x)^4/(8*b*f) + ((a^2
 - b^2)*(e + f*x)^4)/(4*b^3*f) - (2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(
a*d) - (6*f^2*(e + f*x)*Cos[c + d*x])/(a*d^3) - (6*(a^2 - b^2)*f^2*(e + f*x
)*Cos[c + d*x])/(a*b^2*d^3) + ((e + f*x)^3*Cos[c + d*x])/(a*d) + ((a^2 - b^
2)*(e + f*x)^3*Cos[c + d*x])/(a*b^2*d) + (3*f^3*Cos[c + d*x]^2)/(8*b*d^4) -
 (3*f*(e + f*x)^2*Cos[c + d*x]^2)/(4*b*d^2) + (I*(a^2 - b^2)^(3/2)*(e + f*x
)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d) - (I*(a
^2 - b^2)^(3/2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b
^2])])/(a*b^3*d) + ((3*I)*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))])/(a*d^
2) - ((3*I)*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) + (3*(a^2 -
b^2)^(3/2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b
^2])])/(a*b^3*d^2) - (3*(a^2 - b^2)^(3/2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^3*d^2) - (6*f^2*(e + f*x)*PolyLo
g[3, -E^(I*(c + d*x))])/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[3, E^(I*(c + d*x
))])/(a*d^3) + ((6*I)*(a^2 - b^2)^(3/2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*
(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d^3) - ((6*I)*(a^2 - b^2)^(3/2)*
f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b
^3*d^3) - ((6*I)*f^3*PolyLog[4, -E^(I*(c + d*x))])/(a*d^4) + ((6*I)*f^3*Pol
yLog[4, E^(I*(c + d*x))])/(a*d^4) - (6*(a^2 - b^2)^(3/2)*f^3*PolyLog[4, (I*
b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d^4) + (6*(a^2 - b^2)^(3/
2)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^3*d^4)
 + (6*f^3*Sin[c + d*x])/(a*d^4) + (6*(a^2 - b^2)*f^3*Sin[c + d*x])/(a*b^2*d
^4) - (3*f*(e + f*x)^2*Sin[c + d*x])/(a*d^2) - (3*(a^2 - b^2)*f*(e + f*x)^2
*Sin[c + d*x])/(a*b^2*d^2) + (3*f^2*(e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(4
*b*d^3) - ((e + f*x)^3*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 2.10748, antiderivative size = 1138, normalized size of antiderivative = 1., number of steps used = 53, number of rules used = 18, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {4543, 4408, 4405, 3311, 3296, 2637, 2633, 4183, 2531, 6609, 2282, 6589, 4525, 32, 3310, 3323, 2264, 2190}

$$\frac{\left(a^2-b^2\right)(e+fx)^4}{4b^3f}-\frac{(e+fx)^4}{8bf}-\frac{2\tanh^{-1}\left(e^{i(c+dx)}\right)(e+fx)^3}{ad}+\frac{\left(a^2-b^2\right)\cos(c+dx)(e+fx)^3}{ab^2d}+\frac{\cos(c+dx)(e+fx)^3}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*e*f^2*x)/(4*b*d^2) + (3*f^3*x^2)/(8*b*d^2) - (e + f*x)^4/(8*b*f) + ((a^2
 - b^2)*(e + f*x)^4)/(4*b^3*f) - (2*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(
a*d) - (6*f^2*(e + f*x)*Cos[c + d*x])/(a*d^3) - (6*(a^2 - b^2)*f^2*(e + f*x
)*Cos[c + d*x])/(a*b^2*d^3) + ((e + f*x)^3*Cos[c + d*x])/(a*d) + ((a^2 - b^
2)*(e + f*x)^3*Cos[c + d*x])/(a*b^2*d) + (3*f^3*Cos[c + d*x]^2)/(8*b*d^4) -
 (3*f*(e + f*x)^2*Cos[c + d*x]^2)/(4*b*d^2) + (I*(a^2 - b^2)^(3/2)*(e + f*x
)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d) - (I*(a
^2 - b^2)^(3/2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b
^2])])/(a*b^3*d) + ((3*I)*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*x))])/(a*d^
2) - ((3*I)*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) + (3*(a^2 -
b^2)^(3/2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b
^2])])/(a*b^3*d^2) - (3*(a^2 - b^2)^(3/2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(
I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^3*d^2) - (6*f^2*(e + f*x)*PolyLo
g[3, -E^(I*(c + d*x))])/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[3, E^(I*(c + d*x
))])/(a*d^3) + ((6*I)*(a^2 - b^2)^(3/2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*
(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d^3) - ((6*I)*(a^2 - b^2)^(3/2)*
f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b
^3*d^3) - ((6*I)*f^3*PolyLog[4, -E^(I*(c + d*x))])/(a*d^4) + ((6*I)*f^3*Pol
yLog[4, E^(I*(c + d*x))])/(a*d^4) - (6*(a^2 - b^2)^(3/2)*f^3*PolyLog[4, (I*
b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d^4) + (6*(a^2 - b^2)^(3/
2)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^3*d^4)
 + (6*f^3*Sin[c + d*x])/(a*d^4) + (6*(a^2 - b^2)*f^3*Sin[c + d*x])/(a*b^2*d
^4) - (3*f*(e + f*x)^2*Sin[c + d*x])/(a*d^2) - (3*(a^2 - b^2)*f*(e + f*x)^2
*Sin[c + d*x])/(a*b^2*d^2) + (3*f^2*(e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(4
*b*d^3) - ((e + f*x)^3*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
```

```
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\cos^3(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cos^4(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^3\cos(c+dx)\cot(c+dx)\,dx}{a} - \frac{\int (e+fx)^3\cos^2(c+dx)\,dx}{b} + \left(\frac{a}{b} - \frac{b}{a}\right.$$

$$= -\frac{3f(e+fx)^2\cos^2(c+dx)}{4bd^2} - \frac{(e+fx)^3\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int (e+fx)^3 \mathrm{c}}{}$$

$$= -\frac{(e+fx)^4}{8bf} + \frac{(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^3\mathrm{co}}{ad}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} - \frac{(e+fx)^4}{8bf} + \frac{(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx}\right.}{ad}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} - \frac{(e+fx)^4}{8bf} + \frac{(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx}\right.}{ad}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} - \frac{(e+fx)^4}{8bf} + \frac{(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx}\right.}{ad}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} - \frac{(e+fx)^4}{8bf} + \frac{(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx}\right.}{ad}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} - \frac{(e+fx)^4}{8bf} + \frac{(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx}\right.}{ad}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} - \frac{(e+fx)^4}{8bf} + \frac{(a^2-b^2)(e+fx)^4}{4b^3f} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{i(c+dx}\right.}{ad}$$

Mathematica [A] time = 6.63455, size = 1181, normalized size = 1.04

$$\frac{2a(2a^2-3b^2)f^3x^4d^4 + 8a(2a^2-3b^2)ef^2x^3d^4 + 12a(2a^2-3b^2)e^2fx^2d^4 + 8a(2a^2-3b^2)e^3xd^4 - 32b^3(e+fx)^3\tanh^{-1}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (8*a*(2*a^2 - 3*b^2)*d^4*e^3*x + 12*a*(2*a^2 - 3*b^2)*d^4*e^2*f*x^2 + 8*a*(
2*a^2 - 3*b^2)*d^4*e*f^2*x^3 + 2*a*(2*a^2 - 3*b^2)*d^4*f^3*x^4 - 32*b^3*d^3
```

```
*(e + f*x)^3*ArcTanh[Cos[c + d*x] + I*Sin[c + d*x]] - 96*a^2*b*d*f^2*(e + f
*x)*Cos[c + d*x] + 16*a^2*b*d^3*(e + f*x)^3*Cos[c + d*x] + 3*a*b^2*f^3*Cos[
2*(c + d*x)] - 6*a*b^2*d^2*f*(e + f*x)^2*Cos[2*(c + d*x)] + 48*(a^2 - b^2)^
(3/2)*d^2*f*(e + f*x)^2*PolyLog[2, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2
- b^2])] + (16*I)*(a^2 - b^2)^(3/2)*((2*I)*d^3*e^3*ArcTan[(I*a + b*E^(I*(c
+ d*x)))/Sqrt[a^2 - b^2]] + 3*d^3*e^2*f*x*Log[1 + (I*b*E^(I*(c + d*x)))/(-a
 + Sqrt[a^2 - b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (I*b*E^(I*(c + d*x)))/(-a +
Sqrt[a^2 - b^2])] + d^3*f^3*x^3*Log[1 + (I*b*E^(I*(c + d*x)))/(-a + Sqrt[a^
2 - b^2])] - 3*d^3*e^2*f*x*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^
2])] - 3*d^3*e*f^2*x^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]
 - d^3*f^3*x^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] + (3*I)
*d^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]
+ 6*d*f^2*(e + f*x)*PolyLog[3, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^
2])] - 6*d*e*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] -
6*d*f^3*x*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] + (6*I)*f
^3*PolyLog[4, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] - (6*I)*f^3*
PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]) + (48*I)*b^3*f*(d^
2*(e + f*x)^2*PolyLog[2, -Cos[c + d*x] - I*Sin[c + d*x]] + (2*I)*d*f*(e + f
*x)*PolyLog[3, -Cos[c + d*x] - I*Sin[c + d*x]] - 2*f^2*PolyLog[4, -Cos[c +
d*x] - I*Sin[c + d*x]]) - (48*I)*b^3*f*(d^2*(e + f*x)^2*PolyLog[2, Cos[c +
d*x] + I*Sin[c + d*x]] + (2*I)*d*f*(e + f*x)*PolyLog[3, Cos[c + d*x] + I*Si
n[c + d*x]] - 2*f^2*PolyLog[4, Cos[c + d*x] + I*Sin[c + d*x]]) + 96*a^2*b*f
^3*Sin[c + d*x] - 48*a^2*b*d^2*f*(e + f*x)^2*Sin[c + d*x] + 6*a*b^2*d*f^2*(
e + f*x)*Sin[2*(c + d*x)] - 4*a*b^2*d^3*(e + f*x)^3*Sin[2*(c + d*x)])/(16*a
*b^3*d^4)
```

Maple [F] time = 3.191, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\cos(dx+c))^3 \cot(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.69813, size = 9441, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/8*((2*a^3 - 3*a*b^2)*d^4*f^3*x^4 + 4*(2*a^3 - 3*a*b^2)*d^4*e*f^2*x^3 + 24
*I*b^3*f^3*polylog(4, cos(d*x + c) + I*sin(d*x + c)) - 24*I*b^3*f^3*polylog
(4, cos(d*x + c) - I*sin(d*x + c)) + 24*I*b^3*f^3*polylog(4, -cos(d*x + c)
+ I*sin(d*x + c)) - 24*I*b^3*f^3*polylog(4, -cos(d*x + c) - I*sin(d*x + c))
 + 24*I*(a^2*b - b^3)*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(
d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2))/b) - 24*I*(a^2*b - b^3)*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog
(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*si
n(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 24*I*(a^2*b - b^3)*f^3*sqrt(-(a^2
- b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c
) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 24*I*(a^2*b - b^3)*f^3*s
qrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*c
os(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 3*(2*(2*a^3 -
3*a*b^2)*d^4*e^2*f + a*b^2*d^2*f^3)*x^2 - 3*(2*a*b^2*d^2*f^3*x^2 + 4*a*b^2*
d^2*e*f^2*x + 2*a*b^2*d^2*e^2*f - a*b^2*f^3)*cos(d*x + c)^2 + 2*(-6*I*(a^2*
b - b^3)*d^2*f^3*x^2 - 12*I*(a^2*b - b^3)*d^2*e*f^2*x - 6*I*(a^2*b - b^3)*d
^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d
*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*
b)/b + 1) + 2*(6*I*(a^2*b - b^3)*d^2*f^3*x^2 + 12*I*(a^2*b - b^3)*d^2*e*f^2
*x + 6*I*(a^2*b - b^3)*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*
cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(6*I*(a^2*b - b^3)*d^2*f^3*x^2 + 12*I
*(a^2*b - b^3)*d^2*e*f^2*x + 6*I*(a^2*b - b^3)*d^2*e^2*f)*sqrt(-(a^2 - b^2)
/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c
) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(-6*I*(a^2*b
 - b^3)*d^2*f^3*x^2 - 12*I*(a^2*b - b^3)*d^2*e*f^2*x - 6*I*(a^2*b - b^3)*d^
```

2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d
x + c) - 2(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*
b)/b + 1) - 4*((a^2*b - b^3)*d^3*e^3 - 3*(a^2*b - b^3)*c*d^2*e^2*f + 3*(a^2
*b - b^3)*c^2*d*e*f^2 - (a^2*b - b^3)*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(2
*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)
- 4*((a^2*b - b^3)*d^3*e^3 - 3*(a^2*b - b^3)*c*d^2*e^2*f + 3*(a^2*b - b^3)*
c^2*d*e*f^2 - (a^2*b - b^3)*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x
 + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 4*((a^2*
b - b^3)*d^3*e^3 - 3*(a^2*b - b^3)*c*d^2*e^2*f + 3*(a^2*b - b^3)*c^2*d*e*f^
2 - (a^2*b - b^3)*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2
*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 4*((a^2*b - b^3)*
d^3*e^3 - 3*(a^2*b - b^3)*c*d^2*e^2*f + 3*(a^2*b - b^3)*c^2*d*e*f^2 - (a^2*
b - b^3)*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(
d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 4*((a^2*b - b^3)*d^3*f^3*x
^3 + 3*(a^2*b - b^3)*d^3*e*f^2*x^2 + 3*(a^2*b - b^3)*d^3*e^2*f*x + 3*(a^2*b
 - b^3)*c*d^2*e^2*f - 3*(a^2*b - b^3)*c^2*d*e*f^2 + (a^2*b - b^3)*c^3*f^3)*
sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(
b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 4*((a
^2*b - b^3)*d^3*f^3*x^3 + 3*(a^2*b - b^3)*d^3*e*f^2*x^2 + 3*(a^2*b - b^3)*d
^3*e^2*f*x + 3*(a^2*b - b^3)*c*d^2*e^2*f - 3*(a^2*b - b^3)*c^2*d*e*f^2 + (a
^2*b - b^3)*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2
*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) + 2*b)/b) - 4*((a^2*b - b^3)*d^3*f^3*x^3 + 3*(a^2*b - b^3)*d^3*e*f^2*x^
2 + 3*(a^2*b - b^3)*d^3*e^2*f*x + 3*(a^2*b - b^3)*c*d^2*e^2*f - 3*(a^2*b -
b^3)*c^2*d*e*f^2 + (a^2*b - b^3)*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-
2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 4*((a^2*b - b^3)*d^3*f^3*x^3 + 3*(a^2
*b - b^3)*d^3*e*f^2*x^2 + 3*(a^2*b - b^3)*d^3*e^2*f*x + 3*(a^2*b - b^3)*c*d
^2*e^2*f - 3*(a^2*b - b^3)*c^2*d*e*f^2 + (a^2*b - b^3)*c^3*f^3)*sqrt(-(a^2
- b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x
+ c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 24*((a^2*b - b^
3)*d*f^3*x + (a^2*b - b^3)*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(
2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt(-(a^2 - b^2)/b^2))/b) - 24*((a^2*b - b^3)*d*f^3*x + (a^2*b - b^3)*d
*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin
(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b
) - 24*((a^2*b - b^3)*d*f^3*x + (a^2*b - b^3)*d*e*f^2)*sqrt(-(a^2 - b^2)/b^
2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 24*((a^2*b - b^3)*d*f^3*x + (a^2*
b - b^3)*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*
sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/
b) + 2*(2*(2*a^3 - 3*a*b^2)*d^4*e^3 + 3*a*b^2*d^2*e*f^2)*x + 8*(a^2*b*d^3*f
^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + a^2*b*d^3*e^3 - 6*a^2*b*d*e*f^2 + 3*(a^2*b
*d^3*e^2*f - 2*a^2*b*d*f^3)*x)*cos(d*x + c) + (-12*I*b^3*d^2*f^3*x^2 - 24*I
*b^3*d^2*e*f^2*x - 12*I*b^3*d^2*e^2*f)*dilog(cos(d*x + c) + I*sin(d*x + c))

```
 + (12*I*b^3*d^2*f^3*x^2 + 24*I*b^3*d^2*e*f^2*x + 12*I*b^3*d^2*e^2*f)*dilog
(cos(d*x + c) - I*sin(d*x + c)) + (-12*I*b^3*d^2*f^3*x^2 - 24*I*b^3*d^2*e*f
^2*x - 12*I*b^3*d^2*e^2*f)*dilog(-cos(d*x + c) + I*sin(d*x + c)) + (12*I*b^
3*d^2*f^3*x^2 + 24*I*b^3*d^2*e*f^2*x + 12*I*b^3*d^2*e^2*f)*dilog(-cos(d*x +
 c) - I*sin(d*x + c)) - 4*(b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^
3*e^2*f*x + b^3*d^3*e^3)*log(cos(d*x + c) + I*sin(d*x + c) + 1) - 4*(b^3*d^
3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + b^3*d^3*e^3)*log(cos(
d*x + c) - I*sin(d*x + c) + 1) + 4*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3
*c^2*d*e*f^2 - b^3*c^3*f^3)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/
2) + 4*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*
log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) + 4*(b^3*d^3*f^3*x^3 + 3*
b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f
^2 + b^3*c^3*f^3)*log(-cos(d*x + c) + I*sin(d*x + c) + 1) + 4*(b^3*d^3*f^3*
x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c
^2*d*e*f^2 + b^3*c^3*f^3)*log(-cos(d*x + c) - I*sin(d*x + c) + 1) + 24*(b^3
*d*f^3*x + b^3*d*e*f^2)*polylog(3, cos(d*x + c) + I*sin(d*x + c)) + 24*(b^3
*d*f^3*x + b^3*d*e*f^2)*polylog(3, cos(d*x + c) - I*sin(d*x + c)) - 24*(b^3
*d*f^3*x + b^3*d*e*f^2)*polylog(3, -cos(d*x + c) + I*sin(d*x + c)) - 24*(b^
3*d*f^3*x + b^3*d*e*f^2)*polylog(3, -cos(d*x + c) - I*sin(d*x + c)) - 2*(12
*a^2*b*d^2*f^3*x^2 + 24*a^2*b*d^2*e*f^2*x + 12*a^2*b*d^2*e^2*f - 24*a^2*b*f
^3 + (2*a*b^2*d^3*f^3*x^3 + 6*a*b^2*d^3*e*f^2*x^2 + 2*a*b^2*d^3*e^3 - 3*a*b
^2*d*e*f^2 + 3*(2*a*b^2*d^3*e^2*f - a*b^2*d*f^3)*x)*cos(d*x + c))*sin(d*x +
 c))/(a*b^3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.334 $\int \frac{(e+fx)^2 \cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=825

$$\frac{(a^2-b^2)(e+fx)^3}{3b^3f}-\frac{(e+fx)^3}{6bf}-\frac{2\tanh^{-1}\left(e^{i(c+dx)}\right)(e+fx)^2}{ad}+\frac{(a^2-b^2)\cos(c+dx)(e+fx)^2}{ab^2d}+\frac{\cos(c+dx)(e+fx)^2}{ad}+$$

```
[Out] (f^2*x)/(4*b*d^2) - (e + f*x)^3/(6*b*f) + ((a^2 - b^2)*(e + f*x)^3)/(3*b^3*
f) - (2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d) - (2*f^2*Cos[c + d*x])/
(a*d^3) - (2*(a^2 - b^2)*f^2*Cos[c + d*x])/(a*b^2*d^3) + ((e + f*x)^2*Cos[c
 + d*x])/(a*d) + ((a^2 - b^2)*(e + f*x)^2*Cos[c + d*x])/(a*b^2*d) - (f*(e +
 f*x)*Cos[c + d*x]^2)/(2*b*d^2) + (I*(a^2 - b^2)^(3/2)*(e + f*x)^2*Log[1 -
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d) - (I*(a^2 - b^2)^(3
/2)*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^
3*d) + ((2*I)*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2) - ((2*I)*f*
(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) + (2*(a^2 - b^2)^(3/2)*f*(e
+ f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d^2)
 - (2*(a^2 - b^2)^(3/2)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + S
qrt[a^2 - b^2])])/(a*b^3*d^2) - (2*f^2*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3
) + (2*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3) + ((2*I)*(a^2 - b^2)^(3/2)*
f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d^3) -
((2*I)*(a^2 - b^2)^(3/2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/(a*b^3*d^3) - (2*f*(e + f*x)*Sin[c + d*x])/(a*d^2) - (2*(a^2 - b
^2)*f*(e + f*x)*Sin[c + d*x])/(a*b^2*d^2) + (f^2*Cos[c + d*x]*Sin[c + d*x])
/(4*b*d^3) - ((e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 1.6327, antiderivative size = 825, normalized size of antiderivative = 1., number of steps used = 41, number of rules used = 18, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {4543, 4408, 4405, 3310, 3296, 2638, 4183, 2531, 2282, 6589, 4525, 3311, 32, 2635, 8, 3323, 2264, 2190}

$$\frac{(a^2-b^2)(e+fx)^3}{3b^3f}-\frac{(e+fx)^3}{6bf}-\frac{2\tanh^{-1}\left(e^{i(c+dx)}\right)(e+fx)^2}{ad}+\frac{(a^2-b^2)\cos(c+dx)(e+fx)^2}{ab^2d}+\frac{\cos(c+dx)(e+fx)^2}{ad}+$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (f^2*x)/(4*b*d^2) - (e + f*x)^3/(6*b*f) + ((a^2 - b^2)*(e + f*x)^3)/(3*b^3*
f) - (2*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d) - (2*f^2*Cos[c + d*x])/
```

```
(a*d^3) - (2*(a^2 - b^2)*f^2*Cos[c + d*x])/(a*b^2*d^3) + ((e + f*x)^2*Cos[c
 + d*x])/(a*d) + ((a^2 - b^2)*(e + f*x)^2*Cos[c + d*x])/(a*b^2*d) - (f*(e +
 f*x)*Cos[c + d*x]^2)/(2*b*d^2) + (I*(a^2 - b^2)^(3/2)*(e + f*x)^2*Log[1 -
(I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d) - (I*(a^2 - b^2)^(3
/2)*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^
3*d) + ((2*I)*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a*d^2) - ((2*I)*f*
(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^2) + (2*(a^2 - b^2)^(3/2)*f*(e
+ f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d^2)
 - (2*(a^2 - b^2)^(3/2)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + S
qrt[a^2 - b^2])])/(a*b^3*d^2) - (2*f^2*PolyLog[3, -E^(I*(c + d*x))])/(a*d^3
) + (2*f^2*PolyLog[3, E^(I*(c + d*x))])/(a*d^3) + ((2*I)*(a^2 - b^2)^(3/2)*
f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d^3) -
((2*I)*(a^2 - b^2)^(3/2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/(a*b^3*d^3) - (2*f*(e + f*x)*Sin[c + d*x])/(a*d^2) - (2*(a^2 - b
^2)*f*(e + f*x)*Sin[c + d*x])/(a*b^2*d^2) + (f^2*Cos[c + d*x]*Sin[c + d*x])
/(4*b*d^3) - ((e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
```

```
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
```

```
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\cos^3(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cos^4(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^2\cos(c+dx)\cot(c+dx)\,dx}{a} - \frac{\int (e+fx)^2\cos^2(c+dx)\,dx}{b} + \left(\frac{a}{b} - \frac{b}{a}\right.$$

$$= -\frac{f(e+fx)\cos^2(c+dx)}{2bd^2} - \frac{(e+fx)^2\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int (e+fx)^2\operatorname{csc}}{a}$$

$$= -\frac{(e+fx)^3}{6bf} + \frac{\left(a^2-b^2\right)(e+fx)^3}{3b^3f} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+fx)^2\operatorname{co}}{ad}$$

$$= \frac{f^2x}{4bd^2} - \frac{(e+fx)^3}{6bf} + \frac{\left(a^2-b^2\right)(e+fx)^3}{3b^3f} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+f}{}$$

$$= \frac{f^2x}{4bd^2} - \frac{(e+fx)^3}{6bf} + \frac{\left(a^2-b^2\right)(e+fx)^3}{3b^3f} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{2f^2\operatorname{c}}{}$$

$$= \frac{f^2x}{4bd^2} - \frac{(e+fx)^3}{6bf} + \frac{\left(a^2-b^2\right)(e+fx)^3}{3b^3f} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{2f^2\operatorname{c}}{}$$

$$= \frac{f^2x}{4bd^2} - \frac{(e+fx)^3}{6bf} + \frac{\left(a^2-b^2\right)(e+fx)^3}{3b^3f} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{2f^2\operatorname{c}}{}$$

$$= \frac{f^2x}{4bd^2} - \frac{(e+fx)^3}{6bf} + \frac{\left(a^2-b^2\right)(e+fx)^3}{3b^3f} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} - \frac{2f^2\operatorname{c}}{}$$

Mathematica [A] time = 4.98437, size = 1254, normalized size = 1.52

$$-\frac{-24d^2e^2\log\left(1-e^{i(c+dx)}\right)b^3-24d^2f^2x^2\log\left(1-e^{i(c+dx)}\right)b^3-48d^2efx\log\left(1-e^{i(c+dx)}\right)b^3+24d^2e^2\log\left(1+e^{i(c+dx)}\right)b^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(-24*a^3*d^3*e^2*x + 36*a*b^2*d^3*e^2*x - 24*a^3*d^3*e*f*x^2 + 36*a*b^2*d^
3*e*f*x^2 - 8*a^3*d^3*f^2*x^3 + 12*a*b^2*d^3*f^2*x^3 + 48*(a^2 - b^2)^(3/2)
*d^2*e^2*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] - 24*a^2*b*d^2*e
^2*Cos[c + d*x] + 48*a^2*b*f^2*Cos[c + d*x] - 48*a^2*b*d^2*e*f*x*Cos[c + d*
x] - 24*a^2*b*d^2*f^2*x^2*Cos[c + d*x] + 6*a*b^2*d*e*f*Cos[2*(c + d*x)] + 6
*a*b^2*d*f^2*x*Cos[2*(c + d*x)] - 24*b^3*d^2*e^2*Log[1 - E^(I*(c + d*x))] -
 48*b^3*d^2*e*f*x*Log[1 - E^(I*(c + d*x))] - 24*b^3*d^2*f^2*x^2*Log[1 - E^(
I*(c + d*x))] + 24*b^3*d^2*e^2*Log[1 + E^(I*(c + d*x))] + 48*b^3*d^2*e*f*x*
Log[1 + E^(I*(c + d*x))] + 24*b^3*d^2*f^2*x^2*Log[1 + E^(I*(c + d*x))] - (4
8*I)*(a^2 - b^2)^(3/2)*d^2*e*f*x*Log[1 + (I*b*E^(I*(c + d*x)))/(-a + Sqrt[a
^2 - b^2])] - (24*I)*(a^2 - b^2)^(3/2)*d^2*f^2*x^2*Log[1 + (I*b*E^(I*(c + d
*x)))/(-a + Sqrt[a^2 - b^2])] + (48*I)*(a^2 - b^2)^(3/2)*d^2*e*f*x*Log[1 -
(I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] + (24*I)*(a^2 - b^2)^(3/2)*d^2
*f^2*x^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] - (48*I)*b^3*
d*e*f*PolyLog[2, -E^(I*(c + d*x))] - (48*I)*b^3*d*f^2*x*PolyLog[2, -E^(I*(c
 + d*x))] + (48*I)*b^3*d*e*f*PolyLog[2, E^(I*(c + d*x))] + (48*I)*b^3*d*f^2
*x*PolyLog[2, E^(I*(c + d*x))] - 48*(a^2 - b^2)^(3/2)*d*e*f*PolyLog[2, ((-I
)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] - 48*(a^2 - b^2)^(3/2)*d*f^2*x
*PolyLog[2, ((-I)*b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] + 48*(a^2 - b^
2)^(3/2)*d*e*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] + 48
*(a^2 - b^2)^(3/2)*d*f^2*x*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
 b^2])] + 48*b^3*f^2*PolyLog[3, -E^(I*(c + d*x))] - 48*b^3*f^2*PolyLog[3, E
^(I*(c + d*x))] - (48*I)*(a^2 - b^2)^(3/2)*f^2*PolyLog[3, ((-I)*b*E^(I*(c +
 d*x)))/(-a + Sqrt[a^2 - b^2])] + (48*I)*(a^2 - b^2)^(3/2)*f^2*PolyLog[3, (
I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] + 48*a^2*b*d*e*f*Sin[c + d*x] +
 48*a^2*b*d*f^2*x*Sin[c + d*x] + 6*a*b^2*d^2*e^2*Sin[2*(c + d*x)] - 3*a*b^2
*f^2*Sin[2*(c + d*x)] + 12*a*b^2*d^2*e*f*x*Sin[2*(c + d*x)] + 6*a*b^2*d^2*f
^2*x^2*Sin[2*(c + d*x)])/(24*a*b^3*d^3)
```

Maple [F] time = 3.208, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^2 (\cos(dx+c))^3 \cot(dx+c)}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 5.14962, size = 6433, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/12*(2*(2*a^3 - 3*a*b^2)*d^3*f^2*x^3 + 6*(2*a^3 - 3*a*b^2)*d^3*e*f*x^2 + 1
2*b^3*f^2*polylog(3, cos(d*x + c) + I*sin(d*x + c)) + 12*b^3*f^2*polylog(3,
 cos(d*x + c) - I*sin(d*x + c)) - 12*b^3*f^2*polylog(3, -cos(d*x + c) + I*s
in(d*x + c)) - 12*b^3*f^2*polylog(3, -cos(d*x + c) - I*sin(d*x + c)) + 12*(
a^2*b - b^3)*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c)
- 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2))/b) - 12*(a^2*b - b^3)*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*
I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2))/b) - 12*(a^2*b - b^3)*f^2*sqrt(-(a^2 - b^2)/b^2)*p
olylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d
*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 12*(a^2*b - b^3)*f^2*sqrt(-(a^2 - b^2
)/b^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 6*(a*b^2*d*f^2*x + a*b^2*d*e*
```

f)*cos(d*x + c)^2 + 2*(-6*I*(a^2*b - b^3)*d*f^2*x - 6*I*(a^2*b - b^3)*d*e*f
)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c)
+ 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1
) + 2*(6*I*(a^2*b - b^3)*d*f^2*x + 6*I*(a^2*b - b^3)*d*e*f)*sqrt(-(a^2 - b^
2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x +
c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(6*I*(a^2*b
 - b^3)*d*f^2*x + 6*I*(a^2*b - b^3)*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/
2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*(-6*I*(a^2*b - b^3)*d*f^2*x
 - 6*I*(a^2*b - b^3)*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d
*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2) + 2*b)/b + 1) - 6*((a^2*b - b^3)*d^2*e^2 - 2*(a^2*b - b^3)*c
*d*e*f + (a^2*b - b^3)*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c)
 + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 6*((a^2*b - b
^3)*d^2*e^2 - 2*(a^2*b - b^3)*c*d*e*f + (a^2*b - b^3)*c^2*f^2)*sqrt(-(a^2 -
 b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) - 2*I*a) + 6*((a^2*b - b^3)*d^2*e^2 - 2*(a^2*b - b^3)*c*d*e*f + (a^2
*b - b^3)*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin
(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 6*((a^2*b - b^3)*d^2*e^2
- 2*(a^2*b - b^3)*c*d*e*f + (a^2*b - b^3)*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*l
og(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*
I*a) - 6*((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a^2*b - b^3)*d^2*e*f*x + 2*(a^2*b
- b^3)*c*d*e*f - (a^2*b - b^3)*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I
*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*
sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 6*((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a^2*b
- b^3)*d^2*e*f*x + 2*(a^2*b - b^3)*c*d*e*f - (a^2*b - b^3)*c^2*f^2)*sqrt(-(
a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d
*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 6*((a^2*b -
b^3)*d^2*f^2*x^2 + 2*(a^2*b - b^3)*d^2*e*f*x + 2*(a^2*b - b^3)*c*d*e*f - (a
^2*b - b^3)*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) +
2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2) + 2*b)/b) + 6*((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a^2*b - b^3)*d^2*e*f*x +
 2*(a^2*b - b^3)*c*d*e*f - (a^2*b - b^3)*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*lo
g(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin
(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 3*(2*(2*a^3 - 3*a*b^2)*d^3*e^
2 + a*b^2*d*f^2)*x + 12*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + a^2*b*d^2*
e^2 - 2*a^2*b*f^2)*cos(d*x + c) + (-12*I*b^3*d*f^2*x - 12*I*b^3*d*e*f)*dilo
g(cos(d*x + c) + I*sin(d*x + c)) + (12*I*b^3*d*f^2*x + 12*I*b^3*d*e*f)*dilo
g(cos(d*x + c) - I*sin(d*x + c)) + (-12*I*b^3*d*f^2*x - 12*I*b^3*d*e*f)*dil
og(-cos(d*x + c) + I*sin(d*x + c)) + (12*I*b^3*d*f^2*x + 12*I*b^3*d*e*f)*di
log(-cos(d*x + c) - I*sin(d*x + c)) - 6*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x
+ b^3*d^2*e^2)*log(cos(d*x + c) + I*sin(d*x + c) + 1) - 6*(b^3*d^2*f^2*x^2
+ 2*b^3*d^2*e*f*x + b^3*d^2*e^2)*log(cos(d*x + c) - I*sin(d*x + c) + 1) + 6
*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*log(-1/2*cos(d*x + c) + 1/2*I*
sin(d*x + c) + 1/2) + 6*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*log(-1/

```
2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2) + 6*(b^3*d^2*f^2*x^2 + 2*b^3*d^2
*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*log(-cos(d*x + c) + I*sin(d*x + c) +
1) + 6*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*lo
g(-cos(d*x + c) - I*sin(d*x + c) + 1) - 3*(8*a^2*b*d*f^2*x + 8*a^2*b*d*e*f
+ (2*a*b^2*d^2*f^2*x^2 + 4*a*b^2*d^2*e*f*x + 2*a*b^2*d^2*e^2 - a*b^2*f^2)*c
os(d*x + c))*sin(d*x + c))/(a*b^3*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.335 $\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=524

$$\frac{f\left(a^2-b^2\right)^{3/2}\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^3d^2}-\frac{f\left(a^2-b^2\right)^{3/2}\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^3d^2}+\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{if\text{PolyLog}(}{ad^2}$$

```
[Out] -(e*x)/(2*b) + ((a^2 - b^2)*e*x)/b^3 - (f*x^2)/(4*b) + ((a^2 - b^2)*f*x^2)/
(2*b^3) - (2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((e + f*x)*Cos[c +
 d*x])/(a*d) + ((a^2 - b^2)*(e + f*x)*Cos[c + d*x])/(a*b^2*d) - (f*Cos[c +
d*x]^2)/(4*b*d^2) + (I*(a^2 - b^2)^(3/2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d) - (I*(a^2 - b^2)^(3/2)*(e + f*x)*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^3*d) + (I*f*PolyLo
g[2, -E^(I*(c + d*x))])/(a*d^2) - (I*f*PolyLog[2, E^(I*(c + d*x))])/(a*d^2)
 + ((a^2 - b^2)^(3/2)*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(a*b^3*d^2) - ((a^2 - b^2)^(3/2)*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(a*b^3*d^2) - (f*Sin[c + d*x])/(a*d^2) - ((a^2 - b^2
)*f*Sin[c + d*x])/(a*b^2*d^2) - ((e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(2*b*
d)
```

Rubi [A] time = 0.898953, antiderivative size = 524, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 14, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4543, 4408, 4405, 2633, 3296, 2637, 4183, 2279, 2391, 4525, 3310, 3323, 2264, 2190}

$$\frac{f\left(a^2-b^2\right)^{3/2}\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{ab^3d^2}-\frac{f\left(a^2-b^2\right)^{3/2}\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{ab^3d^2}+\frac{if\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{ad^2}-\frac{if\text{PolyLog}(}{ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```


```
[Out] -(e*x)/(2*b) + ((a^2 - b^2)*e*x)/b^3 - (f*x^2)/(4*b) + ((a^2 - b^2)*f*x^2)/
(2*b^3) - (2*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d) + ((e + f*x)*Cos[c +
 d*x])/(a*d) + ((a^2 - b^2)*(e + f*x)*Cos[c + d*x])/(a*b^2*d) - (f*Cos[c +
d*x]^2)/(4*b*d^2) + (I*(a^2 - b^2)^(3/2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d
*x)))/(a - Sqrt[a^2 - b^2])])/(a*b^3*d) - (I*(a^2 - b^2)^(3/2)*(e + f*x)*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a*b^3*d) + (I*f*PolyLo
g[2, -E^(I*(c + d*x))])/(a*d^2) - (I*f*PolyLog[2, E^(I*(c + d*x))])/(a*d^2)
 + ((a^2 - b^2)^(3/2)*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
```

```
2])])/(a*b^3*d^2) - ((a^2 - b^2)^(3/2)*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(
a + Sqrt[a^2 - b^2])])/(a*b^3*d^2) - (f*Sin[c + d*x])/(a*d^2) - ((a^2 - b^2
)*f*Sin[c + d*x])/(a*b^2*d^2) - ((e + f*x)*Cos[c + d*x]*Sin[c + d*x])/(2*b*
d)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\cos^3(c+dx)\cot(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)\cos(c+dx)\cot(c+dx)\,dx}{a} - \frac{\int (e+fx)\cos^2(c+dx)\,dx}{b} + \left(\frac{a}{b}-\frac{b}{a}\right)\int$$

$$= -\frac{f\cos^2(c+dx)}{4bd^2} - \frac{(e+fx)\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int (e+fx)\csc(c+dx)\,dx}{a}$$

$$= -\frac{ex}{2b} + \frac{\left(a^2-b^2\right)ex}{b^3} - \frac{fx^2}{4b} + \frac{\left(a^2-b^2\right)fx^2}{2b^3} - \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+}{}$$

$$= -\frac{ex}{2b} + \frac{\left(a^2-b^2\right)ex}{b^3} - \frac{fx^2}{4b} + \frac{\left(a^2-b^2\right)fx^2}{2b^3} - \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+}{}$$

$$= -\frac{ex}{2b} + \frac{\left(a^2-b^2\right)ex}{b^3} - \frac{fx^2}{4b} + \frac{\left(a^2-b^2\right)fx^2}{2b^3} - \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+}{}$$

$$= -\frac{ex}{2b} + \frac{\left(a^2-b^2\right)ex}{b^3} - \frac{fx^2}{4b} + \frac{\left(a^2-b^2\right)fx^2}{2b^3} - \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+}{}$$

$$= -\frac{ex}{2b} + \frac{\left(a^2-b^2\right)ex}{b^3} - \frac{fx^2}{4b} + \frac{\left(a^2-b^2\right)fx^2}{2b^3} - \frac{2(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad} + \frac{(e+}{}$$

Mathematica [A] time = 11.8142, size = 934, normalized size = 1.78

$$-\frac{(de+dfx)\left(\frac{2(de-cf)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{if\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt{b^2-a^2}}{-ia+b+\sqrt{b^2-a^2}}\right)+\text{PolyLog}\left(2,\frac{a\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a+i\left(b+\sqrt{b^2-a^2}\right)}\right)\right)}{\sqrt{b^2-a^2}}+\frac{if\left(\log \cdots\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((-2*a^2 + 3*b^2)*(c + d*x)*(2*d*e - 2*c*f + f*(c + d*x)))/(4*b^3*d^2) + (
a*(d*e - c*f + f*(c + d*x))*Cos[c + d*x])/(b^2*d^2) - (f*Cos[2*(c + d*x)])/
(8*b*d^2) + (e*Log[Tan[(c + d*x)/2]])/(a*d) - (c*f*Log[Tan[(c + d*x)/2]])/(
a*d^2) + (f*((c + d*x)*(Log[1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c + d*x))]
) + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2, E^(I*(c + d*x))])))/(a*d^2
) - ((a^2 - b^2)^2*(d*e + d*f*x)*((2*(d*e - c*f)*ArcTan[(b + a*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*(Log[1 - I*Tan[(c + d*x)/2]]
*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 +
b^2])] + PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^
2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[
-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2
, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2
+ b^2] + (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2 + b^2] + a*
Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))] + PolyLog[2, (a*(I + Tan[(
c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]))/Sqrt[-a^2 + b^2] - (I*f*(Log[
1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*
a + b - Sqrt[-a^2 + b^2])] + PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(
-b + Sqrt[-a^2 + b^2])]))/Sqrt[-a^2 + b^2]))/(a*b^3*d^2*(d*e - c*f + I*f*L
og[1 - I*Tan[(c + d*x)/2]] - I*f*Log[1 + I*Tan[(c + d*x)/2]])) - (a*f*Sin[c
 + d*x])/(b^2*d^2) - ((d*e - c*f + f*(c + d*x))*Sin[2*(c + d*x)])/(4*b*d^2)
```

Maple [B] time = 1.073, size = 1901, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/16*I*(2*d*f*x+I*f+2*d*e)/b/d^2*exp(2*I*(d*x+c))+1/2*a*(d*f*x-I*f+d*e)/b^2
/d^2*exp(-I*(d*x+c))-1/16*I*(2*d*f*x-I*f+2*d*e)/b/d^2*exp(-2*I*(d*x+c))+1/2
*a*(d*f*x+I*f+d*e)/b^2/d^2*exp(I*(d*x+c))+2*I/d^2*f*c/a*b/(-a^2+b^2)^(1/2)*
arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+1/2*a^2*f*x^2/b^3+2
/b*a/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a
-(-a^2+b^2)^(1/2)))*c-2/b*a/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(
-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-2/b*a/d^2*f/(-a^2+b^2)^(1/2)*ln(
(I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+2/b*a/d*f
/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2
)^(1/2)))*x+2*I*a^3/b^3/d^2*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d
*x+c))-2*a)/(-a^2+b^2)^(1/2))+4*I/b/d*a*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*
b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+2*I/b/d^2*a*f/(-a^2+b^2)^(1/2)*dilo
g((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-2*I/b*a/d
^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-
a^2+b^2)^(1/2)))+I/d^2*f/a*dilog(exp(I*(d*x+c)))+I/d^2*f/a*dilog(exp(I*(d*x
+c))+1)-1/d*f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2
))/(I*a-(-a^2+b^2)^(1/2)))*x-1/d^2*f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(
d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+1/d*f*b/a/(-a^2+b^2)^(1
/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+1/
d^2*f*b/a/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+
(-a^2+b^2)^(1/2)))*c+I/d^2*f*b/a/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c
))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-2*I/d*e/a*b/(-a^2+b^2)^(1/2)*a
rctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))-I/d^2*f*b/a/(-a^2+b^
2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2
)))-1/d^2/a*f*c*ln(exp(I*(d*x+c))-1)-1/d/a*ln(exp(I*(d*x+c))+1)*f*x+1/d/a*e
*ln(exp(I*(d*x+c))-1)-1/d/a*e*ln(exp(I*(d*x+c))+1)-3/4*f*x^2/b-4*I/b/d^2*a*
f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2)
)-3/2*e*x/b+a^2*e*x/b^3-a^3/b^3/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c
))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-a^3/b^3/d^2*f/(-a^2+b^2)^(1/
2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+a^3
/b^3/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(
-a^2+b^2)^(1/2)))*x+a^3/b^3/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))
+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+I*a^3/b^3/d^2*f/(-a^2+b^2)^(1/
2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-2*
I*a^3/b^3/d*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+
b^2)^(1/2))-I*a^3/b^3/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-
a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.03225, size = 3862, normalized size = 7.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/4*((2*a^3 - 3*a*b^2)*d^2*f*x^2 - a*b^2*f*cos(d*x + c)^2 + 2*(2*a^3 - 3*a*
b^2)*d^2*e*x - 2*I*b^3*f*dilog(cos(d*x + c) + I*sin(d*x + c)) + 2*I*b^3*f*d
ilog(cos(d*x + c) - I*sin(d*x + c)) - 2*I*b^3*f*dilog(-cos(d*x + c) + I*sin
(d*x + c)) + 2*I*b^3*f*dilog(-cos(d*x + c) - I*sin(d*x + c)) - 2*I*(a^2*b -
 b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x
 + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)
/b + 1) + 2*I*(a^2*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(
d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(
a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*(a^2*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2)*
dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*
b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*I*(a^2*b - b^3)*f*
sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) -
 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)
 - 2*((a^2*b - b^3)*d*e - (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b
*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) -
2*((a^2*b - b^3)*d*e - (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*co
s(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(
(a^2*b - b^3)*d*e - (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(
d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*((a
^2*b - b^3)*d*e - (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*
x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*((a^2
*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*
cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)
*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*
(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*((
a^2*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*
I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))
```

```
*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*
c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c)
 - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) +
 4*(a^2*b*d*f*x + a^2*b*d*e)*cos(d*x + c) - 2*(b^3*d*f*x + b^3*d*e)*log(cos
(d*x + c) + I*sin(d*x + c) + 1) - 2*(b^3*d*f*x + b^3*d*e)*log(cos(d*x + c)
- I*sin(d*x + c) + 1) + 2*(b^3*d*e - b^3*c*f)*log(-1/2*cos(d*x + c) + 1/2*I
*sin(d*x + c) + 1/2) + 2*(b^3*d*e - b^3*c*f)*log(-1/2*cos(d*x + c) - 1/2*I*
sin(d*x + c) + 1/2) + 2*(b^3*d*f*x + b^3*c*f)*log(-cos(d*x + c) + I*sin(d*x
 + c) + 1) + 2*(b^3*d*f*x + b^3*c*f)*log(-cos(d*x + c) - I*sin(d*x + c) + 1
) - 2*(2*a^2*b*f + (a*b^2*d*f*x + a*b^2*d*e)*cos(d*x + c))*sin(d*x + c))/(a
*b^3*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)\cos^3(c + dx)\cot(c + dx)}{a + b\sin(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cos(c + d*x)**3*cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.336 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^3d}+\frac{x\left(2a^2-3b^2\right)}{2b^3}+\frac{a\cos(c+dx)}{b^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

```
[Out] ((2*a^2 - 3*b^2)*x)/(2*b^3) - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a*b^3*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (a*Cos[
c + d*x])/(b^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.279724, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2895, 3057, 2660, 618, 204, 3770}

$$-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^3d}+\frac{x\left(2a^2-3b^2\right)}{2b^3}+\frac{a\cos(c+dx)}{b^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a^2 - 3*b^2)*x)/(2*b^3) - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a*b^3*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (a*Cos[
c + d*x])/(b^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
```

```
n + 4, 0]
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{\int \frac{\csc(c+dx)\left(-2b^2-ab\sin(c+dx)-\left(2a^2-3b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}}{2b^2}$$

$$= \frac{\left(2a^2-3b^2\right)x}{2b^3} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \csc(c+dx)\,dx}{a} - \frac{\left(a^2-b\right.}{}$$

$$= \frac{\left(2a^2-3b^2\right)x}{2b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{(2}{}$$

$$= \frac{\left(2a^2-3b^2\right)x}{2b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{(4}{}$$

$$= \frac{\left(2a^2-3b^2\right)x}{2b^3} - \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^3d} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{a\cos(}{b}$$

Mathematica [A] time = 0.281121, size = 143, normalized size = 1.15

$$-\frac{8\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) - 4a^2b\cos(c+dx) - 4a^3c - 4a^3dx + ab^2\sin(2(c+dx)) + 6ab^2c + 6ab^2dx - 4b^3\,\mathrm{l}}{4ab^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(-4*a^3*c + 6*a*b^2*c - 4*a^3*d*x + 6*a*b^2*d*x + 8*(a^2 - b^2)^(3/2)*ArcT
an[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 4*a^2*b*Cos[c + d*x] + 4*b^3
*Log[Cos[(c + d*x)/2]] - 4*b^3*Log[Sin[(c + d*x)/2]] + a*b^2*Sin[2*(c + d*x
)])/(4*a*b^3*d)
```

Maple [B] time = 0.083, size = 334, normalized size = 2.7

$$\frac{1}{bd}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2} + 2\,\frac{(\tan(1/2\,dx+c/2))^2\,a}{b^2d\left(1+(\tan(1/2\,dx+c/2))^2\right)^2} - \frac{1}{bd}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+2/d/b^2/(1+tan(1/2*d*
x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a-1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1
/2*d*x+1/2*c)+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*a+2/d/b^3*arctan(tan(1/2*d
*x+1/2*c))*a^2-3/d/b*arctan(tan(1/2*d*x+1/2*c))-2/d*a^3/b^3/(a^2-b^2)^(1/2)
*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d/b*a/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a*b/(a^
2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/a/d
*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.0426, size = 853, normalized size = 6.88

$$\left[-\frac{ab^2\cos(dx+c)\sin(dx+c) - 2\,a^2b\cos(dx+c) + b^3\log\left(\frac{1}{2}\,\cos(dx+c) + \frac{1}{2}\right) - b^3\log\left(-\frac{1}{2}\,\cos(dx+c) + \frac{1}{2}\right) - \left(2\,a^3 - \right.}{2\,ab^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^2*cos(d*x + c)*sin(d*x + c) - 2*a^2*b*cos(d*x + c) + b^3*log(1/2
*cos(d*x + c) + 1/2) - b^3*log(-1/2*cos(d*x + c) + 1/2) - (2*a^3 - 3*a*b^2)
*d*x - (-a^2 + b^2)^(3/2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*
x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(
-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/(a*b^3
*d), -1/2*(a*b^2*cos(d*x + c)*sin(d*x + c) - 2*a^2*b*cos(d*x + c) + b^3*log
(1/2*cos(d*x + c) + 1/2) - b^3*log(-1/2*cos(d*x + c) + 1/2) - (2*a^3 - 3*a*
```

```
b^2)*d*x - 2*(a^2 - b^2)^(3/2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2
)*cos(d*x + c))))/(a*b^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.08771, size = 247, normalized size = 1.99

$$\frac{\frac{2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{\left(2\,a^2-3\,b^2\right)(dx+c)}{b^3}-\frac{4\left(a^4-2\,a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,ab^3}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a + (2*a^2 - 3*b^2)*(d*x + c)/b^3 - 4
*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan(
(a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a*b^3) + 2*
(b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/
2*c) + 2*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

3.337 $\int \frac{(e+fx)^3 \cos(c+dx) \cot^2(c+dx)}{a+b \sin(c+dx)} dx$

Optimal. Leaf size=852

$$\frac{ib(e+fx)^4}{4a^2f}+\frac{i\left(a^2-b^2\right)(e+fx)^4}{4a^2bf}-\frac{\csc(c+dx)(e+fx)^3}{ad}-\frac{\left(a^2-b^2\right)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)(e+fx)^3}{a^2bd}-\frac{\left(a^2-b^2\right)\log\left(1-\frac{ib}{a+}\right.}{a^2ba}$$

```
[Out] ((I/4)*b*(e + f*x)^4)/(a^2*f) + ((I/4)*(a^2 - b^2)*(e + f*x)^4)/(a^2*b*f) -
 (6*f*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d^2) - ((e + f*x)^3*Csc[c +
d*x])/(a*d) - ((a^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(a^2*b*d) - ((a^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c
 + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d) - (b*(e + f*x)^3*Log[1 - E^((2*
I)*(c + d*x))])/(a^2*d) + ((6*I)*f^2*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))]
)/(a*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^3) + ((3
*I)*(a^2 - b^2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^
2 - b^2])])/(a^2*b*d^2) + ((3*I)*(a^2 - b^2)*f*(e + f*x)^2*PolyLog[2, (I*b*
E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d^2) + (((3*I)/2)*b*f*(e +
f*x)^2*PolyLog[2, E^((2*I)*(c + d*x))])/(a^2*d^2) - (6*f^3*PolyLog[3, -E^(I
*(c + d*x))])/(a*d^4) + (6*f^3*PolyLog[3, E^(I*(c + d*x))])/(a*d^4) - (6*(a
^2 - b^2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(a^2*b*d^3) - (6*(a^2 - b^2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c +
d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d^3) - (3*b*f^2*(e + f*x)*PolyLog[3,
E^((2*I)*(c + d*x))])/(2*a^2*d^3) - ((6*I)*(a^2 - b^2)*f^3*PolyLog[4, (I*b*
E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b*d^4) - ((6*I)*(a^2 - b^2)*f
^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d^4) - (
((3*I)/4)*b*f^3*PolyLog[4, E^((2*I)*(c + d*x))])/(a^2*d^4)
```

Rubi [A] time = 1.78215, antiderivative size = 852, normalized size of antiderivative = 1., number of steps used = 48, number of rules used = 19, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.559, Rules used = {4543, 4408, 3296, 2638, 4410, 4183, 2531, 2282, 6589, 4404, 3311, 32, 2635, 8, 3717, 2190, 6609, 4525, 4519}

$$\frac{ib(e+fx)^4}{4a^2f}+\frac{i\left(a^2-b^2\right)(e+fx)^4}{4a^2bf}-\frac{\csc(c+dx)(e+fx)^3}{ad}-\frac{\left(a^2-b^2\right)\log\left(1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)(e+fx)^3}{a^2bd}-\frac{\left(a^2-b^2\right)\log\left(1-\frac{ib}{a+}\right.}{a^2ba}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((I/4)*b*(e + f*x)^4)/(a^2*f) + ((I/4)*(a^2 - b^2)*(e + f*x)^4)/(a^2*b*f) -
 (6*f*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a*d^2) - ((e + f*x)^3*Csc[c +
d*x])/(a*d) - ((a^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - S
qrt[a^2 - b^2])])/(a^2*b*d) - ((a^2 - b^2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c
 + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d) - (b*(e + f*x)^3*Log[1 - E^((2*
I)*(c + d*x))])/(a^2*d) + ((6*I)*f^2*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))]
)/(a*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a*d^3) + ((3
*I)*(a^2 - b^2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^
2 - b^2])])/(a^2*b*d^2) + ((3*I)*(a^2 - b^2)*f*(e + f*x)^2*PolyLog[2, (I*b*
E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d^2) + (((3*I)/2)*b*f*(e +
f*x)^2*PolyLog[2, E^((2*I)*(c + d*x))])/(a^2*d^2) - (6*f^3*PolyLog[3, -E^(I
*(c + d*x))])/(a*d^4) + (6*f^3*PolyLog[3, E^(I*(c + d*x))])/(a*d^4) - (6*(a
^2 - b^2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(a^2*b*d^3) - (6*(a^2 - b^2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c +
d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d^3) - (3*b*f^2*(e + f*x)*PolyLog[3,
E^((2*I)*(c + d*x))])/(2*a^2*d^3) - ((6*I)*(a^2 - b^2)*f^3*PolyLog[4, (I*b*
E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b*d^4) - ((6*I)*(a^2 - b^2)*f
^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d^4) - (
((3*I)/4)*b*f^3*PolyLog[4, E^((2*I)*(c + d*x))])/(a^2*d^4)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
```

```
[{c, d}, x]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
```

```
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\cos(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{\int (e+fx)^3\cos(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\cot(c+dx)\csc(c+dx)\,dx}{a} - \frac{b\int (e \cdots}{}$$

$$= -\frac{(e+fx)^3\csc(c+dx)}{ad} - \frac{(e+fx)^3\sin(c+dx)}{ad} + \frac{\int (e+fx)^3\cos(c+dx)\,dx}{a} \cdots$$

$$= \frac{ib(e+fx)^4}{4a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4bf} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2} - \frac{3f(e+f \cdots}{}$$

$$= \frac{ib(e+fx)^4}{4a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4bf} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2} - \frac{(e+fx) \cdots}{}$$

$$= \frac{ib(e+fx)^4}{4a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4bf} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2} + \frac{6f^3\cos \cdots}{a \cdots}$$

$$= \frac{ib(e+fx)^4}{4a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4bf} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2} - \frac{(e+fx) \cdots}{}$$

$$= \frac{ib(e+fx)^4}{4a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4bf} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2} - \frac{(e+fx) \cdots}{}$$

$$= \frac{ib(e+fx)^4}{4a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4bf} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2} - \frac{(e+fx) \cdots}{}$$

Mathematica [B] time = 46.364, size = 2974, normalized size = 3.49

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((-I)*b*(e + f*x)^4)/((-1 + E^((2*I)*c))*f) + (6*e*f*(b*d*e - 2*a*f)*x*Lo
g[1 - E^((-I)*(c + d*x))])/d^2 + (6*f^2*(b*d*e - a*f)*x^2*Log[1 - E^((-I)*(
c + d*x))])/d^2 + (2*b*f^3*x^3*Log[1 - E^((-I)*(c + d*x))])/d + (6*e*f*(b*d
*e + 2*a*f)*x*Log[1 + E^((-I)*(c + d*x))])/d^2 + (6*f^2*(b*d*e + a*f)*x^2*L
og[1 + E^((-I)*(c + d*x))])/d^2 + (2*b*f^3*x^3*Log[1 + E^((-I)*(c + d*x))])
/d + (2*e^2*(b*d*e - 3*a*f)*((-I)*d*x + Log[1 - E^(I*(c + d*x))]))/d^2 + (2
```

e^2(b*d*e + 3*a*f)*((-I)*d*x + Log[1 + E^(I*(c + d*x))]))/d^2 + ((6*I)*e*
f*(b*d*e + 2*a*f)*PolyLog[2, -E^((-I)*(c + d*x))])/d^3 + ((6*I)*e*f*(b*d*e
- 2*a*f)*PolyLog[2, E^((-I)*(c + d*x))])/d^3 + (12*f^2*(b*d*e + a*f)*(I*d*x
PolyLog[2, -E^((-I)(c + d*x))] + PolyLog[3, -E^((-I)*(c + d*x))]))/d^4 +
(12*f^2*(b*d*e - a*f)*(I*d*x*PolyLog[2, E^((-I)*(c + d*x))] + PolyLog[3, E^
((-I)*(c + d*x))]))/d^4 + (6*b*f^3*(I*d^2*x^2*PolyLog[2, -E^((-I)*(c + d*x)
)] + 2*d*x*PolyLog[3, -E^((-I)*(c + d*x))] - (2*I)*PolyLog[4, -E^((-I)*(c +
d*x))]))/d^4 + (6*b*f^3*(I*d^2*x^2*PolyLog[2, E^((-I)*(c + d*x))] + 2*d*x*
PolyLog[3, E^((-I)*(c + d*x))] - (2*I)*PolyLog[4, E^((-I)*(c + d*x))]))/d^4
)/(2*a^2) + ((a^2 - b^2)*((4*I)*d^4*e^3*E^((2*I)*c)*x + (6*I)*d^4*e^2*E^((2
*I)*c)*f*x^2 + (4*I)*d^4*e*E^((2*I)*c)*f^2*x^3 + I*d^4*E^((2*I)*c)*f^3*x^4
+ (2*I)*d^3*e^3*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))
] - (2*I)*d^3*e^3*E^((2*I)*c)*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I
)*(c + d*x))))] + d^3*e^3*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I
)*(c + d*x)))^2] - d^3*e^3*E^((2*I)*c)*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*
(-1 + E^((2*I)*(c + d*x)))^2] + 6*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))
/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e^2*E^((2*I)*c)*f*
x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c
)])] + 6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-
a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*
c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 2*d^3*f^3*x^3*
Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]
)] - 2*d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) -
Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c + d*
x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e^2*E^((2*I)*c
)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*
I)*c)])] + 6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqr
t[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I
*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 2*d^3*f^3*
x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)
*c)])] - 2*d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*
c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (6*I)*d^2*(-1 + E^((2*I)*c))*f*(e +
f*x)^2*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)
*E^((2*I)*c)])] + (6*I)*d^2*(-1 + E^((2*I)*c))*f*(e + f*x)^2*PolyLog[2, -((
b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 12*
d*e*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)
*E^((2*I)*c)])] - 12*d*e*E^((2*I)*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))
/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*d*f^3*x*PolyLog[3, (I
*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*
d*E^((2*I)*c)*f^3*x*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[
(-a^2 + b^2)*E^((2*I)*c)])] + 12*d*e*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))
/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 12*d*e*E^((2*I)*c)*f^2*
PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I
)*c)]))] + 12*d*f^3*x*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqr
t[(-a^2 + b^2)*E^((2*I)*c)]))] - 12*d*E^((2*I)*c)*f^3*x*PolyLog[3, -((b*E^(

```
I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (12*I)*f
^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((
2*I)*c)])] - (12*I)*E^((2*I)*c)*f^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E
^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (12*I)*f^3*PolyLog[4, -((b*E^
(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (12*I)*
E^((2*I)*c)*f^3*PolyLog[4, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^
2 + b^2)*E^((2*I)*c)]))]))/(2*a^2*b*d^4*(-1 + E^((2*I)*c))) + ((-4*b*e^3 -
12*b*e^2*f*x - 12*b*e*f^2*x^2 - 4*b*f^3*x^3 - 4*a*d*e^3*x*Cos[c] - 6*a*d*e^
2*f*x^2*Cos[c] - 4*a*d*e*f^2*x^3*Cos[c] - a*d*f^3*x^4*Cos[c])*Csc[c/2]*Sec[
c/2])/(8*a*b*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]*(-(e^3*Sin[(d*x)/2]) - 3*e^2
*f*x*Sin[(d*x)/2] - 3*e*f^2*x^2*Sin[(d*x)/2] - f^3*x^3*Sin[(d*x)/2]))/(2*a*
d) + (Csc[c/2]*Csc[c/2 + (d*x)/2]*(e^3*Sin[(d*x)/2] + 3*e^2*f*x*Sin[(d*x)/2
] + 3*e*f^2*x^2*Sin[(d*x)/2] + f^3*x^3*Sin[(d*x)/2]))/(2*a*d)
```

Maple [F] time = 2.659, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cos(dx+c)(\cot(dx+c))^2}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 5.06862, size = 9106, normalized size = 10.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/2*(2*a*b*d^3*f^3*x^3 + 6*a*b*d^3*e*f^2*x^2 + 6*a*b*d^3*e^2*f*x + 2*a*b*d
^3*e^3 + 6*I*b^2*f^3*polylog(4, cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c)
 - 6*I*b^2*f^3*polylog(4, cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - 6*I
*b^2*f^3*polylog(4, -cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 6*I*b^2*
f^3*polylog(4, -cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 6*I*(a^2 - b^
2)*f^3*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x
 + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 6*I*(a^
2 - b^2)*f^3*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*c
os(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 6
*I*(a^2 - b^2)*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(
d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 6*I*
(a^2 - b^2)*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x
 + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - (-3*I*(
a^2 - b^2)*d^2*f^3*x^2 - 6*I*(a^2 - b^2)*d^2*e*f^2*x - 3*I*(a^2 - b^2)*d^2*
e^2*f)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c
) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) - (
-3*I*(a^2 - b^2)*d^2*f^3*x^2 - 6*I*(a^2 - b^2)*d^2*e*f^2*x - 3*I*(a^2 - b^2
)*d^2*e^2*f)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d
*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x +
c) - (3*I*(a^2 - b^2)*d^2*f^3*x^2 + 6*I*(a^2 - b^2)*d^2*e*f^2*x + 3*I*(a^2
- b^2)*d^2*e^2*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b
*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(
d*x + c) - (3*I*(a^2 - b^2)*d^2*f^3*x^2 + 6*I*(a^2 - b^2)*d^2*e*f^2*x + 3*I
*(a^2 - b^2)*d^2*e^2*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c)
- 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1
)*sin(d*x + c) - (3*I*b^2*d^2*f^3*x^2 + 3*I*b^2*d^2*e^2*f - 6*I*a*b*d*e*f^2
 + 6*I*(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*dilog(cos(d*x + c) + I*sin(d*x + c))*
sin(d*x + c) - (-3*I*b^2*d^2*f^3*x^2 - 3*I*b^2*d^2*e^2*f + 6*I*a*b*d*e*f^2
- 6*I*(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*dilog(cos(d*x + c) - I*sin(d*x + c))*s
in(d*x + c) - (-3*I*b^2*d^2*f^3*x^2 - 3*I*b^2*d^2*e^2*f - 6*I*a*b*d*e*f^2 -
 6*I*(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*dilog(-cos(d*x + c) + I*sin(d*x + c))*s
in(d*x + c) - (3*I*b^2*d^2*f^3*x^2 + 3*I*b^2*d^2*e^2*f + 6*I*a*b*d*e*f^2 +
6*I*(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*dilog(-cos(d*x + c) - I*sin(d*x + c))*si
n(d*x + c) + ((a^2 - b^2)*d^3*e^3 - 3*(a^2 - b^2)*c*d^2*e^2*f + 3*(a^2 - b^
2)*c^2*d*e*f^2 - (a^2 - b^2)*c^3*f^3)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x
```

```
+ c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) + ((a^2 - b^2)*d^3*
e^3 - 3*(a^2 - b^2)*c*d^2*e^2*f + 3*(a^2 - b^2)*c^2*d*e*f^2 - (a^2 - b^2)*c
^3*f^3)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b
^2) - 2*I*a)*sin(d*x + c) + ((a^2 - b^2)*d^3*e^3 - 3*(a^2 - b^2)*c*d^2*e^2*
f + 3*(a^2 - b^2)*c^2*d*e*f^2 - (a^2 - b^2)*c^3*f^3)*log(-2*b*cos(d*x + c)
+ 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) + (
(a^2 - b^2)*d^3*e^3 - 3*(a^2 - b^2)*c*d^2*e^2*f + 3*(a^2 - b^2)*c^2*d*e*f^2
 - (a^2 - b^2)*c^3*f^3)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sq
rt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) + ((a^2 - b^2)*d^3*f^3*x^3 + 3*(
a^2 - b^2)*d^3*e*f^2*x^2 + 3*(a^2 - b^2)*d^3*e^2*f*x + 3*(a^2 - b^2)*c*d^2*
e^2*f - 3*(a^2 - b^2)*c^2*d*e*f^2 + (a^2 - b^2)*c^3*f^3)*log(1/2*(2*I*a*cos
(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-
(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) + ((a^2 - b^2)*d^3*f^3*x^3 + 3*(a^2
 - b^2)*d^3*e*f^2*x^2 + 3*(a^2 - b^2)*d^3*e^2*f*x + 3*(a^2 - b^2)*c*d^2*e^2
*f - 3*(a^2 - b^2)*c^2*d*e*f^2 + (a^2 - b^2)*c^3*f^3)*log(1/2*(2*I*a*cos(d*
x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) + ((a^2 - b^2)*d^3*f^3*x^3 + 3*(a^2 -
b^2)*d^3*e*f^2*x^2 + 3*(a^2 - b^2)*d^3*e^2*f*x + 3*(a^2 - b^2)*c*d^2*e^2*f
 - 3*(a^2 - b^2)*c^2*d*e*f^2 + (a^2 - b^2)*c^3*f^3)*log(1/2*(-2*I*a*cos(d*x
 + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2
 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) + ((a^2 - b^2)*d^3*f^3*x^3 + 3*(a^2 - b^
2)*d^3*e*f^2*x^2 + 3*(a^2 - b^2)*d^3*e^2*f*x + 3*(a^2 - b^2)*c*d^2*e^2*f -
3*(a^2 - b^2)*c^2*d*e*f^2 + (a^2 - b^2)*c^3*f^3)*log(1/2*(-2*I*a*cos(d*x +
c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2) + 2*b)/b)*sin(d*x + c) + (b^2*d^3*f^3*x^3 + b^2*d^3*e^3 + 3*a*b*d
^2*e^2*f + 3*(b^2*d^3*e*f^2 + a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d
^2*e*f^2)*x)*log(cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) + (b^2*d^3
*f^3*x^3 + b^2*d^3*e^3 + 3*a*b*d^2*e^2*f + 3*(b^2*d^3*e*f^2 + a*b*d^2*f^3)*
x^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^2)*x)*log(cos(d*x + c) - I*sin(d*x +
 c) + 1)*sin(d*x + c) + (b^2*d^3*e^3 - 3*(b^2*c + a*b)*d^2*e^2*f + 3*(b^2*c
^2 + 2*a*b*c)*d*e*f^2 - (b^2*c^3 + 3*a*b*c^2)*f^3)*log(-1/2*cos(d*x + c) +
1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + (b^2*d^3*e^3 - 3*(b^2*c + a*b)*d^2
*e^2*f + 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 - (b^2*c^3 + 3*a*b*c^2)*f^3)*log(-1/
2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + (b^2*d^3*f^3*x^3
+ 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 + 3*a*b*c^2)
*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f - 2*a*b*d^2*e
*f^2)*x)*log(-cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) + (b^2*d^3*f^
3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 + 3*a*
b*c^2)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f - 2*a*b
*d^2*e*f^2)*x)*log(-cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) + 6*((a
^2 - b^2)*d*f^3*x + (a^2 - b^2)*d*e*f^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c)
 - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^
2)/b^2))/b)*sin(d*x + c) + 6*((a^2 - b^2)*d*f^3*x + (a^2 - b^2)*d*e*f^2)*po
lylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I
*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 6*((a^2 - b^2)*d
```

```
*f^3*x + (a^2 - b^2)*d*e*f^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c
) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x
+ c) + 6*((a^2 - b^2)*d*f^3*x + (a^2 - b^2)*d*e*f^2)*polylog(3, -(I*a*cos(d
*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2))/b)*sin(d*x + c) + 6*(b^2*d*f^3*x + b^2*d*e*f^2 - a*b*f^3)*polyl
og(3, cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 6*(b^2*d*f^3*x + b^2*d*
e*f^2 - a*b*f^3)*polylog(3, cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 6
*(b^2*d*f^3*x + b^2*d*e*f^2 + a*b*f^3)*polylog(3, -cos(d*x + c) + I*sin(d*x
 + c))*sin(d*x + c) + 6*(b^2*d*f^3*x + b^2*d*e*f^2 + a*b*f^3)*polylog(3, -c
os(d*x + c) - I*sin(d*x + c))*sin(d*x + c))/(a^2*b*d^4*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e+fx\right)^{3}\cos\left(c+dx\right)\cot^{2}\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*cos(c + d*x)*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.338 $\int \frac{(e+fx)^2\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=616

$$\frac{2if\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd^2}+\frac{2if\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2bd^2}-\frac{2f^2\left(a^2-b^2\right)\text{PolyLog}\left(3\right.}{a^2bd^3}$$

```
[Out] ((I/3)*b*(e + f*x)^3)/(a^2*f) + ((I/3)*(a^2 - b^2)*(e + f*x)^3)/(a^2*b*f) -
 (4*f*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d^2) - ((e + f*x)^2*Csc[c + d*
x])/(a*d) - ((a^2 - b^2)*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqr
t[a^2 - b^2])])/(a^2*b*d) - ((a^2 - b^2)*(e + f*x)^2*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d) - (b*(e + f*x)^2*Log[1 - E^((2*I)
*(c + d*x))])/(a^2*d) + ((2*I)*f^2*PolyLog[2, -E^(I*(c + d*x))])/(a*d^3) -
((2*I)*f^2*PolyLog[2, E^(I*(c + d*x))])/(a*d^3) + ((2*I)*(a^2 - b^2)*f*(e +
 f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b*d^2)
+ ((2*I)*(a^2 - b^2)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/(a^2*b*d^2) + (I*b*f*(e + f*x)*PolyLog[2, E^((2*I)*(c + d*x)
)])/(a^2*d^2) - (2*(a^2 - b^2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(a^2*b*d^3) - (2*(a^2 - b^2)*f^2*PolyLog[3, (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d^3) - (b*f^2*PolyLog[3, E^((2*I)*(c
 + d*x))])/(2*a^2*d^3)
```

Rubi [A] time = 1.38718, antiderivative size = 616, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 17, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4543, 4408, 3296, 2637, 4410, 4183, 2279, 2391, 4404, 3310, 3717, 2190, 2531, 2282, 6589, 4525, 4519}

$$\frac{2if\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd^2}+\frac{2if\left(a^2-b^2\right)(e+fx)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2bd^2}-\frac{2f^2\left(a^2-b^2\right)\text{PolyLog}\left(3\right.}{a^2bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] ((I/3)*b*(e + f*x)^3)/(a^2*f) + ((I/3)*(a^2 - b^2)*(e + f*x)^3)/(a^2*b*f) -
 (4*f*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a*d^2) - ((e + f*x)^2*Csc[c + d*
x])/(a*d) - ((a^2 - b^2)*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqr
t[a^2 - b^2])])/(a^2*b*d) - ((a^2 - b^2)*(e + f*x)^2*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d) - (b*(e + f*x)^2*Log[1 - E^((2*I)
*(c + d*x))])/(a^2*d) + ((2*I)*f^2*PolyLog[2, -E^(I*(c + d*x))])/(a*d^3) -
```

```
((2*I)*f^2*PolyLog[2, E^(I*(c + d*x))])/(a*d^3) + ((2*I)*(a^2 - b^2)*f*(e +
 f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b*d^2)
+ ((2*I)*(a^2 - b^2)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/(a^2*b*d^2) + (I*b*f*(e + f*x)*PolyLog[2, E^((2*I)*(c + d*x)
)])/(a^2*d^2) - (2*(a^2 - b^2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sq
rt[a^2 - b^2])])/(a^2*b*d^3) - (2*(a^2 - b^2)*f^2*PolyLog[3, (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b*d^3) - (b*f^2*PolyLog[3, E^((2*I)*(c
 + d*x))])/(2*a^2*d^3)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\cos(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{\int (e+fx)^2\cos(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\cot(c+dx)\csc(c+dx)\,dx}{a} - \frac{b\int (e$$

$$= -\frac{(e+fx)^2\csc(c+dx)}{ad} - \frac{(e+fx)^2\sin(c+dx)}{ad} + \frac{\int (e+fx)^2\cos(c+dx)\,dx}{a}$$

$$= \frac{ib(e+fx)^3}{3a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3bf} - \frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2} - \frac{2f(e+f$$

$$= \frac{ib(e+fx)^3}{3a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3bf} - \frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2} - \frac{(e+fx)^2$$

$$= \frac{ib(e+fx)^3}{3a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3bf} - \frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2} - \frac{(e+fx)^2$$

$$= \frac{ib(e+fx)^3}{3a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3bf} - \frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2} - \frac{(e+fx)^2$$

$$= \frac{ib(e+fx)^3}{3a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3bf} - \frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2} - \frac{(e+fx)^2$$

Mathematica [B] time = 14.0071, size = 1833, normalized size = 2.98

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((2*I)*b*(e + f*x)^3)/((-1 + E^((2*I)*c))*f) + (6*f*(-(b*d*e) + a*f)*x*Log
[1 - E^((-I)*(c + d*x))])/d^2 - (3*b*f^2*x^2*Log[1 - E^((-I)*(c + d*x))])/d
 - (6*f*(b*d*e + a*f)*x*Log[1 + E^((-I)*(c + d*x))])/d^2 - (3*b*f^2*x^2*Log
[1 + E^((-I)*(c + d*x))])/d + ((3*I)*e*(b*d*e - 2*a*f)*(d*x + I*Log[1 - E^(
I*(c + d*x))]))/d^2 + ((3*I)*e*(b*d*e + 2*a*f)*(d*x + I*Log[1 + E^(I*(c + d
*x))]))/d^2 - ((6*I)*f*(b*d*e + a*f)*PolyLog[2, -E^((-I)*(c + d*x))])/d^3 +
 ((6*I)*f*(-(b*d*e) + a*f)*PolyLog[2, E^((-I)*(c + d*x))])/d^3 - ((6*I)*b*f
^2*(d*x*PolyLog[2, -E^((-I)*(c + d*x))] - I*PolyLog[3, -E^((-I)*(c + d*x))]
))/d^3 - ((6*I)*b*f^2*(d*x*PolyLog[2, E^((-I)*(c + d*x))] - I*PolyLog[3, E^
```

```
((-I)*(c + d*x))]]))/d^3)/(3*a^2) + ((a^2 - b^2)*((12*I)*d^3*e^2*E^((2*I)*c)
*x + (12*I)*d^3*e*E^((2*I)*c)*f*x^2 + (4*I)*d^3*E^((2*I)*c)*f^2*x^3 + (6*I)
*d^2*e^2*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] - (6*
I)*d^2*e^2*E^((2*I)*c)*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c +
d*x))))] + 3*d^2*e^2*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c
+ d*x)))^2] - 3*d^2*e^2*E^((2*I)*c)*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1
 + E^((2*I)*(c + d*x)))^2] + 12*d^2*e*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*
a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d^2*e*E^((2*I)*c)*f*x*Log
[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
+ 6*d^2*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b
^2)*E^((2*I)*c)])] - 6*d^2*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x))
)/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*d^2*e*f*x*Log[1 + (b
*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d^
2*e*E^((2*I)*c)*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2
 + b^2)*E^((2*I)*c)])] + 6*d^2*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E
^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 6*d^2*E^((2*I)*c)*f^2*x^2*Log[1
 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] +
(12*I)*d*(-1 + E^((2*I)*c))*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/
(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (12*I)*d*(-1 + E^((2*I)*c
))*f*(e + f*x)*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2
 + b^2)*E^((2*I)*c)]))] + 12*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I
*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*E^((2*I)*c)*f^2*PolyLog[3, (I
*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*
f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^(
(2*I)*c)]))] - 12*E^((2*I)*c)*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E
^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]))/(6*a^2*b*d^3*(-1 + E^((2*I)*c)
)) + ((-3*b*e^2 - 6*b*e*f*x - 3*b*f^2*x^2 - 3*a*d*e^2*x*Cos[c] - 3*a*d*e*f*
x^2*Cos[c] - a*d*f^2*x^3*Cos[c])*Csc[c/2]*Sec[c/2])/(6*a*b*d) + (Sec[c/2]*S
ec[c/2 + (d*x)/2]*(-(e^2*Sin[(d*x)/2]) - 2*e*f*x*Sin[(d*x)/2] - f^2*x^2*Sin
[(d*x)/2]))/(2*a*d) + (Csc[c/2]*Csc[c/2 + (d*x)/2]*(e^2*Sin[(d*x)/2] + 2*e*
f*x*Sin[(d*x)/2] + f^2*x^2*Sin[(d*x)/2]))/(2*a*d)
```

Maple [F] time = 2.075, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2}\cos\left(dx+c\right)\left(\cot\left(dx+c\right)\right)^{2}}{a+b\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.5389, size = 6120, normalized size = 9.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/2*(2*a*b*d^2*f^2*x^2 + 4*a*b*d^2*e*f*x + 2*a*b*d^2*e^2 + 2*b^2*f^2*polyl
og(3, cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 2*b^2*f^2*polylog(3, co
s(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 2*b^2*f^2*polylog(3, -cos(d*x +
 c) + I*sin(d*x + c))*sin(d*x + c) + 2*b^2*f^2*polylog(3, -cos(d*x + c) - I
*sin(d*x + c))*sin(d*x + c) + 2*(a^2 - b^2)*f^2*polylog(3, 1/2*(2*I*a*cos(d
*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2))/b)*sin(d*x + c) + 2*(a^2 - b^2)*f^2*polylog(3, 1/2*(2*I*a*c
os(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt
(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 2*(a^2 - b^2)*f^2*polylog(3, -(I*a*co
s(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^
2 - b^2)/b^2))/b)*sin(d*x + c) + 2*(a^2 - b^2)*f^2*polylog(3, -(I*a*cos(d*x
 + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2))/b)*sin(d*x + c) - (-2*I*(a^2 - b^2)*d*f^2*x - 2*I*(a^2 - b^2)*d*e
*f)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) -
 I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) - (-2*
I*(a^2 - b^2)*d*f^2*x - 2*I*(a^2 - b^2)*d*e*f)*dilog(-1/2*(2*I*a*cos(d*x +
c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) - (2*I*(a^2 - b^2)*d*f^2*x + 2*I*(a^2
```

```
- b^2)*d*e*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos
(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x
+ c) - (2*I*(a^2 - b^2)*d*f^2*x + 2*I*(a^2 - b^2)*d*e*f)*dilog(-1/2*(-2*I*a
*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) - (2*I*b^2*d*f^2*x + 2*I*b^
2*d*e*f - 2*I*a*b*f^2)*dilog(cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) -
(-2*I*b^2*d*f^2*x - 2*I*b^2*d*e*f + 2*I*a*b*f^2)*dilog(cos(d*x + c) - I*sin
(d*x + c))*sin(d*x + c) - (-2*I*b^2*d*f^2*x - 2*I*b^2*d*e*f - 2*I*a*b*f^2)*
dilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - (2*I*b^2*d*f^2*x + 2*I
*b^2*d*e*f + 2*I*a*b*f^2)*dilog(-cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c
) + ((a^2 - b^2)*d^2*e^2 - 2*(a^2 - b^2)*c*d*e*f + (a^2 - b^2)*c^2*f^2)*log
(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a
)*sin(d*x + c) + ((a^2 - b^2)*d^2*e^2 - 2*(a^2 - b^2)*c*d*e*f + (a^2 - b^2)
*c^2*f^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)
/b^2) - 2*I*a)*sin(d*x + c) + ((a^2 - b^2)*d^2*e^2 - 2*(a^2 - b^2)*c*d*e*f
+ (a^2 - b^2)*c^2*f^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqr
t(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) + ((a^2 - b^2)*d^2*e^2 - 2*(a^2 -
 b^2)*c*d*e*f + (a^2 - b^2)*c^2*f^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x
+ c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) + ((a^2 - b^2)*d^2*
f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x + 2*(a^2 - b^2)*c*d*e*f - (a^2 - b^2)*c^2
*f^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) -
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) + ((a^2 - b
^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x + 2*(a^2 - b^2)*c*d*e*f - (a^2 -
b^2)*c^2*f^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x
 + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) + (
(a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x + 2*(a^2 - b^2)*c*d*e*f -
 (a^2 - b^2)*c^2*f^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(
b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x
 + c) + ((a^2 - b^2)*d^2*f^2*x^2 + 2*(a^2 - b^2)*d^2*e*f*x + 2*(a^2 - b^2)*
c*d*e*f - (a^2 - b^2)*c^2*f^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x +
 c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b
)*sin(d*x + c) + (b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f + 2*(b^2*d^2*
e*f + a*b*d*f^2)*x)*log(cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) + (
b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f + 2*(b^2*d^2*e*f + a*b*d*f^2)*x
)*log(cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) + (b^2*d^2*e^2 - 2*(b
^2*c + a*b)*d*e*f + (b^2*c^2 + 2*a*b*c)*f^2)*log(-1/2*cos(d*x + c) + 1/2*I*
sin(d*x + c) + 1/2)*sin(d*x + c) + (b^2*d^2*e^2 - 2*(b^2*c + a*b)*d*e*f + (
b^2*c^2 + 2*a*b*c)*f^2)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2)*s
in(d*x + c) + (b^2*d^2*f^2*x^2 + 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c)*f^2 +
2*(b^2*d^2*e*f - a*b*d*f^2)*x)*log(-cos(d*x + c) + I*sin(d*x + c) + 1)*sin(
d*x + c) + (b^2*d^2*f^2*x^2 + 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c)*f^2 + 2*(
b^2*d^2*e*f - a*b*d*f^2)*x)*log(-cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x
 + c))/(a^2*b*d^3*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2 \cos (c+dx) \cot ^2(c+dx)}{a+b \sin (c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*cos(c + d*x)*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.339 $\int \frac{(e+fx)\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=386

$$\frac{if\left(a^2-b^2\right)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd^2}+\frac{if\left(a^2-b^2\right)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2bd^2}+\frac{ibf\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{2a^2d^2}-\frac{\left(a^2-b^2\right)(e+fx)}{a^2}$$

```
[Out] ((I/2)*b*(e + f*x)^2)/(a^2*f) + ((I/2)*(a^2 - b^2)*(e + f*x)^2)/(a^2*b*f) -
 (f*ArcTanh[Cos[c + d*x]])/(a*d^2) - ((e + f*x)*Csc[c + d*x])/(a*d) - ((a^2
 - b^2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^
2*b*d) - ((a^2 - b^2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/(a^2*b*d) - (b*(e + f*x)*Log[1 - E^((2*I)*(c + d*x))])/(a^2*d) +
 (I*(a^2 - b^2)*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/
(a^2*b*d^2) + (I*(a^2 - b^2)*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a
^2 - b^2])])/(a^2*b*d^2) + ((I/2)*b*f*PolyLog[2, E^((2*I)*(c + d*x))])/(a^2
*d^2)
```

Rubi [A] time = 0.784269, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 15, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.469, Rules used = {4543, 4408, 3296, 2638, 4410, 3770, 4404, 2635, 8, 3717, 2190, 2279, 2391, 4525, 4519}

$$\frac{if\left(a^2-b^2\right)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2bd^2}+\frac{if\left(a^2-b^2\right)\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2bd^2}+\frac{ibf\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{2a^2d^2}-\frac{\left(a^2-b^2\right)(e+fx)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((I/2)*b*(e + f*x)^2)/(a^2*f) + ((I/2)*(a^2 - b^2)*(e + f*x)^2)/(a^2*b*f) -
 (f*ArcTanh[Cos[c + d*x]])/(a*d^2) - ((e + f*x)*Csc[c + d*x])/(a*d) - ((a^2
 - b^2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^
2*b*d) - ((a^2 - b^2)*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/(a^2*b*d) - (b*(e + f*x)*Log[1 - E^((2*I)*(c + d*x))])/(a^2*d) +
 (I*(a^2 - b^2)*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/
(a^2*b*d^2) + (I*(a^2 - b^2)*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a
^2 - b^2])])/(a^2*b*d^2) + ((I/2)*b*f*PolyLog[2, E^((2*I)*(c + d*x))])/(a^2
*d^2)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
```

```
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
```

```
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rubi steps

$$\int \frac{(e+fx)\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\cos(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{\int (e+fx)\cos(c+dx)\,dx}{a} + \frac{\int (e+fx)\cot(c+dx)\csc(c+dx)\,dx}{a} - \frac{b\int (e+$$

$$= -\frac{(e+fx)\csc(c+dx)}{ad} - \frac{(e+fx)\sin(c+dx)}{ad} + \frac{\int (e+fx)\cos(c+dx)\,dx}{a} - \frac{b}{}$$

$$= \frac{ib(e+fx)^2}{2a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^2}{2bf} - \frac{f\tanh^{-1}(\cos(c+dx))}{ad^2} - \frac{f\cos(c+dx)}{ad^2} -$$

$$= \frac{ib(e+fx)^2}{2a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^2}{2bf} - \frac{f\tanh^{-1}(\cos(c+dx))}{ad^2} - \frac{(e+fx)\csc(c+}{ad}$$

$$= \frac{ib(e+fx)^2}{2a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^2}{2bf} - \frac{f\tanh^{-1}(\cos(c+dx))}{ad^2} - \frac{(e+fx)\csc(c+}{ad}$$

$$= \frac{ib(e+fx)^2}{2a^2f} + \frac{i\left(1-\frac{b^2}{a^2}\right)(e+fx)^2}{2bf} - \frac{f\tanh^{-1}(\cos(c+dx))}{ad^2} - \frac{(e+fx)\csc(c+}{ad}$$

Mathematica [B] time = 14.7458, size = 2314, normalized size = 5.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-(d*e*Cos[(c + d*x)/2]) + c*f*Cos[(c + d*x)/2] - f*(c + d*x)*Cos[(c + d*x
)/2])*Csc[(c + d*x)/2])/(2*a*d^2) - (b*e*Log[Sin[c + d*x]])/(a^2*d) + (b*c*
f*Log[Sin[c + d*x]])/(a^2*d^2) + (f*Log[Tan[(c + d*x)/2]])/(a*d^2) - (b*f*(
(c + d*x)*Log[1 - E^((2*I)*(c + d*x))] - (I/2)*((c + d*x)^2 + PolyLog[2, E^
((2*I)*(c + d*x))])))/(a^2*d^2) + (Sec[(c + d*x)/2]*(-(d*e*Sin[(c + d*x)/2]
) + c*f*Sin[(c + d*x)/2] - f*(c + d*x)*Sin[(c + d*x)/2]))/(2*a*d^2) + ((f*(
c + d*x)^2 + (2*I)*d*e*Log[Sec[(c + d*x)/2]^2] - (2*I)*c*f*Log[Sec[(c + d*x
)/2]^2] - (2*I)*d*e*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])] + (2*I)*c*
f*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])] - (4*I)*f*(c + d*x)*Log[(-2*
I)/(-I + Tan[(c + d*x)/2])] - 2*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt
[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] + 2*f*Log[
1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(
I*a - b + Sqrt[-a^2 + b^2]))] + 2*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sq
rt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])] - 2*f
*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]
)/(I*a + b + Sqrt[-a^2 + b^2])] + 4*f*PolyLog[2, -Cos[c + d*x] + I*Sin[c +
d*x]] + 2*f*PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 +
 b^2]))] - 2*f*PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^
2 + b^2]))] + 2*f*PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^
2 + b^2])] - 2*f*PolyLog[2, (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a
^2 + b^2]))])*(-((e*Cos[c + d*x])/(a + b*Sin[c + d*x])) + (b^2*e*Cos[c + d*
x])/(a^2*(a + b*Sin[c + d*x])) + (c*f*Cos[c + d*x])/(d*(a + b*Sin[c + d*x])
) - (b^2*c*f*Cos[c + d*x])/(a^2*d*(a + b*Sin[c + d*x])) - (f*(c + d*x)*Cos[
c + d*x])/(d*(a + b*Sin[c + d*x])) + (b^2*f*(c + d*x)*Cos[c + d*x])/(a^2*d*
(a + b*Sin[c + d*x]))))/(d*(2*f*(c + d*x) - (4*I)*f*Log[(-2*I)/(-I + Tan[(c
 + d*x)/2])] - (4*f*Log[1 + Cos[c + d*x] - I*Sin[c + d*x]]*(I*Cos[c + d*x]
+ Sin[c + d*x]))/(-Cos[c + d*x] + I*Sin[c + d*x]) + (I*f*Log[1 - (a*(1 - I*
Tan[(c + d*x)/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 -
 I*Tan[(c + d*x)/2]) - (I*f*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2
])/(I*a - b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/
2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sq
rt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) + (I*f*Log[1
- (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x
)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(
c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(
c + d*x)/2]) - (I*f*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a +
b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) + (2*I)
*d*e*Tan[(c + d*x)/2] - (2*I)*c*f*Tan[(c + d*x)/2] + ((2*I)*f*(c + d*x)*Sec
[(c + d*x)/2]^2)/(-I + Tan[(c + d*x)/2]) - (f*Log[1 - (a*(I + Tan[(c + d*x)
/2]))/(I*a - b + Sqrt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(I + Tan[(c + d*x)/
2]) + (I*a*f*Log[1 - (a + I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^
2]))]*Sec[(c + d*x)/2]^2)/(a + I*a*Tan[(c + d*x)/2]) + (a*f*Log[1 - I*Tan[(
```

```
c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]
) - (a*f*Log[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b - Sqrt[-a^2 + b
^2] + a*Tan[(c + d*x)/2]) + (a*f*Log[1 - I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/
2]^2)/(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) - (a*f*Log[1 + I*Tan[(c +
 d*x)/2]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) -
 ((2*I)*d*e*Cos[(c + d*x)/2]^2*(b*Cos[c + d*x]*Sec[(c + d*x)/2]^2 + Sec[(c
+ d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c + d*x)/2]))/(a + b*Sin[c + d*x]) +
((2*I)*c*f*Cos[(c + d*x)/2]^2*(b*Cos[c + d*x]*Sec[(c + d*x)/2]^2 + Sec[(c +
 d*x)/2]^2*(a + b*Sin[c + d*x])*Tan[(c + d*x)/2]))/(a + b*Sin[c + d*x]))
```

Maple [B] time = 0.381, size = 1732, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2*I/b*f*x^2-2*I*(f*x+e)*exp(I*(d*x+c))/d/a/(exp(2*I*(d*x+c))-1)+1/b/d^2*f
*c*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-2/b/d^2*f*c*ln(exp(I*(d*
x+c)))-I/b*e*x+1/d*b/a^2*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+
I/d^2/b*c^2*f-2*b/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))
/(I*a-(-a^2+b^2)^(1/2)))*x-2*b/d^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-
a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-2*b/d*f/(-a^2+b^2)*ln((I*a+b*exp(
I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-2*b/d^2*f/(-a^2+b^2)
*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-1/b/d
*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+2/b/d*ln(exp(I*(d*x+c)))
*e+1/d^2/a^2*b*f*c*ln(exp(I*(d*x+c))-1)-1/d/a^2*b*f*ln(exp(I*(d*x+c))+1)*x-
I/d^2/a^2*b*f*dilog(exp(I*(d*x+c)))+1/b/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x
+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2*x+1/b/d^2*f/(-a^2+b^2)*l
n((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2*c+1/b
/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^
(1/2)))*a^2*x+1/b/d^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2
))/(I*a-(-a^2+b^2)^(1/2)))*a^2*c-1/d^2*b/a^2*f*c*ln(I*b*exp(2*I*(d*x+c))-2*
a*exp(I*(d*x+c))-I*b)+I/d^2*b/a^2*f*dilog(exp(I*(d*x+c))+1)+2*I/d^2*b*f/(-a
^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)
))+2*I/d^2*b*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*
a+(-a^2+b^2)^(1/2)))+2*I/d/b*c*f*x-1/d/a^2*b*e*ln(exp(I*(d*x+c))-1)-1/d/a^2
*b*e*ln(exp(I*(d*x+c))+1)-I/b/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+
(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2-I/b/d^2*f/(-a^2+b^2)*dilog((I
*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*a^2+1/d^2/a*f
*ln(exp(I*(d*x+c))-1)-1/d^2/a*f*ln(exp(I*(d*x+c))+1)+1/d^2*b^3/a^2*f/(-a^2+
```

```
b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+1
/d^2*b^3/a^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(
-a^2+b^2)^(1/2)))*c+1/d*b^3/a^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2
+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+1/d*b^3/a^2*f/(-a^2+b^2)*ln((I*a+b*e
xp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-I/d^2*b^3/a^2*f/(
-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/
2)))-I/d^2*b^3/a^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2
))/(I*a+(-a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.48655, size = 3592, normalized size = 9.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*a*b*d*f*x - I*b^2*f*dilog(cos(d*x + c) + I*sin(d*x + c))*sin(d*x +
c) + I*b^2*f*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + I*b^2*f*di
log(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - I*b^2*f*dilog(-cos(d*x +
 c) - I*sin(d*x + c))*sin(d*x + c) + 2*a*b*d*e + I*(a^2 - b^2)*f*dilog(-1/2
*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x +
 c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) + I*(a^2 - b^2)*f*di
log(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) - I*(a^2 - b
^2)*f*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c
) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) - I
*(a^2 - b^2)*f*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*co
```

```
s(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x
 + c) + ((a^2 - b^2)*d*e - (a^2 - b^2)*c*f)*log(2*b*cos(d*x + c) + 2*I*b*si
n(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) + ((a^2 - b^2
)*d*e - (a^2 - b^2)*c*f)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sq
rt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) + ((a^2 - b^2)*d*e - (a^2 - b^2)
*c*f)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^
2) + 2*I*a)*sin(d*x + c) + ((a^2 - b^2)*d*e - (a^2 - b^2)*c*f)*log(-2*b*cos
(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*
x + c) + ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(1/2*(2*I*a*cos(d*x + c)
+ 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) + 2*b)/b)*sin(d*x + c) + ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(1
/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x
 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) + ((a^2 - b^2)*d*f*x +
 (a^2 - b^2)*c*f)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*co
s(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c
) + ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(1/2*(-2*I*a*cos(d*x + c) + 2*
a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2) + 2*b)/b)*sin(d*x + c) + (b^2*d*f*x + b^2*d*e + a*b*f)*log(cos(d*x + c)
+ I*sin(d*x + c) + 1)*sin(d*x + c) + (b^2*d*f*x + b^2*d*e + a*b*f)*log(cos(
d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) + (b^2*d*e - (b^2*c + a*b)*f)*l
og(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + (b^2*d*e -
(b^2*c + a*b)*f)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2)*sin(d*x
+ c) + (b^2*d*f*x + b^2*c*f)*log(-cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*
x + c) + (b^2*d*f*x + b^2*c*f)*log(-cos(d*x + c) - I*sin(d*x + c) + 1)*sin(
d*x + c))/(a^2*b*d^2*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \cos(c + dx) \cot^2(c + dx)}{a + b \sin(c + dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cos(c + d*x)*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cos(dx + c) \cot(dx + c)^2}{b \sin(dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((f*x + e)*cos(d*x + c)*cot(d*x + c)^2/(b*sin(d*x + c) + a), x)
```

3.340 $\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=60

$$-\frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\csc(c+dx)}{ad}$$

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) - ((1 - b^2/a^2)*Log[
a + b*Sin[c + d*x]])/(b*d)
```

Rubi [A] time = 0.123461, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$-\frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) - ((1 - b^2/a^2)*Log[
a + b*Sin[c + d*x]])/(b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2(b^2-x^2)}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{b^2}{ax^2}-\frac{b^2}{a^2x}+\frac{-a^2+b^2}{a^2(a+x)}\right)dx, x, b\sin(c+dx)\right)}{bd}\\ &= -\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}\end{aligned}$$

Mathematica [A] time = 0.0937522, size = 54, normalized size = 0.9

$$\frac{\left(b^2-a^2\right)\log(a+b\sin(c+dx))-ab\csc(c+dx)+b^2(-\log(\sin(c+dx)))}{a^2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(a*b*Csc[c + d*x]) - b^2*Log[Sin[c + d*x]] + (-a^2 + b^2)*Log[a + b*Sin[c
 + d*x]])/(a^2*b*d)
```

Maple [A] time = 0.003, size = 72, normalized size = 1.2

$$-\frac{\ln(a+b\sin(dx+c))}{bd}+\frac{b\ln(a+b\sin(dx+c))}{da^2}-\frac{1}{da\sin(dx+c)}-\frac{b\ln(\sin(dx+c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -ln(a+b*sin(d*x+c))/b/d+1/d/a^2*b*ln(a+b*sin(d*x+c))-1/d/a/sin(d*x+c)-b*ln(
sin(d*x+c))/a^2/d
```

Maxima [A] time = 0.973981, size = 77, normalized size = 1.28

$$-\frac{\frac{b\log(\sin(dx+c))}{a^2}+\frac{(a^2-b^2)\log(b\sin(dx+c)+a)}{a^2b}+\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b*log(sin(d*x + c))/a^2 + (a^2 - b^2)*log(b*sin(d*x + c) + a)/(a^2*b) + 1
/(a*sin(d*x + c)))/d
```

Fricas [A] time = 1.95147, size = 166, normalized size = 2.77

$$-\frac{b^2\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+\left(a^2-b^2\right)\log(b\sin(dx+c)+a)\sin(dx+c)+ab}{a^2bd\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(b^2*log(1/2*sin(d*x + c))*sin(d*x + c) + (a^2 - b^2)*log(b*sin(d*x + c) +
 a)*sin(d*x + c) + a*b)/(a^2*b*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.15671, size = 97, normalized size = 1.62

$$-\frac{\frac{b\log(|\sin(dx+c)|)}{a^2}+\frac{(a^2-b^2)\log(|b\sin(dx+c)+a|)}{a^2b}-\frac{b\sin(dx+c)-a}{a^2\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*log(abs(sin(d*x + c)))/a^2 + (a^2 - b^2)*log(abs(b*sin(d*x + c) + a))/(
a^2*b) - (b*sin(d*x + c) - a)/(a^2*sin(d*x + c)))/d
```

3.341 $\int \frac{(e+fx)^3 \cos^2(c+dx) \cot^2(c+dx)}{a+b \sin(c+dx)} \, dx$

Optimal. Leaf size=1144

result too large to display

```
[Out] ((-I)*(e + f*x)^3)/(a*d) - (e + f*x)^4/(4*a*f) - ((a^2 - b^2)*(e + f*x)^4)/
(4*a*b^2*f) + (2*b*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a^2*d) + (6*b*f^2
*(e + f*x)*Cos[c + d*x])/(a^2*d^3) + (6*(a^2 - b^2)*f^2*(e + f*x)*Cos[c + d
*x])/(a^2*b*d^3) - (b*(e + f*x)^3*Cos[c + d*x])/(a^2*d) - ((a^2 - b^2)*(e +
 f*x)^3*Cos[c + d*x])/(a^2*b*d) - ((e + f*x)^3*Cot[c + d*x])/(a*d) - (I*(a^
2 - b^2)^(3/2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(a^2*b^2*d) + (I*(a^2 - b^2)^(3/2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^2*d) + (3*f*(e + f*x)^2*Log[1 - E^((
2*I)*(c + d*x))])/(a*d^2) - ((3*I)*b*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*
x))])/(a^2*d^2) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a^2*
d^2) - (3*(a^2 - b^2)^(3/2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/
(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d^2) + (3*(a^2 - b^2)^(3/2)*f*(e + f*x)^2*
PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^2*d^2) - ((
3*I)*f^2*(e + f*x)*PolyLog[2, E^((2*I)*(c + d*x))])/(a*d^3) + (6*b*f^2*(e +
 f*x)*PolyLog[3, -E^(I*(c + d*x))])/(a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[
3, E^(I*(c + d*x))])/(a^2*d^3) - ((6*I)*(a^2 - b^2)^(3/2)*f^2*(e + f*x)*Pol
yLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d^3) + ((6*I
)*(a^2 - b^2)^(3/2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqr
t[a^2 - b^2])])/(a^2*b^2*d^3) + (3*f^3*PolyLog[3, E^((2*I)*(c + d*x))])/(2*
a*d^4) + ((6*I)*b*f^3*PolyLog[4, -E^(I*(c + d*x))])/(a^2*d^4) - ((6*I)*b*f^
3*PolyLog[4, E^(I*(c + d*x))])/(a^2*d^4) + (6*(a^2 - b^2)^(3/2)*f^3*PolyLog
[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d^4) - (6*(a^2 -
 b^2)^(3/2)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a
^2*b^2*d^4) - (6*b*f^3*Sin[c + d*x])/(a^2*d^4) - (6*(a^2 - b^2)*f^3*Sin[c +
 d*x])/(a^2*b*d^4) + (3*b*f*(e + f*x)^2*Sin[c + d*x])/(a^2*d^2) + (3*(a^2 -
 b^2)*f*(e + f*x)^2*Sin[c + d*x])/(a^2*b*d^2)
```

Rubi [A] time = 2.6563, antiderivative size = 1144, normalized size of antiderivative = 1., number of steps used = 66, number of rules used = 20, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4543, 4408, 3311, 32, 3310, 3720, 3717, 2190, 2531, 2282, 6589, 4405, 3296, 2637, 2633, 4183, 6609, 4525, 3323, 2264}

$$-\frac{\left(a^2-b^2\right)(e+fx)^4}{4ab^2f}-\frac{(e+fx)^4}{4af}+\frac{2b\tanh^{-1}\left(e^{i(c+dx)}\right)(e+fx)^3}{a^2d}-\frac{b\cos(c+dx)(e+fx)^3}{a^2d}-\frac{\left(a^2-b^2\right)\cos(c+dx)(e+f}{a^2bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-I)*(e + f*x)^3)/(a*d) - (e + f*x)^4/(4*a*f) - ((a^2 - b^2)*(e + f*x)^4)/
(4*a*b^2*f) + (2*b*(e + f*x)^3*ArcTanh[E^(I*(c + d*x))])/(a^2*d) + (6*b*f^2
*(e + f*x)*Cos[c + d*x])/(a^2*d^3) + (6*(a^2 - b^2)*f^2*(e + f*x)*Cos[c + d
*x])/(a^2*b*d^3) - (b*(e + f*x)^3*Cos[c + d*x])/(a^2*d) - ((a^2 - b^2)*(e +
 f*x)^3*Cos[c + d*x])/(a^2*b*d) - ((e + f*x)^3*Cot[c + d*x])/(a*d) - (I*(a^
2 - b^2)^(3/2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^
2])])/(a^2*b^2*d) + (I*(a^2 - b^2)^(3/2)*(e + f*x)^3*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^2*d) + (3*f*(e + f*x)^2*Log[1 - E^((
2*I)*(c + d*x))])/(a*d^2) - ((3*I)*b*f*(e + f*x)^2*PolyLog[2, -E^(I*(c + d*
x))])/(a^2*d^2) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, E^(I*(c + d*x))])/(a^2*
d^2) - (3*(a^2 - b^2)^(3/2)*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/
(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d^2) + (3*(a^2 - b^2)^(3/2)*f*(e + f*x)^2*
PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^2*d^2) - ((
3*I)*f^2*(e + f*x)*PolyLog[2, E^((2*I)*(c + d*x))])/(a*d^3) + (6*b*f^2*(e +
 f*x)*PolyLog[3, -E^(I*(c + d*x))])/(a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[
3, E^(I*(c + d*x))])/(a^2*d^3) - ((6*I)*(a^2 - b^2)^(3/2)*f^2*(e + f*x)*Pol
yLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d^3) + ((6*I
)*(a^2 - b^2)^(3/2)*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqr
t[a^2 - b^2])])/(a^2*b^2*d^3) + (3*f^3*PolyLog[3, E^((2*I)*(c + d*x))])/(2*
a*d^4) + ((6*I)*b*f^3*PolyLog[4, -E^(I*(c + d*x))])/(a^2*d^4) - ((6*I)*b*f^
3*PolyLog[4, E^(I*(c + d*x))])/(a^2*d^4) + (6*(a^2 - b^2)^(3/2)*f^3*PolyLog
[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d^4) - (6*(a^2 -
 b^2)^(3/2)*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a
^2*b^2*d^4) - (6*b*f^3*Sin[c + d*x])/(a^2*d^4) - (6*(a^2 - b^2)*f^3*Sin[c +
 d*x])/(a^2*b*d^4) + (3*b*f*(e + f*x)^2*Sin[c + d*x])/(a^2*d^2) + (3*(a^2 -
 b^2)*f*(e + f*x)^2*Sin[c + d*x])/(a^2*b*d^2)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
```

```
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\cos^2(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{\int (e+fx)^3\cos^2(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\cot^2(c+dx)\,dx}{a} - \frac{b\int (e+fx)^3\,\mathrm{co}}{}$$

$$= -\frac{3f(e+fx)^2\cos^2(c+dx)}{4ad^2} - \frac{(e+fx)^3\cot(c+dx)}{ad} - \frac{(e+fx)^3\cos(c+dx)\,\mathrm{si}}{2ad}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{3(e+fx)^4}{8af} + \frac{3f^3\cos^2(c+dx)}{8ad^4} - \frac{(e+fx)^3\cot(c+dx)}{ad} + \frac{3f^2(}{}$$

$$= \frac{3ef^2x}{4ad^2} + \frac{3f^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4b^2f} + \frac{2b(e+fx)}{}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4b^2f} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right.}{a^2d}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4b^2f} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right.}{a^2d}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4b^2f} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right.}{a^2d}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4b^2f} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right.}{a^2d}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4b^2f} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right.}{a^2d}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{(e+fx)^4}{4af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^4}{4b^2f} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{i(c+dx)}\right.}{a^2d}$$

Mathematica [B] time = 42.502, size = 3860, normalized size = 3.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),
```

x]

```
[Out] (((-2*I)*a*d^3*(e + f*x)^3)/(-1 + E^((2*I)*c)) - 3*d^2*e*f*(b*d*e - 2*a*f)*
x*Log[1 - E^((-I)*(c + d*x))] - 3*d^2*f^2*(b*d*e - a*f)*x^2*Log[1 - E^((-I)
*(c + d*x))] - b*d^3*f^3*x^3*Log[1 - E^((-I)*(c + d*x))] + 3*d^2*e*f*(b*d*e
 + 2*a*f)*x*Log[1 + E^((-I)*(c + d*x))] + 3*d^2*f^2*(b*d*e + a*f)*x^2*Log[1
 + E^((-I)*(c + d*x))] + b*d^3*f^3*x^3*Log[1 + E^((-I)*(c + d*x))] + I*d^2*
e^2*(b*d*e - 3*a*f)*(d*x + I*Log[1 - E^(I*(c + d*x))]) + d^2*e^2*(b*d*e + 3
*a*f)*((-I)*d*x + Log[1 + E^(I*(c + d*x))]) + (3*I)*d*e*f*(b*d*e + 2*a*f)*P
olyLog[2, -E^((-I)*(c + d*x))] - (3*I)*d*e*f*(b*d*e - 2*a*f)*PolyLog[2, E^(
(-I)*(c + d*x))] + 6*f^2*(b*d*e + a*f)*(I*d*x*PolyLog[2, -E^((-I)*(c + d*x)
)] + PolyLog[3, -E^((-I)*(c + d*x))]) + 6*f^2*(-(b*d*e) + a*f)*(I*d*x*PolyL
og[2, E^((-I)*(c + d*x))] + PolyLog[3, E^((-I)*(c + d*x))]) + 3*b*f^3*(I*d^
2*x^2*PolyLog[2, -E^((-I)*(c + d*x))] + 2*d*x*PolyLog[3, -E^((-I)*(c + d*x)
)] - (2*I)*PolyLog[4, -E^((-I)*(c + d*x))]) - (3*I)*b*f^3*(d^2*x^2*PolyLog[
2, E^((-I)*(c + d*x))] - (2*I)*d*x*PolyLog[3, E^((-I)*(c + d*x))] - 2*PolyL
og[4, E^((-I)*(c + d*x))]))/(a^2*d^4) + (Sqrt[-(a^2 - b^2)^2]*(-2*Sqrt[-a^2
 + b^2]*d^3*e^3*ArcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] - 3*Sqrt[
a^2 - b^2]*d^3*e^2*f*x*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^
2])] - 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x^2*Log[1 - (b*E^(I*(c + d*x)))/((-I)*a
+ Sqrt[-a^2 + b^2])] - Sqrt[a^2 - b^2]*d^3*f^3*x^3*Log[1 - (b*E^(I*(c + d*x
)))/((-I)*a + Sqrt[-a^2 + b^2])] + 3*Sqrt[a^2 - b^2]*d^3*e^2*f*x*Log[1 + (b
*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] + 3*Sqrt[a^2 - b^2]*d^3*e*f^2*x
^2*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] + Sqrt[a^2 - b^2]*
d^3*f^3*x^3*Log[1 + (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])] + (3*I)*S
qrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a + S
qrt[-a^2 + b^2])] - (3*I)*Sqrt[a^2 - b^2]*d^2*f*(e + f*x)^2*PolyLog[2, -((b
*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] - 6*Sqrt[a^2 - b^2]*d*e*f^2*Po
lyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 6*Sqrt[a^2 - b^
2]*d*f^3*x*PolyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + 6*
Sqrt[a^2 - b^2]*d*e*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 +
 b^2]))] + 6*Sqrt[a^2 - b^2]*d*f^3*x*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a
+ Sqrt[-a^2 + b^2]))] - (6*I)*Sqrt[a^2 - b^2]*f^3*PolyLog[4, (b*E^(I*(c + d
*x)))/((-I)*a + Sqrt[-a^2 + b^2])] + (6*I)*Sqrt[a^2 - b^2]*f^3*PolyLog[4, -
((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))]))/(a^2*b^2*d^4) + Csc[c]*Cs
c[c + d*x]*(Cos[c + d*x]/(16*a*b^2*d^4) - ((I/16)*Sin[c + d*x])/(a*b^2*d^4)
)*((8*I)*b^2*d^3*e^3*Cos[c] + (24*I)*b^2*d^3*e^2*f*x*Cos[c] + (24*I)*b^2*d^
3*e*f^2*x^2*Cos[c] + (8*I)*b^2*d^3*f^3*x^3*Cos[c] - 2*a*b*d^3*e^3*Cos[d*x]
+ (18*I)*a*b*d^2*e^2*f*Cos[d*x] + 12*a*b*d*e*f^2*Cos[d*x] - (36*I)*a*b*f^3*
Cos[d*x] - 6*a*b*d^3*e^2*f*x*Cos[d*x] + (36*I)*a*b*d^2*e*f^2*x*Cos[d*x] + 1
2*a*b*d*f^3*x*Cos[d*x] - 6*a*b*d^3*e*f^2*x^2*Cos[d*x] + (18*I)*a*b*d^2*f^3*
x^2*Cos[d*x] - 2*a*b*d^3*f^3*x^3*Cos[d*x] + 2*a*b*d^3*e^3*Cos[2*c + d*x] -
(18*I)*a*b*d^2*e^2*f*Cos[2*c + d*x] - 12*a*b*d*e*f^2*Cos[2*c + d*x] + (36*I
)*a*b*f^3*Cos[2*c + d*x] + 6*a*b*d^3*e^2*f*x*Cos[2*c + d*x] - (36*I)*a*b*d^
```

2*e*f^2*x*Cos[2*c + d*x] - 12*a*b*d*f^3*x*Cos[2*c + d*x] + 6*a*b*d^3*e*f^2*
x^2*Cos[2*c + d*x] - (18*I)*a*b*d^2*f^3*x^2*Cos[2*c + d*x] + 2*a*b*d^3*f^3*
x^3*Cos[2*c + d*x] - (8*I)*b^2*d^3*e^3*Cos[c + 2*d*x] - 4*a^2*d^4*e^3*x*Cos
[c + 2*d*x] - (24*I)*b^2*d^3*e^2*f*x*Cos[c + 2*d*x] - 6*a^2*d^4*e^2*f*x^2*C
os[c + 2*d*x] - (24*I)*b^2*d^3*e*f^2*x^2*Cos[c + 2*d*x] - 4*a^2*d^4*e*f^2*x
^3*Cos[c + 2*d*x] - (8*I)*b^2*d^3*f^3*x^3*Cos[c + 2*d*x] - a^2*d^4*f^3*x^4*
Cos[c + 2*d*x] + 4*a^2*d^4*e^3*x*Cos[3*c + 2*d*x] + 6*a^2*d^4*e^2*f*x^2*Cos
[3*c + 2*d*x] + 4*a^2*d^4*e*f^2*x^3*Cos[3*c + 2*d*x] + a^2*d^4*f^3*x^4*Cos[
3*c + 2*d*x] - 2*a*b*d^3*e^3*Cos[2*c + 3*d*x] - (6*I)*a*b*d^2*e^2*f*Cos[2*c
 + 3*d*x] + 12*a*b*d*e*f^2*Cos[2*c + 3*d*x] + (12*I)*a*b*f^3*Cos[2*c + 3*d*
x] - 6*a*b*d^3*e^2*f*x*Cos[2*c + 3*d*x] - (12*I)*a*b*d^2*e*f^2*x*Cos[2*c +
3*d*x] + 12*a*b*d*f^3*x*Cos[2*c + 3*d*x] - 6*a*b*d^3*e*f^2*x^2*Cos[2*c + 3*
d*x] - (6*I)*a*b*d^2*f^3*x^2*Cos[2*c + 3*d*x] - 2*a*b*d^3*f^3*x^3*Cos[2*c +
 3*d*x] + 2*a*b*d^3*e^3*Cos[4*c + 3*d*x] + (6*I)*a*b*d^2*e^2*f*Cos[4*c + 3*
d*x] - 12*a*b*d*e*f^2*Cos[4*c + 3*d*x] - (12*I)*a*b*f^3*Cos[4*c + 3*d*x] +
6*a*b*d^3*e^2*f*x*Cos[4*c + 3*d*x] + (12*I)*a*b*d^2*e*f^2*x*Cos[4*c + 3*d*x
] - 12*a*b*d*f^3*x*Cos[4*c + 3*d*x] + 6*a*b*d^3*e*f^2*x^2*Cos[4*c + 3*d*x]
+ (6*I)*a*b*d^2*f^3*x^2*Cos[4*c + 3*d*x] + 2*a*b*d^3*f^3*x^3*Cos[4*c + 3*d*
x] - 8*b^2*d^3*e^3*Sin[c] - (8*I)*a^2*d^4*e^3*x*Sin[c] - 24*b^2*d^3*e^2*f*x
*Sin[c] - (12*I)*a^2*d^4*e^2*f*x^2*Sin[c] - 24*b^2*d^3*e*f^2*x^2*Sin[c] - (
8*I)*a^2*d^4*e*f^2*x^3*Sin[c] - 8*b^2*d^3*f^3*x^3*Sin[c] - (2*I)*a^2*d^4*f^
3*x^4*Sin[c] + (2*I)*a*b*d^3*e^3*Sin[d*x] - 6*a*b*d^2*e^2*f*Sin[d*x] - (12*
I)*a*b*d*e*f^2*Sin[d*x] + 12*a*b*f^3*Sin[d*x] + (6*I)*a*b*d^3*e^2*f*x*Sin[d
*x] - 12*a*b*d^2*e*f^2*x*Sin[d*x] - (12*I)*a*b*d*f^3*x*Sin[d*x] + (6*I)*a*b
*d^3*e*f^2*x^2*Sin[d*x] - 6*a*b*d^2*f^3*x^2*Sin[d*x] + (2*I)*a*b*d^3*f^3*x^
3*Sin[d*x] - (2*I)*a*b*d^3*e^3*Sin[2*c + d*x] + 6*a*b*d^2*e^2*f*Sin[2*c + d
*x] + (12*I)*a*b*d*e*f^2*Sin[2*c + d*x] - 12*a*b*f^3*Sin[2*c + d*x] - (6*I)
*a*b*d^3*e^2*f*x*Sin[2*c + d*x] + 12*a*b*d^2*e*f^2*x*Sin[2*c + d*x] + (12*I
)*a*b*d*f^3*x*Sin[2*c + d*x] - (6*I)*a*b*d^3*e*f^2*x^2*Sin[2*c + d*x] + 6*a
*b*d^2*f^3*x^2*Sin[2*c + d*x] - (2*I)*a*b*d^3*f^3*x^3*Sin[2*c + d*x] + 8*b^
2*d^3*e^3*Sin[c + 2*d*x] - (4*I)*a^2*d^4*e^3*x*Sin[c + 2*d*x] + 24*b^2*d^3*
e^2*f*x*Sin[c + 2*d*x] - (6*I)*a^2*d^4*e^2*f*x^2*Sin[c + 2*d*x] + 24*b^2*d^
3*e*f^2*x^2*Sin[c + 2*d*x] - (4*I)*a^2*d^4*e*f^2*x^3*Sin[c + 2*d*x] + 8*b^2
*d^3*f^3*x^3*Sin[c + 2*d*x] - I*a^2*d^4*f^3*x^4*Sin[c + 2*d*x] + (4*I)*a^2*
d^4*e^3*x*Sin[3*c + 2*d*x] + (6*I)*a^2*d^4*e^2*f*x^2*Sin[3*c + 2*d*x] + (4*
I)*a^2*d^4*e*f^2*x^3*Sin[3*c + 2*d*x] + I*a^2*d^4*f^3*x^4*Sin[3*c + 2*d*x]
- (2*I)*a*b*d^3*e^3*Sin[2*c + 3*d*x] + 6*a*b*d^2*e^2*f*Sin[2*c + 3*d*x] + (
12*I)*a*b*d*e*f^2*Sin[2*c + 3*d*x] - 12*a*b*f^3*Sin[2*c + 3*d*x] - (6*I)*a*
b*d^3*e^2*f*x*Sin[2*c + 3*d*x] + 12*a*b*d^2*e*f^2*x*Sin[2*c + 3*d*x] + (12*
I)*a*b*d*f^3*x*Sin[2*c + 3*d*x] - (6*I)*a*b*d^3*e*f^2*x^2*Sin[2*c + 3*d*x]
+ 6*a*b*d^2*f^3*x^2*Sin[2*c + 3*d*x] - (2*I)*a*b*d^3*f^3*x^3*Sin[2*c + 3*d*
x] + (2*I)*a*b*d^3*e^3*Sin[4*c + 3*d*x] - 6*a*b*d^2*e^2*f*Sin[4*c + 3*d*x]
- (12*I)*a*b*d*e*f^2*Sin[4*c + 3*d*x] + 12*a*b*f^3*Sin[4*c + 3*d*x] + (6*I)
*a*b*d^3*e^2*f*x*Sin[4*c + 3*d*x] - 12*a*b*d^2*e*f^2*x*Sin[4*c + 3*d*x] - (
12*I)*a*b*d*f^3*x*Sin[4*c + 3*d*x] + (6*I)*a*b*d^3*e*f^2*x^2*Sin[4*c + 3*d*

```
x] - 6*a*b*d^2*f^3*x^2*Sin[4*c + 3*d*x] + (2*I)*a*b*d^3*f^3*x^3*Sin[4*c + 3
*d*x])
```

Maple [F] time = 3.043, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\cos(dx+c))^2 (\cot(dx+c))^2}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 8.59119, size = 10645, normalized size = 9.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/4*(12*a^2*b*d^2*f^3*x^2 + 24*a^2*b*d^2*e*f^2*x + 12*a^2*b*d^2*e^2*f - 12*
I*b^3*f^3*polylog(4, cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 12*I*b^3
*f^3*polylog(4, cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - 12*I*b^3*f^3*
```

```
polylog(4, -cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 12*I*b^3*f^3*poly
log(4, -cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - 12*I*(a^2*b - b^3)*f^
3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x +
 c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(
d*x + c) + 12*I*(a^2*b - b^3)*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, 1/2*(2*
I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 12*I*(a^2*b - b^3)*f^3*sqrt(-(a^
2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x +
 c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 12*I*(a^2
*b - b^3)*f^3*sqrt(-(a^2 - b^2)/b^2)*polylog(4, -(I*a*cos(d*x + c) + a*sin(
d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*s
in(d*x + c) - 24*a^2*b*f^3 + 2*(3*I*(a^2*b - b^3)*d^2*f^3*x^2 + 6*I*(a^2*b
- b^3)*d^2*e*f^2*x + 3*I*(a^2*b - b^3)*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*di
log(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) + 2*(-3*I*(a
^2*b - b^3)*d^2*f^3*x^2 - 6*I*(a^2*b - b^3)*d^2*e*f^2*x - 3*I*(a^2*b - b^3)
*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin
(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) +
2*b)/b + 1)*sin(d*x + c) + 2*(-3*I*(a^2*b - b^3)*d^2*f^3*x^2 - 6*I*(a^2*b -
 b^3)*d^2*e*f^2*x - 3*I*(a^2*b - b^3)*d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dil
og(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*s
in(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) + 2*(3*I*(a^
2*b - b^3)*d^2*f^3*x^2 + 6*I*(a^2*b - b^3)*d^2*e*f^2*x + 3*I*(a^2*b - b^3)*
d^2*e^2*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin
(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) +
2*b)/b + 1)*sin(d*x + c) + 2*((a^2*b - b^3)*d^3*e^3 - 3*(a^2*b - b^3)*c*d^2
*e^2*f + 3*(a^2*b - b^3)*c^2*d*e*f^2 - (a^2*b - b^3)*c^3*f^3)*sqrt(-(a^2 -
b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)
/b^2) + 2*I*a)*sin(d*x + c) + 2*((a^2*b - b^3)*d^3*e^3 - 3*(a^2*b - b^3)*c*
d^2*e^2*f + 3*(a^2*b - b^3)*c^2*d*e*f^2 - (a^2*b - b^3)*c^3*f^3)*sqrt(-(a^2
 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b
^2)/b^2) - 2*I*a)*sin(d*x + c) - 2*((a^2*b - b^3)*d^3*e^3 - 3*(a^2*b - b^3)
*c*d^2*e^2*f + 3*(a^2*b - b^3)*c^2*d*e*f^2 - (a^2*b - b^3)*c^3*f^3)*sqrt(-(
a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2
 - b^2)/b^2) + 2*I*a)*sin(d*x + c) - 2*((a^2*b - b^3)*d^3*e^3 - 3*(a^2*b -
b^3)*c*d^2*e^2*f + 3*(a^2*b - b^3)*c^2*d*e*f^2 - (a^2*b - b^3)*c^3*f^3)*sqr
t(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-
(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) + 2*((a^2*b - b^3)*d^3*f^3*x^3 + 3*(
a^2*b - b^3)*d^3*e*f^2*x^2 + 3*(a^2*b - b^3)*d^3*e^2*f*x + 3*(a^2*b - b^3)*
c*d^2*e^2*f - 3*(a^2*b - b^3)*c^2*d*e*f^2 + (a^2*b - b^3)*c^3*f^3)*sqrt(-(a
^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*
x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) -
2*((a^2*b - b^3)*d^3*f^3*x^3 + 3*(a^2*b - b^3)*d^3*e*f^2*x^2 + 3*(a^2*b - b
^3)*d^3*e^2*f*x + 3*(a^2*b - b^3)*c*d^2*e^2*f - 3*(a^2*b - b^3)*c^2*d*e*f^2
 + (a^2*b - b^3)*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c
```

) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b
^2)/b^2) + 2*b)/b)*sin(d*x + c) + 2*((a^2*b - b^3)*d^3*f^3*x^3 + 3*(a^2*b -
 b^3)*d^3*e*f^2*x^2 + 3*(a^2*b - b^3)*d^3*e^2*f*x + 3*(a^2*b - b^3)*c*d^2*e
^2*f - 3*(a^2*b - b^3)*c^2*d*e*f^2 + (a^2*b - b^3)*c^3*f^3)*sqrt(-(a^2 - b^
2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c)
 + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) - 2*((a^
2*b - b^3)*d^3*f^3*x^3 + 3*(a^2*b - b^3)*d^3*e*f^2*x^2 + 3*(a^2*b - b^3)*d^
3*e^2*f*x + 3*(a^2*b - b^3)*c*d^2*e^2*f - 3*(a^2*b - b^3)*c^2*d*e*f^2 + (a^
2*b - b^3)*c^3*f^3)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2
*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2) + 2*b)/b)*sin(d*x + c) - 12*((a^2*b - b^3)*d*f^3*x + (a^2*b - b^3)*d*e*
f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*
x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*s
in(d*x + c) + 12*((a^2*b - b^3)*d*f^3*x + (a^2*b - b^3)*d*e*f^2)*sqrt(-(a^2
 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*c
os(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 1
2*((a^2*b - b^3)*d*f^3*x + (a^2*b - b^3)*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*po
lylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 12*((a^2*b - b^3)*d*f^3*x
 + (a^2*b - b^3)*d*e*f^2)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x +
 c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2))/b)*sin(d*x + c) - 12*(a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2
*b*d^2*e^2*f - 2*a^2*b*f^3)*cos(d*x + c)^2 + (6*I*b^3*d^2*f^3*x^2 + 6*I*b^3
*d^2*e^2*f - 12*I*a*b^2*d*e*f^2 + 12*I*(b^3*d^2*e*f^2 - a*b^2*d*f^3)*x)*dil
og(cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + (-6*I*b^3*d^2*f^3*x^2 - 6*
I*b^3*d^2*e^2*f + 12*I*a*b^2*d*e*f^2 - 12*I*(b^3*d^2*e*f^2 - a*b^2*d*f^3)*x
)*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + (6*I*b^3*d^2*f^3*x^2
+ 6*I*b^3*d^2*e^2*f + 12*I*a*b^2*d*e*f^2 + 12*I*(b^3*d^2*e*f^2 + a*b^2*d*f^
3)*x)*dilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + (-6*I*b^3*d^2*f^
3*x^2 - 6*I*b^3*d^2*e^2*f - 12*I*a*b^2*d*e*f^2 - 12*I*(b^3*d^2*e*f^2 + a*b^
2*d*f^3)*x)*dilog(-cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 2*(b^3*d^3
*f^3*x^3 + b^3*d^3*e^3 + 3*a*b^2*d^2*e^2*f + 3*(b^3*d^3*e*f^2 + a*b^2*d^2*f
^3)*x^2 + 3*(b^3*d^3*e^2*f + 2*a*b^2*d^2*e*f^2)*x)*log(cos(d*x + c) + I*sin
(d*x + c) + 1)*sin(d*x + c) + 2*(b^3*d^3*f^3*x^3 + b^3*d^3*e^3 + 3*a*b^2*d^
2*e^2*f + 3*(b^3*d^3*e*f^2 + a*b^2*d^2*f^3)*x^2 + 3*(b^3*d^3*e^2*f + 2*a*b^
2*d^2*e*f^2)*x)*log(cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) - 2*(b^
3*d^3*e^3 - 3*(b^3*c + a*b^2)*d^2*e^2*f + 3*(b^3*c^2 + 2*a*b^2*c)*d*e*f^2 -
 (b^3*c^3 + 3*a*b^2*c^2)*f^3)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) +
1/2)*sin(d*x + c) - 2*(b^3*d^3*e^3 - 3*(b^3*c + a*b^2)*d^2*e^2*f + 3*(b^3*c
^2 + 2*a*b^2*c)*d*e*f^2 - (b^3*c^3 + 3*a*b^2*c^2)*f^3)*log(-1/2*cos(d*x + c
) - 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) - 2*(b^3*d^3*f^3*x^3 + 3*b^3*c*d
^2*e^2*f - 3*(b^3*c^2 + 2*a*b^2*c)*d*e*f^2 + (b^3*c^3 + 3*a*b^2*c^2)*f^3 +
3*(b^3*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + 3*(b^3*d^3*e^2*f - 2*a*b^2*d^2*e*f^
2)*x)*log(-cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) - 2*(b^3*d^3*f^3
*x^3 + 3*b^3*c*d^2*e^2*f - 3*(b^3*c^2 + 2*a*b^2*c)*d*e*f^2 + (b^3*c^3 + 3*a

```
*b^2*c^2)*f^3 + 3*(b^3*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + 3*(b^3*d^3*e^2*f -
2*a*b^2*d^2*e*f^2)*x)*log(-cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c)
- 12*(b^3*d*f^3*x + b^3*d*e*f^2 - a*b^2*f^3)*polylog(3, cos(d*x + c) + I*si
n(d*x + c))*sin(d*x + c) - 12*(b^3*d*f^3*x + b^3*d*e*f^2 - a*b^2*f^3)*polyl
og(3, cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 12*(b^3*d*f^3*x + b^3*d
*e*f^2 + a*b^2*f^3)*polylog(3, -cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c)
 + 12*(b^3*d*f^3*x + b^3*d*e*f^2 + a*b^2*f^3)*polylog(3, -cos(d*x + c) - I*
sin(d*x + c))*sin(d*x + c) - 4*(a*b^2*d^3*f^3*x^3 + 3*a*b^2*d^3*e*f^2*x^2 +
 3*a*b^2*d^3*e^2*f*x + a*b^2*d^3*e^3)*cos(d*x + c) - (a^3*d^4*f^3*x^4 + 4*a
^3*d^4*e*f^2*x^3 + 6*a^3*d^4*e^2*f*x^2 + 4*a^3*d^4*e^3*x + 4*(a^2*b*d^3*f^3
*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + a^2*b*d^3*e^3 - 6*a^2*b*d*e*f^2 + 3*(a^2*b*d
^3*e^2*f - 2*a^2*b*d*f^3)*x)*cos(d*x + c))*sin(d*x + c))/(a^2*b^2*d^4*sin(d
*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.342 $\int \frac{(e+fx)^2\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=840

$$-\frac{\left(a^2-b^2\right)(e+fx)^3}{3ab^2f}-\frac{(e+fx)^3}{3af}+\frac{2b\tanh^{-1}\left(e^{i(c+dx)}\right)(e+fx)^2}{a^2d}-\frac{b\cos(c+dx)(e+fx)^2}{a^2d}-\frac{\left(a^2-b^2\right)\cos(c+dx)(e+f}{a^2bd}$$

```
[Out] ((-I)*(e + f*x)^2)/(a*d) - (e + f*x)^3/(3*a*f) - ((a^2 - b^2)*(e + f*x)^3)/
(3*a*b^2*f) + (2*b*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a^2*d) + (2*b*f^2
*Cos[c + d*x])/(a^2*d^3) + (2*(a^2 - b^2)*f^2*Cos[c + d*x])/(a^2*b*d^3) - (
b*(e + f*x)^2*Cos[c + d*x])/(a^2*d) - ((a^2 - b^2)*(e + f*x)^2*Cos[c + d*x]
)/(a^2*b*d) - ((e + f*x)^2*Cot[c + d*x])/(a*d) - (I*(a^2 - b^2)^(3/2)*(e +
f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d) +
(I*(a^2 - b^2)^(3/2)*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^
2 - b^2])])/(a^2*b^2*d) + (2*f*(e + f*x)*Log[1 - E^((2*I)*(c + d*x))])/(a*d
^2) - ((2*I)*b*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a^2*d^2) + ((2*I)
*b*f*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a^2*d^2) - (2*(a^2 - b^2)^(3/2
)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2
*b^2*d^2) + (2*(a^2 - b^2)^(3/2)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a^2*b^2*d^2) - (I*f^2*PolyLog[2, E^((2*I)*(c +
d*x))])/(a*d^3) + (2*b*f^2*PolyLog[3, -E^(I*(c + d*x))])/(a^2*d^3) - (2*b*f
^2*PolyLog[3, E^(I*(c + d*x))])/(a^2*d^3) - ((2*I)*(a^2 - b^2)^(3/2)*f^2*Po
lyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d^3) + ((2*
I)*(a^2 - b^2)^(3/2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b
^2])])/(a^2*b^2*d^3) + (2*b*f*(e + f*x)*Sin[c + d*x])/(a^2*d^2) + (2*(a^2 -
 b^2)*f*(e + f*x)*Sin[c + d*x])/(a^2*b*d^2)
```

Rubi [A] time = 2.15194, antiderivative size = 840, normalized size of antiderivative = 1., number of steps used = 53, number of rules used = 22, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {4543, 4408, 3311, 32, 2635, 8, 3720, 3717, 2190, 2279, 2391, 4405, 3310, 3296, 2638, 4183, 2531, 2282, 6589, 4525, 3323, 2264}

$$-\frac{\left(a^2-b^2\right)(e+fx)^3}{3ab^2f}-\frac{(e+fx)^3}{3af}+\frac{2b\tanh^{-1}\left(e^{i(c+dx)}\right)(e+fx)^2}{a^2d}-\frac{b\cos(c+dx)(e+fx)^2}{a^2d}-\frac{\left(a^2-b^2\right)\cos(c+dx)(e+f}{a^2bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-I)*(e + f*x)^2)/(a*d) - (e + f*x)^3/(3*a*f) - ((a^2 - b^2)*(e + f*x)^3)/
(3*a*b^2*f) + (2*b*(e + f*x)^2*ArcTanh[E^(I*(c + d*x))])/(a^2*d) + (2*b*f^2
*Cos[c + d*x])/(a^2*d^3) + (2*(a^2 - b^2)*f^2*Cos[c + d*x])/(a^2*b*d^3) - (
b*(e + f*x)^2*Cos[c + d*x])/(a^2*d) - ((a^2 - b^2)*(e + f*x)^2*Cos[c + d*x]
)/(a^2*b*d) - ((e + f*x)^2*Cot[c + d*x])/(a*d) - (I*(a^2 - b^2)^(3/2)*(e +
f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d) +
(I*(a^2 - b^2)^(3/2)*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^
2 - b^2])])/(a^2*b^2*d) + (2*f*(e + f*x)*Log[1 - E^((2*I)*(c + d*x))])/(a*d
^2) - ((2*I)*b*f*(e + f*x)*PolyLog[2, -E^(I*(c + d*x))])/(a^2*d^2) + ((2*I)
*b*f*(e + f*x)*PolyLog[2, E^(I*(c + d*x))])/(a^2*d^2) - (2*(a^2 - b^2)^(3/2
)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2
*b^2*d^2) + (2*(a^2 - b^2)^(3/2)*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a^2*b^2*d^2) - (I*f^2*PolyLog[2, E^((2*I)*(c +
d*x))])/(a*d^3) + (2*b*f^2*PolyLog[3, -E^(I*(c + d*x))])/(a^2*d^3) - (2*b*f
^2*PolyLog[3, E^(I*(c + d*x))])/(a^2*d^3) - ((2*I)*(a^2 - b^2)^(3/2)*f^2*Po
lyLog[3, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d^3) + ((2*
I)*(a^2 - b^2)^(3/2)*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b
^2])])/(a^2*b^2*d^3) + (2*b*f*(e + f*x)*Sin[c + d*x])/(a^2*d^2) + (2*(a^2 -
 b^2)*f*(e + f*x)*Sin[c + d*x])/(a^2*b*d^2)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\cos^2(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{\int (e+fx)^2\cos^2(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\cot^2(c+dx)\,dx}{a} - \frac{b\int (e+fx)^2\,\mathrm{co}}{}$$

$$= -\frac{f(e+fx)\cos^2(c+dx)}{2ad^2} - \frac{(e+fx)^2\cot(c+dx)}{ad} - \frac{(e+fx)^2\cos(c+dx)\sin(}{2ad}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{2af} - \frac{(e+fx)^2\cot(c+dx)}{ad} + \frac{f^2\cos(c+dx)\sin(c+dx)}{4ad^3}$$

$$= \frac{f^2x}{4ad^2} - \frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3b^2f} + \frac{2b(e+fx)^2\tanh^{-1}\left(\right.}{a^2d}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3b^2f} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right.}{a^2d}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3b^2f} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right.}{a^2d}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3b^2f} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right.}{a^2d}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3b^2f} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right.}{a^2d}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} - \frac{a\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{3b^2f} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right.}{a^2d}$$

Mathematica [A] time = 10.8118, size = 951, normalized size = 1.13

$$\frac{12\left(-bd^2x^2\log\left(1-e^{-i(c+dx)}\right)f^2 + bd^2x^2\log\left(1+e^{-i(c+dx)}\right)f^2 + 2b\left(idx\text{PolyLog}\left(2,-e^{-i(c+dx)}\right) + \text{PolyLog}\left(3,-e^{-i(c+dx)}\right)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),
x]
```

```
[Out] (12*(((-2*I)*a*d^2*(e + f*x)^2)/(-1 + E^((2*I)*c)) - 2*d*f*(b*d*e - a*f)*x*
Log[1 - E^((-I)*(c + d*x))] - b*d^2*f^2*x^2*Log[1 - E^((-I)*(c + d*x))] + 2
*d*f*(b*d*e + a*f)*x*Log[1 + E^((-I)*(c + d*x))] + b*d^2*f^2*x^2*Log[1 + E^
((-I)*(c + d*x))] + I*d*e*(b*d*e - 2*a*f)*(d*x + I*Log[1 - E^(I*(c + d*x))]
) + d*e*(b*d*e + 2*a*f)*((-I)*d*x + Log[1 + E^(I*(c + d*x))]) + (2*I)*f*(b*
d*e + a*f)*PolyLog[2, -E^((-I)*(c + d*x))] + (2*I)*f*(-(b*d*e) + a*f)*PolyL
og[2, E^((-I)*(c + d*x))] + 2*b*f^2*(I*d*x*PolyLog[2, -E^((-I)*(c + d*x))]
+ PolyLog[3, -E^((-I)*(c + d*x))]) - (2*I)*b*f^2*(d*x*PolyLog[2, E^((-I)*(c
 + d*x))] - I*PolyLog[3, E^((-I)*(c + d*x))])) - ((12*I)*Sqrt[-(a^2 - b^2)^
2]*(-2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, (b*E^(I*(c + d*x)))/((-I)*a
 + Sqrt[-a^2 + b^2])] + 2*Sqrt[a^2 - b^2]*d*f*(e + f*x)*PolyLog[2, -((b*E^(
I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))] - I*(d^2*(2*Sqrt[-a^2 + b^2]*e^2*A
rcTan[(I*a + b*E^(I*(c + d*x)))/Sqrt[a^2 - b^2]] + Sqrt[a^2 - b^2]*f*x*(2*e
 + f*x)*(Log[1 - (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - Log[1 +
 (b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2])])) + 2*Sqrt[a^2 - b^2]*f^2*Po
lyLog[3, (b*E^(I*(c + d*x)))/((-I)*a + Sqrt[-a^2 + b^2])] - 2*Sqrt[a^2 - b^
2]*f^2*PolyLog[3, -((b*E^(I*(c + d*x)))/(I*a + Sqrt[-a^2 + b^2]))])))/b^2 +
 (a*Csc[c]*Csc[c + d*x]*(-2*a^2*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Cos[d*x]
+ 2*a^2*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Cos[2*c + d*x] + 3*b*(-(a*(-2*f^2
 + d^2*(e + f*x)^2)*Cos[c + 2*d*x]) + a*(-2*f^2 + d^2*(e + f*x)^2)*Cos[3*c
+ 2*d*x] + 2*d*(e + f*x)*(2*b*d*(e + f*x)*Sin[d*x] + 4*a*f*Sin[c]*Sin[c + d
*x]^2))))/b^2)/(12*a^2*d^3)
```

Maple [F] time = 3.44, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \left(\cos \left(dx+c\right)\right)^{2} \left(\cot \left(dx+c\right)\right)^{2}}{a+b\sin \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 5.3425, size = 7205, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/12*(24*a^2*b*d*f^2*x - 12*b^3*f^2*polylog(3, cos(d*x + c) + I*sin(d*x + c
))*sin(d*x + c) - 12*b^3*f^2*polylog(3, cos(d*x + c) - I*sin(d*x + c))*sin(
d*x + c) + 12*b^3*f^2*polylog(3, -cos(d*x + c) + I*sin(d*x + c))*sin(d*x +
c) + 12*b^3*f^2*polylog(3, -cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 2
4*a^2*b*d*e*f - 12*(a^2*b - b^3)*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*
(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 12*(a^2*b - b^3)*f^2*sqrt(-(a
^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b
*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) +
 12*(a^2*b - b^3)*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x + c)
+ a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^
2))/b)*sin(d*x + c) - 12*(a^2*b - b^3)*f^2*sqrt(-(a^2 - b^2)/b^2)*polylog(3
, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 2*(6*I*(a^2*b - b^3)*d*f^2*x + 6
*I*(a^2*b - b^3)*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x +
c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 -
b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) + 2*(-6*I*(a^2*b - b^3)*d*f^2*x - 6*I*
(a^2*b - b^3)*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c)
+ 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) + 2*b)/b + 1)*sin(d*x + c) + 2*(-6*I*(a^2*b - b^3)*d*f^2*x - 6*I*(a^
2*b - b^3)*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) +
2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/
b^2) + 2*b)/b + 1)*sin(d*x + c) + 2*(6*I*(a^2*b - b^3)*d*f^2*x + 6*I*(a^2*b
 - b^3)*d*e*f)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a
*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2
) + 2*b)/b + 1)*sin(d*x + c) + 6*((a^2*b - b^3)*d^2*e^2 - 2*(a^2*b - b^3)*c
*d*e*f + (a^2*b - b^3)*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c)
 + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) +
```

```
6*((a^2*b - b^3)*d^2*e^2 - 2*(a^2*b - b^3)*c*d*e*f + (a^2*b - b^3)*c^2*f^2)
*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqr
t(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) - 6*((a^2*b - b^3)*d^2*e^2 - 2*(a
^2*b - b^3)*c*d*e*f + (a^2*b - b^3)*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*
b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*s
in(d*x + c) - 6*((a^2*b - b^3)*d^2*e^2 - 2*(a^2*b - b^3)*c*d*e*f + (a^2*b -
 b^3)*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x
 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) + 6*((a^2*b - b^3)
*d^2*f^2*x^2 + 2*(a^2*b - b^3)*d^2*e*f*x + 2*(a^2*b - b^3)*c*d*e*f - (a^2*b
 - b^3)*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*s
in(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2)
+ 2*b)/b)*sin(d*x + c) - 6*((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a^2*b - b^3)*d^2
*e*f*x + 2*(a^2*b - b^3)*c*d*e*f - (a^2*b - b^3)*c^2*f^2)*sqrt(-(a^2 - b^2)
/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) -
I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) + 6*((a^2*b
 - b^3)*d^2*f^2*x^2 + 2*(a^2*b - b^3)*d^2*e*f*x + 2*(a^2*b - b^3)*c*d*e*f -
 (a^2*b - b^3)*c^2*f^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c)
 + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^
2)/b^2) + 2*b)/b)*sin(d*x + c) - 6*((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a^2*b -
b^3)*d^2*e*f*x + 2*(a^2*b - b^3)*c*d*e*f - (a^2*b - b^3)*c^2*f^2)*sqrt(-(a^
2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*
x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) -
24*(a^2*b*d*f^2*x + a^2*b*d*e*f)*cos(d*x + c)^2 + (12*I*b^3*d*f^2*x + 12*I*
b^3*d*e*f - 12*I*a*b^2*f^2)*dilog(cos(d*x + c) + I*sin(d*x + c))*sin(d*x +
c) + (-12*I*b^3*d*f^2*x - 12*I*b^3*d*e*f + 12*I*a*b^2*f^2)*dilog(cos(d*x +
c) - I*sin(d*x + c))*sin(d*x + c) + (12*I*b^3*d*f^2*x + 12*I*b^3*d*e*f + 12
*I*a*b^2*f^2)*dilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + (-12*I*b
^3*d*f^2*x - 12*I*b^3*d*e*f - 12*I*a*b^2*f^2)*dilog(-cos(d*x + c) - I*sin(d
*x + c))*sin(d*x + c) + 6*(b^3*d^2*f^2*x^2 + b^3*d^2*e^2 + 2*a*b^2*d*e*f +
2*(b^3*d^2*e*f + a*b^2*d*f^2)*x)*log(cos(d*x + c) + I*sin(d*x + c) + 1)*sin
(d*x + c) + 6*(b^3*d^2*f^2*x^2 + b^3*d^2*e^2 + 2*a*b^2*d*e*f + 2*(b^3*d^2*e
*f + a*b^2*d*f^2)*x)*log(cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) -
6*(b^3*d^2*e^2 - 2*(b^3*c + a*b^2)*d*e*f + (b^3*c^2 + 2*a*b^2*c)*f^2)*log(-
1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) - 6*(b^3*d^2*e^2
- 2*(b^3*c + a*b^2)*d*e*f + (b^3*c^2 + 2*a*b^2*c)*f^2)*log(-1/2*cos(d*x + c
) - 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) - 6*(b^3*d^2*f^2*x^2 + 2*b^3*c*d
*e*f - (b^3*c^2 + 2*a*b^2*c)*f^2 + 2*(b^3*d^2*e*f - a*b^2*d*f^2)*x)*log(-co
s(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) - 6*(b^3*d^2*f^2*x^2 + 2*b^3*
c*d*e*f - (b^3*c^2 + 2*a*b^2*c)*f^2 + 2*(b^3*d^2*e*f - a*b^2*d*f^2)*x)*log(
-cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) - 12*(a*b^2*d^2*f^2*x^2 +
2*a*b^2*d^2*e*f*x + a*b^2*d^2*e^2)*cos(d*x + c) - 4*(a^3*d^3*f^2*x^3 + 3*a^
3*d^3*e*f*x^2 + 3*a^3*d^3*e^2*x + 3*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x
+ a^2*b*d^2*e^2 - 2*a^2*b*f^2)*cos(d*x + c))*sin(d*x + c))/(a^2*b^2*d^3*sin
(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e+fx\right)^2\cos^2\left(c+dx\right)\cot^2\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x)),
 x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.343 $\int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=517

$$-\frac{f\left(a^2-b^2\right)^{3/2}\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2b^2d^2}+\frac{f\left(a^2-b^2\right)^{3/2}\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2b^2d^2}-\frac{ibf\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{a^2d^2}+\frac{ibf\text{PolyL}}{}$$

```
[Out] -((e*x)/a) + ((1 - a^2/b^2)*e*x)/a - (f*x^2)/(2*a) + ((1 - a^2/b^2)*f*x^2)/
(2*a) + (2*b*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a^2*d) - (b*(e + f*x)*Cos
[c + d*x])/(a^2*d) - ((a^2 - b^2)*(e + f*x)*Cos[c + d*x])/(a^2*b*d) - ((e +
 f*x)*Cot[c + d*x])/(a*d) - (I*(a^2 - b^2)^(3/2)*(e + f*x)*Log[1 - (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d) + (I*(a^2 - b^2)^(3/2)*(e
 + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^2*d) +
 (f*Log[Sin[c + d*x]])/(a*d^2) - (I*b*f*PolyLog[2, -E^(I*(c + d*x))])/(a^2*
d^2) + (I*b*f*PolyLog[2, E^(I*(c + d*x))])/(a^2*d^2) - ((a^2 - b^2)^(3/2)*f
*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d^2) + (
(a^2 - b^2)^(3/2)*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]
)/(a^2*b^2*d^2) + (b*f*Sin[c + d*x])/(a^2*d^2) + ((a^2 - b^2)*f*Sin[c + d*x
])/(a^2*b*d^2)
```

Rubi [A] time = 1.14243, antiderivative size = 517, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 16, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {4543, 4408, 3310, 3720, 3475, 4405, 2633, 3296, 2637, 4183, 2279, 2391, 4525, 3323, 2264, 2190}

$$-\frac{f\left(a^2-b^2\right)^{3/2}\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2b^2d^2}+\frac{f\left(a^2-b^2\right)^{3/2}\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2b^2d^2}-\frac{ibf\text{PolyLog}\left(2,-e^{i(c+dx)}\right)}{a^2d^2}+\frac{ibf\text{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((e*x)/a) + ((1 - a^2/b^2)*e*x)/a - (f*x^2)/(2*a) + ((1 - a^2/b^2)*f*x^2)/
(2*a) + (2*b*(e + f*x)*ArcTanh[E^(I*(c + d*x))])/(a^2*d) - (b*(e + f*x)*Cos
[c + d*x])/(a^2*d) - ((a^2 - b^2)*(e + f*x)*Cos[c + d*x])/(a^2*b*d) - ((e +
 f*x)*Cot[c + d*x])/(a*d) - (I*(a^2 - b^2)^(3/2)*(e + f*x)*Log[1 - (I*b*E^(
I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d) + (I*(a^2 - b^2)^(3/2)*(e
 + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^2*d) +
 (f*Log[Sin[c + d*x]])/(a*d^2) - (I*b*f*PolyLog[2, -E^(I*(c + d*x))])/(a^2*
d^2) + (I*b*f*PolyLog[2, E^(I*(c + d*x))])/(a^2*d^2) - ((a^2 - b^2)^(3/2)*f
```

```
*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^2*d^2) + (
(a^2 - b^2)^(3/2)*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])]
)/(a^2*b^2*d^2) + (b*f*Sin[c + d*x])/(a^2*d^2) + ((a^2 - b^2)*f*Sin[c + d*x
])/(a^2*b*d^2)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
```

```
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\cos^2(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{\int (e+fx)\cos^2(c+dx)\,dx}{a} + \frac{\int (e+fx)\cot^2(c+dx)\,dx}{a} - \frac{b\int (e+fx)\cos^3(}{}$$

$$= -\frac{f\cos^2(c+dx)}{4ad^2} - \frac{(e+fx)\cot(c+dx)}{ad} - \frac{(e+fx)\cos(c+dx)\sin(c+dx)}{2ad} -$$

$$= -\frac{3ex}{2a} - \frac{3fx^2}{4a} - \frac{(e+fx)\cot(c+dx)}{ad} + \frac{f\log(\sin(c+dx))}{ad^2} + \frac{\int (e+fx)\,dx}{2a} -$$

$$= -\frac{ex}{a} - \frac{a\left(1-\frac{b^2}{a^2}\right)ex}{b^2} - \frac{fx^2}{2a} - \frac{a\left(1-\frac{b^2}{a^2}\right)fx^2}{2b^2} + \frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} -$$

$$= -\frac{ex}{a} - \frac{a\left(1-\frac{b^2}{a^2}\right)ex}{b^2} - \frac{fx^2}{2a} - \frac{a\left(1-\frac{b^2}{a^2}\right)fx^2}{2b^2} + \frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} -$$

$$= -\frac{ex}{a} - \frac{a\left(1-\frac{b^2}{a^2}\right)ex}{b^2} - \frac{fx^2}{2a} - \frac{a\left(1-\frac{b^2}{a^2}\right)fx^2}{2b^2} + \frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} -$$

$$= -\frac{ex}{a} - \frac{a\left(1-\frac{b^2}{a^2}\right)ex}{b^2} - \frac{fx^2}{2a} - \frac{a\left(1-\frac{b^2}{a^2}\right)fx^2}{2b^2} + \frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} -$$

$$= -\frac{ex}{a} - \frac{a\left(1-\frac{b^2}{a^2}\right)ex}{b^2} - \frac{fx^2}{2a} - \frac{a\left(1-\frac{b^2}{a^2}\right)fx^2}{2b^2} + \frac{2b(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{a^2d} -$$

Mathematica [A] time = 11.9493, size = 1019, normalized size = 1.97

$$\frac{(de+dfx)\left(\frac{2(de-cf)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{if\left(\log\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\log\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt{b^2-a^2}}{-ia+b+\sqrt{b^2-a^2}}\right)+\text{PolyLog}\left(2,\frac{a\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a+i\left(b+\sqrt{b^2-a^2}\right)}\right)\right)}{\sqrt{b^2-a^2}} + \frac{if\left(\log\left(\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*(c + d*x)*(2*d*e - 2*c*f + f*(c + d*x)))/(2*b^2*d^2) - ((d*e - c*f + f*
(c + d*x))*Cos[c + d*x])/(b*d^2) + ((-(d*e*Cos[(c + d*x)/2]) + c*f*Cos[(c +
```

```
d*x)/2] - f*(c + d*x)*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(2*a*d^2) + (f*L
og[Sin[c + d*x]])/(a*d^2) - (b*e*Log[Tan[(c + d*x)/2]])/(a^2*d) + (b*c*f*Lo
g[Tan[(c + d*x)/2]])/(a^2*d^2) - (b*f*((c + d*x)*(Log[1 - E^(I*(c + d*x))]
- Log[1 + E^(I*(c + d*x))]) + I*(PolyLog[2, -E^(I*(c + d*x))] - PolyLog[2,
E^(I*(c + d*x))])))/(a^2*d^2) + ((a^2 - b^2)^2*(d*e + d*f*x)*((2*(d*e - c*f
)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (I*f*
(Log[1 - I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]
)/((-I)*a + b + Sqrt[-a^2 + b^2])] + PolyLog[2, (a*(1 - I*Tan[(c + d*x)/2])
)/(a + I*(b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 + I*Tan[
(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sq
rt[-a^2 + b^2])] + PolyLog[2, (a*(1 + I*Tan[(c + d*x)/2]))/(a - I*(b + Sqrt
[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2] + (I*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[
-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))
] + PolyLog[2, (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])]))/S
qrt[-a^2 + b^2] - (I*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^
2] + a*Tan[(c + d*x)/2])/(I*a + b - Sqrt[-a^2 + b^2])] + PolyLog[2, (a + I*
a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]))/Sqrt[-a^2 + b^2]))/(
a^2*b^2*d^2*(d*e - c*f + I*f*Log[1 - I*Tan[(c + d*x)/2]] - I*f*Log[1 + I*Ta
n[(c + d*x)/2]])) + (Sec[(c + d*x)/2]*(d*e*Sin[(c + d*x)/2] - c*f*Sin[(c +
d*x)/2] + f*(c + d*x)*Sin[(c + d*x)/2]))/(2*a*d^2) + (f*Sin[c + d*x])/(b*d^
2)
```

Maple [B] time = 1.136, size = 1890, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a
^2+b^2)^(1/2)))*x-2/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b
^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c+2/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I
*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x+2/d^2*f/(-a^2+b^2)^(1
/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-a*
e*x/b^2+2*I/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(
1/2))/(I*a-(-a^2+b^2)^(1/2)))-2*I/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I
*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))-4*I/d*e/(-a^2+b^2)^(1/2
)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+a^2/b^2/d*f/(-a^2
+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2
)))*x+a^2/b^2/d^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1
/2))/(I*a-(-a^2+b^2)^(1/2)))*c-a^2/b^2/d*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I
```

```
*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-a^2/b^2/d^2*f/(-a^2+b
^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2))
)*c+I*a^2/b^2/d^2*f/(-a^2+b^2)^(1/2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)
^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+2*I*a^2/b^2/d*e/(-a^2+b^2)^(1/2)*arctan(1/2
*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))-I*a^2/b^2/d^2*f/(-a^2+b^2)^(1
/2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-2
*I/d^2/a^2*b^2*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(
-a^2+b^2)^(1/2))-1/2*a*f*x^2/b^2+1/d^2/a^2*b*f*c*ln(exp(I*(d*x+c))-1)+1/d/a
^2*b*f*ln(exp(I*(d*x+c))+1)*x-I/d^2/a^2*b*f*dilog(exp(I*(d*x+c))+1)-I/d^2/a
^2*b*f*dilog(exp(I*(d*x+c)))-1/2*(d*f*x-I*f+d*e)/b/d^2*exp(-I*(d*x+c))-2*I*
(f*x+e)/d/a/(exp(2*I*(d*x+c))-1)+4*I/d^2*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2
*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))+I/d^2/a^2*b^2*f/(-a^2+b^2)^(1/2)
*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+1/d/
a^2*b^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-
(-a^2+b^2)^(1/2)))*x+1/d^2/a^2*b^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+
c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-1/d/a^2*b^2*f/(-a^2+b^2)^(1
/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-1/
d^2/a^2*b^2*f/(-a^2+b^2)^(1/2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(
I*a+(-a^2+b^2)^(1/2)))*c+2*I/d/a^2*b^2*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b
*exp(I*(d*x+c))-2*a)/(-a^2+b^2)^(1/2))-I/d^2/a^2*b^2*f/(-a^2+b^2)^(1/2)*dil
og((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))-2*I*a^2/
b^2/d^2*f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*I*b*exp(I*(d*x+c))-2*a)/(-a^2+b^
2)^(1/2))-2/d^2/a*f*ln(exp(I*(d*x+c)))-1/d/a^2*b*e*ln(exp(I*(d*x+c))-1)+1/d
/a^2*b*e*ln(exp(I*(d*x+c))+1)+1/d^2/a*f*ln(exp(I*(d*x+c))-1)+1/d^2/a*f*ln(e
xp(I*(d*x+c))+1)-1/2*(d*f*x+I*f+d*e)/b/d^2*exp(I*(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.27251, size = 4292, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/4*(4*a^2*b*f*cos(d*x + c)^2 - 2*I*b^3*f*dilog(cos(d*x + c) + I*sin(d*x +
 c))*sin(d*x + c) + 2*I*b^3*f*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x
+ c) - 2*I*b^3*f*dilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 2*I*b
^3*f*dilog(-cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - 2*I*(a^2*b - b^3)
*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c)
 + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b +
1)*sin(d*x + c) + 2*I*(a^2*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*
I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))
*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) + 2*I*(a^2*b - b^3)*f*sq
rt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2
*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*s
in(d*x + c) - 2*I*(a^2*b - b^3)*f*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a
*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) - 4*a^2*b*f - 2*((a^2*b - b
^3)*d*e - (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) +
2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) - 2*(
(a^2*b - b^3)*d*e - (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d
*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x
+ c) + 2*((a^2*b - b^3)*d*e - (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*log
(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*
a)*sin(d*x + c) + 2*((a^2*b - b^3)*d*e - (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^
2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/
b^2) - 2*I*a)*sin(d*x + c) - 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)*sq
rt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*
cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x +
 c) + 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*lo
g(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) - 2*((a^2*b - b^3)*
d*f*x + (a^2*b - b^3)*c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x +
 c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 -
 b^2)/b^2) + 2*b)/b)*sin(d*x + c) + 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*
c*f)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c)
 - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*s
in(d*x + c) - 2*(b^3*d*f*x + b^3*d*e + a*b^2*f)*log(cos(d*x + c) + I*sin(d*
x + c) + 1)*sin(d*x + c) - 2*(b^3*d*f*x + b^3*d*e + a*b^2*f)*log(cos(d*x +
c) - I*sin(d*x + c) + 1)*sin(d*x + c) + 2*(b^3*d*e - (b^3*c + a*b^2)*f)*log
(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + 2*(b^3*d*e -
(b^3*c + a*b^2)*f)*log(-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2)*sin(d*
x + c) + 2*(b^3*d*f*x + b^3*c*f)*log(-cos(d*x + c) + I*sin(d*x + c) + 1)*si
```

```
n(d*x + c) + 2*(b^3*d*f*x + b^3*c*f)*log(-cos(d*x + c) - I*sin(d*x + c) + 1
)*sin(d*x + c) + 4*(a*b^2*d*f*x + a*b^2*d*e)*cos(d*x + c) + 2*(a^3*d^2*f*x^
2 + 2*a^3*d^2*e*x + 2*(a^2*b*d*f*x + a^2*b*d*e)*cos(d*x + c))*sin(d*x + c))
/(a^2*b^2*d^2*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)\cos^2(c + dx)\cot^2(c + dx)}{a + b\sin(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.344 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=104

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{ax}{b^2}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

```
[Out] -((a*x)/b^2) + (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^2*b^2*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cos[c + d*x]/(
b*d) - Cot[c + d*x]/(a*d)
```

Rubi [A] time = 0.270288, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2894, 3057, 2660, 618, 204, 3770}

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{ax}{b^2}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) + (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^2*b^2*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cos[c + d*x]/(
b*d) - Cot[c + d*x]/(a*d)
```

Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} - \frac{\int \frac{\csc(c+dx)\left(b^2+2ab\sin(c+dx)+a^2\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{ab}$$

$$= -\frac{ax}{b^2} - \frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} - \frac{b\int \csc(c+dx)\,dx}{a^2} + \frac{\left(a^2-b^2\right)^2 \int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2b^2}$$

$$= -\frac{ax}{b^2} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} + \frac{\left(2\left(a^2-b^2\right)^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{ax}{b^2} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} - \frac{\left(4\left(a^2-b^2\right)^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{ax}{b^2} + \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^2d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{bd} -$$

Mathematica [A] time = 0.802352, size = 146, normalized size = 1.4

$$-\frac{-4\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + 2a^2b\cos(c+dx) + 2a^3c + 2a^3dx - ab^2\tan\left(\frac{1}{2}(c+dx)\right) + ab^2\cot\left(\frac{1}{2}(c+dx)\right) +}{2a^2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(2*a^3*c + 2*a^3*d*x - 4*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]] + 2*a^2*b*Cos[c + d*x] + a*b^2*Cot[(c + d*x)/2] - 2*b^3*L
og[Cos[(c + d*x)/2]] + 2*b^3*Log[Sin[(c + d*x)/2]] - a*b^2*Tan[(c + d*x)/2]
)/(2*a^2*b^2*d)
```

Maple [B] time = 0.076, size = 249, normalized size = 2.4

$$\frac{1}{2da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - 2\frac{1}{bd\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)} - 2\frac{a\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{b^2d} + 2\frac{a^2}{b^2d\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\,\mathrm{t}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/a/d*tan(1/2*d*x+1/2*c)-2/d/b/(1+tan(1/2*d*x+1/2*c)^2)-2/d/b^2*a*arctan(
tan(1/2*d*x+1/2*c))+2/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2
*c)+2*b)/(a^2-b^2)^(1/2))*a^2-4/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d*b^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/2/a/d/tan(1/2*d*x+1/2*c)-1/d/a^
2*b*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.95285, size = 981, normalized size = 9.43

$$\left[\frac{b^3 \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - b^3 \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - 2\,ab^2\cos(dx+c) - \left(a^2 - b^2\right)\sqrt{-a}}{2} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(b^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b^3*log(-1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 2*a*b^2*cos(d*x + c) - (a^2 - b^2)*sqrt(-a^2 + b^
2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(
a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 2*(a^3*d*x + a^2
*b*cos(d*x + c))*sin(d*x + c))/(a^2*b^2*d*sin(d*x + c)), 1/2*(b^3*log(1/2*c
os(d*x + c) + 1/2)*sin(d*x + c) - b^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 2*a*b^2*cos(d*x + c) - 2*(a^2 - b^2)^(3/2)*arctan(-(a*sin(d*x + c) +
 b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 2*(a^3*d*x + a^2*b*cos(d
```

```
*x + c))*sin(d*x + c))/(a^2*b^2*d*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [B] time = 1.81875, size = 298, normalized size = 2.87

$$-\frac{\frac{6(dx+c)a}{b^2}+\frac{6b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{12\left(a^4-2a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}a^2b^2}-\frac{2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a/b^2 + 6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 3*tan(1/
2*d*x + 1/2*c)/a - 12*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi +
1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a
^2 - b^2)*a^2*b^2) - (2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/
2*c)^2 - 12*a^2*tan(1/2*d*x + 1/2*c) + 2*b^2*tan(1/2*d*x + 1/2*c) - 3*a*b)/
((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*a^2*b))/d
```

3.345 $\int \frac{(e+fx)^3 \cos^3(c+dx) \cot^2(c+dx)}{a+b \sin(c+dx)} dx$

Optimal. Leaf size=1432

result too large to display

```
[Out] (3*b*f^3*x)/(8*a^2*d^3) + (3*(a^2 - b^2)*f^3*x)/(8*a^2*b*d^3) - (b*(e + f*x
)^3)/(4*a^2*d) - ((a^2 - b^2)*(e + f*x)^3)/(4*a^2*b*d) + ((I/4)*b*(e + f*x)
^4)/(a^2*f) - ((I/4)*(a^2 - b^2)^2*(e + f*x)^4)/(a^2*b^3*f) - (6*f*(e + f*x
)^2*ArcTanh[E^(I*(c + d*x))])/(a*d^2) + (6*f^3*Cos[c + d*x])/(a*d^4) + (6*(
a^2 - b^2)*f^3*Cos[c + d*x])/(a*b^2*d^4) - (3*f*(e + f*x)^2*Cos[c + d*x])/(
a*d^2) - (3*(a^2 - b^2)*f*(e + f*x)^2*Cos[c + d*x])/(a*b^2*d^2) - ((e + f*x
)^3*Csc[c + d*x])/(a*d) + ((a^2 - b^2)^2*(e + f*x)^3*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d) + ((a^2 - b^2)^2*(e + f*x)^3*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3*d) - (b*(e + f
*x)^3*Log[1 - E^((2*I)*(c + d*x))])/(a^2*d) + ((6*I)*f^2*(e + f*x)*PolyLog[
2, -E^(I*(c + d*x))])/(a*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[2, E^(I*(c + d
*x))])/(a*d^3) - ((3*I)*(a^2 - b^2)^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c
 + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d^2) - ((3*I)*(a^2 - b^2)^2*f*(e
 + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3
*d^2) + (((3*I)/2)*b*f*(e + f*x)^2*PolyLog[2, E^((2*I)*(c + d*x))])/(a^2*d^
2) - (6*f^3*PolyLog[3, -E^(I*(c + d*x))])/(a*d^4) + (6*f^3*PolyLog[3, E^(I*
(c + d*x))])/(a*d^4) + (6*(a^2 - b^2)^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*
(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d^3) + (6*(a^2 - b^2)^2*f^2*(e
 + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3*d
^3) - (3*b*f^2*(e + f*x)*PolyLog[3, E^((2*I)*(c + d*x))])/(2*a^2*d^3) + ((6
*I)*(a^2 - b^2)^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]
)])/(a^2*b^3*d^4) + ((6*I)*(a^2 - b^2)^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3*d^4) - (((3*I)/4)*b*f^3*PolyLog[4, E^((
2*I)*(c + d*x))])/(a^2*d^4) + (6*f^2*(e + f*x)*Sin[c + d*x])/(a*d^3) + (6*(
a^2 - b^2)*f^2*(e + f*x)*Sin[c + d*x])/(a*b^2*d^3) - ((e + f*x)^3*Sin[c + d
*x])/(a*d) - ((a^2 - b^2)*(e + f*x)^3*Sin[c + d*x])/(a*b^2*d) - (3*b*f^3*Co
s[c + d*x]*Sin[c + d*x])/(8*a^2*d^4) - (3*(a^2 - b^2)*f^3*Cos[c + d*x]*Sin[
c + d*x])/(8*a^2*b*d^4) + (3*b*f*(e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(4*
a^2*d^2) + (3*(a^2 - b^2)*f*(e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(4*a^2*b
*d^2) - (3*b*f^2*(e + f*x)*Sin[c + d*x]^2)/(4*a^2*d^3) - (3*(a^2 - b^2)*f^2
*(e + f*x)*Sin[c + d*x]^2)/(4*a^2*b*d^3) + (b*(e + f*x)^3*Sin[c + d*x]^2)/(
2*a^2*d) + ((a^2 - b^2)*(e + f*x)^3*Sin[c + d*x]^2)/(2*a^2*b*d)
```

Rubi [A] time = 2.94674, antiderivative size = 1432, normalized size of antiderivative = 1., number of steps used = 85, number of rules used = 21, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ =

0.583, Rules used = {4543, 4408, 3311, 3296, 2638, 3310, 4410, 4183, 2531, 2282, 6589, 4405, 32, 2635, 8, 4404, 3717, 2190, 6609, 4525, 4519}

$$-\frac{i\left(a^2-b^2\right)^2(e+fx)^4}{4a^2b^3f}+\frac{ib(e+fx)^4}{4a^2f}+\frac{b\sin^2(c+dx)(e+fx)^3}{2a^2d}+\frac{\left(a^2-b^2\right)\sin^2(c+dx)(e+fx)^3}{2a^2bd}-\frac{\csc(c+dx)(e+fx)^3}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b*f^3*x)/(8*a^2*d^3) + (3*(a^2 - b^2)*f^3*x)/(8*a^2*b*d^3) - (b*(e + f*x
)^3)/(4*a^2*d) - ((a^2 - b^2)*(e + f*x)^3)/(4*a^2*b*d) + ((I/4)*b*(e + f*x)
^4)/(a^2*f) - ((I/4)*(a^2 - b^2)^2*(e + f*x)^4)/(a^2*b^3*f) - (6*f*(e + f*x
)^2*ArcTanh[E^(I*(c + d*x))])/(a*d^2) + (6*f^3*Cos[c + d*x])/(a*d^4) + (6*(
a^2 - b^2)*f^3*Cos[c + d*x])/(a*b^2*d^4) - (3*f*(e + f*x)^2*Cos[c + d*x])/(
a*d^2) - (3*(a^2 - b^2)*f*(e + f*x)^2*Cos[c + d*x])/(a*b^2*d^2) - ((e + f*x
)^3*Csc[c + d*x])/(a*d) + ((a^2 - b^2)^2*(e + f*x)^3*Log[1 - (I*b*E^(I*(c +
 d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d) + ((a^2 - b^2)^2*(e + f*x)^3*Lo
g[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3*d) - (b*(e + f
*x)^3*Log[1 - E^((2*I)*(c + d*x))])/(a^2*d) + ((6*I)*f^2*(e + f*x)*PolyLog[
2, -E^(I*(c + d*x))])/(a*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[2, E^(I*(c + d
*x))])/(a*d^3) - ((3*I)*(a^2 - b^2)^2*f*(e + f*x)^2*PolyLog[2, (I*b*E^(I*(c
 + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d^2) - ((3*I)*(a^2 - b^2)^2*f*(e
 + f*x)^2*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3
*d^2) + (((3*I)/2)*b*f*(e + f*x)^2*PolyLog[2, E^((2*I)*(c + d*x))])/(a^2*d^
2) - (6*f^3*PolyLog[3, -E^(I*(c + d*x))])/(a*d^4) + (6*f^3*PolyLog[3, E^(I*
(c + d*x))])/(a*d^4) + (6*(a^2 - b^2)^2*f^2*(e + f*x)*PolyLog[3, (I*b*E^(I*
(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d^3) + (6*(a^2 - b^2)^2*f^2*(e
 + f*x)*PolyLog[3, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3*d
^3) - (3*b*f^2*(e + f*x)*PolyLog[3, E^((2*I)*(c + d*x))])/(2*a^2*d^3) + ((6
*I)*(a^2 - b^2)^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]
)])/(a^2*b^3*d^4) + ((6*I)*(a^2 - b^2)^2*f^3*PolyLog[4, (I*b*E^(I*(c + d*x)
))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3*d^4) - (((3*I)/4)*b*f^3*PolyLog[4, E^((
2*I)*(c + d*x))])/(a^2*d^4) + (6*f^2*(e + f*x)*Sin[c + d*x])/(a*d^3) + (6*(
a^2 - b^2)*f^2*(e + f*x)*Sin[c + d*x])/(a*b^2*d^3) - ((e + f*x)^3*Sin[c + d
*x])/(a*d) - ((a^2 - b^2)*(e + f*x)^3*Sin[c + d*x])/(a*b^2*d) - (3*b*f^3*Co
s[c + d*x]*Sin[c + d*x])/(8*a^2*d^4) - (3*(a^2 - b^2)*f^3*Cos[c + d*x]*Sin[
c + d*x])/(8*a^2*b*d^4) + (3*b*f*(e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(4*
a^2*d^2) + (3*(a^2 - b^2)*f*(e + f*x)^2*Cos[c + d*x]*Sin[c + d*x])/(4*a^2*b
*d^2) - (3*b*f^2*(e + f*x)*Sin[c + d*x]^2)/(4*a^2*d^3) - (3*(a^2 - b^2)*f^2
*(e + f*x)*Sin[c + d*x]^2)/(4*a^2*b*d^3) + (b*(e + f*x)^3*Sin[c + d*x]^2)/(
2*a^2*d) + ((a^2 - b^2)*(e + f*x)^3*Sin[c + d*x]^2)/(2*a^2*b*d)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rubi steps

$$\int \frac{(e+fx)^3\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^3\cos^3(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{\int (e+fx)^3\cos^3(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\cos(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b}{}$$

$$= -\frac{f(e+fx)^2\cos^3(c+dx)}{3ad^2} - \frac{(e+fx)^3\cos^2(c+dx)\sin(c+dx)}{3ad} - \frac{2\int (e+fx}{}$$

$$= \frac{2f^3\cos^3(c+dx)}{27ad^4} - \frac{(e+fx)^3\csc(c+dx)}{ad} - \frac{5(e+fx)^3\sin(c+dx)}{3ad} + \frac{2f^2(e}{}$$

$$= \frac{ib(e+fx)^4}{4a^2f} - \frac{i\left(a^2-b^2\right)^2(e+fx)^4}{4a^2b^3f} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2} - \frac{5f(e}{}$$

$$= \frac{ib(e+fx)^4}{4a^2f} - \frac{i\left(a^2-b^2\right)^2(e+fx)^4}{4a^2b^3f} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2} + \frac{4f^3\,c}{}$$

$$= -\frac{b(e+fx)^3}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{4bd} + \frac{ib(e+fx)^4}{4a^2f} - \frac{i\left(a^2-b^2\right)^2(e+fx)^4}{4a^2b^3f} - \frac{6}{}$$

$$= \frac{3bf^3x}{8a^2d^3} + \frac{3\left(1-\frac{b^2}{a^2}\right)f^3x}{8bd^3} - \frac{b(e+fx)^3}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{4bd} + \frac{ib(e+fx)^4}{4a^2f} -$$

$$= \frac{3bf^3x}{8a^2d^3} + \frac{3\left(1-\frac{b^2}{a^2}\right)f^3x}{8bd^3} - \frac{b(e+fx)^3}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{4bd} + \frac{ib(e+fx)^4}{4a^2f} -$$

$$= \frac{3bf^3x}{8a^2d^3} + \frac{3\left(1-\frac{b^2}{a^2}\right)f^3x}{8bd^3} - \frac{b(e+fx)^3}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)(e+fx)^3}{4bd} + \frac{ib(e+fx)^4}{4a^2f} -$$

Mathematica [B] time = 44.8669, size = 3944, normalized size = 2.75

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),
x]
```

```
[Out] ((-e^3 - 3*e^2*f*x - 3*e*f^2*x^2 - f^3*x^3)*Csc[c + d*x])/(a*d) - (((-I)*b*
(e + f*x)^4)/((-1 + E^((2*I)*c))*f) + (6*e*f*(b*d*e - 2*a*f)*x*Log[1 - E^((
```

-I)*(c + d*x))])/d^2 + (6*f^2*(b*d*e - a*f)*x^2*Log[1 - E^((-I)*(c + d*x))]
)/d^2 + (2*b*f^3*x^3*Log[1 - E^((-I)*(c + d*x))])/d + (6*e*f*(b*d*e + 2*a*f
)*x*Log[1 + E^((-I)*(c + d*x))])/d^2 + (6*f^2*(b*d*e + a*f)*x^2*Log[1 + E^(
(-I)*(c + d*x))])/d^2 + (2*b*f^3*x^3*Log[1 + E^((-I)*(c + d*x))])/d + (2*e^
2*(b*d*e - 3*a*f)*((-I)*d*x + Log[1 - E^(I*(c + d*x))]))/d^2 + (2*e^2*(b*d*
e + 3*a*f)*((-I)*d*x + Log[1 + E^(I*(c + d*x))]))/d^2 + ((6*I)*e*f*(b*d*e +
 2*a*f)*PolyLog[2, -E^((-I)*(c + d*x))])/d^3 + ((6*I)*e*f*(b*d*e - 2*a*f)*P
olyLog[2, E^((-I)*(c + d*x))])/d^3 + (12*f^2*(b*d*e + a*f)*(I*d*x*PolyLog[2
, -E^((-I)*(c + d*x))] + PolyLog[3, -E^((-I)*(c + d*x))]))/d^4 + (12*f^2*(b
*d*e - a*f)*(I*d*x*PolyLog[2, E^((-I)*(c + d*x))] + PolyLog[3, E^((-I)*(c +
 d*x))]))/d^4 + (6*b*f^3*(I*d^2*x^2*PolyLog[2, -E^((-I)*(c + d*x))] + 2*d*x
PolyLog[3, -E^((-I)(c + d*x))] - (2*I)*PolyLog[4, -E^((-I)*(c + d*x))]))/
d^4 + (6*b*f^3*(I*d^2*x^2*PolyLog[2, E^((-I)*(c + d*x))] + 2*d*x*PolyLog[3,
 E^((-I)*(c + d*x))] - (2*I)*PolyLog[4, E^((-I)*(c + d*x))]))/d^4)/(2*a^2)
+ ((a^2 - b^2)^2*((-4*I)*d^4*e^3*E^((2*I)*c)*x - (6*I)*d^4*e^2*E^((2*I)*c)*
f*x^2 - (4*I)*d^4*e*E^((2*I)*c)*f^2*x^3 - I*d^4*E^((2*I)*c)*f^3*x^4 - (2*I)
*d^3*e^3*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c + d*x))))] + (2*
I)*d^3*e^3*E^((2*I)*c)*ArcTan[(2*a*E^(I*(c + d*x)))/(b*(-1 + E^((2*I)*(c +
d*x))))] - d^3*e^3*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E^((2*I)*(c +
d*x)))^2] + d^3*e^3*E^((2*I)*c)*Log[4*a^2*E^((2*I)*(c + d*x)) + b^2*(-1 + E
^((2*I)*(c + d*x)))^2] - 6*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E
^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e^2*E^((2*I)*c)*f*x*Log[1
 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] -
6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b
^2)*E^((2*I)*c)])] + 6*d^3*e*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x
)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 2*d^3*f^3*x^3*Log[1 +
 (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 2*
d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) - Sqrt[(
-a^2 + b^2)*E^((2*I)*c)])] - 6*d^3*e^2*f*x*Log[1 + (b*E^(I*(2*c + d*x)))/(I
*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 6*d^3*e^2*E^((2*I)*c)*f*x*L
og[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])
] - 6*d^3*e*f^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2
 + b^2)*E^((2*I)*c)])] + 6*d^3*e*E^((2*I)*c)*f^2*x^2*Log[1 + (b*E^(I*(2*c +
 d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 2*d^3*f^3*x^3*Log
[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
+ 2*d^3*E^((2*I)*c)*f^3*x^3*Log[1 + (b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sq
rt[(-a^2 + b^2)*E^((2*I)*c)])] - (6*I)*d^2*(-1 + E^((2*I)*c))*f*(e + f*x)^2
*PolyLog[2, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*
I)*c)])] - (6*I)*d^2*(-1 + E^((2*I)*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(I*
(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 12*d*e*f^2
*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*
I)*c)])] + 12*d*e*E^((2*I)*c)*f^2*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(
I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 12*d*f^3*x*PolyLog[3, (I*b*E^(I
*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 12*d*E^((2
*I)*c)*f^3*x*PolyLog[3, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 +

b^2)*E^((2*I)*c)])] - 12*d*e*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 12*d*e*E^((2*I)*c)*f^2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 12*d*f^3*x*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 12*d*E^((2*I)*c)*f^3*x*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (12*I)*f^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (12*I)*E^((2*I)*c)*f^3*PolyLog[4, (I*b*E^(I*(2*c + d*x)))/(a*E^(I*c) + I*Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (12*I)*f^3*PolyLog[4, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (12*I)*E^((2*I)*c)*f^3*PolyLog[4, -((b*E^(I*(2*c + d*x)))/(I*a*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]))/(2*a^2*b^3*d^4*(-1 + E^((2*I)*c))) - (I*(-a^2 + 2*b^2)*e^3*x*(1 + Cos[2*c] + I*Sin[2*c]))/(b^3*(-1 + Cos[2*c] + I*Sin[2*c])) - (((3*I)/2)*(-a^2 + 2*b^2)*e^2*f*x^2*(1 + Cos[2*c] + I*Sin[2*c]))/(b^3*(-1 + Cos[2*c] + I*Sin[2*c])) - (I*(-a^2 + 2*b^2)*e*f^2*x^3*(1 + Cos[2*c] + I*Sin[2*c]))/(b^3*(-1 + Cos[2*c] + I*Sin[2*c])) - ((I/4)*(-a^2 + 2*b^2)*f^3*x^4*(1 + Cos[2*c] + I*Sin[2*c]))/(b^3*(-1 + Cos[2*c] + I*Sin[2*c])) + (((-I/2)*a*f^3*x^3*Cos[c])/(b^2*d) - (a*f^3*x^3*Sin[c])/(2*b^2*d) + ((-I)*d^3*e^3 - 3*d^2*e^2*f + (6*I)*d*e*f^2 + 6*f^3)*((a*Cos[c])/(2*b^2*d^4) - ((I/2)*a*Sin[c])/(b^2*d^4)) + (a*d^2*e^2*f - (2*I)*a*d*e*f^2 - 2*a*f^3)*((((-3*I)/2)*x*Cos[c])/(b^2*d^3) - (3*x*Sin[c])/(2*b^2*d^3)) + (a*d*e*f^2 - I*a*f^3)*((((-3*I)/2)*x^2*Cos[c])/(b^2*d^2) - (3*x^2*Sin[c])/(2*b^2*d^2)))*(Cos[d*x] - I*Sin[d*x]) + (((I/2)*a*f^3*x^3*Cos[c])/(b^2*d) - (a*f^3*x^3*Sin[c])/(2*b^2*d) + (I*d^3*e^3 - 3*d^2*e^2*f - (6*I)*d*e*f^2 + 6*f^3)*((a*Cos[c])/(2*b^2*d^4) + ((I/2)*a*Sin[c])/(b^2*d^4)) + (((3*I)/2)*x^2*(a*d*e*f^2*Cos[c] + I*a*f^3*Cos[c] + I*a*d*e*f^2*Sin[c] - a*f^3*Sin[c]))/(b^2*d^2) + (((3*I)/2)*x*(a*d^2*e^2*f*Cos[c] + (2*I)*a*d*e*f^2*Cos[c] - 2*a*f^3*Cos[c] + I*a*d^2*e^2*f*Sin[c] - 2*a*d*e*f^2*Sin[c] - (2*I)*a*f^3*Sin[c]))/(b^2*d^3))*(Cos[d*x] + I*Sin[d*x]) + (-(f^3*x^3*Cos[2*c])/(8*b*d) + ((I/8)*f^3*x^3*Sin[2*c])/(b*d) + (4*d^3*e^3 - (6*I)*d^2*e^2*f - 6*d*e*f^2 + (3*I)*f^3)*(-Cos[2*c]/(32*b*d^4) + ((I/32)*Sin[2*c])/(b*d^4)) + ((2*I)*d^2*e^2*f + 2*d*e*f^2 - I*f^3)*((((3*I)/16)*x*Cos[2*c])/(b*d^3) + (3*x*Sin[2*c])/(16*b*d^3)) + ((2*I)*d*e*f^2 + f^3)*((((3*I)/16)*x^2*Cos[2*c])/(b*d^2) + (3*x^2*Sin[2*c])/(16*b*d^2)))*(Cos[2*d*x] - I*Sin[2*d*x]) + (-(f^3*x^3*Cos[2*c])/(8*b*d) - ((I/8)*f^3*x^3*Sin[2*c])/(b*d) + (4*d^3*e^3 + (6*I)*d^2*e^2*f - 6*d*e*f^2 - (3*I)*f^3)*(-Cos[2*c]/(32*b*d^4) - ((I/32)*Sin[2*c])/(b*d^4)) - (((3*I)/16)*x*((-2*I)*d^2*e^2*f*Cos[2*c] + 2*d*e*f^2*Cos[2*c] + I*f^3*Cos[2*c] + 2*d^2*e^2*f*Sin[2*c] + (2*I)*d*e*f^2*Sin[2*c] - f^3*Sin[2*c]))/(b*d^3) - (((3*I)/16)*x^2*((-2*I)*d*e*f^2*Cos[2*c] + f^3*Cos[2*c] + 2*d*e*f^2*Sin[2*c] + I*f^3*Sin[2*c]))/(b*d^2))*(Cos[2*d*x] + I*Sin[2*d*x])

Maple [F] time = 4.713, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\cos(dx+c))^3 (\cot(dx+c))^2}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [C] time = 9.77457, size = 11439, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -1/8*(8*(a^3*b + a*b^3)*d^3*f^3*x^3 + 24*(a^3*b + a*b^3)*d^3*e*f^2*x^2 + 24
*I*b^4*f^3*polylog(4, cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - 24*I*b^
4*f^3*polylog(4, cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - 24*I*b^4*f^3
*polylog(4, -cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 24*I*b^4*f^3*pol
ylog(4, -cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - 48*a^3*b*d*e*f^2 + 8
*(a^3*b + a*b^3)*d^3*e^3 - 24*I*(a^4 - 2*a^2*b^2 + b^4)*f^3*polylog(4, 1/2*
(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 24*I*(a^4 - 2*a^2*b^2 + b^4)*
```

f^3*polylog(4, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 24*I*(a^4 - 2*a^2*b^2 + b^4)*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 24*I*(a^4 - 2*a^2*b^2 + b^4)*f^3*polylog(4, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 3*(2*a^2*b^2*d^2*f^3*x^2 + 4*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*d^2*e^2*f - a^2*b^2*f^3)*cos(d*x + c)^3 - 8*(a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + a^3*b*d^3*e^3 - 6*a^3*b*d*e*f^2 + 3*(a^3*b*d^3*e^2*f - 2*a^3*b*d*f^3)*x)*cos(d*x + c)^2 - (12*I*(a^4 - 2*a^2*b^2 + b^4)*d^2*f^3*x^2 + 24*I*(a^4 - 2*a^2*b^2 + b^4)*d^2*e*f^2*x + 12*I*(a^4 - 2*a^2*b^2 + b^4)*d^2*e^2*f)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) - (12*I*(a^4 - 2*a^2*b^2 + b^4)*d^2*f^3*x^2 + 24*I*(a^4 - 2*a^2*b^2 + b^4)*d^2*e*f^2*x + 12*I*(a^4 - 2*a^2*b^2 + b^4)*d^2*e^2*f)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) - (-12*I*(a^4 - 2*a^2*b^2 + b^4)*d^2*f^3*x^2 - 24*I*(a^4 - 2*a^2*b^2 + b^4)*d^2*e*f^2*x - 12*I*(a^4 - 2*a^2*b^2 + b^4)*d^2*e^2*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) - (-12*I*(a^4 - 2*a^2*b^2 + b^4)*d^2*f^3*x^2 - 24*I*(a^4 - 2*a^2*b^2 + b^4)*d^2*e*f^2*x - 12*I*(a^4 - 2*a^2*b^2 + b^4)*d^2*e^2*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) - (12*I*b^4*d^2*f^3*x^2 + 12*I*b^4*d^2*e^2*f - 24*I*a*b^3*d*e*f^2 + 24*I*(b^4*d^2*e*f^2 - a*b^3*d*f^3)*x)*dilog(cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - (-12*I*b^4*d^2*f^3*x^2 - 12*I*b^4*d^2*e^2*f + 24*I*a*b^3*d*e*f^2 - 24*I*(b^4*d^2*e*f^2 - a*b^3*d*f^3)*x)*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - (-12*I*b^4*d^2*f^3*x^2 - 12*I*b^4*d^2*e^2*f - 24*I*a*b^3*d*e*f^2 - 24*I*(b^4*d^2*e*f^2 + a*b^3*d*f^3)*x)*dilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - (12*I*b^4*d^2*f^3*x^2 + 12*I*b^4*d^2*e^2*f + 24*I*a*b^3*d*e*f^2 + 24*I*(b^4*d^2*e*f^2 + a*b^3*d*f^3)*x)*dilog(-cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*d^3*e^3 - 3*(a^4 - 2*a^2*b^2 + b^4)*c*d^2*e^2*f + 3*(a^4 - 2*a^2*b^2 + b^4)*c^2*d*e*f^2 - (a^4 - 2*a^2*b^2 + b^4)*c^3*f^3)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*d^3*e^3 - 3*(a^4 - 2*a^2*b^2 + b^4)*c*d^2*e^2*f + 3*(a^4 - 2*a^2*b^2 + b^4)*c^2*d*e*f^2 - (a^4 - 2*a^2*b^2 + b^4)*c^3*f^3)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*d^3*e^3 - 3*(a^4 - 2*a^2*b^2 + b^4)*c*d^2*e^2*f + 3*(a^4 - 2*a^2*b^2 + b^4)*c^2*d*e*f^2 - (a^4 - 2*a^2*b^2 + b^4)*c^3*f^3)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*d^3*e^3 - 3*(a^4 - 2*a^2*b^2 + b^4)*c*d^2*e^2*f + 3*(a^4 - 2*a^2*b^2 + b^4)*c^2*d*e*f^2 - (a^4 - 2*a^2*b^2 + b^4)*c^3*f^3)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*

```
sin(d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 3*(a^4 - 2*a^2*b^2
+ b^4)*d^3*e*f^2*x^2 + 3*(a^4 - 2*a^2*b^2 + b^4)*d^3*e^2*f*x + 3*(a^4 - 2*a
^2*b^2 + b^4)*c*d^2*e^2*f - 3*(a^4 - 2*a^2*b^2 + b^4)*c^2*d*e*f^2 + (a^4 -
2*a^2*b^2 + b^4)*c^3*f^3)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) +
2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(
d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 3*(a^4 - 2*a^2*b^2 + b^
4)*d^3*e*f^2*x^2 + 3*(a^4 - 2*a^2*b^2 + b^4)*d^3*e^2*f*x + 3*(a^4 - 2*a^2*b
^2 + b^4)*c*d^2*e^2*f - 3*(a^4 - 2*a^2*b^2 + b^4)*c^2*d*e*f^2 + (a^4 - 2*a^
2*b^2 + b^4)*c^3*f^3)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b
*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x
+ c) - 4*((a^4 - 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*d
^3*e*f^2*x^2 + 3*(a^4 - 2*a^2*b^2 + b^4)*d^3*e^2*f*x + 3*(a^4 - 2*a^2*b^2 +
 b^4)*c*d^2*e^2*f - 3*(a^4 - 2*a^2*b^2 + b^4)*c^2*d*e*f^2 + (a^4 - 2*a^2*b^
2 + b^4)*c^3*f^3)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*co
s(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c
) - 4*((a^4 - 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*d^3*
e*f^2*x^2 + 3*(a^4 - 2*a^2*b^2 + b^4)*d^3*e^2*f*x + 3*(a^4 - 2*a^2*b^2 + b^
4)*c*d^2*e^2*f - 3*(a^4 - 2*a^2*b^2 + b^4)*c^2*d*e*f^2 + (a^4 - 2*a^2*b^2 +
 b^4)*c^3*f^3)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d
*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) +
 4*(b^4*d^3*f^3*x^3 + b^4*d^3*e^3 + 3*a*b^3*d^2*e^2*f + 3*(b^4*d^3*e*f^2 +
a*b^3*d^2*f^3)*x^2 + 3*(b^4*d^3*e^2*f + 2*a*b^3*d^2*e*f^2)*x)*log(cos(d*x +
 c) + I*sin(d*x + c) + 1)*sin(d*x + c) + 4*(b^4*d^3*f^3*x^3 + b^4*d^3*e^3 +
 3*a*b^3*d^2*e^2*f + 3*(b^4*d^3*e*f^2 + a*b^3*d^2*f^3)*x^2 + 3*(b^4*d^3*e^2
*f + 2*a*b^3*d^2*e*f^2)*x)*log(cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x +
 c) + 4*(b^4*d^3*e^3 - 3*(b^4*c + a*b^3)*d^2*e^2*f + 3*(b^4*c^2 + 2*a*b^3*c
)*d*e*f^2 - (b^4*c^3 + 3*a*b^3*c^2)*f^3)*log(-1/2*cos(d*x + c) + 1/2*I*sin(
d*x + c) + 1/2)*sin(d*x + c) + 4*(b^4*d^3*e^3 - 3*(b^4*c + a*b^3)*d^2*e^2*f
 + 3*(b^4*c^2 + 2*a*b^3*c)*d*e*f^2 - (b^4*c^3 + 3*a*b^3*c^2)*f^3)*log(-1/2*
cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + 4*(b^4*d^3*f^3*x^3
+ 3*b^4*c*d^2*e^2*f - 3*(b^4*c^2 + 2*a*b^3*c)*d*e*f^2 + (b^4*c^3 + 3*a*b^3*
c^2)*f^3 + 3*(b^4*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 3*(b^4*d^3*e^2*f - 2*a*b
^3*d^2*e*f^2)*x)*log(-cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) + 4*(
b^4*d^3*f^3*x^3 + 3*b^4*c*d^2*e^2*f - 3*(b^4*c^2 + 2*a*b^3*c)*d*e*f^2 + (b^
4*c^3 + 3*a*b^3*c^2)*f^3 + 3*(b^4*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 3*(b^4*d
^3*e^2*f - 2*a*b^3*d^2*e*f^2)*x)*log(-cos(d*x + c) - I*sin(d*x + c) + 1)*si
n(d*x + c) - 24*((a^4 - 2*a^2*b^2 + b^4)*d*f^3*x + (a^4 - 2*a^2*b^2 + b^4)*
d*e*f^2)*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) + 2*(b*cos(d
*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 24*((
a^4 - 2*a^2*b^2 + b^4)*d*f^3*x + (a^4 - 2*a^2*b^2 + b^4)*d*e*f^2)*polylog(3
, 1/2*(2*I*a*cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(
d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 24*((a^4 - 2*a^2*b^2 +
b^4)*d*f^3*x + (a^4 - 2*a^2*b^2 + b^4)*d*e*f^2)*polylog(3, -(I*a*cos(d*x +
c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)
/b^2))/b)*sin(d*x + c) - 24*((a^4 - 2*a^2*b^2 + b^4)*d*f^3*x + (a^4 - 2*a^2
```

```
*b^2 + b^4)*d*e*f^2)*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*co
s(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 24
*(b^4*d*f^3*x + b^4*d*e*f^2 - a*b^3*f^3)*polylog(3, cos(d*x + c) + I*sin(d*
x + c))*sin(d*x + c) + 24*(b^4*d*f^3*x + b^4*d*e*f^2 - a*b^3*f^3)*polylog(3
, cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 24*(b^4*d*f^3*x + b^4*d*e*f
^2 + a*b^3*f^3)*polylog(3, -cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 2
4*(b^4*d*f^3*x + b^4*d*e*f^2 + a*b^3*f^3)*polylog(3, -cos(d*x + c) - I*sin(
d*x + c))*sin(d*x + c) - 24*(2*a^3*b*d*f^3 - (a^3*b + a*b^3)*d^3*e^2*f)*x -
 3*(2*a^2*b^2*d^2*f^3*x^2 + 4*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*d^2*e^2*f - a
^2*b^2*f^3)*cos(d*x + c) - (2*a^2*b^2*d^3*f^3*x^3 + 6*a^2*b^2*d^3*e*f^2*x^2
 + 2*a^2*b^2*d^3*e^3 - 3*a^2*b^2*d*e*f^2 - 2*(2*a^2*b^2*d^3*f^3*x^3 + 6*a^2
*b^2*d^3*e*f^2*x^2 + 2*a^2*b^2*d^3*e^3 - 3*a^2*b^2*d*e*f^2 + 3*(2*a^2*b^2*d
^3*e^2*f - a^2*b^2*d*f^3)*x)*cos(d*x + c)^2 + 3*(2*a^2*b^2*d^3*e^2*f - a^2*
b^2*d*f^3)*x - 24*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*
f - 2*a^3*b*f^3)*cos(d*x + c))*sin(d*x + c))/(a^2*b^3*d^4*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cos(d*x+c)**3*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="giac")

[Out] Timed out
```

3.346 $\int \frac{(e+fx)^2 \cos^3(c+dx) \cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=1051

result too large to display

```
[Out] -(b*e*f*x)/(2*a^2*d) - ((a^2 - b^2)*e*f*x)/(2*a^2*b*d) - (b*f^2*x^2)/(4*a^2
*d) - ((a^2 - b^2)*f^2*x^2)/(4*a^2*b*d) + ((I/3)*b*(e + f*x)^3)/(a^2*f) - (
(I/3)*(a^2 - b^2)^2*(e + f*x)^3)/(a^2*b^3*f) - (4*f*(e + f*x)*ArcTanh[E^(I*
(c + d*x))])/(a*d^2) - (2*f*(e + f*x)*Cos[c + d*x])/(a*d^2) - (2*(a^2 - b^2
)*f*(e + f*x)*Cos[c + d*x])/(a*b^2*d^2) - ((e + f*x)^2*Csc[c + d*x])/(a*d)
+ ((a^2 - b^2)^2*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
b^2])])/(a^2*b^3*d) + ((a^2 - b^2)^2*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x
)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3*d) - (b*(e + f*x)^2*Log[1 - E^((2*I)*(
c + d*x))])/(a^2*d) + ((2*I)*f^2*PolyLog[2, -E^(I*(c + d*x))])/(a*d^3) - ((
2*I)*f^2*PolyLog[2, E^(I*(c + d*x))])/(a*d^3) - ((2*I)*(a^2 - b^2)^2*f*(e +
 f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d^2
) - ((2*I)*(a^2 - b^2)^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a +
Sqrt[a^2 - b^2])])/(a^2*b^3*d^2) + (I*b*f*(e + f*x)*PolyLog[2, E^((2*I)*(c
+ d*x))])/(a^2*d^2) + (2*(a^2 - b^2)^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))
/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d^3) + (2*(a^2 - b^2)^2*f^2*PolyLog[3, (I
*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3*d^3) - (b*f^2*PolyLog[
3, E^((2*I)*(c + d*x))])/(2*a^2*d^3) + (2*f^2*Sin[c + d*x])/(a*d^3) + (2*(a
^2 - b^2)*f^2*Sin[c + d*x])/(a*b^2*d^3) - ((e + f*x)^2*Sin[c + d*x])/(a*d)
- ((a^2 - b^2)*(e + f*x)^2*Sin[c + d*x])/(a*b^2*d) + (b*f*(e + f*x)*Cos[c +
 d*x]*Sin[c + d*x])/(2*a^2*d^2) + ((a^2 - b^2)*f*(e + f*x)*Cos[c + d*x]*Sin
[c + d*x])/(2*a^2*b*d^2) - (b*f^2*Sin[c + d*x]^2)/(4*a^2*d^3) - ((a^2 - b^2
)*f^2*Sin[c + d*x]^2)/(4*a^2*b*d^3) + (b*(e + f*x)^2*Sin[c + d*x]^2)/(2*a^2
*d) + ((a^2 - b^2)*(e + f*x)^2*Sin[c + d*x]^2)/(2*a^2*b*d)
```

Rubi [A] time = 2.24098, antiderivative size = 1051, normalized size of antiderivative = 1., number of steps used = 60, number of rules used = 20, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4543, 4408, 3311, 3296, 2637, 2633, 4410, 4183, 2279, 2391, 4405, 3310, 4404, 3717, 2190, 2531, 2282, 6589, 4525, 4519}

$$-\frac{i\left(a^2-b^2\right)^2(e+fx)^3}{3a^2b^3f}+\frac{ib(e+fx)^3}{3a^2f}+\frac{b\sin^2(c+dx)(e+fx)^2}{2a^2d}+\frac{\left(a^2-b^2\right)\sin^2(c+dx)(e+fx)^2}{2a^2bd}-\frac{\csc(c+dx)(e+fx)^2}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*e*f*x)/(2*a^2*d) - ((a^2 - b^2)*e*f*x)/(2*a^2*b*d) - (b*f^2*x^2)/(4*a^2
*d) - ((a^2 - b^2)*f^2*x^2)/(4*a^2*b*d) + ((I/3)*b*(e + f*x)^3)/(a^2*f) - (
(I/3)*(a^2 - b^2)^2*(e + f*x)^3)/(a^2*b^3*f) - (4*f*(e + f*x)*ArcTanh[E^(I*
(c + d*x))])/(a*d^2) - (2*f*(e + f*x)*Cos[c + d*x])/(a*d^2) - (2*(a^2 - b^2
)*f*(e + f*x)*Cos[c + d*x])/(a*b^2*d^2) - ((e + f*x)^2*Csc[c + d*x])/(a*d)
+ ((a^2 - b^2)^2*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 -
b^2])])/(a^2*b^3*d) + ((a^2 - b^2)^2*(e + f*x)^2*Log[1 - (I*b*E^(I*(c + d*x
)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3*d) - (b*(e + f*x)^2*Log[1 - E^((2*I)*(
c + d*x))])/(a^2*d) + ((2*I)*f^2*PolyLog[2, -E^(I*(c + d*x))])/(a*d^3) - ((
2*I)*f^2*PolyLog[2, E^(I*(c + d*x))])/(a*d^3) - ((2*I)*(a^2 - b^2)^2*f*(e +
 f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d^2
) - ((2*I)*(a^2 - b^2)^2*f*(e + f*x)*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a +
Sqrt[a^2 - b^2])])/(a^2*b^3*d^2) + (I*b*f*(e + f*x)*PolyLog[2, E^((2*I)*(c
+ d*x))])/(a^2*d^2) + (2*(a^2 - b^2)^2*f^2*PolyLog[3, (I*b*E^(I*(c + d*x)))
/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d^3) + (2*(a^2 - b^2)^2*f^2*PolyLog[3, (I
*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b^3*d^3) - (b*f^2*PolyLog[
3, E^((2*I)*(c + d*x))])/(2*a^2*d^3) + (2*f^2*Sin[c + d*x])/(a*d^3) + (2*(a
^2 - b^2)*f^2*Sin[c + d*x])/(a*b^2*d^3) - ((e + f*x)^2*Sin[c + d*x])/(a*d)
- ((a^2 - b^2)*(e + f*x)^2*Sin[c + d*x])/(a*b^2*d) + (b*f*(e + f*x)*Cos[c +
 d*x]*Sin[c + d*x])/(2*a^2*d^2) + ((a^2 - b^2)*f*(e + f*x)*Cos[c + d*x]*Sin
[c + d*x])/(2*a^2*b*d^2) - (b*f^2*Sin[c + d*x]^2)/(4*a^2*d^3) - ((a^2 - b^2
)*f^2*Sin[c + d*x]^2)/(4*a^2*b*d^3) + (b*(e + f*x)^2*Sin[c + d*x]^2)/(2*a^2
*d) + ((a^2 - b^2)*(e + f*x)^2*Sin[c + d*x]^2)/(2*a^2*b*d)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
```

```
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rubi steps

$$\int \frac{(e+fx)^2\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)^2\cos^3(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{\int (e+fx)^2\cos^3(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\cos(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b}{}$$

$$= -\frac{2f(e+fx)\cos^3(c+dx)}{9ad^2} - \frac{(e+fx)^2\cos^2(c+dx)\sin(c+dx)}{3ad} - \frac{2\int (e+f}{}$$

$$= -\frac{(e+fx)^2\csc(c+dx)}{ad} - \frac{5(e+fx)^2\sin(c+dx)}{3ad} + \frac{2\int (e+fx)^2\cos(c+dx)}{3a}$$

$$= \frac{ib(e+fx)^3}{3a^2f} - \frac{i\left(a^2-b^2\right)^2(e+fx)^3}{3a^2b^3f} - \frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2} - \frac{10f(e}{}$$

$$= \frac{ib(e+fx)^3}{3a^2f} - \frac{i\left(a^2-b^2\right)^2(e+fx)^3}{3a^2b^3f} - \frac{4f(e+fx)\tanh^{-1}\left(e^{i(c+dx)}\right)}{ad^2} - \frac{2f(e+}{}$$

$$= -\frac{befx}{2a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)efx}{2bd} - \frac{bf^2x^2}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)f^2x^2}{4bd} + \frac{ib(e+fx)^3}{3a^2f} - \frac{i\left(a^2-b^2\right)^2}{3a^2}$$

$$= -\frac{befx}{2a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)efx}{2bd} - \frac{bf^2x^2}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)f^2x^2}{4bd} + \frac{ib(e+fx)^3}{3a^2f} - \frac{i\left(a^2-b^2\right)^2}{3a^2}$$

$$= -\frac{befx}{2a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)efx}{2bd} - \frac{bf^2x^2}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)f^2x^2}{4bd} + \frac{ib(e+fx)^3}{3a^2f} - \frac{i\left(a^2-b^2\right)^2}{3a^2}$$

Mathematica [B] time = 13.5755, size = 5156, normalized size = 4.91

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),
x]
```

```
[Out] Result too large to show
```

Maple [F] time = 4.864, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \left(\cos \left(dx+c\right)\right)^{3} \left(\cot \left(dx+c\right)\right)^{2}}{a+b\sin \left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [C] time = 5.62446, size = 7509, normalized size = 7.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -1/8*(8*b^4*f^2*polylog(3, cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 8*
b^4*f^2*polylog(3, cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 8*b^4*f^2*
polylog(3, -cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) + 8*b^4*f^2*polylog
(3, -cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) + 8*(a^3*b + a*b^3)*d^2*f^
2*x^2 - 16*a^3*b*f^2 + 16*(a^3*b + a*b^3)*d^2*e*f*x + 8*(a^3*b + a*b^3)*d^2
*e^2 - 8*(a^4 - 2*a^2*b^2 + b^4)*f^2*polylog(3, 1/2*(2*I*a*cos(d*x + c) - 2
*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b
^2))/b)*sin(d*x + c) - 8*(a^4 - 2*a^2*b^2 + b^4)*f^2*polylog(3, 1/2*(2*I*a*
```

```
cos(d*x + c) - 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 8*(a^4 - 2*a^2*b^2 + b^4)*f^2*polylo
g(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) - 8*(a^4 - 2*a^2*b^2 + b^4)*f^2
*polylog(3, -(I*a*cos(d*x + c) + a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin
(d*x + c))*sqrt(-(a^2 - b^2)/b^2))/b)*sin(d*x + c) + 4*(a^2*b^2*d*f^2*x + a
^2*b^2*d*e*f)*cos(d*x + c)^3 - 8*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + a
^3*b*d^2*e^2 - 2*a^3*b*f^2)*cos(d*x + c)^2 - (8*I*(a^4 - 2*a^2*b^2 + b^4)*d
*f^2*x + 8*I*(a^4 - 2*a^2*b^2 + b^4)*d*e*f)*dilog(-1/2*(2*I*a*cos(d*x + c)
+ 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2
)/b^2) + 2*b)/b + 1)*sin(d*x + c) - (8*I*(a^4 - 2*a^2*b^2 + b^4)*d*f^2*x +
8*I*(a^4 - 2*a^2*b^2 + b^4)*d*e*f)*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin
(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) +
2*b)/b + 1)*sin(d*x + c) - (-8*I*(a^4 - 2*a^2*b^2 + b^4)*d*f^2*x - 8*I*(a^4
 - 2*a^2*b^2 + b^4)*d*e*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x +
c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b
+ 1)*sin(d*x + c) - (-8*I*(a^4 - 2*a^2*b^2 + b^4)*d*f^2*x - 8*I*(a^4 - 2*a^
2*b^2 + b^4)*d*e*f)*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*
(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*si
n(d*x + c) - (8*I*b^4*d*f^2*x + 8*I*b^4*d*e*f - 8*I*a*b^3*f^2)*dilog(cos(d*
x + c) + I*sin(d*x + c))*sin(d*x + c) - (-8*I*b^4*d*f^2*x - 8*I*b^4*d*e*f +
 8*I*a*b^3*f^2)*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - (-8*I*b
^4*d*f^2*x - 8*I*b^4*d*e*f - 8*I*a*b^3*f^2)*dilog(-cos(d*x + c) + I*sin(d*x
 + c))*sin(d*x + c) - (8*I*b^4*d*f^2*x + 8*I*b^4*d*e*f + 8*I*a*b^3*f^2)*dil
og(-cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4
)*d^2*e^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*c*d*e*f + (a^4 - 2*a^2*b^2 + b^4)*c^2
*f^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2
) + 2*I*a)*sin(d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*d^2*e^2 - 2*(a^4 - 2*a
^2*b^2 + b^4)*c*d*e*f + (a^4 - 2*a^2*b^2 + b^4)*c^2*f^2)*log(2*b*cos(d*x +
c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c)
- 4*((a^4 - 2*a^2*b^2 + b^4)*d^2*e^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*c*d*e*f +
(a^4 - 2*a^2*b^2 + b^4)*c^2*f^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c)
 + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) - 4*((a^4 - 2*a^2*b^2 +
 b^4)*d^2*e^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*c*d*e*f + (a^4 - 2*a^2*b^2 + b^4)
*c^2*f^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) - 2*I*a)*sin(d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(
a^4 - 2*a^2*b^2 + b^4)*d^2*e*f*x + 2*(a^4 - 2*a^2*b^2 + b^4)*c*d*e*f - (a^4
 - 2*a^2*b^2 + b^4)*c^2*f^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c)
 + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*s
in(d*x + c) - 4*((a^4 - 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^4 - 2*a^2*b^2 +
 b^4)*d^2*e*f*x + 2*(a^4 - 2*a^2*b^2 + b^4)*c*d*e*f - (a^4 - 2*a^2*b^2 + b^
4)*c^2*f^2)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x +
 c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) - 4*(
(a^4 - 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d^2*e*f*x +
 2*(a^4 - 2*a^2*b^2 + b^4)*c*d*e*f - (a^4 - 2*a^2*b^2 + b^4)*c^2*f^2)*log(1
```

```
/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*
x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) - 4*((a^4 - 2*a^2*b^2
 + b^4)*d^2*f^2*x^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d^2*e*f*x + 2*(a^4 - 2*a^2*
b^2 + b^4)*c*d*e*f - (a^4 - 2*a^2*b^2 + b^4)*c^2*f^2)*log(1/2*(-2*I*a*cos(d
*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a
^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) + 4*(b^4*d^2*f^2*x^2 + b^4*d^2*e^2 +
2*a*b^3*d*e*f + 2*(b^4*d^2*e*f + a*b^3*d*f^2)*x)*log(cos(d*x + c) + I*sin(d
*x + c) + 1)*sin(d*x + c) + 4*(b^4*d^2*f^2*x^2 + b^4*d^2*e^2 + 2*a*b^3*d*e*
f + 2*(b^4*d^2*e*f + a*b^3*d*f^2)*x)*log(cos(d*x + c) - I*sin(d*x + c) + 1)
*sin(d*x + c) + 4*(b^4*d^2*e^2 - 2*(b^4*c + a*b^3)*d*e*f + (b^4*c^2 + 2*a*b
^3*c)*f^2)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) +
 4*(b^4*d^2*e^2 - 2*(b^4*c + a*b^3)*d*e*f + (b^4*c^2 + 2*a*b^3*c)*f^2)*log(
-1/2*cos(d*x + c) - 1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + 4*(b^4*d^2*f^2
*x^2 + 2*b^4*c*d*e*f - (b^4*c^2 + 2*a*b^3*c)*f^2 + 2*(b^4*d^2*e*f - a*b^3*d
*f^2)*x)*log(-cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) + 4*(b^4*d^2*
f^2*x^2 + 2*b^4*c*d*e*f - (b^4*c^2 + 2*a*b^3*c)*f^2 + 2*(b^4*d^2*e*f - a*b^
3*d*f^2)*x)*log(-cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) - 4*(a^2*b
^2*d*f^2*x + a^2*b^2*d*e*f)*cos(d*x + c) - (2*a^2*b^2*d^2*f^2*x^2 + 4*a^2*b
^2*d^2*e*f*x + 2*a^2*b^2*d^2*e^2 - a^2*b^2*f^2 - 2*(2*a^2*b^2*d^2*f^2*x^2 +
 4*a^2*b^2*d^2*e*f*x + 2*a^2*b^2*d^2*e^2 - a^2*b^2*f^2)*cos(d*x + c)^2 - 16
*(a^3*b*d*f^2*x + a^3*b*d*e*f)*cos(d*x + c))*sin(d*x + c))/(a^2*b^3*d^3*sin
(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cos(d*x+c)**3*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.347 $\int \frac{(e+fx)\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=641

$$-\frac{if\left(a^2-b^2\right)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2b^3d^2}-\frac{if\left(a^2-b^2\right)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2b^3d^2}+\frac{ibf\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{2a^2d^2}-\frac{f\left(a^2-b^2\right)\cos}{ab^2d^2}$$

```
[Out] -(b*f*x)/(4*a^2*d) - ((a^2 - b^2)*f*x)/(4*a^2*b*d) + ((I/2)*b*(e + f*x)^2)/
(a^2*f) - ((I/2)*(a^2 - b^2)^2*(e + f*x)^2)/(a^2*b^3*f) - (f*ArcTanh[Cos[c
+ d*x]])/(a*d^2) - (f*Cos[c + d*x])/(a*d^2) - ((a^2 - b^2)*f*Cos[c + d*x])/
(a*b^2*d^2) - ((e + f*x)*Csc[c + d*x])/(a*d) + ((a^2 - b^2)^2*(e + f*x)*Log
[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d) + ((a^2 - b^
2)^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b
^3*d) - (b*(e + f*x)*Log[1 - E^((2*I)*(c + d*x))])/(a^2*d) - (I*(a^2 - b^2)
^2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d^2)
 - (I*(a^2 - b^2)^2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]
)])/(a^2*b^3*d^2) + ((I/2)*b*f*PolyLog[2, E^((2*I)*(c + d*x))])/(a^2*d^2) -
 ((e + f*x)*Sin[c + d*x])/(a*d) - ((a^2 - b^2)*(e + f*x)*Sin[c + d*x])/(a*b
^2*d) + (b*f*Cos[c + d*x]*Sin[c + d*x])/(4*a^2*d^2) + ((a^2 - b^2)*f*Cos[c
+ d*x]*Sin[c + d*x])/(4*a^2*b*d^2) + (b*(e + f*x)*Sin[c + d*x]^2)/(2*a^2*d)
 + ((a^2 - b^2)*(e + f*x)*Sin[c + d*x]^2)/(2*a^2*b*d)
```

Rubi [A] time = 1.22528, antiderivative size = 641, normalized size of antiderivative = 1., number of steps used = 45, number of rules used = 17, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4543, 4408, 3310, 3296, 2638, 4410, 3770, 4405, 2635, 8, 4404, 3717, 2190, 2279, 2391, 4525, 4519}

$$-\frac{if\left(a^2-b^2\right)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{a^2b^3d^2}-\frac{if\left(a^2-b^2\right)^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{a^2b^3d^2}+\frac{ibf\text{PolyLog}\left(2,e^{2i(c+dx)}\right)}{2a^2d^2}-\frac{f\left(a^2-b^2\right)\cos}{ab^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] -(b*f*x)/(4*a^2*d) - ((a^2 - b^2)*f*x)/(4*a^2*b*d) + ((I/2)*b*(e + f*x)^2)/
(a^2*f) - ((I/2)*(a^2 - b^2)^2*(e + f*x)^2)/(a^2*b^3*f) - (f*ArcTanh[Cos[c
+ d*x]])/(a*d^2) - (f*Cos[c + d*x])/(a*d^2) - ((a^2 - b^2)*f*Cos[c + d*x])/
(a*b^2*d^2) - ((e + f*x)*Csc[c + d*x])/(a*d) + ((a^2 - b^2)^2*(e + f*x)*Log
[1 - (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d) + ((a^2 - b^
2)^2*(e + f*x)*Log[1 - (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(a^2*b
```

```
^3*d) - (b*(e + f*x)*Log[1 - E^((2*I)*(c + d*x))])/(a^2*d) - (I*(a^2 - b^2)
^2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(a^2*b^3*d^2)
 - (I*(a^2 - b^2)^2*f*PolyLog[2, (I*b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]
)])/(a^2*b^3*d^2) + ((I/2)*b*f*PolyLog[2, E^((2*I)*(c + d*x))])/(a^2*d^2) -
 ((e + f*x)*Sin[c + d*x])/(a*d) - ((a^2 - b^2)*(e + f*x)*Sin[c + d*x])/(a*b
^2*d) + (b*f*Cos[c + d*x]*Sin[c + d*x])/(4*a^2*d^2) + ((a^2 - b^2)*f*Cos[c
+ d*x]*Sin[c + d*x])/(4*a^2*b*d^2) + (b*(e + f*x)*Sin[c + d*x]^2)/(2*a^2*d)
 + ((a^2 - b^2)*(e + f*x)*Sin[c + d*x]^2)/(2*a^2*b*d)
```

Rule 4543

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(p_.)*Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (
f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist
[1/a, Int[(e + f*x)^m*Cos[c + d*x]^p*Cot[c + d*x]^n, x], x] - Dist[b/a, Int
[((e + f*x)^m*Cos[c + d*x]^(p + 1)*Cot[c + d*x]^(n - 1))/(a + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && I
GtQ[p, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4525

```
Int[(Cos[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[a/b^2, Int[(e + f*x)^m*Cos[c +
 d*x]^(n - 2), x], x] + (-Dist[1/b, Int[(e + f*x)^m*Cos[c + d*x]^(n - 2)*Si
n[c + d*x], x], x] - Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Cos[c + d*x]^(n
 - 2))/(a + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGt
Q[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rubi steps

$$\int \frac{(e+fx)\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\int (e+fx)\cos^3(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{\int (e+fx)\cos^3(c+dx)\,dx}{a} + \frac{\int (e+fx)\cos(c+dx)\cot^2(c+dx)\,dx}{a} - \frac{b\int (e-}{}$$

$$= -\frac{f\cos^3(c+dx)}{9ad^2} - \frac{(e+fx)\cos^2(c+dx)\sin(c+dx)}{3ad} - \frac{2\int (e+fx)\cos(c+dx)}{3a}$$

$$= -\frac{(e+fx)\csc(c+dx)}{ad} - \frac{5(e+fx)\sin(c+dx)}{3ad} + \frac{2\int (e+fx)\cos(c+dx)\,dx}{3a} -$$

$$= \frac{ib(e+fx)^2}{2a^2f} - \frac{i\left(a^2-b^2\right)^2(e+fx)^2}{2a^2b^3f} - \frac{f\tanh^{-1}(\cos(c+dx))}{ad^2} - \frac{5f\cos(c+dx)}{3ad^2}$$

$$= \frac{ib(e+fx)^2}{2a^2f} - \frac{i\left(a^2-b^2\right)^2(e+fx)^2}{2a^2b^3f} - \frac{f\tanh^{-1}(\cos(c+dx))}{ad^2} - \frac{f\cos(c+dx)}{ad^2}$$

$$= -\frac{bfx}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)fx}{4bd} + \frac{ib(e+fx)^2}{2a^2f} - \frac{i\left(a^2-b^2\right)^2(e+fx)^2}{2a^2b^3f} - \frac{f\tanh^{-1}(\cos(c}{ad^2}$$

$$= -\frac{bfx}{4a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)fx}{4bd} + \frac{ib(e+fx)^2}{2a^2f} - \frac{i\left(a^2-b^2\right)^2(e+fx)^2}{2a^2b^3f} - \frac{f\tanh^{-1}(\cos(c}{ad^2}$$

Mathematica [B] time = 15.2167, size = 2504, normalized size = 3.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*f*Cos[c + d*x])/(b^2*d^2)) - ((d*e - c*f + f*(c + d*x))*Cos[2*(c + d*x
)])/(4*b*d^2) + ((-(d*e*Cos[(c + d*x)/2]) + c*f*Cos[(c + d*x)/2] - f*(c + d
*x)*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(2*a*d^2) - (b*e*Log[Sin[c + d*x]])
/(a^2*d) + (b*c*f*Log[Sin[c + d*x]])/(a^2*d^2) + (f*Log[Tan[(c + d*x)/2]])/
(a*d^2) - (b*f*((c + d*x)*Log[1 - E^((2*I)*(c + d*x))] - (I/2)*((c + d*x)^2
 + PolyLog[2, E^((2*I)*(c + d*x))])))/(a^2*d^2) + (Sec[(c + d*x)/2]*(-(d*e*
Sin[(c + d*x)/2]) + c*f*Sin[(c + d*x)/2] - f*(c + d*x)*Sin[(c + d*x)/2]))/(
2*a*d^2) - (a*(d*e - c*f + f*(c + d*x))*Sin[c + d*x])/(b^2*d^2) + (f*Sin[2*
(c + d*x)])/(8*b*d^2) + ((f*(c + d*x)^2 + (2*I)*d*e*Log[Sec[(c + d*x)/2]^2]
```

- (2*I)*c*f*Log[Sec[(c + d*x)/2]^2] - (2*I)*d*e*Log[Sec[(c + d*x)/2]^2*(a
+ b*Sin[c + d*x])] + (2*I)*c*f*Log[Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x])]
 - (4*I)*f*(c + d*x)*Log[(-2*I)/(-I + Tan[(c + d*x)/2])] - 2*f*Log[1 + I*Ta
n[(c + d*x)/2]]*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b -
Sqrt[-a^2 + b^2])] + 2*f*Log[1 - I*Tan[(c + d*x)/2]]*Log[-((b - Sqrt[-a^2 +
 b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 + b^2]))] + 2*f*Log[1 - I*
Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a +
 b + Sqrt[-a^2 + b^2])] - 2*f*Log[1 + I*Tan[(c + d*x)/2]]*Log[(b + Sqrt[-a^
2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])] + 4*f*PolyLog[
2, -Cos[c + d*x] + I*Sin[c + d*x]] + 2*f*PolyLog[2, (a*(1 - I*Tan[(c + d*x)
/2]))/(a + I*(b + Sqrt[-a^2 + b^2]))] - 2*f*PolyLog[2, (a*(1 + I*Tan[(c + d
x)/2]))/(a - I(b + Sqrt[-a^2 + b^2]))] + 2*f*PolyLog[2, (a*(I + Tan[(c +
d*x)/2]))/(I*a - b + Sqrt[-a^2 + b^2])] - 2*f*PolyLog[2, (a + I*a*Tan[(c +
d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]])*((-2*e*Cos[c + d*x])/(a + b*Sin[
c + d*x]) + (a^2*e*Cos[c + d*x])/(b^2*(a + b*Sin[c + d*x])) + (b^2*e*Cos[c
+ d*x])/(a^2*(a + b*Sin[c + d*x])) + (2*c*f*Cos[c + d*x])/(d*(a + b*Sin[c +
 d*x])) - (a^2*c*f*Cos[c + d*x])/(b^2*d*(a + b*Sin[c + d*x])) - (b^2*c*f*Co
s[c + d*x])/(a^2*d*(a + b*Sin[c + d*x])) - (2*f*(c + d*x)*Cos[c + d*x])/(d*
(a + b*Sin[c + d*x])) + (a^2*f*(c + d*x)*Cos[c + d*x])/(b^2*d*(a + b*Sin[c
+ d*x])) + (b^2*f*(c + d*x)*Cos[c + d*x])/(a^2*d*(a + b*Sin[c + d*x]))))/(d
*(2*f*(c + d*x) - (4*I)*f*Log[(-2*I)/(-I + Tan[(c + d*x)/2])] - (4*f*Log[1
+ Cos[c + d*x] - I*Sin[c + d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))/(-Cos[c +
 d*x] + I*Sin[c + d*x]) + (I*f*Log[1 - (a*(1 - I*Tan[(c + d*x)/2]))/(a + I*
(b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) - (I*
f*Log[-((b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a - b + Sqrt[-a^2 +
b^2]))]*Sec[(c + d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) - (I*f*Log[(b + Sqrt[-
a^2 + b^2] + a*Tan[(c + d*x)/2])/((-I)*a + b + Sqrt[-a^2 + b^2])]*Sec[(c +
d*x)/2]^2)/(1 - I*Tan[(c + d*x)/2]) + (I*f*Log[1 - (a*(1 + I*Tan[(c + d*x)/
2]))/(a - I*(b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*
x)/2]) - (I*f*Log[(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b - Sq
rt[-a^2 + b^2])]*Sec[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) - (I*f*Log[(b
 + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])/(I*a + b + Sqrt[-a^2 + b^2])]*Sec
[(c + d*x)/2]^2)/(1 + I*Tan[(c + d*x)/2]) + (2*I)*d*e*Tan[(c + d*x)/2] - (2
*I)*c*f*Tan[(c + d*x)/2] + ((2*I)*f*(c + d*x)*Sec[(c + d*x)/2]^2)/(-I + Tan
[(c + d*x)/2]) - (f*Log[1 - (a*(I + Tan[(c + d*x)/2]))/(I*a - b + Sqrt[-a^2
 + b^2])]*Sec[(c + d*x)/2]^2)/(I + Tan[(c + d*x)/2]) + (I*a*f*Log[1 - (a +
I*a*Tan[(c + d*x)/2])/(a + I*(-b + Sqrt[-a^2 + b^2]))]*Sec[(c + d*x)/2]^2)/
(a + I*a*Tan[(c + d*x)/2]) + (a*f*Log[1 - I*Tan[(c + d*x)/2]]*Sec[(c + d*x)
/2]^2)/(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) - (a*f*Log[1 + I*Tan[(c
+ d*x)/2]]*Sec[(c + d*x)/2]^2)/(b - Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2])
+ (a*f*Log[1 - I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2)/(b + Sqrt[-a^2 + b^2
] + a*Tan[(c + d*x)/2]) - (a*f*Log[1 + I*Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]
^2)/(b + Sqrt[-a^2 + b^2] + a*Tan[(c + d*x)/2]) - ((2*I)*d*e*Cos[(c + d*x)/
2]^2*(b*Cos[c + d*x]*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c +
 d*x])*Tan[(c + d*x)/2]))/(a + b*Sin[c + d*x]) + ((2*I)*c*f*Cos[(c + d*x)/2

```
]^2*(b*Cos[c + d*x]*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^2*(a + b*Sin[c +
d*x])*Tan[(c + d*x)/2]))/(a + b*Sin[c + d*x])))
```

Maple [B] time = 2.569, size = 2485, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/b^3/d*a^2*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-2/b^3/d*a^2*e
*ln(exp(I*(d*x+c)))-1/b^3/d*a^4*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2
+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x-1/b^3/d^2*a^4*f/(-a^2+b^2)*ln((I*a+b
*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-1/b^3/d*a^4*f/(
-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2))
)*x-1/b^3/d^2*a^4*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(
I*a+(-a^2+b^2)^(1/2)))*c-2*I*(f*x+e)*exp(I*(d*x+c))/d/a/(exp(2*I*(d*x+c))-1
)+2/b/d^2*f*c*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)-4/b/d^2*f*c*l
n(exp(I*(d*x+c)))+1/d*b/a^2*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*
b)+I/b^3*a^2*e*x-3*b/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/
2))/(I*a-(-a^2+b^2)^(1/2)))*x-3*b/d^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))
-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*c-3*b/d*f/(-a^2+b^2)*ln((I*a+b*e
xp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*x-3*b/d^2*f/(-a^2+b
^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c-2/
b/d*e*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c))-I*b)+4/b/d*ln(exp(I*(d*x+c
)))*e+1/d^2/a^2*b*f*c*ln(exp(I*(d*x+c))-1)-1/d/a^2*b*f*ln(exp(I*(d*x+c))+1)
*x-I/d^2/a^2*b*f*dilog(exp(I*(d*x+c)))+3/b/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(
d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2*x+3/b/d^2*f/(-a^2+b^2
)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2*c+
3/b/d*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^
2)^(1/2)))*a^2*x+3/b/d^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(
1/2))/(I*a-(-a^2+b^2)^(1/2)))*a^2*c-1/d^2*b/a^2*f*c*ln(I*b*exp(2*I*(d*x+c))
-2*a*exp(I*(d*x+c))-I*b)+I/d^2*b/a^2*f*dilog(exp(I*(d*x+c))+1)-1/d/a^2*b*e*
ln(exp(I*(d*x+c))-1)-1/d/a^2*b*e*ln(exp(I*(d*x+c))+1)+2/b^3/d^2*a^2*f*c*ln(
exp(I*(d*x+c)))-1/b^3/d^2*a^2*f*c*ln(I*b*exp(2*I*(d*x+c))-2*a*exp(I*(d*x+c)
)-I*b)+1/2*I*a*(d*f*x+I*f+d*e)/b^2/d^2*exp(I*(d*x+c))+1/d^2/a*f*ln(exp(I*(d
*x+c))-1)-1/d^2/a*f*ln(exp(I*(d*x+c))+1)-I/b^3/d^2*a^2*f*c^2+3*I/d^2*b*f/(-
a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2
)))+3*I/d^2*b*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I
*a+(-a^2+b^2)^(1/2)))+4*I/d/b*c*f*x-1/16*(2*d*f*x+I*f+2*d*e)/b/d^2*exp(2*I*
(d*x+c))-1/16*(2*d*f*x-I*f+2*d*e)/b/d^2*exp(-2*I*(d*x+c))+I/b*f*x^2-2*I/b*e
```

```
*x+1/d^2*b^3/a^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I
*a-(-a^2+b^2)^(1/2)))*c+1/d^2*b^3/a^2*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))
+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*c+1/d*b^3/a^2*f/(-a^2+b^2)*ln((I
*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*x+1/d*b^3/a^2
*f/(-a^2+b^2)*ln((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1
/2)))*x-I/d^2*b^3/a^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(
1/2))/(I*a-(-a^2+b^2)^(1/2)))-I/d^2*b^3/a^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I
*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))+I/d^2/b^3*a^4*f/(-a^2+b
^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))+I
/d^2/b^3*a^4*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*
a+(-a^2+b^2)^(1/2)))-3*I/b/d^2*f/(-a^2+b^2)*dilog((I*a+b*exp(I*(d*x+c))-(-a
^2+b^2)^(1/2))/(I*a-(-a^2+b^2)^(1/2)))*a^2-3*I/b/d^2*f/(-a^2+b^2)*dilog((I*
a+b*exp(I*(d*x+c))+(-a^2+b^2)^(1/2))/(I*a+(-a^2+b^2)^(1/2)))*a^2-2*I/d/b^3*
a^2*c*f*x-1/2*I/b^3*a^2*f*x^2-1/2*I*a*(d*f*x-I*f+d*e)/b^2/d^2*exp(-I*(d*x+c
))+2*I/d^2/b*c^2*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 4.42835, size = 4313, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/4*(a^2*b^2*f*cos(d*x + c)^3 - 2*I*b^4*f*dilog(cos(d*x + c) + I*sin(d*x +
 c))*sin(d*x + c) + 2*I*b^4*f*dilog(cos(d*x + c) - I*sin(d*x + c))*sin(d*x
+ c) + 2*I*b^4*f*dilog(-cos(d*x + c) + I*sin(d*x + c))*sin(d*x + c) - 2*I*b
^4*f*dilog(-cos(d*x + c) - I*sin(d*x + c))*sin(d*x + c) - a^2*b^2*f*cos(d*x
```

```
 + c) + 4*(a^3*b + a*b^3)*d*f*x - 2*I*(a^4 - 2*a^2*b^2 + b^4)*f*dilog(-1/2*
(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x +
c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) - 2*I*(a^4 - 2*a^2*b^
2 + b^4)*f*dilog(-1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x
 + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c)
 + 2*I*(a^4 - 2*a^2*b^2 + b^4)*f*dilog(-1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(
d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2
*b)/b + 1)*sin(d*x + c) + 2*I*(a^4 - 2*a^2*b^2 + b^4)*f*dilog(-1/2*(-2*I*a*
cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqr
t(-(a^2 - b^2)/b^2) + 2*b)/b + 1)*sin(d*x + c) + 4*(a^3*b + a*b^3)*d*e - 4*
(a^3*b*d*f*x + a^3*b*d*e)*cos(d*x + c)^2 - 2*((a^4 - 2*a^2*b^2 + b^4)*d*e -
 (a^4 - 2*a^2*b^2 + b^4)*c*f)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2
*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) - 2*((a^4 - 2*a^2*b^2 + b^4
)*d*e - (a^4 - 2*a^2*b^2 + b^4)*c*f)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x +
 c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) - 2*((a^4 - 2*a^2*b^
2 + b^4)*d*e - (a^4 - 2*a^2*b^2 + b^4)*c*f)*log(-2*b*cos(d*x + c) + 2*I*b*s
in(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a)*sin(d*x + c) - 2*((a^4 -
2*a^2*b^2 + b^4)*d*e - (a^4 - 2*a^2*b^2 + b^4)*c*f)*log(-2*b*cos(d*x + c) -
 2*I*b*sin(d*x + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a)*sin(d*x + c) - 2*
((a^4 - 2*a^2*b^2 + b^4)*d*f*x + (a^4 - 2*a^2*b^2 + b^4)*c*f)*log(1/2*(2*I*
a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*s
qrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) - 2*((a^4 - 2*a^2*b^2 + b^4)*d
*f*x + (a^4 - 2*a^2*b^2 + b^4)*c*f)*log(1/2*(2*I*a*cos(d*x + c) + 2*a*sin(d
*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*
b)/b)*sin(d*x + c) - 2*((a^4 - 2*a^2*b^2 + b^4)*d*f*x + (a^4 - 2*a^2*b^2 +
b^4)*c*f)*log(1/2*(-2*I*a*cos(d*x + c) + 2*a*sin(d*x + c) + 2*(b*cos(d*x +
c) + I*b*sin(d*x + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) - 2*((
a^4 - 2*a^2*b^2 + b^4)*d*f*x + (a^4 - 2*a^2*b^2 + b^4)*c*f)*log(1/2*(-2*I*a
*cos(d*x + c) + 2*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sq
rt(-(a^2 - b^2)/b^2) + 2*b)/b)*sin(d*x + c) + 2*(b^4*d*f*x + b^4*d*e + a*b^
3*f)*log(cos(d*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) + 2*(b^4*d*f*x + b
^4*d*e + a*b^3*f)*log(cos(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) + 2*(
b^4*d*e - (b^4*c + a*b^3)*f)*log(-1/2*cos(d*x + c) + 1/2*I*sin(d*x + c) + 1
/2)*sin(d*x + c) + 2*(b^4*d*e - (b^4*c + a*b^3)*f)*log(-1/2*cos(d*x + c) -
1/2*I*sin(d*x + c) + 1/2)*sin(d*x + c) + 2*(b^4*d*f*x + b^4*c*f)*log(-cos(d
*x + c) + I*sin(d*x + c) + 1)*sin(d*x + c) + 2*(b^4*d*f*x + b^4*c*f)*log(-c
os(d*x + c) - I*sin(d*x + c) + 1)*sin(d*x + c) - (a^2*b^2*d*f*x + a^2*b^2*d
*e - 4*a^3*b*f*cos(d*x + c) - 2*(a^2*b^2*d*f*x + a^2*b^2*d*e)*cos(d*x + c)^
2)*sin(d*x + c))/(a^2*b^3*d^2*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)**3*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.348 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=96

$$\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3d}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{a\sin(c+dx)}{b^2d}-\frac{\csc(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2bd}$$

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) + ((a^2 - b^2)^2*Log[
a + b*Sin[c + d*x]])/(a^2*b^3*d) - (a*Sin[c + d*x])/(b^2*d) + Sin[c + d*x]^
2/(2*b*d)
```

Rubi [A] time = 0.155076, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3d}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{a\sin(c+dx)}{b^2d}-\frac{\csc(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) + ((a^2 - b^2)^2*Log[
a + b*Sin[c + d*x]])/(a^2*b^3*d) - (a*Sin[c + d*x])/(b^2*d) + Sin[c + d*x]^
2/(2*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2(b^2-x^2)^2}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$

$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$

$$= \frac{\text{Subst}\left(\int \left(-a+\frac{b^4}{ax^2}-\frac{b^4}{a^2x}+x+\frac{(a^2-b^2)^2}{a^2(a+x)}\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$

$$= -\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3d}-\frac{a\sin(c+dx}{b^2d}$$

Mathematica [A] time = 0.192026, size = 86, normalized size = 0.9

$$\frac{\frac{2(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3}-\frac{2b\log(\sin(c+dx))}{a^2}-\frac{2a\sin(c+dx)}{b^2}-\frac{2\csc(c+dx)}{a}+\frac{\sin^2(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*Csc[c + d*x])/a - (2*b*Log[Sin[c + d*x]])/a^2 + (2*(a^2 - b^2)^2*Log[a
 + b*Sin[c + d*x]])/(a^2*b^3) - (2*a*Sin[c + d*x])/b^2 + Sin[c + d*x]^2/b)/
(2*d)
```

Maple [A] time = 0.073, size = 124, normalized size = 1.3

$$\frac{(\sin(dx+c))^2}{2bd}-\frac{a\sin(dx+c)}{b^2d}+\frac{\ln(a+b\sin(dx+c))a^2}{db^3}-2\frac{\ln(a+b\sin(dx+c))}{bd}+\frac{b\ln(a+b\sin(dx+c))}{da^2}-\frac{}{da\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2*sin(d*x+c)^2/b/d-a*sin(d*x+c)/b^2/d+1/d/b^3*ln(a+b*sin(d*x+c))*a^2-2*ln
(a+b*sin(d*x+c))/b/d+1/d/a^2*b*ln(a+b*sin(d*x+c))-1/d/a/sin(d*x+c)-b*ln(sin
(d*x+c))/a^2/d
```

Maxima [A] time = 0.982893, size = 123, normalized size = 1.28

$$-\frac{\frac{2\,b\log\left(\sin\left(dx+c\right)\right)}{a^{2}}-\frac{b\sin\left(dx+c\right)^{2}-2\,a\sin\left(dx+c\right)}{b^{2}}+\frac{2}{a\sin\left(dx+c\right)}-\frac{2\,\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^{2}b^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*log(sin(d*x + c))/a^2 - (b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2
 + 2/(a*sin(d*x + c)) - 2*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/(
a^2*b^3))/d
```

Fricas [A] time = 2.16953, size = 317, normalized size = 3.3

$$\frac{4\,a^{3}b\cos\left(dx+c\right)^{2}-4\,b^{4}\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-4\,a^{3}b-4\,ab^{3}+4\,\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\log\left(b\sin\left(dx+c\right)+a\right)\sin}{4\,a^{2}b^{3}d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*b*cos(d*x + c)^2 - 4*b^4*log(1/2*sin(d*x + c))*sin(d*x + c) - 4*
a^3*b - 4*a*b^3 + 4*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)*sin(d*x
 + c) - (2*a^2*b^2*cos(d*x + c)^2 - a^2*b^2)*sin(d*x + c))/(a^2*b^3*d*sin(d
*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.19449, size = 142, normalized size = 1.48

$$-\frac{\frac{2\,b\log(|\sin(dx+c)|)}{a^2}-\frac{b\sin(dx+c)^2-2\,a\sin(dx+c)}{b^2}-\frac{2\,(b\sin(dx+c)-a)}{a^2\sin(dx+c)}-\frac{2\,\left(a^4-2\,a^2b^2+b^4\right)\log(|b\sin(dx+c)+a|)}{a^2b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*log(abs(sin(d*x + c)))/a^2 - (b*sin(d*x + c)^2 - 2*a*sin(d*x + c)
)/b^2 - 2*(b*sin(d*x + c) - a)/(a^2*sin(d*x + c)) - 2*(a^4 - 2*a^2*b^2 + b^
4)*log(abs(b*sin(d*x + c) + a))/(a^2*b^3))/d
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```